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Quiz 7 :

① $\frac{1}{2}n^2 + 3n = \theta(n^2)$

$f(n) = \theta(g(n)) \Leftrightarrow f(n) = O(g(n)) \wedge f(n) = \Omega(g(n))$

$\frac{7}{2}n^2$

$n \geq 1 : \frac{1}{2}n^2 + 3n \leq \frac{1}{2}n^2 + 3n^2 \longrightarrow f(n) = O(n^2)$ (I)

$n \geq 0 : \frac{1}{2}n^2 + 3n \geq \frac{1}{2}n^2 \longrightarrow f(n) = \Omega(n^2)$ (II)

(I), (II) $\rightarrow \theta(n^2) = \frac{1}{2}n^2 + 3n$

② $(n\log n - 2n + 13) = \Omega(n\log n)$

$0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \quad , \quad n \geq n_0$

$0 \leq c_1 n\log n \leq n\log n - 2n + 13 \leq c_2 n\log n \quad , \quad n \geq n_0$

$n\log n - 2n \leq n\log n - 2n + 13$

$\Rightarrow c_1 n\log n \leq n\log n - 2n \longrightarrow c_1 \leq 1 - \frac{2}{\log n}$ $(n > 1)$ $\log n$

$n \geq 8 : \frac{2}{\log n} \leq \frac{2}{3} \rightarrow c_1 \leq 1 - \frac{2}{\log n} = \frac{1}{3} \rightarrow c_1 = \frac{1}{3}$

$c = \frac{1}{3}, n_0 = 8 \Rightarrow 0 \leq c_1 n\log n \leq n\log n - 2n \leq n\log n - 2n + 13$

$\Rightarrow n\log n - 2n + 13 = \Omega(n\log n)$

②

	Cost	Times
line 1	—	—
line 2	c_1	1
line 3	c_2	n
line 4	c_3	n−1

$$T(n) = c_1 + n c_2 + (n-1) c_3 = (c_2 + c_3) n + (c_1 - c_3)$$
$$= an + b$$

↳ Best case, Average case, Worst case

③

	Cost	Times
int binarysearch (int a[], int n, int val)	—	—
{	—	—
int l=1, r=n, m;	c_1	1
while (r≥l) {	c_2	n
m = (l+r)/2	c_3	n−1
if (a[m] == val) return m;	c_4	n−1
if (a[m] > val) r = m−1; else l = m+1; }	c_5	n−1
return −1;	c_6	1
}		

$$N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \cdots \rightarrow \frac{N}{2^k}$$

الرسم درخط کلام اکام شود:

$$\frac{N}{2^k} = 1 \rightarrow k = \log_2^N \longrightarrow T(n) = \log^n \rightarrow \text{Average \& Worst Case}$$

Best case: $N = 1 \rightarrow T(n) = 1$

(4)

	Cost	Times
`int *compute_sums(int A[], int n){`	—	—
`int M[n][n];`	c_1	1
`int i,j;`	c_2	1
`for(i=0; i<n; i++)`	c_3	n
`for(j=0; j<n; j++)`	c_4	$n(n-1)$
`M[i][j] = A[i] + A[j];`	c_5	$n(n-1)-1$
`return M;`	c_6	1
`}`		

$$T(n) = c_1 + c_2 + n c_3 + (n^2 - n) c_4 + (n^2 - n - 1) c_5 + c_6$$

$$T(n) = (c_4 + c_5) n^2 + (c_3 - c_4 - c_5) n + (c_1 + c_2 - c_5 + c_6)$$

↳ Best case, Average case, Worst case