Quiz 6

الناز رضائی و حوریه سبزواری

98412034 98411387

Let $T(1), T(2), T(3), \dots, T(n)$ be a sequence of numbers such that for all $n \geq 2$ then

$$T(n) = 2T(n-1) + 2^{n+1}$$

If $T(1) = 4$ then prove that for all $n \geq 1$,

$$T(n) = n\,2^{n+1}$$

By induction ;

① Base case : $T(1) = 4$

$$T(n) = n\,2^{n+1} \rightarrow T(1) = 1 \times 2^{2} = 4 \checkmark$$

② Induction step :

- We should prove that for every n, if the statement holds for n, then it holds for n+1.

It means : $T(n) \rightarrow T(n+1)$

$$n\,2^{n+1} \rightarrow (n+1)\,2^{n+2}$$

We know that $T(n) = 2T(n-1) + 2^{n+1}$, then we choose $n+1$ for the input of $T(n)$:

$$T(n+1) = 2T(n+1-1) + 2^{n+1+1}$$

$$\Rightarrow T(n+1) = 2T(n) + 2^{n+2}$$

According to induction assumption : $T(n) = n\,2^{n+1}$

Then we have : $T(n+1) = 2(n\,2^{n+1}) + 2^{n+2}$

$\rightarrow T(n+1) = n\,2^{n+2} + 2^{n+2} = (n+1)\,2^{n+2}$ ✓