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Analysis of a Mathematical Model of Smoking

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Abstract

This study presents a mathematical model that represents the population growth dynamics of tobacco consumers based on a system of ordinary nonlinear differential equations. The model is used to determine the Basic Reproductive Number (R_0). Two points of equilibrium are found and their local stability is classified. Finally, the Matlab software is used to present numerical simulations using the fourth-order Runge-Kutta method, and it is shown that the solutions approach an asymptotically stable point, under the variation of R_0.

Key words: Smoking, Basic Reproductive Number, Dynamical Systems

1 Introduction

In Colombia and around the world, one of the public health problems that has been recognised in recent years is smoking addict, which has developed into an epidemic causing many deaths [8]. Environmental pressure, curiosity and stress are factors that influence the development of a smoking habit [2]. The emergence of the habit occurs in stages such as: preparation, trying tobacco for the first time, experimentation and beginning repeated but irregular use, habitual use of tobacco and finally the development of dependence and addiction [6].

Most smokers who know the dangers of tobacco wish to quit and, in theory, after taking up smoking it is possible to stop voluntarily and permanently [6, 1], but counseling and medication can double the likelihood that a smoker who wants to quit will do so. The process of giving up smoking occurs in five phases: pre-contemplation, contemplation, preparation, action and maintenance [6].

Mathematical models are used to interpret the increase in smoking and anticipate the impact of smokers on society [1]. This is how some studies related to the subject have appeared in the literature. The principal characteristics shared by these studies are that a constant population is divided into three groups: potential smokers; smokers; and ex-smokers. The studies consider neither death by illnesses caused by tobacco, nor the possiblilty that a person completely stops being exposed to the comsumption of tobacco. Amongst these, in 1997, C. Castillo-Garsow, G. Jordan-Salivia and A. Rodriguez-Herrera [3] consider a basic model of three ordinary nonlinear differential equations, which consider the use of drugs in general in adolescents and where the factors involved are contagion, recovery and relapse. In [7, 4] a model of four nonlinear differential equations is constructed where groups of people are: potential smokers; smokers; and temporary or permanent ex-smokers. While in [1] a model of five nonlinear differential equations is formulated, distinguishing between to classes of smokers according the the frequency of consumption, high or low. Similar to [1], Sintayehu Matintu [9] studies a non-constant population but holding the other characteristics constant that make a great difference to the model when varied as proposed in this article.

2 Approach of the Model

For this population behavior model of people in the presence of tobacco consumption the following assumptions are taken into account:

- There are deaths as a result of smoking.
- The average number of healthy and completely recovered people are not directly considered in the system.

- People can be in one of two main states: Exposed (people who are not able to pass on the habit, but are at risk of becoming active smokers); and Infected (people who are addicted to tobacco and can pass on the habit). The first state is divided into two sub-states: passive smokers or those at risk of smoking, E_1; and people who have stopped smoking but are at risk of relapsing, E_2.

- There is a constant flow, Δ, of healthy people to the state E_1.

- People leave from the states under study due to factors such as: living in a completely non-smoking population; or the death of the individual.

Table 1 describes the variables and parameters used create the mathematical model that represents the dynamics of transmission of the habit of smoking. The flow diagram in Figure 1 shows the state transitions possible under the model.

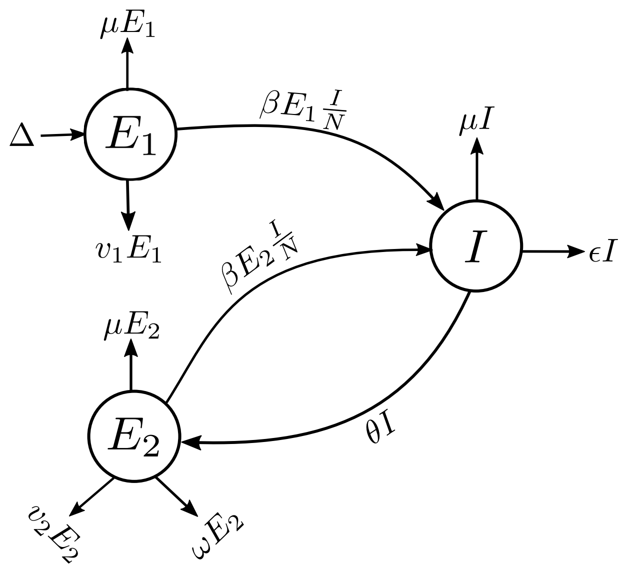

Figure 1: State transmission diagram for the model of smoking habits.

Variable/ parameter	**Description**
E_1	Average number of people over a time t that are at risk of becoming active smokers.
E_2	Average number of people over a time t who have been active smokers and quit, but are at risk of relapsing.
I	Average number of active smokers over a time t.
N	The total size of the population, calculated as $N = E_1+E_2+I$, over each unit of time t.
Δ	Average number of healthy people who become people at risk of becoming active smokers.
μ	The natural death rate over the population.
v_1	The exit rate of people from being at risk of becoming smokers to the healthy population.
v_2	The exit rate of ex-smokers to the healthy population.
ω	Death rate of ex-smokers as a consequence of tobacco consumption.
β	The infection rate of smoking.
ϵ	Death rate of active smokers as a consequence of tobacco consumption.
θ	The exit rate of people from being active smokers to being at risk of relapsing.

Table 1: Variables and parameters of the model

In order to describe the mathematical model by means of a system of differential equations, the following dynamics are taken into account:

- There is a constant inflow of people at risk of becoming an active smoker E_1 given by Δ.
- The number E_1 of people decreases due to: natural death (μE_1 people per unit time); the influence of factors that cause a person to move from the population at risk of being an active smoker to the population of healthy people ($v_1 E_1$ people per unit time); and the impact of smoking ($\beta E_1 \frac{I}{N}$ people per unit time), where $\frac{I}{N}$ is the probablility of meeting an active smoker in the total population.
- The number of active smokers I decreases due to: natural death (μI people per unit time); death as a consequence of smoking (ϵI people per unit time); and the influence of factore that cause a person to leave the population active smokers and join the population of people at risk of relapsing E_2 (θI people per unit time).