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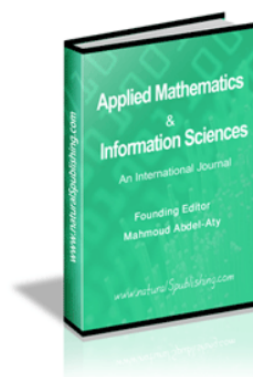
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Stability Analysis of a Model with Integrated Control for Population Growth of the *Aedes Aegypti* Mosquito

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Abstract: We present a system of ordinary nonlinear differential equations describing the population growth dynamics of the *Aedes aegypti* mosquito, the main transmitter of the dengue virus in Colombia. This model incorporates the three types of known control for mosquito eradication: mechanical, biological and chemical, focusing on biological control through the use of the *Wolbachia* bacterium, which is the new hope for the control of the diseases transmitted by this mosquito. A local stability analysis of the model is performed on the three equilibrium points that are found, determining the conditions under which those points become stable or unstable. Finally, we present numerical simulations implemented in Matlab, where the numerical results are obtained using hypothetical values of the parameters obtained from the literature.

Keywords: *Aedes aegypti*; integrated control; *Wolbachia*; stability.

1 Introduction

Aedes aegypti is a mosquito that mainly lives close to human populations. It flies only short distances and requires blood (primarily human), to reproduce [1], [7].

During their lifetime the mosquitoes go through two stages: immature and mature. In the immature phase, the mosquito is aquatic and undergoes a metamorphosis from egg to an adult. It feeds mainly on residues in the water where they were laid by the female. Adult mosquitoes are airborne and while the males feed on plant nectar, the females feed on blood. [4].

By feeding on blood, in order to mature and deposit her eggs, the female mosquito promotes the transmission of viruses and pathogens that cause various diseases including Dengue fever. For this reason global campaigns have been founded to eradicate the mosquito. So far, the struggle has been unsuccessful because although some countries have achieved temporary the extinction, the mosquito soon returns due to the infestation of neighbouring countries [2], [7].

There are three main mechanisms to control the propagation of mosquitoes, namely: mechanical control, focused on preventing the reproduction of the mosquito

using traps, destroying breeding grounds, etc.; chemical control based on insecticides or larvicides; and biological control that makes use of other living organisms such as the *Wolbachia* bacterium which reduces the life span of the mosquito and also, in the case of dengue, almost eliminates the probability of transmitting the virus to humans [5], [7], [8].

The present article demonstrates a mathematical model that describes the population growth of the female mosquito in the adult phase. The model incorporates all three control mechanisms for the mosquito. A stability analysis is performed and we show how the population growth dynamics change in response to a program of biological control via the introduction of the *Wolbachia* bacterium into the population. In this way, the model will serve as a tool for the those who wish to determine the way in which mechanical, chemical and biological controls should be applied to diminish the breeding of mosquitoes and, therefore, the propagation of diseases like dengue that are transmitted by them.

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2 The Model

The following hypotheses are considered in the creation of the model:

- The population of interest is that of the adult female *Aedes aegypti* mosquitoes.
- There are two populations of adult female mosquitoes: those that are infected by the *Wolbachia* bacterium and those that are not.
- The death rate of an adult mosquito infected with the bacterium is greater than the death rate of an uninfected mosquito.
- Three controls on the mosquito population are used: traps that prevent the eggs from reaching adulthood (mechanical control); in the immature state a proportion of the eggs are infected by the *Wolbachia* bacterium, which genetically manipulates the mosquito and is transmitted vertically (biological control); a proportion of adult mosquitoes die from the use of insecticides (chemical control).

Taking these hypotheses into account, let us consider b , the total number of adult female mosquitoes that are *not* infected with the bacterium at time t ; and B , the total number of adult female mosquitoes that *are* infected with the bacterium in a time t . We also consider the parameters in Table 1. In this analysis we use the week as the period of time because it is short enough that a single female mosquito will lay eggs at most once during the period.

Table 1: Parameters of the model

Parameter	Description
ξ	Rate of development of a mosquito from the immature phase to the adult phase
f	Proportion of immature mosquitoes that develop into adult females
ϕ	The probability that an adult female mosquito will lay eggs in the week
δ	The average number of eggs laid at a time by an adult female mosquito
π	The natural death rate of immature mosquitoes
κ	The maximum number of mosquitoes that the environment can support
ε	The natural death rate of the mosquito without <i>Wolbachia</i>
ν	The death rate of mosquitoes infected by <i>Wolbachia</i>
u_1	The proportion of immature mosquito deaths caused by traps
u_2	The proportion of immature mosquito deaths caused by insecticides
u_3	The proportion of eggs infected by the <i>Wolbachia</i> bacterium through micro-injection

If we consider the variation of b with over time, we recognise that this population grows continuously as a result of the development of immature mosquitoes. It also decreases as a result of the natural death of mosquitoes or the use of insecticides. Therefore, in terms of the parameters shown in Table 1, we see that the expression $(1 - u_3) \frac{\xi f \phi \delta}{\pi + u_1}$ represents, for each adult female that lays eggs in the week, the average number of eggs that survive to adulthood without being infected by the *Wolbachia* bacterium. Furthermore, the expression $1 - \frac{b+B}{\kappa}$ represents the probability that a mosquito that develops to the mature phase finds space available in the environment. From these, we have the number of mosquitoes that enter the population of adult females without being infected by the bacterium is given by $(1 - u_3) \frac{\xi f \phi \delta}{\pi + u_1} (1 - \frac{b+B}{\kappa}) b$. Similarly, the number of female mosquitoes in this state that die in each instant is given by $(\varepsilon + u_2)b$. Thus we have that:

$$\frac{db}{dt} = (1 - u_3) \frac{\xi f \phi \delta}{\pi + u_1} \left(1 - \frac{b+B}{\kappa}\right) b - (\varepsilon + u_2)b.$$

Now, the mosquitoes that enter the population of adult females infected by the *Wolbachia* bacterium are those that develop from eggs that have been laid by an infected female mosquito (because the bacterium is transmitted vertically) and those that develop from eggs that have been laid by uninfected females but are infected through micro-injection. Therefore the change in this population is given by:

$$\frac{dB}{dt} = \frac{\xi f \phi \delta}{\pi + u_1} \left(1 - \frac{b+B}{\kappa}\right) B + \frac{u_3 \xi f \phi \delta}{\pi + u_1} \left(1 - \frac{b+B}{\kappa}\right) b - (\nu + u_2)B.$$

Thus the system of ordinary non-linear equations representing the growth dynamic of the population of adult female mosquitoes both with and without *Wolbachia* infection is given by:

$$\begin{aligned} \frac{db}{dt} &= (1 - u_3) \frac{\xi f \phi \delta}{\pi + u_1} \left(1 - \frac{b+B}{\kappa}\right) b - (\varepsilon + u_2)b \quad (1) \\ \frac{dB}{dt} &= \frac{\xi f \phi \delta}{\pi + u_1} \left(1 - \frac{b+B}{\kappa}\right) (B + u_3b) - (\nu + u_2)B. \end{aligned}$$

3 Points of Equilibrium

Setting the right hand side of the system's differential equations to zero, we find that the model has three points of equilibrium:

$$P_1 = (0, 0), \quad P_2 = \left(0, k \left(\frac{H-1}{H}\right)\right) \quad \text{and} \quad P_3 = (b_1, B_1)$$