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Tabla: Álgebra y Trigonometría
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Álgebra

- Operaciones básicas con números racionales.

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd} \qquad \frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd} \qquad \frac{a}{b}\frac{c}{d}=\frac{ac}{bd} \qquad \frac{a}{b}\div\frac{c}{d}=\frac{ad}{bc}$$

- Propiedades de potenciación y radicación.

$$a^n a^m = a^{n+m} \qquad (a^n)^m = a^{nm} \qquad \tfrac{a^n}{a^m} = a^{n-m} \qquad a^0 = 1$$

$$a^{-n} = \tfrac{1}{a^n} \qquad a^n b^n = (ab)^n \qquad \tfrac{a^n}{b^n} = \left(\tfrac{a}{b}\right)^n \qquad a^{n/m} = \sqrt[m]{a^n}$$

- Propiedades de logaritmos.

$$\log_n a = \tfrac{\log_m a}{\log_m n} \qquad \log_e a = \ln a \qquad \log_{10} a = \log a \qquad \log_n n = 1$$

$$\log_n (ab) = \log_n a + \log_n b \qquad \log_n \left(\tfrac{a}{b}\right) = \log_n a - \log_n b \qquad \log_n (b^a) = a \log_n b$$

$$\log_n a = c \Leftrightarrow a = n^c$$

- Productos notables.

$(a-b)(a+b) = a^2 - b^2$ $\qquad$ $(a\pm b)^2 = a^2 \pm 2ab + b^2$ $\qquad$ $(a\pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

Término $r-$ésimo de $(a\pm b)^n$:

$$T_r = (\pm 1)^{r-1}\, \frac{n(n-1)(n-2)\cdots(n-(r-2))}{(r-1)!} a^{n-(r-1)} b^{r-1}, \quad r > 2$$

- Factorización.

$\mathbf{A}^2 - \mathbf{B}^2 = (\mathbf{A}-\mathbf{B})(\mathbf{A}+\mathbf{B})$ $\qquad$ $\mathbf{E^2} + (a+b)\mathbf{E} + ab = (\mathbf{E}+a)(\mathbf{E}+b)$

$\mathbf{A}^3 - \mathbf{B}^3 = (\mathbf{A}-\mathbf{B})(\mathbf{A}^2 + \mathbf{AB} + \mathbf{B}^2)$ $\qquad$ $\mathbf{A}^3 + \mathbf{B}^3 = (\mathbf{A}+\mathbf{B})(\mathbf{A}^2 - \mathbf{AB} + \mathbf{B}^2)$

Para $a\mathbf{E^2} + b\mathbf{E} + c = 0$, $\qquad \mathbf{E} = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$, $a \neq 0$.

- Conjugados.

$$\begin{array}{rcl}
\sqrt[n]{A} & \Leftrightarrow & \sqrt[n]{A^{n-1}} \\
\sqrt{A}-\sqrt{B} & \Leftrightarrow & \sqrt{A}+\sqrt{B} \\
\sqrt[3]{A}-\sqrt[3]{B} & \Leftrightarrow & \sqrt[3]{A^2}+\sqrt[3]{AB}+\sqrt[3]{B^2} \\
\sqrt[3]{A}+\sqrt[3]{B} & \Leftrightarrow & \sqrt[3]{A^2}-\sqrt[3]{AB}+\sqrt[3]{B^2}
\end{array}$$

- Desigualdades.

Si $a < b \;\Rightarrow\; b > a$. Es decir, $b - a > 0$ (positivo)

$$\begin{array}{lllll}
\text{Si } a < b & \Rightarrow & a+c < b+c & & (\text{se cumple también para } \leq, >, \geq) \\
 & \Rightarrow & ac < bc & \text{siempre que} & c > 0 \\
 & \Rightarrow & ac > bc & \text{siempre que} & c < 0
\end{array}$$

Si $a < b$ y $c < d$, entonces se cumple que: $a + c < b + d$

Si $ab>0 \;\Rightarrow\; (a>0 \wedge b>0) \vee (a>0 \wedge b>0)$

Si $ab<0 \;\Rightarrow\; (a>0 \wedge b<0) \vee (a<0 \wedge b>0)$

- Valor absoluto.

$$|a| = a, \quad \text{si} \quad a \geq 0. \quad |a| = -a, \quad \text{si} \quad a < 0.$$

Si $|a| > b \;\Leftrightarrow\; a > b \vee a < -b$

Si $|a| \geq b \;\Leftrightarrow\; a \geq b \vee a \leq -b$

Si $|a| < b \;\Leftrightarrow\; a > -b \wedge a < b$

Si $|a| \leq b \;\Leftrightarrow\; a \geq -b \wedge a \leq b$

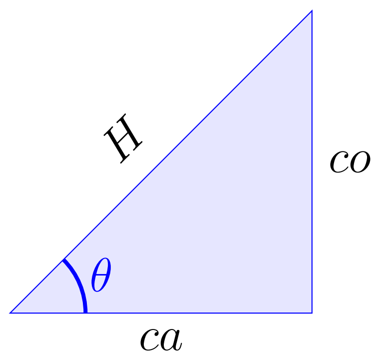
- Otras desigualdades.

$$\frac{A}{A+B} \leq 1, \qquad AB \leq \frac{A^2+B^2}{2}, \qquad A, B \in \mathbb{R}_+$$

$$|A+B| \leq |A| + |B|, \qquad |A| \leq \sqrt{A^2+B^2}, \qquad A, B \in \mathbb{R}$$

$$|\text{sen} A| \leq |A|, \qquad |\cos A| \leq |A|$$

Trigonometría



$$\text{sen}\theta = \frac{co}{H} \qquad \cos\theta = \frac{ca}{H}$$

$$\tan\theta = \frac{co}{ca} \qquad \cot\theta = \frac{ca}{co}$$

$$\sec\theta = \frac{H}{ca} \qquad \csc\theta = \frac{H}{co}$$

- Identidades lineales

$$\tan\theta = \frac{\text{sen}\theta}{\cos\theta} \qquad \tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{\cos\theta}{\text{sen}\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\text{sen}\theta} \qquad \text{sen}\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

- Identidades cuadricas

$$\text{sen}^2\theta + \cos^2\theta = 1 \qquad \text{sen}^2\theta = 1 - \cos^2\theta \qquad \cos^2\theta = 1 - \text{sen}^2\theta$$

$$\tan^2\theta = \sec^2\theta - 1 \qquad \cot^2\theta = \csc^2\theta - 1 \qquad \sec^2\theta = 1 + \tan^2\theta$$

$$\csc^2\theta = 1 + \cot^2\theta$$

- Suma (resta) de ángulos

$$\text{sen}(a+b) = \text{sen}a\cos b + \text{sen}b\cos a \qquad \text{sen}(a-b) = \text{sen}a\cos b - \text{sen}b\cos a$$

$$\cos(a+b) = \cos a\cos b - \text{sen}a\,\text{sen}b \qquad \cos(a-b) = \cos a\cos b + \text{sen}a\,\text{sen}b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a\tan b} \qquad \tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a\tan b}$$

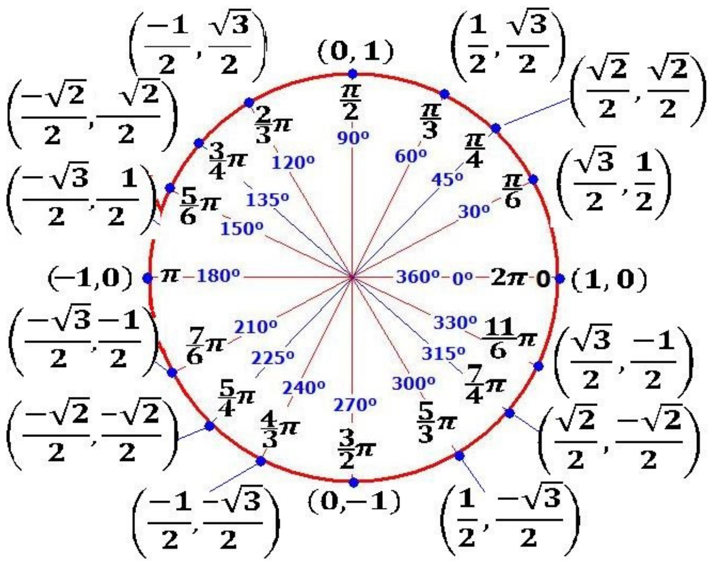
- Ángulos dobles

$$\text{sen}(2a) = 2\text{sen}a\cos a \qquad \cos(2a) = \cos^2 a - \text{sen}^2 a \qquad \tan(2a) = \frac{2\tan a}{1 - \tan^2 a}$$

- Reducción de Orden

$$\operatorname{sen}^2 a = \frac{1-\cos(2a)}{2} \qquad\qquad \cos^2 a = \frac{1+\cos(2a)}{2}$$

- Circulo Trigonométrico



- Ley de Senos y Cosenos

Ley de Senos:

$$\frac{\operatorname{sen}\alpha}{a} = \frac{\operatorname{sen}\beta}{b} = \frac{\operatorname{sen}\gamma}{c}$$

Ley de Cosenos:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$