Notes for Paper 1

Paper 1

Title: Data-driven decision making in power systems with probabilistic guarantees: Theory and applications of chance-constrained optimization

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Notes:

1. Problem description:

The problem of **Chance-constrained optimization(CCO)** can usually be expressed like this:

$$\begin{aligned} \min_x \quad & c^T x \\ s.t. \quad & P_\varepsilon(f(x,\varepsilon) \le 0) \ge 1-\epsilon \\ & x \in \chi \end{aligned}$$

This kind of problem can be converted into another type:
Violation Probability: $V(x^o) := P_\varepsilon(f(x,\varepsilon) \ge 0)$
where x^o is a feasible solution to CCO.
Define:

$$F_\epsilon := \{x \in R^n : V(x) \le \epsilon\} = \{x \in R^n : P_\varepsilon(f(x,\varepsilon) \ge 0) \le 1-\epsilon\}$$

Then, x^o is feasible to CCO if $x \in \chi \cap F_\epsilon$.
So CCO can be equivalently written as:

$$\begin{aligned} \min_x \quad & c^T x \\ s.t. \quad & V(x) \le \epsilon \end{aligned}$$

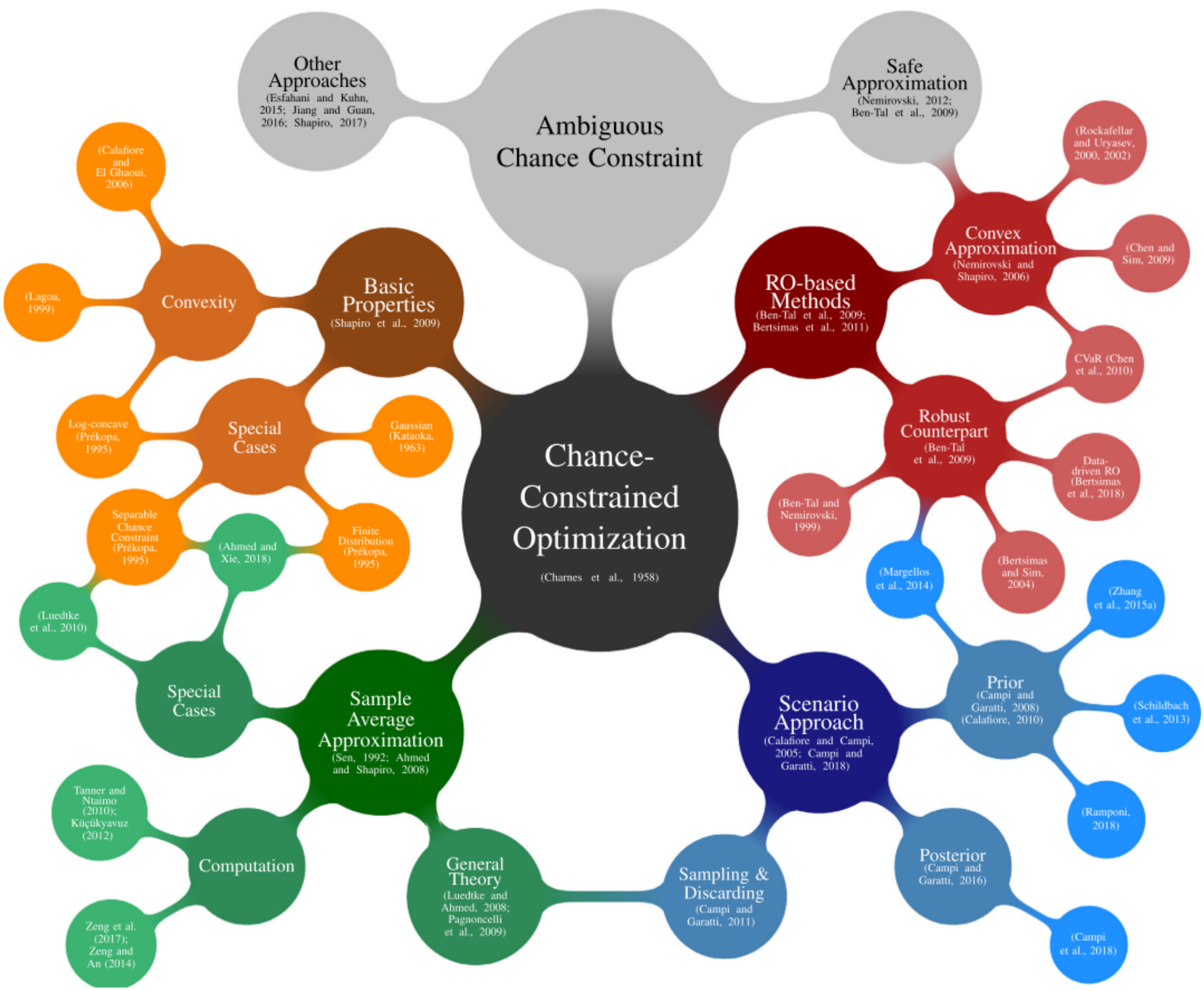
$x \in \chi$

So, the first step is checking its **feasibility** (x^o satisfies the constraint), then is checking its **optimality** (the distance between the corresponding o^o and the best optimal value o^*).

2. difficulties or issues:

There are **two main difficulties** to solve the problems:
(D1) calculating the probability involves multivariate inte- gration, which is NP-Hard.
(D2) feasible region F_ϵ is often non-convex

3. Methods



Scenario approach

Scenario approach utilizes a dataset witg N scenarios $\{\varepsilon_i\}_{i=1}^N$ to approximate CCO:

$$(SP_N)\min_{x\in\chi} \quad c^T x$$

$$s.t. \quad f(x,\varepsilon^1) \leq 0, ..., f(x,\varepsilon^N) \leq 0$$

Because massive scenario number N will cause huge computation problem, so it involves another question: how to find the suitable scenario number (sample complexity N)?

a. A-priori feasibility guarantees

steps:

1. exploring the problem structure and obtain the upper bound $\overline{h}$ on the number of support scenarios;
2. choosing a good sample complexity N
3. Solving SP_N and obtain x_N^* and o_N^*

Many scholars make efforts on how to find the upper bound $\overline{h}$ and the good sample comlexity N

b. A-posteriori feasibility guarantees

steps:

1. given dataset $\{\varepsilon_i\}_{i=1}^N$ solve SP_N and obtain x_N^*;
2. find the support scenarios number (denoted as s_N^*)
3. calculate the posterior violation probability $\epsilon(\beta, s_N^*, N)$
4. if $\epsilon(\beta, s_N^*, N) \geq \epsilon$, repeat step 1 to 3 with more scenarios until reaching $\epsilon(\beta, s_N^*, N) \leq \epsilon$. If all available scenarios are used but still fails to reach the condition, then it might be impossible to obtain a solution x_N^*.

$\epsilon(\beta, s_N^*, N)$ is the solution in the interval $(0, 1)$ of the equality below:

$$\frac{\beta}{N+1} \sum_{i=k}^{N} \binom{i}{k} t^{i-k} - \binom{N}{k} t^{i-k} = 0$$

where $\beta \in (0, 1)$

Many scholars make efforts on how to find the upper bound $\overline{\epsilon}$. If $\overline{\epsilon} < \epsilon$, then x^o is feasible.

c. Optimality guarateens

The optimality checking is conducted by constructing lower bounds $\underline{o}$ on o^*

Sample average approximation

Sample average approximation converts CCO into:

$(SAA) : \min\limits_{x} \quad c^T x$

$\quad s.t. \frac{1}{N} \sum_{i=1}^{N} 1_{\overline{f}(x, \varepsilon^i) > 0} \leq \varepsilon$

where
$\overline{f}(x, \varepsilon^i) := \max\{f_1(x, \varepsilon), ..., f_m(x, \varepsilon)\}$ and the violation probability ε differs from ϵ in CCO.
Data-driven: SAA approximates the true distribution from N samples $\{\varepsilon_i\}_{i=1}^N$.

SAA further converts CCO into:
$$\begin{aligned} &\min_{x,z} c^T x \\ &\quad s.t. \quad f(x, \varepsilon^i) - M z_i 1_m \le 0 \\ &\qquad\qquad \tfrac{1}{N}\sum_{i=1}^N z_i \le \varepsilon \\ &\qquad x \in \chi, z_i \in \{0, 1\}, i = 1, 2, ..., N \end{aligned}$$
Because M are big coefficients (weak formulations), it may cause numerical issues. Many scholars pay their attention on finding strong formulations without big coeffiecients M.

a. feasibility guarantees
The feasible region of SAA is defined as:

$$F_{\varepsilon,\gamma}^N := \{x \in \chi : \frac{1}{N}\sum_{i=1}^N 1_{\overline{f}(x,\varepsilon^i)+\gamma \le 0} \ge 1 - \varepsilon\}$$

Solutions of SAA is feasible to CCO with high probability $1 - \beta$:

$$P(F_{\varepsilon,\gamma}^N \subseteq F_\epsilon) \ge 1 - \beta$$

b. optimality guarantees
use SAA to generate lower bounds of CCO o_L^* with probability at least $1 - \delta$.

Robust optimization related methods

Robust optimization's typical form is:

$$\begin{aligned} (RC) : \min_{x \in \chi} \quad & c^T x \\ s.t. \quad & f(x, \varepsilon) \le 0, \in U_\epsilon \end{aligned}$$

RC finds the optimal solution which is feasible all realizations of uncertainties. The key point is how to construct an good uncertainty set U_ϵ.
Two points for an good uncertainty set:

1. RC is computationally tractable.
2. The optimal solution of RC is not too conservative.
   Many scholars made efforts on the second point.

safe approximation

$$(SA): \min_{x \in \chi} c^T x$$
$$s.t. \quad x \in \underline{F}$$

where $underline F \subseteq F_\epsilon$. F_ϵ is the feasible region of CCO.

The authors used a large space to talk how to find the uncertainty sets by applying safe approximation in **individual chance constraints**. However, it is hard for a rookie like me to figure out the mathematical derivation in each step.
Authors introduced several approaches to apply safe approximation into joint chance constrains:

1. convert joint chance constrains to individual chance constrains. For example, use
   $P\left(f_i(x, \varepsilon) \leq 1-\epsilon_i, \quad i=1, \ldots, m\right)$
   or use the pointwise maximum
   $\overline{f}(x, \varepsilon):=\max \{f_1(x, \varepsilon), \ldots, f_m(x, \varepsilon)\}$
2. directly deal with joint chance constraints.Authors introduced three typical approaches.

(The mathmatic part is over, next is application part.)

Applications in power systems

Pivotal task: maintain the real-time balance of supply and demand while ensuring the system is low-cost and reliable.

Security-constrained economic dispatch(SCED)

a. Deterministic SCED: no uncertainties.
Example: direct current optinam power flow (DCOPF)

$$(det-DCOPF): \min_{g} c(g)$$
$$s.t \quad 1^T g = 1^T d - 1^T \hat{w}$$
$$f = H_g g + H_w W - H_d d$$
$$\underline{f} \leq f \leq \overline{f}$$
$$\underline{g} \leq g \leq \overline{g}$$

where decision varibles are generation output levels $g \in R^{n_g}$. $c(g)$ is the total generation cost, $1^T d - 1^T \hat{w}$ is net demands, H is power transfer distribution factor (PTDF) matrix, f is transmission line flows, $\hat{w}$ is wind generation.

b. Chance-constrained SCED

Treat wind generation w as a random vector.

$$
\begin{gathered}
(cc-DCOPF): \min_{g,\eta} c(g) \\
s.t \quad 1^T g = 1^T d - 1^T w \\
f(\hat{w},\widetilde{w}) = H_g(g - 1^T \widetilde{w}\eta) - H_d d + H_w(\hat{w}+\widetilde{w}) \\
P_{\hat{w}}(\underline{f} \leq f(\hat{w},\widetilde{w}) \leq \overline{f} \quad and \quad \underline{g} \leq g - 1^T \widetilde{w}\eta \leq \overline{g}) \geq 1-\epsilon \\
1^T \eta = 1 \\
\underline{g} \leq g \leq \overline{g} \\
-1 \leq \eta \geq 1
\end{gathered}
$$

where η is affine control policy $\eta \in [-1,1]^{n_g}$ (proportionally allocate total wind fluctuations to each generate), it's also called participation factor or dustribution vector.

Security-constrained unit commitment(SCUC)

SCUC is used to minimize the generation cost.

Deterministic SCUC

$$
\begin{aligned}
&(det-SCUC): \min_{z,u,v,g,s} \textstyle\sum_{t=1}^{n^t} c_n^T z^t + c_u^T u^t + c_v^T v^t + c_g^T g^{t,0} + c_s^T s^t \\
&s.t. \quad 1^T g^{t,k} \geq 1^T \hat{d}^t - 1^T \hat{w}^t
\end{aligned}
$$

$$\underline{f} \leq H_g^{t,k} g^{t,k} - H_d^{t,k} \hat{d}^t + H_w^{t,k} \hat{w}^t \leq \overline{f} \quad (76c)$$

$$\underline{r} \leq g^{t,k} - g^{t-1,k} \leq \overline{r} \quad (76d)$$

$$a^k \circ (g^{t,0} - s^t) \leq g^{t,k} \leq a^k \circ (g^{t,0} + s^t) \quad (76e)$$

$$k \in [0, n_k], t \in [1, n_t]$$

$$\underline{g} \circ z^t \leq g^{t,0} \leq \overline{g} \circ z^t \quad (76f)$$

$$\underline{s} \circ z^t \leq s^t \leq \overline{s} \circ z^t \quad (76g)$$

$$\underline{g} \circ z^t \leq g^{t,0} - s^t \leq g^{t,0} + s^t \leq \overline{g} \circ z^t \quad (76h)$$

$$z^{t-1} - z^t + u^t \geq 0 \quad (76i)$$

$$z^t - z^{t-1} + v^t \geq 0 \quad (76j)$$

$$t \in [1, n_t]$$

$$z_i^t - z_i^{t-1} \leq z_i^\iota, \;\; \iota \in [t+1, \min\{t + \underline{u}_i - 1, n_t\}] \quad (76k)$$

$$z_i^{t-1} - z_i^t \leq 1 - z_i^\iota, \;\; \iota \in [t+1, \min\{t + \underline{v}_i - 1, n_t\}] \quad (76l)$$

$$i \in [1, n_g], \;\; t \in [2, n_t]$$

Objective function is total operation costs, including no-load costs $c_n^T z^t$, startup costs $c_u^T u^t$, shutdown costs $c_v^T v^t$, generation costs $c_g^T g^{t,0}$ and reserve costs $c_s^T s^t$.

The first constraint assures enough supply to meet net demand.

Constraints (76c), (76d) and (76g) are about transmission capacity, generation ramping capability and reserve limit in contingency scenario k at time t.

Constraints (76f) and (76g) are generation and reserve capacity.

(76i)-(76j) are logistic constraints about commitment status, startup and shutdown decisions.

Constraints (76k)-(76l) are minimum on/off time constraints for all generators.

Deterministic SCUC has no uncertain variables

Chance-constrained SCUC

(cc-SCUC):

$$\min_{z,u,v,g,s} \sum_{t=1}^{n_t} c_n^\intercal z^t + c_u^\intercal u^t + c_v^\intercal v^t + c_g^\intercal g^{t,0} + c_s^\intercal s^t \tag{77a}$$

$$\begin{aligned}
\text{s.t. } & (76b), (76c), (76d), (76e), \;\; k \in [0, n_k], t \in [1, n_t] \\
& (76f), (76g), (76h)), (76i), (76j), \;\; t \in [1, n_t] \\
& (76k), (76l), \;\; i \in [1, n_g], \;\; t \in [2, n_t]
\end{aligned}$$

$$\mathbb{P}\Big(\mathbf{1}^\intercal g^{t,k} \geq \mathbf{1}^\intercal(\hat{d}^t + \tilde{d}^t) - \mathbf{1}^\intercal(\hat{w}^t + \tilde{w}^t), \tag{77b}$$

$$\underline{f} \leq H_g^{t,k} g^{t,k} - H_d^{t,k}(\hat{d}^t + \tilde{d}^t) + H_w^{t,k}(\hat{w}^t + \tilde{w}^t) \leq \overline{f}, \tag{77c}$$

$$k \in [0, n_k], t \in [1, n_t]\Big) \geq 1 - \epsilon \tag{77d}$$

Wind generation os modeled as a random vector consisting of a deterministic predicted component $\hat{w} \in R^{n_w}$ and a stochastic error component $\widetilde{w} \in R^{n_w}$
The demand is also treated as a random vector with deterministic component $\hat{d}$ and a stochastic error component $\tilde{d}$.

solving chance-constrained SCUC
Sample average approximation is commonly used. There is no upper bound on the number of support scenarios for non-convex problems, so the scenario approach cannot be directly applied on cc-SCUC.

Generation and transmission expansion Generation

a. Purposes:

1. when to invest on new elements such as transmission lines and generators;
2. what types of new elements are necessary;
3. how much capacity is needed and where the best location would be for those new elements.

b. Objective function:

1. total cost of investment in new generators and transmission line;
2. environmental impacts;
3. cost of generation.

c. Constrains:

1. total or individual costs within budget;
2. capacity constriant;
3. reliability requirement;

4. supply-demand balance;
5. power flow equations;
6. operation requirements such as generation or transmission limits.

d. Uncertainties:

1. demand;
2. generation;
3. transmission outages;
4. renewables.