Chapter 5 -> Theory of Production

The prodn function indicates the physical, technological relnship betn input & output. It shows that with the given state of technological knowledge & during a particular period of ~~the~~ time the maximum possible output that consumer be produce with given amt of input or alteratives minimum quantity of inputs necessary to produce a given level of output.

Some basic points to be noted with respect to prodn function are as follows.

1 Prodn function is expressed with reference to the particular period of time.
2. It establishes functional relnship betn physical input & output.
3 It assumes a given state of technology. Improvement in technology creates new prodn function that produces larger output before with same amt of inputs.
4. It shows a flow of output resulting from a flow of input

Production function can be explained as follows

$Qx = f(La, L, K, M, T, ---)$

Prodn function thus shows prodn is the result of the various inputs such as land, labour, capital, etc. which go into the prodn of commodity x.

For simplicity, we can take only 2 most essential factors in prodn function labour (L) & capital (K).

$Qx = f(L, K)$

Thus, two inputs are substitute to some extend. There are 2 types of prodⁿ function.

1. Short-run prodⁿ function / Single variable prodⁿ function.

It is the study of the change in one variable factor on the output where other factors are fixed. In the short-run, output can be increased only by increasing the variable factor like labour, raw materials, etc. other fixed factors such as land, building, machine, etc can't be varied. Law of variable proportions studies the short run input output relation.

2. Long-run prodⁿ function / Multi variable prodⁿ function.

It is the study of the effect of change in all inputs on outputs in the long-run. Thus, in the long run all inputs are variable. It is expressed by law of return to scale. It refers to the functional relⁿship betⁿ the quantities of all inputs & output.

Cobb-Douglas Production Function

In 1928 American scholars C.W. Cobb & P.H. Douglas made a statistical enquiry into into same manufacturing industries in America & other countries to studies the emperical relation betⁿ change in physical input & resulting in physical output. From their studies, they introduce a prodⁿ function with two variable input labour & capital, this prodⁿ function is known as Cobb-Douglas Prodⁿ function.

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$Q = AL^a K^b$ or $Q = AL^{\alpha} K^{(1-a)}$

$Q = AL^{\alpha} K^{\beta}$ or $Q = AL^{\alpha} K^{(1-\alpha)}$

where, Q = Total output of the product

A = Input productivity / efficiency parameter

L = Quantity of labour employed

K = Quantity of capital employed

'a' & 'b' or 'α' & 'β' = Exponents / Powers.

+ve constant parameters of the prodⁿ function which measures the responsiveness of output to change in labour & capital or output elasticity of labour & capital with given technology.

The sum of the power of inputs (a+b) measures returns to scale. If $(a+b) = 1 \rightarrow$ Returns to scale are constant.

Also known as Linear Homogeneous P.F

If $(a+b) > 1 \rightarrow$ Returns to scale are increasing

If $(a+b) < 1 \rightarrow$ Returns to scale are decreasing.

Considered the Production function (P.F) $Q = 5L^{0.5} K^{0.3}$ –i Does it represent increasing, decreasing or constant returns to scale. Solⁿ,

Inc. both labour & capital input by m. we have.

$$Q = 5(m.L)^{0.5} \cdot (m.K)^{0.3}$$
$$= 5 \times m^{0.5} \times L^{0.5} \times m^{0.3} \times K^{0.3}$$
$$= 5 \times m^{0.8} \times L^{0.5} \times K^{0.3}$$
$$= m^{0.8} (L^{0.5} \times K^{0.3}) 5$$
$$= m^{0.8} Q \quad \leftarrow \text{from (i)}$$

Thus, by increasing both L & K by $m^{0.8}$ it is the case of decreasing returns to scale.

Total Production (TP), Marginal Production (MP) & Average Production (AP)

The quantity of good produced by using factors or inputs can be expressed on the basis of TP, MP & AP.

TP → Total volume of goods produced during the specify period of time is known as TP.

$$TP = AP \times \text{units of variable factor}$$

or, $TP = \Sigma MP$

MP → It is the addition to the TP due to a unit increase in variable factor.

$$MP = TP_{(n+1)} - TP_n$$

or, $MP = \frac{\Delta TP}{\Delta \text{ variable factor}}$

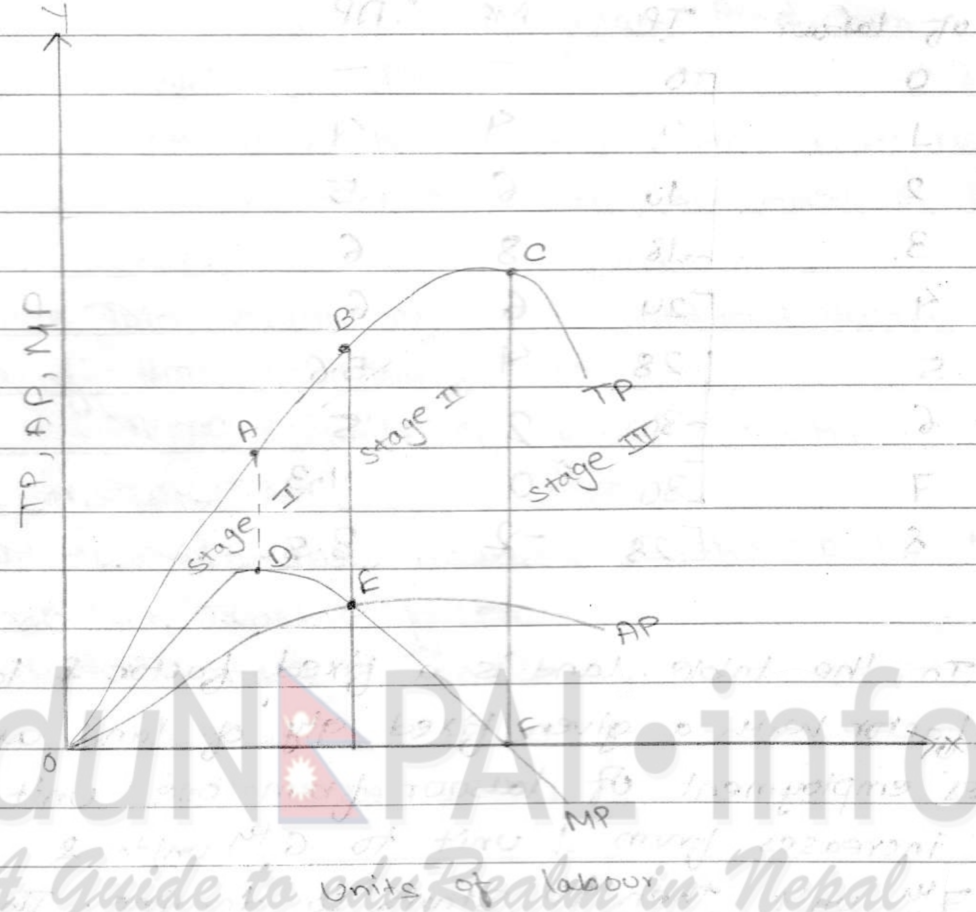
$$AP = \frac{TP}{\text{Units of variable factor}}$$

Law of variable proportions

The law of variable proportions concerns with short-run production function. This law examines the prodⁿ. with one factor variable, keeping the quantities of other factor fixed. TP increases at increasing rate, but after a point increases at decreasing rate becomes maximum & starts to decline.

Land	Unit of labour	TP	MP	AP
5	0	0	–	–
5	1	4	4	4
5	2	10	6	5
5	3	18	8	6
5	4	24	6	6
5	5	28	4	5.6
5	6	30	2	5
5	7	30	0	4.3
5	8	28	−2	3.5

In the table, land is a fixed factor & labour is a variable factor. With a given fixed qty of land as a farmer raises employment of labour from one unit to 6th unit, TP increases from 1 unit to 6th unit & become maximum at 7th unit then TP starts to decline. This fact can be viewed clearly from MP column. MP upto 3rd unit of labour increases & starts to diminish & at 7th unit it become zero & then it is negative. Hence, TP increase at an increasing rate up to 3rd unit and the increase at a decreasing rate upto 6th unit. Becomes maximum at unit 7 & then diminishes. The AP reach maximum at the 4th unit of labour & then starts to diminish.



Stage I (Increasing Return)

In this stage, TP increases at an increasing rate to a certain point & then rate of increasing switches from increasing to diminishing. TP is increasing upto point A at an increasing rate & then started to increase at diminishing rate. The AP increases throughout this stage & reaches to maximum at point E. The boundary of this stage is point E where marginal & average products are equal.

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Stage II

In this stage, TP continues to increase at a diminishing rate until it reaches it's maximum at point C where second stage ends. In this stage, both the MP & AP are diminishing but are +ve. At the end of second stage, i.e at point F MP is zero.

Stage III

This stage starts from the point C. In this stage total product is declining & MP is negative & MP goes below the x-axis. In this stage, variable factor is too much relative to the fixed factor.

Chapter 5 Theory of Production.

Causes of Increasing Returns in the first Stage

1. Increase in efficiency of fixed factor: In the initial stage of prodⁿ, the quantity of fixed factor are large than that of variable factor. Therefore, when more & more units of the variable factor are used to the constant quantity of the fixed factor, the fixed factor is more efficiently utilized so, the efficiency of the fixed factor increase as additional units of the variable factor are added to it. This causes the prodⁿ to increase at a rapid rate. At the beginning, when the variable factor is relatively smaller in quantity, some amt of fixed factor may remain unutilized. Therefore, when the variable factor is increased, the fuller utilization of the fixed factor becomes possible. As a result, the increasing returns are obtained.

2. Increase in the efficiency or productivity of variable factor: As variable factors are increased in the first stage, the efficiency or productivity capacity of variable factor itself increases. And in result it become possible to have division of labour which increases the efficiery of variable factors. Thus, in the initial stage prodⁿ increases.

Causes of Diminishing Returns in the second stage

1. Indivisibility of the fixed factor: As more & more quantities of variable factors are added the combination of fixed factor & Variable factors will be in the best proportion at a certain level of output. This is the optimum factor of proportion. But

beyond this point a quantities of variable factor are further added, the efficiency of the fixed indivisible factor decreases. AP diminishes & MP also goes on diminishing & become zero.

2. Imperfect substitutability of the factor: According to John Robbinson, factors can be substituted upto a limit beyond this limit, if more variable factor is added to the fixed factor diminishing returns sets in imperfect substitutability.

Causes of Negative Returns in the Third Stage.

1. Excessive variable factor: This stage sets in because quantity of ~~fo~~ variable factor becomes too excessive relative to the fixed factor. Too large no. of variable factor reduces the efficiency. Hence, MP of variable factors becomes negative & TP decline.

Stage of operation

A rational producer will never choose to produce in stage III where MP of variable factor is -ve. Even if the variable factor is free, a rational producer will not produce in stage III coz TP declines in this stage.

A rational producer also doesn't choose to produce in stage I although TP increases & MP of variable factor is +ve in this stage. This is coz the MP of fixed factor in this stage is -ve. Thus, there is opportunities of increasing prodn by increasing quantity of the variable factor where AP is continue to rise throughout the stage I.

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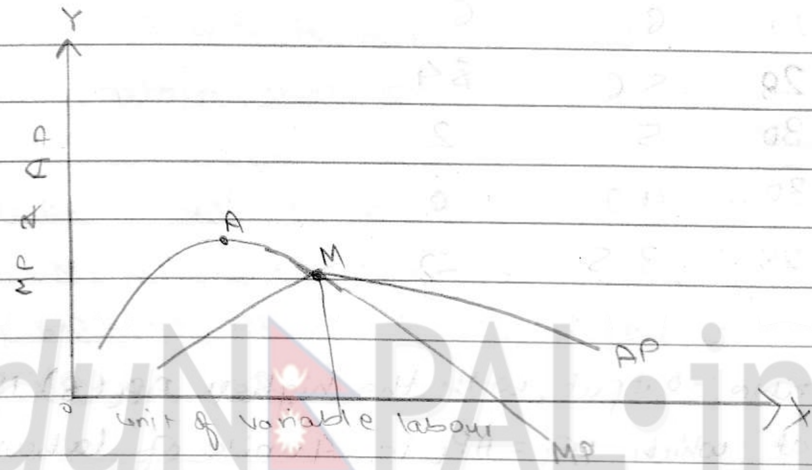
Thus, it is clear that a rational producer will produce in stage II where both MP & AP of variable factors are diminishing. At which particular point in this stage, the producer will decide to produce depends upon the prices of factors.

Relation betn ~~TP,~~ MP & AP.



AP & MP both are increasing. MP is highest at point A. AP is maximum at point M where MP & AP are equal & coincides each other. MP is less when AP diminishes. MP becomes -ve but AP remains +ve.

suppose a prodn schedule is given as follows complete the table & find the following.

Labour	0	1	2	3	4	5	6	7	2
TP	0	4	10	18	24	28	30	30	28

(a.) Output with the highest AP_L. (b) Employment at which $MP_L = AP_L$. (c) Employment at which TP is highest. (d) Construct a fig. (e) Enumerate law of Variable Proportion based on the fig. (f) Exp. the rel'ship betn MP & AP.

Soln,

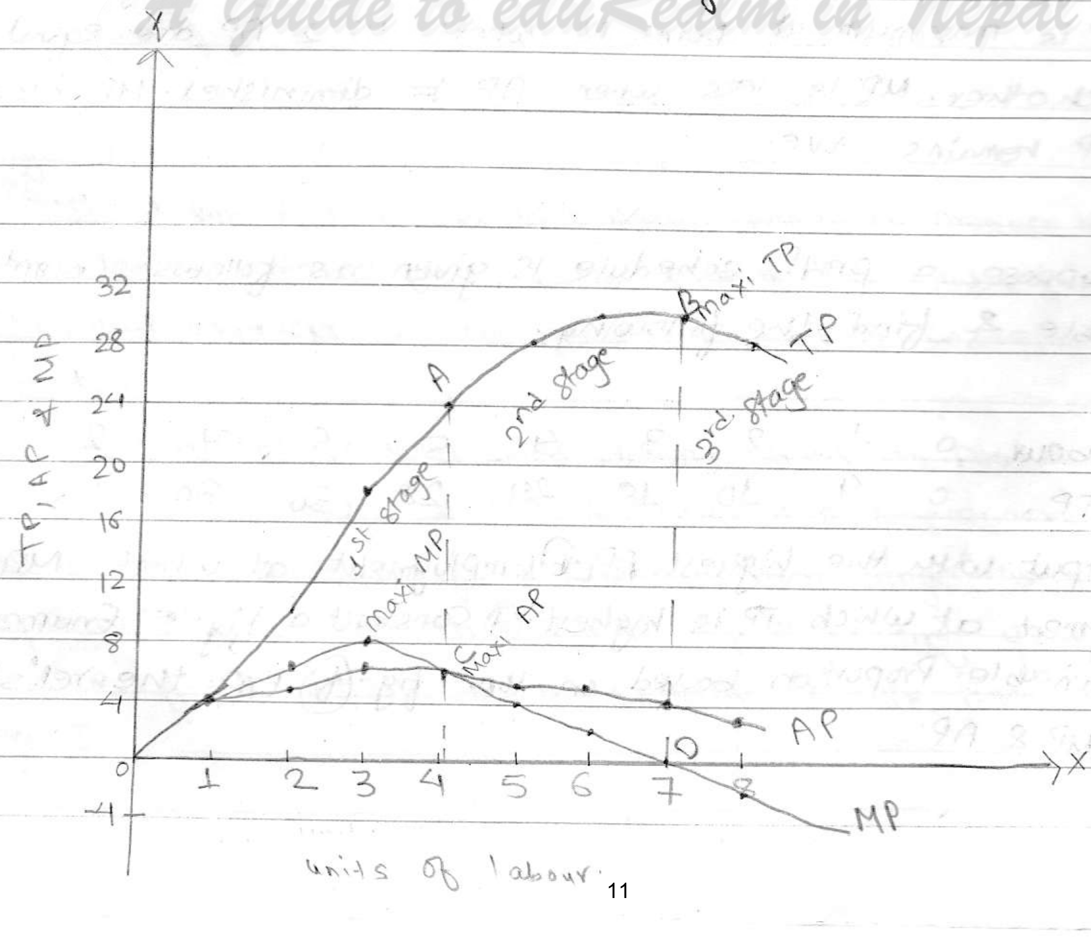
Labour	TP	AP	MP
0	0	–	–
1	4	4	4
2	10	5	6
3	18	6	8
4	24	6	6
5	28	5.6	~~8~~ 4
6	30	5	2
7	30	4.2	0
8	28	3.5	-2

a) From the table, output with the highest AP_L(6) is 24.

b) Emplayment at which $MP_L = AP_L$ is 4 units of labour.

c) " " " TP_L is highest & constant is 7 unit of labo

d)



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c. In the fig. units of labour is measured along x-axis & TP, AP & MP are measured along Y-axis. TP starts to increase at increasing rate from the ~~begg~~ beginning upto 3rd unit of labour. Then TP increase at decreasing rate upto 6th unit of labour. Become maximum at 7th unit where MP is 0 then TP starts to fall. The AP reaches maximum at 4th unit of labour & the AP starts to diminish but never attain 0 & -ve but MP attain zero & also reaches -ve.

b. Relationship betn. AP & MP

i When MP > AP, AP is rising.

ii When MP = AP, AP is at its maximum.

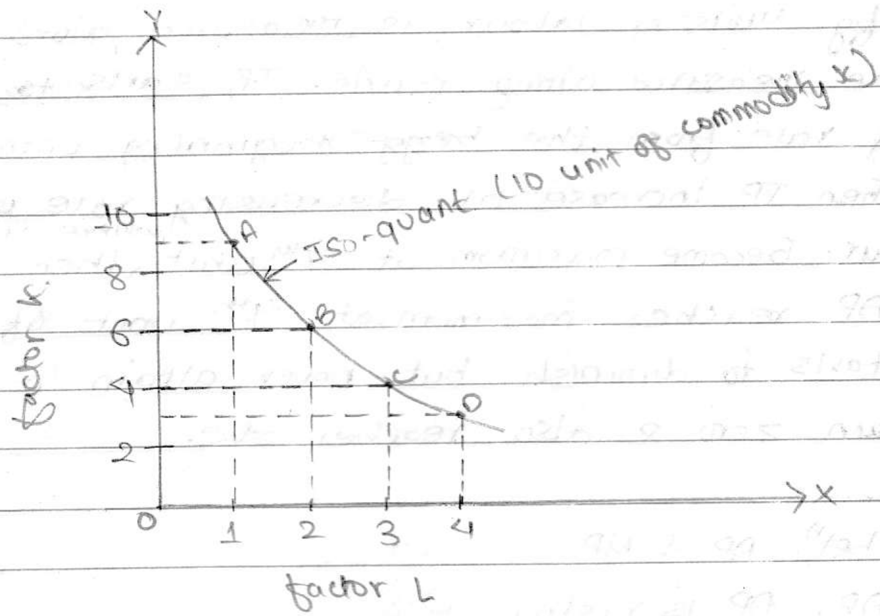
iii When MP < AP, AP is falling.

Iso-quant, Iso-product curve, Equal-product curve

The term 'isoquant' has been derived from a Greek word 'iso' means equal & Latin word 'quant' means quantity. Therefore, the isoquant curve is also known as equal product curve & prodn. indifference curve. An isoquant is the locus of points representing different combinations of two inputs (labour & capital) yielding the same level of output. The producer is indifferent among them.

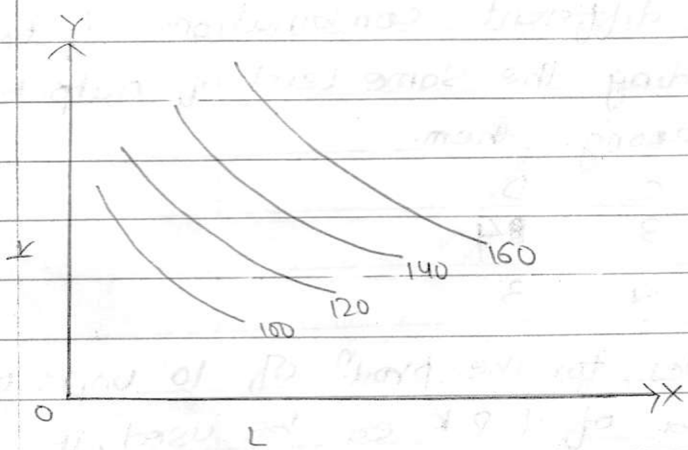
Combination	A	B	C	D
Labour (L)	1	2	3	4
Capital (K)	9	6	4	3

The schedule indicates, for the prodn of 10 units of x A or B or C or D combi. of L & K can be used, if we plot all these combi. graphically we get a curve known as iso-curve.



In the fig. points A, B, C & D on the isoquand IQ shows four different combination of capital & labour. But all these combinations yield the same output i.e 10 units. Movement from A to B indicates decreasing quantity of K & increasing quantity of L. This implies substitution of labour for capital yield the same level of output along the isoquant IQ.

Iso-quant Map.



The fig. shows a set of iso-quant. The combinations lying on the same isoquant shows an equal level of output But the higher isoquant represents the higher level of output than the lower isoquant

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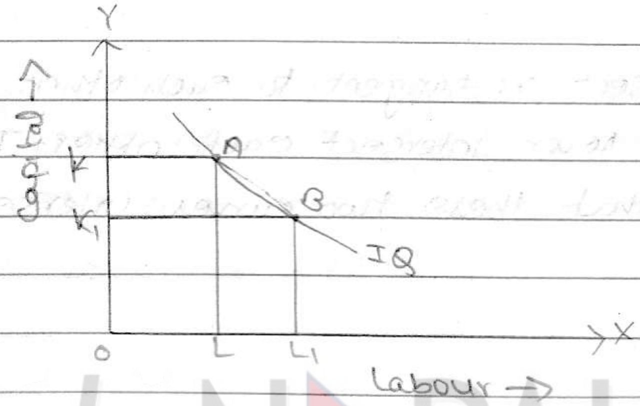
Properties of Isoquants

1. Slope of Isoquant curve is negative or downward sloping

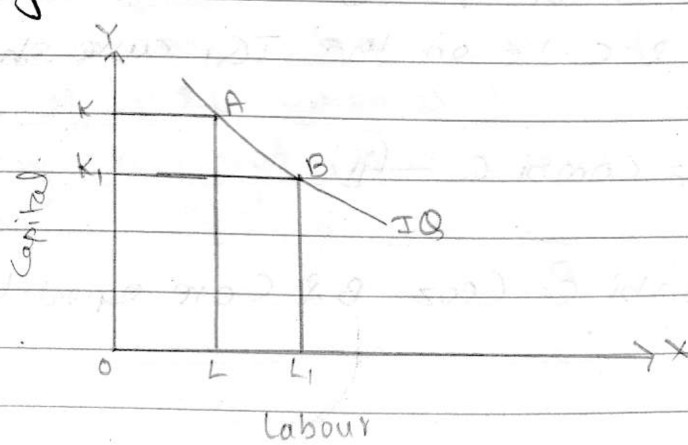
An isoquant curve is negatively sloped. This is so because when unit of labor is increased, the units of capital must be reduced so as to keep the amount of output constant.



In the fig, the movement from A to B on IQ shows that KK_1 unit of capital is reduced from the prodn process, LL_1 unit of labor is increase to maintain to same level of output.

2. Isoquant curve is convex to ~~be~~ bent the origin

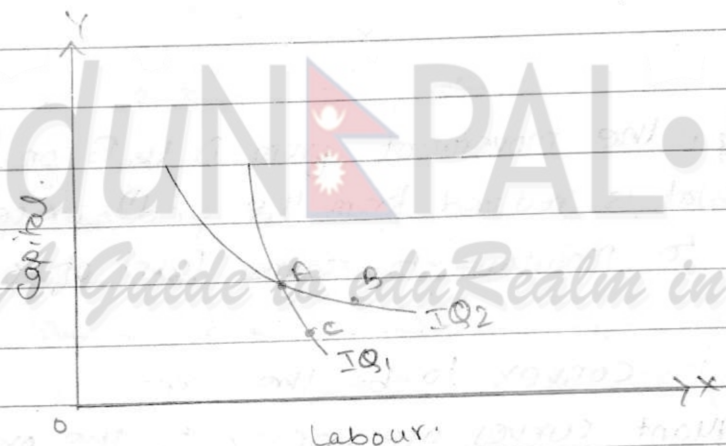
All the isoquant curves are convex to the origin. It is due to the fact that MRTS falls as more & more unit of labor is substitute for capital. It means, less & less unit of K is substituted by labor so as to keep the level of output unchanged.



In the fig., units of labour are increased & unit of capital are decreased. But both labour & capital are increasing & decreasing in same rate respectively. It means that initially more & less & ~~less~~ unit of K are substituted for each unit of L. This gives us a convex IQ curve.

3. IQ curve never intersect or tangent to each other

Two IQ curves never intersect each other. To prove this property, let's suppose that these two curves intersect each other.



In the fig., two IQ curves IQ1 & IQ2 are intersected each other at poin A. Point A & B lie on the same IQ curve IQ2. They represent same level of output

Combi. A = Combi. B —(i)

Similarly, points A & C lie on the IQ1 curve showing equal level of output.

Combi A = Combi C —(ii)

From i & ii

Combi C = Combi B (coz B & C are equal to A)

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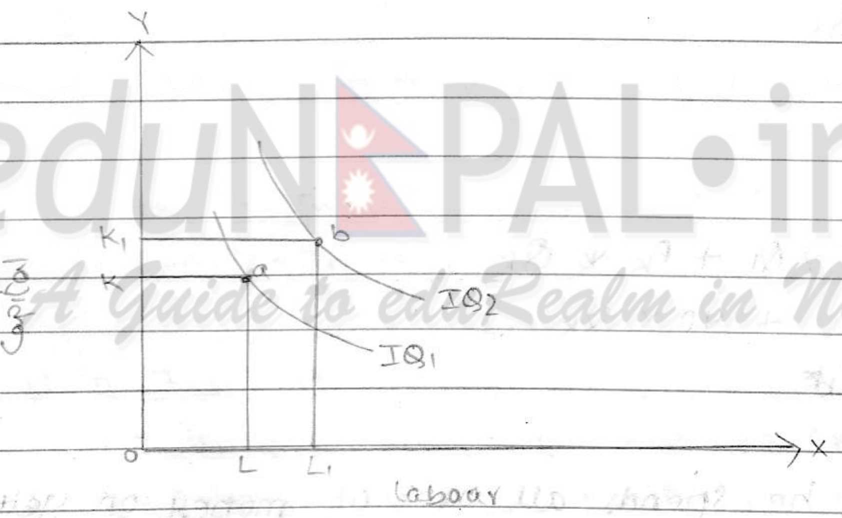
But this result is absurd since B lies on the higher IQ curve i.e IQ2 & thus represents higher level of output. While point C is located on the lower IQ i.e, IQ2 shows the smaller amount of output. In this way, B can't be equal to C. Hence, two IQ curves never intersect or tangent to each other.

4. Higher Iso-quant represents higher level of output

The higher IQ curve represents higher level of output than the lower one because higher IQ consists of more of both labours & capital.



In the fig., the higher IQ curves IQ2 always represents a higher level of output than lower IQ1. At any point on IQ2 consists of more of either capital or labour or both. Therefore IQ2 represents a higher level of output. For eg: 'a' is the combi. of K & L whereas 'b' is the combi. of K1 & L1 which clearly show that there is more amt of K & capital & labour in combi. b.

Iso - cost Line.

An Iso cost line shows all possible combination of two factors that the producer can get by spending a given amt of money on two factors, L & K given their prices.

It is the total cost using the total amt of money for spending L & K.

Suppose a producer wants to spend Rs 200 on factor L & K. Price of L is Rs 20 & K is Rs 40.

$C = P_L \times Q_L + P_K \times Q_K$

$200 = 20 \times Q_L + 0$

$\frac{200}{20} = Q_L$

$10 = Q_L$

$C = P_L \times Q_L + P_K * Q_K$

$200 = 0 + 40 \times Q_K$

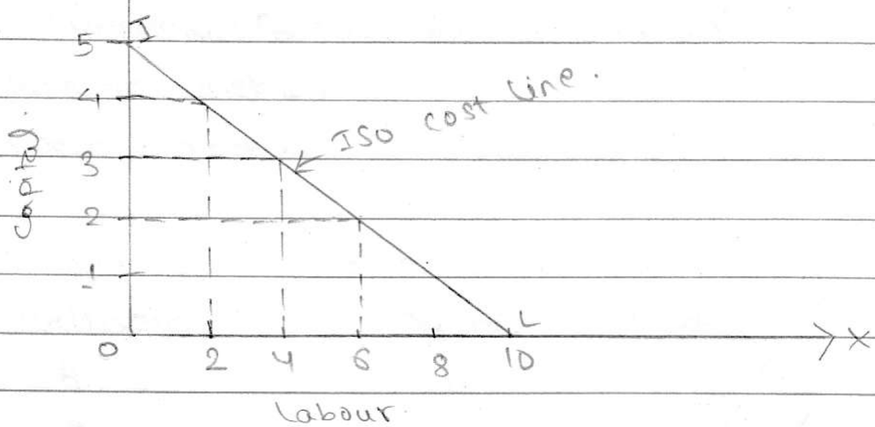
$5 = Q_K$

If he spends all amt of money on getting factor L, he can get 10 units of it.

If he spends all amt of money on getting factor K, he can get 5 units of it.

But he can get some other combination of L & K too. Such as (2L + 4K), (4K + 3K), (6L + 2K).

L	K
2	4
4	3
6	2



Slope of Iso-cost line.

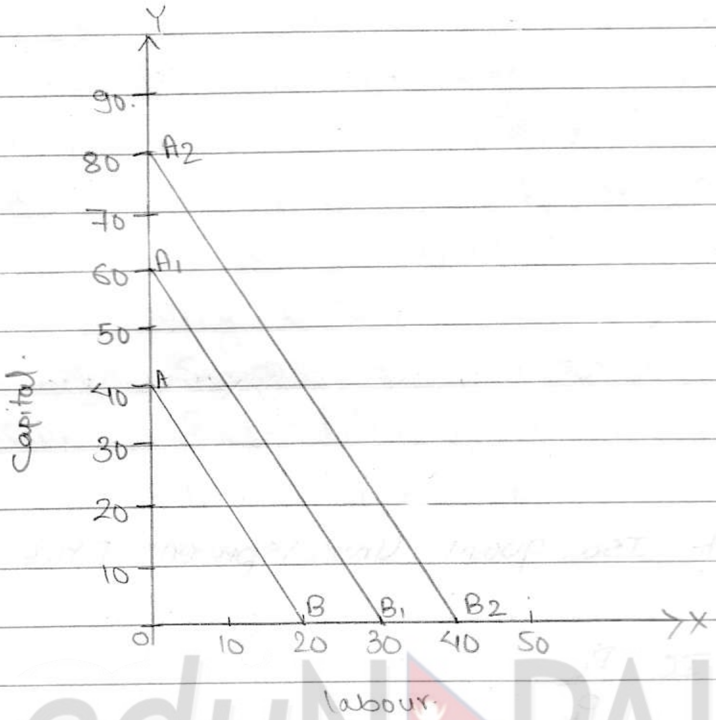
The slope of Iso quant line represent price of factor K & factor L.

$$\text{Slope of IC} = \frac{P_L}{P_K}$$

$$\text{or, } \frac{\text{Y-Intercept}}{\text{X-Intercept}} = \frac{OI}{OL} = \frac{C/P_K}{C/P_L} = \frac{P_L}{P_K}$$

Shift in Iso-Cost line.

The iso-cost line shifts with the change in total outlay. If the price of labour & capital remain constant but the producer decides to spend Rs 300 rather than Rs 200, the Iso-cost line shifts outward to A_1B_1. It shows that now 60 units of capital or 30 units of labour can be purchase Similarly, when the producer wants to spend Rs.400, the iso-cost line shifts upward to A_2B_2. Therefore, if the prices of both inputs remain the same but the expenditure to be done on the purchase of factors increases, we get a higher Iso-cost line. ~~ed~~ Similarly, if the proposed expenditure to be done by the producer falls, the Iso cost line shifts downwart but remain parallel to the original Iso-cost line.



The slope of Iso-cost line can also change when the outlay remains the same but the price of one input or both the inputs changes. The change is similar to the rotation in the price line as was explained in the IC analysis.

Fill up the following IQ schedule

Combination	labour	Capital	MRTSLK	ΔK/ΔL
A	1	14	–	
B	2	10	4	$= \frac{14-10}{2-1}$
C	3	7	3	=
D	4	5	2	
E	5	4	1	

Least cost Combination of Inputs
(Optimal Combination of Inputs)

1. Arithemetic method
2. Geometric method

1. Least cost of L & K are calculated from total cost.

Combination	Labour	Capital	Total cost of L	Total cost of K	Total cost
A	3	12	21	60	81
B	4	9	28	45	73
C	5	7	35	35	70
D	6	6	42	30	72
E	7	5	49	25	74

Suppose a producer has decided to produce 100 units of comm X. There are 5 diff. combi of L & K. Per unit Labour cost (P_L) = Rs 7 & capital cost (P_K) = Rs 5.

⇒ In the table among the 5 alternative combi, the producer will choose the combi. 'C' where 5 units of labour & 7 units of capital are capable of producing 100 units of comm. X at the least cost of Rs 70. Thus, 'C' is the least or optimal combi. of L & K for producing 100 units of X.

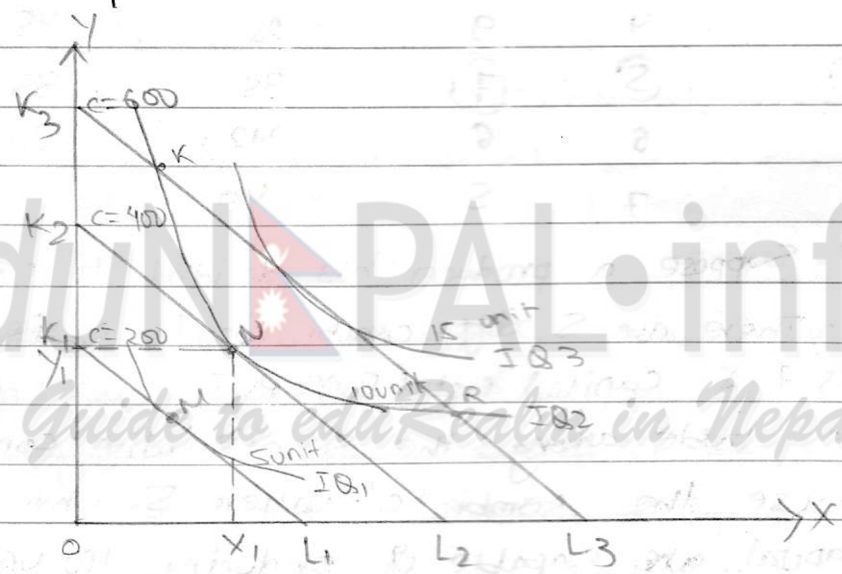
2. A rational consumer always tries to maximize profit by producing a given output at least cost combi. of factors. It is determined on the basis of ISO cost line & ISO-quants. There are two essential conditions for determining least cost combi. of factors.

a. Iso-cost line should be tangent to IQ. In other words slope of the Iso-cost line & IQ should be equal.

MRTSLK

$$\frac{\Delta K}{\Delta L} = \frac{P_K}{P_L} = \frac{(\text{Price of } K)}{(\text{Price of } L)} \frac{r}{w}$$

b. MRTSLK must be diminishing i.e IQ convex to origin at the point of equilibrium.



In the fig. K_1L_1, K_2L_2 & K_3L_3 are Iso cost line representing total cost c 200, 400 & 600 respectively. IQ are represented by IQ_1 & IQ_2 representing 5 unit & 10 units of comm. X respectively. Among the diff. combination the producer chooses 'N' combination at which K_2L_2 is tangent to IQ_2. Combi 'N' will cost him least for producing 10 units of commodity X. He will not choose any other combi. such as K & R coz it lies in higher Iso cost line K_3L_3, it means higher

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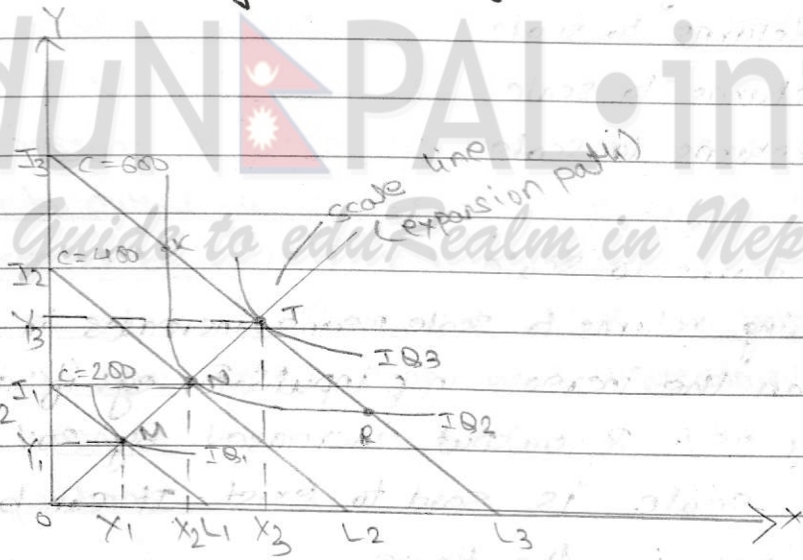
total cost Rs 600. for producing 10 units of commodity x. The producer doesn't choose any other combination of L & K on IQ1, because it represent less unit of comm. x (5 units). Therefore, the producer will be equilibrium at the point N where the amt of money spent will be least by choosing OX1 of labour & OY1 ~~amt~~ of capital.

At the equilibrium point, the slope of IQ represented by $MRTS_{LK}$ & the slope of Iso cost line represented by Price ratio of two factor L & K will be equal.

At the equilibrium point, N, IQ is convex to the origin indicating decreasing $MRTS_{LK}$.



In the fig. M, N & T are the equilibrium points representing 5 units, 10 units & 15 units respectively. By joining these equilibrium points we get a like known as scale line or expansion path. Scale line shows the points of tangency betn Iso cost line & IQ. It indicates how the producer will change combi. of two factors L & K to expand the output of the firm. It shows the cheapest method of

Returns to factor → law of variable proportion (change in proportion)
Returns to scale → long run input-out relation (change in scale)

2L + 1M
4L + 2M
6L + 3M

All the inputs are variable, No change in the proportion

each of goods at the given prices of two factors.

Returns to scale.

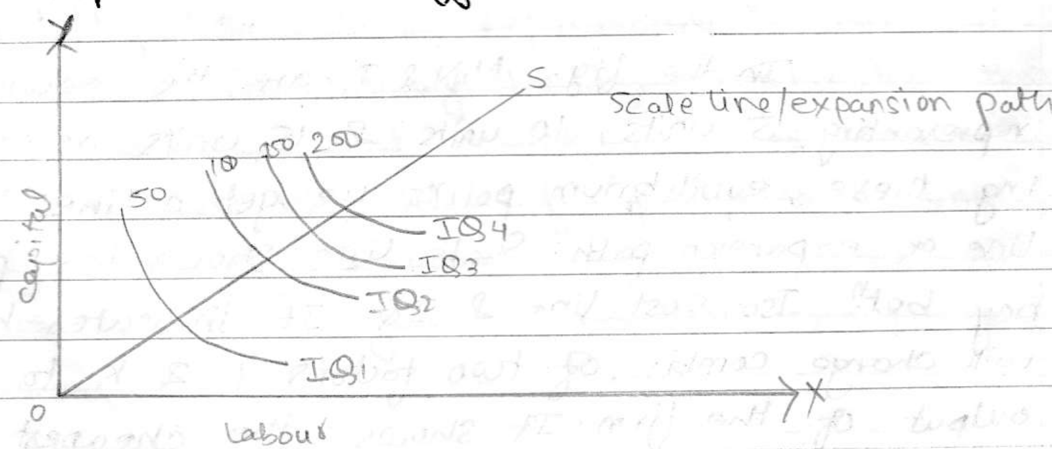
Long Run Input-Output relation

In the long run all the factors are variable distinction betn. fixed & variable factors doesn't exist in long run by changing all the factors in the same (given) proportion the firm can change scale of production. When a firm change its scale of prodn. what would be the real effect on the prodn. in the long run is explains by the returns to scale. Prodn. is subject to 3 types of Returns to scales.

1. Increasing Returns to scale.
2. Constant Returns to scale
3. Decreasing Returns to scale

1. Increasing Returns to Scale.

Increasing returns to scale means increases in output a greater proportion than the increase in inputs. For eg: if all inputs are increased by 25% & output increased by 30% then increasing returns to scale is said to exist. It can be shown by using Isoquant in the figure.



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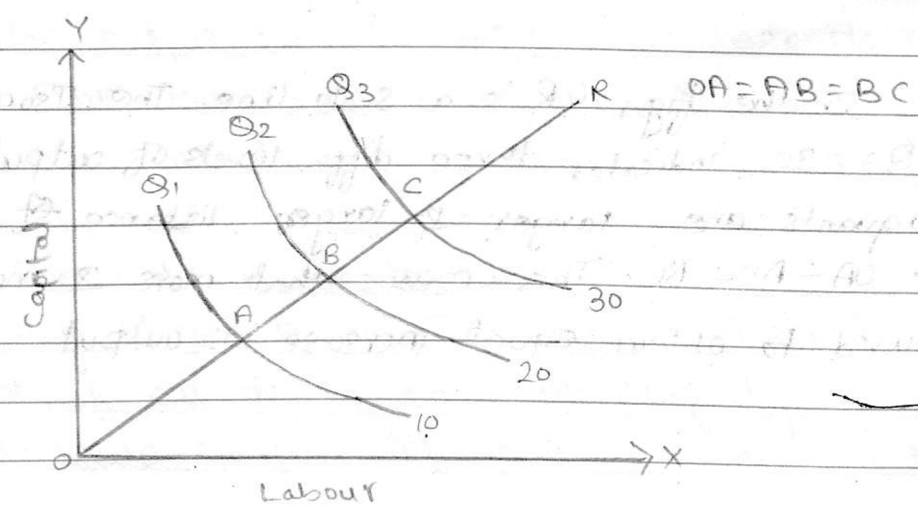
The fig shows, the prodⁿ function with increasing returns to scale. The line OS drawn from the origin is the scale line which are the same slope throughout because L & K are increased in a given/fixed proportion indicating the changes in the scale of production. IQ1, IQ2, IQ3 & IQ4 represents Iso-quants indicating 50, 100, 150 & 200 units of output respectively. In the case of Increasing returns to scale, the distance betⁿ IQs are decreasing. It indicates that as firm expanses the output, it requires small increases in quantity of L & K to produces the equal increment of output OA > AB > BC > CD. It is cause due to large scale economies (advantages).

2. Constant Returns to scale.

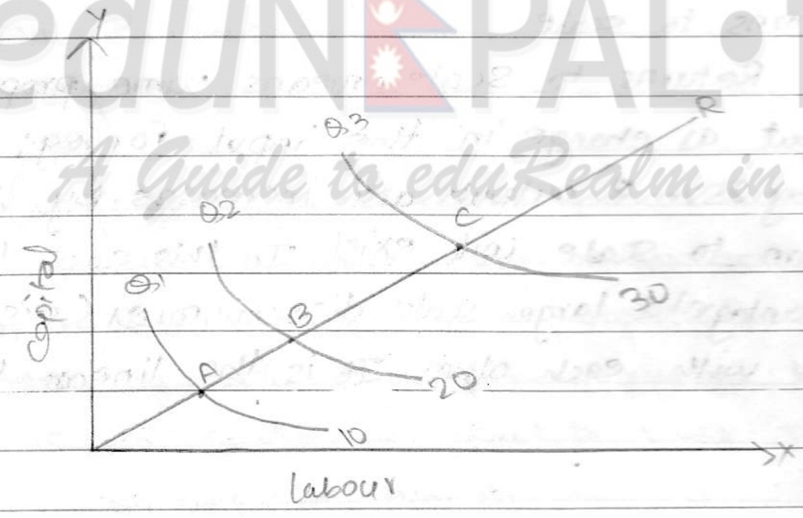
Constant Returns to scale means same proportion of change in output as change in the input. For eg: if all inputs are increased by 25% & output also increases by 25% then constant ~~increase~~ in return to scale will exist. In this case, large scale economies (advantage) & large scale diseconomies (disadvantage) counter balance with each other. It is the linear homogenous prodⁿ function.



In the fig. OR is a scale line. The Isoquants, $Q_1 = 10$, $Q_2 = 20$ & $Q_3 = 30$ indicate three different levels of output. The successive isoquants are equi-distance from each other i.e $OA = AB = BC$. This means if both labour & Capital are increased in a given proportion then the output expands by the same proportion.

3. Decreasing Returns to Scale.

When output increases less than proportionately to increase in inputs it is called decreasing returns to scale. For eg: if all inputs are increased by 25% & output increased by 20% then decreasing returns to scale will prevail.



In the fig; OR is a scale line. The Isoquants, $Q_1 = 10$, $Q_2 = 20$ & $Q_3 = 30$ indicates three diff. levels of output. The successive isoquants are larger & larger distance ~~of~~ on the scale line. i.e $OA < AB < BC$. This means that more & more of inputs are required to obtain equal increase in output.

1. Cause of Increasing Returns to scale.

a. Indivisibility of factors of prodn.

Some of the factors such as enterpreneur & heavy machine are indivisible even in the long run. By increasing the scale of output these factors are fully used, their efficiency increases & increaseing returns to scale can be obtained. If all the factors are perfectly divisible increasing returns to scale do not operate.

b. Specialization: In the view of chamberlin, even if all the factors are perfectly divisible increasing returns to scale will operate because of the greater possibility of specialization of factors such as L & K.

c. Managerial economies: When ~~single~~ scale of prodn is increases management work can be subdivided in diff units or functional basis - such as prodn, finance, marketing, advertisement, etc. They can be entrusted to different specialized personnels & efficiency increases, increasing returns to scale applied.

d. Dimensional relation: According to Baumal increasing returns to scale operate due to dimensional reln where the L & K are double the output will be~~come~~ more than double. Output increases more than proportionatly. When the diameter of water pipe is double the flow of water is more than double.

2. Cause of Constant Returns to Scale

According to the economist John Robinson, Kaldor, Stigler & Knight, if all the factors are perfectly divisible constant returns to scale operate. If some factors are scarce of indivisible, constant returns to scale do not ~~even~~ exist but in the opinion of chamberlin even if all the factors are divisible due to greater specialization of L & K, as scale of prod^n. increased in a fixed proportion increasing returns to scale operate. But empirical evidence shows that in the expansion of a firm, after a certain phase of Increasing R.T.S there is a long phase of constant return to scale covering a wide range of output.

3 Causes of Decreasing Returns to Scale (Diseconomies)

a. Indivisibility of factors: According to Kaldor, decreasing returns to scale occur when indivisible factors such as entrepreneur becomes inefficient & less productive due to over expansion of the scale of production.

b. Difficulty in management
According to chamberlin, because of the difficulty in supervison & cordination in the prod^n. decreasing R.T.S operate.

c. Scarcity of Natural resources
When a firm doubles its scale of prod^n. natural resources such as coal deposits, petroleum products etc can't be doubled.

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Chapter 6 Cost & Revenue Curves

1. Total Revenue (TR) → TR is the total amt of money received by the firm from the sales of its own product at the given period of time. $TR = P \times Q$ P = Price Q = Quantity Sold.

2. Average Revenue (AR) → Average revenue is the price per ~~unit~~ unit. It is obtaind by dividing total revenue by the no. of units sold.

$$AR = \frac{TR}{Q} = \frac{P \times Q}{Q}.$$

3. Marginal Revenue (MR) → MR is the addition to total revenue from the sales of an additional unit of the commodity.

$$MR = \frac{\Delta TR}{\Delta Q} \text{ or, } MR = TR(n+1) - TRn$$

Derivation of Revenue Curve under Perfect competition.

Perfect competition is the market structure where there are large number of buyers & sellers producing a homogeneous product. In the perfect competition, firm is a 'price-taker'. A firm can sell whatever output it produces at ~~price~~ the given price. Therefore, price remains constant at any level of output. The price is determined by market mechanism.

Units sold	AR / Price (RS) = TR/Q	TR = P×Q	$MR = TR_{n+1} - TR_n$
0	0	0	—
1	10	10	10
2	10	20	10
3	10	30	10
4	10	40	10