

For an irrotational flow, the vorticity is zero. Therefore,

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

Substituting the definitions of the stream function into the above equations yields

$$\frac{\partial}{\partial x} \left(-\frac{\partial \Psi}{\partial x} \right) - \frac{\partial}{\partial y} \left(\frac{\partial \Psi}{\partial y} \right) = 0$$

or

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = 0$$

The goal in this problem is to obtain the solution of this elliptic partial differential equation using the various numerical techniques discussed earlier. The solution will provide the streamline pattern within the chamber.

Since the chamber walls are streamlines, i.e. lines of constant Ψ , we will assign values for these streamlines, as shown in the figure. Solve this problem with codes using the following techniques:

- | | |
|-----------------------|--------------|
| a) Point Gauss-Seidel | c) Point SOR |
| b) Line Gauss-Seidel | d) Line SOR |

For all methods, the step sizes are specified as

$$\Delta x = 0.2, \quad \Delta y = 0.2 \quad \text{ERRORMAX} = 0.01$$

with convergence criterion $ERROR < ERRORMAX$, where

$$ERROR = \sum_{\substack{j=JM-1 \\ i=IM-1 \\ i=2 \\ j=2}} \left| \Psi_{i,j}^{k+1} - \Psi_{i,j}^k \right|$$

Use an initial data distribution of $\Psi = 0.0$. Plot the streamline pattern (lines of constant Ψ). Rerun the SOR codes for several values of the relaxation parameter and plot the relaxation parameter versus the number of iterations for these two schemes. In each case, determine the optimal value of the relaxation parameter.

Submit a both a hardcopy of your code and an electronic copy on Canvas.

Note: Except for the purpose of creating plots, MATLAB, Excel, or other commercially available software may not be used for this assignment. You may otherwise program in any language you wish.