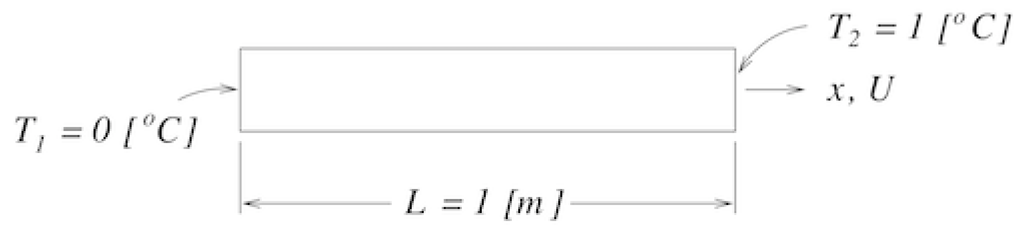
Assignment 3 - Convection of a Scalar

Solve the following problems and explain your results.

Problem 1

Consider a convection/diffusion problem that has Dirichlet conditions on temperature imposed on both ends (see the figure below). This is an unusual problem that would be difficult to reproduce in a laboratory, but it is an interesting problem by which to observe the performance of advection schemes.



The exact solution to this problem is given by:

$$T(x) = T_1 + \frac{e^{xPe/L} - 1}{e^{Pe} - 1}(T_2 - T_1)$$

where Pe is defined as $Pe = uL/\alpha$. Solve this problem numerically for $Pe = 50$ by imposing:

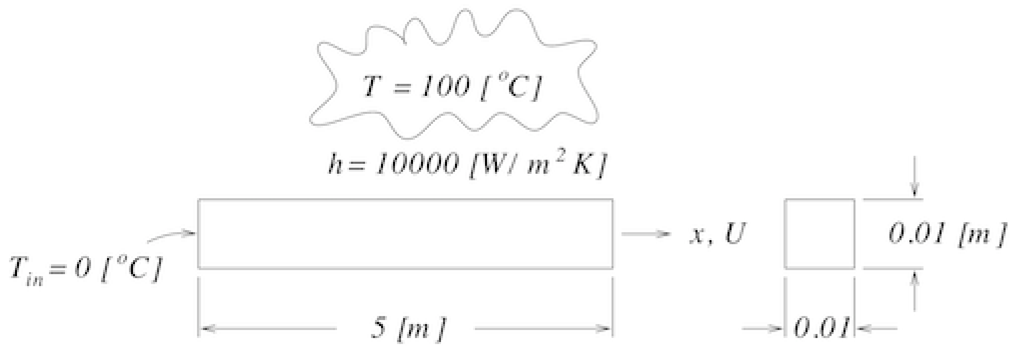
- L = 1 [m]
- ρ = 1 [kg/m^3]
- c_p = 1 [J/kg·K]
- u = 1 [m/s]
- k = 0.02 [[W/m·K]]
- T_1 = 0 [˚C]
- T_2 = 1 [˚C]

Discretize the one-dimensional domain using 10 equal sized control-volumes. Initialize the field variables as T = 0.0 [K] and u = 1 [m/s]. Then, carry out the following:

1. Solve the problem using UDS, CDS, and QUICK (use $\Delta t = 10^{10}$ [s]). Modify the value of α_e on the domain boundaries such that the value of temperature on the face is equal to the specified boundary temperature.
2. Plot the results for T for all cases along with the exact solution. Discuss your results.
3. Re-run the case using 20, 40 and 80 uniformly spaced control volumes. Discuss your results.

Problem 2

Consider the problem of water flowing through a heated square duct (see the figure below).



The properties of water are ρ = 1000 [kg/m^3], k = 0.590 [W/m·K], and c_p = 4189 [J/kg·K]. The exact solution for this problem is:

$$\frac{T_\infty - T(x)}{T_\infty - T_{in}} = e^{-\frac{hP_0 x}{\dot{m}c_p}}$$

To solve this problem, start with 5 equal-length control volumes, initialize the temperature and velocity fields as T = 0 [°C] and u = 1.5 [m/s], and use a time-step size of 10^{10} [s]. At the left boundary, set $\alpha_e = 1$ to ensure that the correct value of T gets carried into the domain. Then, carry out the following:

1. Solve the problem using UDS, CDS and QUICK and plot T for all schemes along with T_{exact} vs. x with α_e = -1.0 at the right boundary. Discuss the results.
2. Test the effect of α_e at the right boundary. That is, set α_e = 1.0 at the right boundary, run the cases of (1) again and discuss the differences that occur. What physical effect does setting $\alpha_e = \pm 1.0$ at the right boundary have? Which is realistic?
3. Reverse the flow direction and the boundary conditions re-run parts (1) and (2). Show plots of T vs. x. Make sure you use appropriate values for α_e on the boundaries. Your solutions should be the same as those from parts (1) and (2), except opposite.

Based on your results for this problem, make a general statement about how α_e should be set on the boundaries and why.

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