

Pricing and profit testing of life insurance

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by

Lemin Wu
Department of Mathematics
University of Leicester

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Contents

Declaration	ii
Abstract	iii
Background	1
Gauss's Work	2
Hilbert's Work	2
1 Results of Gauss	4
1.1 Gauss's youthful work	4
1.2 Gauss's mature work	4
2 Unit-linked insurance	5
2.1 Background	5
2.1.1 Some assumptions *** change variables!!	6
2.2 Deterministic pricing and reserving	7
2.2.1 Profitability	7
2.3 Stochastic pricing	7
2.3.1 time series and stochastic process	7
2.3.2 Monte Carlo simulation	7
2.3.3 Pricing	7
2.3.4 Reserving	7
2.4 Deterministic VS Stochastic test	7
Bibliography	8
Appendix 1: Optional Extra	9
Appendix 2: More Extra	10

Declaration

All sentences or passages quoted in this project dissertation from other people's work have been specifically acknowledged by clear cross referencing to author, work and page(s). I understand that failure to do this amounts to plagiarism and will be considered grounds for failure in this module and the degree examination as a whole.

Name:

Lemin Wu

Signed:

Date:

Abstract

A summary of the thesis in about 200 words.

Introduction

In this thesis we consider the work of Gauss [1, ch 2] and Hilbert [?] on the subject of the title.

1. why doing the project
2. what have I done
3. what have I achieve
4. what work can be done in extension

Conventions

The following variables are used throughout the project

$$\ddot{a}$$

$$\ddot{A}$$

$$\ddot{A}_{[x]}^{(m)}$$

$$\ddot{a}_{[x]}^{(m)}$$

Notation

assurance?annuity

We will discuss Gauss's work in Chapter ??.

unit-linked

We will discuss Hilbert's work in Chapter ??.

Background

?Do I need background? mathematical background

Chapter 1

Results of Gauss

In [1] Gauss proved the following very important result.

THEOREM 1.1 (Gauss) *Some very profound result.*

Later on in Chapter ?? we will have more to say about Theorem 1.1.

1.1 Gauss's youthful work

1.2 Gauss's mature work

Chapter 2

Unit-linked insurance

In this chapter we introduce the unit-linked insurance contract. We start from some assumptions to establish a deterministic pricing model, and demonstrate that the deterministic pricing test is not accurate enough for this contract, which is caused by the uncertainty, or in other words risk, in the investment is not diversifiable.

To solve this problem, we consider to use a stochastic pricing test with future investment return as an random variable, then testify the stochastic test will determine a better premium and reserve.

2.1 Background

In some countries unit-linked contract is also called equity-linked contract, it is because the single or regular premium paid by the policy holder will be invested on units of Equity or bonds on their own behalf.

Once the insurer receive the premium, it will be payed into policy holder's fund after deduction of expenses and management costs, and the deduction goes to insurer's account. In UK, there is an un-allocated percentage, which is another agreed regular deduction from policy holder's fund to insurer's account.

[2]If the policy holder survival to the maturity date, he/she will receive the greater value of total premiums payed in or the fund in policy holder's account. This is called

guaranteed minimum maturity benefit (GMMB). But the if the death happens first, there is a guaranteed minimum death benefit, which allows policy holder's estate to receive the policy holder's fund with an extra amount.

2.1.1 Some assumptions *** change variables!!

Before we establish the model, we need some assumptions, the following are some ideas from [3] book *Actuarial Mathematics for Life Contingent Risks*, and some online sources [4].

A company is going to issue a new 10 years unit-linked contract to people from 55 to 60 years old. In the contract, it stated that the annual premium is £5200, with an un-allocated rate of 5% in the first year and 1% in the subsequent years. Expenses occurs the same time as the premium paid in, which is 10% (including commision) and 0.5% repectively. At the end of each year, there is a further management charge of 0.8% of policy holder's fund. All the above deductions from policy holder's account will be transferred to insurer's account.

When the contract mature, the policy holder can received the greater of policy holder's fund and accumulation of premiums paid in. But if the death happens before the mature date, policy holder's estate will receive 110% of the policy holder's fund where the extra 10% is from insurer's account.

The insurance company is also prepared to have 10% of policy holders surrender the contract in the first year and 5% in the second year, but no more in the subsequent years. The policy holders who surrender the contract will received their premium(s) after deduction of management cost at the end of the year.

In the Chapter 1, we had introduce different methods to calculate the mortality rate, but here we are going to use a constant force of mortality, $\mu_x = 2.2218$, for people aged 55 to 60, which means the mortality rate of policy holders $q_x = 0.006$ is a constant in each year. The reason we set the mortality rate to a constant will be explained later in this chapter.

BABE!!! HOW CAN I MAKR A EQUATION AT THE BOTTOM OF THE PAGE? ??

2.2 Deterministic pricing and reserving

Last section we introduce the unit-linked contract and have some assumptions to help us establish the model. But here is something we did not count in, the uncertainty in the investment return in both policy holder's account and insurer's account. For deterministic pricing, we used conservative interest rates for both accounts, 8% for policy holder's account and 5% for insurer's account, and assume no reserve holds in this case. Since the mortality rate is a constant, as well as and conservative interest rates, the cash flow for different policy holders will be exactly the same.

2.2.1 Profitability

2.3 Stochastic pricing

2.3.1 time series and stochastic process

2.3.2 Monte Carlo simulation

2.3.3 Pricing

2.3.4 Reserving

2.4 Deterministic VS Stochastic test

Bibliography

- [1] Gauss, C.F., “Disquisitiones Arithmeticae”, Leipzig, 1801.
- [2] David C. M. Dickson, Mary R. Hardy and Howard R. Waters, *Actuarial Mathematics for Life Contingent Risks*,, 3rd edition, 2011, page 374-375.
- [3] David C. M. Dickson, Mary R. Hardy and Howard R. Waters, *Actuarial Mathematics for Life Contingent Risks*,, 3rd edition, 2011, page 375.
- [4] Mrs Giselle du Toit *Actuarial Mathematics for Life Contingent Risks*,, 2nd edition, 2011, page 375.

Appendix 1: Optional Extra

A PROGRAM TO COMPUTE EIGENVALUES

```
100 GOTO 200
```

```
200 END
```

Appendix 2: More Extra

Table 2.1: Lowercase Greek Letters.

α	<code>\alpha</code>	θ	<code>\theta</code>	o	<code>o</code>	v	<code>\upsilon</code>
β	<code>\beta</code>	ϑ	<code>\vartheta</code>	π	<code>\pi</code>	ϕ	<code>\phi</code>
γ	<code>\gamma</code>	ι	<code>\iota</code>	ϖ	<code>\varpi</code>	φ	<code>\varphi</code>
δ	<code>\delta</code>	κ	<code>\kappa</code>	ρ	<code>\rho</code>	χ	<code>\chi</code>
ϵ	<code>\epsilon</code>	λ	<code>\lambda</code>	ϱ	<code>\varrho</code>	ψ	<code>\psi</code>
ε	<code>\varepsilon</code>	μ	<code>\mu</code>	σ	<code>\sigma</code>	ω	<code>\omega</code>
ζ	<code>\zeta</code>	ν	<code>\nu</code>	ς	<code>\varsigma</code>		
η	<code>\eta</code>	ξ	<code>\xi</code>	τ	<code>\tau</code>		

Table 2.2: Uppercase Greek Letters.

Γ	<code>\Gamma</code>	Λ	<code>\Lambda</code>	Σ	<code>\Sigma</code>	Ψ	<code>\Psi</code>
Δ	<code>\Delta</code>	Ξ	<code>\Xi</code>	Υ	<code>\Upsilon</code>	Ω	<code>\Omega</code>
Θ	<code>\Theta</code>	Π	<code>\Pi</code>	Φ	<code>\Phi</code>		

Table 2.3: Math Alphabets.

Example	Command	Required package
$ABCDEabcde1234$	<code>\mathrm{ABCDE abcde 1234}</code>	
$ABCDEabcde1234$	<code>\mathit{ABCDE abcde 1234}</code>	
$ABCDEabcde1234$	<code>\mathnormal{ABCDE abcde 1234}</code>	
$ABCDE\infty\epsilon\Delta$	<code>\mathcal{ABCDE abcde 1234}</code>	
$\frac{ABCDEabcde1234}{}$	<code>\mathfrak{ABCDE abcde 1234}</code>	amsfonts or amssymb
$\mathbb{ABCDE\omega\mu\xi\tau}$	<code>\mathbb{ABCDE abcde 1234}</code>	amsfonts or amssymb