Pricing and profit testing of life insurance

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in partial fulfilment of the requirements for
the MSc degree in Financial Mathematics and Computation

by

Lemin Wu
Department of Mathematics
University of Leicester

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Contents

Declaration

All sentences or passages quoted in this project dissertation from other people's work have been specifically acknowledged by clear cross referencing to author, work and page(s). I understand that failure to do this amounts to plagiarism and will be considered grounds for failure in this module and the degree examination as a whole.

Name:

Lemin Wu

Signed:

Date:

Abstract

A summary of the thesis in about 200 words.

Introduction

In this thesis we consider the work of Gauss [1, ch 2] and Hilbert [**?**] on the subject of the title.

1. why doing the project
2. what have I done
3. what have I achieve
4. what work can be done in extension

Convensions

The following variables are used though out the project

$\ddot{a}$

$\ddot{A}$

$\ddot{A}_{[x]}^{(m)}$

$\ddot{a}_{[x]}^{(m)}$

${}_tp_x$

Notation

assurance?annuity

unit-linked

Background

?Do I need background? mathematical background

Chapter 1

Annuity and Assurance

1.1 Background

1.2 Pricing

1.2.1 Mortality rate

1.2.2 annuity

1.2.3 Premium

1.3 Reserving

1.4 Sensitivity test

Chapter 2

Unit-linked insurance

In this chapter we introduce the unit-linked insurance contract. We start from some assumptions to establish a deterministic pricing model, and demonstrate that the deterministic pricing test is not accurate enough for this contract, which is caused by the uncertainty, or in other words risk, in the investment is not diversifiable.

To solve this problem, we consider to use a stochastic pricing test with future investment return as an random variable, then testify the stochastic test will determine a better premium and reserve.

2.1 Background

In some countries unit-linked contract is also called equity-linked contract, it is because the single or regular premium paid by the policy holder will be invested on units of Equity or bonds on their own behalf.

Once the insurer receive the premium, it will be payed into policy holder's fund after deduction of expenses and management costs, and the deduction goes to insurer's account. In UK, there is an un-allocated percentage, which is another agreed regular deduction from policy holder's fund to insurer's account.

[2]If the policy holder survival to the maturity date, he/she will receive the greater value of total premiums payed in or the fund in policy holder's account. This is called

guranteed minimum maturity benefit (GMMB). But the if the death happens first, there is a guaranteed minimum death benefit, which allows policy holder's estate to receive the policy holder's fund with an extra amount.

2.1.1 Some assumptions

Before we establish the model, we need some assumptions, the following are some ideas from [3] book *Actuarial Mathematics for Life Contingent Risks,* and some online sources [4].

A company is going to issue a new 10 years unit-linked contract to people from 55 to 60 years old. In the contract, it stated that the annual premium is £5200, with an un-allocated rate of 5% in the first year and 1% in the subsequent years. Expenses occurs the same time as the premium payed in, which is 13% (including commision) and 0.7% repectively. At the end of each year, there is a further management charge of 0.8% of policy holder's fund. All the above deductions from policy holder's account will be transferred to insurer's account.

When the contract mature, the policy holder can received the greater of policy holder's fund and accumulation of premiums payed in. But if the death happens before the mature date, policy holder's estate will receive 110% of the policy holder's fund where the extra 10% is from insurer's account.

The insurance company is also prepared to have 10% of policy holders surrender the contract in the first year and 5% in the second year, but no more in the subsequent years. The policy holders who surrender the contract will received their premium(s) after deduction of management cost at the end of the year.

In the Chapter 1, we had introduce different methods to calculate the mortality rate, but here we are going to use a constant force of mortality, $\mu_x = 2.2218$, for people aged 55 to 60, which means the mortality rate of policy holders $q_x = 0.006$ is a constant in each year. The reason we set the mortality rate to a constant will be explained later in this chapter.

BABE!!! HOW CAN I MAKR A EQUATION AT THE BOTTOM OF THE PAGE? ??

2.2 Deterministic pricing and reserving

Last section we introduce the unit-linked contract and have some assumptions to help us establish the model. But here is something we did not count in, the uncertainty in the investment return in both policy holder's account ans insurer's account.

For deterministic pricing, we used conservative interest rates for both accounts, 8% for policy holder's account and 5% for insurer's account, and assume no reserve holds in this case. Since the mortality rate is a constant, as well as the conservative interest rates, the cash flow for different policy holders will be exactly the same. Now we set one as an example.

2.2.1 Cash flow

Now from all the assumptions, we can create the cash flow tables for both policy holder and insurer's account for the 10 years term. First let us explore the policy holder's account.

		Cash flow for policy holder's account					
Year t	Annual premium	Allocated premium	Fund brought forward	Interest	Fund at time t^-	Management cost	Fund bring forward
1	5,200	4,940	0	395.2	5,335.2	42.68	5,292.52
2	5,200	5,148	5,292.52	835.24	11,275.76	90.21	11,185.55
3	5,200	5,148	11,185.55	1,306.68	17,640.24	141.12	17,499.12
4	5,200	5,148	17,499.12	1,811.77	24,458.89	195.67	24,263.21
5	5,20,0	5,148	24,263.21	2,352.89	31,764.11	254.11	31,509.99
6	5,200	5,148	31,509.99	2,932.64	39,590.64	316.73	39,273.91
7	5,200	5,148	39,273.91	3,553.75	47,975.67	383.81	47,591.86
8	5,200	5,148	47,591.86	4,219.19	56,959.05	455.67	56,503.38
9	5,200	5,148	56,503.38	4,932.11	66,583.49	532.67	66,050.82
10	5,200	5,148	66,050.82	5,695.91	76,894.73	615.16	76,279.57

Figure 2.1: Cash flow

Here are the method used to calculate the table above from colum to column. (BABE, I DON'T MEAN METHOD HERE, WHAT WORD CAN I USE?)

Allocated premium: For the first year, the premium allocated in the policy holder's fund will be (1-5%) of the premium payed, where 5% is the agreed un-allocated premium which will be payed into insurer's account. For the subsequent years, the un-allocated rate decrease to 1%, hence there will be 99% premium payed into policy holder's fund.

BABE!! CAN YOU PLEASE LINE UP THE TWO EQUATIONS BELOW? THX!! XXX

First year allocated premium: 5,200 $\times$ 0.95= 4,940

Subsequent years allocated premium: 5,200 $\times$ 0.99= 5,148

Interest: It is the profit policy holder obtain from the the accumulation of premium payed in at the beginning of the year and fund brought forward from end of last year with the assumed interest rate 8%.

eg. year t=5, interest = (5,148 + 24263.21) $\times$ 0.08 = 2,352.89

Fund at time t^-: It is the accumulation of premium payed in at the beginning of the year, fund brought forward from end of last year and interest earned.

Management cost: In our assumption, the management cost is 0.08% of the policy holder's fund, which is the fund at time t^- here.

Fund bring forward: It is the fund left in policy holder's account after deduction of management cost.

Table SIMON!!!! HOW TO NUMBER THE TABLE???!? is showing the insurer's account cash flow for one policy holder.

		Cash flow for insurer's account						
Year t	Annual premium	Un-allocated premium	Expenses	Interest	Management cost	Expected death benefit	Profit	Π_t
0	0	0	676	0	0	0	-676	-676
1	5,200	260	36.4	11.18	42.686	3.18	274.29	274.29
2	5,200	52	36.4	0.78	90.21	6.71	99.87	89.35
3	5,200	52	36.4	0.78	141.12	10.50	147.00	124.14
4	5,200	52	36.4	0.78	195.67	14.56	197.49	165.78
5	5,200	52	36.4	0.78	254.11	18.91	251.59	209.92
6	5,200	52	36.4	0.78	316.73	23.56	309.54	256.73
7	5,200	52	36.4	0.78	383.81	28.56	371.63	306.38
8	5,200	52	36.4	0.78	455.67	33.90	438.15	359.05
9	5,200	52	36.4	0.78	532.67	39.63	509.42	414.95
10	5,200	52	36.4	0.78	615.16	45.77	585.77	474.28

In insurer's account, un-allocated premium, expense, interest and management cost are calculated in the same way as in policy holder's account. You may notice there is an extra row in the table, year 0 in insurer's cash flow, this is because the expenses will always occurs before receive the premium, eg. renting, stationary cost and employees' salary.

For expected death benefit, profit and profit signature are calculated as below:

Expected death benefit: If the death occurs before the contract mature, the policy holder's estate will receive the fund from policy holder's account with extra 10% from insurer account. The death benefit we calculate here is the 10% from insurer's account.

At time 0, there policy holder just enter the contract, so the probability in force is 1.

In year 1, policy holders pay the first premium right after sign the contract, so the probability in force is still 1.

During the first year, by our assumption, there will be 10% policy holders surender the contract, and the mortality rate for the other 90% is 0.006. Therefore, at the beginning of the second calandar year, insurer will be expecting p_2 = (1-0.006) $\times$(1-0.1) =0.8946 of policy holders renew their contracts.

Same as in the first year, the insurer is expecting p_3= (1-0.006-0.5) $\times p_2$= 0.8445024 will renew the contract, since by assumption there are 5% policy holder will surender during the second year.

From the third year, the probability in force will be p_t = (1-0.006) $\times$ p_{t-1}. (Showing in table !!!SIMON ADD TABLE NUMBER FOR ME!!)

probability in force	
t=	${}_tp_x$
0	1
1	1
2	0.8946
3	0.8445024
4	0.839435386
5	0.834398773
6	0.829392381
7	0.824416026
8	0.81946953
9	0.814552713
10	0.809665397

Profit: For each year, profit is calculated by the fomula

Profit = Un-allocated premium - Expenses + Interest + Management cost - Death benefit

Profit signature: This is profit the insurer can make with probality the contracts are still in force, which is defined as $\Pi_t = p_x \times$ Profit$_t$

2.2.2 Profitability

From the table !!!!!! and table!!!!!! we can see that policy holder's fund is increasing every year, and on the date the contract mature, it has £76,279, which is approximately 1.47 times of the accumulation of all premiums. This means, if the policy holder survive to the mature date, he/she will withdraw the fund from policy holder fund since it will always be the greater one compares to the accumulation of all the premiums. [1] This means the insurer will have the probability of 0.81 [2] to make profit of £585.77 on each policy holder. And if we discount the profit signature with discount rate of interest of 15%, the net present value of the profit will be

$$NPV = \sum_{i=1}^{10} \Pi_t \times (1 + 0.15)^{-t} = \pounds 489.59$$

And the profit margin is

$$Profit\ Margin = \frac{NPV}{P\ddot{a}_x}^{3} = \frac{489.59}{5200 \times 7.619285226} = 1.24\%$$

Since in the very beginning, we used conservative assumption, such as expenses, mortality rate, interest rate for insurer's fund etc., so if in reality the expense and mortality rate are lower than modelled, but interest rate is higher then the profit the insurer can obtain will be higher. Sensitivity test can be used to predict the profit the insurance company can obtain with changes in different factors.

[1] GMMB policy: on the mature date policy holders can withdraw the greater value between policy holder's fund and the accumulation of all the premiums.

[2] the probability of this contract in force till the mature date, table!?!?!?!?!?!!?

[3] $\ddot{a}_x = \sum_{i=1}^{10} v^t {}_tp_x = 7.619285226$, where ${}_tp_x$ read from table ?!?!?!? and $v = \frac{1}{1+0.05}$

2.2.3 Sensitivity test

Now we change our previous assumptions one by one and see what will happen.

Scenario 1: Lower expenses

We had

Scenario 2: Lower mortality rate

Constant mortality rate change from 0.006 to 0.005.

		Cash flow for insurer's account						
Year t	Annual premium	Un-allocated premium	Expenses	Interest	Management cost	Expected death benefit	Profit	Π_t
1	5,200	260	36.4	11.18	42.68	2.65	274.82	274.81
2	5,200	52	36.4	0.78	90.21	5.59	100.99	90.44
3	5,200	52	36.4	0.78	141.12	8.75	148.75	125.88
4	5,200	52	36.4	0.78	195.67	12.13	199.92	168.34
5	5,200	52	36.4	0.78	254.11	15.75	254.74	213.42
6	5,200	52	36.4	0.78	316.73	19.64	313.47	261.31
7	5,200	52	36.4	0.78	383.81	23.80	376.39	312.20
8	5,200	52	36.4	0.78	455.67	28.25	443.80	366.27
9	5,200	52	36.4	0.78	532.67	33.03	516.02	423.74
10	5,200	52	36.4	0.78	615.16	38.14	**593.40**	**484.85**

Here are the new net present value of profit and profit margin obtianed from the new cash flow:

$$NPV = \sum_{i=1}^{10} \Pi_t \times (1 + 0.15)^{-t} = £491.43$$

$$Profit\ Margin = \frac{NPV}{P\ddot{a}_x}^4 = \frac{491.43}{5200 \times 7.619285226} = 1.24\%$$

The lower mortality rate will only affect the insurer's account. From the table we can see that the profit will increase from £585.77 to £593.40, but the net present value of the profit will increase less than £2, and the profit margin will not change. This means the change in mortality rate is not sensitive, and should not be count as a important factor in our model, so a constant assumption for the mortality rate is acceptable.

[4] $\ddot{a}_x = \sum_{i=1}^{10} v^t \, {}_tp_x = 7.619285226$, where ${}_tp_x$ read from table ?!?!?!? and $v = \frac{1}{1+0.05}$

Scenario 3: Higher interest rate

Interest rate change from 8% to 9%.

		Cash flow for policy holder and insurer's account				
Year t	Interest	Fund at time t^-	Management cost	Fund bring forward	Profit	Π_t
0					-676	-676
1	444.6	5,384.6	43.08	5,341.5232	274.65	274.65
2	944.06	11,433.58	91.47	11,342.11	101.04	90.39
3	1,484.11	17,974.22	143.79	17,830.43	149.48	126.23
4	2,068.06	25,046.49	200.37	24,846.11	201.84	169.44
5	2,699.47	32,693.58	261.55	32,432.04	258.47	215.67
6	3,382.20	40,962.24	327.7	40,634.54	319.7	265.15
7	4,120.43	49,902.97	399.22	49,503.75	385.9	318.14
8	4,918.66	59,570.4	476.56	59,093.84	457.49	374.9
9	5,781.77	70,023.6	560.19	69,463.42	534.89	435.67
10	6,715.03	81,326.44	650.61	**80,675.83**	618.59	500.85

In the table, it shows that once interest change 1%, the fund in the policy holder's account will increase about 4.6% every year, and the profit for insurer will increase 5.3%. And the new net present value of profit and profit margin will be as following:

$$NPV = \sum_{i=1}^{10} \Pi_t \times (1+0.15)^{-t} = £522.72$$

And the profit margin is

$$Profit\ Margin = \frac{NPV}{P\ddot{a}_x} = \frac{522.72}{5200 \times 7.619285226} = 1.3\%$$

There is a big raise for both of the factors, NPV of profit increases from £489.59 to £522.72, and the profit margin shoots up from 1.24% to 1.3%. This is a very sensitive case, little change in interest will have huge impact in profit. A good interest model will be very essential in this case.

2.2.4 Little summary for deterministic test

So far everything looks fine, but is this a good model for unit-linked contract? The answer will be no, from our sensitivity test, we can see that the interest rate change will lead to an fluctuate in profit, so the asssumption of constant interest rate is not good enough in this case. This is because the constant interest rate does not contain enough information in our model, the uncertainty of investment on equity and bond is not diversifiable. By law, we do not have an arbitrage market, no one can make profit without risk, and the interest on return will be affected by a few different factors, [5] eg. inflation, quality of information, governmental policy etc., change in bond market or equity market etc., and none of the factors is predictable. So now we should consider to model the interest rate non-linearly, eg. a random variable.

2.3 Stochastic pricing

From the sensitivity test in last section, we conclude that the change in interest rate is non-diversifiable and extremely sensitive, which a constant assumption is not suitable. So now we are going to introduce stochastic test which can model the interest rate as a random variable, such that it can realistically reflect the reality. (BABE! I DON'T LIKE THE LAST SENTENSE, CAN YOU FIGURE OUT SOMETHING BETTER??? XXX)

2.3.1 Central Limit Theorem and Monte Carlo simulation

In our basic assumptions [5], we assume that the policy holders will be able to choose different units of equities or bonds or portfolios [6] to invest their premium. Which means the the more equities and bonds available, the less similarity in the interest return each year for different policy holders.

By [6]Central Limit Theorem, which obtained by Mood, Graybill, and Boes in 1974, if we have a number of independent identically distributed random variables [7], we can observe

[5]assumption in 2.1.1

[6]Portfolio: mixed equities and bonds

[7]the random variables can follow any distribution

a new series of random variables following normal distribution by summing or multiplying the old ones. The theorem can be used in finance, compound return or equity return follows a log-normal distribution[8], which means the logarithm of interest return I_t follows a normal distribution with mean μ and variance σ^2. We can apply this theorem in our model, then we will have

$$logI_t = \xi \sim N(\mu, \sigma^2)$$

$$where\ \mu = 0.74928, \sigma = 0.15$$

Now we can use Monte Carlo simulation[7], generate some random numbers [9] ξ_t' from normal distribution with mean 0 and variance 1, and transform them to the specified Normal distribution $N(0.74928, 0.15^2)$ via

$$\xi_t = 0.74928 + 0.15 \times \xi_t'$$

then, our interest will be simulated by

$$I_t = exp\{\xi_t\}$$

2.3.2 Modelling

Now we assume there are 1,000 policy holders [10] purchased unit-linked contract with different portfolios. Using the generated random variable from Standard Normal distribution, we can construct our new cash flow tables. I used C^{++} to help me with constructing the tables [11] since there are 1,000 of them and the interest rate will be simulated differently for each policy holder and different years.

Here is an example we get from the program. [12]

[8]log-normal distribution is also called Galton's distribution, after Francis Galton

[9]I used C^{++} boost random number generator,

[10]The larger number the better statistic result we will get, here we just set 1,000 as an example.

[11]codes will be included in the appendix

[12]To curious readers, this is the 1st cash flow table we get from the program.

Year t	ξ_t	Cash flow for policy holder and insurer's fund R_t	Premium	Allocated Premium	Fund at t^-	Management	Fund at t	Profit	Π_t
0	0	0	0	0	0	0	0	-676	-676
1	0.213436	1.11287	5,200	4,940	5,497.58	43.9807	5,453.6	313.709	313.71
2	-0.49558	1.00059	5,200	5,148	10,607.9	84.863	10,523	94.9292	84.92
3	1.57538	1.36511	5,200	5,148	21,392.6	171.141	21,221.5	174.788	147.61
4	-1.0592	**0.919475**	5,200	5,148	**24,246.1**	193.969	**24,052.1**	195.917	164.46
5	1.83927	1.42023	5,200	5,148	41,470.8	331.767	41,139.1	323.463	269.90
6	1.88577	1.43017	5,200	5,148	66,198.4	529.587	65,668.8	506.566	420.14
7	0.604675	1.18014	5,200	5,148	83,573.4	668.588	82,904.9	635.225	523.69
8	-0.365983	1.02023	5,200	5,148	89,834.4	718.675	89,115.7	681.586	558.54
9	-0.578264	0.988258	5,200	5,148	93,156.9	745.255	92,411.6	706.188	575.22
10	-0.634376	0.979975	5,200	5,148	95,606	764.848	94,841.1	724.323	586.46

Notes:

ξ_t**:** This column is the generated random variables from Standard Normal distribution.

R_t**:** This is simulated annual accumulation factor, **the annual interest rate is R_t-1**.

Other columns in the table: The table was calculated using the same method in deterministic cash flow, the only difference is the interest rate.

From this single table we can see that, the annual accumulation factor is sometimes less than one, which indecates that annual interest R_t-1 is negative in that year, then there is a loss in the policy holder's fund. But for this policy holder, the net present value of profit and profit margin[13] are:

$$NPV = \pounds 1,011.86 \qquad \& \qquad Profitmargin = 2.6\%$$

which is ideal for the insure. Now showing another example from the 1,000 which is not that idea.

[13] Profit margin is calculated using risk discount factor 15%

Year t	ξ_t	Cash flow for policy holder and insurer's fund R_t	Prem- ium	Alloc- ated Premium	Fund at t^-	Manage- ment	Fund at t	Profit	Π_t
0	0	0	0	0	0	0	0	-676	-676
1	-1.33578	0.88211	5,200	4,940	4357.62	34.86	4,322.76	305.27	305.27
2	-0.412251	1.01318	5,200	5,148	9,595.55	76.76	9518.79	87.43	78.22
3	-1.09414	0.91467	5,200	5,148	13,415.3	107.32	13,307.9	115.72	97.72
4	0.847198	1.22386	5,200	5,148	22,587.4	180.7	22,406.7	183.64	154.15
5	-0.581987	0.987706	5,200	5,148	27,216	217.73	26,998.3	217.91	181.82
6	0.43257	1.15006	5,200	5148	36,970.1	295.76	36,674.3	290.14	240.64
7	0.0178033	1.08069	5,200	5148	45,196.9	361.58	44,835.4	351.05	289.412
8	-2.10093	0.786462	5,200	5148	39,310	314.48	38,995.5	307.46	251.96
9	-1.4693	0.864618	5,200	5148	38,167.3	305.34	37,862	299.00	243.55
10*	-0.207703	1.04474	5,200	5148	44,934.4	359.48	**44,575**	**349.11**	-4,609.47
10	-0.207703	1.04474	5,200	5148	44,934.4	359.48	**44,575**	**-7,031.38**	-5,693.06

Note:

Row 10* & 10: It is showing that the fund in policy holder's account at the mature date, £4,4575, is less than the accumalation of 10 premiums, £52,000, therefore the difference will be deducted to pay off the insurer's liability with probability 0.994[14].

$$(52,000 - 44,575) \times 0.994 = £7,380.45$$

The profit showing in row 10* is before the deduction, the actual profit the insurer obtaind in year 10 is £349.11 - £7,380.45 = -£7031.3 showing in row 10. And the net present value of the profit in this case is -£995.422.

Now there are two examples for both situation, making profit and lossing, it will be really important for the company to know the statistic numbers, eg. how many contracts have negative profit for insurer, how much profit can the company make, what is the probability for the company to lose etc..

2.3.3 Statistic analysis

In the C^{++} program, the NPV can be sorted in order, and counted how many negative values are there, the table below showing the statistic results.

[14]The probability that the contract is still in force from year 9

```
NPV median = 677.104
NPV fifth percentile = -927.485
NPV ninety fifth percentile = 1035.04
NPV mean = 534.232
NPV sd = 687.702
NPV count negative values = 85
95% CI for NPV = (491.607,576.856)
```

Notes:

NPV median: This is the sample median of all NPV values.

NPV 5^{th} percentile: The 50^{th} in the ascending sorted 1,000 NPV values.

NPV 95^{th} percentile: The 950^{th} in the ascending sorted 1,000 NPV values.

NPV mean: This is the sample mean, which calculated by $\frac{\sum_{i=1}^{1000} NPV_i}{1000}$

NPV sd: This is the sample standard deviation.

NPV count negative values: This is the number of negative net present value of profit out of 1,000.

95% CI for NPV: The 95% of all NPV of profit will drop in this interval.

There are 85 negative NPV of profit out of 1,000, which means the probability for this company to lose on this contract is 8.5%, which is too high. And the 5^{th} percentile, $-£927.485$, is a very large lose as well. In conclusion, this contract has a high probability of losing a large amount of money, which should not be issued in reality. This is caused by the gauranteed benefit increasing the risk in the contracts. In the next section we are going to disscuss how to lower the risk of loss and make a profit.

2.3.4 Stochastic pricing

Recall in Chapter 1, the equivalence principle were used to find the premium, but it can not be used in this unit-linked contract since there is a high non-diversifiable risk. But instead, we can use stochastic simulation with quantile premium principle[8].

For quantile premium principle, the idea is to obtain a profit with a certain probability. If the 5^{th} percentile is positive and probability of loss is no greater than 5% then it is a good model and can be taken to practice. Based on the previous model, changing in some assumptions may let us achieve this target.

Scenario 1: Increase the premium

Increase the premium from £5,200 to £5,400

```
NPV median = 703.147
NPV fifth percentile = -963.16
NPV ninety fifth percentile = 1074.85
NPV mean = 554.779
NPV sd = 714.152
NPV count negative values = 85
95% CI for NPV = (510.515,599.042)
```

There is an increase on the NPV sample mean and median but the number of negative NPV of profit did not change, and there is even a decrease on 5^{th} percentile. This is because when we increase the premium, we increase the guaranteed minimum maturity benefit as well, which means we increase the liability on the mature date of the contract.

Scenario 2: Increase the expenses deduction

Initial expenses increase from 13% to 15%, renewal expenses increase from 0.7% to 0.9%

```
NPV median = 638.943
NPV fifth percentile = -965.646
NPV ninety fifth percentile = 996.881
NPV mean = 496.07
NPV sd = 687.702
NPV count negative values = 86
95% CI for NPV = (453.446,538.695)
```

Scenario 3: Lower the mortality rate

Mortality rate decreases from 0.006 to 0.005

```
NPV median = 689.194
NPV fifth percentile = -919.938
NPV ninety fifth percentile = 1051.93
NPV mean = 546.463
NPV sd = 689.761
NPV count negative values = 85
95% CI for NPV = (503.711,589.214)
```

In the above two sensitivity tests, we can see a growth in the sample mean and median too, but the number of negative NPV of profit did not change or even raise a little. The 5^{th} percentiles are still negative, or even decreased.

Scenario 4: Lower the GMMB

The guaranteed minimum maturity benefit decrease from 100% to 91% premiums paid.

```
NPV median = 677.337
NPV fifth percentile = 3.53572
NPV ninety fifth percentile = 1035.04
NPV mean = 617.827
NPV sd = 480.784
NPV count negative values = 50
95% CI for NPV = (588.028,647.627)
```

In this scenario, we lowered the guaranteed minimum maturity benefit in order to decrease the liability to meet on the date of mature. The new assumption will enable the insurer to have 5% to lose with a positive 5^{th} percentile. This is a ideal contract for the insuer, but may not be ideal for the policy holders since the decrement on maturity benefit.

2.3.5 Reserving

From the scenario test 4 in last section, we have a model which enable the insurer have less risk to make profit on the contract. Even the risk is much lower but if the number of contracts issued is large, the insurer may still have a probability to lose. In this case, the insurer may have the following solutions[9]:

1. Hedge the risk in the financial market.

2. Reinsure (purchase insurance from one or more other insurance companies to pass on the risk) [10].

3. Reserving.

In this section, we are going to disscuss the third option, reserving. To calculate the reserve for the contract, the non-diversifiable risk should be taken into account, the methodology which enable us to do this is called risk measure[11]. One popular type is called Value at Risk, it is defined in terms of a parameter α which is the probability that the loss L will not exceed ℓ. Therefore, we have

$$P[L \leq \ell] = \alpha$$

Continue on scenario 4, set $\alpha = 0.95$, the reserve ${}_0V$ at time t = 0 will equal to the 95^{th} percentile[15] of the distribution of L_i where

$$L_i = -\sum_{\mathbf{t=1}}^{10} \frac{{}_{t-1}p_x Pr_{t,i}}{(1+j)^t}$$

It is calculated similar to NPV of profit, but with different discount rate $j = 0.05$ and the first acquisition cost will not be included.

$${}_0V = £1,552.99$$

In practice, the insurer may review the reserve annually, and make adjust based on change in different factors.

2.4 Deterministic VS Stochastic test

For unit-linked contract, there is an uncertainty factor on the investment return, where the interest rate is non-diversifiable, so the simple deterministic test will not be able to model the profit realistically. But instead, the stochastic model will fit better. In the sensitivity test, we can also see that the mortality rate, initial/renewal expence, management cost changes can only affect the profit in a certain amount, but the little change in interest rate is very sensitive, which means a better model is essential in this case.

[15] if the 95^{th} percentile is negative, the reserve will be set to zero.

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Appendix 1: Optional Extra

A PROGRAM TO COMPUTE EIGENVALUES

```
100 GOTO 200

200 END
```

Appendix 2: More Extra

Table 2.1: Lowercase Greek Letters.

α	`\alpha`	θ	`\theta`	o	`o`	υ	`\upsilon`
β	`\beta`	ϑ	`\vartheta`	π	`\pi`	ϕ	`\phi`
γ	`\gamma`	ι	`\iota`	ϖ	`\varpi`	φ	`\varphi`
δ	`\delta`	κ	`\kappa`	ρ	`\rho`	χ	`\chi`
ϵ	`\epsilon`	λ	`\lambda`	ϱ	`\varrho`	ψ	`\psi`
ε	`\varepsilon`	μ	`\mu`	σ	`\sigma`	ω	`\omega`
ζ	`\zeta`	ν	`\nu`	ς	`\varsigma`		
η	`\eta`	ξ	`\xi`	τ	`\tau`		

Table 2.2: Uppercase Greek Letters.

Γ	`\Gamma`	Λ	`\Lambda`	Σ	`\Sigma`	Ψ	`\Psi`
Δ	`\Delta`	Ξ	`\Xi`	Υ	`\Upsilon`	Ω	`\Omega`
Θ	`\Theta`	Π	`\Pi`	Φ	`\Phi`		

Table 2.3: Math Alphabets.

Example	Command	Required package
$\mathrm{ABCDEabcde1234}$	`\mathrm{ABCDE abcde 1234}`	
$\mathit{ABCDEabcde1234}$	`\mathit{ABCDE abcde 1234}`	
$\mathnormal{ABCDEabcde1234}$	`\mathnormal{ABCDE abcde 1234}`	
$\mathcal{ABCDE}\dashv\lfloor\rfloor\lceil\rceil\infty\in\ni\triangle$	`\mathcal{ABCDE abcde 1234}`	
$\mathfrak{ABCDEabcde1234}$	`\mathfrak{ABCDE abcde 1234}`	amsfonts or amssymb
$\mathbb{ABCDE}\Game\nVdash\nvDash\nVDash\ntrianglerighteq$	`\mathbb{ABCDE abcde 1234}`	amsfonts or amssymb