Pricing and profit testing of life insurance

Thesis submitted at the University of Leicester
in partial fulfilment of the requirements for
the MSc degree in Financial Mathematics and Computation

by

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September 2012

Contents

Declaration

All sentences or passages quoted in this project dissertation from other people's work have been specifically acknowledged by clear cross referencing to author, work and page(s). I understand that failure to do this amounts to plagiarism and will be considered grounds for failure in this module and the degree examination as a whole.

Name:

Lemin Wu

Signed:

Date:

Abstract

A summary of the thesis in about 200 words.

Introduction

In this thesis we consider the work of Gauss [1, ch 2] and Hilbert [**?**] on the subject of the title.

1. why doing the project
2. what have I done
3. what have I achieve
4. what work can be done in extension

Convensions

The following variables are used though out the project

$\ddot{a}$

$\ddot{A}$

$\ddot{A}^{(m)}_{[x]}$

$\ddot{a}^{(m)}_{[x]}$

Notation

assurance?annuity

We will discuss Gauss's work in Chapter **??**.

unit-linked

We will discuss Hilbert's work in Chapter **??**.

Background

?Do I need background? mathematical background

Chapter 1

Results of Gauss

In [1] Gauss proved the following very important result.

THEOREM 1.1 (Gauss) *Some very profound result.*

Later on in Chapter **??** we will have more to say about Theorem 1.1.

1.1 Gauss's youthful work

1.2 Gauss's mature work

Chapter 2

Unit-linked insurance

In this chapter we introduce the unit-linked insurance contract. We start from some assumptions to establish a deterministic pricing model, and demonstrate that the deterministic pricing test is not accurate enough for this contract, which is caused by the uncertainty, or in other words risk, in the investment is not diversifiable.

To solve this problem, we consider to use a stochastic pricing test with future investment return as an random variable, then testify the stochastic test will determine a better premium and reserve.

2.1 Background

In some countries unit-linked contract is also called equity-linked contract, it is because the single or regular premium paid by the policy holder will be invested on units of Equity or bonds on their own behalf.

Once the insurer receive the premium, it will be payed into policy holder's fund after deduction of expenses and management costs, and the deduction goes to insurer's account. In UK, there is an un-allocated percentage, which is another agreed regular deduction from policy holder's fund to insurer's account.

[2]If the policy holder survival to the maturity date, he/she will receive the greater value of total premiums payed in or the fund in policy holder's account. This is called

guranteed minimum maturity benefit (GMMB). But the if the death happens first, there is a guaranteed minimum death benefit, which allows policy holder's estate to receive the policy holder's fund with an extra amount.

2.1.1 Some assumptions *** change variables!!

Before we establish the model, we need some assumptions, the following are some ideas from [3] book *Actuarial Mathematics for Life Contingent Risks,* and some online sources [4].

A company is going to issue a new 10 years unit-linked contract to people from 55 to 60 years old. In the contract, it stated that the annual premium is £5200, with an un-allocated rate of 5% in the first year and 1% in the subsequent years. Expenses occurs the same time as the premium payed in, which is 10% (including commision) and 0.5% repectively. At the end of each year, there is a further management charge of 0.8% of policy holder's fund. All the above deductions from policy holder's account will be transferred to insurer's account.

When the contract mature, the policy holder can received the greater of policy holder's fund and accumulation of premiums payed in. But if the death happens before the mature date, policy holder's estate will receive 110% of the policy holder's fund where the extra 10% is from insurer's account.

The insurance company is also prepared to have 10% of policy holders surrender the contract in the first year and 5% in the second year, but no more in the subsequent years. The policy holders who surrender the contract will received their premium(s) after deduction of management cost at the end of the year.

In the Chapter 1, we had introduce different methods to calculate the mortality rate, but here we are going to use a constant force of mortality, $\mu_x = 2.2218$, for people aged 55 to 60, which means the mortality rate of policy holders $q_x = 0.006$ is a constant in each year. The reason we set the mortality rate to a constant will be explained later in this chapter.

BABE!!! HOW CAN I MAKR A EQUATION AT THE BOTTOM OF THE PAGE? ??

2.2 Deterministic pricing and reserving

Last section we introduce the unit-linked contract and have some assumptions to help us establish the model. But here is something we did not count in, the uncertainty in the investment return in both policy holder's account ans insurer's account.

For deterministic pricing, we used conservative interest rates for both accounts, 8% for policy holder's account and 5% for insurer's account, and assume no reserve holds in this case. Since the mortality rate is a constant, as well as the conservative interest rates, the cash flow for different policy holders will be exactly the same. Now we set one as an example.

2.2.1 Cash flow analysis

Now from all the assumptions, we can create the cash flow tables for both policy holder and insurer's account for the 10 years term.

		Cash flow for policy holder's account					
Year t	A	All	Fur	int	Fu t^-	mast	Fundr ($)
1	5200	4940	0	395.2	5335.2	42.68	5292.52
2	5200	5148	5292.52	835.24	11275.76	90.21	11185.55
3	5200	5148	11185.55	1306.68	17640.24	141.12	17499.12
4	5200	5148	17499.12	1811.77	24458.89	195.67	24263.21
5	5200	5148	24263.21	2352.89	31764.11	254.11	31509.99
6	5200	5148	31509.99	2932.64	39590.64	316.73	39273.91
7	5200	5148	39273.91	3553.75	47975.67	383.81	47591.86
8	5200	5148	47591.86	4219.189	56959.05	455.67	56503.38
9	5200	5148	56503.38	4932.11	66583.49	532.67	66050.82
10	5200	5148	66050.82	5695.91	76894.73	615.16	76279.57

Here are the method used to calculate the table from colum to column. (BABE, I DON'T MEAN METHOD HERE, WHAT WORD CAN I USE?)

Allocated premium: For the first year, the premium allocated in the policy holder's fund will be (1-5%) of the premium payed, where 5% is the agreed un-allocated premium which will be payed into insurer's account. For the subsequent years, the un-allocated rate decrease to 1%, hence there will be 99% premium payed into policy holder's fund.

First year allocated premium: 5,200 $\times$ 0.95= 4,940

Subsequent years allocated premium: 5,200 $\times$ 0.99= 5,148

Interest: It is the profit policy holder obtain from the the accumulation of premium payed in at the beginning of the year and fund brought forward from end of last year with the assumed interest rate 8%.

eg. year t=5, interest = (5,148 + 24263.21) $\times$ 0.08 = 2,352.89

Fund at time t^-: It is the accumulation of premium payed in at the beginning of the year, fund brought forward from end of last year and interest earned.

Management cost: In our assumption, the management cost is 0.08% of the policy holder's fund, which is the fund at time t^- here.

Fund bring forward: It is the fund left in policy holder's account after deduction of management cost.

2.2.2 Profitability

2.3 Stochastic pricing

2.3.1 time series and stochastic process

2.3.2 Monte Carlo simulation

2.3.3 Pricing

2.3.4 Reserving

2.4 Deterministic VS Stochastic test

Bibliography

[1] Gauss, C.F., "Disquisitiones Arithmeticae", Leipzig, 1801.

[2] David C. M. Dickson, Mary R. Hardy and Howard R. Waters, *Actuarial Mathematics for Life Contingent Risks,*, 3rd edition, 2011, page 374-375.

[3] David C. M. Dickson, Mary R. Hardy and Howard R. Waters, *Actuarial Mathematics for Life Contingent Risks,*, 3rd edition, 2011, page 375.

[4] Mrs Giselle du Toit *Actuarial Mathematics for Life Contingent Risks,*, 2nd edition, 2011, page 375.

Appendix 1: Optional Extra

A PROGRAM TO COMPUTE EIGENVALUES

```
100 GOTO 200

200 END
```

Appendix 2: More Extra

Table 2.1: Lowercase Greek Letters.

α	`\alpha`	θ	`\theta`	o	`o`	υ	`\upsilon`
β	`\beta`	ϑ	`\vartheta`	π	`\pi`	ϕ	`\phi`
γ	`\gamma`	ι	`\iota`	ϖ	`\varpi`	φ	`\varphi`
δ	`\delta`	κ	`\kappa`	ρ	`\rho`	χ	`\chi`
ϵ	`\epsilon`	λ	`\lambda`	ϱ	`\varrho`	ψ	`\psi`
ε	`\varepsilon`	μ	`\mu`	σ	`\sigma`	ω	`\omega`
ζ	`\zeta`	ν	`\nu`	ς	`\varsigma`		
η	`\eta`	ξ	`\xi`	τ	`\tau`		

Table 2.2: Uppercase Greek Letters.

Γ	`\Gamma`	Λ	`\Lambda`	Σ	`\Sigma`	Ψ	`\Psi`
Δ	`\Delta`	Ξ	`\Xi`	Υ	`\Upsilon`	Ω	`\Omega`
Θ	`\Theta`	Π	`\Pi`	Φ	`\Phi`		

Table 2.3: Math Alphabets.

Example	Command	Required package
$\mathrm{ABCDE abcde 1234}$	`\mathrm{ABCDE abcde 1234}`	
$\mathit{ABCDE abcde 1234}$	`\mathit{ABCDE abcde 1234}`	
$\mathnormal{ABCDE abcde 1234}$	`\mathnormal{ABCDE abcde 1234}`	
$\mathcal{ABCDE}\dashv\lfloor\rfloor\lceil\rceil\infty\in\ni\triangle$	`\mathcal{ABCDE abcde 1234}`	
$\mathfrak{ABCDE abcde 1234}$	`\mathfrak{ABCDE abcde 1234}`	amsfonts or amssymb
$\mathbb{ABCDE}\Game\nvdash\nvDash\nVdash\ntrianglerighteq$	`\mathbb{ABCDE abcde 1234}`	amsfonts or amssymb