Assignment 4
MATH 3423 - Numerical Methods 2
Acadia University

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Winter 2002
Due April 3rd 2002

1 Requirements

You can do this assignment in teams of two or alone. 5% of the mark will be based on presentation. All required plots should be computer-generated. A title page is required. All computer code must be properly commented and should be computationally efficient. Feel free to use the computer language you prefer. Maple and Java are both fine choices. You are expected to reuse the same routines as you go from one problem to the next.

2 Solving Differential Equations

2.1 Runge-Kutta Method

1. Using a Computer Algebra System (Maple, Maxima, Mathematica...), solve $u' = e^x u + \ln x$. Only give the command you used to solve the differential equation (not the answer) and specify which tool you used. You may try to evaluate $u(2)$ if $u(1) = 0$.

2. Write a Runge-Kutta routine that solves $u' = e^x u + \ln x$, $u(1) = 0$ numerically over the interval $[1,2]$ starting at 1 and going up to 2 with step size h. Give $u(2)$ for $h = 1, 0.5, 0.25, 0.125\ldots$ until the value appears to stabilize. Plot your answer as $u(2)$ versus h, Runge-Kutta is a fourth order method, does the result reflect that? Does the problem appear stable? Explain. (Don't do a full stability analysis, but try to determine if for all h, the answer is reasonable. If your solution blows up for h above a threshold, then it is not stable.)

3. Explain why we chose the interval $[1,2]$ instead of $[0,1]$. What would you do if you were asked to solve the problem over $[0,1]$? Assume your boss doesn't know anything about mathematics and you need to find a way to make it work.

2.2 Multistep Method

1. Replace the full Runge-Kutta solution by a multistep 4^{th} order Adams-Bashforth method. Solve $u' = e^x u + \ln x$, $u(1) = 0$ numerically over the interval $[1,2]$ starting at 0 and going up to 1 with step size h. Initialize your problem with Runge-Kutta. Give $u(2)$ for $h = 1/8, 1/16...$ until the value appears to stabilize. Plot your answer as $u(2)$ versus h, Your approach is 4^{th} order method, does the result reflect that? Does the problem appear stable? Is Adams-Bashforth better or worse than Runge-Kutta? Which one would you choose in practice?

2. Improve the Adams-Bashforth method by adding a 5^{th} order corrector (Adams-Moulton). Solve $u' = e^x u + \ln x$, $u(1) = 0$ numerically over the interval $[1,2]$ starting at 0 and going up to 1 with step size h. Initialize your problem with Runge-Kutta and use both Adams-Bashforth and Adams-Moulton according to the predictor-corrector routine. Give $u(2)$ for $h = 1/8, 1/16...$ until the value appears to stabilize. Plot your answer as $u(2)$ versus h. Does the problem appear stable? How does the corrector-predictor method compares to the other two approaches in terms of accuracy and computational cost (roughly)?

2.3 Shooting

Solve the linear second-order differential equation $y'' = e^x y + \ln x$ with boundary conditions $y(1) = 0$, $y(2) = \pi$. For $h = 1/8, 1/16, \ldots$, give the values of $y(1.25), y(1.5)$, and $y(1.75)$ on three separate plots. Show all of your derivations and verify numerically that $y(2) = \pi$ (π =3.141592653589793).