

NAME: _____

Please Print

Id. No.: _____

Note: You are allowed to one sheet (both sides) with handwritten notes and a calculator. You have 50 minutes to write the test.

1. A vector field is given by $\vec{F}(x, y, z) = x^2 y \vec{i} + \left(\frac{x^3}{3} + x\right) \vec{j} + \vec{k}$.

(a) [10 marks] Compute $\nabla \cdot \vec{F}$ (the divergence). Show your work.

(b) [10 marks] Compute $\nabla \times \vec{F}$ (the curl). Show your work.

(c) [20 marks] Compute the line integral $\oint_C \vec{F} \cdot d\vec{r}$ where C is the unit circle centered at 0 on the plane ($x^2 + y^2 = 1, z = 0$) using Stoke's theorem. State any assumption you make in integrating.

2. Series and sequences

(a) [10 marks] Give an example of a monotone sequence that does not converge.

(b) [10 marks] Does the series $\sum_{k=1}^{\infty} \frac{e^k}{(k^2)!}$ converges? Does it converge absolutely? Prove your result.

(c) [10 marks] Show that the series $\sum_{k=1}^{\infty} \frac{(-1)^2}{\sqrt{k}}$ converges.

(d) [10 marks] A convergent series is absolutely convergent. Prove or give a counter-example.

(e) [10 marks] Let $S_n = \sum_{k=1}^n \frac{(-1)^2}{\sqrt{k}}$ and $L = \sum_{k=1}^{\infty} \frac{(-1)^2}{\sqrt{k}}$, find N such that $|S_N - L| < \frac{1}{3}$.