NAME: ______________________________
Please Print

Id. No.: ______________________________

**Department of Mathematics & Statistics**
**Acadia University**

**MATH 2023** **FINAL EXAMINATION** **Winter 2002**

**Instructor: Daniel Lemire, Ph.D.**

Time: 3 hours April 13th 2002 2:00 p.m.-5:00 p.m.

**Instructions:**

1. Put your name and ID number at the top of this page.
2. Answer the questions in the spaces provided, using the backs of pages for overflow or rough work.
3. ONE CHEAT SHEET ALLOWED (BOTH SIDES, HANDWRITTEN).
4. For true or false questions, you don't have to show your work.

| Question | Mark |
|---|---|
| 1 | /20 |
| 2 | /20 |
| 3 | /20 |
| 4 | /20 |
| 5 | /20 |
| Total | /100 |

[20] 1. (First-Order Differential Equations) The equation for the current $I$ going through a simple RL circuit with electromotive force $E$ is given by

$$\frac{dI(t)}{dt} + \frac{R}{L} I(t) = \frac{E}{L}.$$

Suppose that the electromotive force is given by $E(t) = \cos mt$ where $m$ is non-zero. $R$ and $L$ are unknown non-zero parameters.

(a) [2 marks] This differential equation is linear? True □ False □

(b) [2 marks] State the corresponding homogeneous differential equation.

(c) [2 marks] How many linearily independent solutions must the corresponding homogeneous problem have?

(d) [4 marks] Find all such solutions.

(e) [5 marks] Give the general solution to $\frac{dI(t)}{dt} + \frac{R}{L}I(t) = \frac{E}{L}$ with $E(t) = \cos mt$.

(f) [5 marks] Solve the differential equation with the initial condition $I(0) = 0$.

[20] 2. (Vector Calculus)

(a) [2 marks] The curl of the gradient of $\sin(xyz)$ is zero.
True ☐ False ☐

(b) [2 marks] The line integral of $y\vec{i}+x\vec{j}$ around the positively oriented unit circle is $2\pi$.
True ☐ False ☐

(c) [2 marks] The line integral of $-y\vec{i}+x\vec{j}$ around the positively oriented unit circle is $2\pi$.
True ☐ False ☐

(d) [2 marks] Given $\phi(x,y,z) = x^2y^2z^3$ compute $\nabla\phi(x,y,z)$.

(e) [3 marks] Compute the line integral $\int_P \nabla\phi(x,y,z)\cdot d\vec{r}$ where $P$ is the parametric curve $\underline{x}(t) = \left(t^2, t^3, t^4\right)$ with $t : 0 \to 1$ and where $\phi$ is given in part (a).

(f) [2 marks] Compute $\nabla^2\phi(x,y,z)$ where $\phi$ is given in part (a).

(g) [2 marks] Given $\vec{F}(x,y,z) = (y,x,0)$ compute $\nabla\times\vec{F}(x,y)$.

(h) [5 marks] Compute the line integral $\oint_Q \vec{F} \cdot d\vec{r}$ where $Q$ is the triangle defined by the three points $\underline{x_1} = (0,0)$, $\underline{x_2} = (1,0)$, and $\underline{x_3} = (1,1)$ and where $\vec{F}(x,y) = (y,x)$. Assume that the path $Q$ is positively oriented (counterclock-wise).

[20] 3. (Series)

(a) [2 marks] A bounded sequence is always a Cauchy sequence. True ☐ False ☐

(b) [2 marks] An alternating series will always converge. True ☐ False ☐

(c) [4 marks] Let $f(x) = \sum_{k=0}^{\infty} \frac{(2k)!x^k}{(4k)!}$. Find the radius of convergence of this power series.

(d) [2 marks] What is $f'(0)$ if $f(x) = \sum_{k=0}^{\infty} \frac{(2k)!x^k}{(4k)!}$?

(e) [4 marks] Without using your calculator, estimate $L = \sum_{k=0}^{\infty} \frac{(-1)^k}{10^k}$ within 0.001.

(f) [2 marks] Define a “monotone increasing sequence”.

(g) [4 marks] Prove that a bounded monotone increasing sequence must converge.

[20] 4. (Higher-Order Differential Equations)

(a) [2 marks] A $n^{th}$ order linear differential equation has $n+1$ solutions. True ☐ False ☐

(b) [2 marks] The function $\sin(x)$ can never be the solution of a homogeneous linear differential equation with constant coefficients. True ☐ False ☐

(c) [4 marks] Solve $y'' - 4y = 0$.

(d) [6 marks] Solve $y'' - 4y = e^{2x}$.

(e) [6 marks] Solve $y'' - 4y = xe^{2x}$.

[20] 5. (Variable Coefficients) Consider the differential equation $y'' - xy' + 2y = 0$.

(a) [2 marks] This differential equation is homogeneous. True ☐ False ☐

(b) [2 marks] An analytic function can have at most one singular point. True ☐ False ☐

(c) [2 marks] What are the singular points of this differential equation?

(d) [6 marks] Find the recurrence formula for power series solution of $y'' - xy' + 2y = 0$ around $x = 0$.

(e) [6 marks] Find the general solution for $y'' - xy' + 2y = 0$ by computing explicitely the first 7 terms in the power series expansion solution.

(f) [2 marks] Use the general solution in part (e) to solve for the initial values $y(0) = 1$ and $y'(0) = 1$.