Acadia University
Department of Mathematics and Statistics

INTRODUCTORY CALCULUS 1
(MATH 1013)

ASSIGNMENT 1 Solutions

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Contents

1 Functions and Models

1.5 Exponential Functions

14. (a) We say that two functions f and g are reflected about a line $y = a$ if $f(x) - a$ and $g(x) - a$ are equal but have opposite signs for all x. Therefore, in the present case, we need to find a function $f(x)$ such that $f(x) - 4 = 4 - e^x$ and that's $f(x) = 8 - e^x$ (see Fig. **??**).

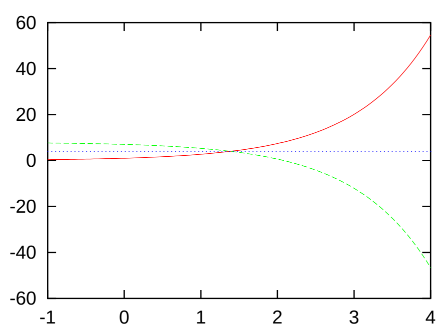

Figure 1: Reflection about the line $y = 4$

(b) Two functions f and g are reflected about the line $x = a$ if $f(a + x) = g(a - x)$ for all x. The function we are looking for is $f(x + 2) = e^{2-x}$ because $g(x) = e^x$. We finally substitute $z = x + 2$ (or $x = z - 2$) to obtain $f(z) = e^{4-z}$ (see Fig. **??**).

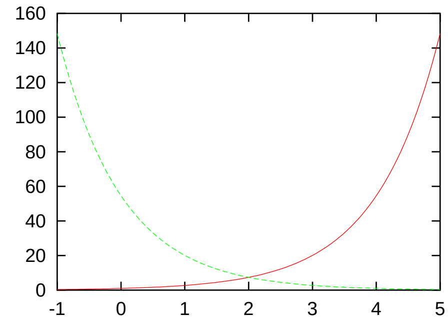

Figure 2: Reflection about the line $x = 2$

16. Looking at the graph, we can observe that $f(0) = 2$ and $f(2) = \frac{2}{9}$. Since $f(x) = Ca^x$, we have that

$$Ca^0 = 2 \tag{1}$$

and

$$Ca^2 = \frac{2}{9}. \tag{2}$$

From equation **??**, we see right away that $C = 2$ and therefore, equation **??** becomes

$$a^2 = \frac{1}{9}$$

which implies that $a = \pm\frac{1}{3}$. However, we must reject the solution $a = -\frac{1}{3}$ because the function $f(x) = 2 \times \left(\frac{-1}{3}\right)^x$ implies $f(1) = \frac{-2}{3}$ which doesn't agree with the graph. We have one remaining solution and it is

$$f(x) = 2 \times \left(\frac{1}{3}\right)^x.$$

18. We will assume that the highest paying job at the end of the month must be chosen. The first job pays $1,000,000$ over a month. How much pays the second one? Setting up the equation is not very difficult, we readily see that it pays[1]

$$pay = \sum_{n=1}^{30} 2^{n-1}. \tag{3}$$

How much is it? The formula **??** is called a *geometric series*. We can solve it rather easily, we simply evaluate $pay - 2 \times pay$ by using formula **??** which gives

$$\begin{aligned} pay - 2 \times pay = & \sum_{n=1}^{30} 2^{n-1} - 2 \times \sum_{n=1}^{30} 2^{n-1} \\ = & \sum_{n=1}^{30} 2^{n-1} - \sum_{n=2}^{31} 2^{n-1} \\ = & 2^0 - 2^{30} \end{aligned}$$

[1] We chose a month with 30 days.

We then solve for *pay* in $pay - 2 \times pay = 1 - 2^{30}$ which is very easy since $pay - 2 \times pay = -pay$, and we finally have $pay = 2^{30} - 1$. It should be noted here that 2^{30} is a very big number (about one billion or $1,073,741,824$). What is the lesson here? Exponential functions can increase **very fast**[2].

24. (a) Given that the half-life is 15 hours, after 15 hours, we have $2/2 = 1$ g left (by definition). After another 15 hours (for a total of 30 hours), we have $1/2$ g left, and after yet another 15 hours (for a total of 30 hours), we have $1/4$ g left. Finally, after 60 hours, we'll have $1/8$ g left[3].

(b) Since the decay is exponential, we have that

$$m(x) = Ca^x. \tag{4}$$

First of all, $m(0) = 2$ and therefore $C = 2$ (see equation **??** on this page). We have that after 15 hours, the mass must be half, so

$$m(15) = 1. \tag{5}$$

Combining equations **??** and **??**, we get that

$$2a^{15} = 1 \tag{6}$$

$2a^{15} = 1$ and we must now solve for a. One way to do it (assuming we don't know about logarithms) is to take the equation **??**to the power $\frac{1}{15}$ to get

$$a = \frac{1}{2^{1/15}} \cong \frac{1}{1.047} \cong 0.9548.$$

This means that the mass of our sample goes down according to the equation[4]

$$m(x) = 2\left(\frac{1}{2}\right)^{\frac{x}{15}} = \left(\frac{1}{2}\right)^{\frac{x}{15}-1}. \tag{7}$$

(c) Amount after 4 days? What you must not do here is substitute 4 in equation **??**. The correct reasoning is to first convert 4 days in hours. We have 24 hours for every day, so 4 days is $4 \times 24 = 96$ hours. Therefore, the remaining mass will be

$$m(96) = \left(\frac{1}{2}\right)^{\frac{96}{15}-1} \cong 0.02.$$

[2]Most modern computer architectures are 32 bits (Windows, Linux and even the new gaming consoles). Therefore, the highest integer value most programmers will ever expect is $2^{32} - 1$. Of course, newer computer architectures will probably be 64 bits and integers will go up to $2^{64} - 1$. Will that be a big improvement? How much bigger is 2^{64} with respect to 2^{32}?

[3]It is also possible to deduce this result from equation **??**.

[4]Actually, your calculator or mathematical software is likely to evaluate this function using the formula $e^{(1-x/15)\ln 2}$ instead. We will come back to logarithms and their applications later in the course.

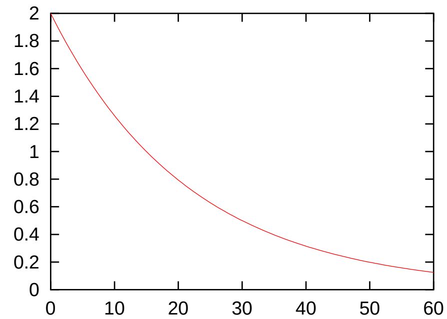

Figure 3: Mass of sodium vs time according to equation **??**

1.6 Inverse Functions and Logarithms

18. (a) We have to solve the equation

$$3 = 3 + x^2 + \tan(\pi x/2)$$

which simplifies to

$$\tan(\pi x/2) = -x^2.$$

Now, some of you might think that life is pretty hard (and it sometimes is!). But wait! What is $\tan(0)$? 0 of course! And what is x^2 evaluated at 0? 0 of course! So we found a solution which we will assume to be unique and we conclude that $f^{-1}(3) = 0$ (See Fig. **??**).

(b) You must not try to evaluate $f^{-1}(5)$! We write $x_5 = f^{-1}(5)$ and x_5 is defined by the equation $f(x_5) = 5$. And thus

$$f\left(f^{-1}(5)\right) = f(x_5) = 5.$$

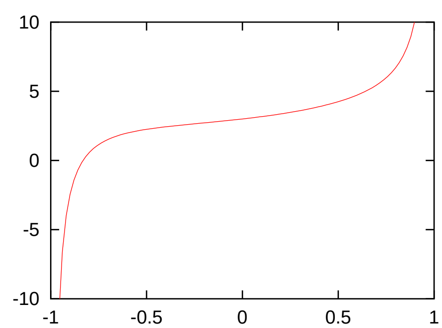

Figure 4: $3 + x^2 + \tan(\pi x/2)$

28. We want to compute the inverse of the following function

$$f(x) = (1 + e^x)/(1 - e^x) \tag{8}$$

It is worth noting that when $x = 0$, we get $f(0) = 2/0 = \infty$. Since we know that $\ln(e^x) = x$, we have

$$\begin{aligned}
\frac{1+e^x}{1-e^x} = y \quad &\Longrightarrow \quad (1-e^x)y = 1+e^x \\
&\Longrightarrow \quad (1+y)e^x = y-1 \\
&\Longrightarrow \quad e^x = \frac{y-1}{y+1} \\
&\Longrightarrow \quad x = \ln\left(\frac{y-1}{y+1}\right).
\end{aligned}$$

Of course, this function isn't defined for $\frac{y-1}{y+1} \leq 0$ or $-1 \leq y \leq 1$. One could verify that $f(x)$ in equation **??** never takes these values (see also Fig. **??**).

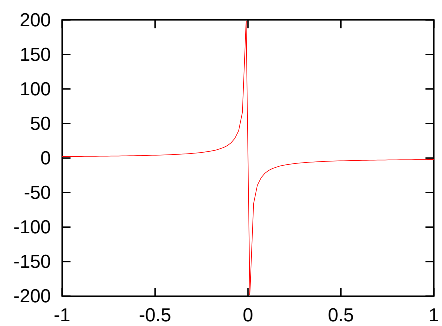

Figure 5: $f(x) = (1+e^x)/(1-e^x)$

36. Recall that $\log a^b = b\log a$ and $\log_a a = 1$.

  (a) $\log_8 2 = \log_8\left(8^{\frac{1}{3}}\right) = \frac{1}{3} \times \log_8 8$ because $\log a^b = b\log a$, and since $\log_a a = 1$,

$$\frac{1}{3} \times \log_8 8 = \frac{1}{3}$$

  so that $\log_8 2 = \frac{1}{3}$.

  (b) $\ln e^{\sqrt{2}} = \sqrt{2}\ln e$ because $\ln a^b = b\ln a$, and since $\ln e = 1$, we have $\ln e^{\sqrt{2}} = \sqrt{2}$.

38. Recall that $a^{b+c} = a^b a^c$ and $a^{\log_a b} = b$ (see equation 7 in [**?**] on page 68).

  (a) $2^{\log_2 3 + \log_2 5} = 2^{\log_2 3} \times 2^{\log_2 5} = 3 \times 5 = 15$

  (b) $e^{3\ln 2} = e^{\ln(2^3)} = 2^3 = 8$

42. The logarithm has $(0,\infty)$ for its domain (it isn't defined elsewhere, at least in this course). Therefore, in $\ln(4-x^2)$, we need to have $4-x^2 \geq 0$ or

$$x^2 < 4. \tag{9}$$

At this point, one must be careful. We may think of taking the square root on both sides of equation **??** (getting $x<2$) and indeed, it is correct to do so... as long as you realize that if $a^2=b$ then so does $(-a)^2=b$! That is, you have to consider negative values as well... Therefore, we have

$$x<2$$

and

$$x>-2.$$

We conclude that the domain of $\ln\left(4-x^2\right)$ is $(-2,2)$.
As for the range of the function... we first have to look at the range of $4-x^2$ over $(-2,2)$. Clearly, the polynomial is at its maximum when $x=0$ and so its range has 4 as an upper bound. On the other hand, it has 0 as its lower bound and therefore the range of $4-x^2$ over $(-2,2)$ is $(0,4]$. Since the logarithm is a strictly increasing function, we can conclude that the range of $\ln\left(4-x^2\right)$ is $(\ln 0,\ln 4]$ or $(-\infty,\ln 4]$ or approximately $(-\infty,1.386]$ (see Fig. **??**).

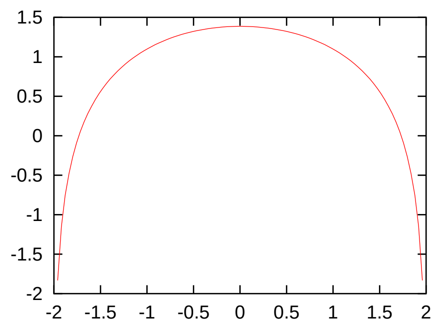

Figure 6: $f(x)=(1+e^x)/(1-e^x)$

52. (a) Starting from $\ln\ln x=1$, we write $e^1=e^{\ln\ln x}$ but since $e^{\ln w}=w$, we have $e=\ln x$ and finally, we write

$$e^e=e^{\ln x}=x$$

so that $x=e^e$.

(b) Starting from $e^{ax}=Ce^{bx}$, we divide both sides of the equation by e^{bx} (which we know to be a non-zero positive number! why?) to get

$$e^{(a-b)x}=C.$$

We can then simply take the logarithm of both sides of the equation to get

$$(a-b)x=\ln C$$

and therefore $x=\frac{\ln C}{a-b}$ because $a\neq b$.

58. (a) We have to solve for t in

$$Q_0\left(1-e^{-t/a}\right)=Q.$$

We begin by dividing by Q_0 both sides of the equation and after some algebra, we get

$$e^{-t/a} = 1 - \frac{Q}{Q_0}.$$

Taking the logarithm, we get

$$\frac{-t}{a} = \ln\left(1 - \frac{Q}{Q_0}\right)$$

or

$$t = -a\ln\left(1 - \frac{Q}{Q_0}\right)$$

or

$$t = a\ln\frac{Q_0}{Q_0 - Q} \tag{10}$$

because $\ln c/b = -\ln b/c$. It should be noted that if $Q_0 = Q$ (at time $t = 0$), then we have the logarithm of 0 or of ∞ which is not very good! We must therefore require that $Q < Q_0$ in which case

$$\frac{Q_0}{Q_0 - Q} > 1$$

and therefore $t > 0$ (because $\ln(1) = 0$). When $Q = 0$, we have $t = 0$. What does equation **??** mean? Well, it gives us time as a function of the charge Q. When the charge is at its minimum (0), then $t = 0$ and we know that we just started recharging... as Q increases (getting closer to Q_0), we can compute the corresponding number of seconds we waited.

(b) Setting $Q = 0.9Q_0$ and substituting in equation **??**, we have

$$\begin{aligned} t &= a\ln\frac{Q_0}{Q_0 - 0.9Q_0} \\ &= a\ln\frac{Q_0}{0.1Q_0} \\ &= a\ln\frac{1}{0.1} \\ &= a\ln 10 \end{aligned}$$

and because $a = 2$, we have $t = 2\ln 10 \cong 4.6$ seconds. Bonus: how long for 99% of the charge? We substitute again to get $t = 2\ln\frac{1}{0.01} \cong 9.2$ seconds... how long for 99.9% of the charge? We have $t = 2\ln\frac{1}{0.001} \cong 13.8$ seconds... What is happening? The logarithm grows very slowly! We can get very, very close to full charge without waiting all that long!

2 Limits and Derivatives

2.1 The Tangent and Velocity Problems

2. It is always useful to begin by plotting the points we are given. You can do it very easily using Maple[5] (see Fig. **??**).

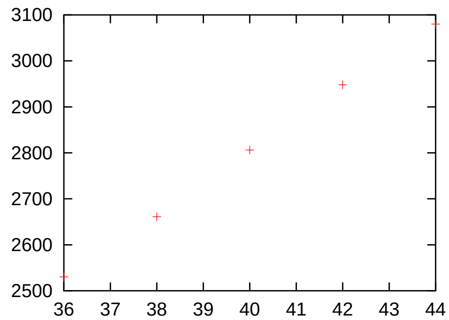

Figure 7: Heartbeats vs minutes

(a) We want to compute the slope of the secant line passing through $(36,2530)$ and $(42,2948)$. Recall that the slope of the line passing through (x_1,y_1) and (x_1,y_1) is given my

$$m = \frac{y_1 - y_2}{x_1 - x_2}.$$

In the present case, we therefore have

$$m = \frac{2530 - 2948}{36 - 42} \cong 69.7.$$

(b) We have to compute the slope of the secant line passing through $(38,2661)$ and $(42,2948)$ which is

$$m = \frac{2661 - 2948}{38 - 42} = 71.75.$$

(c) We have to compute the slope of the secant line passing through $(40,2806)$ and $(42,2948)$ which is

$$m = \frac{2806 - 2948}{40 - 42} = 71.$$

(d) We have to compute the slope of the secant line passing through $(42,2948)$ and $(44,3080)$ which is

$$m = \frac{2948 - 3080}{42 - 44} = 66.$$

If we want to approximate the slope of the tangent line at 42, we can look at the two closest secant lines (c and d) and expect the slope of the tangent line to be somewhere

[5] Many different plotting tools can be used including Mathematica, gnuplot, Matlab, Mathcad, EasyCalc (for Palm Pilot or Visor users). While you'll learn to use Maple in this course, you can use any tool that suits your needs. Some of them are free (gnuplot and EasyCalc for example).

between 66 and 71. We can use the average of the two values $\frac{66+71}{2} \cong 68.5$ as a good estimation for the heart rate in beats per minute at time $t = 72$.

4. (a) We have have to compute the slope of the line passing through $(2, \ln 2)$ and $(x, \ln x)$ for some values of x. The general formula for the slope is going to be

$$m = \frac{\ln 2 - \ln x}{2 - x}.$$

Note that while the question in Steward require 6 decimal places values for the slope, we give only 3 decimal places below.

i. For $x = 1.5$, we have
$$m = \frac{\ln 2 - \ln 1.5}{2 - 1.5} \cong 0.575.$$

ii. For $x = 0.9$, we have
$$m = \frac{\ln 2 - \ln 0.9}{2 - 0.9} \cong 0.726.$$

iii. For $x = 1.99$, we have
$$m = \frac{\ln 2 - \ln 1.99}{2 - 1.99} \cong 0.501.$$

iv. For $x = 1.999$, we have
$$m = \frac{\ln 2 - \ln 1.999}{2 - 1.999} \cong 0.5.$$

v. For $x = 2.5$, we have
$$m = \frac{\ln 2 - \ln 2.5}{2 - 2.5} \cong 0.446.$$

vi. For $x = 2.1$, we have
$$m = \frac{\ln 2 - \ln 2.1}{2 - 2.1} \cong 0.488.$$

vii. For $x = 2.01$, we have
$$m = \frac{\ln 2 - \ln 2.01}{2 - 2.01} \cong 0.499.$$

viii. For $x = 2.001$, we have
$$m = \frac{\ln 2 - \ln 1.5}{2 - 1.5} \cong 0.5.$$

(b) To guess the value of the slope, we look at the secant lines we get with $x = 1.999$ and $x = 2.001$, they both have a slope of ≈ 0.5 and therefore, $m = 0.5$ is our best guess.

(c) The tangent line will have a slope of 0.5 and therefore, its equation is going to be

$$y = 0.5x + b \tag{11}$$

and it remains to solve for b. We know that the tangent line must pass through $(2, \ln 2)$ and so we substitute these values in equation **??**. Solving for b, we get

$$b = \ln 2 - 0.5 \times 2 = \ln 2 - 1 \cong -0.306.$$

We therefore have the equation

$$y = 0.5x - \ln 2 + 1.$$

(d) Before sketching, we may compute the actual equations for the second lines. Choosing $x = 1.5$ and $x = 2.5$ from part a, we have to solve for b in the following equations

$$0.575 \times 2 + b = \ln 2$$

and

$$0.446 \times 2 + b = \ln 2.$$

It is, of course, very easy. We get $b \cong -0.457$ and $b \cong -0.198$ respectively. For sketch, see Fig. **??**

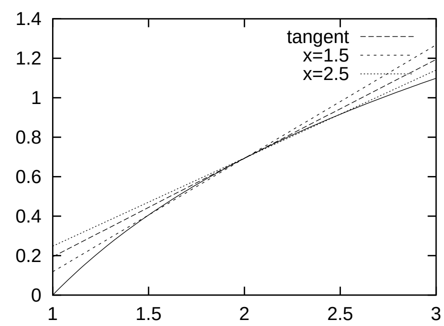

Figure 8: The tangent line and two secant lines at $x = 2$ for the curve $y = \ln x$.

6. We plot the height as a function in time (see Fig. **??**).

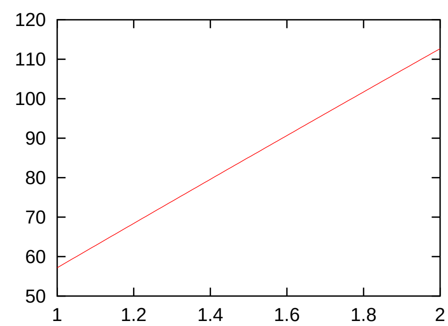

Figure 9: $h = 58t - 0.83t^2$

(a) Computing the average velocity is very similar to computing the slope of a secant line as we shall see.

i. The average velocity is given by

$$\frac{\left(58\times 1-0.83\times 1^2\right)-\left(58\times 2-0.83\times 2^2\right)}{1-2}=55.51.$$

ii. The average velocity is given by

$$\frac{\left(58\times 1-0.83\times 1^2\right)-\left(58\times 1.5-0.83\times 1.5^2\right)}{1-1.5}=55.925.$$

iii. The average velocity is given by

$$\frac{\left(58\times 1-0.83\times 1^2\right)-\left(58\times 1.1-0.83\times 1.1^2\right)}{1-1.1}=56.257.$$

iv. The average velocity is given by

$$\frac{\left(58\times 1-0.83\times 1^2\right)-\left(58\times 2-0.83\times 1.01^2\right)}{1-1.01}=56.3317.$$

v. The average velocity is given by

$$\frac{\left(58\times 1-0.83\times 1^2\right)-\left(58\times 1.001-0.83\times 1.001^2\right)}{1-1.001}=56.33917.$$

(b) We can estimate from the previous results that the velocity at $t=1$ is going to be 56.34.

2.2 The Limit of a Function

4. (a) We have a continuous function at $x=0$ and the limit from both sides will tend to 3.

(b) If we come from the left, the limit will tend to 4 even though it might not agree with the limit from the right or the value of the function at $x=3$.

(c) This time, the limit tends to 2 even though it doesn't agree with the limit from the left and the value of the function at $x=3$.

(d) From our previous comments, the limits from the left and from the right don't agree so that the limit doesn't exist (see [**?**, section 2.2, equation 3])

(e) Even though, it might seem strange, the value of $f(3)$ is simply 3.

6. We sketch the graph of the function f using our favorite software package (in this case, we are using gnuplot, see Fig. **??**). For the graph, it is clear that $\lim_{x\to -1} f(x)$ and $\lim_{x\to 1} f(x)$ do not exist. Other than than, all limits will exist so that the set we are looking for is $\mathbb{R}-\{-1,1\}$.

16. (a) We plot $\frac{6^x-2^x}{x}$ near $x=0$ using our favorite plotting software (see Fig. **??**). Clearly we have that the limit must be closed to 1.10.

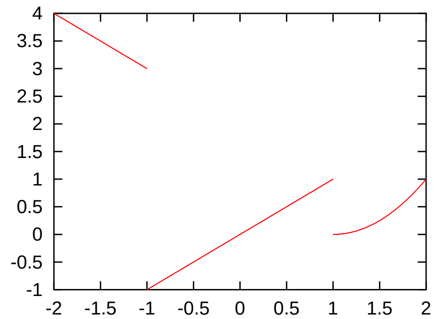

Figure 10: The discontinuous function $f(x)$.

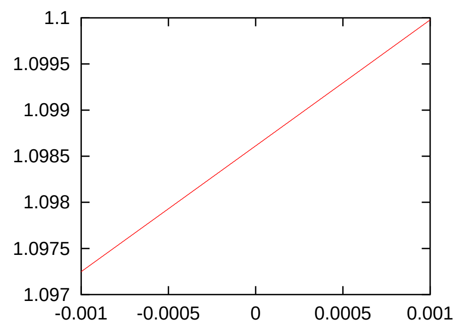

Figure 11: The function $\frac{6^x-2^x}{x}$ near $x=0$.

(b) Evaluating $\frac{6^x-2^x}{x}$ at $x=0.0001$, we get ≈ 1.098 and at $x=-0.0001$, we get ≈ 1.098 and therefore 1.10 is clearly a good choice.

18. (a) We use our calculator to get

$$\frac{\tan 1 - 1}{1^3} \cong 0.5574,$$
$$\frac{\tan 0.5 - 0.5}{0.5^3} \cong 0.3704,$$
$$\frac{\tan 0.1 - 0.1}{0.1^3} \cong 0.3347,$$
$$\frac{\tan 0.05 - 0.05}{0.05^3} \cong 0.3337,$$
$$\frac{\tan 0.01 - 0.01}{0.01^3} \cong 0.3333,$$
$$\frac{\tan 0.005 - 0.005}{0.005^3} \cong 0.3333.$$

(b) Since values tend to go to $1/3$ when x goes down to 0, we must assume that the limit will tend to 0.

(c) Using a calculator, we get 0 for values around 1×10^{-8}[6]. Looking closely at the problem, we see that $\tan x - x$ has value 0 on the calculator for $x = 1 \times 10^{-8}$, whereas $x^3 = 1 \times 10^{-24}$. Why do we get $\tan x - x = 0$ instead of $\sim x^3/3 \cong 3 \times 10^{-25}$? First of all, what is $\tan x$ when $x = 1 \times 10^{-8}$? Simply 1×10^{-8}, of course, which explain why we get $\tan x - x = 0$ in the first place! We might expect $\tan x = 1 \times 10^{-8} + 3 \times 10^{-25}$ instead... but wait! Can your calculator manage a number like $1 \times 10^{-8} + 3 \times 10^{-25}$. Of course not! For most calculator, 3×10^{-25} is just too small with respect to 1×10^{-8} and it will just be discarded. Therefore, we are still confident about the value of our limit being $1/3$ and that the zero values come from numerical error.

(d) We use our favorite plotting package... See Fig. **??** and **??**. We can see on Fig. **??** how the graphing tool has problems with values very close to 0 since the curve doesn't seem to be as smooth as one would expect and it eventually goes to 0. Notice that scientists often get around these problems by changing the units they are using.

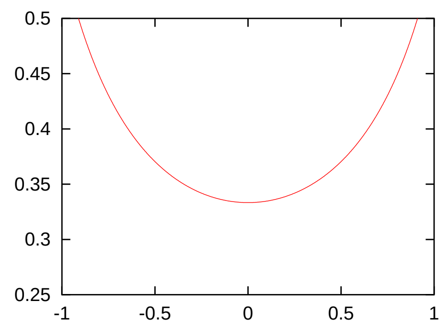

Figure 12: The function $\frac{\tan x - x}{x^3}$.

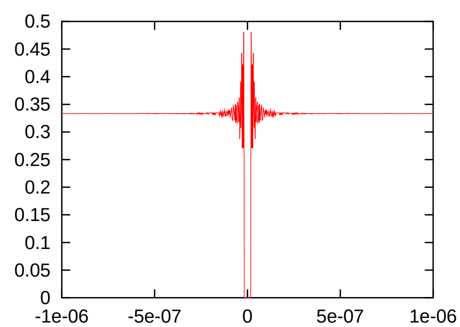

Figure 13: The function $\frac{\tan x - x}{x^3}$ very near $x = 0$. Notice how we stress the floating point lower limit of our graphing tool as x tends to 0.

[6] Smarter calculators may still print a non-zero value, but no matter what, you'll eventually get small enough values for your calculator to print 0. Just try smaller values for x.

2.3 Calculating Limits Using the Limit Laws

6. We have that

$$\begin{aligned}
\lim_{u\to-2}\sqrt{u^4+3u+6} &= \sqrt{\lim_{u\to-2}u^4+3u+6} && (12)\\
&= \sqrt{\lim_{u\to-2}u^4+\lim_{u\to-2}3u+\lim_{u\to-2}6} && (13)\\
&= \sqrt{\left(\lim_{u\to-2}u\right)^4+3\lim_{u\to-2}u+\lim_{u\to-2}6} && (14)\\
&= \sqrt{(-2)^4+3\times-2+6} && (15)\\
&= \sqrt{16}=4 && (16)
\end{aligned}$$

by using respectively the Root Law (equation **??**), Law 1 (equation **??**), the Power Law and Law 3 (equation **??**) and Laws 7 and 8 (equation **??**).

8. (a) It is not true when $x=2$ because the right hand side isn't defined (division by 0).

   (b) The limit of a function at $x=2$ doesn't depend on the value of the function at $x=2$ itself, but only on the values nearby. Since the equality presented in question a is true everywhere but at $x=2$, this new equation is correct.

10. We have that $\lim_{x\to-4}x^2+3x-4=0$ and therefore, we cannot use the quotient rule. Notice however that

$$\frac{x^2+5x+4}{x^2+3x-4}=\frac{(x+4)(x+1)}{(x+4)(x-1)}$$

and so, for $x\neq-4$, we can consider

$$f(x)=\frac{(x+1)}{(x-1)} \tag{17}$$

instead. Now, it is much easier because we can use the quotient rule.

$$\begin{aligned}
\lim_{x\to-4}\frac{(x+1)}{(x-1)} &= \frac{\lim_{x\to-4}(x+1)}{\lim_{x\to-4}(x-1)}\\
&= \frac{\lim_{x\to-4}x+\lim_{x\to-4}1}{\lim_{x\to-4}x-\lim_{x\to-4}1}\\
&= \frac{-4+1}{-4-1}=\frac{3}{5}.
\end{aligned}$$

12. We must first simplify the function for $x\neq1$,

$$\frac{x^3-1}{x^2-1}=\frac{(x-1)\left(x^2+x+1\right)}{(x-1)(x+1)}=\frac{x^2+x+1}{x+1}$$

and now, we can use the Quotient Law,

$$\begin{aligned}\lim_{x\to 1}\frac{x^3-1}{x^2-1} &= \lim_{x\to 1}\frac{x^2+x+1}{x+1}\\ &= \frac{\lim_{x\to 1} x^2+x+1}{\lim_{x\to 1} x+1}\\ &= \frac{3}{2}.\end{aligned}$$

26. We use the Squeeze Theorem (see [**?**, Theorem 3 in section 2.3]). We have that

$$\lim_{x\to 1} 3x \le \lim_{x\to 1} f(x) \le \lim_{x\to 1} x^3+2$$

and therefore

$$3 \le \lim_{x\to 1} f(x) \le 3$$

which implies

$$\lim_{x\to 1} f(x) = 3.$$

30. On the one hand, we have

$$\lim_{x\to 2^-}\frac{|x-2|}{x-2} = \lim_{x\to 2^-}\frac{2-x}{x-2} = -1$$

and on the other, we have

$$\lim_{x\to 2^+}\frac{|x-2|}{x-2} = \lim_{x\to 2^+}\frac{x-2}{x-2} = 1.$$

Clearly, the limit doesn't exist!

References

[Stewart] James Stewart, *Calculus: Concepts and Contexts* (Second Edition), Brooks/Cole, 2001.