

Numerical Methods 2 (MATH 3423) - MidTerm

Daniel Lemire, Ph.D.

February 25th 2002

Note: You are allowed to one sheet (both sides) with handwritten notes and a calculator. You have 50 minutes to write the test.

1. [25 marks] Given the following (x, y) couples, find an $O((\Delta x)^2)$ estimation for the derivative at 0 using Taylor series: $(-1, 2), (0, 1), (3, 0)$. Show your work.

2. [25 marks] Using the Gaussian Quadrature, how many nodes ($n = ?$) do I need to integrate $\int_a^b x^2 + x^5 + x^7 dx$ exactly (except for unavoidable numerical errors)? How many nodes do I need if I use the composite Simpson's method? Which method is best?

3. [25 marks] You want to integrate

$$f(x) = \begin{cases} x^4 & x > 0 \\ x^2 + 1 & x \leq 0 \end{cases}$$

from -1 to 1 . Which type of method is best, composite Newton-Cotes or Gaussian Quadrature? Explain.

4. [25 marks] Show that if f satisfies $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(x+y)}{2}$ for any x, y and $\max_x \{|f(x) - f(x+1)|\} = M < \infty$, then f must be continuous (a sketch of the proof is sufficient).