Power Systems Flexibility from District Heating Networks - Online Appendix

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NOMENCLATURE

Sets and Indexes

$\mathcal{T}$	Set of time periods
$\mathcal{I}^{HS}$	Set of heat stations
$\mathcal{I}^{HES}$	Set of heat exchanger stations
$\mathcal{I}^{CHP}$	Set of combined heat and power plants $\mathcal{I}^{CHP} \subset \mathcal{I}^{HS}$
$\mathcal{I}^{HP}$	Set of heat pumps $\mathcal{I}^{HP} \subset \mathcal{I}^{HS}$
$\mathcal{I}^{E}$	Set of electricity generators
$\mathcal{I}^{N}$	Set of nodes in the district heating network
$\mathcal{I}^{B}$	Set of electricity buses
$\mathcal{I}^{P}$	Set of heat pipelines
S_n^{P+}/S_n^{P-}	Set of pipes starting/ending at node n
S_n^{HS}	Set of heat stations connected to node n
S_n^{HES}	Set of heat exchanger stations connected to node n
S_n^{E}	Set of electricity generators connected to bus n
S_n^{B}	Set of electricity buses connected to bus n

Input Parameters

R_p/L_p	Radius/Length of pipe p (in m)
μ_p	Thermal loss coefficient in pipe p (in J.m^{-2}.s^{-2}.K^{-1})
ρ	Relative density of water (in Kg.m^{-3})
ν_p	Pressure loss coefficient in pipeline p
C_p	Specific heat capacity of water in pipeline p (in J.Kg^{-1}.K^{-1})
η_j^{pump}	Efficiency of water pump in heat station j
Δt	Time intervals (in s)
$\underline{mf}_{it}^{HES}/\overline{mf}_{it}^{HES}$	Minimum/maximum mass flow rates at the heat exchanger station i (in Kg/s)
$\underline{mf}_{jt}^{HS}/\overline{mf}_{jt}^{HS}$	Minimum/maximum mass flow rates at the heat station j (in Kg/s)
$\underline{mf}_{pt}^{S}/\overline{mf}_{pt}^{S}$	Minimum/maximum mass flow rates in the supply network (in Kg/s)
$\underline{mf}_{pt}^{R}/\overline{mf}_{pt}^{R}$	Minimum/maximum mass flow rates in the return network (in Kg/s)
$\underline{T}_{nt}^{S}/\overline{T}_{nt}^{S}$	Supply temperature bounds at node n (in K)
$\underline{T}_{nt}^{R}/\overline{T}_{nt}^{R}$	Return temperature bounds at node n (in K)
$\underline{pr}_{nt}^{S}/\overline{pr}_{nt}^{S}$	Pressure bounds at node n in the supply network (in Pa)
$\underline{pr}_{nt}^{R}/\overline{pr}_{nt}^{R}$	Pressure bounds at node n in the return network (in Pa)
$\underline{pr}_{it}^{HES}$	Minimum pressure difference at heat exchanger station i (in Pa)
α_j/α_i	Marginal cost parameter of heat station j/electricity producer i (in \$/MWh)
ρ_j^H/ρ_j^E	Heat/electricity fuel efficiency of CHP j
r_j^0/r_j	Heat and electricity outputs ratio of CHP j
$\overline{F}_j$	Maximum fuel consumption of CHP j (in MWh)
L_{nt}^{E}	Electricity load at bus n (in MWh)
L_{it}^{H}	Heat load at heat exchanger station i (in MWh)
B_{nm}	Susceptance of line connecting buses n and m (in S)
$\overline{P}_{jt}$	Maximum power output of electricity generator j (in MWh)
$\overline{f}_{nm}$	Maximum flow in line connecting buses n and m (in MWh)

Decision variables

$T_{pt}^{S,in}/T_{pt}^{S,out}$	Inlet/outlet temperatures in the supply network (in K)
$T_{pt}^{R,in}/T_{pt}^{R,out}$	Inlet/outlet temperatures in the return network (in K)
T_{nt}^{S}/T_{nt}^{R}	Supply/return temperatures at node n (in K)
mf_{pt}^{S}/mf_{pt}^{R}	Mass flow rate in the supply/return network (in Kg.m^{-3})
mf_{jt}^{HS}	Mass flow rate at heat station j (in Kg.m^{-3})

mf^{HES}_{it}	Mass flow rate at heat exchanger station i (in Kg.m^{-3})
pr^S_{nt}/pr^R_{nt}	Pressure at node n in the supply/return network (in Pa)
τ^S_{pt}/τ^R_{pt}	Time delay in the supply/return network (in h)
Q_{jt}	Heat production of heat station j (in MWh)
P_{jt}	Electricity production of electricity generator or CHP j (in MWh)
L^E_{jt}	Electricity consumption of heat pump j (in MWh)
L^{pump}_{jt}	Electricity consumption of water pump in heat station j (in MWh)
θ_{nt}	Voltage angle at bus n (in rad)

APPENDIX A: MCCORMICK RELAXATIONS

For supply and return temperature mixing equations at each node, we introduce the auxiliary variables w^S_{pt} and w^R_{pt} such that

$$w^S_{pt} = mf^S_{pt}\left(T^S_{nt} - T^{S,out}_{pt}\right) \quad \forall n \in \mathcal{I}^N, p \in S^{P-}_n \tag{1}$$

$$w^R_{pt} = mf^R_{pt}\left(T^R_{nt} - T^{R,out}_{pt}\right) \quad \forall n \in \mathcal{I}^N, p \in S^{P+}_n. \tag{2}$$

The products in (1)-(2) can be linearized using a McCormick envelopes

$$w^S_{pt} \geq \overline{mf}^S_{pt}\left(T^S_{nt} - T^{S,out}_{pt}\right) + mf^S_{pt}\left(\overline{T}^S_{nt} - \underline{T}^{S,out}_{pt}\right) - \overline{mf}^S_{pt}\left(\overline{T}^S_{nt} - \underline{T}^{S,out}_{pt}\right), \ \forall n \in \mathcal{I}^N, p \in S^-_n, t \in \mathcal{T} \tag{3a}$$

$$w^S_{pt} \geq \underline{mf}^S_{pt}\left(T^S_{nt} - T^{S,out}_{pt}\right) + mf^S_{pt}\left(\underline{T}^S_{nt} - \overline{T}^{S,out}_{pt}\right) - \underline{mf}^S_{pt}\left(\underline{T}^S_{nt} - \overline{T}^{S,out}_{pt}\right), \ \forall n \in \mathcal{I}^N, p \in S^-_n, t \in \mathcal{T} \tag{3b}$$

$$w^S_{pt} \leq \overline{mf}^S_{pt}\left(T^S_{nt} - T^{S,out}_{pt}\right) + mf^S_{pt}\left(\underline{T}^S_{nt} - \overline{T}^{S,out}_{pt}\right) - \overline{mf}^S_{pt}\left(\underline{T}^S_{nt} - \overline{T}^{S,out}_{pt}\right), \ \forall n \in \mathcal{I}^N, p \in S^-_n, t \in \mathcal{T} \tag{3c}$$

$$w^S_{pt} \leq \underline{mf}^S_{pt}\left(T^S_{nt} - T^{S,out}_{pt}\right) + mf^S_{pt}\left(\overline{T}^S_{nt} - \underline{T}^{S,out}_{pt}\right) - \underline{mf}^S_{pt}\left(\overline{T}^S_{nt} - \underline{T}^{S,out}_{pt}\right), \ \forall n \in \mathcal{I}^N, p \in S^-_n, t \in \mathcal{T}, \tag{3d}$$

and

$$w^R_{pt} \geq \overline{mf}^R_{pt}\left(T^R_{nt} - T^{R,out}_{pt}\right) + mf^R_{pt}\left(\overline{T}^R_{nt} - \underline{T}^{R,out}_{pt}\right) - \overline{mf}^R_{pt}\left(\overline{T}^R_{nt} - \underline{T}^{R,out}_{pt}\right), \ \forall n \in \mathcal{I}^N, p \in S^+_n, t \in \mathcal{T} \tag{4a}$$

$$w^R_{pt} \geq \underline{mf}^R_{pt}\left(T^R_{nt} - T^{R,out}_{pt}\right) + mf^R_{pt}\left(\underline{T}^R_{nt} - \overline{T}^{R,out}_{pt}\right) - \underline{mf}^R_{pt}\left(\underline{T}^R_{nt} - \overline{T}^{R,out}_{pt}\right), \ \forall n \in \mathcal{I}^N, p \in S^+_n, t \in \mathcal{T} \tag{4b}$$

$$w^R_{pt} \leq \overline{mf}^R_{pt}\left(T^R_{nt} - T^{R,out}_{pt}\right) + mf^R_{pt}\left(\underline{T}^R_{nt} - \overline{T}^{R,out}_{pt}\right) - \overline{mf}^R_{pt}\left(\underline{T}^R_{nt} - \overline{T}^{R,out}_{pt}\right), \ \forall n \in \mathcal{I}^N, p \in S^+_n, t \in \mathcal{T} \tag{4c}$$

$$w^R_{pt} \leq \underline{mf}^R_{pt}\left(T^R_{nt} - T^{R,out}_{pt}\right) + mf^R_{pt}\left(\overline{T}^R_{nt} - \underline{T}^{R,out}_{pt}\right) - \underline{mf}^R_{pt}\left(\overline{T}^R_{nt} - \underline{T}^{R,out}_{pt}\right), \ \forall n \in \mathcal{I}^N, p \in S^+_n, t \in \mathcal{T}. \tag{4d}$$

The temperature mixing equations at each node can be reformulated as

$$\sum_{p \in S^-_n} w^S_{pt} = 0, \ \sum_{p \in S^+_n} w^R_{pt} = 0 \quad \forall n \in \mathcal{I}^N, t \in \mathcal{T}. \tag{5}$$

APPENDIX B: CASE STUDY DATA

As shown in Fig. 1, the integrated heat and electricity system considered comprises a conventional thermal generator, a wind producer with an installed capacity of 500MW, n extraction CHP plant and a HP. The technical characteristics

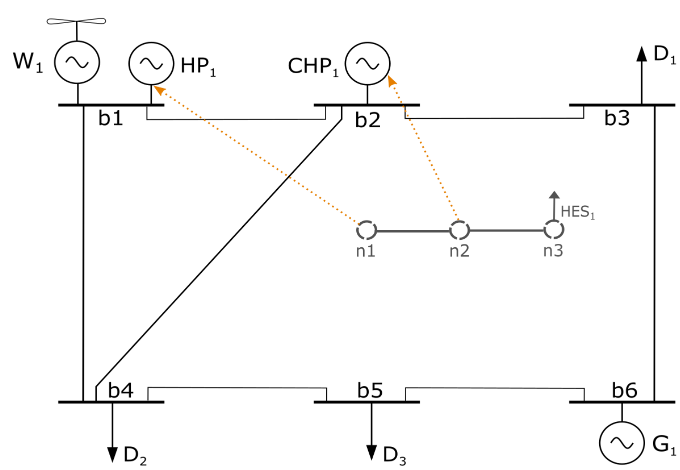

Figure 1. Integrated heat and electricity system

of these units are detailed in Table I.

TABLE I
GENERATION UNITS PARAMETERS

		G_1	W_1	CHP_1	HP_1
$\overline{P}$	MWh	180	500	-	-
$\overline{Q}$	MWh	-	-	250	150
$\overline{mf}^{HS}$	$Kg.s^{-1}$	-	-	300	300
$\overline{F}$	MWh	-	-	250	-
COP	-	-	-	-	2.5
r	-	-	-	0.6	-
ρ^E	-	-	-	2.4	-
ρ^H	-	-	-	0.25	-
α	$\$/MWh$	11	0	12.5	-

The technical parameters of the power transmission network and DHN are presented in Tables II, III, and IV.

TABLE II
ELECTRICITY TRANSMISSION NETWORK PARAMETERS

		l_{12}	l_{23}	l_{34}	l_{45}	l_{56}	l_{16}	l_{35}
$\overline{f}$	MWh	400	200	200	200	200	200	200
X	$10^{-1}\Omega$	1.70	0.37	2.58	1.97	0.37	1.40	0.18

TABLE III
DHN PARAMETERS

		p_{12}	p_{23}
R	m	0.80	0.80
L	m	500	500
C	$Wh.Kg^{-1}.K^{-1}$	1.17	1.17
μ	$W.m^{-2}.K^{-1}$	20	20
ν	(10^{-3})	1.93	1.93
η	-	0.9	0.9
$\overline{mf}^S/\overline{mf}^R$	$Kg.s^{-1}$	50	50
$\underline{mf}^S/\underline{mf}^R$	$Kg.s^{-1}$	300	300
$\underline{T}^{S,in}$	C	60	60
$\overline{T}^{S,in}$	C	30	30
$\underline{T}^{R,in}$	C	90	90
$\overline{T}^{R,in}$	C	120	120

TABLE IV
DHN NODAL PARAMETERS

		n_1	n_2	n_3 (HES_1)
$\overline{mf}^{HES}$	$Kg.s^{-1}$	-	-	50
$\underline{mf}^{HES}$	$Kg.s^{-1}$	-	-	300
$\underline{T}^S$	C	30	30	30
$\overline{T}^S$	C	60	60	60
$\underline{T}^R$	C	90	90	90
$\overline{T}^R$	C	120	120	120
$\underline{pr}^S/\underline{pr}^R$	Pa	0	0	0
$\overline{pr}^S/\overline{pr}^S$	Pa	1000	1000	1000

Heat and electricity loads and available wind production are represented in Fig. 2. table **??** shows the repartition of the electric loads at each bus

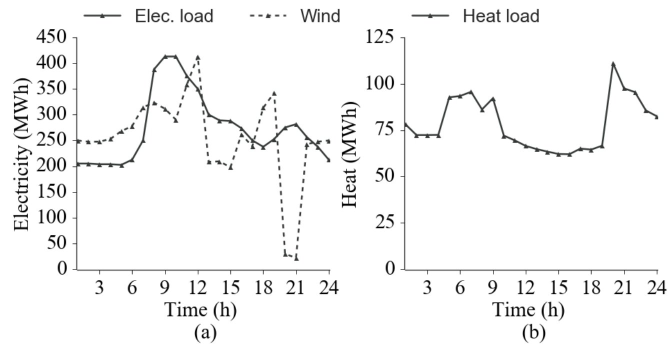

Figure 2. Case study setup: (a) Electricity load and available wind power and (b) Heat load

TABLE V
ELECTRIC LOADS AT EACH BUS

	b_1	b_2	b_3	b_4	b_5	b_6
% of load	0	0	20	40	40	0

APPENDIX C: CONVENTIONAL ECONOMIC DISPATCH

The Conventional Economic Dispatch (CED) is formulated as follows

$$\min_{\tilde{\Omega}} \sum_{j \in \mathcal{I}^H, t \in \mathcal{T}} \alpha_j Q_{jt} + \sum_{j \in \mathcal{I}^E, t \in \mathcal{T}} \alpha_j P_{jt} \tag{6a}$$

$$+ \sum_{j \in \mathcal{I}^{CHP}, t \in \mathcal{T}} \alpha_j \left(\rho_j^E P_{jt} + \rho_j^H Q_{jt} \right) \tag{6b}$$

$$s.t. \sum_{i \in \mathcal{I}^{HES}} L_{it}^H = \sum_{j \in \mathcal{I}^{HS}} Q_{jt} \quad \forall t \in \mathcal{T} \tag{6c}$$

$$L_{nt}^E = \sum_{j \in S_n^E} P_{jt} + \sum_{m \in S_n^B} B_{nm} \left(\theta_{mt} - \theta_{nt} \right)$$

$$\forall n \in \mathcal{I}^B, t \in \mathcal{T} \tag{6d}$$

$$-\overline{f}_{nm} \leq B_{nm} \left(\theta_{mt} - \theta_{nt} \right) \leq \overline{f}_{nm}$$

$$\forall n \in \mathcal{I}^B, m \in S_n^B, t \in \mathcal{T} \tag{6e}$$

$$0 \leq P_j \leq \overline{P}_{jt} \quad \forall i \in \mathcal{I}^E, t \in \mathcal{T} \tag{6f}$$

$$\underline{Q}_{jt} \leq Q_{jt} \leq \overline{Q}_{jt} \quad \forall j \in \mathcal{I}^{HS}, t \in \mathcal{T} \tag{6g}$$

$$P_{jt} \geq r_j^0 + r_c Q_{jt} \quad \forall j \in \mathcal{I}^{CHP}, t \in \mathcal{T} \tag{6h}$$

$$0 \leq \rho_c^E P_{jt} + \rho_c^H Q_{jt} \leq \overline{F}_j \quad \forall j \in \mathcal{I}^{CHP}, t \in \mathcal{T} \tag{6i}$$

$$Q_{jt} = \boldsymbol{COP}_{jt} L_{jt}^{HS} \quad \forall j \in \mathcal{I}^{HP}, t \in \mathcal{T}, \tag{6j}$$

where the set of optimization variables $\tilde{\Omega} = \{P, Q, L^{HS}, B, \theta\}$.