Power Systems Flexibility from District Heating Networks - Online Appendix

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NOMENCLATURE

Sets and Indexes

$\mathcal{T}$	Set of time periods
$\mathcal{I}^{HS}$	Set of heat stations
$\mathcal{I}^{HES}$	Set of heat exchanger stations
$\mathcal{I}^{CHP}$	Set of combined heat and power plants $\mathcal{I}^{CHP} \subset \mathcal{I}^{HS}$
$\mathcal{I}^{HP}$	Set of heat pumps $\mathcal{I}^{HP} \subset \mathcal{I}^{HS}$
$\mathcal{I}^{E}$	Set of electricity generators
$\mathcal{I}^{N}$	Set of nodes in the district heating network
$\tilde{\mathcal{I}}^{N-}$	Set of nodes with a single pipe arriving $\tilde{\mathcal{I}}^{N-} \subset \mathcal{I}^{N}$
$\tilde{\mathcal{I}}^{N+}$	Set of nodes with a single pipe departing $\tilde{\mathcal{I}}^{N+} \subset \mathcal{I}^{N}$
$\mathcal{I}^{B}$	Set of electricity buses
$\mathcal{I}^{P}$	Set of heat pipelines
S_n^{P+}/S_n^{P-}	Set of pipes starting/ending at node n
S_n^{HS}	Set of heat stations connected to node n
S_n^{HES}	Set of heat exchanger stations connected to node n
S_n^{HP}	Set of heat pumps connected to node n
S_n^{E}	Set of electricity generators connected to bus n
S_n^{B}	Set of electricity buses connected to bus n

Input Parameters

R_p/L_p	Radius/Length of pipe p (m)
μ_p	Thermal loss coefficient in pipe p (J.m^{-2}.s^{-2}.K^{-1})
ρ	Density of water (kg.m^{-3})
ν_p	Pressure loss coefficient in pipeline p
c	Specific heat capacity of water (J.kg^{-1}.K^{-1})
η_j^{pump}	Efficiency of water pump in heat station j
Δt	Time intervals (s)
$\underline{mf}_{it}^{HES}/\overline{mf}_{it}^{HES}$	Minimum/maximum mass flow rates at heat exchanger station i (kg/s)
$\underline{mf}_{jt}^{HS}/\overline{mf}_{jt}^{HS}$	Minimum/maximum mass flow rates at heat station j (kg/s)
$\underline{mf}_{pt}^{S}/\overline{mf}_{pt}^{S}$	Minimum/maximum mass flow rates in the supply network (kg/s)
$\underline{mf}_{pt}^{R}/\overline{mf}_{pt}^{R}$	Minimum/maximum mass flow rates in the return network (kg/s)
$\underline{T}_{nt}^{S}/\overline{T}_{nt}^{S}$	Supply temperature bounds at node n (K)
$\underline{T}_{nt}^{R}/\overline{T}_{nt}^{R}$	Return temperature bounds at node n (K)
$\underline{pr}_{nt}^{S}/\overline{pr}_{nt}^{S}$	Pressure bounds at node n in the supply network (Pa)
$\underline{pr}_{nt}^{R}/\overline{pr}_{nt}^{R}$	Pressure bounds at node n in the return network (Pa)
$\underline{pr}_{i}^{HES}$	Minimum pressure difference at heat exchanger station i (Pa)
$\overline{Q}_{jt}$	Maximum heat output of heat station j (Wh)
α_j	Marginal cost parameter of heat station or electricity producer j (\$/Wh)
ρ_j^H/ρ_j^E	Heat/electricity fuel efficiency of CHP j
r_j^0/r_j	Heat and electricity outputs ratio of CHP j
$\overline{F}_j$	Maximum fuel consumption of CHP j (Wh)
L_{nt}^{E}	Electricity load at bus n (Wh)
L_{it}^{H}	Heat load at heat exchanger station i (Wh)
B_{nm}	Susceptance of line connecting buses n and m (S)
$\overline{P}_{jt}$	Maximum power output of electricity generator j (Wh)
$\overline{f}_{nm}$	Maximum flow in line connecting buses n and m (Wh)
COP_{jt}	Coefficient of performance of heat pump j

Decision variables

$T_{pt}^{S,in}/T_{pt}^{S,out}$	Inlet/outlet temperatures in the supply network (K)
$T_{pt}^{R,in}/T_{pt}^{R,out}$	Inlet/outlet temperatures in the return net-

	work (K)
T^S_{nt}/T^R_{nt}	Supply/return temperatures at node n (K)
mf^S_{pt}/mf^R_{pt}	Mass flow rate in the supply/return network (kg.m^{-3})
mf^{HS}_{jt}	Mass flow rate at heat station j (kg.m^{-3})
mf^{HES}_{it}	Mass flow rate at heat exchanger station i (kg.m^{-3})
pr^S_{nt}/pr^R_{nt}	Pressure at node n in the supply/return network (Pa)
τ^S_{pt}/τ^R_{pt}	Time delay in the supply/return network (h)
Q_{jt}	Heat production of heat station j (Wh)
P_{jt}	Electricity production of electricity generator or CHP j (Wh)
L^{HP}_{jt}	Electricity consumption of heat pump j (Wh)
L^{pump}_{jt}	Electricity consumption of water pump in heat station j (Wh)
θ_{nt}	Voltage angle at bus n (rad)

APPENDIX A: MCCORMICK RELAXATIONS

For each HES, we relax the bilinear terms $mf^{HES}_{it}\left(T^S_{nt} - T^R_{nt}\right)$ by introducing the following upper bounding and lower bounding linear functions

$$L^H_{it} \geq \frac{c}{\Delta t}\overline{mf}^{HES}_{it}\left(T^S_{nt} - T^R_{nt}\right) + \frac{c}{\Delta t}mf^{HES}_{it}\left(\overline{T}^S_{nt} - \underline{T}^R_{nt}\right) - \frac{c}{\Delta t}\overline{mf}^{HES}_{it}\left(\overline{T}^S_{nt} - \underline{T}^R_{nt}\right) \quad \forall n \in \mathcal{I}^N, i \in S^{HES}_n, t \in \mathcal{T} \tag{1a}$$

$$L^H_{it} \geq \frac{c}{\Delta t}\underline{mf}^{HES}_{it}\left(T^S_{nt} - T^R_{nt}\right) + \frac{c}{\Delta t}mf^{HES}_{it}\left(\underline{T}^S_{nt} - \overline{T}^R_{nt}\right) - \frac{c}{\Delta t}\underline{mf}^{HES}_{it}\left(\underline{T}^S_{nt} - \overline{T}^R_{nt}\right) \quad \forall n \in \mathcal{I}^N, i \in S^{HES}_n, t \in \mathcal{T} \tag{1b}$$

$$L^H_{it} \leq \frac{c}{\Delta t}\overline{mf}^{HES}_{it}\left(T^S_{nt} - T^R_{nt}\right) + \frac{c}{\Delta t}mf^{HES}_{it}\left(\underline{T}^S_{nt} - \overline{T}^R_{nt}\right) - \frac{c}{\Delta t}\overline{mf}^{HES}_{it}\left(\underline{T}^S_{nt} - \overline{T}^R_{nt}\right) \quad \forall n \in \mathcal{I}^N, i \in S^{HES}_n, t \in \mathcal{T} \tag{1c}$$

$$L^H_{it} \leq \frac{c}{\Delta t}\underline{mf}^{HES}_{it}\left(T^S_{nt} - T^R_{nt}\right) + \frac{c}{\Delta t}mf^{HES}_{it}\left(\overline{T}^S_{nt} - \underline{T}^R_{nt}\right) - \frac{c}{\Delta t}\underline{mf}^{HES}_{it}\left(\overline{T}^S_{nt} - \underline{T}^R_{nt}\right) \quad \forall n \in \mathcal{I}^N, i \in S^{HES}_n, t \in \mathcal{T}. \tag{1d}$$

For supply and return temperature mixing equations at each node, we introduce the auxiliary variables w^S_{pt} and w^R_{pt} such that

$$w^S_{pt} = mf^S_{pt}\left(T^S_{nt} - T^{S,out}_{pt}\right) \quad \forall n \in \mathcal{I}^N, p \in S^{P-}_n \tag{2}$$

$$w^R_{pt} = mf^R_{pt}\left(T^R_{nt} - T^{R,out}_{pt}\right) \quad \forall n \in \mathcal{I}^N, p \in S^{P+}_n. \tag{3}$$

The products in (2)-(3) can be linearized using a McCormick envelopes

$$w^S_{pt} \geq \overline{mf}^S_{pt}\left(T^S_{nt} - T^{S,out}_{pt}\right) + mf^S_{pt}\left(\overline{T}^S_{nt} - \underline{T}^{S,out}_{pt}\right) - \overline{mf}^S_{pt}\left(\overline{T}^S_{nt} - \underline{T}^{S,out}_{pt}\right), \ \forall n \in \mathcal{I}^N, p \in S^-_n, t \in \mathcal{T} \tag{4a}$$

$$w^S_{pt} \geq \underline{mf}^S_{pt}\left(T^S_{nt} - T^{S,out}_{pt}\right) + mf^S_{pt}\left(\underline{T}^S_{nt} - \overline{T}^{S,out}_{pt}\right) - \underline{mf}^S_{pt}\left(\underline{T}^S_{nt} - \overline{T}^{S,out}_{pt}\right), \ \forall n \in \mathcal{I}^N, p \in S^-_n, t \in \mathcal{T} \tag{4b}$$

$$w^S_{pt} \leq \overline{mf}^S_{pt}\left(T^S_{nt} - T^{S,out}_{pt}\right) + mf^S_{pt}\left(\underline{T}^S_{nt} - \overline{T}^{S,out}_{pt}\right) - \overline{mf}^S_{pt}\left(\underline{T}^S_{nt} - \overline{T}^{S,out}_{pt}\right), \ \forall n \in \mathcal{I}^N, p \in S^-_n, t \in \mathcal{T} \tag{4c}$$

$$w^S_{pt} \leq \underline{mf}^S_{pt}\left(T^S_{nt} - T^{S,out}_{pt}\right) + mf^S_{pt}\left(\overline{T}^S_{nt} - \underline{T}^{S,out}_{pt}\right) - \underline{mf}^S_{pt}\left(\overline{T}^S_{nt} - \underline{T}^{S,out}_{pt}\right), \ \forall n \in \mathcal{I}^N, p \in S^-_n, t \in \mathcal{T}, \tag{4d}$$

and

$$w^R_{pt} \geq \overline{mf}^R_{pt}\left(T^R_{nt} - T^{R,out}_{pt}\right) + mf^R_{pt}\left(\overline{T}^R_{nt} - \underline{T}^{R,out}_{pt}\right) - \overline{mf}^R_{pt}\left(\overline{T}^R_{nt} - \underline{T}^{R,out}_{pt}\right), \ \forall n \in \mathcal{I}^N, p \in S^+_n, t \in \mathcal{T} \tag{5a}$$

$$w^R_{pt} \geq \underline{mf}^R_{pt}\left(T^R_{nt} - T^{R,out}_{pt}\right) + mf^R_{pt}\left(\underline{T}^R_{nt} - \overline{T}^{R,out}_{pt}\right) - \underline{mf}^R_{pt}\left(\underline{T}^R_{nt} - \overline{T}^{R,out}_{pt}\right), \ \forall n \in \mathcal{I}^N, p \in S^+_n, t \in \mathcal{T} \tag{5b}$$

$$w^R_{pt} \leq \overline{mf}^R_{pt}\left(T^R_{nt} - T^{R,out}_{pt}\right) + mf^R_{pt}\left(\underline{T}^R_{nt} - \overline{T}^{R,out}_{pt}\right) - \overline{mf}^R_{pt}\left(\underline{T}^R_{nt} - \overline{T}^{R,out}_{pt}\right), \ \forall n \in \mathcal{I}^N, p \in S^+_n, t \in \mathcal{T} \tag{5c}$$

$$w^R_{pt} \leq \underline{mf}^R_{pt}\left(T^R_{nt} - T^{R,out}_{pt}\right) + mf^R_{pt}\left(\overline{T}^R_{nt} - \underline{T}^{R,out}_{pt}\right) - \underline{mf}^R_{pt}\left(\overline{T}^R_{nt} - \underline{T}^{R,out}_{pt}\right), \ \forall n \in \mathcal{I}^N, p \in S^+_n, t \in \mathcal{T}. \tag{5d}$$

The temperature mixing equations at each node can be reformulated as

$$\sum_{p \in S^-_n} w^S_{pt} = 0, \ \sum_{p \in S^+_n} w^R_{pt} = 0 \quad \forall n \in \mathcal{I}^N, t \in \mathcal{T}. \tag{6}$$

APPENDIX B: CASE STUDY DATA

As shown in Fig. 1, the integrated heat and electricity system considered comprises a conventional thermal generator, a wind producer with an installed capacity of 500MW, n extraction CHP plant and a HP. The technical characteristics of these units are detailed in Table I.

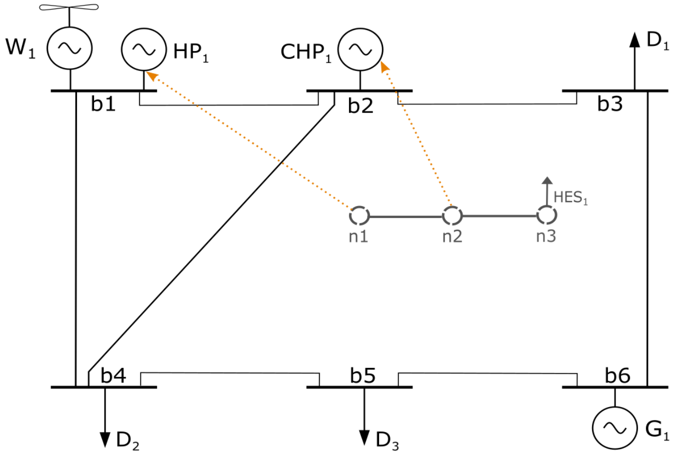

Figure 1. Integrated heat and electricity system

TABLE I
GENERATION UNITS PARAMETERS

		G_1	W_1	CHP_1	HP_1
$\overline{P}$	MWh	180	500	-	-
$\overline{Q}$	MWh	-	-	250	150
$\overline{mf}^{HS}$	kg/s	-	-	300	300
$\overline{F}$	MWh	-	-	250	-
COP	-	-	-	-	2.5
r	-	-	-	0.6	-
ρ^E	-	-	-	2.4	-
ρ^H	-	-	-	0.25	-
η	-	-	-	0.9	0.9
α	\$/MWh	11	0	12.5	-

The technical parameters of the power transmission network and DHN are presented in Tables II, III, and IV.

TABLE II
ELECTRICITY TRANSMISSION NETWORK PARAMETERS

		l_{12}	l_{23}	l_{34}	l_{45}	l_{56}	l_{16}	l_{35}
$\overline{f}$	MWh	400	200	200	200	200	200	200
X	$10^{-1}\Omega$	1.70	0.37	2.58	1.97	0.37	1.40	0.18

TABLE III
DHN PARAMETERS

		p_{12}	p_{23}
R	m	0.80	0.80
L	m	500	500
μ	W.m^{-2}.K^{-1}	20	20
ν	(10^{-3})	1.93	1.93
$\underline{mf}^S/\underline{mf}^R$	kg/s	50	50
$\overline{mf}^S/\overline{mf}^R$	kg/s	300	300

TABLE IV
DHN NODAL PARAMETERS

		n_1	n_2	n_3 (HES_1)
$\underline{mf}^{HES}$	kg/s	-	-	50
$\overline{mf}^{HES}$	kg/s	-	-	300
$\underline{T}^R$	C	30	30	30
$\overline{T}^R$	C	60	60	60
$\underline{T}^S$	C	90	90	90
$\overline{T}^S$	C	120	120	120
$\underline{pr}^S/\underline{pr}^R$	kPa	0	0	0
$\overline{pr}^S/\overline{pr}^S$	kPa	100	100	100

We consider a specific heat capacity of water of 1.17Wh.kg^{-1}.K^{-1} and a water density of 988kg.m^{-3} through the whole DHN. And the time interval considered for optimization is $\delta t = 3600$s.

Heat and electricity loads and available wind production are represented in Fig. 2 and Table V shows the repartition of the electric loads at each bus.

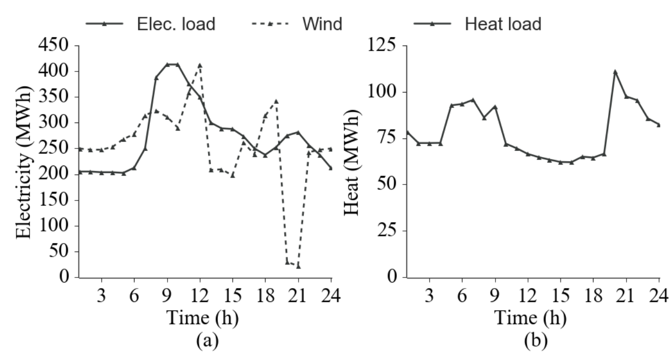

Figure 2. Case study setup: (a) Electricity load and available wind power and (b) Heat load

TABLE V
ELECTRIC LOADS AT EACH BUS

	b_1	b_2	b_3	b_4	b_5	b_6
% of load	0	0	20	40	40	0

APPENDIX C: CONVENTIONAL ECONOMIC DISPATCH

The Conventional Economic Dispatch (CED) is formulated as follows

$$\min_{\tilde{\Omega}} \sum_{j\in\mathcal{I}^H, t\in\mathcal{T}} \alpha_j Q_{jt} + \sum_{j\in\mathcal{I}^E, t\in\mathcal{T}} \alpha_j P_{jt} \tag{7a}$$

$$+ \sum_{j\in\mathcal{I}^{CHP}, t\in\mathcal{T}} \alpha_j \left(\rho_j^E P_{jt} + \rho_j^H Q_{jt}\right) \tag{7b}$$

$$\textit{s.t.} \sum_{i\in\mathcal{I}^{HES}} L_{it}^H = \sum_{j\in\mathcal{I}^{HS}} Q_{jt} \quad \forall t\in\mathcal{T} \tag{7c}$$

$$L_{nt}^E + \sum_{j\in S_n^{HP}} L_{jt}^{HP} = \sum_{j\in S_n^E} P_{jt} + \sum_{m\in S_n^B} B_{nm}\left(\theta_{mt} - \theta_{nt}\right)$$

$$\forall n\in\mathcal{I}^B, t\in\mathcal{T} \tag{7d}$$

$$-\overline{f}_{nm} \le B_{nm}\left(\theta_{mt} - \theta_{nt}\right) \le \overline{f}_{nm}$$

$$\forall n\in\mathcal{I}^B, m\in S_n^B, t\in\mathcal{T} \tag{7e}$$

$$0 \le P_j \le \overline{P}_{jt} \quad \forall i\in\mathcal{I}^E, t\in\mathcal{T} \tag{7f}$$

$$\underline{Q}_{jt} \le Q_{jt} \le \overline{Q}_{jt} \quad \forall j\in\mathcal{I}^{HS}, t\in\mathcal{T} \tag{7g}$$

$$P_{jt} \ge r_j^0 + r_c Q_{jt} \quad \forall j\in\mathcal{I}^{CHP}, t\in\mathcal{T} \tag{7h}$$

$$0 \le \rho_c^E P_{jt} + \rho_c^H Q_{jt} \le \overline{F}_j \quad \forall j\in\mathcal{I}^{CHP}, t\in\mathcal{T} \tag{7i}$$

$$Q_{jt} = \mathit{COP}_{jt} L_{jt}^{HP} \quad \forall j\in\mathcal{I}^{HP}, t\in\mathcal{T}, \tag{7j}$$

where the set of optimization variables $\tilde{\Omega} = \{P, Q, L^{HS}, B, \theta\}$.