Visual inertial wheel Odometry System

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0 前沿

本文研究对象为室内扫地机，研究课题是依靠视觉+IMU+轮式里程计实现室内扫地机的精确定位。本文参考开源 VIO 项目 VINS-Fusion 方案，采用滑动窗口方式的非线性优化 VIO 算法框架，并结合论文《Tightly-coupled Monocular Visual-odometric SLAM using Wheels and a MEMS Gyroscope》中用轮式里程计信息替代加速度信息方法实现视觉+陀螺仪+轮式里程计 VIWO 算法。

1 算法框架

2 预积分

VIWO 预积分部分相对于 VINS-Fusion 的 VIO 不同之处在于用轮式里程计数据替代加速度数据，原因有二：1）扫地机经常无法避免的发生碰撞，这种情形极有可能让 VIO 系统崩溃,而里程计信息发生碰撞测量信息一般不会有较大变化;2)里程计可以提供更加精准的里程信息。然而，VIWO 系统需要处理轮子打滑的问题，这个后续再讨论。

VIWO 预积分将利用陀螺仪数据和里程计数据计算两帧图像间的相对位姿和置信度信息，下面将给出具体推导公式，在此之前先定义坐标系和相关符号。定义世界坐标系 w，轮式里程计坐标系为 o，陀螺仪坐标系为 b，摄像头坐标系为 c。我们用里程计坐标系相对于世界坐标系的位姿来表示机器人的位姿即P_o^w R_o^w。对于连续两帧图像 i 帧和 j 帧，可以得到一些列的陀螺仪数据ω_k以及左右轮的线速度φ_k^l和φ_k^r。为了获取更加精确的预积分状态量，采用中值积分法，定义

$$\omega = \frac{1}{2}R_b^o[(\omega_k - b_k^g + n_k^g) + (\omega_{k+1} - b_k^g + n_{k+1}^g)] \tag{1}$$

$$\phi = \frac{1}{2}[R_{o_k}^{o_i}(\phi_k + n_k^{\phi}) + R_{o_{k+1}}^{o_i}(\phi_{k+1} + n_{k+1}^{\phi})] \text{ 其中} \phi_m = \frac{1}{2}(\phi_m^l + \phi_m^r) \tag{2}$$

R_b^o是 IMU 相对于里程计坐标系的旋转矩阵，n_m^g定义为 m 时刻陀螺仪的测量噪声，n_m^{φ}为

m 时刻里程计测量噪声，b_m^g为陀螺仪零偏噪声，预积分过程中认为其值不变。

1）预积分主体部分推算

那么用里程计和陀螺仪更新 j 帧时刻的位姿如下：

$$\begin{cases} P_{o_j}^{w} = P_{o_i}^{w} + R_{o_i}^{w} \int_i^{j-1} \phi \Delta t \\ R_{o_j}^{w} = R_{o_i}^{w} \prod_i^{j-1} Exp(\omega \Delta t) \end{cases} \tag{3}$$

上式中Exp(.)是李群和李代数一种映射关系，表示将$\omega\Delta t$映射为相应的旋转。对公式（3）两边同乘$R_w^{o_i}$，可得公式（4）

$$\begin{cases} P_{o_i o_j} = \int_i^{j-1} \phi \Delta t \\ R_{o_j}^{o_i} = \prod_i^{j-1} Exp(\omega \Delta t) \end{cases} \tag{4}$$

公式（4）的含义在于将（3）中绝对的位姿量转化在里程计坐标系相对 i 帧的相对位姿量，而相对位姿量是完全可以只依靠 i 和 j 帧之间的陀螺仪数据和里程计数据即可计算的状态，即预积分。

2）预积分置信度推算

在后续进行非线性优化的时候，需要用到预积分状态量的信息矩阵，因此需要计算其协方差。下面来推导从第 k 次到 k+1 次误差项的传递过程：

首先，明确误差项的数学模型，如公式（5）。

$$\begin{bmatrix} \delta P_{o_i o_{k+1}} \\ \delta\theta_{o_i o_{k+1}} \\ \delta b_{k+1}^{g} \end{bmatrix} = F \begin{bmatrix} \delta P_{o_i o_k} \\ \delta\theta_{o_i o_k} \\ \delta b_k^{g} \end{bmatrix} + G \begin{bmatrix} n_k^{g} \\ n_{k+1}^{g} \\ n_k^{\phi} \\ n_{k+1}^{\phi} \\ n_k^{b^g} \end{bmatrix} \tag{5}$$

需要计算公式中的 F 和 G，即可得到协方差矩阵的传递过程如公式（6），其中N为噪声矩阵。

$$\Sigma_{k+1} = F\Sigma_k F^T + GNG^T \tag{6}$$

现推导公式中的 F 和 G 矩阵，首先根据公式（4），得 k 和 k+1 状态递推过程如下：

$$\begin{cases} P_{o_i o_{k+1}} = P_{o_i o_k} + \phi \Delta t \\ R_{o_{k+1}}^{o_i} = R_{o_k}^{o_i} Exp(\omega \Delta t) \\ b_{k+1}^{g} = b_k^{g} + n_k^{b^g} \Delta t \end{cases} \tag{7}$$

误差项由历史误差传递和量测噪声两部分组成，具体推导如下：

2-1）$\delta P_{o_i o_{k+1}}$的推导

$$\frac{\partial \delta P_{o_i o_{k+1}}}{\partial \delta P_{o_i o_k}} = I_{3\times 3}$$

$$\frac{\partial \delta P_{o_i o_{k+1}}}{\partial \delta \theta_{o_i o_k}} = \frac{\partial \delta \frac{1}{2}[R_{o_k}^{o_i} Exp(\delta\theta_{o_i o_k}) \phi_k + R_{o_k}^{o_i} Exp(\delta\theta_{o_i o_k}) Exp(\omega \Delta t) \phi_{k+1}] \Delta t}{\partial \delta \theta_{o_i o_k}}$$

$$= -\frac{1}{2} R_{o_k}^{o_i} [\phi_k]_\times \Delta t - \frac{1}{2} R_{o_k}^{o_i} [Exp(\omega \Delta t) \phi_{k+1}]_\times \Delta t$$

$$= -\frac{1}{2} R_{o_k}^{o_i} [\varphi_k]_\times \Delta t - \frac{1}{2} R_{o_{k+1}}^{o_i} [\varphi_{k+1}]_\times (I - [\omega]_\times \Delta t) \Delta t$$

$$\frac{\partial \delta P_{o_i o_{k+1}}}{\partial \delta b_k^g} = \frac{\partial \delta \frac{1}{2} R_{o_k}^{o_i} Exp(\omega \Delta t) \phi_{k+1} \Delta t}{\partial \delta b_k^g}$$

$$= \frac{\partial \delta \frac{1}{2} R_{o_k}^{o_i} Exp(\overline{\omega} \Delta t - R_b^o \delta b_k^g \Delta t) \phi_{k+1} \Delta t}{\partial \delta b_k^g}$$

$$= \frac{\partial \delta \frac{1}{2} R_{o_k}^{o_i} Exp(\overline{\omega} \Delta t) Exp(-J_r R_b^o \delta b_k^g \Delta t) \phi_{k+1} \Delta t}{\partial \delta b_k^g}$$

$$= -\frac{1}{2} R_{o_{k+1}}^{o_i} [\varphi_{k+1}]_\times (-J_r R_b^o \Delta t) \Delta t$$

$$\approx -\frac{1}{2} R_{o_{k+1}}^{o_i} [\varphi_{k+1}]_\times (-R_b^o \Delta t) \Delta t$$

$$\frac{\partial \delta P_{o_i o_{k+1}}}{\partial n_k^g} = \frac{\partial \delta \frac{1}{2} R_{o_k}^{o_i} Exp(\omega \Delta t) \phi_{k+1} \Delta t}{\partial n_k^g}$$

$$= \frac{\partial \delta \frac{1}{2} R_{o_k}^{o_i} Exp(\overline{\omega} \Delta t + \frac{1}{2} R_b^o n_k^g \Delta t) \phi_{k+1} \Delta t}{\partial n_k^g}$$

$$= \frac{\partial \delta \frac{1}{2} R_{o_k}^{o_i} Exp(\overline{\omega} \Delta t) Exp(\frac{1}{2} J_r R_b^o n_k^g \Delta t) \phi_{k+1} \Delta t}{\partial n_k^g}$$

$$= -\frac{1}{2} R_{o_{k+1}}^{o_i} [\varphi_{k+1}]_\times (\frac{1}{2} J_r R_b^o \Delta t) \Delta t$$

$$\approx -\frac{1}{2} R_{o_{k+1}}^{o_i} [\varphi_{k+1}]_\times (\frac{1}{2} R_b^o \Delta t) \Delta t$$

$$\frac{\partial \delta P_{o_i o_{k+1}}}{\partial n_{k+1}^g} = \frac{\partial \delta P_{o_i o_{k+1}}}{\partial n_k^g}$$

$$\frac{\partial \delta \mathrm{P}_{\mathrm{o_i o_{k+1}}}}{\partial \mathrm{n}_\mathrm{k}^{\phi}} = \frac{\partial \delta \frac{1}{2} R_{o_k}^{o_i} (\phi_\mathrm{k} + \mathrm{n}_\mathrm{k}^{\phi}) \Delta \mathrm{t}}{\partial \mathrm{n}_\mathrm{k}^{\phi}}$$

$$= \frac{1}{2} R_{o_k}^{o_i} \Delta \mathrm{t}$$

$$\frac{\partial \delta \mathrm{P}_{\mathrm{o_i o_{k+1}}}}{\partial \mathrm{n}_\mathrm{k+1}^{\phi}} = \frac{\partial \delta \frac{1}{2} R_{o_{k+1}}^{o_i} (\phi_\mathrm{k+1} + \mathrm{n}_\mathrm{k+1}^{\phi}) \Delta \mathrm{t}}{\partial \mathrm{n}_\mathrm{k+1}^{\phi}}$$

$$= \frac{1}{2} R_{o_{k+1}}^{o_i} \Delta \mathrm{t}$$

$$\frac{\partial \delta \mathrm{P}_{\mathrm{o_i o_{k+1}}}}{\partial \mathrm{n}_\mathrm{k}^{\mathrm{b^g}}} = O_{3\times 3}$$

2-2）$\boldsymbol{\delta\theta_{o_i o_{k+1}}}$的推导

$$\frac{\partial \delta\theta_{o_i o_{k+1}}}{\partial \delta p_{o_i o_k}} = O_{3\times 3}$$

$$\frac{\partial \delta\theta_{o_i o_{k+1}}}{\partial \delta\theta_{o_i o_k}} = R_{o_k}^{o_{k+1}} \ :$$

$$R_{o_{k+1}}^{o_i} = R_{o_k}^{o_i} Exp(\omega \Delta \mathrm{t})$$

$$R_{o_{k+1}}^{o_i} Exp(\delta\theta_{o_i o_{k+1}}) = R_{o_k}^{o_i} Exp(\delta\theta_{o_i o_k}) Exp(\omega \Delta \mathrm{t})$$

$$Exp(\delta\theta_{o_i o_{k+1}}) = R_{o_i}^{o_{k+1}} R_{o_k}^{o_i} Exp(\delta\theta_{o_i o_k}) Exp(\omega \Delta \mathrm{t})$$

$$Exp(\delta\theta_{o_i o_{k+1}}) = R_{o_k}^{o_{k+1}} Exp(\delta\theta_{o_i o_k}) R_{o_{k+1}}^{o_k}$$

$$Exp(\delta\theta_{o_i o_{k+1}}) = Exp(R_{o_k}^{o_{k+1}} \delta\theta_{o_i o_k})$$

$$\frac{\partial \delta\theta_{o_i o_{k+1}}}{\partial \delta b_k^{g}} = -\mathrm{R_b^o} \Delta \mathrm{t} \quad :$$

$$R_{o_{k+1}}^{o_i} = R_{o_k}^{o_i} Exp(\omega \Delta \mathrm{t})$$

$$R_{o_{k+1}}^{o_i} Exp(\delta\theta_{o_i o_{k+1}}) = R_{o_k}^{o_i} Exp(\overline{\omega} \Delta \mathrm{t} - \mathrm{R_b^o} \delta b_k^{g} \Delta \mathrm{t})$$

$$Exp(\delta\theta_{o_i o_{k+1}}) = R_{o_k}^{o_{k+1}} Exp(\overline{\omega} \Delta \mathrm{t}) Exp(-\mathrm{J_r} \mathrm{R_b^o} \delta b_k^{g} \Delta \mathrm{t})$$

$$Exp(\delta\theta_{o_i o_{k+1}}) = Exp(-\mathrm{J_r} \mathrm{R_b^o} \delta b_k^{g} \Delta \mathrm{t})$$

$$\delta\theta_{o_i o_{k+1}} = -\mathrm{J_r} \mathrm{R_b^o} \Delta \mathrm{t} \delta b_k^{g}$$

$$\delta\theta_{o_i o_{k+1}} \approx -\mathrm{R_b^o} \Delta \mathrm{t} \delta b_k^{g}$$

$\frac{\partial \delta\theta_{o_i o_{k+1}}}{\partial n_k^g} = \frac{1}{2} \mathrm{R_b^o} \Delta \mathrm{t}$ ：

$R_{o_{k+1}}^{o_i} = R_{o_k}^{o_i} Exp(\omega \Delta \mathrm{t})$

$R_{o_{k+1}}^{o_i} Exp(\delta\theta_{o_i o_{k+1}}) = R_{o_k}^{o_i} Exp(\overline{\omega}\Delta \mathrm{t} + \frac{1}{2} \mathrm{R_b^o} n_k^g \Delta \mathrm{t})$

$Exp(\delta\theta_{o_i o_{k+1}}) = R_{o_k}^{o_{k+1}} Exp(\overline{\omega}\Delta \mathrm{t}) Exp(\frac{1}{2} \mathrm{J_r} \mathrm{R_b^o} n_k^g \Delta \mathrm{t})$

$Exp(\delta\theta_{o_i o_{k+1}}) = Exp(\frac{1}{2} \mathrm{J_r} \mathrm{R_b^o} n_k^g \Delta \mathrm{t})$

$\delta\theta_{o_i o_{k+1}} = \frac{1}{2} \mathrm{J_r} \mathrm{R_b^o} \Delta \mathrm{t} n_k^g$

$\delta\theta_{o_i o_{k+1}} \approx \frac{1}{2} \mathrm{R_b^o} \Delta \mathrm{t} n_k^g$

$\frac{\partial \delta\theta_{o_i o_{k+1}}}{\partial n_{k+1}^g} = \frac{\partial \delta\theta_{o_i o_{k+1}}}{\partial n_k^g} = \frac{1}{2} \mathrm{R_b^o} \Delta \mathrm{t}$

$\frac{\partial \delta\theta_{o_i o_{k+1}}}{\partial \mathrm{n_k^\phi}} = O_{3\times 3}$

$\frac{\partial \delta\theta_{o_i o_{k+1}}}{\partial \mathrm{n_{k+1}^\phi}} = O_{3\times 3}$

$\frac{\partial \delta\theta_{o_i o_{k+1}}}{\partial \mathrm{n_k^{b^g}}} = O_{3\times 3}$

2-3）$\boldsymbol{\delta b_{k+1}^g}$的推导

$\frac{\partial \delta b_{k+1}^g}{\partial \delta p_{o_i o_k}} = O_{3\times 3}$

$\frac{\partial \delta b_{k+1}^g}{\partial \delta\theta_{o_i o_k}} = O_{3\times 3}$

$\frac{\partial \delta b_{k+1}^g}{\partial \delta b_k^g} = I_{3\times 3}$

$\frac{\partial \delta b_{k+1}^g}{\partial n_k^g} = O_{3\times 3}$

$\frac{\partial \delta b_{k+1}^g}{\partial n_{k+1}^g} = O_{3\times 3}$

$\frac{\partial \delta b_{k+1}^g}{\partial \mathrm{n_k^\phi}} = O_{3\times 3}$

$$\frac{\partial \delta b_{k+1}^{g}}{\partial \mathrm{n}_{k+1}^{\phi}} = O_{3\times 3}$$

$$\frac{\partial \delta\theta_{o_i o_{k+1}}}{\partial \mathrm{n}_{k}^{\mathrm{b}^{\mathrm{g}}}} = I_{3\times 3}\Delta \mathrm{t}$$

3）b^{g}变化更新预积分的推导

后续针对状态量b^{g}的优化调整势必会影响预积分的数值，如果用更性后的b^{g}重新计算预积分则显得过于麻烦，这里利用一阶泰勒展开，近似得：

$$P_{o_i o_j} = \bar{P}_{o_i o_j} + J_{\mathrm{b}^{\mathrm{g}}}^{P}\delta \mathrm{b}^{\mathrm{g}}$$

$$R_{o_j}^{o_i} = \bar{R}_{o_j}^{o_i} Exp(J_{\mathrm{b}^{\mathrm{g}}}^{q}\delta \mathrm{b}^{\mathrm{g}})$$

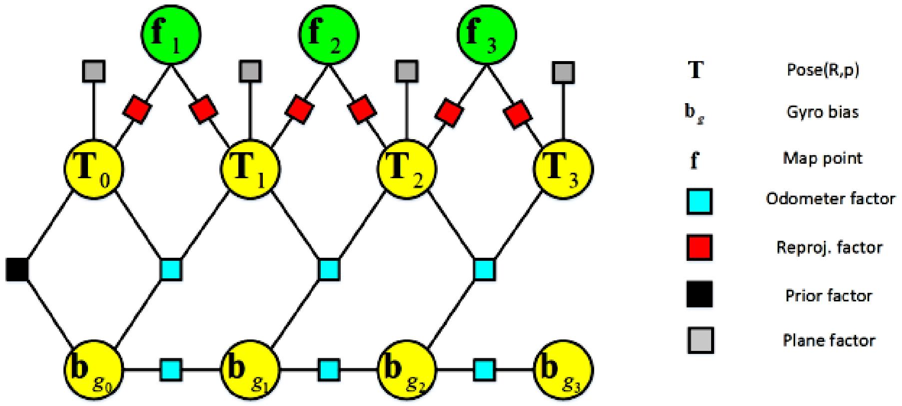
其中$J_{b^g}^{P}$、$J_{b^g}^{q}$是从 i 帧到 j 帧位置和角度误差相对于陀螺仪零偏误差的雅可比矩阵，其值可通过每一次$\frac{\partial \delta \mathrm{P}_{\mathrm{o_i o_{k+1}}}}{\partial \mathrm{b}_{\mathrm{k}}^{\mathrm{g}}}$、$\frac{\partial \delta \theta_{\mathrm{o_i o_{k+1}}}}{\partial \mathrm{b}_{\mathrm{k}}^{\mathrm{g}}}$值连乘得到。

3 非线性优化

定义滑窗优化的变量为：

$$\chi = [\mathrm{T}_0, \mathrm{T}_1 \ldots \mathrm{T}_n, \mathrm{f}_1, \mathrm{f}_2 \ldots \mathrm{f}_m, \mathrm{b}_0^{\mathrm{g}}, \mathrm{b}_1^{\mathrm{g}}, \ldots \mathrm{b}_n^{\mathrm{g}}] \text{其中} \mathrm{T}_i = \left[R_{o_i}^{w} | P_{o_i}^{w}\right]$$

直接用参考论文的图优化示意图，如下所示。绿色圆圈f_m为待优化的路标点逆深度，黄色圆圈为扫地机器人的位姿T_i以及陀螺仪零偏，它们之间通过重投影误差进行约束，如图中的红色方框;同时，因为考虑扫地机是二维空间运动，所以每一个位姿T_i它的俯仰角、横滚角以及高度都应受到约束，如图中灰色方框所示；而蓝色方框则表示预积分与相邻帧位姿变换的约束；黑色方框是初始状态约束。



用数学语言描述：$\chi^* = \text{argmax}\ p(\chi|Z)$，其中后验概率用贝叶斯公式将其转化为先验乘似然的形式，最终得下式。

$$p(\chi|Z) \propto p(\chi_0)p(Z|\chi) = p(\chi_0)\prod_{(i,j)\in K} p(Z_{C_i}, O_{ij}, \widetilde{pl}_i|\chi)$$

$$= p(\chi_0)\prod_{(i,j)\in K} p(O_{ij}|x_i, x_j)\prod_{i\in K}\prod_{l\in L} p(\tilde{z}_{il}|x_i, f_l^w)\prod_{i\in K} p(\widetilde{pl}_i|x_i)$$

用负对数形式求解

$$\chi^* = \text{argmax}\ -\log p(\chi|Z)$$

$$= \text{argmin}\|r_0\|^2_{\Sigma_0} + \sum_{(i,j)\in K}\rho\left(\left\|r_{O_{ij}}\right\|^2_{\Sigma_{O_{ij}}}\right) + \sum_{i\in K}\sum_{l\in L}\rho\left(\left\|r_{C_{il}}\right\|^2_{\Sigma_{C_{il}}}\right) + \sum_{i\in K}\rho\left(\left\|r_{pl_i}\right\|^2_{\Sigma_{pl}}\right)$$

下面逐一给出重投影误差、预积分误差以及平面约束误差的形式极其对应雅可比矩阵的推导。

1）重投影误差

重投影误差计算公式如下：

$$r_c = \begin{bmatrix} \frac{x_{c_j}}{z_{c_j}} - u_{c_j} \\ \frac{y_{c_j}}{z_{c_j}} - v_{c_j} \end{bmatrix}$$

其中$f_{c_j} = [x_{c_j} \quad y_{c_j} \quad z_{c_j} \quad 1]^T$是经过 i 帧像素点重投影到 j 帧的像素点预测位置，其重投影公式如下：

$$f_{c_j} = T_{oc}^T T_{wo_j}^T T_{wo_i} T_{oc} f_{c_i} = T_{oc}^T T_{wo_j}^T T_{wo_i} T_{oc} \begin{bmatrix} \frac{1}{\lambda} u_{c_i} \\ \frac{1}{\lambda} v_{c_i} \\ \frac{1}{\lambda} \\ 1 \end{bmatrix}$$

其中T_{wo_j}、T_{wo_i}、T_{oc}、λ都是待优化的状态量，λ是该像素点对应的逆深度。迭代寻优时需要计算误差相对于它们的雅可比， 为了区分旋转和平移首先将上式进行分解如下：

$$f_{c_j} = R_{oc}^T R_{wo_j}^T R_{wo_i} R_{oc} f_{c_i} + R_{oc}^T (R_{wo_j}^T (R_{wo_i} P_{oc} + P_{wo_i} - P_{wo_j}) - P_{oc})$$

同时，利用链式法则，首先计算r_c关于f_{c_j}的雅可比为：

$$\frac{\partial r_c}{\partial f_{c_j}} = \begin{bmatrix} \frac{1}{z_{c_j}} & 0 & -\frac{x_{c_j}}{z_{c_j}^2} \\ 0 & \frac{1}{z_{c_j}} & -\frac{y_{c_j}}{z_{c_j}^2} \end{bmatrix}$$

1-1)f_{c_j}关于$\mathbf{T}_{\mathbf{wo_i}}$雅可比

$$\frac{\partial f_{c_j}}{\partial P_{wo_i}} = R_{oc}^T R_{wo_j}^T$$

$$\frac{\partial f_{c_j}}{\partial \delta\theta_{wo_i}} = \frac{\partial R_{oc}^T R_{wo_j}^T R_{wo_i} (R_{oc} f_{c_i} + P_{oc})}{\partial \delta\theta_{wo_i}}$$

$$= \frac{\partial R_{oc}^T R_{wo_j}^T R_{wo_i} Exp(\delta\theta_{wo_i}) f_{o_i}}{\partial \delta\theta_{wo_i}}$$

$$= -R_{oc}^T R_{wo_j}^T R_{wo_i} [f_{o_i}]_\times$$

1-2)f_{c_j}关于$\mathbf{T}_{\mathbf{wo_j}}$雅可比

$$\frac{\partial f_{c_j}}{\partial P_{wo_j}} = -R_{oc}^T R_{wo_j}^T$$

$$\frac{\partial f_{c_j}}{\partial \delta\theta_{wo_j}} = \frac{\partial R_{oc}^T R_{wo_j}^T (R_{wo_i}(R_{oc} f_{c_i} + P_{oc}) + P_{wo_i} - P_{wo_j})}{\partial \delta\theta_{wo_i}}$$

$$= \frac{\partial R_{oc}^T \left(R_{wo_j} Exp(\delta\theta_{wo_j})\right)^T (f_{w_i} - P_{wo_j})}{\partial \delta\theta_{wo_j}}$$

$$= \frac{\partial R_{oc}^T (I - \left[\delta\theta_{wo_j}\right]_\times) R_{wo_j}^T (f_{w_i} - P_{wo_j})}{\partial \delta\theta_{wo_j}}$$

$$= -\frac{\partial R_{oc}^T \left[\delta\theta_{wo_j}\right]_\times f_{O_j}}{\partial \delta\theta_{wo_j}}$$

$$= R_{oc}^T \left[f_{O_j}\right]_\times$$

1-3)f_{c_j}关于T_{oc}雅可比

$$\frac{\partial f_{c_j}}{\partial P_{oc}} = R_{oc}^T (R_{wo_j}^T R_{wo_i} - I)$$

$$\frac{\partial f_{c_j}}{\partial \delta\theta_{oc}} = \frac{\partial R_{oc}^T R_{wo_j}^T R_{wo_i} R_{oc} f_{c_i}}{\partial \delta\theta_{oc}} + \frac{\partial R_{oc}^T (R_{wo_j}^T (R_{wo_i} P_{oc} + P_{wo_i} - P_{wo_j}) - P_{oc})}{\partial \delta\theta_{oc}}$$

$$\frac{\partial R_{oc}^T R_{wo_j}^T R_{wo_i} R_{oc} f_{c_i}}{\partial \delta\theta_{oc}} = \frac{\partial (I - [\delta\theta_{oc}]_\times) R_{oc}^T R_{wo_j}^T R_{wo_i} R_{oc} (I + [\delta\theta_{oc}]_\times) f_{c_i}}{\partial \delta\theta_{oc}}$$

$$= \frac{\partial - [\delta\theta_{oc}]_\times R_{oc}^T R_{wo_j}^T R_{wo_i} R_{oc} f_{c_i} + R_{oc}^T R_{wo_j}^T R_{wo_i} R_{oc} [\delta\theta_{oc}]_\times f_{c_i}}{\partial \delta\theta_{oc}}$$

$$= \left[R_{oc}^T R_{wo_j}^T R_{wo_i} R_{oc} f_{c_i}\right]_\times - R_{oc}^T R_{wo_j}^T R_{wo_i} R_{oc} \left[f_{c_i}\right]_\times$$

$$\frac{\partial R_{oc}^T (R_{wo_j}^T (R_{wo_i} P_{oc} + P_{wo_i} - P_{wo_j}) - P_{oc})}{\partial \delta\theta_{oc}}$$

$$= \frac{\partial (I - [\delta\theta_{oc}]_\times) R_{oc}^T (R_{wo_j}^T (R_{wo_i} P_{oc} + P_{wo_i} - P_{wo_j}) - P_{oc})}{\partial \delta\theta_{oc}}$$

$$= \left[R_{oc}^T (R_{wo_j}^T (R_{wo_i} P_{oc} + P_{wo_i} - P_{wo_j}) - P_{oc})\right]_\times$$

1-4)r_c关于λ雅可比

$$\frac{\partial f_{c_j}}{\partial \lambda}=\frac{\partial f_{c_j}}{\partial f_{c_i}}\frac{\partial f_{c_i}}{\partial \lambda}$$

$$=R_{oc}^T R_{wo_j}^T R_{wo_i} R_{oc}\frac{\partial f_{c_i}}{\partial \lambda}$$

$$=-\frac{1}{\lambda}R_{oc}^T R_{wo_j}^T R_{wo_i} R_{oc} f_{c_i}$$

2）**预积分误差**

预积分误差公式如下：

$$r_o=\begin{bmatrix} r_p \\ r_q \\ r_{bg} \end{bmatrix}=\begin{bmatrix} (R_{wo_i}^T(p_{wo_j}-p_{wo_i})-p_{o_io_j} \\ 2[q_{o_io_j}\otimes q_{wo_j}^*\otimes q_{wo_i}]_{xyz} \\ b_j^g-b_i^g \end{bmatrix}$$

其中r_q取虚部，理论上预积分两帧见的旋转和预测的值一样的话，那么虚部应该为 ***0***。涉及到的待优化的量有R_{wo_i}、p_{wo_i}、R_{wo_j}、p_{wo_j}、b_i^g、b_j^g，下面给出r_o关于它们的雅可比矩阵。

2-1-1）r_p关于p_{wo_i}、R_{wo_i}雅可比

$$\frac{\partial r_p}{\partial P_{wo_i}}=-R_{wo_i}^T$$

$$\frac{\partial r_p}{\partial \delta\theta_{wo_i}}=\frac{\partial (R_{wo_i}Exp(\delta\theta_{wo_i}))^T(p_{wo_j}-p_{wo_i})}{\partial \delta\theta_{wo_i}}$$

$$=\frac{\partial (I-[\delta\theta_{wo_i}]_\times)R_{wo_i}^T(p_{wo_j}-p_{wo_i})}{\partial \delta\theta_{wo_i}}$$

$$=\left[R_{wo_i}^T(p_{wo_j}-p_{wo_i})\right]_\times$$

2-1-2）r_p关于p_{wo_j}、R_{wo_j}雅可比

$$\frac{\partial r_p}{\partial P_{wo_j}}=R_{wo_i}^T$$

$$\frac{\partial r_p}{\partial \delta\theta_{wo_j}} = O_{3\times3}$$

2-1-3）r_p关于b_i^g、b_j^g雅可比

$$\frac{\partial r_p}{\partial b_i^g} = \frac{\partial - p_{o_io_j}}{\partial \delta b^g} = -J_{b^g}^P$$

$$\frac{\partial r_p}{\partial b_j^g} = O_{3\times3}$$

2-2-1）r_q关于p_{wo_i}、R_{wo_i}雅可比

$$\frac{\partial r_q}{\partial P_{wo_i}} = O_{3\times3}$$

$$\frac{\partial r_q}{\partial \delta\theta_{wo_i}} = \frac{\partial 2[q_{o_io_j}\otimes q_{wo_j}^*\otimes q_{wo_i}\otimes(q_1 + \begin{bmatrix} 0 \\ \frac{1}{2}\delta\theta_{wo_i}\end{bmatrix})]_{xyz}}{\partial \delta\theta_{wo_i}}$$

$$= \frac{\partial 2\left([q_{o_io_j}\otimes q_{wo_j}^*\otimes q_{wo_i}]_L\right)_{3\times3}\frac{1}{2}\delta\theta_{wo_i}}{\partial \delta\theta_{wo_i}}$$

$$= \left([q_{o_io_j}\otimes q_{wo_j}^*\otimes q_{wo_i}]_L\right)_{3\times3}$$

2-2-2）r_q关于p_{wo_j}、R_{wo_j}雅可比

$$\frac{\partial r_q}{\partial P_{wo_j}} = O_{3\times3}$$

$$\frac{\partial r_q}{\partial \delta\theta_{wo_j}} = \frac{\partial 2[q_{o_io_j}\otimes\left(q_{wo_j}\otimes(q_1 + \begin{bmatrix} 0 \\ \frac{1}{2}\delta\theta_{wo_j}\end{bmatrix})\right)^*\otimes q_{wo_i}]_{xyz}}{\partial \delta\theta_{wo_j}}$$

$$= \frac{\partial 2[q_{o_io_j}\otimes\left(q_1 - \begin{bmatrix} 0 \\ \frac{1}{2}\delta\theta_{wo_j}\end{bmatrix}\right)\otimes q_{wo_j}^*\otimes q_{wo_i}]_{xyz}}{\partial \delta\theta_{wo_j}}$$

$$= \frac{\partial 2[q_{o_io_j}\otimes\begin{bmatrix} 0 \\ -\frac{1}{2}\delta\theta_{wo_j}\end{bmatrix}\otimes q_{wo_j}^*\otimes q_{wo_i}]_{xyz}}{\partial \delta\theta_{wo_j}}$$

$$=\frac{\partial 2\left(\left[q_{o_i o_j}\right]_L\left[q_{wo_j}^{*}\otimes q_{wo_i}\right]_R\right)_{3\times 3}(-\frac{1}{2}\delta\theta_{wo_j})}{\partial\delta\theta_{wo_j}}$$

$$=-\left(\left[q_{o_i o_j}\right]_L\left[q_{wo_j}^{*}\otimes q_{wo_i}\right]_R\right)_{3\times 3}$$

2-2-3）$\boldsymbol{r_q}$关于b_i^g、b_j^g雅可比

$$\frac{\partial r_q}{\partial b_i^g}=\frac{\partial 2[q_{o_i o_j}\otimes(q_1+\begin{bmatrix}0\\ \frac{1}{2}J_{b^g}^{q}\delta b_i^g\end{bmatrix})\otimes q_{wo_j}^{*}\otimes q_{wo_i}]_{xyz}}{\partial\delta b_i^g}$$

$$=\frac{\partial 2[q_{o_i o_j}\otimes\begin{bmatrix}0\\ \frac{1}{2}J_{b^g}^{q}\delta b_i^g\end{bmatrix}\otimes q_{wo_j}^{*}\otimes q_{wo_i}]_{xyz}}{\partial\delta b_i^g}$$

$$=\frac{\partial 2\left(\left[q_{o_i o_j}\right]_L\left[q_{wo_j}^{*}\otimes q_{wo_i}\right]_R\right)_{3\times 3}(\frac{1}{2}J_{b^g}^{q}\delta b_i^g)}{\partial\delta b_i^g}$$

$$=\left(\left[q_{o_i o_j}\right]_L\left[q_{wo_j}^{*}\otimes q_{wo_i}\right]_R\right)_{3\times 3}J_{b^g}^{q}$$

$$\frac{\partial r_q}{\partial b_j^g}=O_{3\times 3}$$

2-3-1）$\boldsymbol{r_{bg}}$关于$\boldsymbol{p_{wo_i}}$、$\boldsymbol{R_{wo_i}}$雅可比

$$\frac{\partial r_{bg}}{\partial P_{wo_i}}=O_{3\times 3}$$

$$\frac{\partial r_{bg}}{\partial\delta\theta_{wo_i}}=O_{3\times 3}$$

2-3-2）$\boldsymbol{r_{bg}}$关于$\boldsymbol{p_{wo_j}}$、$\boldsymbol{R_{wo_j}}$雅可比

$$\frac{\partial r_{bg}}{\partial P_{wo_j}}=O_{3\times 3}$$

$$\frac{\partial r_{bg}}{\partial\delta\theta_{wo_j}}=O_{3\times 3}$$

2-3-3）$\boldsymbol{r_{bg}}$关于b_i^g、b_j^g雅可比

$\frac{\partial r_{bg}}{\partial b_i^g} = -I_{3\times3}$

$\frac{\partial r_{bg}}{\partial b_j^g} = I_{3\times3}$

3）**平面约束误差**

平面约束主要指，相对于开始处的位姿，扫地机器人在高度方向以及水平姿态上变化都不应该很大，可用下式表示二者误差为：

$$r_{p_{li}} = \begin{bmatrix} [e_1 \quad e_2]^T R_{wo_1}^T R_{wo_i} e_3 \\ e_3^T(-p_{wo_1} - R_{wo_1}^T p_{wo_i}) \end{bmatrix} - \mathbf{0}_{3\times1}$$

其中待优化的量为p_{wo_i}、R_{wo_i}，其对应的雅可比如下：

3-1-1）$r_{pa_{li}}$关于p_{wo_i}、R_{wo_i}雅可比

$\frac{\partial r_{pa_{li}}}{\partial p_{wo_i}} = O_{3\times3}$

$\frac{\partial r_{pa_{li}}}{\partial \delta\theta_{wo_i}} = \frac{\partial [e_1 \quad e_2]^T R_{wo_1}^T R_{wo_i} Exp(\delta\theta_{wo_i}) e_3}{\partial \delta\theta_{wo_i}}$

$= -[e_1 \quad e_2]^T R_{wo_1}^T R_{wo_i} [e_3]_\times$

3-1-2）$r_{pz_{li}}$关于p_{wo_i}、R_{wo_i}雅可比

$\frac{\partial r_{pz_{li}}}{\partial p_{wo_i}} = \frac{\partial e_3^T(-p_{wo_1} - R_{wo_1}^T p_{wo_i})}{\partial p_{wo_i}}$

$= -e_3^T R_{wo_1}^T$

$\frac{\partial r_{pz_{li}}}{\partial \delta\theta_{wo_i}} = O_{3\times3}$