Contents

1 Basic

1.1 .vimrc

```
set ru nu cin cul sc so=3 ts=4 sw=4 bs=2 ls=2 mouse=a
inoremap {<CR> {<CR>}<C-o>O
map <F7> :w<CR>:!g++
    "%" -Wall -Wextra -Wshadow -Wconversion -fsanitize
   =address,=undefined -D_GLIBCXX_DEBUG && ./a.out<CR>
```

1.2 PBDS

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
tree<int, null_type, less<int>, rb_tree_tag
    , tree_order_statistics_node_update> bst;
// order_of_key(n): # of elements <= n
// find_by_order(n): 0-indexed

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
__gnu_pbds::priority_queue
   <int, greater<int>, pairing_heap_tag> pq;
```

1.3 pargma

```
#pragma GCC optimize("Ofast,unroll-loops")
#pragma GCC target("avx,avx2,sse,sse2
   ,sse3,ssse3,sse4,popcnt,abm,mmx,fma,tune=native")
```

2 Graph

2.1 2SAT/SCC

```
struct SAT { // 0-base
  int low[N], dfn[N], bln[N], n, Time, nScc;
  bool instack[N], istrue[N];
  stack<int> st;
  vector<int> G[N], SCC[N];
  void init(int _n) {
    n = _n; // assert(n * 2 <= N);
    for (int i = 0; i < n + n; ++i) G[i].clear();
  }
  void add_edge(int a, int b) { G[a].emplace_back(b); }
  int rv(int a) {
    if (a >= n) return a - n;
    return a + n;
  }
  void add_clause(int a, int b) {
    add_edge(rv(a), b), add_edge(rv(b), a);
  }
  void dfs(int u) {
    dfn[u] = low[u] = ++Time;
    instack[u] = 1, st.push(u);
    for (int i : G[u])
      if (!dfn[i])
        dfs(i), low[u] = min(low[i], low[u]);
      else if (instack[i] && dfn[i] < dfn[u])
        low[u] = min(low[u], dfn[i]);
    if (low[u] == dfn[u]) {
      int tmp;
      do {
        tmp = st.top(), st.pop();
        instack[tmp] = 0, bln[tmp] = nScc;
      } while (tmp != u);
      ++nScc;
    }
  }
  bool solve() {
    Time = nScc = 0;
    for (int i = 0; i < n + n; ++i)
      SCC[i].clear(), low[i] = dfn[i] = bln[i] = 0;
    for (int i = 0; i < n + n; ++i)
      if (!dfn[i]) dfs(i);
    for (int i =
         0; i < n + n; ++i) SCC[bln[i]].emplace_back(i);
    for (int i = 0; i < n; ++i) {
      if (bln[i] == bln[i + n]) return false;
      istrue[i] = bln[i] < bln[i + n];
      istrue[i + n] = !istrue[i];
    }
    return true;
  }
};
```

2.2 BCC Vertex

```
int n, m, dfn[N], low[N], is_cut[N], nbcc = 0, t = 0;
vector<int> g[N], bcc[N], G[2 * N];
stack<int> st;
void tarjan(int p, int lp) {
  dfn[p] = low[p] = ++t;
  st.push(p);
  for (auto i : g[p]) {
    if (!dfn[i]) {
      tarjan(i, p);
      low[p] = min(low[p], low[i]);
      if (dfn[p] <= low[i]) {
        nbcc++;
        is_cut[p] = 1;
        for (int x = 0; x != i; st.pop()) {
          x = st.top();
          bcc[nbcc].push_back(x);
        }
        bcc[nbcc].push_back(p);
      }
    } else low[p] = min(low[p], dfn[i]);
  }
}
void build() { // [n+1,n+nbcc] cycle, [1,n] vertex
  for (int i = 1; i <= nbcc; i++) {
    for (auto j : bcc[i]) {
      G[i + n].push_back(j);
      G[j].push_back(i + n);
    }
  }
}
```

2.3 MinimumMeanCycle

```
ll road[N][N]; // input here
struct MinimumMeanCycle {
  ll dp[N + 5][N], n;
  pll solve() {
    ll a = -1, b = -1, L = n + 1;
    for (int i = 2; i <= L; ++i)
      for (int k = 0; k < n; ++k)
        for (int j = 0; j < n; ++j)
          dp[i][j] =
            min(dp[i - 1][k] + road[k][j], dp[i][j]);
    for (int i = 0; i < n; ++i) {
      if (dp[L][i] >= INF) continue;
      ll ta = 0, tb = 1;
      for (int j = 1; j < n; ++j)
        if (dp[j][i] < INF &&
          ta * (L - j) < (dp[L][i] - dp[j][i]) * tb)
          ta = dp[L][i] - dp[j][i], tb = L - j;
      if (ta == 0) continue;
      if (a == -1 || a * tb > ta * b) a = ta, b = tb;
    }
    if (a != -1) {
      ll g = __gcd(a, b);
      return pll(a / g, b / g);
    }
    return pll(-1LL, -1LL);
  }
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i)
      for (int j = 0; j < n; ++j) dp[i + 2][j] = INF;
  }
};
```

2.4 MaximumCliqueDyn

```
struct MaxClique { // fast when N <= 100
  bitset<N> G[N], cs[N];
  int ans, sol[N], q, cur[N], d[N], n;
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) G[i].reset();
  }
  void add_edge(int u, int v) {
    G[u][v] = G[v][u] = 1;
  }
  void pre_dfs(vector<int> &r, int l, bitset<N> mask) {
    if (l < 4) {
      for (int i : r) d[i] = (G[i] & mask).count();
      sort(all(r)
        , [&](int x, int y) { return d[x] > d[y]; });
    }
    vector<int> c(r.size());
    int lft = max(ans - q + 1, 1), rgt = 1, tp = 0;
    cs[1].reset(), cs[2].reset();
    for (int p : r) {
      int k = 1;
      while ((cs[k] & G[p]).any()) ++k;
      if (k > rgt) cs[++rgt + 1].reset();
      cs[k][p] = 1;
      if (k < lft) r[tp++] = p;
    }
    for (int k = lft; k <= rgt; ++k)
      for (int p = cs[k]._Find_first
        (); p < N; p = cs[k]._Find_next(p))
        r[tp] = p, c[tp] = k, ++tp;
    dfs(r, c, l + 1, mask);
  }
  void dfs(vector<
    int> &r, vector<int> &c, int l, bitset<N> mask) {
    while (!r.empty()) {
      int p = r.back();
      r.pop_back(), mask[p] = 0;
      if (q + c.back() <= ans) return;
      cur[q++] = p;
      vector<int> nr;
      for (int i : r) if (G[p][i]) nr.emplace_back(i);
      if (!nr.empty()) pre_dfs(nr, l, mask & G[p]);
      else if (q > ans) ans = q, copy_n(cur, q, sol);
      c.pop_back(), --q;
    }
  }
  int solve() {
    vector<int> r(n);
    ans = q = 0, iota(all(r), 0);
    pre_dfs(r, 0, bitset<N>(string(n, '1')));
    return ans;
  }
};
```

2.5 MinimumSteinerTree

```
struct SteinerTree { // 0-base
  int n, dst[N][N], dp[1 << T][N], tdst[N];
  int vcst[N]; // the cost of vertexs
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) {
      fill_n(dst[i], n, INF);
      dst[i][i] = vcst[i] = 0;
    }
  }
  void chmin(int &x, int val) {
    x = min(x, val);
  }
  void add_edge(int ui, int vi, int wi) {
    chmin(dst[ui][vi], wi);
  }
  void shortest_path() {
    for (int k = 0; k < n; ++k)
      for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
          chmin(dst[i][j], dst[i][k] + dst[k][j]);
  }
  int solve(const vector<int>& ter) {
    shortest_path();
    int t = SZ(ter), full = (1 << t) - 1;
    for (int i = 0; i <= full; ++i)
      fill_n(dp[i], n, INF);
    copy_n(vcst, n, dp[0]);
    for (int msk = 1; msk <= full; ++msk) {
      if (!(msk & (msk - 1))) {
        int who = __lg(msk);
        for (int i = 0; i < n; ++i)
          dp[msk
              ][i] = vcst[ter[who]] + dst[ter[who]][i];
      }
      for (int i = 0; i < n; ++i)
        for (int sub = (
            msk - 1) & msk; sub; sub = (sub - 1) & msk)
          chmin(dp[msk][i],
              dp[sub][i] + dp[msk ^ sub][i] - vcst[i]);
      for (int i = 0; i < n; ++i) {
        tdst[i] = INF;
        for (int j = 0; j < n; ++j)
          chmin(tdst[i], dp[msk][j] + dst[j][i]);
      }
      copy_n(tdst, n, dp[msk]);
    }
    return *min_element(dp[full], dp[full] + n);
  }
}; // O(V 3^T + V^2 2^T)
```

2.6 DMST(slow)

```
struct zhu_liu { // O(VE)
  struct edge {
    int u, v;
    ll w;
  };
  vector<edge> E; // 0-base
  int pe[N], id[N], vis[N];
  ll in[N];
  void init() { E.clear(); }
  void add_edge(int u, int v, ll w) {
    if (u != v) E.emplace_back(edge{u, v, w});
  }
  ll build(int root, int n) {
    ll ans = 0;
    for (;;) {
      fill_n(in, n, INF);
      for (int i = 0; i < E.size(); ++i)
        if (E[i].u != E[i].v && E[i].w < in[E[i].v])
          pe[E[i].v] = i, in[E[i].v] = E[i].w;
      for (int u = 0; u < n; ++u) // no solution
        if (u != root && in[u] == INF) return -INF;
      int cntnode = 0;
      fill_n(id, n, -1), fill_n(vis, n, -1);
      for (int u = 0; u < n; ++u) {
        if (u != root) ans += in[u];
        int v = u;
```

```
        while (vis[v] != u && !~id[v] && v != root)
          vis[v] = u, v = E[pe[v]].u;
        if (v != root && !~id[v]) {
          for (int x = E[pe[v]].u; x != v;
               x = E[pe[x]].u)
            id[x] = cntnode;
          id[v] = cntnode++;
        }
      }
      if (!cntnode) break; // no cycle
      for (int u = 0; u < n; ++u)
        if (!~id[u]) id[u] = cntnode++;
      for (int i = 0; i < E.size(); ++i) {
        int v = E[i].v;
        E[i].u = id[E[i].u], E[i].v = id[E[i].v];
        if (E[i].u != E[i].v) E[i].w -= in[v];
      }
      n = cntnode, root = id[root];
    }
    return ans;
  }
};
```

2.7 DMST

```
#define rep(i, a, b) for (int i = a; i < (b); ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
typedef vector<int> vi;
struct RollbackUF {
  vi e;
  vector<pii> st;
  RollbackUF(int n) : e(n, -1) {}
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : find(e[x]); }
  int time() { return sz(st); }
  void rollback(int t) {
    for (int i = time(); i-- > t;)
      e[st[i].first] = st[i].second;
    st.resize(t);
  }
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    st.push_back({a, e[a]});
    st.push_back({b, e[b]});
    e[a] += e[b];
    e[b] = a;
    return true;
  }
};
struct Edge {
  int a, b;
  ll w;
};
struct Node { /// lazy skew heap node
  Edge key;
  Node *l, *r;
  ll delta;
  void prop() {
    key.w += delta;
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  }
  Edge top() {
    prop();
    return key;
  }
};
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, (a->r = merge(b, a->r)));
  return a;
}
void pop(Node *&a) {
  a->prop();
  a = merge(a->l, a->r);
}

pair<ll, vi> dmst(int n, int r, vector<Edge> &g) {
  RollbackUF uf(n);
  vector<Node *> heap(n);
  for (Edge e : g)
    heap[e.b] = merge(heap[e.b], new Node{e});
  ll res = 0;
  vi seen(n, -1), path(n), par(n);
  seen[r] = r;
  vector<Edge> Q(n), in(n, {-1, -1}), comp;
  deque<tuple<int, int, vector<Edge>>> cycs;
  rep(s, 0, n) {
    int u = s, qi = 0, w;
    while (seen[u] < 0) {
      if (!heap[u]) return {-1, {}};
      Edge e = heap[u]->top();
      heap[u]->delta -= e.w, pop(heap[u]);
      Q[qi] = e, path[qi++] = u, seen[u] = s;
      res += e.w, u = uf.find(e.a);
      if (seen[u] == s) { /// found cycle, contract
        Node *cyc = 0;
        int end = qi, time = uf.time();
        do cyc = merge(cyc, heap[w = path[--qi]]);
        while (uf.join(u, w));
        u = uf.find(u), heap[u] = cyc, seen[u] = -1;
        cycs.push_front({u, time, {&Q[qi], &Q[end]}});
      }
    }
    rep(i, 0, qi) in[uf.find(Q[i].b)] = Q[i];
  }

  for (auto &[u, t, comp] :
    cycs) { // restore sol (optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto &e : comp) in[uf.find(e.b)] = e;
    in[uf.find(inEdge.b)] = inEdge;
  }
  rep(i, 0, n) par[i] = in[i].a;
  return {res, par};
}
```

2.8 VizingTheorem

```
namespace Vizing { // Edge coloring
                   // G: coloring adjM
int C[N][N], G[N][N];
void clear(int n) {
  for (int i = 0; i <= n; i++) {
    for (int j = 0; j <= n; j++) C[i][j] = G[i][j] = 0;
  }
}
void solve(vector<pii> &E, int n, int m) {
  int X[n] = {}, a;
  auto update = [&](int u) {
    for (X[u] = 1; C[u][X[u]]; X[u]++);
  };
  auto color = [&](int u, int v, int c) {
    int p = G[u][v];
    G[u][v] = G[v][u] = c;
    C[u][c] = v;
    C[v][c] = u;
    C[u][p] = C[v][p] = 0;
    if (p) X[u] = X[v] = p;
    else update(u), update(v);
    return p;
  };
  auto flip = [&](int u, int c1, int c2) {
    int p = C[u][c1];
    swap(C[u][c1], C[u][c2]);
    if (p) G[u][p] = G[p][u] = c2;
    if (!C[u][c1]) X[u] = c1;
    if (!C[u][c2]) X[u] = c2;
    return p;
  };
  for (int i = 1; i <= n; i++) X[i] = 1;
  for (int t = 0; t < E.size(); t++) {
    int u = E[t].first, v0 = E[t].second, v = v0,
        c0 = X[u], c = c0, d;
    vector<pii> L;
    int vst[n] = {};
    while (!G[u][v0]) {
      L.emplace_back(v, d = X[v]);
      if (!C[v][c])
        for (a = (int)L.size() - 1; a >= 0; a--)
          c = color(u, L[a].first, c);
      else if (!C[u][d])
        for (a = (int)L.size() - 1; a >= 0; a--)
          color(u, L[a].first, L[a].second);
      else if (vst[d]) break;
```

```
      else vst[d] = 1, v = C[u][d];
    }
    if (!G[u][v0]) {
      for (; v; v = flip(v, c, d), swap(c, d));
      if (C[u][c0]) {
        for (a = (int)L.size() - 2;
             a >= 0 && L[a].second != c; a--)
          ;
        for (; a >= 0; a--)
          color(u, L[a].first, L[a].second);
      } else t--;
    }
  }
}
} // namespace Vizing
```

2.9 MinimumCliqueCover

```
struct Clique_Cover { // 0-base, O(n2^n)
  int co[1 << N], n, E[N];
  int dp[1 << N];
  void init(int _n) {
    n = _n, fill_n(dp, 1 << n, 0);
    fill_n(E, n, 0), fill_n(co, 1 << n, 0);
  }
  void add_edge(int u, int v) {
    E[u] |= 1 << v, E[v] |= 1 << u;
  }
  int solve() {
    for (int i = 0; i < n; ++i)
      co[1 << i] = E[i] | (1 << i);
    co[0] = (1 << n) - 1;
    dp[0] = (n & 1) * 2 - 1;
    for (int i = 1; i < (1 << n); ++i) {
      int t = i & -i;
      dp[i] = -dp[i ^ t];
      co[i] = co[i ^ t] & co[t];
    }
    for (int i = 0; i < (1 << n); ++i)
      co[i] = (co[i] & i) == i;
    fwt(co, 1 << n, 1);
    for (int ans = 1; ans < n; ++ans) {
      int sum = 0; // probabilistic
      for (int i = 0; i < (1 << n); ++i)
        sum += (dp[i] *= co[i]);
      if (sum) return ans;
    }
    return n;
  }
};
```

2.10 CountMaximalClique

```
struct BronKerbosch { // 1-base
  int n, a[N], g[N][N];
  int S, all[N][N], some[N][N], none[N][N];
  void init(int _n) {
    n = _n;
    for (int i = 1; i <= n; ++i)
      for (int j = 1; j <= n; ++j) g[i][j] = 0;
  }
  void add_edge(int u, int v) {
    g[u][v] = g[v][u] = 1;
  }
  void dfs(int d, int an, int sn, int nn) {
    if (S > 1000) return; // pruning
    if (sn == 0 && nn == 0) ++S;
    int u = some[d][0];
    for (int i = 0; i < sn; ++i) {
      int v = some[d][i];
      if (g[u][v]) continue;
      int tsn = 0, tnn = 0;
      copy_n(all[d], an, all[d + 1]);
      all[d + 1][an] = v;
      for (int j = 0; j < sn; ++j)
        if (g[v][some[d][j]])
          some[d + 1][tsn++] = some[d][j];
      for (int j = 0; j < nn; ++j)
        if (g[v][none[d][j]])
          none[d + 1][tnn++] = none[d][j];
      dfs(d + 1, an + 1, tsn, tnn);
      some[d][i] = 0, none[d][nn++] = v;
    }
  }
  int solve() {
    iota(some[0], some[0] + n, 1);
    S = 0, dfs(0, 0, n, 0);
    return S;
  }
};
```

2.11 Theorems

|Maximum independent edge set| $=$ $|V|$ $-$ |Minimum edge cover|
|Maximum independent set| $=$ $|V|$ $-$ |Minimum vertex cover|

3 Flow-Matching

3.1 KM

```
ll w[N][N], hl[N], hr[N], slk[N];
int fl[N], fr[N], pre[N], qu[N], ql, qr, n;
bool vl[N], vr[N];
void init(int _n) {
  n = _n;
  for (int i = 0; i < n; ++i)
    fill_n(w[i], n, -INF);
}
void add_edge(int a, int b, ll wei) {
  w[a][b] = wei;
}
bool Check(int x) {
  if (vl[x] = 1, ~fl[x])
    return vr[qu[qr++] = fl[x]] = 1;
  while (~x) swap(x, fr[fl[x] = pre[x]]);
  return 0;
}
void bfs(int s) {
  fill_n(slk
    , n, INF), fill_n(vl, n, 0), fill_n(vr, n, 0);
  ql = qr = 0, qu[qr++] = s, vr[s] = 1;
  for (ll d;;) {
    while (ql < qr)
      for (int x = 0, y = qu[ql++]; x < n; ++x)
        if (!vl[x] && slk
            [x] >= (d = hl[x] + hr[y] - w[x][y])) {
          if (pre[x] = y, d) slk[x] = d;
          else if (!Check(x)) return;
        }
    d = INF;
    for (int x = 0; x < n; ++x)
      if (!vl[x] && d > slk[x]) d = slk[x];
    for (int x = 0; x < n; ++x) {
      if (vl[x]) hl[x] += d;
      else slk[x] -= d;
      if (vr[x]) hr[x] -= d;
    }
    for (int x = 0; x < n; ++x)
      if (!vl[x] && !slk[x] && !Check(x)) return;
  }
}
ll solve() {
  fill_n(fl
    , n, -1), fill_n(fr, n, -1), fill_n(hr, n, 0);
  for (int i = 0; i < n; ++i)
    hl[i] = *max_element(w[i], w[i] + n);
  for (int i = 0; i < n; ++i) bfs(i);
  ll res = 0;
  for (int i = 0; i < n; ++i) res += w[i][fl[i]];
  return res;
}
};
```

3.2 MCMF

```
struct Edge {
  ll from, to, cap, flow, cost, rev;
} *past[N];
vector<Edge> G[N];
int inq[N], n, s, t;
ll dis[N], up[N], pot[N];
bool BellmanFord() {
  fill_n(dis, n, INF), fill_n(inq, n, 0);
  queue<int> q;
  auto relax = [&](int u, ll d, ll cap, Edge *e) {
    if (cap > 0 && dis[u] > d) {
      dis[u] = d, up[u] = cap, past[u] = e;
      if (!inq[u]) inq[u] = 1, q.push(u);
    }
  };
```

```
    relax(s, 0, INF, 0);
    while (!q.empty()) {
      int u = q.front();
      q.pop(), inq[u] = 0;
      for (auto &e : G[u]) {
        ll d2 = dis[u] + e.cost + pot[u] - pot[e.to];
        relax
            (e.to, d2, min(up[u], e.cap - e.flow), &e);
      }
    }
    return dis[t] != INF;
  }
  void solve(int _s
      , int _t, ll &flow, ll &cost, bool neg = true) {
    s = _s, t = _t, flow = 0, cost = 0;
    if (neg) BellmanFord(), copy_n(dis, n, pot);
    for (; BellmanFord(); copy_n(dis, n, pot)) {
      for (int
          i = 0; i < n; ++i) dis[i] += pot[i] - pot[s];
      flow += up[t], cost += up[t] * dis[t];
      for (int i = t; past[i]; i = past[i]->from) {
        auto &e = *past[i];
        e.flow += up[t], G[e.to][e.rev].flow -= up[t];
      }
    }
  }
  void init(int _n) {
    n = _n, fill_n(pot, n, 0);
    for (int i = 0; i < n; ++i) G[i].clear();
  }
  void add_edge(ll a, ll b, ll cap, ll cost) {
    G[a].pb(Edge{a, b, cap, 0, cost, SZ(G[b])});
    G[b].pb(Edge{b, a, 0, 0, -cost, SZ(G[a]) - 1});
  }
};
```

3.3 GeneralGraphMatching

```
  queue<int> q; int n;
  vector<int> fa, s, vis, pre, match;
  vector<vector<int>> G;
  int Find(int u)
  { return u == fa[u] ? u : fa[u] = Find(fa[u]); }
  int LCA(int x, int y) {
    static int tk = 0; tk++; x = Find(x); y = Find(y);
    for (;; swap(x, y)) if (x != n) {
      if (vis[x] == tk) return x;
      vis[x] = tk;
      x = Find(pre[match[x]]);
    }
  }
  void Blossom(int x, int y, int l) {
    for (; Find(x) != l; x = pre[y]) {
      pre[x] = y, y = match[x];
      if (s[y] == 1) q.push(y), s[y] = 0;
      for (int z: {x, y}) if (fa[z] == z) fa[z] = l;
    }
  }
  bool Bfs(int r) {
    iota(ALL(fa), 0); fill(ALL(s), -1);
    q = queue<int>(); q.push(r); s[r] = 0;
    for (; !q.empty(); q.pop()) {
      for (int x = q.front(); int u : G[x])
        if (s[u] == -1) {
          if (pre[u] = x, s[u] = 1, match[u] == n) {
            for (int a = u, b = x, last;
                b != n; a = last, b = pre[a])
              last =
                  match[b], match[b] = a, match[a] = b;
            return true;
          }
          q.push(match[u]); s[match[u]] = 0;
        } else if (!s[u] && Find(u) != Find(x)) {
          int l = LCA(u, x);
          Blossom(x, u, l); Blossom(u, x, l);
        }
    }
    return false;
  }
  Matching(int _n) : n(_n), fa(n + 1), s(n + 1), vis
      (n + 1), pre(n + 1, n), match(n + 1, n), G(n) {}
  void add_edge(int u, int v)
  { G[u].pb(v), G[v].pb(u); }
  int solve() {
    int ans = 0;
    for (int x = 0; x < n; ++x)
      if (match[x] == n) ans += Bfs(x);
    return ans;
  } // match[x] == n means not matched
};
```

3.4 MaxWeightMaching

```
struct WeightGraph { // 1-based
  struct edge { int u, v, w; }; int n, nx;
  vector<int> lab; vector<vector<edge>> g;
  vector<int> slack, match, st, pa, S, vis;
  vector<vector<int>> flo, flo_from; queue<int> q;
  WeightGraph(int n_) : n(n_), nx(n * 2), lab(nx + 1),
    g(nx + 1, vector<edge>(nx + 1)),slack(nx + 1),
    flo(nx + 1), flo_from(nx + 1, vector(n + 1, 0)) {
    match = st = pa = S = vis = slack;
    rep(u, 1, n) rep(v, 1, n) g[u][v] = {u, v, 0};
  }
  int ED(edge e)
  { return lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2; }
  void update_slack(int u, int x, int &s)
  { if (!s || ED(g[u][x]) < ED(g[s][x])) s = u; }
  void set_slack(int x) {
    slack[x] = 0;
    for (int u = 1; u <= n; ++u)
      if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
        update_slack(u, x, slack[x]);
  }
  void q_push(int x) {
    if (x <= n) q.push(x);
    else for (int y : flo[x]) q_push(y);
  }
  void set_st(int x, int b) {
    st[x] = b;
    if (x > n) for (int y : flo[x]) set_st(y, b);
  }
  vector<int> split_flo(auto &f, int xr) {
    auto it = find(ALL(f), xr);
    if (auto pr = it - f.begin(); pr % 2 == 1)
      reverse(1 + ALL(f)), it = f.end() - pr;
    auto res = vector(f.begin(), it);
    return f.erase(f.begin(), it), res;
  }
  void set_match(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    int xr = flo_from[u][g[u][v].u];
    auto &f = flo[u], z = split_flo(f, xr);
    rep(i, 0, SZ(z)-1) set_match(z[i], z[i ^ 1]);
    set_match(xr, v); f.insert(f.end(), ALL(z));
  }
  void augment(int u, int v) {
    for (;;) {
      int xnv = st[match[u]]; set_match(u, v);
      if (!xnv) return;
      set_match(xnv, st[pa[xnv]]);
      u = st[pa[xnv]], v = xnv;
    }
  }
  int lca(int u, int v) {
    static int t = 0; ++t;
    for (++t; u || v; swap(u, v)) if (u) {
      if (vis[u] == t) return u;
      vis[u] = t; u = st[match[u]];
      if (u) u = st[pa[u]];
    }
    return 0;
  }
  void add_blossom(int u, int o, int v) {
    int b = find(n + 1 + ALL(st), 0) - begin(st);
    lab[b] = 0, S[b] = 0; match[b] = match[o];
    vector<int> f = {o};
    for (int x = u, y; x != o; x = st[pa[y]])
      f.pb(x), f.pb(y = st[match[x]]), q_push(y);
    reverse(1 + ALL(f));
    for (int x = v, y; x != o; x = st[pa[y]])
      f.pb(x), f.pb(y = st[match[x]]), q_push(y);
    flo[b] = f; set_st(b, b);
    for (int x = 1; x <= nx; ++x)
      g[b][x].w = g[x][b].w = 0;
    fill(ALL(flo_from[b]), 0);
    for (int xs : flo[b]) {
      for (int x = 1; x <= nx; ++x)
        if (g[
              b][x].w == 0 || ED(g[xs][x]) < ED(g[b][x]))
          g[b][x] = g[xs][x], g[x][b] = g[x][xs];
```

```
      for (int x = 1; x <= n; ++x)
        if (flo_from[xs][x]) flo_from[b][x] = xs;
    }
    set_slack(b);
  }
  void expand_blossom(int b) {
    for (int x : flo[b]) set_st(x, x);
    int xr = flo_from[b][g[b][pa[b]].u], xs = -1;
    for (int x : split_flo(flo[b], xr)) {
      if (xs == -1) { xs = x; continue; }
      pa[xs] = g[x][xs].u; S[xs] = 1, S[x] = 0;
      slack[xs] = 0; set_slack(x); q_push(x); xs = -1;
    }
    for (int x : flo[b])
      if (x == xr) S[x] = 1, pa[x] = pa[b];
      else S[x] = -1, set_slack(x);
    st[b] = 0;
  }
  bool on_found_edge(const edge &e) {
    if (int u = st[e.u], v = st[e.v]; S[v] == -1) {
      int nu = st[match[v]]; pa[v] = e.u; S[v] = 1;
      slack[v] = slack[nu] = 0; S[nu] = 0; q_push(nu);
    } else if (S[v] == 0) {
      if (int o = lca(u, v)) add_blossom(u, o, v);
      else return augment(u, v), augment(v, u), true;
    }
    return false;
  }
  bool matching() {
    fill(ALL(S), -1), fill(ALL(slack), 0);
    q = queue<int>();
    for (int x = 1; x <= nx; ++x)
      if (st[x] == x && !match[x])
        pa[x] = 0, S[x] = 0, q_push(x);
    if (q.empty()) return false;
    for (;;) {
      while (SZ(q)) {
        int u = q.front(); q.pop();
        if (S[st[u]] == 1) continue;
        for (int v = 1; v <= n; ++v)
          if (g[u][v].w > 0 && st[u] != st[v]) {
            if (ED(g[u][v]) != 0)
              update_slack(u, st[v], slack[st[v]]);
            else if
                 (on_found_edge(g[u][v])) return true;
          }
      }
      int d = INF;
      for (int b = n + 1; b <= nx; ++b)
        if (st[b] == b && S[b] == 1)
          d = min(d, lab[b] / 2);
      for (int x = 1; x <= nx; ++x)
        if (int
              s = slack[x]; st[x] == x && s && S[x] <= 0)
          d = min(d, ED(g[s][x]) / (S[x] + 2));
      for (int u = 1; u <= n; ++u)
        if (S[st[u]] == 1) lab[u] += d;
        else if (S[st[u]] == 0) {
          if (lab[u] <= d) return false;
          lab[u] -= d;
        }
      rep(b, n + 1, nx) if (st[b] == b && S[b] >= 0)
        lab[b] += d * (2 - 4 * S[b]);
      for (int x = 1; x <= nx; ++x)
        if (int s = slack[x]; st[x] == x &&
              s && st[s] != x && ED(g[s][x]) == 0)
          if (on_found_edge(g[s][x])) return true;
      for (int b = n + 1; b <= nx; ++b)
        if (st[b] == b && S[b] == 1 && lab[b] == 0)
          expand_blossom(b);
    }
    return false;
  }
  pair<ll, int> solve() {
    fill(ALL(match), 0);
    rep(u, 0, n) st[u] = u, flo[u].clear();
    int w_max = 0;
    rep(u, 1, n) rep(v, 1, n) {
      flo_from[u][v] = (u == v ? u : 0);
      w_max = max(w_max, g[u][v].w);
    }
    fill(ALL(lab), w_max);
    int n_matches = 0; ll tot_weight = 0;
    while (matching()) ++n_matches;
    rep(u, 1, n) if (match[u] && match[u] < u)
      tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
  }
  void add_edge(int u, int v, int w)
  { g[u][v].w = g[v][u].w = w; }
};
```

3.5 GlobalMinCut

```
  #define REP for (int i = 0; i < n; ++i)
  static const int MXN = 514, INF = 2147483647;
  int vst[MXN], edge[MXN][MXN], wei[MXN];
  void init(int n) {
    REP fill_n(edge[i], n, 0);
  }
  void addEdge(int u, int v, int w){
    edge[u][v] += w; edge[v][u] += w;
  }
  int search(int &s, int &t, int n){
    fill_n(vst, n, 0), fill_n(wei, n, 0);
    s = t = -1;
    int mx, cur;
    for (int j = 0; j < n; ++j) {
      mx = -1, cur = 0;
      REP if (wei[i] > mx) cur = i, mx = wei[i];
      vst[cur] = 1, wei[cur] = -1;
      s = t; t = cur;
      REP if (!vst[i]) wei[i] += edge[cur][i];
    }
    return mx;
  }
  int solve(int n) {
    int res = INF;
    for (int x, y; n > 1; n--){
      res = min(res, search(x, y, n));
      REP edge[i][x] = (edge[x][i] += edge[y][i]);
      REP {
        edge[y][i] = edge[n - 1][i];
        edge[i][y] = edge[i][n - 1];
      } // edge[y][y] = 0;
    }
    return res;
  }
} sw;
```

3.6 BoundedFlow(Dinic)

```
  struct edge {
    int to, cap, flow, rev;
  };
  vector<edge> G[N];
  int n, s, t, dis[N], cur[N], cnt[N];
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n + 2; ++i)
      G[i].clear(), cnt[i] = 0;
  }
  void add_edge(int u, int v, int lcap, int rcap) {
    cnt[u] -= lcap, cnt[v] += lcap;
    G[u].pb(edge{v, rcap, lcap, SZ(G[v])});
    G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
  }
  void add_edge(int u, int v, int cap) {
    G[u].pb(edge{v, cap, 0, SZ(G[v])});
    G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
  }
  int dfs(int u, int cap) {
    if (u == t || !cap) return cap;
    for (int &i = cur[u]; i < SZ(G[u]); ++i) {
      edge &e = G[u][i];
      if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
        int df = dfs(e.to, min(e.cap - e.flow, cap));
        if (df) {
          e.flow += df, G[e.to][e.rev].flow -= df;
          return df;
        }
      }
    }
    dis[u] = -1;
    return 0;
  }
  bool bfs() {
    fill_n(dis, n + 3, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
      int u = q.front();
```

```
      q.pop();
      for (edge &e : G[u])
        if (!~dis[e.to] && e.flow != e.cap)
          q.push(e.to), dis[e.to] = dis[u] + 1;
    }
    return dis[t] != -1;
  }
  int maxflow(int _s, int _t) {
    s = _s, t = _t;
    int flow = 0, df;
    while (bfs()) {
      fill_n(cur, n + 3, 0);
      while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
  }
  bool solve() {
    int sum = 0;
    for (int i = 0; i < n; ++i)
      if (cnt[i] > 0)
        add_edge(n + 1, i, cnt[i]), sum += cnt[i];
      else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
    if (sum != maxflow(n + 1, n + 2)) sum = -1;
    for (int i = 0; i < n; ++i)
      if (cnt[i] > 0)
        G[n + 1].pop_back(), G[i].pop_back();
      else if (cnt[i] < 0)
        G[i].pop_back(), G[n + 2].pop_back();
    return sum != -1;
  }
  int solve(int _s, int _t) {
    add_edge(_t, _s, INF);
    if (!solve()) return -1; // invalid flow
    int x = G[_t].back().flow;
    return G[_t].pop_back(), G[_s].pop_back(), x;
  }
};
```

3.7 GomoryHuTree

```
int g[MAXN];
void GomoryHu(int n) { // 0-base
  fill_n(g, n, 0);
  for (int i = 1; i < n; ++i) {
    Dinic.reset();
    add_edge(i, g[i], Dinic.maxflow(i, g[i]));
    for (int j = i + 1; j <= n; ++j)
      if (g[j] == g[i] && ~Dinic.dis[j])
        g[j] = i;
  }
}
```

3.8 MinCostCirculation

```
  struct Edge {
    ll from, to, cap, fcap, flow, cost, rev;
  } *past[N];
  vector<Edge> G[N];
  ll dis[N], inq[N], n;
  void BellmanFord(int s) {
    fill_n(dis, n, INF), fill_n(inq, n, 0);
    queue<int> q;
    auto relax = [&](int u, ll d, Edge *e) {
      if (dis[u] > d) {
        dis[u] = d, past[u] = e;
        if (!inq[u]) inq[u] = 1, q.push(u);
      }
    };
    relax(s, 0, 0);
    while (!q.empty()) {
      int u = q.front();
      q.pop(), inq[u] = 0;
      for (auto &e : G[u])
        if (e.cap > e.flow)
          relax(e.to, dis[u] + e.cost, &e);
    }
  }
  void try_edge(Edge &cur) {
    if (cur.cap > cur.flow) return ++cur.cap, void();
    BellmanFord(cur.to);
    if (dis[cur.from] + cur.cost < 0) {
      ++cur.flow, --G[cur.to][cur.rev].flow;
      for (int
             i = cur.from; past[i]; i = past[i]->from) {
        auto &e = *past[i];
        ++e.flow, --G[e.to][e.rev].flow;
      }
    }
    ++cur.cap;
  }
  void solve(int mxlg) {
    for (int b = mxlg; b >= 0; --b) {
      for (int i = 0; i < n; ++i)
        for (auto &e : G[i])
          e.cap *= 2, e.flow *= 2;
      for (int i = 0; i < n; ++i)
        for (auto &e : G[i])
          if (e.fcap >> b & 1)
            try_edge(e);
    }
  }
  void init(int _n) { n = _n;
    for (int i = 0; i < n; ++i) G[i].clear();
  }
  void add_edge(ll a, ll b, ll cap, ll cost) {
    G[a].pb(Edge
        {a, b, 0, cap, 0, cost, SZ(G[b]) + (a == b)});
    G[b].pb(Edge{b, a, 0, 0, 0, -cost, SZ(G[a]) - 1});
  }
} mcmf; // O(VE * ElogC)
```

3.9 FlowModelsBuilding

- Maximum/Minimum flow with lower bound / Circulation problem
  1. Construct super source S and sink T.
  2. For each edge (x,y,l,u), connect $x \to y$ with capacity $u-l$.
  3. For each vertex v, denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  4. If $in(v)>0$, connect $S \to v$ with capacity $in(v)$, otherwise, connect $v \to T$ with capacity $-in(v)$.
     - To maximize, connect $t \to s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T. If $f \neq \sum_{v\in V,in(v)>0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
     - To minimize, let f be the maximum flow from S to T. Connect $t \to s$ with capacity ∞ and let the flow from S to T be f'. If $f+f' \neq \sum_{v\in V,in(v)>0} in(v)$, there's no solution. Otherwise, f' is the answer.
  5. The solution of each edge e is l_e+f_e, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X,Y)
  1. Redirect every edge: $y \to x$ if $(x,y)\in M$, $x \to y$ otherwise.
  2. DFS from unmatched vertices in X.
  3. $x \in X$ is chosen iff x is unvisited.
  4. $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
  1. Consruct super source S and sink T
  2. For each edge (x,y,c), connect $x \to y$ with $(cost,cap)=(c,1)$ if $c>0$, otherwise connect $y \to x$ with $(cost,cap)=(-c,1)$
  3. For each edge with $c<0$, sum these cost as K, then increase $d(y)$ by 1, decrease $d(x)$ by 1
  4. For each vertex v with $d(v)>0$, connect $S \to v$ with $(cost,cap)=(0,d(v))$
  5. For each vertex v with $d(v)<0$, connect $v \to T$ with $(cost,cap)=(0,-d(v))$
  6. Flow from S to T, the answer is the cost of the flow $C+K$
- Maximum density induced subgraph
  1. Binary search on answer, suppose we're checking answer T
  2. Construct a max flow model, let K be the sum of all weights
  3. Connect source $s \to v$, $v \in G$ with capacity K
  4. For each edge (u,v,w) in G, connect $u \to v$ and $v \to u$ with capacity w
  5. For $v \in G$, connect it with sink $v \to t$ with capacity $K+2T-(\sum_{e\in E(v)} w(e))-2w(v)$
  6. T is a valid answer if the maximum flow $f<K|V|$
- Minimum weight edge cover
  1. For each $v \in V$ create a copy v', and connect $u' \to v'$ with weight $w(u,v)$.
  2. Connect $v \to v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v.
  3. Find the minimum weight perfect matching on G'.
- Project selection problem
  1. If $p_v>0$, create edge (s,v) with capacity p_v; otherwise, create edge (v,t) with capacity $-p_v$.
  2. Create edge (u,v) with capacity w with w being the cost of choosing u without choosing v.
  3. The mincut is equivalent to the maximum profit of a subset of projects.
- Dual of minimum cost maximum flow
  1. Capacity c_{uv}, Flow f_{uv}, Cost w_{uv}, Required Flow difference for vertex b_u.

2. If all w_{uv} are integers, then optimal solution can happen when all p_u are integers.

$$\min \sum_{uv} w_{uv} f_{uv} \qquad \min \sum_u b_u p_u + \sum_{uv} c_{uv} \max(0, p_v - p_u - w_{uv})$$

$$-f_{uv} \ge -c_{uv} \Leftrightarrow$$

$$\sum_v f_{vu} - \sum_v f_{uv} = -b_u \qquad p_u \ge 0$$

4 Data Struture

4.1 LCT

```
  static Splay nil;
  Splay *ch[2], *f;
  int val, sum, rev, size;
  Splay (int
      _val = 0) : val(_val), sum(_val), rev(0), size(1)
  { f = ch[0] = ch[1] = &nil; }
  bool isr()
  { return f->ch[0] != this && f->ch[1] != this; }
  int dir()
  { return f->ch[0] == this ? 0 : 1; }
  void setCh(Splay *c, int d) {
    ch[d] = c;
    if (c != &nil) c->f = this;
    pull();
  }
  void give_tag(int r) {
    if (r) swap(ch[0], ch[1]), rev ^= 1;
  }
  void push() {
    if (ch[0] != &nil) ch[0]->give_tag(rev);
    if (ch[1] != &nil) ch[1]->give_tag(rev);
    rev = 0;
  }
  void pull() {
    // take care of the nil!
    size = ch[0]->size + ch[1]->size + 1;
    sum = ch[0]->sum ^ ch[1]->sum ^ val;
    if (ch[0] != &nil) ch[0]->f = this;
    if (ch[1] != &nil) ch[1]->f = this;
  }
} Splay::nil;
Splay *nil = &Splay::nil;
void rotate(Splay *x) {
  Splay *p = x->f;
  int d = x->dir();
  if (!p->isr()) p->f->setCh(x, p->dir());
  else x->f = p->f;
  p->setCh(x->ch[!d], d);
  x->setCh(p, !d);
  p->pull(), x->pull();
}
void splay(Splay *x) {
  vector<Splay*> splayVec;
  for (Splay *q = x;; q = q->f) {
    splayVec.pb(q);
    if (q->isr()) break;
  }
  reverse(ALL(splayVec));
  for (auto it : splayVec) it->push();
  while (!x->isr()) {
    if (x->f->isr()) rotate(x);
    else if (x->dir() == x->f->dir())
      rotate(x->f), rotate(x);
    else rotate(x), rotate(x);
  }
}
Splay* access(Splay *x) {
  Splay *q = nil;
  for (; x != nil; x = x->f)
    splay(x), x->setCh(q, 1), q = x;
  return q;
}
void root_path(Splay *x) { access(x), splay(x); }
void chroot(Splay *x){
  root_path(x), x->give_tag(1);
  x->push(), x->pull();
}
void split(Splay *x, Splay *y) {
  chroot(x), root_path(y);
}
void link(Splay *x, Splay *y) {
  root_path(x), chroot(y);
  x->setCh(y, 1);
}
void cut(Splay *x, Splay *y) {
  split(x, y);
  if (y->size != 5) return;
  y->push();
  y->ch[0] = y->ch[0]->f = nil;
}
Splay* get_root(Splay *x) {
  for (root_path(x); x->ch[0] != nil; x = x->ch[0])
    x->push();
  splay(x);
  return x;
}
bool conn(Splay *x, Splay *y) {
  return get_root(x) == get_root(y);
}
Splay* lca(Splay *x, Splay *y) {
  access(x), root_path(y);
  if (y->f == nil) return y;
  return y->f;
}
void change(Splay *x, int val) {
  splay(x), x->val = val, x->pull();
}
int query(Splay *x, Splay *y) {
  split(x, y);
  return y->sum;
}
```

5 String

5.1 KMP

```
  int n = t.size(), ans = 0;
  vector<int> f(t.size(), 0);
  f[0] = -1;
  for (int i = 1, j = -1; i < t.size(); i++) {
    while (j >= 0)
      if (t[j + 1] == t[i]) break;
      else j = f[j];
    f[i] = ++j;
  }
  for (int i = 0, j = 0; i < s.size(); i++) {
    while (j >= 0)
      if (t[j + 1] == s[i]) break;
      else j = f[j];
    if (++j + 1 == t.size()) ans++, j = f[j];
  }
  return ans;
}
```

5.2 Z

```
void z(string s) {
  for (int i = 1, mx = 0; i < s.size(); i++) {
    if (i < Z[mx] + mx)
      Z[i] = min(Z[mx] - i + mx, Z[i - mx]);
    while (
      Z[i] + i < s.size() && s[i + Z[i]] == s[Z[i]])
      Z[i]++;
    if (Z[i] + i > Z[mx] + mx) mx = i;
  }
}
```

5.3 Manacher

```
int manacher(string s) {
  string t;
  for (int i = 0; i < s.size(); i++) {
    if (i) t.push_back('$');
    t.push_back(s[i]);
  }
  int mx = 0, ans = 0;
  for (int i = 0; i < t.size(); i++) {
    man[i] = 1;
    man[i] = min(man[mx] + mx - i, man[2 * mx - i]);
    while (man[i] + i < t.size() && i - man[i] >= 0 &&
      t[i + man[i]] == t[i - man[i]])
      man[i]++;
    if (i + man[i] > mx + man[mx]) mx = i;
  }
  for (int i = 0; i < t.size(); i++)
    ans = max(ans, man[i] - 1);
  return ans;
}
```

5.4 SuffixArray

```
void SA(string s) {
  int n = s.size();
  sa.resize(n), cnt.resize(n), rk.resize(n),
    tmp.resize(n);
  iota(sa.begin(), sa.end(), 0);
  sort(sa.begin(), sa.end(),
    [&](int i, int j) { return s[i] < s[j]; });
  rk[0] = 0;
  for (int i = 1; i < n; i++)
    rk[sa[i]] =
      rk[sa[i - 1]] + (s[sa[i - 1]] != s[sa[i]]);
  for (int k = 1; k <= n; k <<= 1) {
    fill(cnt.begin(), cnt.end(), 0);
    for (int i = 0; i < n; i++)
      cnt[rk[(sa[i] - k + n) % n]]++;
    for (int i = 1; i < n; i++) cnt[i] += cnt[i - 1];
    for (int i = n - 1; i >= 0; i--)
      tmp[--cnt[rk[(sa[i] - k + n) % n]]] =
        (sa[i] - k + n) % n;
    sa.swap(tmp);
    tmp[sa[0]] = 0;
    for (int i = 1; i < n; i++)
      tmp[sa[i]] = tmp[sa[i - 1]] +
        (rk[sa[i - 1]] != rk[sa[i]] ||
          rk[(sa[i - 1] + k) % n] !=
            rk[(sa[i] + k) % n]);
    rk.swap(tmp);
  }
}

void LCP(string s) {
  int n = s.size(), k = 0;
  lcp.resize(n);
  for (int i = 0; i < n; i++)
    if (rk[i] == 0) lcp[rk[i]] = 0;
    else {
      if (k) k--;
      int j = sa[rk[i] - 1];
      while (
        i + k < n && j + k < n && s[i + k] == s[j + k])
        k++;
      lcp[rk[i]] = k;
    }
}
```

5.5 SAIS

```
bool _t[N * 2];
int SA[N * 2], H[N], RA[N];
int _s[N * 2], _c[N * 2], x[N], _p[N], _q[N * 2];
// zero based, string content MUST > 0
// SA[i]: SA[i]-th
    suffix is the i-th lexigraphically smallest suffix.
// H[i]: longest
    common prefix of suffix SA[i] and suffix SA[i - 1].
void pre(int *sa, int *c, int n, int z)
{ fill_n(sa, n, 0), copy_n(c, z, x); }
void induce
    (int *sa, int *c, int *s, bool *t, int n, int z) {
  copy_n(c, z - 1, x + 1);
  for (int i = 0; i < n; ++i)
    if (sa[i] && !t[sa[i] - 1])
      sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
  copy_n(c, z, x);
  for (int i = n - 1; i >= 0; --i)
    if (sa[i] && t[sa[i] - 1])
      sa[--x[s[sa[i] - 1]]] = sa[i] - 1;
}
void sais(int *s, int *sa
    , int *p, int *q, bool *t, int *c, int n, int z) {
  bool uniq = t[n - 1] = true;
  int nn = 0,
      nmxz = -1, *nsa = sa + n, *ns = s + n, last = -1;
  fill_n(c, z, 0);
  for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
  partial_sum(c, c + z, c);
  if (uniq) {
    for (int i = 0; i < n; ++i) sa[--c[s[i]]] = i;
    return;
  }
  for (int i = n - 2; i >= 0; --i)
    t[i] = (
        s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
  pre(sa, c, n, z);
  for (int i = 1; i <= n - 1; ++i)
    if (t[i] && !t[i - 1])
      sa[--x[s[i]]] = p[q[i] = nn++] = i;
  induce(sa, c, s, t, n, z);
  for (int i = 0; i < n; ++i)
    if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
      bool neq = last < 0 || !equal
          (s + sa[i], s + p[q[sa[i]] + 1], s + last);
      ns[q[last = sa[i]]] = nmxz += neq;
    }
  sais(ns,
      nsa, p + nn, q + n, t + n, c + z, nn, nmxz + 1);
  pre(sa, c, n, z);
  for (int i = nn - 1; i >= 0; --i)
    sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
  induce(sa, c, s, t, n, z);
}
void mkhei(int n) {
  for (int i = 0, j = 0; i < n; ++i) {
    if (RA[i])
      for (; _s[i + j] == _s[SA[RA[i] - 1] + j]; ++j);
    H[RA[i]] = j, j = max(0, j - 1);
  }
}
void build(int *s, int n) {
  copy_n(s, n, _s), _s[n] = 0;
  sais(_s, SA, _p, _q, _t, _c, n + 1, 256);
  copy_n(SA + 1, n, SA);
  for (int i = 0; i < n; ++i) RA[SA[i]] = i;
  mkhei(n);
}}
```

5.6 ACAutomaton

```
#define sigma 26
#define base 'a'
struct AhoCorasick {
  int ch[sumS][sigma] = {{}}, f[sumS] = {-1},
      tag[sumS], mv[sumS][sigma], jump[sumS],
      cnt[sumS];
  int idx = 0;
  int insert(string &s) {
    int j = 0;
    for (int i = 0; i < (int)s.size(); i++) {
      if (!ch[j][s[i] - base])
        ch[j][s[i] - base] = ++idx;
      j = ch[j][s[i] - base];
    }
    tag[j] = 1;
    return j;
  }
  int next(int u, int c) {
    return u < 0 ? 0 : mv[u][c];
  }
  void build() {
    queue<int> q;
    q.push(0);
    while (!q.empty()) {
      int u = q.front();
      q.pop();
      for (int v = 0; v < sigma; v++) {
        if (ch[u][v]) {
          f[ch[u][v]] = next(f[u], v);
          q.push(ch[u][v]);
        }
        mv[u][v] =
          (ch[u][v] ? ch[u][v] : next(f[u], v));
      }
      if (u) jump[u] = (tag[f[u]] ? f[u] : jump[f[u]]);
    }
  }
  void match(string &s) {
    for (int i = 0; i <= idx; i++) cnt[i] = 0;
    for (int i = 0, j = 0; i < (int)s.size(); i++) {
      j = next(j, s[i] - base);
      cnt[j]++;
    }
    vector<int> v;
    v.emplace_back(0);
    for (int i = 0; i < (int)v.size(); i++)
      for (int j = 0; j < sigma; j++)
        if (ch[v[i]][j]) v.emplace_back(ch[v[i]][j]);
    reverse(v.begin(), v.end());
    for (int i : v)
      if (f[i] > 0) cnt[f[i]] += cnt[i];
  }
```

```
} ac;
```

5.7 MinRotation

```
  int n = s.size();
  s = s + s;
  int i = 0, ans = 0;
  while (i < n) {
    ans = i;
    int j = i + 1, k = i;
    while (j < s.size() && s[j] >= s[k]) {
      k = (s[j] > s[k] ? i : k + 1);
      ++j;
    }
    while (i <= k) i += j - k;
  }
  return ans;
}
```

5.8 ExtSAM

```
  int len[N * 2], link[N * 2]; // maxlength, suflink
  int next[N * 2][CNUM], tot; // [0, tot), root = 0
  int lenSorted[N * 2]; // topo. order
  int cnt[N * 2]; // occurence
  int newnode() {
    fill_n(next[tot], CNUM, 0);
    len[tot] = cnt[tot] = link[tot] = 0;
    return tot++;
  }
  void init() { tot = 0, newnode(), link[0] = -1; }
  int insertSAM(int last, int c) {
    int cur = next[last][c];
    len[cur] = len[last] + 1;
    int p = link[last];
    while (p != -1 && !next[p][c])
      next[p][c] = cur, p = link[p];
    if (p == -1) return link[cur] = 0, cur;
    int q = next[p][c];
    if (len
          [p] + 1 == len[q]) return link[cur] = q, cur;
    int clone = newnode();
    for (int i = 0; i < CNUM; ++i)
      next[
          clone][i] = len[next[q][i]] ? next[q][i] : 0;
    len[clone] = len[p] + 1;
    while (p != -1 && next[p][c] == q)
      next[p][c] = clone, p = link[p];
    link[link[cur] = clone] = link[q];
    link[q] = clone;
    return cur;
  }
  void insert(const string &s) {
    int cur = 0;
    for (auto ch : s) {
      int &nxt = next[cur][int(ch - 'a')];
      if (!nxt) nxt = newnode();
      cnt[cur = nxt] += 1;
    }
  }
  void build() {
    queue<int> q;
    q.push(0);
    while (!q.empty()) {
      int cur = q.front();
      q.pop();
      for (int i = 0; i < CNUM; ++i)
        if (next[cur][i])
          q.push(insertSAM(cur, i));
    }
    vector<int> lc(tot);
    for (int i = 1; i < tot; ++i) ++lc[len[i]];
    partial_sum(ALL(lc), lc.begin());
    for (int i
          = 1; i < tot; ++i) lenSorted[--lc[len[i]]] = i;
  }
  void solve() {
    for (int i = tot - 2; i >= 0; --i)
      cnt[link[lenSorted[i]]] += cnt[lenSorted[i]];
  }
};
```

5.9 PalindromeTree

```
  struct node {
    int next[26], fail, len;
    int cnt, num; // cnt: appear times, num: number of
                  // pal. suf.
    node(int l = 0) : fail(0), len(l), cnt(0), num(0) {
      for (int i = 0; i < 26; ++i) next[i] = 0;
    }
  };
  vector<node> St;
  vector<char> s;
  int last, n;
  palindromic_tree() : St(2), last(1), n(0) {
    St[0].fail = 1, St[1].len = -1, s.pb(-1);
  }
  inline void clear() {
    St.clear(), s.clear(), last = 1, n = 0;
    St.pb(0), St.pb(-1);
    St[0].fail = 1, s.pb(-1);
  }
  inline int get_fail(int x) {
    while (s[n - St[x].len - 1] != s[n])
      x = St[x].fail;
    return x;
  }
  inline void add(int c) {
    s.push_back(c -= 'a'), ++n;
    int cur = get_fail(last);
    if (!St[cur].next[c]) {
      int now = SZ(St);
      St.pb(St[cur].len + 2);
      St[now].fail =
        St[get_fail(St[cur].fail)].next[c];
      St[cur].next[c] = now;
      St[now].num = St[St[now].fail].num + 1;
    }
    last = St[cur].next[c], ++St[last].cnt;
  }
  inline void count() { // counting cnt
    auto i = St.rbegin();
    for (; i != St.rend(); ++i) {
      St[i->fail].cnt += i->cnt;
    }
  }
  inline int size() { // The number of diff. pal.
    return SZ(St) - 2;
  }
};
```

6 Number Theory

6.1 Primes

6.2 ExtGCD

```
// beware of negative numbers!
void extgcd(ll a, ll b, ll c, ll &x, ll &y) {
  if (b == 0) x = c / a, y = 0;
  else {
    extgcd(b, a % b, c, y, x);
    y -= x * (a / b);
  }
} // |x| <= b/2, |y| <= a/2
```

6.3 FloorCeil

```
{ return a / b - (a % b && (a < 0) ^ (b < 0)); }
int ceil(int a, int b)
{ return a / b + (a % b && (a < 0) ^ (b > 0)); }
```

6.4 FloorSum

```
  if (A == 0) return (N + 1) * (B / C);
  if (A > C || B > C)
    return (N + 1) * (B / C) +
      N * (N + 1) / 2 * (A / C) +
      floorsum(A % C, B % C, C, N);
  ll M = (A * N + B) / C;
  return N * M - floorsum(C, C - B - 1, A, M - 1);
} // \sum^{n}_0 floor((ai + b) / m)
```

6.5 MillerRabin

```
// n < 1,122,004,669,633   4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383   6 : primes <= 13
// n < 2^64                7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool Miller_Rabin(ll a, ll n) {
```

```
  if ((a = a % n) == 0) return 1;
  if (n % 2 == 0) return n == 2;
  ll tmp = (n - 1) / ((n - 1) & (1 - n));
  ll t = __lg(((n - 1) & (1 - n))), x = 1;
  for (; tmp; tmp >>= 1, a = mul(a, a, n))
    if (tmp & 1) x = mul(x, a, n);
  if (x == 1 || x == n - 1) return 1;
  while (--t)
    if ((x = mul(x, x, n)) == n - 1) return 1;
  return 0;
}
```

6.6 PollardRho

```
void PollardRho(ll n) {
  if (n == 1) return;
  if (prime(n)) return ++cnt[n], void();
  if (n % 2
      == 0) return PollardRho(n / 2), ++cnt[2], void();
  ll x = 2, y = 2, d = 1, p = 1;
  #define f(x, n, p) ((mul(x, x, n) + p) % n)
  while (true) {
    if (d != n && d != 1) {
      PollardRho(n / d);
      PollardRho(d);
      return;
    }
    if (d == n) ++p;
    x = f(x, n, p), y = f(f(y, n, p), n, p);
    d = gcd(abs(x - y), n);
  }
}
```

6.7 Fraction

```
  ll n, d;
  fraction
      (const ll &_n=0, const ll &_d=1): n(_n), d(_d) {
    ll t = gcd(n, d);
    n /= t, d /= t;
    if (d < 0) n = -n, d = -d;
  }
  fraction operator-() const
  { return fraction(-n, d); }
  fraction operator+(const fraction &b) const
  { return fraction(n * b.d + b.n * d, d * b.d); }
  fraction operator-(const fraction &b) const
  { return fraction(n * b.d - b.n * d, d * b.d); }
  fraction operator*(const fraction &b) const
  { return fraction(n * b.n, d * b.d); }
  fraction operator/(const fraction &b) const
  { return fraction(n * b.d, d * b.n); }
  void print() {
    cout << n;
    if (d != 1) cout << "/" << d;
  }
};
```

6.8 Simplex

Standard form: maximize $\mathbf{c}^T\mathbf{x}$ subject to $A\mathbf{x}\le\mathbf{b}$ and $\mathbf{x}\ge 0$.
Dual LP: minimize $\mathbf{b}^T\mathbf{y}$ subject to $A^T\mathbf{y}\ge\mathbf{c}$ and $\mathbf{y}\ge 0$.
$\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ are optimal if and only if for all $i\in[1,n]$, either $\bar{x}_i=0$ or $\sum_{j=1}^{m}A_{ji}\bar{y}_j=c_i$ holds and for all $i\in[1,m]$ either $\bar{y}_i=0$ or $\sum_{j=1}^{n}A_{ij}\bar{x}_j=b_j$ holds.

1. In case of minimization, let $c'_i=-c_i$
2. $\sum_{1\le i\le n}A_{ji}x_i\ge b_j\to\sum_{1\le i\le n}-A_{ji}x_i\le -b_j$
3. $\sum_{1\le i\le n}A_{ji}x_i=b_j$
   - $\sum_{1\le i\le n}A_{ji}x_i\le b_j$
   - $\sum_{1\le i\le n}A_{ji}x_i\ge b_j$
4. If x_i has no lower bound, replace x_i with $x_i-x'_i$

```
// infeasible < 0, unbounded = inf, Ax <= b, max
struct simplex {
  const double inf = 1 / .0, eps = 1e-9;
  int n, m = 0;
  double A[205][205], B[205];
  void init(int _n) { n = _n; }
  void equation(vector<double> a, double b) {
    ++m;
    for (int i = 0; i < n; i++) A[m][i] = a[i];
    A[m][n + m] = 1, B[m] = b;
  }
  double solve(vector<double> c) {
    for (int i = 0; i < n; i++) A[0][i] = -c[i];
    A[0][n] = 1;
    int flag = 1;
    while (flag--)
      for (int i = 0; i <= n + m; i++)
        if (A[0][i] < -eps) {
          double bx = inf;
          int piv = -1;

          for (int j = 1; j <= m; j++)
            if (0 <= A[j][i] && B[j] / A[j][i] <= bx)
              piv = j, bx = B[j] / A[j][i];
          if (piv == -1) continue;
          if (bx == inf) return inf;
          flag = 1;
          for (int j = 0; j <= m; j++)
            if (j != piv) {
              for (int k = 0; k <= n + m; k++)
                if (k != i)
                  A[j][k] -=
                    A[piv][k] * A[j][i] / A[piv][i];
              B[j] -= B[piv] * A[j][i] / A[piv][i];
              A[j][i] = 0;
            }
        }
    for (int i = 0; i <= m; i++)
      if (B[i] < -eps) return -inf;
    return B[0] / A[0][n];
  }
} lp;
```

6.9 GuassianElimination

```
  int n, m;
  fraction M[MAXN][MAXN + 1], sol[MAXN];
  int solve() { //-1: inconsistent, >= 0: rank
    for (int i = 0; i < n; ++i) {
      int piv = 0;
      while (piv < m && !M[i][piv].n) ++piv;
      if (piv == m) continue;
      for (int j = 0; j < n; ++j) {
        if (i == j) continue;
        fraction tmp = -M[j][piv] / M[i][piv];
        for (int k = 0; k <=
             m; ++k) M[j][k] = tmp * M[i][k] + M[j][k];
      }
    }
    int rank = 0;
    for (int i = 0; i < n; ++i) {
      int piv = 0;
      while (piv < m && !M[i][piv].n) ++piv;
      if (piv == m && M[i][m].n) return -1;
      else if (piv
             < m) ++rank, sol[piv] = M[i][m] / M[i][piv];
    }
    return rank;
  }
};
```

6.10 ChineseRemainder

```
  ll g = gcd(m1, m2);
  if ((x2 - x1) % g) return -1; // no sol
  m1 /= g; m2 /= g;
  pll p = exgcd(m1, m2);
  ll lcm = m1 * m2 * g;
  ll res = p.first * (x2 - x1) * m1 + x1;
  // be careful with overflow
  return (res % lcm + lcm) % lcm;
}
```

6.11 FactorialModp^k

```
ll prod[MAXP];
ll fac_no_p(ll n, ll p, ll pk) {
  prod[0] = 1;
  for (int i = 1; i <= pk; ++i)
    if (i % p) prod[i] = prod[i - 1] * i % pk;
    else prod[i] = prod[i - 1];
  ll rt = 1;
  for (; n; n /= p) {
    rt = rt * mpow(prod[pk], n / pk, pk) % pk;
    rt = rt * prod[n % pk] % pk;
  }
  return rt;
} // (n! without factor p) % p^k
```

6.12 QuadraticResidue

```
ll trial(ll y, ll z, ll m) {
  ll a0 = 1, a1 = 0, b0 = z, b1 = 1, p = (m - 1) / 2;
  while (p) {
    if (p & 1)
      tie(a0, a1) =
        make_pair((a1 * b1 % m * y + a0 * b0) % m,
          (a0 * b1 + a1 * b0) % m);
    tie(b0, b1) =
      make_pair((b1 * b1 % m * y + b0 * b0) % m,
        (2 * b0 * b1) % m);
    p >>= 1;
  }
  if (a1) return inv(a1, m);
  return -1;
}

mt19937 rd(49);

ll psqrt(ll y, ll p) {
  if (fpow(y, (p - 1) / 2, p) != 1) return -1;
  for (int i = 0; i < 30; i++) {
    ll z = rd() % p;
    if (z * z % p == y) return z;
    ll x = trial(y, z, p);
    if (x == -1) continue;
    return x;
  }
  return -1;
}
```

6.13 MeisselLehmer

```
  if (n <= 1) return 0;
  int v = sqrt(n), s = (v + 1) / 2, pc = 0;
  vector<int> smalls(v + 1), skip(v + 1), roughs(s);
  vector<ll> larges(s);
  for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
  for (int i = 0; i < s; ++i) {
    roughs[i] = 2 * i + 1;
    larges[i] = (n / (2 * i + 1) + 1) / 2;
  }
  for (int p = 3; p <= v; ++p) {
    if (smalls[p] > smalls[p - 1]) {
      int q = p * p;
      ++pc;
      if (1LL * q * q > n) break;
      skip[p] = 1;
      for (int i = q; i <= v; i += 2 * p) skip[i] = 1;
      int ns = 0;
      for (int k = 0; k < s; ++k) {
        int i = roughs[k];
        if (skip[i]) continue;
        ll d = 1LL * i * p;
        larges[ns] = larges[k] - (d <= v ? larges
            [smalls[d] - pc] : smalls[n / d]) + pc;
        roughs[ns++] = i;
      }
      s = ns;
      for (int j = v / p; j >= p; --j) {
        int c =
            smalls[j] - pc, e = min(j * p + p, v + 1);
        for (int i = j * p; i < e; ++i) smalls[i] -= c;
      }
    }
  }
  for (int k = 1; k < s; ++k) {
    const ll m = n / roughs[k];
    ll t = larges[k] - (pc + k - 1);
    for (int l = 1; l < k; ++l) {
      int p = roughs[l];
      if (1LL * p * p > m) break;
      t -= smalls[m / p] - (pc + l - 1);
    }
    larges[0] -= t;
  }
  return larges[0];
}
```

6.14 DiscreteLog

```
  constexpr int kStep = 32000;
  unordered_map<int, int> p;
  int b = 1;
  for (int i = 0; i < kStep; ++i) {
    p[y] = i;
    y = 1LL * y * x % m;
    b = 1LL * b * x % m;
  }
  for (int i = 0; i < m + 10; i += kStep) {
    s = 1LL * s * b % m;
    if (p.find(s) != p.end()) return i + kStep - p[s];
  }
  return -1;
}
int DiscreteLog(int x, int y, int m) {
  if (m == 1) return 0;
  int s = 1;
  for (int i = 0; i < 100; ++i) {
    if (s == y) return i;
    s = 1LL * s * x % m;
  }
  if (s == y) return 100;
  int p = 100 + DiscreteLog(s, x, y, m);
  if (fpow(x, p, m) != y) return -1;
  return p;
}
```

6.15 BerlekampMassey

```
vector<T> BerlekampMassey(const vector<T> &output) {
  vector<T> d(SZ(output) + 1), me, he;
  for (int f = 0, i = 1; i <= SZ(output); ++i) {
    for (int j = 0; j < SZ(me); ++j)
      d[i] += output[i - j - 2] * me[j];
    if ((d[i] -= output[i - 1]) == 0) continue;
    if (me.empty()) {
      me.resize(f = i);
      continue;
    }
    vector<T> o(i - f - 1);
    T k = -d[i] / d[f]; o.pb(-k);
    for (T x : he) o.pb(x * k);
    o.resize(max(SZ(o), SZ(me)));
    for (int j = 0; j < SZ(me); ++j) o[j] += me[j];
    if (i - f + SZ(he) >= SZ(me)) he = me, f = i;
    me = o;
  }
  return me;
}
```

6.16 Theorems

- Cramer's rule

$$\begin{array}{l} ax+by=e \\ cx+dy=f \end{array} \Rightarrow \begin{array}{l} x=\dfrac{ed-bf}{ad-bc} \\ y=\dfrac{af-ec}{ad-bc} \end{array}$$

- Vandermonde's Identity

$$C(n+m,k)=\sum_{i=0}^{k} C(n,i)C(m,k-i)$$

- Kirchhoff's Theorem
  Denote L be a $n\times n$ matrix as the Laplacian matrix of graph G, where $L_{ii}=d(i)$, $L_{ij}=-c$ where c is the number of edge (i,j) in G.
  - The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
  - The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.
- Tutte's Matrix
  Let D be a $n\times n$ matrix, where $d_{ij}=x_{ij}$ (x_{ij} is chosen uniformly at random) if $i<j$ and $(i,j)\in E$, otherwise $d_{ij}=-d_{ji}$. $\frac{rank(D)}{2}$ is the maximum matching on G.
- Cayley's Formula
  - Given a degree sequence $d_1,d_2,\ldots,d_n$ for each *labeled* vertices, there are $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!}$ spanning trees.
  - Let $T_{n,k}$ be the number of *labeled* forests on n vertices with k components, such that vertex $1,2,\ldots,k$ belong to different components. Then $T_{n,k}=kn^{n-k-1}$.
- Erdős–Gallai theorem
  A sequence of nonnegative integers $d_1\geq\cdots\geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1+\cdots+d_n$ is even and $\sum_{i=1}^{k} d_i \leq k(k-1)+\sum_{i=k+1}^{n}\min(d_i,k)$ holds for every $1\leq k\leq n$.

- Gale–Ryser theorem
  A pair of sequences of nonnegative integers $a_1 \geq \cdots \geq a_n$ and $b_1,\ldots,b_n$ is bigraphic if and only if $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i$ and $\sum_{i=1}^{k} a_i \leq \sum_{i=1}^{n} \min(b_i,k)$ holds for every $1 \leq k \leq n$.
- Fulkerson–Chen–Anstee theorem
  A sequence $(a_1,b_1),\ldots,(a_n,b_n)$ of nonnegative integer pairs with $a_1 \geq \cdots \geq a_n$ is digraphic if and only if $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i$ and $\sum_{i=1}^{k} a_i \leq \sum_{i=1}^{k} \min(b_i,k-1) + \sum_{i=k+1}^{n} \min(b_i,k)$ holds for every $1 \leq k \leq n$.
- Pick's theorem
  For simple polygon, when points are all integer, we have $A = \#\{\text{lattice points in the interior}\} + \frac{\#\{\text{lattice points on the boundary}\}}{2} - 1$.
- Möbius inversion formula
  - $f(n)=\sum_{d|n} g(d) \Leftrightarrow g(n)=\sum_{d|n} \mu(d) f(\frac{n}{d})$
  - $f(n)=\sum_{n|d} g(d) \Leftrightarrow g(n)=\sum_{n|d} \mu(\frac{d}{n}) f(d)$
- Spherical cap
  - A portion of a sphere cut off by a plane.
  - r: sphere radius, a: radius of the base of the cap, h: height of the cap, θ: arcsin(a/r).
  - Volume $= \pi h^2(3r-h)/3 = \pi h(3a^2+h^2)/6 = \pi r^3(2+\cos\theta)(1-\cos\theta)^2/3$.
  - Area $= 2\pi r h = \pi(a^2+h^2) = 2\pi r^2(1-\cos\theta)$.
- Lagrange multiplier
  - Optimize $f(x_1,\ldots,x_n)$ when k constraints $g_i(x_1,\ldots,x_n)=0$.
  - Lagrangian function $\mathcal{L}(x_1,\ldots,x_n,\lambda_1,\ldots,\lambda_k) = f(x_1,\ldots,x_n) - \sum_{i=1}^{k} \lambda_i g_i(x_1,\ldots,x_n)$.
  - The solution corresponding to the original constrained optimization is always a saddle point of the Lagrangian function.
- Nearest points of two skew lines
  - Line 1: $\boldsymbol{v}_1 = \boldsymbol{p}_1 + t_1 \boldsymbol{d}_1$
  - Line 2: $\boldsymbol{v}_2 = \boldsymbol{p}_2 + t_2 \boldsymbol{d}_2$
  - $\boldsymbol{n} = \boldsymbol{d}_1 \times \boldsymbol{d}_2$
  - $\boldsymbol{n}_1 = \boldsymbol{d}_1 \times \boldsymbol{n}$
  - $\boldsymbol{n}_2 = \boldsymbol{d}_2 \times \boldsymbol{n}$
  - $\boldsymbol{c}_1 = \boldsymbol{p}_1 + \frac{(\boldsymbol{p}_2-\boldsymbol{p}_1)\cdot\boldsymbol{n}_2}{\boldsymbol{d}_1\cdot\boldsymbol{n}_2}\boldsymbol{d}_1$
  - $\boldsymbol{c}_2 = \boldsymbol{p}_2 + \frac{(\boldsymbol{p}_1-\boldsymbol{p}_2)\cdot\boldsymbol{n}_1}{\boldsymbol{d}_2\cdot\boldsymbol{n}_1}\boldsymbol{d}_2$

6.17 Estimation

- Estimation
  - The number of divisors of n is at most around 100 for $n<5e4$, 500 for $n<1e7$, 2000 for $n<1e10$, 200000 for $n<1e19$.
  - The number of ways of writing n as a sum of positive integers, disregarding the order of the summands. $1,1,2,3,5,7,11,15,22,30$ for $n=0\sim 9$, 627 for $n=20$, $\sim 2e5$ for $n=50$, $\sim 2e8$ for $n=100$.
  - Total number of partitions of n distinct elements: $B(n)=1,1,2,5,15,52,203,877,4140,21147,115975,678570,4213597,27644437,190899322,\ldots$.

6.18 EuclideanAlgorithms

- $m=\lfloor\frac{an+b}{c}\rfloor$
- Time complexity: $O(\log n)$

$$f(a,b,c,n)=\sum_{i=0}^{n}\lfloor\frac{ai+b}{c}\rfloor$$
$$=\begin{cases}\lfloor\frac{a}{c}\rfloor\cdot\frac{n(n+1)}{2}+\lfloor\frac{b}{c}\rfloor\cdot(n+1) & \\ +f(a \bmod c, b \bmod c, c, n), & a\geq c \vee b\geq c \\ 0, & n<0 \vee a=0 \\ nm-f(c,c-b-1,a,m-1), & \text{otherwise}\end{cases}$$

$$g(a,b,c,n)=\sum_{i=0}^{n} i\lfloor\frac{ai+b}{c}\rfloor$$
$$=\begin{cases}\lfloor\frac{a}{c}\rfloor\cdot\frac{n(n+1)(2n+1)}{6}+\lfloor\frac{b}{c}\rfloor\cdot\frac{n(n+1)}{2} & \\ +g(a \bmod c, b \bmod c, c, n), & a\geq c \vee b\geq c \\ 0, & n<0 \vee a=0 \\ \frac{1}{2}\cdot(n(n+1)m-f(c,c-b-1,a,m-1) & \\ -h(c,c-b-1,a,m-1)), & \text{otherwise}\end{cases}$$

$$h(a,b,c,n)=\sum_{i=0}^{n}\lfloor\frac{ai+b}{c}\rfloor^2$$
$$=\begin{cases}\lfloor\frac{a}{c}\rfloor^2\cdot\frac{n(n+1)(2n+1)}{6}+\lfloor\frac{b}{c}\rfloor^2\cdot(n+1) & \\ +\lfloor\frac{a}{c}\rfloor\cdot\lfloor\frac{b}{c}\rfloor\cdot n(n+1) & \\ +h(a \bmod c, b \bmod c, c, n) & \\ +2\lfloor\frac{a}{c}\rfloor\cdot g(a \bmod c, b \bmod c, c, n) & \\ +2\lfloor\frac{b}{c}\rfloor\cdot f(a \bmod c, b \bmod c, c, n), & a\geq c \vee b\geq c \\ 0, & n<0 \vee a=0 \\ nm(m+1)-2g(c,c-b-1,a,m-1) & \\ -2f(c,c-b-1,a,m-1)-f(a,b,c,n), & \text{otherwise}\end{cases}$$

6.19 Numbers

- Bernoulli numbers
  $B_0-1, B_1^{\pm}=\pm\frac{1}{2}, B_2=\frac{1}{6}, B_3=0$
  $\sum_{j=0}^{m}\binom{m+1}{j}B_j=0$, EGF is $B(x)=\frac{x}{e^x-1}=\sum_{n=0}^{\infty}B_n\frac{x^n}{n!}$.
  $S_m(n)=\sum_{k=1}^{n}k^m=\frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k}B_k^{+}n^{m+1-k}$
- Stirling numbers of the second kind Partitions of n distinct elements into exactly k groups.
  $S(n,k)=S(n-1,k-1)+kS(n-1,k), S(n,1)=S(n,n)=1$
  $S(n,k)=\frac{1}{k!}\sum_{i=0}^{k}(-1)^{k-i}\binom{k}{i}i^n$
  $x^n=\sum_{i=0}^{n}S(n,i)(x)_i$
- Pentagonal number theorem
  $\prod_{n=1}^{\infty}(1-x^n)=1+\sum_{k=1}^{\infty}(-1)^k\left(x^{k(3k+1)/2}+x^{k(3k-1)/2}\right)$
- Catalan numbers
  $C_n^{(k)}=\frac{1}{(k-1)n+1}\binom{kn}{n}$
  $C^{(k)}(x)=1+x[C^{(k)}(x)]^k$
- Eulerian numbers
  Number of permutations $\pi\in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j)>\pi(j+1)$, $k+1$ j:s s.t. $\pi(j)\geq j$, k j:s s.t. $\pi(j)>j$.
  $E(n,k)=(n-k)E(n-1,k-1)+(k+1)E(n-1,k)$
  $E(n,0)=E(n,n-1)=1$
  $E(n,k)=\sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$

6.20 GenerationFunctions

- Ordinary Generating Function $A(x)=\sum_{i\geq 0}a_i x^i$
  - $A(rx)\Rightarrow r^n a_n$
  - $A(x)+B(x)\Rightarrow a_n+b_n$
  - $A(x)B(x)\Rightarrow\sum_{i=0}^{n}a_i b_{n-i}$
  - $A(x)^k\Rightarrow\sum_{i_1+i_2+\cdots+i_k=n}a_{i_1}a_{i_2}\ldots a_{i_k}$
  - $xA(x)'\Rightarrow na_n$
  - $\frac{A(x)}{1-x}\Rightarrow\sum_{i=0}^{n}a_i$
- Exponential Generating Function $A(x)=\sum_{i\geq 0}\frac{a_i}{i!}x_i$
  - $A(x)+B(x)\Rightarrow a_n+b_n$
  - $A^{(k)}(x)\Rightarrow a_{n+k}$
  - $A(x)B(x)\Rightarrow\sum_{i=0}^{n}\binom{n}{i}a_i b_{n-i}$
  - $A(x)^k\Rightarrow\sum_{i_1+i_2+\cdots+i_k=n}\binom{n}{i_1,i_2,\ldots,i_k}a_{i_1}a_{i_2}\ldots a_{i_k}$
  - $xA(x)\Rightarrow na_n$
- Special Generating Function
  - $(1+x)^n=\sum_{i\geq 0}\binom{n}{i}x^i$
  - $\frac{1}{(1-x)^n}=\sum_{i\geq 0}\binom{i}{n-1}x^i$
  - $S_k=\sum_{x=1}^{n}x^k$: $S=\sum_{p=0}^{\infty}x^p=\frac{e^x-e^{x(n+1)}}{1-e^x}$

7 Polynomials

7.1 NTT (FFT)

```
#define base ll // complex<double>
#define N 524288
// const double PI = acosl(-1);
const ll mod = 998244353, g = 3;
base omega[4 * N], omega_[4 * N];
int rev[4 * N];

ll fpow(ll b, ll p);

ll inverse(ll a) { return fpow(a, mod - 2); }

void calcW(int n) {
  ll r = fpow(g, (mod - 1) / n), invr = inverse(r);
  omega[0] = omega_[0] = 1;
  for (int i = 1; i < n; i++) {
    omega[i] = omega[i - 1] * r % mod;
    omega_[i] = omega_[i - 1] * invr % mod;
```

```
  }
  // double arg = 2.0 * PI / n;
  // for (int i = 0; i < n; i++)
  // {
  //     omega[i] = base(cos(i * arg), sin(i * arg));
  //     omega_[i] = base(cos(-i * arg), sin(-i *
  //     arg));
  // }
}

void calcrev(int n) {
  int k = __lg(n);
  for (int i = 0; i < n; i++) rev[i] = 0;
  for (int i = 0; i < n; i++)
    for (int j = 0; j < k; j++)
      if (i & (1 << j)) rev[i] ^= 1 << (k - j - 1);
}

vector<base> NTT(vector<base> poly, bool inv) {
  base *w = (inv ? omega_ : omega);
  int n = poly.size();
  for (int i = 0; i < n; i++)
    if (rev[i] > i) swap(poly[i], poly[rev[i]]);

  for (int len = 1; len < n; len <<= 1) {
    int arg = n / len / 2;
    for (int i = 0; i < n; i += 2 * len)
      for (int j = 0; j < len; j++) {
        base odd =
          w[j * arg] * poly[i + j + len] % mod;
        poly[i + j + len] =
          (poly[i + j] - odd + mod) % mod;
        poly[i + j] = (poly[i + j] + odd) % mod;
      }
  }
  if (inv)
    for (auto &a : poly) a = a * inverse(n) % mod;
  return poly;
}

vector<base> mul(vector<base> f, vector<base> g) {
  int sz = 1 << (__lg(f.size() + g.size() - 1) + 1);
  f.resize(sz), g.resize(sz);
  calcrev(sz);
  calcW(sz);
  f = NTT(f, 0), g = NTT(g, 0);
  for (int i = 0; i < sz; i++)
    f[i] = f[i] * g[i] % mod;
  return NTT(f, 1);
}
```

7.2 FHWT

```
or: (y += x * op), and: (x += y * op)
xor: (x, y = (x + y) * op, (x - y) * op)
invop: or, and, xor = -1, -1, 1/2 */
void fwt(int *a, int n, int op) { //or
  for (int L = 2; L <= n; L <<= 1)
    for (int i = 0; i < n; i += L)
      for (int j = i; j < i + (L >> 1); ++j)
        a[j + (L >> 1)] += a[j] * op;
}
const int N = 21;
int f[
    N][1 << N], g[N][1 << N], h[N][1 << N], ct[1 << N];
void
    subset_convolution(int *a, int *b, int *c, int L) {
  // c_k = \sum_{i | j = k, i & j = 0} a_i * b_j
  int n = 1 << L;
  for (int i = 1; i < n; ++i)
    ct[i] = ct[i & (i - 1)] + 1;
  for (int i = 0; i < n; ++i)
    f[ct[i]][i] = a[i], g[ct[i]][i] = b[i];
  for (int i = 0; i <= L; ++i)
    fwt(f[i], n, 1), fwt(g[i], n, 1);
  for (int i = 0; i <= L; ++i)
    for (int j = 0; j <= i; ++j)
      for (int x = 0; x < n; ++x)
        h[i][x] += f[j][x] * g[i - j][x];
  for (int i = 0; i <= L; ++i)
    fwt(h[i], n, -1);
  for (int i = 0; i < n; ++i)
    c[i] = h[ct[i]][i];
}
```

7.3 PolynomialOperations

```
poly inv(poly A) {
  A.resize(1 << (__lg(A.size() - 1) + 1));
  poly B = {inverse(A[0])};
  for (int n = 1; n < A.size(); n += n) {
    poly pA(A.begin(), A.begin() + 2 * n);
    calcrev(4 * n);
    calcW(4 * n);
    pA.resize(4 * n);
    B.resize(4 * n);
    pA = NTT(pA, 0);
    B = NTT(B, 0);
    for (int i = 0; i < 4 * n; i++)
      B[i] =
        ((B[i] * 2 - pA[i] * B[i] % mod * B[i]) % mod +
          mod) %
        mod;
    B = NTT(B, 1);
    B.resize(2 * n);
  }
  return B;
}

pair<poly, poly> div(poly A, poly B) {
  if (A.size() < B.size()) return make_pair(poly(), A);
  int n = A.size(), m = B.size();
  poly revA = A, invrevB = B;
  reverse(revA.begin(), revA.end());
  reverse(invrevB.begin(), invrevB.end());
  revA.resize(n - m + 1);
  invrevB.resize(n - m + 1);
  invrevB = inv(invrevB);

  poly Q = mul(revA, invrevB);
  Q.resize(n - m + 1);
  reverse(Q.begin(), Q.end());
  poly R = mul(Q, B);
  R.resize(m - 1);
  for (int i = 0; i < m - 1; i++)
    R[i] = (A[i] - R[i] + mod) % mod;
  return make_pair(Q, R);
}

ll fast_kitamasa(ll k, poly A, poly C) {
  int n = A.size();
  C.emplace_back(mod - 1);
  poly Q, R = {0, 1}, F = {1};
  R = div(R, C);
  while (k) {
    if (k & 1) F = div(mul(F, R), C);
    R = div(mul(R, R), C);
    k >>= 1;
  }
  ll ans = 0;
  for (int i = 0; i < F.size(); i++)
    ans = (ans + A[i] * F[i]) % mod;
  return ans;
}

vector<ll> fpow(vector<ll> f, ll p, ll m) {
  int b = 0;
  while (b < f.size() && f[b] == 0) b++;
  f = vector<ll>(f.begin() + b, f.end());
  int n = f.size();
  f.emplace_back(0);
  vector<ll> q(min(m, b * p), 0);
  q.emplace_back(fpow(f[0], p));
  for (int k = 0; q.size() < m; k++) {
    ll res = 0;
    for (int i = 0; i < min(n, k + 1); i++)
      res = (res +
              p * (i + 1) % mod * f[i + 1] % mod *
                q[k - i + b * p]) %
        mod;
    for (int i = 1; i < min(n, k + 1); i++)
      res = (res -
              f[i] * (k - i + 1) % mod *
                q[k - i + 1 + b * p]) %
        mod;
    res = (res < 0 ? res + mod : res) *
      inv(f[0] * (k + 1) % mod) % mod;
    q.emplace_back(res);
  }
  return q;
```

```
}
```

7.4 NewtonMethod+MiscGF

Given $F(x)$ where

$$F(x)=\sum_{i=0}^{\infty}\alpha_i(x-\beta)^i$$

for β being some constant. Polynomial P such that $F(P)=0$ can be found iteratively. Denote by Q_k the polynomial such that $F(Q_k)=0$ (mod x^{2^k}), then

$$Q_{k+1}=Q_k-\frac{F(Q_k)}{F'(Q_k)} \pmod{x^{2^{k+1}}}$$

- A^{-1}: $B_{k+1}=B_k(2-AB_k) \mod x^{2^{k+1}}$
- $\ln A$: $(\ln A)'=\frac{A'}{A}$
- $\exp A$: $B_{k+1}=B_k(1+A-\ln B_k) \mod x^{2^{k+1}}$
- $\sqrt{A}$: $B_{k+1}=\frac{1}{2}(B_k+AB_k^{-1}) \mod x^{2^{k+1}}$

8 Geometry

8.1 Basic

```
typedef pair<pdd, pdd> Line;
struct Cir{ pdd O; double R; };
const double eps = 1e-8;
pdd operator+(pdd a, pdd b)
{ return pdd(a.X + b.X, a.Y + b.Y); }
pdd operator-(pdd a, pdd b)
{ return pdd(a.X - b.X, a.Y - b.Y); }
pdd operator*(pdd a, double b)
{ return pdd(a.X * b, a.Y * b); }
pdd operator/(pdd a, double b)
{ return pdd(a.X / b, a.Y / b); }
double dot(pdd a, pdd b)
{ return a.X * b.X + a.Y * b.Y; }
double cross(pdd a, pdd b)
{ return a.X * b.Y - a.Y * b.X; }
double abs2(pdd a)
{ return dot(a, a); }
double abs(pdd a)
{ return sqrt(dot(a, a)); }
int sign(double a)
{ return fabs(a) < eps ? 0 : a > 0 ? 1 : -1; }
int ori(pdd a, pdd b, pdd c)
{ return sign(cross(b - a, c - a)); }
bool collinearity(pdd p1, pdd p2, pdd p3)
{ return sign(cross(p1 - p3, p2 - p3)) == 0; }
bool btw(pdd p1, pdd p2, pdd p3) {
  if (!collinearity(p1, p2, p3)) return 0;
  return sign(dot(p1 - p3, p2 - p3)) <= 0;
}
bool seg_intersect(pdd p1, pdd p2, pdd p3, pdd p4) {
  int a123 = ori(p1, p2, p3);
  int a124 = ori(p1, p2, p4);
  int a341 = ori(p3, p4, p1);
  int a342 = ori(p3, p4, p2);
  if (a123 == 0 && a124 == 0)
    return btw(p1, p2, p3) || btw(p1, p2, p4) ||
      btw(p3, p4, p1) || btw(p3, p4, p2);
  return a123 * a124 <= 0 && a341 * a342 <= 0;
}
pdd intersect(pdd p1, pdd p2, pdd p3, pdd p4) {
  double a123 = cross(p2 - p1, p3 - p1);
  double a124 = cross(p2 - p1, p4 - p1);
  return (p4
      * a123 - p3 * a124) / (a123 - a124); // C^3 / C^2
}
pdd perp(pdd p1)
{ return pdd(-p1.Y, p1.X); }
pdd projection(pdd p1, pdd p2, pdd p3)
{ return p1 + (
    p2 - p1) * dot(p3 - p1, p2 - p1) / abs2(p2 - p1); }
pdd reflection(pdd p1, pdd p2, pdd p3)
{ return p3 + perp(p2 - p1
    ) * cross(p3 - p1, p2 - p1) / abs2(p2 - p1) * 2; }
pdd linearTransformation
    (pdd p0, pdd p1, pdd q0, pdd q1, pdd r) {
  pdd dp = p1 - p0
      , dq = q1 - q0, num(cross(dp, dq), dot(dp, dq));
  return q0 + pdd(
      cross(r - p0, num), dot(r - p0, num)) / abs2(dp);
} // from line p0--p1 to q0--q1, apply to r
```

8.2 ConvexHull

```
  sort(dots.begin(), dots.end());
  vector<pll> ans(1, dots[0]);
  for (int ct = 0; ct < 2; ++ct, reverse(ALL(dots)))
    for (int i = 1,
         t = SZ(ans); i < SZ(dots); ans.pb(dots[i++]))
      while (SZ(ans) > t && ori
           (ans[SZ(ans) - 2], ans.back(), dots[i]) <= 0)
        ans.pop_back();
  ans.pop_back(), ans.swap(dots);
}
```

8.3 SortByAngle

```
#define is_neg(k) (
    sign(k.Y) < 0 || (sign(k.Y) == 0 && sign(k.X) < 0))
  int A = is_neg(a), B = is_neg(b);
  if (A != B)
    return A < B;
  if (sign(cross(a, b)) == 0)
    return same ? abs2(a) < abs2(b) : -1;
  return sign(cross(a, b)) > 0;
}
```

8.4 DisPointSegment

```
  if (sign(abs(q0 - q1)) == 0) return abs(q0 - p);
  if (sign(dot(q1 - q0,
      p - q0)) >= 0 && sign(dot(q0 - q1, p - q1)) >= 0)
    return fabs(cross(q1 - q0, p - q0) / abs(q0 - q1));
  return min(abs(p - q0), abs(p - q1));
}
```

8.5 PointInCircle

```
bool in_cc(const array<pll, 3> &p, pll q) {
  __int128 det = 0;
  for (int i = 0; i < 3; ++i)
    det += __int128(abs2(p[i]) - abs2(q)) *
         cross(p[(i + 1) % 3] - q, p[(i + 2) % 3] - q);
  return det > 0; // in: >0, on: =0, out: <0
}
```

8.6 PointInConvex

```
  int a = 1, b = SZ(C) - 1, r = !strict;
  if (SZ(C) == 0) return false;
  if (SZ(C) < 3) return r && btw(C[0], C.back(), p);
  if (ori(C[0], C[a], C[b]) > 0) swap(a, b);
  if (ori
      (C[0], C[a], p) >= r || ori(C[0], C[b], p) <= -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (ori(C[0], C[c], p) > 0 ? b : a) = c;
  }
  return ori(C[a], C[b], p) < r;
}
```

8.7 PointTangentConvex

```
  return arbitrary point on the tangent line */
pii get_tangent(vector<pll> &C, pll p) {
  auto gao = [&](int s) {
    return cyc_tsearch(SZ(C), [&](int x, int y)
    { return ori(p, C[x], C[y]) == s; });
  };
  return pii(gao(1), gao(-1));
} // return (a, b), ori(p, C[a], C[b]) >= 0
```

8.8 CircTangentCirc

```
  // sign1 = 1 for outer tang, -1 for inter tang
  vector<Line> ret;
  double d_sq = abs2(c1.O - c2.O);
  if (sign(d_sq) == 0) return ret;
  double d = sqrt(d_sq);
  pdd v = (c2.O - c1.O) / d;
  double c = (c1.R - sign1 * c2.R) / d;
  if (c * c > 1) return ret;
  double h = sqrt(max(0.0, 1.0 - c * c));
  for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
    pdd n = pdd(v.X * c - sign2 * h * v.Y,
      v.Y * c + sign2 * h * v.X);
    pdd p1 = c1.O + n * c1.R;
    pdd p2 = c2.O + n * (c2.R * sign1);
```

```
    if (sign(p1.X - p2.X) == 0 and
        sign(p1.Y - p2.Y) == 0)
      p2 = p1 + perp(c2.O - c1.O);
    ret.pb(Line(p1, p2));
  }
  return ret;
}
```

8.9 LineConvexIntersect

```
  return cyc_tsearch(SZ(C), [&](int a, int b) {
    return cross(dir, C[a]) > cross(dir, C[b]);
  });
}
#define cmpL(i) sign(cross(C[i] - a, b - a))
pii lineHull(pll a, pll b, vector<pll> &C) {
  int A = TangentDir(C, a - b);
  int B = TangentDir(C, b - a);
  int n = SZ(C);
  if (cmpL(A) < 0 || cmpL(B) > 0)
    return pii(-1, -1); // no collision
  auto gao = [&](int l, int r) {
    for (int t = l; (l + 1) % n != r; ) {
      int m = ((l + r + (l < r ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(t) ? l : r) = m;
    }
    return (l + !cmpL(r)) % n;
  };
  pii res = pii(gao(B, A), gao(A, B)); // (i, j)
  if (res.X == res.Y) // touching the corner i
    return pii(res.X, -1);
  if (!
      cmpL(res.X) && !cmpL(res.Y)) // along side i, i+1
    switch ((res.X - res.Y + n + 1) % n) {
      case 0: return pii(res.X, res.X);
      case 2: return pii(res.Y, res.Y);
    }
  /* crossing sides (i, i+1) and (j, j+1)
  crossing corner i is treated as side (i, i+1)
  returned
       in the same order as the line hits the convex */
  return res;
} // convex cut: (r, l]
```

8.10 CircIntersectCirc

```
  pdd o1 = a.O, o2 = b.O;
  double r1 =
       a.R, r2 = b.R, d2 = abs2(o1 - o2), d = sqrt(d2);
  if(d < max
      (r1, r2) - min(r1, r2) || d > r1 + r2) return 0;
  pdd u = (o1 + o2) * 0.5
       + (o1 - o2) * ((r2 * r2 - r1 * r1) / (2 * d2));
  double A = sqrt((r1 + r2 + d) *
       (r1 - r2 + d) * (r1 + r2 - d) * (-r1 + r2 + d));
  pdd v
       = pdd(o1.Y - o2.Y, -o1.X + o2.X) * A / (2 * d2);
  p1 = u + v, p2 = u - v;
  return 1;
}
```

8.11 PolyIntersectCirc

```
const double PI=acos(-1);
double _area(pdd pa, pdd pb, double r){
  if(abs(pa)<abs(pb)) swap(pa, pb);
  if(abs(pb)<eps) return 0;
  double S, h, theta;
  double a=abs(pb),b=abs(pa),c=abs(pb-pa);
  double cosB = dot(pb,pb-pa) / a / c, B = acos(cosB);
  double cosC = dot(pa,pb) / a / b, C = acos(cosC);
  if(a > r){
    S = (C/2)*r*r;
    h = a*b*sin(C)/c;
    if (h < r && B
        < PI/2) S -= (acos(h/r)*r*r - h*sqrt(r*r-h*h));
  }
  else if(b > r){
    theta = PI - B - asin(sin(B)/r*a);
    S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
  }
  else S = .5*sin(C)*a*b;
  return S;
}
double area_poly_circle(const
     vector<pdd> poly,const pdd &O,const double r){
  double S=0;
  for(int i=0;i<SZ(poly);++i)
    S+=_area(poly[i]-O,poly[(i+1)%SZ(poly
        )]-O,r)*ori(O,poly[i],poly[(i+1)%SZ(poly)]);
  return fabs(S);
}
```

8.12 MinMaxEnclosingRect

```
pdd solve(vector<pll> &dots) {
#define diff(u, v) (dots[u] - dots[v])
#define vec(v) (dots[v] - dots[i])
  hull(dots);
  double Max = 0, Min = INF, deg;
  int n = SZ(dots);
  dots.pb(dots[0]);
  for (int i = 0, u = 1, r = 1, l = 1; i < n; ++i) {
    pll nw = vec(i + 1);
    while (cross(nw, vec(u + 1)) > cross(nw, vec(u)))
      u = (u + 1) % n;
    while (dot(nw, vec(r + 1)) > dot(nw, vec(r)))
      r = (r + 1) % n;
    if (!i) l = (r + 1) % n;
    while (dot(nw, vec(l + 1)) < dot(nw, vec(l)))
      l = (l + 1) % n;
    Min = min(Min, (double)(dot(nw, vec(r)) - dot
        (nw, vec(l))) * cross(nw, vec(u)) / abs2(nw));
    deg = acos(dot(diff(r
        , l), vec(u)) / abs(diff(r, l)) / abs(vec(u)));
    deg = (qi - deg) / 2;
    Max = max(Max, abs(diff
        (r, l)) * abs(vec(u)) * sin(deg) * sin(deg));
  }
  return pdd(Min, Max);
}
```

8.13 MinEnclosingCircle

```
  pdd cent;
  random_shuffle(ALL(dots));
  cent = dots[0], r = 0;
  for (int i = 1; i < SZ(dots); ++i)
    if (abs(dots[i] - cent) > r) {
      cent = dots[i], r = 0;
      for (int j = 0; j < i; ++j)
        if (abs(dots[j] - cent) > r) {
          cent = (dots[i] + dots[j]) / 2;
          r = abs(dots[i] - cent);
          for(int k = 0; k < j; ++k)
            if(abs(dots[k] - cent) > r)
              cent = excenter
                  (dots[i], dots[j], dots[k], r);
        }
    }
  return cent;
}
```

8.14 CircleCover

```
struct CircleCover {
  int C;
  Cir c[N];
  bool g[N][N], overlap[N][N];
  // Area[i] : area covered by at least i circles
  double Area[ N ];
  void init(int _C){ C = _C;}
  struct Teve {
    pdd p; double ang; int add;
    Teve() {}
    Teve(pdd _a
        , double _b, int _c):p(_a), ang(_b), add(_c){}
    bool operator<(const Teve &a)const
    {return ang < a.ang;}
  }eve[N * 2];
  // strict: x = 0, otherwise x = -1
  bool disjuct(Cir &a, Cir &b, int x)
  {return sign(abs(a.O - b.O) - a.R - b.R) > x;}
  bool contain(Cir &a, Cir &b, int x)
  {return sign(a.R - b.R - abs(a.O - b.O)) > x;}
  bool contain(int i, int j) {
    /* c[j] is non-strictly in c[i]. */
    return (sign
        (c[i].R - c[j].R) > 0 || (sign(c[i].R - c[j].
        R) == 0 && i < j)) && contain(c[i], c[j], -1);
  }
  void solve(){
```

```
    fill_n(Area, C + 2, 0);
    for(int i = 0; i < C; ++i)
      for(int j = 0; j < C; ++j)
        overlap[i][j] = contain(i, j);
    for(int i = 0; i < C; ++i)
      for(int j = 0; j < C; ++j)
        g[i][j] = !(overlap[i][j] || overlap[j][i] ||
            disjuct(c[i], c[j], -1));
    for(int i = 0; i < C; ++i){
      int E = 0, cnt = 1;
      for(int j = 0; j < C; ++j)
        if(j != i && overlap[j][i])
          ++cnt;
      for(int j = 0; j < C; ++j)
        if(i != j && g[i][j]) {
          pdd aa, bb;
          CCinter(c[i], c[j], aa, bb);
          double A =
              atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
          double B =
              atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
          eve[E++] = Teve
              (bb, B, 1), eve[E++] = Teve(aa, A, -1);
          if(B > A) ++cnt;
        }
      if(E == 0) Area[cnt] += pi * c[i].R * c[i].R;
      else{
        sort(eve, eve + E);
        eve[E] = eve[0];
        for(int j = 0; j < E; ++j){
          cnt += eve[j].add;
          Area[cnt
              ] += cross(eve[j].p, eve[j + 1].p) * .5;
          double theta = eve[j + 1].ang - eve[j].ang;
          if (theta < 0) theta += 2. * pi;
          Area[cnt] += (theta
              - sin(theta)) * c[i].R * c[i].R * .5;
        }
      }
    }
  }
};
```

8.15 Trapezoidalization

```
struct SweepLine {
  struct cmp {
    cmp(const SweepLine &_swp): swp(_swp) {}
    bool operator()(int a, int b) const {
      if (abs(swp.get_y(a) - swp.get_y(b)) <= swp.eps)
        return swp.slope_cmp(a, b);
      return swp.get_y(a) + swp.eps < swp.get_y(b);
    }
    const SweepLine &swp;
  } _cmp;
  T curTime, eps, curQ;
  vector<Line> base;
  multiset<int, cmp> sweep;
  multiset<pair<T, int>> event;
  vector<typename multiset<int, cmp>::iterator> its;
  vector
      <typename multiset<pair<T, int>>::iterator> eits;
  bool slope_cmp(int a, int b) const {
    assert(a != -1);
    if (b == -1) return 0;
    return sign(cross(base
        [a].Y - base[a].X, base[b].Y - base[b].X)) < 0;
  }
  T get_y(int idx) const {
    if (idx == -1) return curQ;
    Line l = base[idx];
    if (l.X.X == l.Y.X) return l.Y.Y;
    return ((curTime - l.X.X) * l.Y.Y
        + (l.Y.X - curTime) * l.X.Y) / (l.Y.X - l.X.X);
  }
  void insert(int idx) {
    its[idx] = sweep.insert(idx);
    if (its[idx] != sweep.begin())
      update_event(*prev(its[idx]));
    update_event(idx);
    event.emplace(base[idx].Y.X, idx + 2 * SZ(base));
  }
  void erase(int idx) {
    assert(eits[idx] == event.end());
    auto p = sweep.erase(its[idx]);
    its[idx] = sweep.end();
    if (p != sweep.begin())
      update_event(*prev(p));
  }
  void update_event(int idx) {
    if (eits[idx] != event.end())
      event.erase(eits[idx]);
    eits[idx] = event.end();
    auto nxt = next(its[idx]);
    if (nxt ==
        sweep.end() || !slope_cmp(idx, *nxt)) return;
    auto t = intersect(base[idx].
        X, base[idx].Y, base[*nxt].X, base[*nxt].Y).X;
    if (t + eps < curTime || t
        >= min(base[idx].Y.X, base[*nxt].Y.X)) return;
    eits[idx] = event.emplace(t, idx + SZ(base));
  }
  void swp(int idx) {
    assert(eits[idx] != event.end());
    eits[idx] = event.end();
    int nxt = *next(its[idx]);
    swap((int&)*its[idx], (int&)*its[nxt]);
    swap(its[idx], its[nxt]);
    if (its[nxt] != sweep.begin())
      update_event(*prev(its[nxt]));
    update_event(idx);
  }
  // only expected to call the functions below
  SweepLine(T t, T e, vector
      <Line> vec): _cmp(*this), curTime(t), eps(e)
      , curQ(), base(vec), sweep(_cmp), event(), its(SZ
      (vec), sweep.end()), eits(SZ(vec), event.end()) {
    for (int i = 0; i < SZ(base); ++i) {
      auto &[p, q] = base[i];
      if (p > q) swap(p, q);
      if (p.X <= curTime && curTime <= q.X)
        insert(i);
      else if (curTime < p.X)
        event.emplace(p.X, i);
    }
  }
  void setTime(T t, bool ers = false) {
    assert(t >= curTime);
    while (!event.empty() && event.begin()->X <= t) {
      auto [et, idx] = *event.begin();
      int s = idx / SZ(base);
      idx %= SZ(base);
      if (abs(et - t) <= eps && s == 2 && !ers) break;
      curTime = et;
      event.erase(event.begin());
      if (s == 2) erase(idx);
      else if (s == 1) swp(idx);
      else insert(idx);
    }
    curTime = t;
  }
  T nextEvent() {
    if (event.empty()) return INF;
    return event.begin()->X;
  }
  int lower_bound(T y) {
    curQ = y;
    auto p = sweep.lower_bound(-1);
    if (p == sweep.end()) return -1;
    return *p;
  }
};
```

8.16 TriangleHearts

```
  p1 = p1 - p0, p2 = p2 - p0;
  double x1 = p1.X, y1 = p1.Y, x2 = p2.X, y2 = p2.Y;
  double m = 2. * (x1 * y2 - y1 * x2);
  center.X = (x1 * x1
      * y2 - x2 * x2 * y1 + y1 * y2 * (y1 - y2)) / m;
  center.Y = (x1 * x2
      * (x2 - x1) - y1 * y1 * x2 + x1 * y2 * y2) / m;
  return center + p0;
}
pdd incenter
    (pdd p1, pdd p2, pdd p3) { // radius = area / s * 2
  double a =
      abs(p2 - p3), b = abs(p1 - p3), c = abs(p1 - p2);
  double s = a + b + c;
  return (a * p1 + b * p2 + c * p3) / s;
}
pdd masscenter(pdd p1, pdd p2, pdd p3)
```

```
{ return (p1 + p2 + p3) / 3; }
pdd orthcenter(pdd p1, pdd p2, pdd p3)
{ return masscenter
    (p1, p2, p3) * 3 - circenter(p1, p2, p3) * 2; }
```

8.17 HalfPlaneIntersect

```
{ return pll(cross(a.Y
     - a.X, b.X - a.X), cross(a.Y - a.X, b.Y - a.X)); }
bool isin(Line l0, Line l1, Line l2) {
  // Check inter(l1, l2) strictly in l0
  auto [a02X, a02Y] = area_pair(l0, l2);
  auto [a12X, a12Y] = area_pair(l1, l2);
  if (a12X - a12Y < 0) a12X *= -1, a12Y *= -1;
  return (__int128)
       a02Y * a12X - (__int128) a02X * a12Y > 0; // C^4
}
/* Having solution, check size > 2 */
/* --^-- Line.X --^-- Line.Y --^-- */
vector<Line> halfPlaneInter(vector<Line> arr) {
  sort(ALL(arr), [&](Line a, Line b) -> int {
    if (cmp(a.Y - a.X, b.Y - b.X, 0) != -1)
      return cmp(a.Y - a.X, b.Y - b.X, 0);
    return ori(a.X, a.Y, b.Y) < 0;
  });
  deque<Line> dq(1, arr[0]);
  for (auto p : arr) {
    if (cmp(
        dq.back().Y - dq.back().X, p.Y - p.X, 0) == -1)
      continue;
    while (SZ(dq
        ) >= 2 && !isin(p, dq[SZ(dq) - 2], dq.back()))
      dq.pop_back();
    while (SZ(dq) >= 2 && !isin(p, dq[0], dq[1]))
      dq.pop_front();
    dq.pb(p);
  }
  while (SZ(dq)
       >= 3 && !isin(dq[0], dq[SZ(dq) - 2], dq.back()))
    dq.pop_back();
  while (SZ(dq) >= 3 && !isin(dq.back(), dq[0], dq[1]))
    dq.pop_front();
  return vector<Line>(ALL(dq));
}
```

8.18 RotatingSweepLine

```
  int n = SZ(ps), m = 0;
  vector<int> id(n), pos(n);
  vector<pii> line(n * (n - 1));
  for (int i = 0; i < n; ++i)
    for (int j = 0; j < n; ++j)
      if (i != j) line[m++] = pii(i, j);
  sort(ALL(line), [&](pii a, pii b) {
    return cmp(ps[a.Y] - ps[a.X], ps[b.Y] - ps[b.X]);
  }); // cmp(): polar angle compare
  iota(ALL(id), 0);
  sort(ALL(id), [&](int a, int b) {
    if (ps[a].Y != ps[b].Y) return ps[a].Y < ps[b].Y;
    return ps[a] < ps[b];
  }); // initial order, since (1, 0) is the smallest
  for (int i = 0; i < n; ++i) pos[id[i]] = i;
  for (int i = 0; i < m; ++i) {
    auto l = line[i];
    // do something
    tie(pos[l.X], pos[l.Y], id[pos[l.X]], id[pos[l.Y
        ]]) = make_tuple(pos[l.Y], pos[l.X], l.Y, l.X);
  }
}
```

8.19 DelaunayTriangulation

```
Given a sets of points on 2D plane, find a
triangulation such that no points will strictly
inside circumcircle of any triangle.
find : return a triangle contain given point
add_point : add a point into triangulation
A Triangle is in triangulation iff. its has_chd is 0.
Region of triangle u: iterate each u.edge[i].tri,
each points are u.p[(i+1)%3], u.p[(i+2)%3]
Voronoi diagram: for each triangle in triangulation,
the bisector of all its edges will split the region.
nearest point will belong to the triangle containing it
 */
const
     ll inf = MAXC * MAXC * 100; // lower_bound unknown
struct Tri;
struct Edge {
  Tri* tri; int side;
  Edge(): tri(0), side(0){}
  Edge(Tri* _tri, int _side): tri(_tri), side(_side){}
};
struct Tri {
  pll p[3];
  Edge edge[3];
  Tri* chd[3];
  Tri() {}
  Tri(const pll& p0, const pll& p1, const pll& p2) {
    p[0] = p0; p[1] = p1; p[2] = p2;
    chd[0] = chd[1] = chd[2] = 0;
  }
  bool has_chd() const { return chd[0] != 0; }
  int num_chd() const {
    return !!chd[0] + !!chd[1] + !!chd[2];
  }
  bool contains(pll const& q) const {
    for (int i = 0; i < 3; ++i)
      if (ori(p[i], p[(i + 1) % 3], q) < 0)
        return 0;
    return 1;
  }
} pool[N * 10], *tris;
void edge(Edge a, Edge b) {
  if(a.tri) a.tri->edge[a.side] = b;
  if(b.tri) b.tri->edge[b.side] = a;
}
struct Trig { // Triangulation
  Trig() {
    the_root
        = // Tri should at least contain all points
      new(tris++) Tri(pll(-inf, -inf),
          pll(inf + inf, -inf), pll(-inf, inf + inf));
  }
  Tri* find(pll p) { return find(the_root, p); }
  void add_point(const
       pll &p) { add_point(find(the_root, p), p); }
  Tri* the_root;
  static Tri* find(Tri* root, const pll &p) {
    while (1) {
      if (!root->has_chd())
        return root;
      for (int i = 0; i < 3 && root->chd[i]; ++i)
        if (root->chd[i]->contains(p)) {
          root = root->chd[i];
          break;
        }
    }
    assert(0); // "point not found"
  }
  void add_point(Tri* root, pll const& p) {
    Tri* t[3];
    /* split it into three triangles */
    for (int i = 0; i < 3; ++i)
      t[i] = new(tris
          ++) Tri(root->p[i], root->p[(i + 1) % 3], p);
    for (int i = 0; i < 3; ++i)
      edge(Edge(t[i], 0), Edge(t[(i + 1) % 3], 1));
    for (int i = 0; i < 3; ++i)
      edge(Edge(t[i], 2), root->edge[(i + 2) % 3]);
    for (int i = 0; i < 3; ++i)
      root->chd[i] = t[i];
    for (int i = 0; i < 3; ++i)
      flip(t[i], 2);
  }
  void flip(Tri* tri, int pi) {
    Tri* trj = tri->edge[pi].tri;
    int pj = tri->edge[pi].side;
    if (!trj) return;
    if (!in_cc(tri->p
        [0], tri->p[1], tri->p[2], trj->p[pj])) return;
    /* flip edge between tri,trj */
    Tri* trk = new(tris++) Tri
        (tri->p[(pi + 1) % 3], trj->p[pj], tri->p[pi]);
    Tri* trl = new(tris++) Tri
        (trj->p[(pj + 1) % 3], tri->p[pi], trj->p[pj]);
    edge(Edge(trk, 0), Edge(trl, 0));
    edge(Edge(trk, 1), tri->edge[(pi + 2) % 3]);
    edge(Edge(trk, 2), trj->edge[(pj + 1) % 3]);
    edge(Edge(trl, 1), trj->edge[(pj + 2) % 3]);
    edge(Edge(trl, 2), tri->edge[(pi + 1) % 3]);
    tri->chd
        [0] = trk; tri->chd[1] = trl; tri->chd[2] = 0;
```

```
    trj->chd
        [0] = trk; trj->chd[1] = trl; trj->chd[2] = 0;
    flip(trk, 1); flip(trk, 2);
    flip(trl, 1); flip(trl, 2);
  }
};
vector<Tri*> triang; // vector of all triangle
set<Tri*> vst;
void go(Tri* now) { // store all tri into triang
  if (vst.find(now) != vst.end())
    return;
  vst.insert(now);
  if (!now->has_chd())
    return triang.pb(now);
  for (int i = 0; i < now->num_chd(); ++i)
    go(now->chd[i]);
}
void build(int n, pll* ps) { // build triangulation
  tris = pool; triang.clear(); vst.clear();
  random_shuffle(ps, ps + n);
  Trig tri; // the triangulation structure
  for (int i = 0; i < n; ++i)
    tri.add_point(ps[i]);
  go(tri.the_root);
}
```

8.20 VonoroiDiagram

```
vector<vector<Line>> vec;
void build_voronoi_line(int n, pll *arr) {
  tool.init(n, arr); // Delaunay
  vec.clear(), vec.resize(n);
  for (int i = 0; i < n; ++i)
    for (auto e : tool.head[i]) {
      int u = tool.oidx[i], v = tool.oidx[e.id];
      pll m = (arr[v
          ] + arr[u]) / 2LL, d = perp(arr[v] - arr[u]);
      vec[u].pb(Line(m, m + d));
    }
}
```

8.21 3DConeDistance

```
const double eps = 1e-12, pi = acos(-1);
struct point {
  ld x, y, z;
};
ld r, h;

bool on_edge(point p) {
  return abs(p.z) < eps &&
    abs(p.x * p.x + p.y * p.y - r * r) < eps;
}

bool on_bottom(point p) { return abs(p.z) < eps; }

ld straight(point a, point b) {
  return sqrtl((a.x - b.x) * (a.x - b.x) +
    (a.y - b.y) * (a.y - b.y) +
    (a.z - b.z) * (a.z - b.z));
}

ld arc(point a, point b) {
  if (abs(a.x) < eps && abs(a.y) < eps &&
    abs(a.z - h) < eps)
    return straight(a, b);
  swap(a, b);
  if (abs(a.x) < eps && abs(a.y) < eps &&
    abs(a.z - h) < eps)
    return straight(a, b);
  swap(a, b);

  ld ans = 1e18;

  for (int t = 0; t <= 1; t++) {
    double alpha = (a.x < 0 ? pi + atan2l(-a.y, -a.x)
                            : atan2l(a.y, a.x));
    if (alpha < 0) alpha += 2 * pi;
    alpha *= r / sqrtl(h * h + r * r);
    double lA = sqrtl(
      a.x * a.x + a.y * a.y + (a.z - h) * (a.z - h));

    double beta = (b.x < 0 ? pi + atan2l(-b.y, -b.x)
                           : atan2l(b.y, b.x));
    if (beta < 0) beta += 2 * pi;
    beta *= r / sqrtl(h * h + r * r);
    double lB = sqrtl(
      b.x * b.x + b.y * b.y + (b.z - h) * (b.z - h));

    double Ax = lA * cosl(alpha),
           Ay = lA * sinl(alpha), Bx = lB * cosl(beta),
           By = lB * sinl(beta);
    ans = min(ans,
      sqrtl((Ax - Bx) * (Ax - Bx) +
        (Ay - By) * (Ay - By)));
    a.x = -a.x;
    b.x = -b.x;
  }
  return ans;
}

ld distance(point a, point b) {
  if (on_bottom(a) && !on_edge(a) && !on_bottom(b))
    return 1e18;
  if (on_bottom(b) && !on_edge(b) && !on_bottom(a))
    return 1e18;

  if (on_bottom(a) && on_bottom(b))
    return straight(a, b);
  return arc(a, b);
}

const int s = 90;
ld ans = 1e18, theta = pi / 180.0;
point a, b;
vector<point> p;
vector<ld> ar;
vector<ld> dis, from, center = {0.0, pi};
vector<int> vis;

void iterate() {

  p = {a, b};
  ar = {-1e9, -1e9};
  dis.clear();
  from.clear();
  vis.clear();

  for (auto d : center)
    for (int i = -s; i <= s; i++) {
      p.emplace_back(point{r * cosl(d + i * theta),
        r * sinl(d + i * theta), 0});
      ar.emplace_back(d + i * theta);
    }
  int n = p.size();
  dis.resize(n);
  from.resize(n);
  vis.resize(n);

  for (int i = 1; i < n; i++) dis[i] = 1e18;

  for (int t = 0; t < n; t++) {
    int u = 0;
    ld d = 1e18;
    for (int i = 0; i < n; i++)
      if (!vis[i] && dis[i] < d) u = i, d = dis[i];
    vis[u] = 1;
    for (int v = 0; v < n; v++)
      if (d + distance(p[u], p[v]) < dis[v]) {
        from[v] = u;
        dis[v] = distance(p[u], p[v]) + d;
      }
  }
  ans = min(ans, dis[1]);
  center.clear();
  if (from[1] > 1) center.emplace_back(ar[from[1]]);
  if (from[from[1]] > 1)
    center.emplace_back(ar[from[from[1]]]);
  theta /= 20.0;
}

double cone_distance(
  double r, double h, point a, point b) {
  if (on_bottom(a) && on_bottom(b))
    return straight(a, b);
  else {
    for (int t = 1; t <= 10; t++) iterate();
    return ans;
  }
}
```

9 Misc

9.1 MoAlgoWithModify

```
Mo's Algorithm With modification
Block: N^{2/3}, Complexity: N^{5/3}
*/
struct Query {
  int L, R, LBid, RBid, T;
  Query(int l, int r, int t):
    L(l), R(r), LBid(l / blk), RBid(r / blk), T(t) {}
  bool operator<(const Query &q) const {
    if (LBid != q.LBid) return LBid < q.LBid;
    if (RBid != q.RBid) return RBid < q.RBid;
    return T < b.T;
  }
};
void solve(vector<Query> query) {
  sort(ALL(query));
  int L=0, R=0, T=-1;
  for (auto q : query) {
    while (T < q.T) addTime(L, R, ++T); // TODO
    while (T > q.T) subTime(L, R, T--); // TODO
    while (R < q.R) add(arr[++R]); // TODO
    while (L > q.L) add(arr[--L]); // TODO
    while (R > q.R) sub(arr[R--]); // TODO
    while (L < q.L) sub(arr[L++]); // TODO
    // answer query
  }
}
```

9.2 MoAlgoOnTree

```
Mo's Algorithm On Tree
Preprocess:
1) LCA
2) dfs with in[u] = dft++, out[u] = dft++
3) ord[in[u]] = ord[out[u]] = u
4) bitset<MAXN> inset
*/
struct Query {
  int L, R, LBid, lca;
  Query(int u, int v) {
    int c = LCA(u, v);
    if (c == u || c == v)
      q.lca = -1, q.L = out[c ^ u ^ v], q.R = out[c];
    else if (out[u] < in[v])
      q.lca = c, q.L = out[u], q.R = in[v];
    else
      q.lca = c, q.L = out[v], q.R = in[u];
    q.Lid = q.L / blk;
  }
  bool operator<(const Query &q) const {
    if (LBid != q.LBid) return LBid < q.LBid;
    return R < q.R;
  }
};
void flip(int x) {
    if (inset[x]) sub(arr[x]); // TODO
    else add(arr[x]); // TODO
    inset[x] = ~inset[x];
}
void solve(vector<Query> query) {
  sort(ALL(query));
  int L = 0, R = 0;
  for (auto q : query) {
    while (R < q.R) flip(ord[++R]);
    while (L > q.L) flip(ord[--L]);
    while (R > q.R) flip(ord[R--]);
    while (L < q.L) flip(ord[L++]);
    if (~q.lca) add(arr[q.lca]);
    // answer query
    if (~q.lca) sub(arr[q.lca]);
  }
}
```

9.3 MoAlgoAdvanced

- Mo's Algorithm With Addition Only
  - Sort querys same as the normal Mo's algorithm.
  - For each query $[l,r]$:
  - If $l/blk = r/blk$, brute-force.
  - If $l/blk \neq curL/blk$, initialize $curL := (l/blk+1) \cdot blk, curR := curL - 1$
  - If $r > curR$, increase $curR$
  - decrease $curL$ to fit l, and then undo after answering
- Mo's Algorithm With Offline Second Time
  - Require: Changing answer $\equiv$ adding $f([l,r],r+1)$.
  - Require: $f([l,r],r+1) = f([1,r],r+1) - f([1,l),r+1)$.
  - Part1: Answer all $f([1,r],r+1)$ first.
  - Part2: Store $curR \to R$ for $curL$ (reduce the space to $O(N)$), and then answer them by the second offline algorithm.
  - Note: You must do the above symmetrically for the left boundaries.

9.4 HilbertCurve

```
  ll res = 0;
  for (int s = n / 2; s; s >>= 1) {
    int rx = (x & s) > 0;
    int ry = (y & s) > 0;
    res += s * 1ll * s * ((3 * rx) ^ ry);
    if (ry == 0) {
      if (rx == 1) x = s - 1 - x, y = s - 1 - y;
      swap(x, y);
    }
  }
  return res;
} // n = 2^k
```

9.5 SternBrocotTree

- Construction: Root $\frac{1}{1}$, left/right neighbor $\frac{0}{1}, \frac{1}{0}$, each node is sum of last left/right neighbor: $\frac{a}{b}, \frac{c}{d} \to \frac{a+c}{b+d}$
- Property: Adjacent (mid-order DFS) $\frac{a}{b}, \frac{c}{d} \Rightarrow bc - ad = 1$.
- Search known $\frac{p}{q}$: keep L-R alternative. Each step can calcaulated in $O(1) \Rightarrow$ total $O(\log C)$.
- Search unknown $\frac{p}{q}$: keep L-R alternative. Each step can calcaulated in $O(\log C)$ checks $\Rightarrow$ total $O(\log^2 C)$ checks.

9.6 CyclicLCS

```
#define LU 1
#define U 2
const int mov[3][2] = {0, -1, -1, -1, -1, 0};
int al, bl;
char a[MAXL * 2], b[MAXL * 2]; // 0-indexed
int dp[MAXL * 2][MAXL];
char pred[MAXL * 2][MAXL];
inline int lcs_length(int r) {
  int i = r + al, j = bl, l = 0;
  while (i > r) {
    char dir = pred[i][j];
    if (dir == LU) l++;
    i += mov[dir][0];
    j += mov[dir][1];
  }
  return l;
}
inline void reroot(int r) { // r = new base row
  int i = r, j = 1;
  while (j <= bl && pred[i][j] != LU) j++;
  if (j > bl) return;
  pred[i][j] = L;
  while (i < 2 * al && j <= bl) {
    if (pred[i + 1][j] == U) {
      i++;
      pred[i][j] = L;
    } else if (j < bl && pred[i + 1][j + 1] == LU) {
      i++;
      j++;
      pred[i][j] = L;
    } else {
      j++;
    }
  }
}
int cyclic_lcs() {
  // a, b, al, bl should be properly filled
  // note: a WILL be altered in process
  //         -- concatenated after itself
  char tmp[MAXL];
  if (al > bl) {
    swap(al, bl);
    strcpy(tmp, a);
    strcpy(a, b);
    strcpy(b, tmp);
  }
  strcpy(tmp, a);
  strcat(a, tmp);
  // basic lcs
  for (int i = 0; i <= 2 * al; i++) {
    dp[i][0] = 0;
    pred[i][0] = U;
```

```
  }
  for (int j = 0; j <= bl; j++) {
    dp[0][j] = 0;
    pred[0][j] = L;
  }
  for (int i = 1; i <= 2 * al; i++) {
    for (int j = 1; j <= bl; j++) {
      if (a[i - 1] == b[j - 1])
        dp[i][j] = dp[i - 1][j - 1] + 1;
      else dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
      if (dp[i][j - 1] == dp[i][j]) pred[i][j] = L;
      else if (a[i - 1] == b[j - 1]) pred[i][j] = LU;
      else pred[i][j] = U;
    }
  }
  // do cyclic lcs
  int clcs = 0;
  for (int i = 0; i < al; i++) {
    clcs = max(clcs, lcs_length(i));
    reroot(i + 1);
  }
  // recover a
  a[al] = '\0';
  return clcs;
}
```

9.7 AllLCS

```
  vector<int> h(SZ(t));
  iota(ALL(h), 0);
  for (int a = 0; a < SZ(s); ++a) {
    int v = -1;
    for (int c = 0; c < SZ(t); ++c)
      if (s[a] == t[c] || h[c] < v)
        swap(h[c], v);
    // LCS(s[0, a], t[b, c]) =
    // c - b + 1 - sum([h[i] >= b] | i <= c)
    // h[i] might become -1 !!
  }
}
```

9.8 SimulatedAnnealing

```
const int base = 1e9; // remember to run ~ 10 times
for (int it = 1; it <= 1000000; ++it) {
    // ans:
        answer, nw: current value, rnd(): mt19937 rnd()
    if (exp(-(nw - ans
        ) / factor) >= (double)(rnd() % base) / base)
        ans = nw;
    factor *= 0.99995;
}
```

9.9 Python

```
math.isqrt(2) # integer sqrt
```