Contents

1 Basic

1.1 .vimrc

```
sy on
set ru nu cin cul sc so=3 ts=4 sw=4 bs=2 ls=2 mouse=a
inoremap {<CR> {<CR>}<C-o>O
map <F7> :w<CR>:!g++
     "%" -std=c++17 -Wall -Wextra -Wshadow -Wconversion
     -fsanitize=address,undefined -g && ./a.out<CR>
```

1.2 PBDS

```
// Tree and fast PQ
#include <bits/extc++.h>
using namespace __gnu_pbds;

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
tree<int, null_type, less<int>, rb_tree_tag
    , tree_order_statistics_node_update> bst;
// order_of_key(n): # of elements <= n
// find_by_order(n): 0-indexed

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
__gnu_pbds::priority_queue
    <int, greater<int>, thin_heap_tag> pq;
```

1.3 pragma

```
#pragma GCC optimize("Ofast,unroll-loops")
#pragma GCC target("avx,avx2,sse,sse2
    ,sse3,ssse3,sse4,popcnt,abm,mmx,fma,tune=native")
// chrono
    ::steady_clock::now().time_since_epoch().count()
```

1.4 LambdaCompare

```
auto cmp = [](int x, int y){ /* return x < y; */ };
std::set<int, decltype(cmp)> st(cmp);
```

2 Graph

2.1 2SAT/SCC

```
struct SAT { // 0-base
  int low[N], dfn[N], bln[N], n, Time, nScc;
  bool instack[N], istrue[N];
  stack<int> st;
  vector<int> G[N], SCC[N];
  void init(int _n) {
    n = _n; // assert(n * 2 <= N);
    for (int i = 0; i < n + n; ++i) G[i].clear();
  }
  void add_edge(int a, int b) { G[a].emplace_back(b); }
  int rv(int a) {
    if (a >= n) return a - n;
    return a + n;
  }
  void add_clause(int a, int b) {
    add_edge(rv(a), b), add_edge(rv(b), a);
  }
  void dfs(int u) {
    dfn[u] = low[u] = ++Time;
    instack[u] = 1, st.push(u);
    for (int i : G[u])
      if (!dfn[i])
        dfs(i), low[u] = min(low[i], low[u]);
      else if (instack[i] && dfn[i] < dfn[u])
        low[u] = min(low[u], dfn[i]);
    if (low[u] == dfn[u]) {
      int tmp;
      do {
        tmp = st.top(), st.pop();
        instack[tmp] = 0, bln[tmp] = nScc;
      } while (tmp != u);
      ++nScc;
    }
  }
  bool solve() {
    Time = nScc = 0;
    for (int i = 0; i < n + n; ++i)
      SCC[i].clear(), low[i] = dfn[i] = bln[i] = 0;
    for (int i = 0; i < n + n; ++i)
      if (!dfn[i]) dfs(i);
    for (int i =
         0; i < n + n; ++i) SCC[bln[i]].emplace_back(i);
    for (int i = 0; i < n; ++i) {
      if (bln[i] == bln[i + n]) return false;
      istrue[i] = bln[i] < bln[i + n];
      istrue[i + n] = !istrue[i];
    }
    return true;
  }
};
```

2.2 BCC Vertex

```
int n, m, dfn[N], low[N], is_cut[N], nbcc = 0, t = 0;
vector<int> g[N], bcc[N], G[2 * N];
stack<int> st;
void tarjan(int p, int lp) {
  dfn[p] = low[p] = ++t;
  st.push(p);
  for (auto i : g[p]) {
    if (!dfn[i]) {
```

```
        tarjan(i, p);
        low[p] = min(low[p], low[i]);
        if (dfn[p] <= low[i]) {
          nbcc++;
          is_cut[p] = 1;
          for (int x = 0; x != i; st.pop()) {
            x = st.top();
            bcc[nbcc].push_back(x);
          }
          bcc[nbcc].push_back(p);
        }
      } else low[p] = min(low[p], dfn[i]);
    }
}
void build() { // [n+1,n+nbcc] cycle, [1,n] vertex
  for (int i = 1; i <= nbcc; i++) {
    for (auto j : bcc[i]) {
      G[i + n].push_back(j);
      G[j].push_back(i + n);
    }
  }
}
```

2.3 MinimumMeanCycle

```
/* O(V^3)
let dp[i][j] = min length from 1 to j exactly i edges
ans = min (dp[n + 1][u] - dp[i][u]) / (n + 1 - i) */
```

2.4 MaximumCliqueDyn

```
struct MaxClique { // fast when N <= 100
  bitset<N> G[N], cs[N];
  int ans, sol[N], q, cur[N], d[N], n;
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) G[i].reset();
  }
  void add_edge(int u, int v) {
    G[u][v] = G[v][u] = 1;
  }
  void pre_dfs(vector<int> &r, int l, bitset<N> mask) {
    if (l < 4) {
      for (int i : r) d[i] = (G[i] & mask).count();
      sort(all(r)
          , [&](int x, int y) { return d[x] > d[y]; });
    }
    vector<int> c(r.size());
    int lft = max(ans - q + 1, 1), rgt = 1, tp = 0;
    cs[1].reset(), cs[2].reset();
    for (int p : r) {
      int k = 1;
      while ((cs[k] & G[p]).any()) ++k;
      if (k > rgt) cs[++rgt + 1].reset();
      cs[k][p] = 1;
      if (k < lft) r[tp++] = p;
    }
    for (int k = lft; k <= rgt; ++k)
      for (int p = cs[k]._Find_first
          (); p < N; p = cs[k]._Find_next(p))
        r[tp] = p, c[tp] = k, ++tp;
    dfs(r, c, l + 1, mask);
  }
  void dfs(vector<
      int> &r, vector<int> &c, int l, bitset<N> mask) {
    while (!r.empty()) {
      int p = r.back();
      r.pop_back(), mask[p] = 0;
      if (q + c.back() <= ans) return;
      cur[q++] = p;
      vector<int> nr;
      for (int i : r) if (G[p][i]) nr.emplace_back(i);
      if (!nr.empty()) pre_dfs(nr, l, mask & G[p]);
      else if (q > ans) ans = q, copy_n(cur, q, sol);
      c.pop_back(), --q;
    }
  }
  int solve() {
    vector<int> r(n);
    ans = q = 0, iota(all(r), 0);
    pre_dfs(r, 0, bitset<N>(string(n, '1')));
    return ans;
  }
};
```

2.5 DominatorTree

```
struct DominatorTree { // 1-base
  vector<int> G[N], rG[N];
  int n, pa[N], dfn[N], id[N], Time;
  int semi[N], idom[N], best[N];
  vector<int> tree[N]; // dominator_tree
  void init(int _n) {
    n = _n;
    for (int i = 1; i <= n; ++i)
      G[i].clear(), rG[i].clear();
  }
  void add_edge(int u, int v) {
    G[u].emplace_back(v), rG[v].emplace_back(u);
  }
  void dfs(int u) {
    id[dfn[u] = ++Time] = u;
    for (auto v : G[u])
      if (!dfn[v]) dfs(v), pa[dfn[v]] = dfn[u];
  }
  int find(int y, int x) {
    if (y <= x) return y;
    int tmp = find(pa[y], x);
    if (semi[best[y]] > semi[best[pa[y]]])
      best[y] = best[pa[y]];
    return pa[y] = tmp;
  }
  void tarjan(int root) {
    Time = 0;
    for (int i = 1; i <= n; ++i) {
      dfn[i] = idom[i] = 0;
      tree[i].clear();
      best[i] = semi[i] = i;
    }
    dfs(root);
    for (int i = Time; i > 1; --i) {
      int u = id[i];
      for (auto v : rG[u])
        if (v = dfn[v]) {
          find(v, i);
          semi[i] = min(semi[i], semi[best[v]]);
        }
      tree[semi[i]].emplace_back(i);
      for (auto v : tree[pa[i]]) {
        find(v, pa[i]);
        idom[v] =
          semi[best[v]] == pa[i] ? pa[i] : best[v];
      }
      tree[pa[i]].clear();
    }
    for (int i = 2; i <= Time; ++i) {
      if (idom[i] != semi[i]) idom[i] = idom[idom[i]];
      tree[id[idom[i]]].emplace_back(id[i]);
    }
  }
};
```

2.6 DMST(slow)

```
struct DMST { // O(VE)
  struct edge {
    int u, v;
    ll w;
  };
  vector<edge> E; // 0-base
  int pe[N], id[N], vis[N];
  ll in[N];
  void init() { E.clear(); }
```

```
  void add_edge(int u, int v, ll w) {
    if (u != v) E.emplace_back(edge{u, v, w});
  }
  ll build(int root, int n) {
    ll ans = 0;
    for (;;) {
      fill_n(in, n, INF);
      for (int i = 0; i < E.size(); ++i)
        if (E[i].u != E[i].v && E[i].w < in[E[i].v])
          pe[E[i].v] = i, in[E[i].v] = E[i].w;
      for (int u = 0; u < n; ++u) // no solution
        if (u != root && in[u] == INF) return -INF;
      int cntnode = 0;
      fill_n(id, n, -1), fill_n(vis, n, -1);
      for (int u = 0; u < n; ++u) {
        if (u != root) ans += in[u];
        int v = u;
        while (vis[v] != u && !~id[v] && v != root)
          vis[v] = u, v = E[pe[v]].u;
        if (v != root && !~id[v]) {
          for (int x = E[pe[v]].u; x != v;
               x = E[pe[x]].u)
            id[x] = cntnode;
          id[v] = cntnode++;
        }
      }
      if (!cntnode) break; // no cycle
      for (int u = 0; u < n; ++u)
        if (!~id[u]) id[u] = cntnode++;
      for (int i = 0; i < E.size(); ++i) {
        int v = E[i].v;
        E[i].u = id[E[i].u], E[i].v = id[E[i].v];
        if (E[i].u != E[i].v) E[i].w -= in[v];
      }
      n = cntnode, root = id[root];
    }
    return ans;
  }
};
```

2.7 DMST

```
#define rep(i, a, b) for (int i = a; i < (b); ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
typedef vector<int> vi;
struct RollbackUF {
  vi e;
  vector<pii> st;
  RollbackUF(int n) : e(n, -1) {}
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : find(e[x]); }
  int time() { return sz(st); }
  void rollback(int t) {
    for (int i = time(); i-- > t;)
      e[st[i].first] = st[i].second;
    st.resize(t);
  }
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    st.push_back({a, e[a]});
    st.push_back({b, e[b]});
    e[a] += e[b];
    e[b] = a;
    return true;
  }
};
struct Edge {
  int a, b;
  ll w;
};
struct Node { /// lazy skew heap node
  Edge key;
  Node *l, *r;
  ll delta;
  void prop() {
    key.w += delta;
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  }
  Edge top() {
    prop();
    return key;
  }
};
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, (a->r = merge(b, a->r)));
  return a;
}
void pop(Node *&a) {
  a->prop();
  a = merge(a->l, a->r);
}

pair<ll, vi> dmst(int n, int r, vector<Edge> &g) {
  RollbackUF uf(n);
  vector<Node *> heap(n);
  for (Edge e : g)
    heap[e.b] = merge(heap[e.b], new Node{e});
  ll res = 0;
  vi seen(n, -1), path(n), par(n);
  seen[r] = r;
  vector<Edge> Q(n), in(n, {-1, -1}), comp;
  deque<tuple<int, int, vector<Edge>>> cycs;
  rep(s, 0, n) {
    int u = s, qi = 0, w;
    while (seen[u] < 0) {
      if (!heap[u]) return {-1, {}};
      Edge e = heap[u]->top();
      heap[u]->delta -= e.w, pop(heap[u]);
      Q[qi] = e, path[qi++] = u, seen[u] = s;
      res += e.w, u = uf.find(e.a);
      if (seen[u] == s) { /// found cycle, contract
        Node *cyc = 0;
        int end = qi, time = uf.time();
        do cyc = merge(cyc, heap[w = path[--qi]]);
        while (uf.join(u, w));
        u = uf.find(u), heap[u] = cyc, seen[u] = -1;
        cycs.push_front({u, time, {&Q[qi], &Q[end]}});
      }
    }
    rep(i, 0, qi) in[uf.find(Q[i].b)] = Q[i];
  }

  for (auto &[u, t, comp] :
    cycs) { // restore sol (optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto &e : comp) in[uf.find(e.b)] = e;
    in[uf.find(inEdge.b)] = inEdge;
  }
  rep(i, 0, n) par[i] = in[i].a;
  return {res, par};
}
```

2.8 VizingTheorem

```
namespace Vizing { // Edge coloring
                   // G: coloring adjM
int C[N][N], G[N][N];
void clear(int n) {
  for (int i = 0; i <= n; i++) {
    for (int j = 0; j <= n; j++) C[i][j] = G[i][j] = 0;
  }
}
void solve(vector<pii> &E, int n, int m) {
  int X[n] = {}, a;
  auto update = [&](int u) {
```

```
    for (X[u] = 1; C[u][X[u]]; X[u]++);
  };
  auto color = [&](int u, int v, int c) {
    int p = G[u][v];
    G[u][v] = G[v][u] = c;
    C[u][c] = v;
    C[v][c] = u;
    C[u][p] = C[v][p] = 0;
    if (p) X[u] = X[v] = p;
    else update(u), update(v);
    return p;
  };
  auto flip = [&](int u, int c1, int c2) {
    int p = C[u][c1];
    swap(C[u][c1], C[u][c2]);
    if (p) G[u][p] = G[p][u] = c2;
    if (!C[u][c1]) X[u] = c1;
    if (!C[u][c2]) X[u] = c2;
    return p;
  };
  for (int i = 1; i <= n; i++) X[i] = 1;
  for (int t = 0; t < E.size(); t++) {
    int u = E[t].first, v0 = E[t].second, v = v0,
        c0 = X[u], c = c0, d;
    vector<pii> L;
    int vst[n] = {};
    while (!G[u][v0]) {
      L.emplace_back(v, d = X[v]);
      if (!C[v][c])
        for (a = (int)L.size() - 1; a >= 0; a--)
          c = color(u, L[a].first, c);
      else if (!C[u][d])
        for (a = (int)L.size() - 1; a >= 0; a--)
          color(u, L[a].first, L[a].second);
      else if (vst[d]) break;
      else vst[d] = 1, v = C[u][d];
    }
    if (!G[u][v0]) {
      for (; v; v = flip(v, c, d), swap(c, d));
      if (C[u][c0]) {
        for (a = (int)L.size() - 2;
             a >= 0 && L[a].second != c; a--)
          ;
        for (; a >= 0; a--)
          color(u, L[a].first, L[a].second);
      } else t--;
    }
  }
}
} // namespace Vizing
```

2.9 MinimumCliqueCover

```
struct CliqueCover { // 0-base, O(n2^n)
  int co[1 << N], n, E[N];
  int dp[1 << N];
  void init(int _n) {
    n = _n, fill_n(dp, 1 << n, 0);
    fill_n(E, n, 0), fill_n(co, 1 << n, 0);
  }
  void add_edge(int u, int v) {
    E[u] |= 1 << v, E[v] |= 1 << u;
  }
  int solve() {
    for (int i = 0; i < n; ++i)
      co[1 << i] = E[i] | (1 << i);
    co[0] = (1 << n) - 1;
    dp[0] = (n & 1) * 2 - 1;
    for (int i = 1; i < (1 << n); ++i) {
      int t = i & -i;
      dp[i] = -dp[i ^ t];
      co[i] = co[i ^ t] & co[t];
    }
    for (int i = 0; i < (1 << n); ++i)
      co[i] = (co[i] & i) == i;
    fwt(co, 1 << n, 1);
    for (int ans = 1; ans < n; ++ans) {
      int sum = 0; // probabilistic
      for (int i = 0; i < (1 << n); ++i)
        sum += (dp[i] *= co[i]);
      if (sum) return ans;
    }
    return n;
  }
};
```

2.10 CountMaximalClique

```
struct BronKerbosch { // 1-base
  int n, a[N], g[N][N];
  int S, all[N][N], some[N][N], none[N][N];
  void init(int _n) {
    n = _n;
    for (int i = 1; i <= n; ++i)
      for (int j = 1; j <= n; ++j) g[i][j] = 0;
  }
  void add_edge(int u, int v) {
    g[u][v] = g[v][u] = 1;
  }
  void dfs(int d, int an, int sn, int nn) {
    if (S > 1000) return; // pruning
    if (sn == 0 && nn == 0) ++S;
    int u = some[d][0];
    for (int i = 0; i < sn; ++i) {
      int v = some[d][i];
      if (g[u][v]) continue;
      int tsn = 0, tnn = 0;
      copy_n(all[d], an, all[d + 1]);
      all[d + 1][an] = v;
      for (int j = 0; j < sn; ++j)
        if (g[v][some[d][j]])
          some[d + 1][tsn++] = some[d][j];
      for (int j = 0; j < nn; ++j)
        if (g[v][none[d][j]])
          none[d + 1][tnn++] = none[d][j];
      dfs(d + 1, an + 1, tsn, tnn);
      some[d][i] = 0, none[d][nn++] = v;
    }
  }
  int solve() {
    iota(some[0], some[0] + n, 1);
    S = 0, dfs(0, 0, n, 0);
    return S;
  }
};
```

2.11 Theorems

|max independent edge set$|=|V|-|$min edge cover|
|max independent set$|=|V|-|$min vertex cover|

3 Flow-Matching

3.1 HopcroftKarp

```
struct HopcroftKarp
    { // 0-based, return btoa to get matching
  bool dfs(int a, int L, vector<vector<int>> &g,
    vector<int> &btoa, vector<int> &A,
    vector<int> &B) {
    if (A[a] != L) return 0;
    A[a] = -1;
    for (int b : g[a])
      if (B[b] == L + 1) {
        B[b] = 0;
        if (btoa[b] == -1 ||
          dfs(btoa[b], L + 1, g, btoa, A, B))
          return btoa[b] = a, 1;
      }
    return 0;
  }
  int solve(vector<vector<int>> &g, int m) {
    int res = 0;
    vector<int> btoa(m, -1), A(g.size()),
      B(btoa.size()), cur, next;
```

```
    for (;;) {
      fill(all(A), 0), fill(all(B), 0);
      cur.clear();
      for (int a : btoa)
        if (a != -1) A[a] = -1;
      for (int a = 0; a < g.size(); a++)
        if (A[a] == 0) cur.push_back(a);
      for (int lay = 1;; lay++) {
        bool islast = 0;
        next.clear();
        for (int a : cur)
          for (int b : g[a]) {
            if (btoa[b] == -1) {
              B[b] = lay;
              islast = 1;
            } else if (btoa[b] != a && !B[b]) {
              B[b] = lay;
              next.push_back(btoa[b]);
            }
          }
        if (islast) break;
        if (next.empty()) return res;
        for (int a : next) A[a] = lay;
        cur.swap(next);
      }
      for (int a = 0; a < g.size(); a++)
        res += dfs(a, 0, g, btoa, A, B);
    }
  }
};
```

3.2 KM

```
struct KM { // 0-base
  ll w[N][N], hl[N], hr[N], slk[N];
  int fl[N], fr[N], pre[N], qu[N], ql, qr, n;
  bool vl[N], vr[N];
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i)
      fill_n(w[i], n, -INF);
  }
  void add_edge(int a, int b, ll wei) {
    w[a][b] = wei;
  }
  bool Check(int x) {
    if (vl[x] = 1, ~fl[x])
      return vr[qu[qr++] = fl[x]] = 1;
    while (~x) swap(x, fr[fl[x] = pre[x]]);
    return 0;
  }
  void bfs(int s) {
    fill_n(slk
        , n, INF), fill_n(vl, n, 0), fill_n(vr, n, 0);
    ql = qr = 0, qu[qr++] = s, vr[s] = 1;
    for (ll d;;) {
      while (ql < qr)
        for (int x = 0, y = qu[ql++]; x < n; ++x)
          if (!vl[x] && slk
              [x] >= (d = hl[x] + hr[y] - w[x][y])) {
            if (pre[x] = y, d) slk[x] = d;
            else if (!Check(x)) return;
          }
      d = INF;
      for (int x = 0; x < n; ++x)
        if (!vl[x] && d > slk[x]) d = slk[x];
      for (int x = 0; x < n; ++x) {
        if (vl[x]) hl[x] += d;
        else slk[x] -= d;
        if (vr[x]) hr[x] -= d;
      }
      for (int x = 0; x < n; ++x)
        if (!vl[x] && !slk[x] && !Check(x)) return;
    }
  }
  ll solve() {
    fill_n(fl
        , n, -1), fill_n(fr, n, -1), fill_n(hr, n, 0);
    for (int i = 0; i < n; ++i)
      hl[i] = *max_element(w[i], w[i] + n);
    for (int i = 0; i < n; ++i) bfs(i);
    ll res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
  }
};
```

3.3 MCMF

```
struct MinCostMaxFlow { // 0-base
  struct Edge {
    ll from, to, cap, flow, cost, rev;
  } *past[N];
  vector<Edge> G[N];
  int inq[N], n, s, t;
  ll dis[N], up[N], pot[N];
  bool BellmanFord() {
    fill_n(dis, n, INF), fill_n(inq, n, 0);
    queue<int> q;
    auto relax = [&](int u, ll d, ll cap, Edge *e) {
      if (cap > 0 && dis[u] > d) {
        dis[u] = d, up[u] = cap, past[u] = e;
        if (!inq[u]) inq[u] = 1, q.push(u);
      }
    };
    relax(s, 0, INF, 0);
    while (!q.empty()) {
      int u = q.front();
      q.pop(), inq[u] = 0;
      for (auto &e : G[u]) {
        ll d2 = dis[u] + e.cost + pot[u] - pot[e.to];
        relax(
          e.to, d2, min(up[u], e.cap - e.flow), &e);
      }
    }
    return dis[t] != INF;
  }
  bool Dijkstra() {
    fill_n(dis, n, INF);
    priority_queue<pll, vector<pll>, greater<pll>> pq;
    auto relax = [&](int u, ll d, ll cap, Edge *e) {
      if (cap > 0 && dis[u] > d) {
        dis[u] = d, up[u] = cap, past[u] = e;
        pq.push(pll(d, u));
      }
    };
    relax(s, 0, INF, 0);
    while (!pq.empty()) {
      auto [d, u] = pq.top();
      pq.pop();
      if (dis[u] != d) continue;
      for (auto &e : G[u]) {
        ll d2 = dis[u] + e.cost + pot[u] - pot[e.to];
        relax(
          e.to, d2, min(up[u], e.cap - e.flow), &e);
      }
    }
    return dis[t] != INF;
  }
  void solve(int _s, int _t, ll &flow, ll &cost,
    bool neg = true) {
    s = _s, t = _t, flow = 0, cost = 0;
    if (neg) BellmanFord(), copy_n(dis, n, pot);
    // do BellmanFord() if time isn't tight
    for (; Dijkstra(); copy_n(dis, n, pot)) {
      for (int i = 0; i < n; ++i)
        dis[i] += pot[i] - pot[s];
      flow += up[t], cost += up[t] * dis[t];
      for (int i = t; past[i]; i = past[i]->from) {
        auto &e = *past[i];
        e.flow += up[t], G[e.to][e.rev].flow -= up[t];
      }
```

```
    }
  }
  void init(int _n) {
    n = _n, fill_n(pot, n, 0);
    for (int i = 0; i < n; ++i) G[i].clear();
  }
  void add_edge(ll a, ll b, ll cap, ll cost) {
    G[a].emplace_back(
      Edge{a, b, cap, 0, cost, (int)G[b].size()});
    G[b].emplace_back(
      Edge{b, a, 0, 0, -cost, (int)G[a].size() - 1});
  }
};
```

3.4 GeneralGraphMatching

```
struct Matching { // 0-base
  queue<int> q; int n;
  vector<int> fa, s, vis, pre, match;
  vector<vector<int>> G;
  int Find(int u)
  { return u == fa[u] ? u : fa[u] = Find(fa[u]); }
  int LCA(int x, int y) {
    static int tk = 0; tk++; x = Find(x); y = Find(y);
    for (;; swap(x, y)) if (x != n) {
      if (vis[x] == tk) return x;
      vis[x] = tk;
      x = Find(pre[match[x]]);
    }
  }
  void Blossom(int x, int y, int l) {
    for (; Find(x) != l; x = pre[y]) {
      pre[x] = y, y = match[x];
      if (s[y] == 1) q.push(y), s[y] = 0;
      for (int z: {x, y}) if (fa[z] == z) fa[z] = l;
    }
  }
  bool Bfs(int r) {
    iota(all(fa), 0); fill(all(s), -1);
    q = queue<int>(); q.push(r); s[r] = 0;
    for (; !q.empty(); q.pop()) {
      for (int x = q.front(); int u : G[x])
        if (s[u] == -1) {
          if (pre[u] = x, s[u] = 1, match[u] == n) {
            for (int a = u, b = x, last;
                b != n; a = last, b = pre[a])
              last =
                  match[b], match[b] = a, match[a] = b;
            return true;
          }
          q.push(match[u]); s[match[u]] = 0;
        } else if (!s[u] && Find(u) != Find(x)) {
          int l = LCA(u, x);
          Blossom(x, u, l); Blossom(u, x, l);
        }
    }
    return false;
  }
  Matching(int _n) : n(_n), fa(n + 1), s(n + 1), vis
      (n + 1), pre(n + 1, n), match(n + 1, n), G(n) {}
  void add_edge(int u, int v)
  { G[u].emplace_back(v), G[v].emplace_back(u); }
  int solve() {
    int ans = 0;
    for (int x = 0; x < n; ++x)
      if (match[x] == n) ans += Bfs(x);
    return ans;
  } // match[x] == n means not matched
};
```

3.5 MaxWeightMaching

```
#define rep(i, l, r) for (int i = (l); i <= (r); ++i)
struct WeightGraph { // 1-based
  struct edge {
    int u, v, w;
  };
  int n, nx;
  vector<int> lab;
  vector<vector<edge>> g;
  vector<int> slack, match, st, pa, S, vis;
  vector<vector<int>> flo, flo_from;
  queue<int> q;
  WeightGraph(int n_)
    : n(n_), nx(n * 2), lab(nx + 1),
      g(nx + 1, vector<edge>(nx + 1)), slack(nx + 1),
      flo(nx + 1), flo_from(nx + 1, vector(n + 1, 0)) {
    match = st = pa = S = vis = slack;
    rep(u, 1, n) rep(v, 1, n) g[u][v] = {u, v, 0};
  }
  int ED(edge e) {
    return lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2;
  }
  void update_slack(int u, int x, int &s) {
    if (!s || ED(g[u][x]) < ED(g[s][x])) s = u;
  }
  void set_slack(int x) {
    slack[x] = 0;
    for (int u = 1; u <= n; ++u)
      if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
        update_slack(u, x, slack[x]);
  }
  void q_push(int x) {
    if (x <= n) q.push(x);
    else
      for (int y : flo[x]) q_push(y);
  }
  void set_st(int x, int b) {
    st[x] = b;
    if (x > n)
      for (int y : flo[x]) set_st(y, b);
  }
  vector<int> split_flo(auto &f, int xr) {
    auto it = find(ALL(f), xr);
    if (auto pr = it - f.begin(); pr % 2 == 1)
      reverse(1 + ALL(f)), it = f.end() - pr;
    auto res = vector(f.begin(), it);
    return f.erase(f.begin(), it), res;
  }
  void set_match(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    int xr = flo_from[u][g[u][v].u];
    auto &f = flo[u], z = split_flo(f, xr);
    rep(i, 0, (int)z.size() - 1)
      set_match(z[i], z[i ^ 1]);
    set_match(xr, v);
    f.insert(f.end(), all(z));
  }
  void augment(int u, int v) {
    for (;;) {
      int xnv = st[match[u]];
      set_match(u, v);
      if (!xnv) return;
      set_match(xnv, st[pa[xnv]]);
      u = st[pa[xnv]], v = xnv;
    }
  }
  int lca(int u, int v) {
    static int t = 0;
    ++t;
    for (++t; u || v; swap(u, v))
      if (u) {
        if (vis[u] == t) return u;
        vis[u] = t;
        u = st[match[u]];
        if (u) u = st[pa[u]];
      }
    return 0;
  }
  void add_blossom(int u, int o, int v) {
    int b = find(n + 1 + all(st), 0) - begin(st);
    lab[b] = 0, S[b] = 0;
```

```
    match[b] = match[o];
    vector<int> f = {o};
    for (int x = u, y; x != o; x = st[pa[y]])
      f.emplace_back(x),
        f.emplace_back(y = st[match[x]]), q_push(y);
    reverse(1 + all(f));
    for (int x = v, y; x != o; x = st[pa[y]])
      f.emplace_back(x),
        f.emplace_back(y = st[match[x]]), q_push(y);
    flo[b] = f;
    set_st(b, b);
    for (int x = 1; x <= nx; ++x)
      g[b][x].w = g[x][b].w = 0;
    fill(all(flo_from[b]), 0);
    for (int xs : flo[b]) {
      for (int x = 1; x <= nx; ++x)
        if (g[b][x].w == 0 ||
          ED(g[xs][x]) < ED(g[b][x]))
          g[b][x] = g[xs][x], g[x][b] = g[x][xs];
      for (int x = 1; x <= n; ++x)
        if (flo_from[xs][x]) flo_from[b][x] = xs;
    }
    set_slack(b);
  }
  void expand_blossom(int b) {
    for (int x : flo[b]) set_st(x, x);
    int xr = flo_from[b][g[b][pa[b]].u], xs = -1;
    for (int x : split_flo(flo[b], xr)) {
      if (xs == -1) {
        xs = x;
        continue;
      }
      pa[xs] = g[x][xs].u;
      S[xs] = 1, S[x] = 0;
      slack[xs] = 0;
      set_slack(x);
      q_push(x);
      xs = -1;
    }
    for (int x : flo[b])
      if (x == xr) S[x] = 1, pa[x] = pa[b];
      else S[x] = -1, set_slack(x);
    st[b] = 0;
  }
  bool on_found_edge(const edge &e) {
    if (int u = st[e.u], v = st[e.v]; S[v] == -1) {
      int nu = st[match[v]];
      pa[v] = e.u;
      S[v] = 1;
      slack[v] = slack[nu] = 0;
      S[nu] = 0;
      q_push(nu);
    } else if (S[v] == 0) {
      if (int o = lca(u, v)) add_blossom(u, o, v);
      else return augment(u, v), augment(v, u), true;
    }
    return false;
  }
  bool matching() {
    fill(all(S), -1), fill(all(slack), 0);
    q = queue<int>();
    for (int x = 1; x <= nx; ++x)
      if (st[x] == x && !match[x])
        pa[x] = 0, S[x] = 0, q_push(x);
    if (q.empty()) return false;
    for (;;) {
      while (q.size()) {
        int u = q.front();
        q.pop();
        if (S[st[u]] == 1) continue;
        for (int v = 1; v <= n; ++v)
          if (g[u][v].w > 0 && st[u] != st[v]) {
            if (ED(g[u][v]) != 0)
              update_slack(u, st[v], slack[st[v]]);
            else if (on_found_edge(g[u][v]))
                return true;
          }
      }
      int d = INF;
      for (int b = n + 1; b <= nx; ++b)
        if (st[b] == b && S[b] == 1)
          d = min(d, lab[b] / 2);
      for (int x = 1; x <= nx; ++x)
        if (int s = slack[x];
            st[x] == x && s && S[x] <= 0)
          d = min(d, ED(g[s][x]) / (S[x] + 2));
      for (int u = 1; u <= n; ++u)
        if (S[st[u]] == 1) lab[u] += d;
        else if (S[st[u]] == 0) {
          if (lab[u] <= d) return false;
          lab[u] -= d;
        }
      rep(b, n + 1, nx) if (st[b] == b && S[b] >= 0)
        lab[b] += d * (2 - 4 * S[b]);
      for (int x = 1; x <= nx; ++x)
        if (int s = slack[x]; st[x] == x && s &&
            st[s] != x && ED(g[s][x]) == 0)
          if (on_found_edge(g[s][x])) return true;
      for (int b = n + 1; b <= nx; ++b)
        if (st[b] == b && S[b] == 1 && lab[b] == 0)
          expand_blossom(b);
    }
    return false;
  }
  pair<ll, int> solve() {
    fill(all(match), 0);
    rep(u, 0, n) st[u] = u, flo[u].clear();
    int w_max = 0;
    rep(u, 1, n) rep(v, 1, n) {
      flo_from[u][v] = (u == v ? u : 0);
      w_max = max(w_max, g[u][v].w);
    }
    fill(all(lab), w_max);
    int n_matches = 0;
    ll tot_weight = 0;
    while (matching()) ++n_matches;
    rep(u, 1, n) if (match[u] && match[u] < u)
      tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
  }
  void add_edge(int u, int v, int w) {
    g[u][v].w = g[v][u].w = w;
  }
};
```

3.6 GlobalMinCut

```
struct StoerWagner { // O(V^3), is it O(VE + V log V)?
  int vst[N], edge[N][N], wei[N];
  void init(int n) {
    for (int i = 0; i < n; ++i) fill_n(edge[i], n, 0);
  }
  void addEdge(int u, int v, int w) {
    edge[u][v] += w;
    edge[v][u] += w;
  }
  int search(int &s, int &t, int n) {
    fill_n(vst, n, 0), fill_n(wei, n, 0);
    s = t = -1;
    int mx, cur;
    for (int j = 0; j < n; ++j) {
      mx = -1, cur = 0;
      for (int i = 0; i < n; ++i)
        if (wei[i] > mx) cur = i, mx = wei[i];
      vst[cur] = 1, wei[cur] = -1;
      s = t;
      t = cur;
      for (int i = 0; i < n; ++i)
        if (!vst[i]) wei[i] += edge[cur][i];
    }
    return mx;
  }
```

```cpp
  int solve(int n) {
    int res = INF;
    for (int x, y; n > 1; n--) {
      res = min(res, search(x, y, n));
      for (int i = 0; i < n; ++i)
        edge[i][x] = (edge[x][i] += edge[y][i]);
      for (int i = 0; i < n; ++i) {
        edge[y][i] = edge[n - 1][i];
        edge[i][y] = edge[i][n - 1];
      } // edge[y][y] = 0;
    }
    return res;
  }
} sw;
```

3.7 BoundedFlow(Dinic)

```cpp
struct BoundedFlow { // 0-base
  struct edge {
    int to, cap, flow, rev;
  };
  vector<edge> G[N];
  int n, s, t, dis[N], cur[N], cnt[N];
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n + 2; ++i)
      G[i].clear(), cnt[i] = 0;
  }
  void add_edge(int u, int v, int lcap, int rcap) {
    cnt[u] -= lcap, cnt[v] += lcap;
    G[u].emplace_back
        (edge{v, rcap, lcap, (int)G[v].size()});
    G[v].emplace_back
        (edge{u, 0, 0, (int)G[u].size() - 1});
  }
  void add_edge(int u, int v, int cap) {
    G[u].emplace_back
        (edge{v, cap, 0, (int)G[v].size()});
    G[v].emplace_back
        (edge{u, 0, 0, (int)G[u].size() - 1});
  }
  int dfs(int u, int cap) {
    if (u == t || !cap) return cap;
    for (int &i = cur[u]; i < G[u].size(); ++i) {
      edge &e = G[u][i];
      if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
        int df = dfs(e.to, min(e.cap - e.flow, cap));
        if (df) {
          e.flow += df, G[e.to][e.rev].flow -= df;
          return df;
        }
      }
    }
    dis[u] = -1;
    return 0;
  }
  bool bfs() {
    fill_n(dis, n + 3, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
      int u = q.front();
      q.pop();
      for (edge &e : G[u])
        if (!~dis[e.to] && e.flow != e.cap)
          q.push(e.to), dis[e.to] = dis[u] + 1;
    }
    return dis[t] != -1;
  }
  int maxflow(int _s, int _t) {
    s = _s, t = _t;
    int flow = 0, df;
    while (bfs()) {
      fill_n(cur, n + 3, 0);
      while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
  }
  bool solve() {
    int sum = 0;
    for (int i = 0; i < n; ++i)
      if (cnt[i] > 0)
        add_edge(n + 1, i, cnt[i]), sum += cnt[i];
      else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
    if (sum != maxflow(n + 1, n + 2)) sum = -1;
    for (int i = 0; i < n; ++i)
      if (cnt[i] > 0)
        G[n + 1].pop_back(), G[i].pop_back();
      else if (cnt[i] < 0)
        G[i].pop_back(), G[n + 2].pop_back();
    return sum != -1;
  }
  int solve(int _s, int _t) {
    add_edge(_t, _s, INF);
    if (!solve()) return -1; // invalid flow
    int x = G[_t].back().flow;
    return G[_t].pop_back(), G[_s].pop_back(), x;
  }
};
```

3.8 GomoryHuTree

```cpp
MaxFlow Dinic;
int g[N];
void GomoryHu(int n) { // 0-base
  fill_n(g, n, 0);
  for (int i = 1; i < n; ++i) {
    Dinic.reset();
    add_edge(i, g[i], Dinic.maxflow(i, g[i]));
    for (int j = i + 1; j <= n; ++j)
      if (g[j] == g[i] && ~Dinic.dis[j])
        g[j] = i;
  }
}
```

3.9 MinCostCirculation

```cpp
struct MinCostCirculation { // 0-base
  struct Edge {
    ll from, to, cap, fcap, flow, cost, rev;
  } *past[N];
  vector<Edge> G[N];
  ll dis[N], inq[N], n;
  void BellmanFord(int s) {
    fill_n(dis, n, INF), fill_n(inq, n, 0);
    queue<int> q;
    auto relax = [&](int u, ll d, Edge *e) {
      if (dis[u] > d) {
        dis[u] = d, past[u] = e;
        if (!inq[u]) inq[u] = 1, q.push(u);
      }
    };
    relax(s, 0, 0);
    while (!q.empty()) {
      int u = q.front();
      q.pop(), inq[u] = 0;
      for (auto &e : G[u])
        if (e.cap > e.flow)
          relax(e.to, dis[u] + e.cost, &e);
    }
  }
  void try_edge(Edge &cur) {
    if (cur.cap > cur.flow) return ++cur.cap, void();
    BellmanFord(cur.to);
    if (dis[cur.from] + cur.cost < 0) {
      ++cur.flow, --G[cur.to][cur.rev].flow;
      for (int
           i = cur.from; past[i]; i = past[i]->from) {
        auto &e = *past[i];
        ++e.flow, --G[e.to][e.rev].flow;
      }
    }
    ++cur.cap;
  }
```

```
  void solve(int mxlg) {
    for (int b = mxlg; b >= 0; --b) {
      for (int i = 0; i < n; ++i)
        for (auto &e : G[i])
          e.cap *= 2, e.flow *= 2;
      for (int i = 0; i < n; ++i)
        for (auto &e : G[i])
          if (e.fcap >> b & 1)
            try_edge(e);
    }
  }
  void init(int _n) { n = _n;
    for (int i = 0; i < n; ++i) G[i].clear();
  }
  void add_edge(ll a, ll b, ll cap, ll cost) {
    G[a].emplace_back(Edge{a, b,
         0, cap, 0, cost, (ll)G[b].size() + (a == b)});
    G[b].emplace_back(Edge
        {b, a, 0, 0, 0, -cost, (ll)G[a].size() - 1});
  }
} mcmf; // O(VE * ElogC)
```

3.10 FlowModelsBuilding

- Maximum/Minimum flow with lower bound / Circulation problem
  1. Construct super source S and sink T.
  2. For each edge (x,y,l,u), connect $x \to y$ with capacity $u-l$.
  3. For each vertex v, denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  4. If $in(v) > 0$, connect $S \to v$ with capacity $in(v)$, otherwise, connect $v \to T$ with capacity $-in(v)$.
     - To maximize, connect $t \to s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T. If $f \neq \sum_{v \in V, in(v)>0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
     - To minimize, let f be the maximum flow from S to T. Connect $t \to s$ with capacity ∞ and let the flow from S to T be f'. If $f + f' \neq \sum_{v \in V, in(v)>0} in(v)$, there's no solution. Otherwise, f' is the answer.
  5. The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X,Y)
  1. Redirect every edge: $y \to x$ if $(x,y) \in M$, $x \to y$ otherwise.
  2. DFS from unmatched vertices in X.
  3. $x \in X$ is chosen iff x is unvisited.
  4. $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
  1. Consruct super source S and sink T
  2. For each edge (x,y,c), connect $x \to y$ with $(cost,cap) = (c,1)$ if $c > 0$, otherwise connect $y \to x$ with $(cost,cap) = (-c,1)$
  3. For each edge with $c < 0$, sum these cost as K, then increase $d(y)$ by 1, decrease $d(x)$ by 1
  4. For each vertex v with $d(v) > 0$, connect $S \to v$ with $(cost,cap) = (0,d(v))$
  5. For each vertex v with $d(v) < 0$, connect $v \to T$ with $(cost,cap) = (0,-d(v))$
  6. Flow from S to T, the answer is the cost of the flow $C+K$
- Maximum density induced subgraph
  1. Binary search on answer, suppose we're checking answer T
  2. Construct a max flow model, let K be the sum of all weights
  3. Connect source $s \to v$, $v \in G$ with capacity K
  4. For each edge (u,v,w) in G, connect $u \to v$ and $v \to u$ with capacity w
  5. For $v \in G$, connect it with sink $v \to t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
  6. T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
  1. For each $v \in V$ create a copy v', and connect $u' \to v'$ with weight $w(u,v)$.
  2. Connect $v \to v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v.
  3. Find the minimum weight perfect matching on G'.
- Project selection problem
  1. If $p_v > 0$, create edge (s,v) with capacity p_v; otherwise, create edge (v,t) with capacity $-p_v$.
  2. Create edge (u,v) with capacity w with w being the cost of choosing u without choosing v.
  3. The mincut is equivalent to the maximum profit of a subset of projects.
- Dual of minimum cost maximum flow
  1. Capacity c_{uv}, Flow f_{uv}, Cost w_{uv}, Required Flow difference for vertex b_u.
  2. If all w_{uv} are integers, then optimal solution can happen when all p_u are integers.

$$\begin{aligned} \min \sum_{uv} w_{uv} f_{uv} & \\ -f_{uv} \geq -c_{uv} & \Leftrightarrow \min \sum_u b_u p_u + \sum_{uv} c_{uv} \max(0, p_v - p_u - w_{uv}) \\ \sum_v f_{vu} - \sum_v f_{uv} = -b_u & \qquad p_u \geq 0 \end{aligned}$$

4 Data Struture

4.1 LinkCutTree

```
#define ls(x) Tree[x].son[0]
#define rs(x) Tree[x].son[1]
#define fa(x) Tree[x].fa
struct node {
  int son[2], Min, id, fa, lazy;
} Tree[N];
int n, m, q, w[N], Min;
struct Node {
  int u, v, w;
} a[N];
inline bool IsRoot(int x) {
  return (ls(fa(x)) == x || rs(fa(x)) == x) ? false
                                             : true;
}
inline void PushUp(int x) {
  Tree[x].Min = w[x], Tree[x].id = x;
  if (ls(x) && Tree[ls(x)].Min < Tree[x].Min) {
    Tree[x].Min = Tree[ls(x)].Min;
    Tree[x].id = Tree[ls(x)].id;
  }
  if (rs(x) && Tree[rs(x)].Min < Tree[x].Min) {
    Tree[x].Min = Tree[rs(x)].Min;
    Tree[x].id = Tree[rs(x)].id;
  }
}
inline void Update(int x) {
  Tree[x].lazy ^= 1;
  swap(ls(x), rs(x));
}
inline void PushDown(int x) {
  if (!Tree[x].lazy) return;
  if (ls(x)) Update(ls(x));
  if (rs(x)) Update(rs(x));
  Tree[x].lazy = 0;
}
inline void Rotate(int x) {
  int y = fa(x), z = fa(y), k = rs(y) == x,
      w = Tree[x].son[!k];
  if (!IsRoot(y)) Tree[z].son[rs(z) == y] = x;
  fa(x) = z, fa(y) = x;
  if (w) fa(w) = y;
  Tree[x].son[!k] = y, Tree[y].son[k] = w;
  PushUp(y);
}
inline void Splay(int x) {
  stack<int> Stack;
  int y = x, z;
  Stack.push(y);
  while (!IsRoot(y)) Stack.push(y = fa(y));
  while (!Stack.empty())
    PushDown(Stack.top()), Stack.pop();
  while (!IsRoot(x)) {
    y = fa(x), z = fa(y);
    if (!IsRoot(y))
      Rotate((ls(y) == x) ^ (ls(z) == y) ? x : y);
    Rotate(x);
  }
  PushUp(x);
```

```cpp
}
inline void Access(int root) {
  for (int x = 0; root; x = root, root = fa(root))
    Splay(root), rs(root) = x, PushUp(root);
}
inline void MakeRoot(int x) {
  Access(x), Splay(x), Update(x);
}
inline int FindRoot(int x) {
  Access(x), Splay(x);
  while (ls(x)) x = ls(x);
  return Splay(x), x;
}
inline void Link(int u, int v) {
  MakeRoot(u);
  if (FindRoot(v) != u) fa(u) = v;
}
inline void Cut(int u, int v) {
  MakeRoot(u);
  if (FindRoot(v) != u || fa(v) != u || ls(v)) return;
  fa(v) = rs(u) = 0;
}
inline void Split(int u, int v) {
  MakeRoot(u), Access(v), Splay(v);
}
inline bool Check(int u, int v) {
  return MakeRoot(u), FindRoot(v) == u;
}
inline int LCA(int root, int u, int v) {
  MakeRoot(root), Access(u), Access(v), Splay(u);
  if (!fa(u)) {
    Access(u), Splay(v);
    return fa(v);
  }
  return fa(u);
}
/* ETT
每次進入節點和走邊都放入一次共 3n - 2
node(u) 表示進入節點 u 放入 treap 的位置
edge(u, v) 表示 u -> v 的邊放入 treap 的位置 (push v)
Makeroot u  :
  L1 = [begin, node(u) - 1], L2 = [node(u), end]
  -> L2 + L1

Insert u, v :
  Tu -> L1 = [begin, node(u) - 1], L2 = [node(u), end]
  Tv -> L3 = [begin, node(v) - 1], L4 = [node(v), end]
  -> L2 + L1 + edge(u, v) + L4 + L3 + edge(v, u)

Delect u, v :
  maybe need swap u, v
  T -> L1 + edge(u, v) + L2 + edge(v, u) + L3
  -> L1 + L3, L2
*/
```

5 String

5.1 KMP

```cpp
int KMP(string s, string t) {
  t = " "s + t; // consistency with ACa
  int n = t.size(), ans = 0;
  vector<int> f(t.size(), 0);
  f[0] = -1;
  for (int i = 1, j = -1; i < t.size(); i++) {
    while (j >= 0 && t[j + 1] != t[i])
      j = f[j];
    f[i] = ++j;
  }
  for (int i = 0, j = 0; i < s.size(); i++) {
    while (j >= 0 && t[j + 1] != s[i])
      j = f[j];
    if (++j + 1 == t.size()) ans++, j = f[j];
  }
  return ans;
}
```

5.2 Z

```cpp
int Z[N];
void z(string s) {
  for (int i = 1, mx = 0; i < s.size(); i++) {
    if (i < Z[mx] + mx)
      Z[i] = min(Z[mx] - i + mx, Z[i - mx]);
    while (
      Z[i] + i < s.size() && s[i + Z[i]] == s[Z[i]])
      Z[i]++;
    if (Z[i] + i > Z[mx] + mx) mx = i;
  }
}
```

5.3 Manacher

```cpp
int man[N]; // len: man[i] - 1
void manacher(string s) { // uses 2|s|+1
  string t;
  for (int i = 0; i < (int)s.size(); i++) {
    t.push_back('$');
    t.push_back(s[i]);
  }
  t.push_back('$');
  int mx = 1;
  for (int i = 0; i < (int)t.size(); i++) {
    man[i] = 1;
    man[i] = min(man[mx] + mx - i, man[2 * mx - i]);
    while (man[i] + i < (int)t.size() && i >= man[i] &&
      t[i + man[i]] == t[i - man[i]])
      man[i]++;
    if (i + man[i] > mx + man[mx]) mx = i;
  }
}
```

5.4 SuffixArray

```cpp
struct SuffixArray {
#define add(x, k) (x + k + n) % n
  vector<int> sa, cnt, rk, tmp, lcp;
  // sa: order, rk[i]: pos of s[i..],
  // lcp[i]: LCP of sa[i], sa[i-1]
  void SA(string s) { // remember to append '\1'
    int n = s.size();
    sa.resize(n), cnt.resize(n);
    rk.resize(n), tmp.resize(n);
    iota(all(sa), 0);
    sort(all(sa),
      [&](int i, int j) { return s[i] < s[j]; });
    rk[0] = 0;
    for (int i = 1; i < n; i++)
      rk[sa[i]] =
        rk[sa[i - 1]] + (s[sa[i - 1]] != s[sa[i]]);
    for (int k = 1; k <= n; k <<= 1) {
      fill(all(cnt), 0);
      for (int i = 0; i < n; i++)
        cnt[rk[add(sa[i], -k)]]++;
      for (int i = 1; i < n; i++) cnt[i] += cnt[i - 1];
      for (int i = n - 1; i >= 0; i--)
        tmp[--cnt[rk[add(sa[i], -k)]]] =
          add(sa[i], -k);
      sa.swap(tmp);
      tmp[sa[0]] = 0;
      for (int i = 1; i < n; i++)
        tmp[sa[i]] = tmp[sa[i - 1]] +
          (rk[sa[i - 1]] != rk[sa[i]] ||
            rk[add(sa[i - 1], k)] !=
              rk[add(sa[i], k)]);
      rk.swap(tmp);
    }
  }
  void LCP(string s) {
    int n = s.size(), k = 0;
    lcp.resize(n);
    for (int i = 0; i < n; i++)
      if (rk[i] == 0) lcp[rk[i]] = 0;
      else {
```

```
        if (k) k--;
        int j = sa[rk[i] - 1];
        while (
          max(i, j) + k < n && s[i + k] == s[j + k])
          k++;
        lcp[rk[i]] = k;
      }
  }
};
```

5.5 SAIS

```
auto sais(const auto &s) {
  const int n = SZ(s), z = ranges::max(s) + 1;
  if (n == 1) return vector{0};
  vector<int> c(z); for (int x : s) ++c[x];
  partial_sum(ALL(c), begin(c));
  vector<int> sa(n); auto I = views::iota(0, n);
  vector<bool> t(n, true);
  for (int i = n - 2; i >= 0; --i)
    t[i] = (
        s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
  auto is_lms = views::filter([&t](int x) {
    return x && t[x] && !t[x - 1];
  });
  auto induce = [&] {
    for (auto x = c; int y : sa)
      if (y--) if (!t[y]) sa[x[s[y] - 1]++] = y;
    for (auto x = c; int y : sa | views::reverse)
      if (y--) if (t[y]) sa[--x[s[y]]] = y;
  };
  vector<int> lms, q(n); lms.reserve(n);
  for (auto x = c; int i : I | is_lms)
    q[i] = SZ(lms), lms.pb(sa[--x[s[i]]] = i);
  induce(); vector<int> ns(SZ(lms));
  for (int j = -1, nz = 0; int i : sa | is_lms) {
    if (j >= 0) {
      int len = min({n - i, n - j, lms[q[i] + 1] - i});
      ns[q[i]] = nz += lexicographical_compare(
          begin(s) + j, begin(s) + j + len,
          begin(s) + i, begin(s) + i + len);
    }
    j = i;
  }
  fill(ALL(sa), 0); auto nsa = sais(ns);
  for (auto x = c; int y : nsa | views::reverse)
    y = lms[y], sa[--x[s[y]]] = y;
  return induce(), sa;
}
// sa[i]: sa[i]-th suffix
     is the i-th lexicographically smallest suffix.
// hi[i]: LCP of suffix sa[i] and suffix sa[i - 1].
struct Suffix {
  int n; vector<int> sa, hi, ra;
  Suffix
      (const auto &_s, int _n) : n(_n), hi(n), ra(n) {
    vector<int> s(n + 1); // s[n] = 0;
    copy_n(_s, n, begin(s)); // _s shouldn't contain 0
    sa = sais(s); sa.erase(sa.begin());
    for (int i = 0; i < n; ++i) ra[sa[i]] = i;
    for (int i = 0, h = 0; i < n; ++i) {
      if (!ra[i]) { h = 0; continue; }
      for (int j = sa[ra[i] - 1]; max
          (i, j) + h < n && s[i + h] == s[j + h];) ++h;
      hi[ra[i]] = h ? h-- : 0;
    }
  }
};
```

5.6 ACAutomaton

```
#define sigma 26
#define base 'a'
struct AhoCorasick { // N: sum of length
  int ch[N][sigma] = {{}}, f[N] = {-1}, tag[N],
      mv[N][sigma], jump[N], cnt[N];
  int idx = 0, t = -1;
  vector<int> E[N], q;
  pii o[N];
  int insert(string &s, int t) {
    int j = 0;
    for (int i = 0; i < (int)s.size(); i++) {
      if (!ch[j][s[i] - base])
        ch[j][s[i] - base] = ++idx;
      j = ch[j][s[i] - base];
    }
    tag[j] = 1;
    return j;
  }
  int next(int u, int c) {
    return u < 0 ? 0 : mv[u][c];
  }
  void dfs(int u) {
    o[u].F = ++t;
    for (auto v : E[u]) dfs(v);
    o[u].S = t;
  }
  void build() {
    int k = -1;
    q.emplace_back(0);
    while (++k < q.size()) {
      int u = q[k];
      for (int v = 0; v < sigma; v++) {
        if (ch[u][v]) {
          f[ch[u][v]] = next(f[u], v);
          q.emplace_back(ch[u][v]);
        }
        mv[u][v] =
          (ch[u][v] ? ch[u][v] : next(f[u], v));
      }
      if (u) jump[u] = (tag[f[u]] ? f[u] : jump[f[u]]);
    }
    reverse(q.begin(), q.end());
    for (int i = 1; i <= idx; i++)
      E[f[i]].emplace_back(i);
    dfs(0);
  }
  void match(string &s) {
    fill(cnt, cnt + idx + 1, 0);
    for (int i = 0, j = 0; i < (int)s.size(); i++)
      cnt[j = next(j, s[i] - base)]++;
    for (int i : q)
      if (f[i] > 0) cnt[f[i]] += cnt[i];
  }
} ac;
```

5.7 MinRotation

```
int mincyc(string s) {
  int n = s.size();
  s = s + s;
  int i = 0, ans = 0;
  while (i < n) {
    ans = i;
    int j = i + 1, k = i;
    while (j < s.size() && s[j] >= s[k]) {
      k = (s[j] > s[k] ? i : k + 1);
      ++j;
    }
    while (i <= k) i += j - k;
  }
  return ans;
}
```

5.8 ExtSAM

```
#define CNUM 26
struct exSAM {
  int len[N * 2], link[N * 2]; // maxlength, suflink
  int next[N * 2][CNUM], tot; // [0, tot), root = 0
  int lenSorted[N * 2]; // topo. order
  int cnt[N * 2]; // occurence
  int newnode() {
    fill_n(next[tot], CNUM, 0);
```

```
    len[tot] = cnt[tot] = link[tot] = 0;
    return tot++;
  }
  void init() { tot = 0, newnode(), link[0] = -1; }
  int insertSAM(int last, int c) {
    int cur = next[last][c];
    len[cur] = len[last] + 1;
    int p = link[last];
    while (p != -1 && !next[p][c])
      next[p][c] = cur, p = link[p];
    if (p == -1) return link[cur] = 0, cur;
    int q = next[p][c];
    if (len
        [p] + 1 == len[q]) return link[cur] = q, cur;
    int clone = newnode();
    for (int i = 0; i < CNUM; ++i)
      next[
          clone][i] = len[next[q][i]] ? next[q][i] : 0;
    len[clone] = len[p] + 1;
    while (p != -1 && next[p][c] == q)
      next[p][c] = clone, p = link[p];
    link[link[cur] = clone] = link[q];
    link[q] = clone;
    return cur;
  }
  void insert(const string &s) {
    int cur = 0;
    for (auto ch : s) {
      int &nxt = next[cur][int(ch - 'a')];
      if (!nxt) nxt = newnode();
      cnt[cur = nxt] += 1;
    }
  }
  void build() {
    queue<int> q;
    q.push(0);
    while (!q.empty()) {
      int cur = q.front();
      q.pop();
      for (int i = 0; i < CNUM; ++i)
        if (next[cur][i])
          q.push(insertSAM(cur, i));
    }
    vector<int> lc(tot);
    for (int i = 1; i < tot; ++i) ++lc[len[i]];
    partial_sum(all(lc), lc.begin());
    for (int i
        = 1; i < tot; ++i) lenSorted[--lc[len[i]]] = i;
  }
  void solve() {
    for (int i = tot - 2; i >= 0; --i)
      cnt[link[lenSorted[i]]] += cnt[lenSorted[i]];
  }
};
```

5.9 PalindromeTree

```
struct PalindromicTree {
  struct node {
    int next[26], fail, len;
    int cnt, num; // cnt: appear times, num: number of
                  // pal. suf.
    node(int l = 0) : fail(0), len(l), cnt(0), num(0) {
      for (int i = 0; i < 26; ++i) next[i] = 0;
    }
  };
  vector<node> St;
  vector<char> s;
  int last, n;
  PalindromicTree() : St(2), last(1), n(0) {
    St[0].fail = 1, St[1].len = -1, s.emplace_back(-1);
  }
  inline void clear() {
    St.clear(), s.clear(), last = 1, n = 0;
    St.emplace_back(0), St.emplace_back(-1);
    St[0].fail = 1, s.emplace_back(-1);
  }
  inline int get_fail(int x) {
    while (s[n - St[x].len - 1] != s[n])
      x = St[x].fail;
    return x;
  }
  inline void add(int c) {
    s.push_back(c -= 'a'), ++n;
    int cur = get_fail(last);
    if (!St[cur].next[c]) {
      int now = St.size();
      St.emplace_back(St[cur].len + 2);
      St[now].fail =
        St[get_fail(St[cur].fail)].next[c];
      St[cur].next[c] = now;
      St[now].num = St[St[now].fail].num + 1;
    }
    last = St[cur].next[c], ++St[last].cnt;
  }
  inline void count() { // counting cnt
    auto i = St.rbegin();
    for (; i != St.rend(); ++i) {
      St[i->fail].cnt += i->cnt;
    }
  }
  inline int size() { // The number of diff. pal.
    return (int)St.size() - 2;
  }
};
```

6 Number Theory

6.1 Primes

12721 13331 14341 75577 123457 222557 556679 999983 1097774749 1076767633 100102021 999997771 1001010013 1000512343 987654361 999991231 999888733 98789101 987777733 999991921 1010101333 1010102101 1000000000039 1000000000000037 2305843009213693951 4611686018427387847 9223372036854775783 18446744073709551557

6.2 ExtGCD

```
// beware of negative numbers!
void extgcd(ll a, ll b, ll c, ll &x, ll &y) {
  if (b == 0) x = c / a, y = 0;
  else {
    extgcd(b, a % b, c, y, x);
    y -= x * (a / b);
  }
} // |x| <= b/2, |y| <= a/2
```

6.3 FloorCeil

```
int floor(int a, int b)
{ return a / b - (a % b && (a < 0) ^ (b < 0)); }
int ceil(int a, int b)
{ return a / b + (a % b && (a < 0) ^ (b > 0)); }
```

6.4 FloorSum

Computes

$$f(a,b,c,n)=\sum_{i=0}^{n}\left\lfloor\frac{a\cdot i+b}{m}\right\rfloor$$

Furthermore, Let $m=\left\lfloor\frac{an+b}{c}\right\rfloor$:

$$g(a,b,c,n)=\sum_{i=0}^{n}i\left\lfloor\frac{ai+b}{c}\right\rfloor$$

$$=\begin{cases}\left\lfloor\frac{a}{c}\right\rfloor\cdot\frac{n(n+1)(2n+1)}{6}+\left\lfloor\frac{b}{c}\right\rfloor\cdot\frac{n(n+1)}{2} & \\ +g(a \bmod c, b \bmod c, c, n), & a\geq c\vee b\geq c\\ 0, & n<0\vee a=0\\ \frac{1}{2}\cdot(n(n+1)m-f(c,c-b-1,a,m-1) & \\ -h(c,c-b-1,a,m-1)), & \text{otherwise}\end{cases}$$

$$h(a,b,c,n)=\sum_{i=0}^{n}\left\lfloor\frac{ai+b}{c}\right\rfloor^2$$

$$=\begin{cases}\left\lfloor\frac{a}{c}\right\rfloor^2\cdot\frac{n(n+1)(2n+1)}{6}+\left\lfloor\frac{b}{c}\right\rfloor^2\cdot(n+1) & \\ +\left\lfloor\frac{a}{c}\right\rfloor\cdot\left\lfloor\frac{b}{c}\right\rfloor\cdot n(n+1) & \\ +h(a \bmod c, b \bmod c, c, n) & \\ +2\left\lfloor\frac{a}{c}\right\rfloor\cdot g(a \bmod c, b \bmod c, c, n) & \\ +2\left\lfloor\frac{b}{c}\right\rfloor\cdot f(a \bmod c, b \bmod c, c, n), & a\ge c \lor b\ge c \\ 0, & n<0 \lor a=0 \\ nm(m+1)-2g(c,c-b-1,a,m-1) & \\ -2f(c,c-b-1,a,m-1)-f(a,b,c,n), & \text{otherwise}\end{cases}$$

```
ll floorsum(ll A, ll B, ll C, ll N) {
  if (A == 0) return (N + 1) * (B / C);
  if (A > C || B > C)
    return (N + 1) * (B / C) +
      N * (N + 1) / 2 * (A / C) +
      floorsum(A % C, B % C, C, N);
  ll M = (A * N + B) / C;
  return N * M - floorsum(C, C - B - 1, A, M - 1);
}
```

6.5 MillerRabin

```
// n < 4,759,123,141        3 : 2, 7, 61
// n < 1,122,004,669,633    4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383    6 : primes <= 13
// n < 2^64                 7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool Miller_Rabin(ll a, ll n) {
  if ((a = a % n) == 0) return 1;
  if (n % 2 == 0) return n == 2;
  ll tmp = (n - 1) / ((n - 1) & (1 - n));
  ll t = __lg(((n - 1) & (1 - n))), x = 1;
  for (; tmp; tmp >>= 1, a = mul(a, a, n))
    if (tmp & 1) x = mul(x, a, n);
  if (x == 1 || x == n - 1) return 1;
  while (--t)
    if ((x = mul(x, x, n)) == n - 1) return 1;
  return 0;
}
```

6.6 PollardRho

```
map<ll, int> cnt;
void PollardRho(ll n) {
  if (n == 1) return;
  if (prime(n)) return ++cnt[n], void();
  if (n % 2
      == 0) return PollardRho(n / 2), ++cnt[2], void();
  ll x = 2, y = 2, d = 1, p = 1;
  #define f(x, n, p) ((mul(x, x, n) + p) % n)
  while (true) {
    if (d != n && d != 1) {
      PollardRho(n / d);
      PollardRho(d);
      return;
    }
    if (d == n) ++p;
    x = f(x, n, p), y = f(f(y, n, p), n, p);
    d = gcd(abs(x - y), n);
  }
}
```

6.7 Fraction

```
struct fraction {
  ll n, d;
  fraction
      (const ll &_n=0, const ll &_d=1): n(_n), d(_d) {
    ll t = gcd(n, d);
    n /= t, d /= t;
    if (d < 0) n = -n, d = -d;
  }
  fraction operator-() const
  { return fraction(-n, d); }
  fraction operator+(const fraction &b) const
  { return fraction(n * b.d + b.n * d, d * b.d); }
  fraction operator-(const fraction &b) const
  { return fraction(n * b.d - b.n * d, d * b.d); }
  fraction operator*(const fraction &b) const
  { return fraction(n * b.n, d * b.d); }
  fraction operator/(const fraction &b) const
  { return fraction(n * b.d, d * b.n); }
  void print() {
    cout << n;
    if (d != 1) cout << "/" << d;
  }
};
```

6.8 ChineseRemainder

```
ll solve(ll x1, ll m1, ll x2, ll m2) {
  ll g = gcd(m1, m2);
  if ((x2 - x1) % g) return -1; // no sol
  m1 /= g; m2 /= g;
  ll x, y;
  extgcd(m1, m2, __gcd(m1, m2), x, y);
  ll lcm = m1 * m2 * g;
  ll res = x * (x2 - x1) * m1 + x1;
  // be careful with overflow
  return (res % lcm + lcm) % lcm;
}
```

6.9 FactorialModp^k

```
// O(p^k + log^2 n), pk = p^k
ll prod[MAXP];
ll fac_no_p(ll n, ll p, ll pk) {
  prod[0] = 1;
  for (int i = 1; i <= pk; ++i)
    if (i % p) prod[i] = prod[i - 1] * i % pk;
    else prod[i] = prod[i - 1];
  ll rt = 1;
  for (; n; n /= p) {
    rt = rt * mpow(prod[pk], n / pk, pk) % pk;
    rt = rt * prod[n % pk] % pk;
  }
  return rt;
} // (n! without factor p) % p^k
```

6.10 QuadraticResidue

```
// Berlekamp-Rabin, log^2(p)
ll trial(ll y, ll z, ll m) {
  ll a0 = 1, a1 = 0, b0 = z, b1 = 1, p = (m - 1) / 2;
  while (p) {
    if (p & 1)
      tie(a0, a1) =
        make_pair((a1 * b1 % m * y + a0 * b0) % m,
          (a0 * b1 + a1 * b0) % m);
    tie(b0, b1) =
      make_pair((b1 * b1 % m * y + b0 * b0) % m,
        (2 * b0 * b1) % m);
    p >>= 1;
  }
  if (a1) return inv(a1, m);
  return -1;
}
mt19937 rd(49);
ll psqrt(ll y, ll p) {
  if (fpow(y, (p - 1) / 2, p) != 1) return -1;
  for (int i = 0; i < 30; i++) {
    ll z = rd() % p;
    if (z * z % p == y) return z;
    ll x = trial(y, z, p);
    if (x == -1) continue;
    return x;
  }
  return -1;
}
```

6.11 MeisselLehmer

```
ll PrimeCount(ll n) { // n ~ 10^13 => < 2s
  if (n <= 1) return 0;
  int v = sqrt(n), s = (v + 1) / 2, pc = 0;
  vector<int> smalls(v + 1), skip(v + 1), roughs(s);
  vector<ll> larges(s);
  for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
  for (int i = 0; i < s; ++i) {
    roughs[i] = 2 * i + 1;
    larges[i] = (n / (2 * i + 1) + 1) / 2;
  }
  for (int p = 3; p <= v; ++p) {
    if (smalls[p] > smalls[p - 1]) {
      int q = p * p;
      ++pc;
      if (1LL * q * q > n) break;
      skip[p] = 1;
      for (int i = q; i <= v; i += 2 * p) skip[i] = 1;
      int ns = 0;
      for (int k = 0; k < s; ++k) {
        int i = roughs[k];
        if (skip[i]) continue;
        ll d = 1LL * i * p;
        larges[ns] = larges[k] - (d <= v ? larges
            [smalls[d] - pc] : smalls[n / d]) + pc;
        roughs[ns++] = i;
      }
      s = ns;
      for (int j = v / p; j >= p; --j) {
        int c =
             smalls[j] - pc, e = min(j * p + p, v + 1);
        for (int i = j * p; i < e; ++i) smalls[i] -= c;
      }
    }
  }
  for (int k = 1; k < s; ++k) {
    const ll m = n / roughs[k];
    ll t = larges[k] - (pc + k - 1);
    for (int l = 1; l < k; ++l) {
      int p = roughs[l];
      if (1LL * p * p > m) break;
      t -= smalls[m / p] - (pc + l - 1);
    }
    larges[0] -= t;
  }
  return larges[0];
}
```

6.12 DiscreteLog

```
int DiscreteLog(int s, int x, int y, int m) {
  constexpr int kStep = 32000;
  unordered_map<int, int> p;
  int b = 1;
  for (int i = 0; i < kStep; ++i) {
    p[y] = i;
    y = 1LL * y * x % m;
    b = 1LL * b * x % m;
  }
  for (int i = 0; i < m + 10; i += kStep) {
    s = 1LL * s * b % m;
    if (p.find(s) != p.end()) return i + kStep - p[s];
  }
  return -1;
}
int DiscreteLog(int x, int y, int m) {
  if (m == 1) return 0;
  int s = 1;
  for (int i = 0; i < 100; ++i) {
    if (s == y) return i;
    s = 1LL * s * x % m;
  }
  if (s == y) return 100;
  int p = 100 + DiscreteLog(s, x, y, m);
  if (fpow(x, p, m) != y) return -1;
  return p;
}
```

6.13 Theorems

- **Cramer's Rule**

$$\begin{array}{l} ax+by=e \\ cx+dy=f \end{array} \Rightarrow \begin{array}{l} x=\dfrac{ed-bf}{ad-bc} \\ y=\dfrac{af-ec}{ad-bc} \end{array}$$

- **Vandermonde's Identity**

$$C(n+m,k)=\sum_{i=0}^{k}C(n,i)C(m,k-i)$$

- **Kirchhoff's Theorem**
  Denote L be a $n\times n$ matrix as the Laplacian matrix of graph G, where $L_{ii}=d(i)$, $L_{ij}=-c$ where c is the number of edge (i,j) in G.
  - The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
  - The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.
- **Tutte's Matrix**
  Let D be a $n\times n$ matrix, where $d_{ij}=x_{ij}$ (x_{ij} is chosen uniformly at random) if $i<j$ and $(i,j)\in E$, otherwise $d_{ij}=-d_{ji}$. $\frac{rank(D)}{2}$ is the maximum matching on G.
- **Cayley's Formula**
  - Given a degree sequence $d_1,d_2,\dots,d_n$ for each *labeled* vertices, there are $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!}$ spanning trees.
  - Let $T_{n,k}$ be the number of *labeled* forests on n vertices with k components, such that vertex $1,2,\dots,k$ belong to different components. Then $T_{n,k}=kn^{n-k-1}$.
- **Erdős-Gallai Theorem**
  A sequence of nonnegative integers $d_1\ge\cdots\ge d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1+\cdots+d_n$ is even and $\sum_{i=1}^{k}d_i\le k(k-1)+\sum_{i=k+1}^{n}\min(d_i,k)$ holds for every $1\le k\le n$.
- **Gale-Ryser Theorem**
  A pair of sequences of nonnegative integers $a_1\ge\cdots\ge a_n$ and $b_1,\dots,b_n$ is bigraphic (degree seqence of bipartie graph) if and only if $\sum_{i=1}^{n}a_i=\sum_{i=1}^{n}b_i$ and $\sum_{i=1}^{k}a_i\le\sum_{i=1}^{n}\min(b_i,k)$ holds for every $1\le k\le n$.
- **Fulkerson-Chen-Anstee Theorem**
  A sequence $(a_1,b_1),\dots,(a_n,b_n)$ of nonnegative integer pairs with $a_1\ge\cdots\ge a_n$ is digraphic (in, out degree of a directed graph) if and only if $\sum_{i=1}^{n}a_i=\sum_{i=1}^{n}b_i$ and $\sum_{i=1}^{k}a_i\le\sum_{i=1}^{k}\min(b_i,k-1)+\sum_{i=k+1}^{n}\min(b_i,k)$ holds for every $1\le k\le n$.
- **Möbius Inversion Formula**
  - $f(n)=\sum_{d|n}g(d)\Leftrightarrow g(n)=\sum_{d|n}\mu(d)f(\frac{n}{d})$
  - $f(n)=\sum_{n|d}g(d)\Leftrightarrow g(n)=\sum_{n|d}\mu(\frac{d}{n})f(d)$
- **Lagrange Multiplier**
  - Optimize $f(x_1,\dots,x_n)$ when k constraints $g_i(x_1,\dots,x_n)=0$.
  - Lagrangian function $\mathcal{L}(x_1,\dots,x_n,\lambda_1,\dots,\lambda_k)=f(x_1,\dots,x_n)-\sum_{i=1}^{k}\lambda_i g_i(x_1,\dots,x_n)$.
  - The solution corresponding to the original constrained optimization is always a saddle point of the Lagrangian function.

6.14 Estimation

- Estimation
  - Number of divisors

$n\le$	100	10^3	10^6	10^9	10^{12}	10^{15}	10^{18}
max$d(n)$	12	32	240	1344	6720	26880	103680

  - Unordered integer partition

n	2	3	4	5	6	7	8	9	20	30	40	50	100
$p(n)$	2	3	5	7	11	15	22	30	627	5604	$4\cdot10^4$	$2\cdot10^5$	$2\cdot10^8$

  - Ways of partitions of n distinct elements

n	2	3	4	5	6	7	8	9	10	11	12	13
B_n	2	5	15	52	203	877	4140	21147	115975	$7\cdot10^5$	$4\cdot10^6$	$3\cdot10^7$

6.15 Numbers

- Bernoulli numbers
  $B_0-1, B_1^{\pm}=\pm\frac{1}{2}, B_2=\frac{1}{6}, B_3=0$
  $\sum_{j=0}^{m}\binom{m+1}{j}B_j=0$, EGF is $B(x)=\frac{x}{e^x-1}=\sum_{n=0}^{\infty}B_n\frac{x^n}{n!}$.
  $S_m(n)=\sum_{k=1}^{n}k^m=\frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k}B_k^{+}n^{m+1-k}$
- Stirling numbers of the second kind Partitions of n distinct elements into exactly k groups.
  $S(n,k)=S(n-1,k-1)+kS(n-1,k), S(n,1)=S(n,n)=1$
  $S(n,k)=\frac{1}{k!}\sum_{i=0}^{k}(-1)^{k-i}\binom{k}{i}i^n$
  $x^n=\sum_{i=0}^{n}S(n,i)(x)_i$
- Pentagonal number theorem
  $\prod_{n=1}^{\infty}(1-x^n)=1+\sum_{k=1}^{\infty}(-1)^k\left(x^{k(3k+1)/2}+x^{k(3k-1)/2}\right)$
- Catalan numbers
  $C_n^{(k)}=\frac{1}{(k-1)n+1}\binom{kn}{n}$
  $C^{(k)}(x)=1+x[C^{(k)}(x)]^k$
- Eulerian numbers
  Number of permutations $\pi\in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j)>\pi(j+1)$, $k+1$ j:s s.t. $\pi(j)\geq j$, k j:s s.t. $\pi(j)>j$.
  $E(n,k)=(n-k)E(n-1,k-1)+(k+1)E(n-1,k)$
  $E(n,0)=E(n,n-1)=1$
  $E(n,k)=\sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$

6.16 GeneratingFunctions

- Ordinary Generating Function $A(x)=\sum_{i\geq 0}a_ix^i$
  - $A(rx)\Rightarrow r^na_n$
  - $A(x)+B(x)\Rightarrow a_n+b_n$
  - $A(x)B(x)\Rightarrow\sum_{i=0}^{n}a_ib_{n-i}$
  - $A(x)^k\Rightarrow\sum_{i_1+i_2+\cdots+i_k=n}a_{i_1}a_{i_2}\ldots a_{i_k}$
  - $xA(x)'\Rightarrow na_n$
  - $\frac{A(x)}{1-x}\Rightarrow\sum_{i=0}^{n}a_i$
- Exponential Generating Function $A(x)=\sum_{i\geq 0}\frac{a_i}{i!}x_i$
  - $A(x)+B(x)\Rightarrow a_n+b_n$
  - $A^{(k)}(x)\Rightarrow a_{n+k}$
  - $A(x)B(x)\Rightarrow\sum_{i=0}^{n}\binom{n}{i}a_ib_{n-i}$
  - $A(x)^k\Rightarrow\sum_{i_1+i_2+\cdots+i_k=n}\binom{n}{i_1,i_2,\ldots,i_k}a_{i_1}a_{i_2}\ldots a_{i_k}$
  - $xA(x)\Rightarrow na_n$
- Special Generating Function
  - $(1+x)^n=\sum_{i\geq 0}\binom{n}{i}x^i$
  - $\frac{1}{(1-x)^n}=\sum_{i\geq 0}\binom{i}{n-1}x^i$
  - $S_k=\sum_{x=1}^{n}x^k$: $S=\sum_{p=0}^{\infty}x^p=\frac{e^x-e^{x(n+1)}}{1-e^x}$

7 Linear Algebra

7.1 GuassianElimination

```
struct matrix { // m variables, n equations
  int n, m;
  fraction A[N][N + 1], sol[N];
  int solve() { //-1: inconsistent, >= 0: rank
    for (int i = 0; i < n; ++i) {
      int piv = 0;
      while (piv < m && !A[i][piv].n) ++piv;
      if (piv == m) continue;
      for (int j = 0; j < n; ++j) {
        if (i == j) continue;
        fraction tmp = -A[j][piv] / A[i][piv];
        for (int k = 0; k <= m; ++k)
          A[j][k] = tmp * A[i][k] + A[j][k];
      }
    }
    int rank = 0;
    for (int i = 0; i < n; ++i) {
      int piv = 0;
      while (piv < m && !A[i][piv].n) ++piv;
      if (piv == m && A[i][m].n) return -1;
      else if (piv < m)
        ++rank, sol[piv] = A[i][m] / A[i][piv];
    }
    return rank;
  }
};
```

7.2 BerlekampMassey

```
template <typename T>
vector<T> BerlekampMassey(const vector<T> &output) {
  vector<T> d(output.size() + 1), me, he;
  for (int f = 0, i = 1; i <= output.size(); ++i) {
    for (int j = 0; j < me.size(); ++j)
      d[i] += output[i - j - 2] * me[j];
    if ((d[i] -= output[i - 1]) == 0) continue;
    if (me.empty()) {
      me.resize(f = i);
      continue;
    }
    vector<T> o(i - f - 1);
    T k = -d[i] / d[f];
    o.emplace_back(-k);
    for (T x : he) o.emplace_back(x * k);
    o.resize(max(o.size(), me.size()));
    for (int j = 0; j < me.size(); ++j) o[j] += me[j];
    if (i - f + (int
        )he.size()) >= (int)me.size()) he = me, f = i;
    me = o;
  }
  return me;
}
```

7.3 Simplex

Standard form: maximize $\mathbf{c}^T\mathbf{x}$ subject to $A\mathbf{x}\leq\mathbf{b}$ and $\mathbf{x}\geq 0$.
Dual LP: minimize $\mathbf{b}^T\mathbf{y}$ subject to $A^T\mathbf{y}\geq\mathbf{c}$ and $\mathbf{y}\geq 0$.
$\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ are optimal if and only if for all $i\in[1,n]$, either $\bar{x}_i=0$ or $\sum_{j=1}^{m}A_{ji}\bar{y}_j=c_i$ holds and for all $i\in[1,m]$ either $\bar{y}_i=0$ or $\sum_{j=1}^{n}A_{ij}\bar{x}_j=b_j$ holds.

1. In case of minimization, let $c_i'=-c_i$
2. $\sum_{1\leq i\leq n}A_{ji}x_i\geq b_j\rightarrow\sum_{1\leq i\leq n}-A_{ji}x_i\leq -b_j$
3. $\sum_{1\leq i\leq n}A_{ji}x_i=b_j$
   - $\sum_{1\leq i\leq n}A_{ji}x_i\leq b_j$
   - $\sum_{1\leq i\leq n}A_{ji}x_i\geq b_j$
4. If x_i has no lower bound, replace x_i with x_i-x_i'

```
// n variable, m constraints, M >= n + 2m
struct simplex {
  const double inf = 1 / .0, eps = 1e-9;
  int n, m, k, var[N], inv[N], art[N];
  double A[M][N], B[M], x[N];
  void init(int _n) { n = _n, m = 0; }
  void equation(vector<double> a, double b) {
    for (int i = 0; i < n; i++) A[m][i] = a[i];
    B[m] = b, var[m] = n + m, ++m;
  }
  void pivot(int r, int c, double bx) {
    for (int i = 0; i <= m + 1; i++)
      if (i != r && abs(A[i][c]) > eps) {
        x[var[i]] -= bx * A[i][c] / A[i][var[i]];
        double f = A[i][c] / A[r][c];
        for (int j = 0; j <= n + m + k; j++)
          A[i][j] -= A[r][j] * f;
        B[i] -= B[r] * f;
      }
  }
  double phase(int p) {
    while (true) {
      int in = min_element(
                 A[m + p], A[m + p] + n + m + k + 1) -
        A[m + p];
      if (A[m + p][in] >= -eps) break;
      double bx = inf;
      int piv = -1;
      for (int i = 0; i < m; i++)
        if (A[i][in] > eps && B[i] / A[i][in] <= bx)
          piv = i, bx = B[i] / A[i][in];
      if (piv == -1) return inf;
      int out = var[piv];
      pivot(piv, in, bx);
      x[out] = 0, x[in] = bx, var[piv] = in;
    }
    return x[n + m];
  }
```

```
  double solve(vector<double> c) {
    auto invert = [&](int r) {
      for (int j = 0; j <= n + m; j++) A[r][j] *= -1;
      B[r] *= -1;
    };
    k = 1;
    for (int i = 0; i < n; i++) A[m][i] = -c[i];
    fill(A[m + 1], A[m + 1] + N, 0.0);
    for (int i = 0; i <= m + 1; i++)
      fill(A[i] + n, A[i] + n + m + 2, 0.0),
        var[i] = n + i, A[i][n + i] = 1;

    for (int i = 0; i < m; i++) {
      if (B[i] < 0) {
        ++k;
        for (int j = 0; j <= n + m; j++)
          A[m + 1][j] += A[i][j];
        invert(i);
        var[i] = n + m + k, A[i][var[i]] = 1,
        art[var[i]] = n + i;
      }
      x[var[i]] = B[i];
    }

    phase(1);
    if (*max_element(
          x + (n + m + 2), x + (n + m + k + 1)) > eps)
      return .0 / .0;
    for (int i = 0; i <= m; i++)
      if (var[i] > n + m)
        var[i] = art[var[i]], invert(i);
    k = 0;
    return phase(0);
  }
} lp;
```

8 Polynomials

8.1 NTT (FFT)

Mod	g	Form
65 537	3	$2^{16}+1$
998 244 353	3	$119\cdot2^{23}+1$
1 315 962 881	3	$1255\cdot2^{20}+1$
1 711 276 033	29	$51\cdot2^{25}+1$
9 223 372 036 737 335 297	29	$549755813881\cdot2^{24}+1$

```
#define base ll // complex<double>
// const double PI = acosl(-1);
const ll mod = 998244353, g = 3;
base omega[4 * N], omega_[4 * N];
int rev[4 * N];

ll fpow(ll b, ll p);

ll inverse(ll a) { return fpow(a, mod - 2); }

void calcW(int n) {
  ll r = fpow(g, (mod - 1) / n), invr = inverse(r);
  omega[0] = omega_[0] = 1;
  for (int i = 1; i < n; i++) {
    omega[i] = omega[i - 1] * r % mod;
    omega_[i] = omega_[i - 1] * invr % mod;
  }
  // double arg = 2.0 * PI / n;
  // for (int i = 0; i < n; i++)
  // {
  //   omega[i] = base(cos(i * arg), sin(i * arg));
  //   omega_[i] = base(cos(-i * arg), sin(-i * arg));
  // }
}

void calcrev(int n) {
  int k = __lg(n);
  for (int i = 0; i < n; i++) rev[i] = 0;
  for (int i = 0; i < n; i++)
    for (int j = 0; j < k; j++)
      if (i & (1 << j)) rev[i] ^= 1 << (k - j - 1);
}

vector<base> NTT(vector<base> poly, bool inv) {
  base *w = (inv ? omega_ : omega);
  int n = poly.size();
  for (int i = 0; i < n; i++)
    if (rev[i] > i) swap(poly[i], poly[rev[i]]);

  for (int len = 1; len < n; len <<= 1) {
    int arg = n / len / 2;
    for (int i = 0; i < n; i += 2 * len)
      for (int j = 0; j < len; j++) {
        base odd =
          w[j * arg] * poly[i + j + len] % mod;
        poly[i + j + len] =
          (poly[i + j] - odd + mod) % mod;
        poly[i + j] = (poly[i + j] + odd) % mod;
      }
  }
  if (inv)
    for (auto &a : poly) a = a * inverse(n) % mod;
  return poly;
}

vector<base> mul(vector<base> f, vector<base> g) {
  int sz = 1 << (__lg(f.size() + g.size() - 1) + 1);
  f.resize(sz), g.resize(sz);
  calcrev(sz);
  calcW(sz);
  f = NTT(f, 0), g = NTT(g, 0);
  for (int i = 0; i < sz; i++)
    f[i] = f[i] * g[i] % mod;
  return NTT(f, 1);
}
```

8.2 FHWT

```
/* x: a[j], y: a[j + (L >> 1)]
or: (y += x * op), and: (x += y * op)
xor: (x, y = (x + y) * op, (x - y) * op)
invop: or, and, xor = -1, -1, 1/2 */
void fwt(int *a, int n, int op) { //or
  for (int L = 2; L <= n; L <<= 1)
    for (int i = 0; i < n; i += L)
      for (int j = i; j < i + (L >> 1); ++j)
        a[j + (L >> 1)] += a[j] * op;
}
const int N = 21;
int f[
    N][1 << N], g[N][1 << N], h[N][1 << N], ct[1 << N];
void
    subset_convolution(int *a, int *b, int *c, int L) {
  // c_k = \sum_{i | j = k, i & j = 0} a_i * b_j
  int n = 1 << L;
  for (int i = 1; i < n; ++i)
    ct[i] = ct[i & (i - 1)] + 1;
  for (int i = 0; i < n; ++i)
    f[ct[i]][i] = a[i], g[ct[i]][i] = b[i];
  for (int i = 0; i <= L; ++i)
    fwt(f[i], n, 1), fwt(g[i], n, 1);
  for (int i = 0; i <= L; ++i)
    for (int j = 0; j <= i; ++j)
      for (int x = 0; x < n; ++x)
        h[i][x] += f[j][x] * g[i - j][x];
  for (int i = 0; i <= L; ++i)
    fwt(h[i], n, -1);
  for (int i = 0; i < n; ++i)
    c[i] = h[ct[i]][i];
}
```

8.3 PolynomialOperations

```
#define poly vector<ll>
poly inv(poly A) {
  A.resize(1 << (__lg(A.size() - 1) + 1));
  poly B = {inverse(A[0])};
  for (int n = 1; n < (int)A.size(); n <<= 1) {
```

```
    poly pA(A.begin(), A.begin() + 2 * n);
    calcrev(4 * n), calcW(4 * n);
    pA.resize(4 * n), B.resize(4 * n);
    pA = NTT(pA, 0);
    B = NTT(B, 0);
    for (int i = 0; i < 4 * n; i++)
      B[i] =
        ((B[i] * 2 - pA[i] * B[i] % mod * B[i]) % mod +
          mod) %
        mod;
    B = NTT(B, 1);
    B.resize(2 * n);
  }
  return B;
}
pair<poly, poly> div(poly A, poly B) {
  if (A.size() < B.size()) return make_pair(poly(), A);
  int n = A.size(), m = B.size();
  poly revA = A, invrevB = B;
  reverse(all(revA)), reverse(all(invrevB));
  revA.resize(n - m + 1);
  invrevB.resize(n - m + 1);
  invrevB = inv(invrevB);
  poly Q = mul(revA, invrevB);
  Q.resize(n - m + 1);
  reverse(all(Q));
  poly R = mul(Q, B);
  R.resize(m - 1);
  for (int i = 0; i < m - 1; i++)
    R[i] = (A[i] - R[i] + mod) % mod;
  return make_pair(Q, R);
}
poly modulo(poly A, poly B) { return div(A, B).S; }
ll fast_kitamasa(ll k, poly A, poly C) {
  int n = A.size();
  C.emplace_back(mod - 1);
  poly Q, R = {0, 1}, F = {1};
  R = modulo(R, C);
  for (; k; k >>= 1) {
    if (k & 1) F = modulo(mul(F, R), C);
    R = modulo(mul(R, R), C);
    k >>= 1;
  }
  ll ans = 0;
  for (int i = 0; i < F.size(); i++)
    ans = (ans + A[i] * F[i]) % mod;
  return ans;
}
vector<ll> fpow(vector<ll> f, ll p, ll m) {
  int b = 0;
  while (b < f.size() && f[b] == 0) b++;
  f = vector<ll>(f.begin() + b, f.end());
  int n = f.size();
  f.emplace_back(0);
  vector<ll> q(min(m, b * p), 0);
  q.emplace_back(fpow(f[0], p));
  for (int k = 0; q.size() < m; k++) {
    ll res = 0;
    for (int i = 0; i < min(n, k + 1); i++)
      res = (res +
              p * (i + 1) % mod * f[i + 1] % mod *
                q[k - i + b * p]) %
        mod;
    for (int i = 1; i < min(n, k + 1); i++)
      res = (res -
              f[i] * (k - i + 1) % mod *
                q[k - i + 1 + b * p]) %
        mod;
    res = (res < 0 ? res + mod : res) *
      inv(f[0] * (k + 1) % mod) % mod;
    q.emplace_back(res);
  }
  return q;
}
```

8.4 NewtonMethod+MiscGF

Given $F(x)$ where

$$F(x)=\sum_{i=0}^{\infty}\alpha_i(x-\beta)^i$$

for β being some constant. Polynomial P such that $F(P)=0$ can be found iteratively. Denote by Q_k the polynomial such that $F(Q_k)=0 \pmod{x^{2^k}}$, then

$$Q_{k+1}=Q_k-\frac{F(Q_k)}{F'(Q_k)} \pmod{x^{2^{k+1}}}$$

- A^{-1}: $B_{k+1}=B_k(2-AB_k)$ mod$x^{2^{k+1}}$
- ln$A$: $(\ln A)'=\frac{A'}{A}$
- exp$A$: $B_{k+1}=B_k(1+A-\ln B_k)$ mod$x^{2^{k+1}}$
- $\sqrt{A}$: $B_{k+1}=\frac{1}{2}(B_k+AB_k^{-1})$ mod$x^{2^{k+1}}$

9 Geometry

9.1 Basic

```
typedef pair<pdd, pdd> Line;
struct Cir{ pdd O; double R; };
const double eps = 1e-8;
pll operator+(pll a, pll b)
{ return pll(a.F + b.F, a.S + b.S); }
pll operator-(pll a, pll b)
{ return pll(a.F - b.F, a.S - b.S); }
pll operator-(pll a)
{ return pll(-a.F, -a.S); }
pll operator*(pll a, ll b)
{ return pll(a.F * b, a.S * b); }
pdd operator/(pll a, double b)
{ return pdd(a.F / b, a.S / b); }
ll dot(pll a, pll b)
{ return a.F * b.F + a.S * b.S; }
ll cross(pll a, pll b)
{ return a.F * b.S - a.S * b.F; }
ll abs2(pll a)
{ return dot(a, a); }
double abs(pll a)
{ return sqrt(dot(a, a)); }
int sign(ll a)
{ return fabs(a) < eps ? 0 : a > 0 ? 1 : -1; }
int ori(pll a, pll b, pll c)
{ return sign(cross(b - a, c - a)); }
bool collinearity(pll p1, pll p2, pll p3)
{ return sign(cross(p1 - p3, p2 - p3)) == 0; }
bool btw(pll a, pll b, pll c) {
  return collinearity
      (a, b, c) && sign(dot(a - c, b - c)) <= 0;
}
bool seg_intersect(pdd p1, pdd p2, pdd p3, pdd p4) {
  int a123 = ori(p1, p2, p3);
  int a124 = ori(p1, p2, p4);
  int a341 = ori(p3, p4, p1);
  int a342 = ori(p3, p4, p2);
  if (a123 == 0 && a124 == 0)
    return btw(p1, p2, p3) || btw(p1, p2, p4) ||
      btw(p3, p4, p1) || btw(p3, p4, p2);
  return a123 * a124 <= 0 && a341 * a342 <= 0;
}
pdd intersect(pdd p1, pdd p2, pdd p3, pdd p4) {
  double a123 = cross(p2 - p1, p3 - p1);
  double a124 = cross(p2 - p1, p4 - p1);
  return (p4
      * a123 - p3 * a124) / (a123 - a124); // C^3 / C^2
}
pdd perp(pdd p1)
{ return pdd(-p1.S, p1.F); }
pdd projection(pdd p1, pdd p2, pdd p3)
{ return p1 + (
    p2 - p1) * dot(p3 - p1, p2 - p1) / abs2(p2 - p1); }
pdd reflection(pdd p1, pdd p2, pdd p3)
{ return p3 + perp(p2 - p1
    ) * cross(p3 - p1, p2 - p1) / abs2(p2 - p1) * 2; }
pdd linearTransformation
    (pdd p0, pdd p1, pdd q0, pdd q1, pdd r) {
```

```
  pdd dp = p1 - p0
      , dq = q1 - q0, num(cross(dp, dq), dot(dp, dq));
  return q0 + pdd(
      cross(r - p0, num), dot(r - p0, num)) / abs2(dp);
} // from line p0--p1 to q0--q1, apply to r
```

9.2 ConvexHull

```
void hull(vector<pll> &dots) { // n=1 => ans = {}
  sort(dots.begin(), dots.end());
  vector<pll> ans(1, dots[0]);
  for (int ct = 0; ct < 2; ++ct, reverse(all(dots)))
    for (int i = 1, t = ans.size()
        ; i < dots.size(); ans.emplace_back(dots[i++]))
      while (ans.size() > t &&
          ori(ans.end()[-2], ans.back(), dots[i]) <= 0)
        ans.pop_back();
  ans.pop_back(), ans.swap(dots);
}
```

9.3 SortByAngle

```
int cmp(pll a, pll b, bool same = true) {
#define is_neg(k) (
    sign(k.S) < 0 || (sign(k.S) == 0 && sign(k.F) < 0))
  int A = is_neg(a), B = is_neg(b);
  if (A != B)
    return A < B;
  if (sign(cross(a, b)) == 0)
    return same ? abs2(a) < abs2(b) : -1;
  return sign(cross(a, b)) > 0;
}
```

9.4 Formulas

- **Rotation**

$$M(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

  90 degree: $(x,y)=(Y-y,x)$
- **Pick's theorem**
  For simple integer-coordinate polygon,

$$A=B+\frac{I}{2}-1$$

  Where A is the area; B,I is #lattice points in the interior, on the boundary.
- **Spherical Cap**
  - A portion of a sphere cut off by a plane.
  - r: sphere radius, a: radius of the base of the cap, h: height of the cap, θ: $\arcsin(a/r)$.
  - Volume $=\pi h^2(3r-h)/3=\pi h(3a^2+h^2)/6=\pi r^3(2+\cos\theta)(1-\cos\theta)^2/3$.
  - Area $=2\pi rh=\pi(a^2+h^2)=2\pi r^2(1-\cos\theta)$.
- **Nearest points of two skew lines**
  - Line 1: $\boldsymbol{v}_1=\boldsymbol{p}_1+t_1\boldsymbol{d}_1$
  - Line 2: $\boldsymbol{v}_2=\boldsymbol{p}_2+t_2\boldsymbol{d}_2$
  - $\boldsymbol{n}=\boldsymbol{d}_1\times\boldsymbol{d}_2$
  - $\boldsymbol{n}_1=\boldsymbol{d}_1\times\boldsymbol{n}$
  - $\boldsymbol{n}_2=\boldsymbol{d}_2\times\boldsymbol{n}$
  - $\boldsymbol{c}_1=\boldsymbol{p}_1+\frac{(\boldsymbol{p}_2-\boldsymbol{p}_1)\cdot\boldsymbol{n}_2}{\boldsymbol{d}_1\cdot\boldsymbol{n}_2}\boldsymbol{d}_1$
  - $\boldsymbol{c}_2=\boldsymbol{p}_2+\frac{(\boldsymbol{p}_1-\boldsymbol{p}_2)\cdot\boldsymbol{n}_1}{\boldsymbol{d}_2\cdot\boldsymbol{n}_1}\boldsymbol{d}_2$

9.5 TriangleHearts

```
pdd excenter(
  pdd p0, pdd p1, pdd p2) { // radius = abs(center)
  p1 = p1 - p0, p2 = p2 - p0;
  auto [x1, y1] = p1;
  auto [x2, y2] = p2;
  double m = 2. * cross(p1, p2);
  pdd center = pdd((x1 * x1 * y2 - x2 * x2 * y1 +
                     y1 * y2 * (y1 - y2)),
    (x1 * x2 * (x2 - x1) - y1 * y1 * x2 +
      x1 * y2 * y2)) / m;
  return center + p0;
}
pdd incenter(
  pdd p1, pdd p2, pdd p3) { // radius = area / s * 2
  double a = abs(p2 - p3), b = abs(p1 - p3),
         c = abs(p1 - p2);
  double s = a + b + c;
  return (a * p1 + b * p2 + c * p3) / s;
}
pdd masscenter(pdd p1, pdd p2, pdd p3) {
  return (p1 + p2 + p3) / 3;
}
pdd orthcenter(pdd p1, pdd p2, pdd p3) {
  return masscenter(p1, p2, p3) * 3 -
    excenter(p1, p2, p3) * 2;
}
```

9.6 PointSegmentDist

```
double PointSegDist(pdd q0, pdd q1, pdd p) {
  if (abs(q0 - q1) <= eps) return abs(q0 - p);
  if (dot(q1 - q0,
       p - q0) >= -eps && dot(q0 - q1, p - q1) >= -eps)
    return fabs(cross(q1 - q0, p - q0) / abs(q0 - q1));
  return min(abs(p - q0), abs(p - q1));
}
```

9.7 PointInCircle

```
// return q'
    s relation with circumcircle of tri(p[0],p[1],p[2])
bool in_cc(const array<pll, 3> &p, pll q) {
  __int128 det = 0;
  for (int i = 0; i < 3; ++i)
    det += __int128(abs2(p[i]) - abs2(q)) *
           cross(p[(i + 1) % 3] - q, p[(i + 2) % 3] - q);
  return det > 0; // in: >0, on: =0, out: <0
}
```

9.8 PointInConvex

```
bool PointInConvex
    (const vector<pll> &C, pll p, bool strict = true) {
  int a = 1, b = (int)C.size() - 1, r = !strict;
  if ((int)C.size() == 0) return false;
  if ((int)
       C.size() < 3) return r && btw(C[0], C.back(), p);
  if (ori(C[0], C[a], C[b]) > 0) swap(a, b);
  if (ori
       (C[0], C[a], p) >= r || ori(C[0], C[b], p) <= -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (ori(C[0], C[c], p) > 0 ? b : a) = c;
  }
  return ori(C[a], C[b], p) < r;
}
```

9.9 PointTangentConvex

```
/* The point should be strictly out of hull
  return arbitrary point on the tangent line */
/* bool pred(int a, int b);
f(0) ~ f(n - 1) is a cyclic-shift U-function
return idx s.t. pred(x, idx) is false forall x*/
int cyc_tsearch(int n, auto pred) {
  if (n == 1) return 0;
  int l = 0, r = n; bool rv = pred(1, 0);
  while (r - l > 1) {
    int m = (l + r) / 2;
    if (pred(0, m) ? rv: pred(m, (m + 1) % n)) r = m;
    else l = m;
  }
  return pred(l, r % n) ? l : r % n;
}
pii get_tangent(vector<pll> &C, pll p) {
  auto gao = [&](int s) {
    return cyc_tsearch((int)C.size(), [&](int x, int y)
    { return ori(p, C[x], C[y]) == s; });
  };
  return pii(gao(1), gao(-1));
} // return (a, b), ori(p, C[a], C[b]) >= 0
```

9.10 CircTangentCirc

```
vector<Line
    > go( const Cir& c1 , const Cir& c2 , int sign1 ){
  // sign1 = 1 for outer tang, -1 for inter tang
  vector<Line> ret;
  double d_sq = abs2(c1.O - c2.O);
  if (sign(d_sq) == 0) return ret;
  double d = sqrt(d_sq);
  pdd v = (c2.O - c1.O) / d;
  double c = (c1.R - sign1 * c2.R) / d;
  if (c * c > 1) return ret;
  double h = sqrt(max(0.0, 1.0 - c * c));
  for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
    pdd n = pdd(v.F * c - sign2 * h * v.S,
      v.S * c + sign2 * h * v.F);
    pdd p1 = c1.O + n * c1.R;
    pdd p2 = c2.O + n * (c2.R * sign1);
    if (sign(p1.F - p2.F) == 0 and
        sign(p1.S - p2.S) == 0)
      p2 = p1 + perp(c2.O - c1.O);
    ret.emplace_back(Line(p1, p2));
  }
  return ret;
}
```

9.11 LineCircleIntersect

```
vector<pdd> circleLine(pdd c, double r, pdd a, pdd b) {
  pdd p
     = a + (b - a) * dot(c - a, b - a) / abs2(b - a);
  double s = cross
      (b - a, c - a), h2 = r * r - s * s / abs2(b - a);
  if (h2 < 0) return {};
  if (h2 == 0) return {p};
  pdd h = (b - a) / abs(b - a) * sqrt(h2);
  return {p - h, p + h};
}
```

9.12 LineConvexIntersect

```
int TangentDir(vector<pll> &C, pll dir) {
  return cyc_tsearch((int)C.size(), [&](int a, int b) {
    return cross(dir, C[a]) > cross(dir, C[b]);
  });
}
#define cmpL(i) sign(cross(C[i] - a, b - a))
pii lineHull(pll a, pll b, vector<pll> &C) {
  int A = TangentDir(C, a - b);
  int B = TangentDir(C, b - a);
  int n = (int)C.size();
  if (cmpL(A) < 0 || cmpL(B) > 0)
    return pii(-1, -1); // no collision
  auto gao = [&](int l, int r) {
    for (int t = l; (l + 1) % n != r; ) {
      int m = ((l + r + (l < r ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(t) ? l : r) = m;
    }
    return (l + !cmpL(r)) % n;
  };
  pii res = pii(gao(B, A), gao(A, B)); // (i, j)
  if (res.F == res.S) // touching the corner i
    return pii(res.F, -1);
  if (!
      cmpL(res.F) && !cmpL(res.S)) // along side i, i+1
    switch ((res.F - res.S + n + 1) % n) {
      case 0: return pii(res.F, res.F);
      case 2: return pii(res.S, res.S);
    }
  /* crossing sides (i, i+1) and (j, j+1)
  crossing corner i is treated as side (i, i+1)
  returned
       in the same order as the line hits the convex */
  return res;
} // convex cut: (r, l]
```

9.13 CircIntersectCirc

```
bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
  pdd o1 = a.O, o2 = b.O;
  double r1 =
       a.R, r2 = b.R, d2 = abs2(o1 - o2), d = sqrt(d2);
  if(d < max
      (r1, r2) - min(r1, r2) || d > r1 + r2) return 0;
  pdd u = (o1 + o2) * 0.5
       + (o1 - o2) * ((r2 * r2 - r1 * r1) / (2 * d2));
  double A = sqrt((r1 + r2 + d) *
       (r1 - r2 + d) * (r1 + r2 - d) * (-r1 + r2 + d));
  pdd v
       = pdd(o1.S - o2.S, -o1.F + o2.F) * A / (2 * d2);
  p1 = u + v, p2 = u - v;
  return 1;
}
```

9.14 PolyIntersectCirc

```
// Divides into multiple triangle, and sum up
const double PI = acos(-1);
double _area(pdd pa, pdd pb, double r) {
  if (abs(pa) < abs(pb)) swap(pa, pb);
  if (abs(pb) < eps) return 0;
  double S, h, theta;
  double a = abs(pb), b = abs(pa), c = abs(pb - pa);
  double cosB = dot(pb, pb - pa) / a / c,
         B = acos(cosB);
  double cosC = dot(pa, pb) / a / b, C = acos(cosC);
  if (a > r) {
    S = (C / 2) * r * r;
    h = a * b * sin(C) / c;
    if (h < r && B < PI / 2)
      S -= (acos(h / r) * r * r -
        h * sqrt(r * r - h * h));
  } else if (b > r) {
    theta = PI - B - asin(sin(B) / r * a);
    S = .5 * a * r * sin(theta) +
      (C - theta) / 2 * r * r;
  } else S = .5 * sin(C) * a * b;
  return S;
}
double area_poly_circle(const vector<pdd> poly,
  const pdd &O, const double r) {
  double S = 0;
  for (int i = 0; i < (int)poly.size(); ++i)
    S += _area(poly[i] - O,
           poly[(i + 1) % (int)poly.size()] - O, r) *
      ori(
        O, poly[i], poly[(i + 1) % (int)poly.size()]);
  return fabs(S);
}
```

9.15 MinkowskiSum

```
vector<pll> Minkowski
    (vector<pll> A, vector<pll> B) { // |A|,|B|>=3
  hull(A), hull(B);
  vector<pll> C(1, A[0] + B[0]), s1, s2;
  for (int i = 0; i < A.size(); ++i)
    s1.emplace_back(A[(i + 1) % A.size()] - A[i]);
  for (int i = 0; i < B.size(); i++)
    s2.emplace_back(B[(i + 1) % B.size()] - B[i]);
  for (int i = 0, j = 0; i < A.size() || j < B.size();)
    if (j >= B.size()
        || (i < A.size() && cross(s1[i], s2[j]) >= 0))
      C.emplace_back(B[j % B.size()] + A[i++]);
    else
      C.emplace_back(A[i % A.size()] + B[j++]);
  return hull(C), C;
}
```

9.16 MinMaxEnclosingRect

```
const double INF = 1e18, qi = acos(-1) / 2 * 3;
pdd solve(vector<pll> &dots) {
#define diff(u, v) (dots[u] - dots[v])
```

```
#define vec(v) (dots[v] - dots[i])
  hull(dots);
  double Max = 0, Min = INF, deg;
  int n = (int)dots.size();
  dots.emplace_back(dots[0]);
  for (int i = 0, u = 1, r = 1, l = 1; i < n; ++i) {
    pll nw = vec(i + 1);
    while (cross(nw, vec(u + 1)) > cross(nw, vec(u)))
      u = (u + 1) % n;
    while (dot(nw, vec(r + 1)) > dot(nw, vec(r)))
      r = (r + 1) % n;
    if (!i) l = (r + 1) % n;
    while (dot(nw, vec(l + 1)) < dot(nw, vec(l)))
      l = (l + 1) % n;
    Min = min(Min, (double)(dot(nw, vec(r)) - dot
        (nw, vec(l))) * cross(nw, vec(u)) / abs2(nw));
    deg = acos(dot(diff(r
        , l), vec(u)) / abs(diff(r, l)) / abs(vec(u)));
    deg = (qi - deg) / 2;
    Max = max(Max, abs(diff
        (r, l)) * abs(vec(u)) * sin(deg) * sin(deg));
  }
  return pdd(Min, Max);
}
```

9.17 MinEnclosingCircle

```
pdd Minimum_Enclosing_Circle
    (vector<pdd> dots, double &r) {
  pdd cent;
  random_shuffle(all(dots));
  cent = dots[0], r = 0;
  for (int i = 1; i < (int)dots.size(); ++i)
    if (abs(dots[i] - cent) > r) {
      cent = dots[i], r = 0;
      for (int j = 0; j < i; ++j)
        if (abs(dots[j] - cent) > r) {
          cent = (dots[i] + dots[j]) / 2;
          r = abs(dots[i] - cent);
          for(int k = 0; k < j; ++k)
            if(abs(dots[k] - cent) > r)
              cent = excenter
                  (dots[i], dots[j], dots[k], r);
        }
    }
  return cent;
}
```

9.18 CircleCover

```
const int N = 1021;
struct CircleCover {
  int C;
  Cir c[N];
  bool g[N][N], overlap[N][N];
  // Area[i] : area covered by at least i circles
  double Area[ N ];
  void init(int _C){ C = _C;}
  struct Teve {
    pdd p; double ang; int add;
    Teve() {}
    Teve(pdd _a
        , double _b, int _c):p(_a), ang(_b), add(_c){}
    bool operator<(const Teve &a)const
    {return ang < a.ang;}
  }eve[N * 2];
  // strict: x = 0, otherwise x = -1
  bool disjuct(Cir &a, Cir &b, int x)
  {return sign(abs(a.O - b.O) - a.R - b.R) > x;}
  bool contain(Cir &a, Cir &b, int x)
  {return sign(a.R - b.R - abs(a.O - b.O)) > x;}
  bool contain(int i, int j) {
    /* c[j] is non-strictly in c[i]. */
    return (sign
        (c[i].R - c[j].R) > 0 || (sign(c[i].R - c[j].
        R) == 0 && i < j)) && contain(c[i], c[j], -1);
  }
  void solve(){
    fill_n(Area, C + 2, 0);
    for(int i = 0; i < C; ++i)
      for(int j = 0; j < C; ++j)
        overlap[i][j] = contain(i, j);
    for(int i = 0; i < C; ++i)
      for(int j = 0; j < C; ++j)
        g[i][j] = !(overlap[i][j] || overlap[j][i] ||
            disjuct(c[i], c[j], -1));
    for(int i = 0; i < C; ++i){
      int E = 0, cnt = 1;
      for(int j = 0; j < C; ++j)
        if(j != i && overlap[j][i])
          ++cnt;
      for(int j = 0; j < C; ++j)
        if(i != j && g[i][j]) {
          pdd aa, bb;
          CCinter(c[i], c[j], aa, bb);
          double A =
              atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
          double B =
              atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
          eve[E++] = Teve
              (bb, B, 1), eve[E++] = Teve(aa, A, -1);
          if(B > A) ++cnt;
        }
      if(E == 0) Area[cnt] += pi * c[i].R * c[i].R;
      else{
        sort(eve, eve + E);
        eve[E] = eve[0];
        for(int j = 0; j < E; ++j){
          cnt += eve[j].add;
          Area[cnt
              ] += cross(eve[j].p, eve[j + 1].p) * .5;
          double theta = eve[j + 1].ang - eve[j].ang;
          if (theta < 0) theta += 2. * pi;
          Area[cnt] += (theta
              - sin(theta)) * c[i].R * c[i].R * .5;
        }
      }
    }
  }
};
```

9.19 LineCmp

```
using Line = pair<pll, pll>;
struct lineCmp { // coordinates should be even!
  bool operator()(Line l1, Line l2) const {
    int X =
      (max(l1.F.F, l2.F.F) + min(l1.S.F, l2.S.F)) / 2;
    ll p1 =
         (X - l1.F.F) * l1.S.S + (l1.S.F - X) * l1.F.S,
       p2 =
         (X - l2.F.F) * l2.S.S + (l2.S.F - X) * l2.F.S,
       q1 = (l1.S.F - l1.F.F), q2 = (l2.S.F - l2.F.F);
    if (q1 == 0) p1 = l1.F.S + l1.S.S, q1 = 2;
    if (q2 == 0) p2 = l2.F.S + l2.S.S, q2 = 2;
    if (l1.F == l2.F || l2.F == l2.S) l1 = l2;
    return make_tuple((__int128)(p1 * q2), l1) <
      make_tuple((__int128)(p2 * q1), l2);
  }
};
```

9.20 Trapezoidalization

```
template<class T>
struct SweepLine {
  struct cmp {
    cmp(const SweepLine &_swp): swp(_swp) {}
    bool operator()(int a, int b) const {
      if (abs(swp.get_y(a) - swp.get_y(b)) <= swp.eps)
        return swp.slope_cmp(a, b);
      return swp.get_y(a) + swp.eps < swp.get_y(b);
    }
    const SweepLine &swp;
  } _cmp;
```

```
  T curTime, eps, curQ;
  vector<Line> base;
  multiset<int, cmp> sweep;
  multiset<pair<T, int>> event;
  vector<typename multiset<int, cmp>::iterator> its;
  vector
      <typename multiset<pair<T, int>>::iterator> eits;
  bool slope_cmp(int a, int b) const {
    assert(a != -1);
    if (b == -1) return 0;
    return sign(cross(base
        [a].Y - base[a].X, base[b].Y - base[b].X)) < 0;
  }
  T get_y(int idx) const {
    if (idx == -1) return curQ;
    Line l = base[idx];
    if (l.X.X == l.Y.X) return l.Y.Y;
    return ((curTime - l.X.X) * l.Y.Y
        + (l.Y.X - curTime) * l.X.Y) / (l.Y.X - l.X.X);
  }
  void insert(int idx) {
    its[idx] = sweep.insert(idx);
    if (its[idx] != sweep.begin())
      update_event(*prev(its[idx]));
    update_event(idx);
    event.emplace
        (base[idx].Y.X, idx + 2 * (int)base.size());
  }
  void erase(int idx) {
    assert(eits[idx] == event.end());
    auto p = sweep.erase(its[idx]);
    its[idx] = sweep.end();
    if (p != sweep.begin())
      update_event(*prev(p));
  }
  void update_event(int idx) {
    if (eits[idx] != event.end())
      event.erase(eits[idx]);
    eits[idx] = event.end();
    auto nxt = next(its[idx]);
    if (nxt ==
         sweep.end() || !slope_cmp(idx, *nxt)) return;
    auto t = intersect(base[idx].
        X, base[idx].Y, base[*nxt].X, base[*nxt].Y).X;
    if (t + eps < curTime || t
         >= min(base[idx].Y.X, base[*nxt].Y.X)) return;
    eits[idx
        ] = event.emplace(t, idx + (int)base.size());
  }
  void swp(int idx) {
    assert(eits[idx] != event.end());
    eits[idx] = event.end();
    int nxt = *next(its[idx]);
    swap((int&)*its[idx], (int&)*its[nxt]);
    swap(its[idx], its[nxt]);
    if (its[nxt] != sweep.begin())
      update_event(*prev(its[nxt]));
    update_event(idx);
  }
  // only expected to call the functions below
  SweepLine(T t, T e, vector<Line> vec): _cmp
      (*this), curTime(t), eps(e), curQ(), base(vec),
       sweep(_cmp), event(), its((int)vec.size(), sweep
      .end()), eits((int)vec.size(), event.end()) {
    for (int i = 0; i < (int)base.size(); ++i) {
      auto &[p, q] = base[i];
      if (p > q) swap(p, q);
      if (p.X <= curTime && curTime <= q.X)
        insert(i);
      else if (curTime < p.X)
        event.emplace(p.X, i);
    }
  }
  void setTime(T t, bool ers = false) {
    assert(t >= curTime);
    while (!event.empty() && event.begin()->X <= t) {
      auto [et, idx] = *event.begin();
      int s = idx / (int)base.size();
      idx %= (int)base.size();
      if (abs(et - t) <= eps && s == 2 && !ers) break;
      curTime = et;
      event.erase(event.begin());
      if (s == 2) erase(idx);
      else if (s == 1) swp(idx);
      else insert(idx);
    }
    curTime = t;
  }
  T nextEvent() {
    if (event.empty()) return INF;
    return event.begin()->X;
  }
  int lower_bound(T y) {
    curQ = y;
    auto p = sweep.lower_bound(-1);
    if (p == sweep.end()) return -1;
    return *p;
  }
};
```

9.21 HalfPlaneIntersect

```
pll area_pair(Line a, Line b)
{ return pll(cross(a.S
    - a.F, b.F - a.F), cross(a.S - a.F, b.S - a.F)); }
bool isin(Line l0, Line l1, Line l2) {
  // Check inter(l1, l2) strictly in l0
  auto [a02X, a02Y] = area_pair(l0, l2);
  auto [a12X, a12Y] = area_pair(l1, l2);
  if (a12X - a12Y < 0) a12X *= -1, a12Y *= -1;
  return (__int128)
      a02Y * a12X - (__int128) a02X * a12Y > 0; // C^4
}
/* Having solution, check size > 2 */
/* --^-- Line.X --^-- Line.Y --^-- */
vector<Line> halfPlaneInter(vector<Line> arr) {
  sort(all(arr), [&](Line a, Line b) -> int {
    if (cmp(a.S - a.F, b.S - b.F, 0) != -1)
      return cmp(a.S - a.F, b.S - b.F, 0);
    return ori(a.F, a.S, b.S) < 0;
  });
  deque<Line> dq(1, arr[0]);
  for (auto p : arr) {
    if (cmp(
        dq.back().S - dq.back().F, p.S - p.F, 0) == -1)
      continue;
    while ((int)dq.size() >= 2
        && !isin(p, dq[(int)dq.size() - 2], dq.back()))
      dq.pop_back();
    while
        ((int)dq.size() >= 2 && !isin(p, dq[0], dq[1]))
      dq.pop_front();
    dq.emplace_back(p);
  }
  while ((int)dq.size() >= 3 &&
      !isin(dq[0], dq[(int)dq.size() - 2], dq.back()))
    dq.pop_back();
  while ((int)
      dq.size() >= 3 && !isin(dq.back(), dq[0], dq[1]))
    dq.pop_front();
  return vector<Line>(all(dq));
}
```

9.22 RotatingSweepLine

```
void rotatingSweepLine(vector<pii> &ps) {
  int n = (int)ps.size(), m = 0;
  vector<int> id(n), pos(n);
  vector<pii> line(n * (n - 1));
  for (int i = 0; i < n; ++i)
    for (int j = 0; j < n; ++j)
      if (i != j) line[m++] = pii(i, j);
```

```
  sort(all(line), [&](pii a, pii b) {
    return cmp(ps[a.S] - ps[a.F], ps[b.S] - ps[b.F]);
  }); // cmp(): polar angle compare
  iota(all(id), 0);
  sort(all(id), [&](int a, int b) {
    if (ps[a].S != ps[b].S) return ps[a].S < ps[b].S;
    return ps[a] < ps[b];
  }); // initial order, since (1, 0) is the smallest
  for (int i = 0; i < n; ++i) pos[id[i]] = i;
  for (int i = 0; i < m; ++i) {
    auto l = line[i];
    // do something
    tie(pos[l.F], pos[l.S], id[pos[l.F]], id[pos[l.S
        ]]) = make_tuple(pos[l.S], pos[l.F], l.S, l.F);
  }
}
```

9.23 DelaunayTriangulation

```
/* Delaunay Triangulation:
Given a sets of points on 2D plane, find a
triangulation such that no points will strictly
inside circumcircle of any triangle.
find : return a triangle contain given point
add_point : add a point into triangulation
A Triangle is in triangulation iff. its has_chd is 0.
Region of triangle u: iterate each u.edge[i].tri,
each points are u.p[(i+1)%3], u.p[(i+2)%3]
Voronoi diagram: for each triangle in triangulation,
the bisector of all its edges will split the region.
nearest point will belong to the triangle containing it
 */
const
     ll inf = MAXC * MAXC * 100; // lower_bound unknown
struct Tri;
struct Edge {
  Tri* tri; int side;
  Edge(): tri(0), side(0){}
  Edge(Tri* _tri, int _side): tri(_tri), side(_side){}
};
struct Tri {
  pll p[3];
  Edge edge[3];
  Tri* chd[3];
  Tri() {}
  Tri(const pll& p0, const pll& p1, const pll& p2) {
    p[0] = p0; p[1] = p1; p[2] = p2;
    chd[0] = chd[1] = chd[2] = 0;
  }
  bool has_chd() const { return chd[0] != 0; }
  int num_chd() const {
    return !!chd[0] + !!chd[1] + !!chd[2];
  }
  bool contains(pll const& q) const {
    for (int i = 0; i < 3; ++i)
      if (ori(p[i], p[(i + 1) % 3], q) < 0)
        return 0;
    return 1;
  }
} pool[N * 10], *tris;
void edge(Edge a, Edge b) {
  if(a.tri) a.tri->edge[a.side] = b;
  if(b.tri) b.tri->edge[b.side] = a;
}
struct Trig { // Triangulation
  Trig() {
    the_root
        = // Tri should at least contain all points
      new(tris++) Tri(pll(-inf, -inf),
           pll(inf + inf, -inf), pll(-inf, inf + inf));
  }
  Tri* find(pll p) { return find(the_root, p); }
  void add_point(const
       pll &p) { add_point(find(the_root, p), p); }
  Tri* the_root;
  static Tri* find(Tri* root, const pll &p) {
    while (1) {
      if (!root->has_chd())
        return root;
      for (int i = 0; i < 3 && root->chd[i]; ++i)
        if (root->chd[i]->contains(p)) {
          root = root->chd[i];
          break;
        }
    }
    assert(0); // "point not found"
  }
  void add_point(Tri* root, pll const& p) {
    Tri* t[3];
    /* split it into three triangles */
    for (int i = 0; i < 3; ++i)
      t[i] = new(tris
          ++) Tri(root->p[i], root->p[(i + 1) % 3], p);
    for (int i = 0; i < 3; ++i)
      edge(Edge(t[i], 0), Edge(t[(i + 1) % 3], 1));
    for (int i = 0; i < 3; ++i)
      edge(Edge(t[i], 2), root->edge[(i + 2) % 3]);
    for (int i = 0; i < 3; ++i)
      root->chd[i] = t[i];
    for (int i = 0; i < 3; ++i)
      flip(t[i], 2);
  }
  void flip(Tri* tri, int pi) {
    Tri* trj = tri->edge[pi].tri;
    int pj = tri->edge[pi].side;
    if (!trj) return;
    if (!in_cc(tri->p
        [0], tri->p[1], tri->p[2], trj->p[pj])) return;
    /* flip edge between tri,trj */
    Tri* trk = new(tris++) Tri
        (tri->p[(pi + 1) % 3], trj->p[pj], tri->p[pi]);
    Tri* trl = new(tris++) Tri
        (trj->p[(pj + 1) % 3], tri->p[pi], trj->p[pj]);
    edge(Edge(trk, 0), Edge(trl, 0));
    edge(Edge(trk, 1), tri->edge[(pi + 2) % 3]);
    edge(Edge(trk, 2), trj->edge[(pj + 1) % 3]);
    edge(Edge(trl, 1), trj->edge[(pj + 2) % 3]);
    edge(Edge(trl, 2), tri->edge[(pi + 1) % 3]);
    tri->chd
        [0] = trk; tri->chd[1] = trl; tri->chd[2] = 0;
    trj->chd
        [0] = trk; trj->chd[1] = trl; trj->chd[2] = 0;
    flip(trk, 1); flip(trk, 2);
    flip(trl, 1); flip(trl, 2);
  }
};
vector<Tri*> triang; // vector of all triangle
set<Tri*> vst;
void go(Tri* now) { // store all tri into triang
  if (vst.find(now) != vst.end())
    return;
  vst.insert(now);
  if (!now->has_chd())
    return triang.emplace_back(now);
  for (int i = 0; i < now->num_chd(); ++i)
    go(now->chd[i]);
}
void build(int n, pll* ps) { // build triangulation
  tris = pool; triang.clear(); vst.clear();
  random_shuffle(ps, ps + n);
  Trig tri; // the triangulation structure
  for (int i = 0; i < n; ++i)
    tri.add_point(ps[i]);
  go(tri.the_root);
}
```

9.24 VonoroiDiagram

```
// all coord. is even
  , you may want to call halfPlaneInter after then
vector<vector<Line>> vec;
void build_voronoi_line(int n, pll *arr) {
  tool.init(n, arr); // Delaunay
  vec.clear(), vec.resize(n);
```

```
  for (int i = 0; i < n; ++i)
    for (auto e : tool.head[i]) {
      int u = tool.oidx[i], v = tool.oidx[e.id];
      pll m = (arr[v
          ] + arr[u]) / 2LL, d = perp(arr[v] - arr[u]);
      vec[u].emplace_back(Line(m, m + d));
    }
}
```

10 Misc

10.1 MoAlgoWithModify

```
// Mo's Algorithm With modification
// Block: N^{2/3}, Complexity: N^{5/3}
struct Query {
  const int blk = 2000;
  int L, R, LBid, RBid, T;
  Query(int l, int r, int t):
    L(l), R(r), LBid(l / blk), RBid(r / blk), T(t) {}
  bool operator<(const Query &q) const {
    if (LBid != q.LBid) return LBid < q.LBid;
    if (RBid != q.RBid) return RBid < q.RBid;
    return T < q.T;
  }
};
void solve(vector<Query> query) {
  sort(all(query));
  int L=0, R=0, T=-1;
  for (auto q : query) {
    while (T < q.T) addTime(L, R, ++T); // TODO
    while (T > q.T) subTime(L, R, T--); // TODO
    while (R < q.R) add(arr[++R]); // TODO
    while (L > q.L) add(arr[--L]); // TODO
    while (R > q.R) sub(arr[R--]); // TODO
    while (L < q.L) sub(arr[L++]); // TODO
    // answer query
  }
}
```

10.2 MoAlgoOnTree

```
/*
Mo's Algorithm On Tree
Preprocess:
1) LCA
2) dfs with in[u] = dft++, out[u] = dft++
3) ord[in[u]] = ord[out[u]] = u
4) bitset<MAXN> inset
*/
struct Query {
  int L, R, LBid, lca;
  Query(int u, int v) {
    int c = LCA(u, v);
    if (c == u || c == v)
      q.lca = -1, q.L = out[c ^ u ^ v], q.R = out[c];
    else if (out[u] < in[v])
      q.lca = c, q.L = out[u], q.R = in[v];
    else
      q.lca = c, q.L = out[v], q.R = in[u];
    q.Lid = q.L / blk;
  }
  bool operator<(const Query &q) const {
    if (LBid != q.LBid) return LBid < q.LBid;
    return R < q.R;
  }
};
void flip(int x) {
    if (inset[x]) sub(arr[x]); // TODO
    else add(arr[x]); // TODO
    inset[x] = ~inset[x];
}
void solve(vector<Query> query) {
  sort(ALL(query));
  int L = 0, R = 0;
  for (auto q : query) {
    while (R < q.R) flip(ord[++R]);
    while (L > q.L) flip(ord[--L]);
    while (R > q.R) flip(ord[R--]);
    while (L < q.L) flip(ord[L++]);
    if (~q.lca) add(arr[q.lca]);
    // answer query
    if (~q.lca) sub(arr[q.lca]);
  }
}
```

10.3 MoAlgoAdvanced

- Mo's Algorithm With Addition Only
  - Sort querys same as the normal Mo's algorithm.
  - For each query $[l,r]$:
  - If $l/blk = r/blk$, brute-force.
  - If $l/blk \neq curL/blk$, initialize $curL := (l/blk+1) \cdot blk, curR := curL-1$
  - If $r > curR$, increase $curR$
  - decrease $curL$ to fit l, and then undo after answering
- Mo's Algorithm With Offline Second Time
  - Require: Changing answer $\equiv$ adding $f([l,r],r+1)$.
  - Require: $f([l,r],r+1) = f([1,r],r+1) - f([1,l),r+1)$.
  - Part1: Answer all $f([1,r],r+1)$ first.
  - Part2: Store $curR \to R$ for $curL$ (reduce the space to $O(N)$), and then answer them by the second offline algorithm.
  - Note: You must do the above symmetrically for the left boundaries.

10.4 HilbertCurve

```
ll hilbert(int n, int x, int y) {
  ll res = 0;
  for (int s = n / 2; s; s >>= 1) {
    int rx = (x & s) > 0;
    int ry = (y & s) > 0;
    res += s * 1ll * s * ((3 * rx) ^ ry);
    if (ry == 0) {
      if (rx == 1) x = s - 1 - x, y = s - 1 - y;
      swap(x, y);
    }
  }
  return res;
} // n = 2^k
```

10.5 SternBrocotTree

- Construction: Root $\frac{1}{1}$, left/right neighbor $\frac{0}{1}, \frac{1}{0}$, each node is sum of last left/right neighbor: $\frac{a}{b}, \frac{c}{d} \to \frac{a+c}{b+d}$
- Property: Adjacent (mid-order DFS) $\frac{a}{b}, \frac{c}{d} \Rightarrow bc - ad = 1$.
- Search known $\frac{p}{q}$: keep L-R alternative. Each step can calcaulated in $O(1) \Rightarrow$ total $O(\log C)$.
- Search unknown $\frac{p}{q}$: keep L-R alternative. Each step can calcaulated in $O(\log C)$ checks $\Rightarrow$ total $O(\log^2 C)$ checks.

10.6 AllLCS

```
void all_lcs(string s, string t) { // 0-base
  vector<int> h((int)t.size());
  iota(all(h), 0);
  for (int a = 0; a < (int)s.size(); ++a) {
    int v = -1;
    for (int c = 0; c < (int)t.size(); ++c)
      if (s[a] == t[c] || h[c] < v)
        swap(h[c], v);
    // LCS(s[0, a], t[b, c]) =
    // c - b + 1 - sum([h[i] >= b] | i <= c)
    // h[i] might become -1 !!
  }
}
```

10.7 SimulatedAnnealing

```
double factor = 100000;
const int base = 1e9; // remember to run ~ 10 times
for (int it = 1; it <= 1000000; ++it) {
    // ans:
        answer, nw: current value, rnd(): mt19937 rnd()
    if (exp(-(nw - ans
        ) / factor) >= (double)(rnd() % base) / base)
        ans = nw;
    factor *= 0.99995;
}
```

10.8 SMAWK

```
int opt[N];
ll A(int x, int y); // target func
void smawk(vector<int> &r, vector<int> &c);
void interpolate(vector<int> &r, vector<int> &c) {
  int n = (int)r.size();
  vector<int> er;
  for (int i = 1; i < n; i += 2) er.emplace_back(r[i]);
  smawk(er, c);
  for (int i = 0, j = 0; j < c.size(); j++) {
    if (A(r[i], c[j]) < A(r[i], opt[r[i]]))
      opt[r[i]] = c[j];
    if (i + 2 < n && c[j] == opt[r[i + 1]])
      j--, i += 2;
  }
}
void reduce(vector<int> &r, vector<int> &c) {
  int n = (int)r.size();
  vector<int> nc;
  for (int i : c) {
    int j = (int)nc.size();
    while (
      j && A(r[j - 1], nc[j - 1]) > A(r[j - 1], i))
      nc.pop_back(), j--;
    if (nc.size() < n) nc.emplace_back(i);
  }
  smawk(r, nc);
}
void smawk(vector<int> &r, vector<int> &c) {
  if (r.size() == 1 && c.size() == 1) opt[r[0]] = c[0];
  else if (r.size() >= c.size()) interpolate(r, c);
  else reduce(r, c);
}
```

10.9 Python

```
import math
math.isqrt(2) # integer sqrt
```

10.10 LineContainer

```

struct Line {
  mutable ll k, m, p;
  bool operator<(const Line &o) const {
    return k < o.k;
  }
  bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  static const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b);
  }
  bool isect(iterator x, iterator y) {
    if (y == end()) return x->p = inf, 0;
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y))
      isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```