THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON, DC

Machine Learning II
DATS 6203
Due: *Sep-12-2018*

Homework 1:

- **Show ALL Work, Neatly and in Order.**
- **No** credit for Answers Without Work.
- Solution of each problems should be in a separate page.
- Use Your Own Paper (Do Not Write on Homework Paper).
- Write Only on the Front of Each Sheet.

Note 1: Please read chapter 5 of neural network design book (listed in the syllabus) and then answer the following questions.

E.1:

Practice finding the derivatives of these functions:

i. $f(x) = sin(6x-1)$.

ii. $f(x) = x^8 + 30 + \frac{1}{x^4}$.

iii. $f(x) = e^{(\frac{1}{x})+(\frac{1}{x^2})}$.

iv. $f(x) = sin^2(6x-1)$.

E.2:

Finding when a function is increasing/decreasing and concave up/down. When is the function $f(x) = 2x^3 + 24x^2 - 54x$ decreasing? When is it concave up? Plot the function and find your check your answer?.

E.3:

Finding critical points, local max/min, global max/min, and inflection points. Find all critical points and inflection points of $f(x) = 2x^3 + 24x^2 - 54x$. Classify the critical points as local min, local max, or neither. Find the global max and min of this function on $[-3,3]$ and on $(-\infty,0)$. Plot the function and find your check your answer?.

E.4:

i. Find the gradient vector of $f(x,y) = x^2 + y^2$.

ii. What are the gradient vectors at $(1,2),(2,1) and (0,0)$? Plot the function in 3D space and check your answers?

E.5:

i.Find the gradient vector of $f(x,y) = 2xy + x^2 + y$.

ii.What are the gradient vectors at $(1,1),(0,-1) and (0,0)$? Plot the function in 3D space and check your answers?

iii. Find the gradient vector of $f(x_1,x_2) = x_1^2 + 2x_1x_2 + x_2^2 + x_1x_2^2$.

E.6:

i. Find equation of the line: has slope 3 and y-intercept (0, -0.5)

ii. Find equation of the line: passes through (4, 8) and (6, 14).

iii. Find equation of the line: passes through (3, 2) and is perpendicular to y = 5x + 3.

iv. Find equation of the line: has b = 3 and passes through (2, 1).

v. Find equation of the line: has passes through (6, 4) and (1, -1).

E.7:

Find the eigenvalues and eigenvectors of the given matrix by hand and check results by the computer (use Python to check your results).

i. $\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$

ii. $\begin{bmatrix} 5 & 1 \\ 4 & 5 \end{bmatrix}$

iii. $\begin{bmatrix} 3 & 5 \\ 3 & 1 \end{bmatrix}$

E.8:

i. Consider the set of all continuous functions that satisfy the condition $f(0) = 0$. show that this is a vector space.

ii. Show that the set of $2x2$ matrices is a vector space.

E.9:

Which of the following sets of vectors are independent? Find the dimension of the vector space spanned by each set. (Verify your answers using Python).)

i. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

ii. $sin(t), cos(t), cos(2t)$

iii. $1+t, 1-t$

iv. $\begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 4 \\ 3 \end{bmatrix}$

E.10:

Using the following basis vectors, find an orthogonal set using Gram-Schmidt orthogonalization.

$y_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, y_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, y_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

E.11:

Expand $x = \begin{bmatrix} 1 & 2 & 2 \end{bmatrix}^T$ in terms of the following basis set.

$v_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$