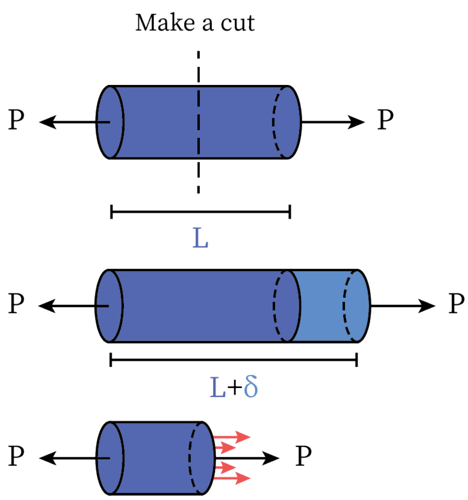
Latex 1.1 Practice

Brandon Page

May 2022

1 Normal Stress and Strain



1.1 Equations

$$\sigma = \frac{P}{A} \tag{1}$$

$$\varepsilon = \frac{\delta}{L} \tag{2}$$

1.2 Vocabulary

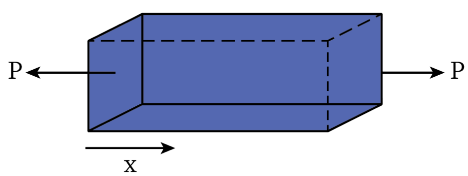
1. $(+)$ Tensile stress - when bar is pulled by P.
2. $(-)$ Compression - when bar is compressed by P.

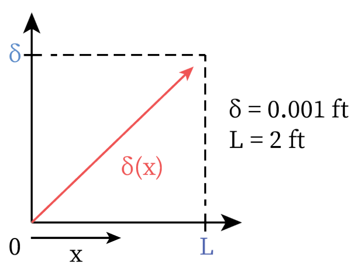
3. Normal Stress - stress which acts perpendicular to the cut surface.
4. σ - symbol for stress.
5. P - resultant force.
6. A - cross-sectional area.
7. δ - elongation of bar or member.
8. ε - engineering strain and only valid for small deformations.
9. L - original length

1.3 Limitations

1. Axial force must act through the centroid of a cross-sectional area
2. The bar must be prismatic (meaning it has the same cross-section everywhere)
3. The material must be homogeneous (same composition at every point) and isotropic (same in all directions)

1.4 Practice Problem 1





What is the strain at $x = 6in$? Strain is the slope of the line $\delta(x)$. The slope of the line is $\frac{\delta}{L}$.

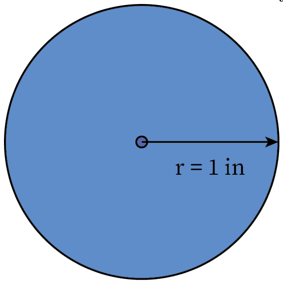
$$\delta(x) = \frac{\delta}{L}(x)$$

$$\varepsilon = \frac{\delta}{L} = \frac{0.001ft}{2ft} = 0.0005$$

Since strain is the slope of the line and the slope is constant. The strain is constant everywhere in the bar. Therefore, the strain at $L = 6$ in is just 0.0005.

1.5 Practice Problem 2

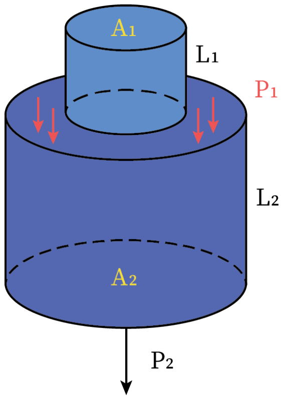
If the stress σ is $1ksi$ $(\frac{kips}{in^2})$ of the following member:



What is the resultant force P?

$$\sigma = \frac{P}{A} \Rightarrow P = \sigma A = (1\frac{kip}{in^2})(\pi(1)^2) = \pi kips$$

1.6 Practice Problem 3



1. Find P_2 so that the stress in the upper part is σ_1 . What is σ_2?

$$\sigma_1 = \frac{P_1 + P_2}{A_1} \Rightarrow P_2 = A_1\sigma_1 - P_1$$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{A_1\sigma_1 - P_1}{A_2}$$

2. If P_1 remains unchanged, find the new value of P_2 such that $\sigma_1 = \sigma_2$.

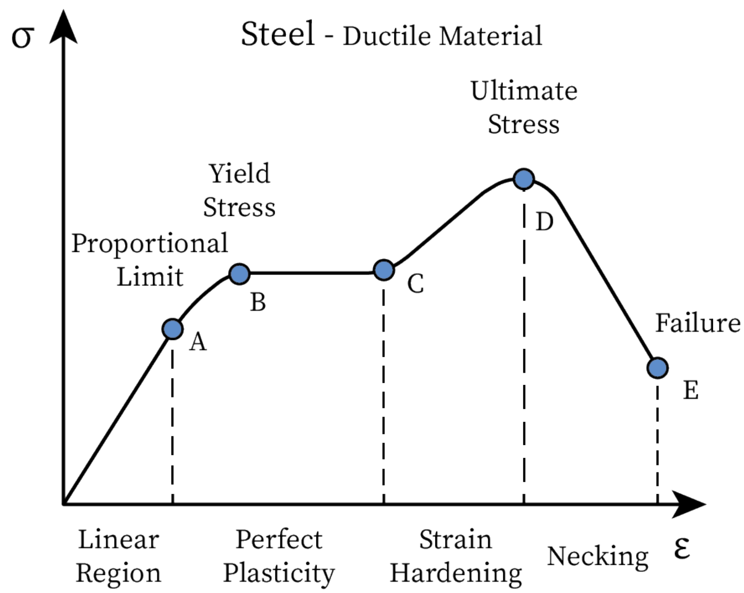
$$\sigma_1 = \frac{P_1 + P_2}{A_1}$$

$$\sigma_2 = \frac{P_2}{A_2}$$

$$\Rightarrow A_2(P_1 + P_2) = A_1 P_2$$

$$\Rightarrow A_2P_1 + A_2P_2 = A_1P_2$$
$$\Rightarrow A_2P_1 = A_1P_2 - A_2P_2$$
$$\Rightarrow A_2P_1 = P_2(A_1 - A_2)$$
$$\Rightarrow P_2 = \frac{A_2}{(A_1 - A_2)}P_1$$

2 Stress Strain Diagrams

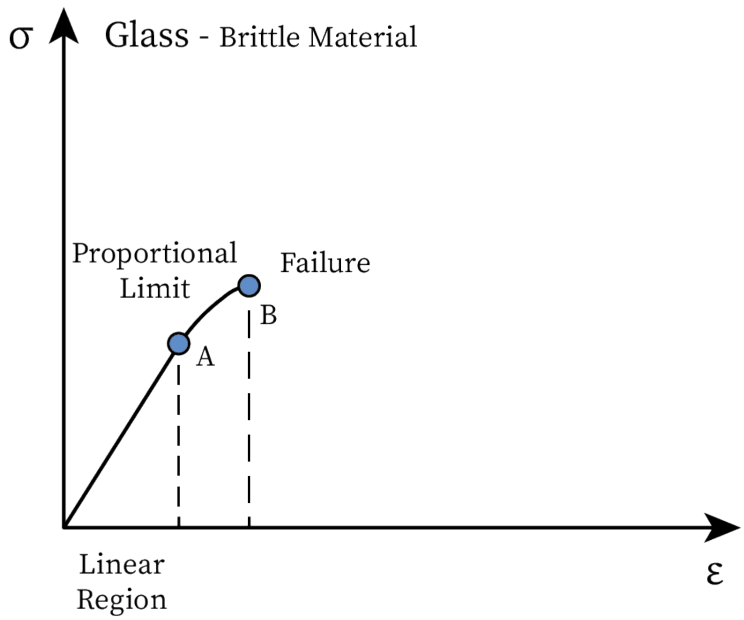


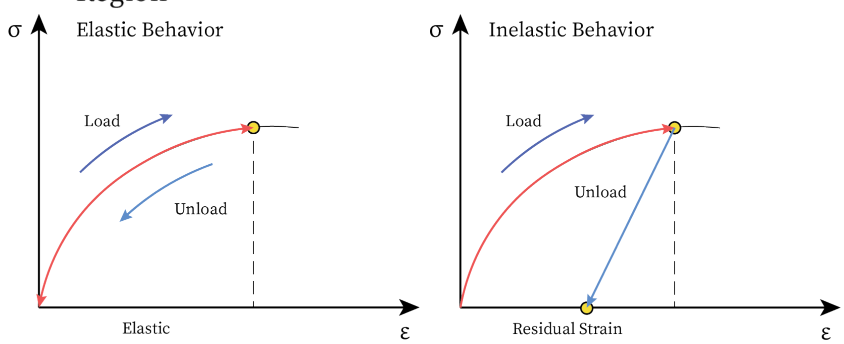
$$\sigma = E\varepsilon \longleftarrow \text{Hooke's Law}$$

E is the Modulus of Elasticity or Young's Modulus

2.1 Thought Experiment

Draw the stress strain diagram for glass.





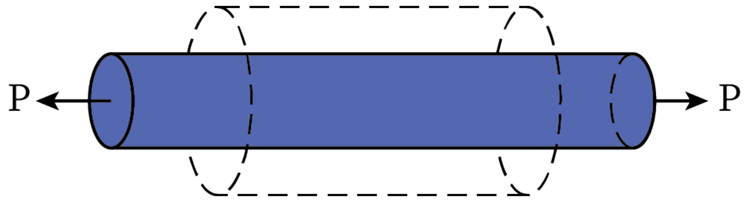
In this class we study linear elasticity. Under this assumption we can use the equation

$$\sigma = E\varepsilon$$

2.2 Thought Experiment

Is the modulus of elasticity higher for rubber or steel? Why?

3 Poisson's Ratio



$$\nu = -\frac{\varepsilon'}{\varepsilon} = \frac{\text{lateral strain}}{\text{axial strain}}$$
$$\Rightarrow \varepsilon' = -\nu\varepsilon$$

*Poisson's Ratio is a measured quantity like E

3.1 Thought Experiment

Why is linear elasticity such an important concept in engineering design?

4 Revisiting and Refining Assumptions

1. Homogeneous: The material must have the same composition and elastic properties at every point.
2. Isotropic: Materials having the same properties in all directions.
3. Linearly Elastic: A material which behaves elastically and also exhibits a linear relationship between stress and strain.

4.1 Examples

4.1.1 Wood Pole

1. Yes
2. No
3. Yes

4.1.2 Metal Rod Composed of $\frac{1}{2}$ Steel and $\frac{1}{2}$ Aluminum

1. No
2. Yes
3. Yes

4.1.3 A Ball of Silly Putty

1. Yes
2. Yes
3. No

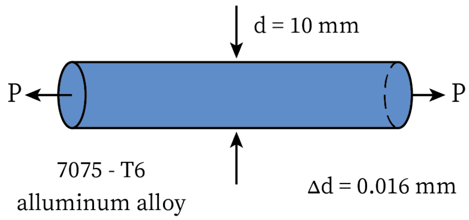
4.1.4 The Prismatic Bar from the Previous Section

1. Yes
2. Yes
3. Yes

4.1.5 Steel Warship Propeller (*at a given loading)

1. Yes
2. Yes
3. Yes

Everything in the remainder of the book satisfies assumptions 1-3.



1. Are all assumptions satisfied? Yes, so I can apply the theory emanating from the model.

2. look up material properties in table I-2.

$$E = 72\ GPa$$
$$1\ Pa = 1\ \frac{N}{m^2}$$
$$\nu = 0.33$$

Yield Stress, Table I-3

$$\sigma_Y = 480\ MPa$$

3. Lateral Strain

$$\varepsilon' = \frac{\Delta\ d}{d} = \frac{-0.016\ mm}{10\ mm} = -0.0016$$

Axial Strain

$$\nu = -\frac{\varepsilon'}{\varepsilon} \Rightarrow \varepsilon = -\frac{1}{\nu}\ \varepsilon' = \frac{0.0016}{0.33} = 0.004848$$

Axial Stress

$$\sigma = E\varepsilon = (72\ GPa)(0.004848) = 0.3491\ GPa = 349.1\ MPa$$

4. Since $\sigma < \sigma_Y$, Hooke's Law is valid as are prismatic beam assumptions.

$$P = \sigma A = \sigma\pi r^2 = (349.1\ MPa)(\pi)(5)^2 =$$

$(\frac{349{,}100{,}000\ N}{m^2})(\frac{m^2}{1{,}000{,}000\ mm^2)}(25\ mm)^2 = 27.42\ KN$