Energy shifts of photons crossing an Alcubierre warp drive

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Abstract

We study the energy shifts of light propagating through an incoming Alcubierre warpdrive spacetime by numerically integrating the null geodesic equations derived from the warpdrive metric. We find that from the perspective of a distant observer in a co-moving reference frame with respect to the warpdrive, light passing through the Alcubierre warpdrive experiences an overall blueshift whose magnitude depends on the photon's impact parameter and the warp drive's velocity. The maximum net blueshift occurs at an impact parameter $b \sim 3.46 - 0.098v$ with a maximum that scales as $\sim 1+v$. Thus, greater net blueshifts result from interacting with faster warpdrives, with the maximum blueshift occurring at smaller impact parameters.

Keywords: Alcubierre warp drive, null geodesics, light propagation

1 Introduction

Faster-than-light (FTL) travel has occupied the public imagination probably for as long as science fiction has been written. Quite simply, this idea allows a space ship to travel great distances in a short amount of time, thus allowing efficient interstellar space travel and exploration. However, Albert Einstein's theory of special relativity (SR) tells us that it is impossible for any massive object to travel faster than light making FTL travel impossible to physically realize. In general relativity (GR), this restriction remains true; locally the speed of light is still an upper bound. However, there exist special solutions to the Einstein's field equations that allow an apparent FTL travel without violating the restrictions of special relativity.

In this paper, we focus on the specific solution proposed by Miguel Alcubierre [1] whose features resembles the "hyper drive" concept in science fiction lore. The Alcubierre solution allows a spaceship to have an apparent speed that surpasses the speed of light by a factor relative to a distant observer. This phenomenon can be understood as the effect of a wave-like distortion in spacetime. The distortion is such that the space ahead of the spaceship to contract while the space behind the spaceship to expand making it seem like the spaceship is being pushed and pulled by the distortion towards a specific direction. Passengers of this spacetime can then travel great distances over a short amount of time. Furthermore, between the expansion and contraction of space, there is a locally flat region that we call as the "warp bubble". The warp bubble accommodates our spaceship and allows it to experience no time dilation between the space-time distortion and a clock in flat space. Notably however, Pfenning and Ford [2] have discussed that the total integrated energy density required to maintain the warp drive is physically unattainable as the this would require the use of "exotic" matter that violates the conditions that states that we cannot have a negative energy density.

Despite its challenges and limitations, it is still interesting to ask how the properties of light changes as it propagates through a warpdrive. Beyond the intrinsic merit of better understanding peculiar physics in curved spacetime, one strong motivation arises from the resurgence of interest in "exotic" metrics as analogue spacetimes in transformation optics and metamaterials as done in [3]. In this paper, we consider a distant co-moving observer relative to an Alcubierre warpdrive and study the null geodesic equations for a photon that traverses the warp-drive distortion. We shall define the warp bubble to have a radius and thickness of $R = 3$ light seconds and $\sigma = 1$, where light seconds corresponds to the number of meters light travels in a second, centered around the bridge of the warpdrive.

The Alcubierre warpdrive is defined by

$$ds^2 = -dt^2 + (dx - v_s f(r_s)dt)^2 + dy^2 + dz^2, \tag{1}$$

where r_s is the center of the warp-drive defined by

$$r_s = \sqrt{(x - x_s)^2 + y^2 + z^2}. \tag{2}$$

We notice that the warpdrive follows the trajectory described by $(x_s(t), 0, 0)$ with a warp speed v_s defined by $v_s = dx_s/dt$. Meanwhile, the function $f(r)$ can be any function that is unit valued at the center of the warpdrive, and reduces rapidly to zero as the value of the r-component reaches $r = R$ and extends to infinity. The specific function that we will be using would be the one prescribed by Alcubierre in his paper:

$$f(r_s) = \frac{\tanh(\sigma(r_s + R)) - \tanh(\sigma(r_s - R))}{2\tanh(\sigma R)}. \tag{3}$$

σ and R are the thickness and radius of the warp bubble, respectively. It is important to note that we use geometric units in this study.

2 Integrating the geodesic equations

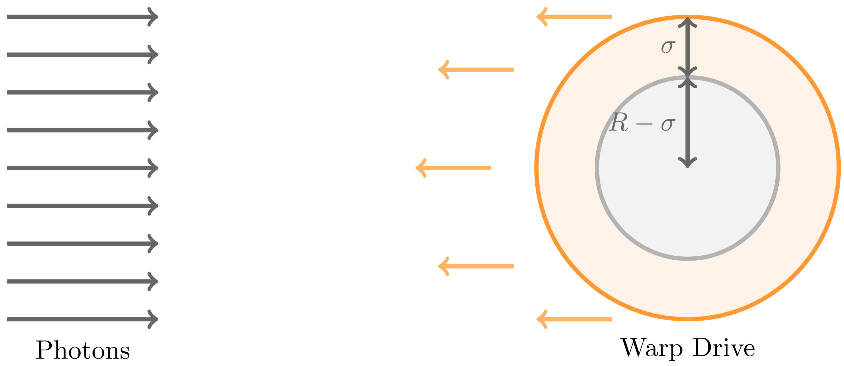


Figure 1: Light or photons starting from spatial infinity collide and propagate through an Alcubierre warp drive distortion of bubble thickness σ and radius R. The resulting light trajectories will then be studied.

We start by solving the trajectories that photons would follow as they move through the spacetime curvature. We then consider the geodesic equations given by

$$\frac{dp^\alpha}{d\lambda} + \Gamma^\alpha_{\mu\nu} p^\mu p^\nu = 0, \tag{4}$$

where the Christoffel symbols $\Gamma^\alpha_{\mu\nu}$ is given by

$$\Gamma^\delta_{\alpha\beta} = \frac{1}{2} g^{\delta\gamma} \left(\frac{\partial g_{\alpha\gamma}}{\partial x^\beta} + \frac{\partial g_{\gamma\beta}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\gamma} \right). \tag{5}$$

Here α, μ, and ν are indices that cover the range $0, 1, 2, 3$. They represent the $t-$, $x-$, $y-$, $z-$ components, respectively.

We consider the case wherein the $z-$ component of the trajectory and momenta equals zero so that our photons only travel in the $x - y$ plane. In this way the geodesic equations would simplify into the following equations

$$\frac{dp^t}{d\lambda} + \Gamma^t_{tt}(p^t)^2 + \Gamma^t_{xx}(p^x)^2 + 2\Gamma^t_{tx}p^t p^x + 2\Gamma^t_{ty}p^t p^y + 2\Gamma^t_{yx}p^y p^x = 0, \tag{6}$$

$$\frac{dp^x}{d\lambda} + \Gamma^x_{tt}(p^t)^2 + \Gamma^x_{xx}(p^x)^2 + 2\Gamma^x_{tx}p^t p^x + 2\Gamma^x_{ty}p^t p^y + 2\Gamma^x_{yx}p^y p^x = 0, \tag{7}$$

$$\frac{dp^y}{d\lambda} + \Gamma^y_{tt}(p^t)^2 + 2\Gamma^y_{tx}p^t p^x = 0. \tag{8}$$

We supplement these equations with momenta relations to provide us information about the shifts in the momenta of light passing through the warp drive

$$\frac{dt}{d\lambda} - p^t = 0 \quad , \quad \frac{dx}{d\lambda} - p^x = 0 \quad , \quad \frac{dy}{d\lambda} - p^y = 0. \tag{9}$$

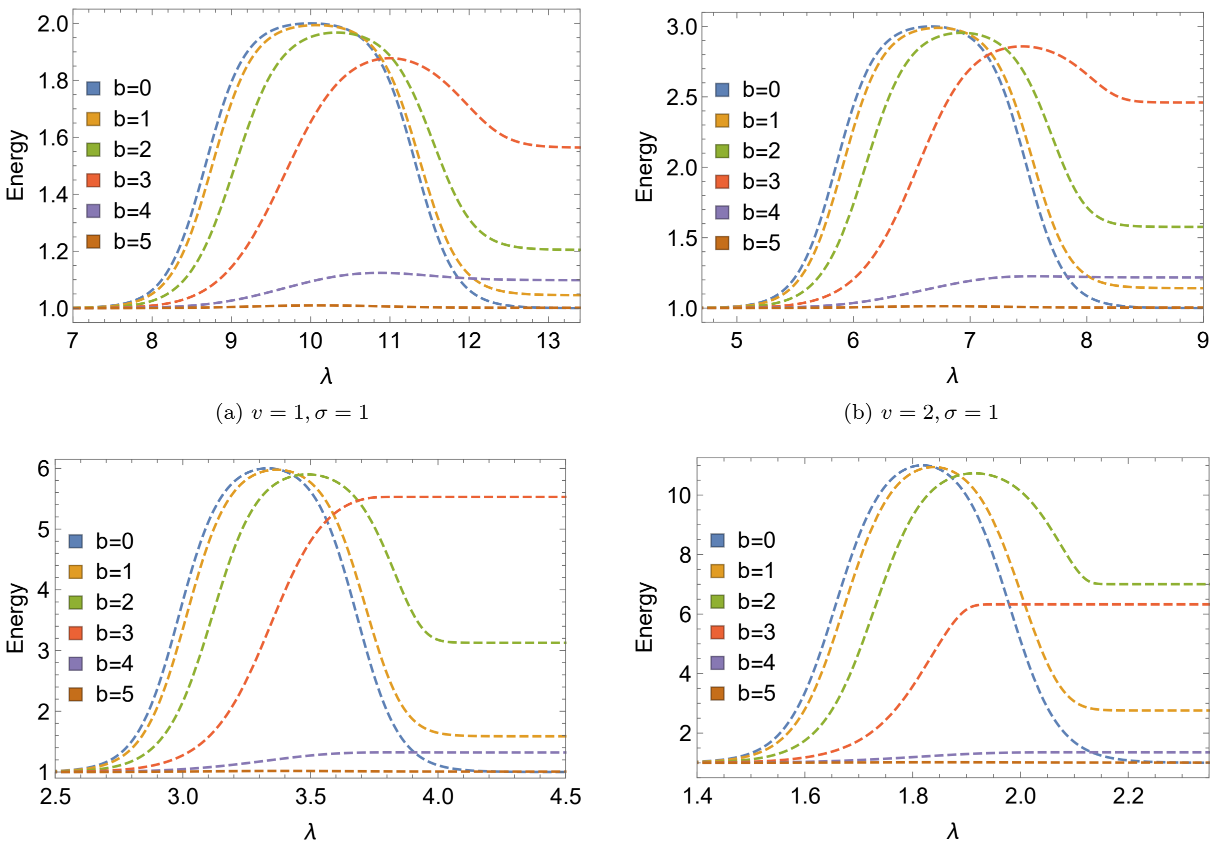
We then consider the setup where a beam of photons initially positioned at infinity are directed towards an incoming warpdrive. The nonlinear coupled ordinary differential equations Eqns. (6)-(9) are numerically integrated to yield equations of motion for the $t-$, $x-$, $y-$ components of the trajectory and momenta of the photon propagation. Saret [4] used the resulting ray tracing to compute for the angular deflections experienced by photons traversing a warpdrive. The momenta of light would be studied for varying warp speeds, with $v_s = v =$ constant, as different warp speeds create different spacetime configurations. Photons are set to start from a region of flat spacetime and are directed from a distance of 20 light seconds away, each with initial momenta $p^t = 1$, $p^x = 1$, $p^y = 0$. These values ensure that the photons collides with the warpdrive head-on. Finally, for the impact parameter or the initial $y-$ component value of the trajectory, we consider values in the range $-6 \leq y \leq 6$.

3 Results and discussion

We obtain the p^t- component of our photon four-momentum and study the shifts in energy of photons of different impact parameters in the vicinity of our warpdrive as the energy-momentum four-velocity is given by:

$$p^\mu = (E, p^x, p^y, p^z)\,. \tag{10}$$

Fig. 2a - 2d shows how the energies of the photons shift as it interacts with the Alcubierre warp drive for different values of warp speed and a constant value of $\sigma = 1$.



(a) $v = 1, \sigma = 1$ (b) $v = 2, \sigma = 1$ (c) $v = 5, \sigma = 1$ (d) $v = 10, \sigma = 1$

Figure 2: Energy vs arbitrary time variable, λ, of photons of different impact parameters for warp speeds values $v = 1, 2, 5, 10$ and warp bubble width $\sigma = 1$.

We observe that photons moving towards the Alcubierre warpdrive experience a blueshift, and a redshift as it moves away as shown by the gradual increase and decrease in energy, respectively. Interestingly, except for the $b = 0$ photon, the energy of the photons does not return back to its original value and is left with an overall blueshift after its interaction with the warpdrive. The shifts in the photon energy are greatly affected by the photon's impact parameter. Photons with impact parameter $b \leq R$ experience strong lensing that changes the photons energy greatly. Meanwhile, photons with impact parameter $b \geq R$ experience weak lensing that does not affect the photon energy as much as the strong lensing case. These are the photons that are almost outside or completely outside of the warp drive vicinity. The photon energy peaks when the $b = 0$ photon is located at the "bridge" or the center of the warp bubble.

The value of the energy at the bridge confirms the results of Clark et al. [5] and Kinach et al. [6] for an incoming warpdrive wherein the energy at the bridge and at infinity are related by the equation

$$E_{\text{bridge}} = (1+v)E_0, \tag{11}$$

where E_{bridge} is the measurement of the energy of photon at the bridge of the warp bubble, and E_0 is the measurement of the energy of the photon at infinity before being beamed towards the warpdrive.

Fig. 3 shows the overall blueshifts of photons after it passes through an Alcubierre warp drive for different warp speeds and impact parameter values but constant warp bubble width. We see that as the impact parameter increases the overall blueshifts gradually increase, which peaks with a value $\sim (1+v)$ at a critical impact parameter value, $b_{\max}(v) \sim 3.46 - 0.098v$, and decreases as the impact parameter moves outside the vicinity of the warpdrive. We also observe that as the warp speed is increased, the overall blueshifts also increase with peaks that are found closer to zero.

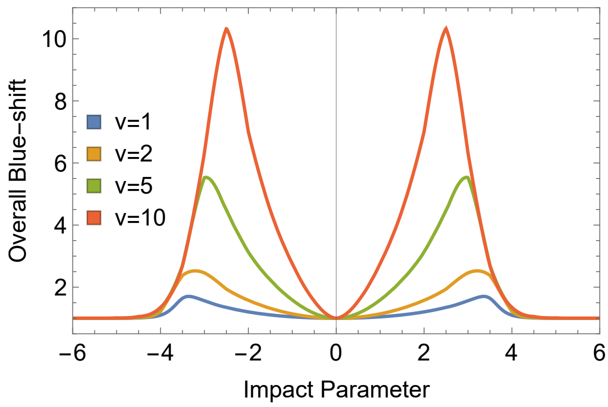


Figure 3: The overall blue-shifts in photon energies vs impact parameter for different warp speed values and constant warp bubble width, $\sigma = 1$.

4 Conclusions

In this paper, we numerically integrated the null geodesic equations in an Alcubierre warpdrive representing the motion of photons traversing the spacetime curvature. We paid particular attention to the $t-$component of the momentum in order to study the energy shifts of our test photons. The photons increase their energies as they enter the warpdrive and correspondingly decrease their energies as they leave it. Increasing the warp speed results in greater, more abrupt changes in the photon energy. Interestingly, this interaction with the warpdrive leaves an overall blueshift in the test photons. For a warpdrive of speed v, the maximum net blueshift scales as $\sim (1+v)$ and is achieved at an impact parameter $b_{\max}(v) \sim 3.46 - 0.098v$. This means that stronger net blueshifts can be achieved with faster warpdrives, with the peak impact parameter getting closer to zero. It will be interesting to confirm this with an analytic calculation.

Acknowledgments

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