Photon scattering by an Alcubierre warp drive

Lemuel Gavin G. Saret* and Michael Francis Ian G. Vega II

National Institute of Physics, University of the Philippines Diliman, Quezon City 1101, Philippines
*Corresponding author: lsaret@nip.upd.edu.ph

Abstract

The null geodesic equations in the Alcubierre warp drive space-time are numerically integrated to determine light propagation and angular deflection of photons originating from infinity that collide with the warp drive distortion. We find that for a distant observer in a co-moving reference frame alongside the warp drive distortion, light rays colliding with the warp drive would have resulting angular deflections that changes for different values of the impact parameter and warp velocity. Meanwhile, images that propagate through the warp drive distortion would experience gravitational lensing that would invert a radial area of the image from the center outward as seen by an observer following the said warp drive. This lensing effect is caused by the negative energy density matter that supports the warp drive space-time.

Keywords: Alcubierre warp drive, null geodesics, photon propagation, gravitational lensing

1 Introduction

In special relativity, we learn that nothing can ever move faster than light. This restricts possible faster-than-light (FTL) space travel or exploration. However, in general relativity we learn a modified version of this fact: nothing can locally travel faster than the speed of light. This raises the possibility of FTL travel if space-times were to be engineered correctly.

Alcubierre [1] provides an explicit example of such an engineered space-time which has features similar to the "warpdrive" which is more commonly known as the "hyperdrive" that we usually read about in common science-fiction lore. This solution allows a "spaceship" to have an apparent speed which is much greater than the speed of light relative to a distant observer. This phenomena causes an apparent contraction and expansion in the front and at the back of the spaceship. In this way, the spaceship will be pushed away from its source and pulled towards some location by the space-time curvature. In between this expansion and contraction of space, there is a locally flat area that houses our spaceship. This prevents our spaceship to experience tidal forces due to the curvature of space-time. However, in order to create this specific curvature, "exotic matter" is required, which violates energy conditions that states that the energy density cannot be negative. Pfenning and Ford [2] discussed that the total integrated energy density required to maintain the warp drive curvature is physically unattainable. Similarly, a study by Lobo [3] suggested that even at low warp speeds, the net negative energy required to sustain the warp drive curvature must be a significant factor of the mass of the space ship.

Despite the difficulties in the construction of the Alcubierre warp drive, it is interesting to ask how light gets distorted as it moves through the Alcubierre warp drive from the perspective of an observer in a co-moving reference frame as the warp bubble, as is done in [4]. In this work, we consider a different thought related problem: How does an Alcubierre warp drive scatter light? We change viewpoints from a co-moving observer to an observer also far away. Throughout this paper, we shall define a warp bubble to be the space-time region with radius, $R = 3$ light seconds, centered around the bridge of the warp drive distortion.

The Alcubierre warp drive space-time is defined by the line element

$$ds^2 = -dt^2 + (dx - v_s f(r_s)dt)^2 + dy^2 + dz^2, \tag{1}$$

where the center of the bridge of the warp bubble is given by

$$r_s = \sqrt{(x - x_s)^2 + y^2 + z^2}, \tag{2}$$

and follows the trajectory described by the function of time, $(x_s(t), 0, 0)$. Meanwhile, $v_s = dx_s/dt$ represents the rate of change in the trajectory of the warp bubble with respect to time.

The function $f(r)$ can be any function that would have a unit value at the center of the space-time distortion, and falls rapidly to zero as the value of r reaches $r = R$ and proceeds to infinity. The particular $f(r)$ function that we would use is one prescribed by Alcubierre:

$$f(r_s) = \frac{\tanh(\sigma(r_s + R) - \tanh(\sigma(r_s - R)}{2\tanh(\sigma R)} \tag{3}$$

where σ and R are the thickness and radius of the warp bubble, respectively. For this study we set $\sigma = 1$ and $R = 3$ light seconds.

2 Methodology

2.1 Equations of motion

To solve for the trajectories of light moving through our warp drive, we follow the work of Clark [4] and first consider the geodesic equations given by

$$\frac{dp^\alpha}{d\lambda} + \Gamma^\alpha_{\mu\nu} p^\mu p^\nu = 0, \tag{4}$$

where $\Gamma^\alpha_{\mu\nu}$ are the Christoffel symbols given by

$$\Gamma^\delta_{\alpha\beta} = \frac{1}{2} g^{\delta\gamma} \left(\frac{\partial g_{\alpha\gamma}}{\partial x^\beta} + \frac{\partial g_{\alpha\beta}}{\partial x^\gamma} - \frac{\partial g_{\alpha\beta}}{\partial x^\gamma} \right), \tag{5}$$

where α, μ, ν are indices taking the range $\{0, 1, 2, 3\}$ which represents the $t-$, $x-$, $y-$, $z-$ components, respectively.

We consider the case wherein our photons travel only on the $x-$ and $y-$ axes. In this way, it is understood that the null geodesics of the photons that would reach our warp bubble would have a $z-$ trajectory and p^z momentum component equal to zero. The geodesic equation and the Christoffel symbols are then evaluated for the remaining trajectory and momenta components resulting to the following equations

$$\frac{dp^t}{d\lambda} + \Gamma^t_{tt}(p^t)^2 + \Gamma^t_{xx}(p^x)^2 + 2\Gamma^t_{tx} p^t p^x + 2\Gamma^t_{ty} p^t p^y + 2\Gamma^t_{yx} p^y p^x = 0, \tag{6}$$

$$\frac{dp^x}{d\lambda} + \Gamma^x_{tt}(p^t)^2 + \Gamma^x_{xx}(p^x)^2 + 2\Gamma^x_{tx} p^t p^x + 2\Gamma^x_{ty} p^t p^y + 2\Gamma^x_{yx} p^y p^x = 0, \tag{7}$$

$$\frac{dp^y}{d\lambda} + \Gamma^y_{tt}(p^t)^2 + 2\Gamma^y_{tx} p^t p^x = 0. \tag{8}$$

We also introduce the following relations necessary to provide us knowledge of the shift in momenta of light that would pass through our system.

$$\frac{dt}{d\lambda} - p^t = 0 \quad , \quad \frac{dx}{d\lambda} - p^x = 0 \quad , \quad \frac{dy}{d\lambda} - p^y = 0 \tag{9}$$

where p^t, p^x, and p^y are the momenta associated with the $t-$, $x-$, and $y-$ coordinate of our system, respectively. Together with Eqns. 6-8, these forms a nonlinear coupled ordinary differential equation that can be numerically integrated to yield expressions for the light trajectories and momenta as it moves through the warp drive distortion. For our study we consider the warp speed of our distortion to have a constant value so that $v_s = v =$ constant and $x_s = vt$.

2.2 Initial setup

We consider an initial system wherein a beam of light is shone to a warp drive from a position at infinity. The setup is shown in Fig. 1. The nonlinear coupled differential equation composed of Eqns. 6 - 9, will be numerically integrated to yield equations for the t-,x-,y- component trajectories of the photon propagation. The trajectories of light in the Alcubierre warp-drive system will be studied for different warp speeds. Different warp velocities constitutes different space-time configurations so in this paper, we talk about different Alcubierre warp drive curvature configurations. We considered photons that is 20 light seconds away from our space ship with initial momenta $p^t = 1$, $p^x = 1$, and $p^y = 0$. For the impact parameter or the initial value for the $y-$ coordinate, we consider the values that are equidistant from each other in the range $-6 \leq y \leq 6$. We vary the impact parameter to see the effects on the photon trajectories as it moves through our warp distortion.

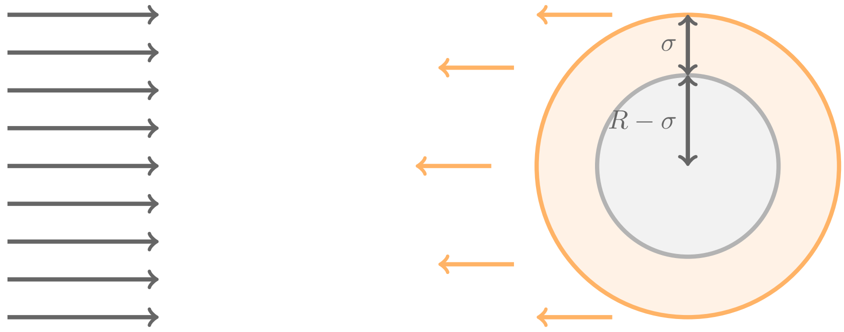
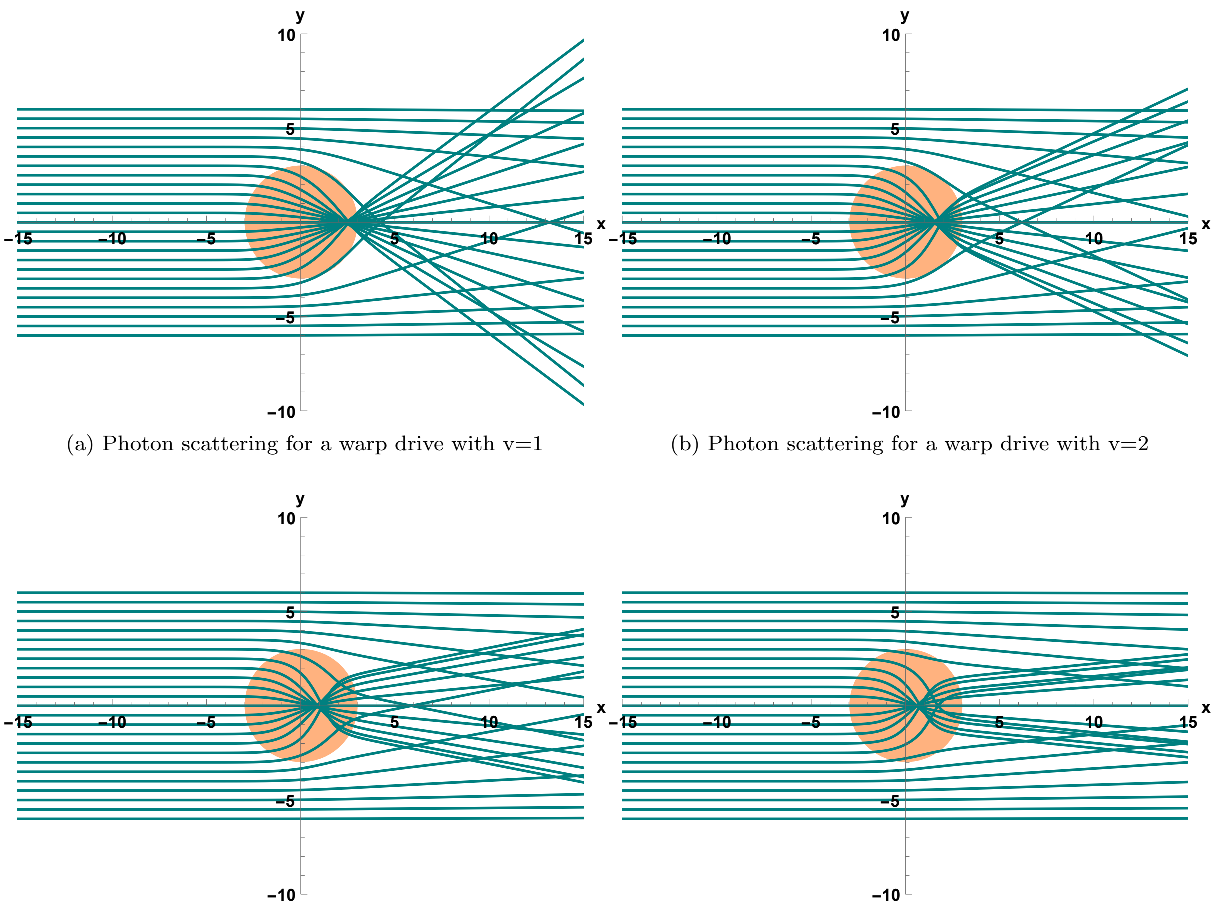


Figure 1: Light or photons starting from spatial infinity collide and propagate through an Alcubierre warp drive distortion of bubble thickness σ and radius R. The resulting light trajectories will then be studied.

3 Results and discussion



(a) Photon scattering for a warp drive with v=1

(b) Photon scattering for a warp drive with v=2

(c) Photon scattering for a warp drive with v=5

(d) Photon scattering for a warp drive with v=10

Figure 2: Ray trajectories for light in the x-y plane of the Alcubierre warp drive for warp speeds $v = \{1, 2, 5, 10\}$. Light from infinity is shone parallel to the $x-$ axis with light rays equally spaced with impact parameters ranging from -6 to 6. Parameter values: $\sigma = 1$ and $R = 3$.

Fig. 2a - 2d show the light trajectories of a light beam directed towards an incoming Alcubierre warp drive with warp speeds $v = \{1, 2, 5, 10\}$ as seen by an observer in a co-moving reference frame to the warp drive. In this frame the warp bubble is stationary. The horizontal axis would show how light would move to the positive x-direction and the vertical axis would show how light is distorted in the y-direction. Notice that for photons with impact parameter $b \leq R$, there is a corresponding strong gravitational lensing effect wherein light would cross the x-axis and be deflected on the opposite region. However, for photons with impact parameter $b \geq R$, there is a corresponding weak lensing effect that we normally see happen around the vicinity of other massive objects. However, light that has impact parameter equal to 0 would show no deflection since there $y-$ trajectory and momenta component would just be 0 due to symmetry. These deflections mean that an image beamed through the warp drive would have a radial area from the center of the image that would have an inverting distortion while the rest of the image would seem to tend to the center of the image. This distortion is the result of the negative energy density requirements necessary to support the Alcubierre warp drive.

Fig. 3 shows a plot of the deflection angles of light for different warp velocities and impact parameters. First, as discussed earlier, photons of zero impact parameter experience no distortion due to the warp drive, therefore they have a zero angular deflection. Next, we notice that as we increase the magnitude of the impact parameter inside the vicinity of the warp bubble, we observe a gradual increase in the angular deflection. However, as we move away from the vicinity of the warp bubble we would observe a steep decrease in the value of the angular deflections. This is because we experience less and less curvature as we move away from the vicinity of the curvature. Finally, we notice that as the value of the warp speed is increased, the overall deflections increases wherein the maximum value for the angular deflections asymptotically approach the value of $\pi/2$ as the warp speed is continually increased.

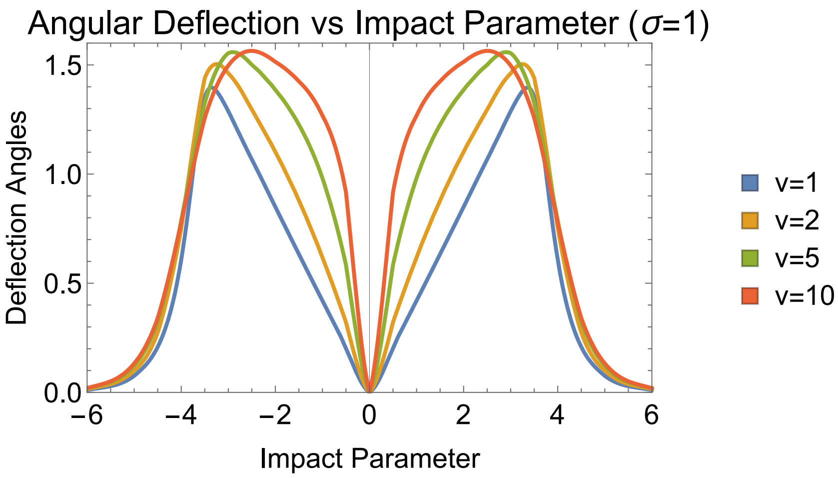


Figure 3: Surface plot of photon angular deflections as a function of warp speed and impact parameter

4 Conclusion

We successfully visualized the trajectories of light of different impact parameters that propagates through the Alcubierre warp drive of varying warp speeds. For warp speeds greater than zero and impact parameters in the vicinity of the warp bubble, photons get increasingly deflected as we increase the warp speed. The deflection angle then steeply decreases as we move away from the area of influence of the warp drive. For further studies, we shall consider relativistic warp speed such as $v = 0.1c, 0.9c$, and $0.99c$ and faster warp speeds such as $v = 20c, 20c$ and $100c$. It is also interesting to ask how photons would scatter in a three dimensional system – that is, a system that considers the photon $z-$ trajectory and momenta propagation, or how charged particles, such as electrons and protons, and how other massive objects, such as stars, would move around the warp drive distortion.

Acknowledgments

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