CS 344: Design and Analysis of Computer Algorithms

Instructor: Kangning Wang

Homework 5: Due on **May 3, 2025**

Acknowledgment: Our team members are Alexander Lin (al1655), Pranav Tikkawar (pt422), and Ivan Zheng (iz72) .

Problem 1

There are n children forming a circle. The i-th child currently has a_i apples. Let $\bar{a} = \frac{\sum_{i=1}^{n} a_i}{n}$ be the average number of apples that a child has, and we assume that it is an integer.

In one move, we can take one apple from a child and give it to an *adjacent* child. What is the minimum number of moves we have to make to make every child have exactly $\bar{a}$ apples? Model this problem by a flow network with costs, and therefore show that this problem can be solved in polynomial time.

Solution. Here is my solution.

Problem 2

Explicitly construct a bijection between each of the following pairs of sets and therefore show that they have the same cardinality.

1. Natural numbers and perfect squares.
2. $(0, 1)$ and $(1, +\infty)$.
3. $(-1, 1)$ and $\mathbb{R}$.
4. $[0, 1]$ and $[0, 1)$.

Solution.

1. Here is my solution.
2.
3.
4.

Problem 3

Review the halting problem, and prove that the following problem is also undecidable: Given a Turing machine, decide whether it always correctly decides the set cover problem. (Note that the given Turing machine may not halt on a valid input, in which case it does not correctly decide the set cover problem.)

Solution. Here is my solution.

Problem 4

Prove that the following problem is NP-complete: Given n positive integers that sum to $2W$, decide whether we can partition the integers into two parts with equal sum (W each).

Solution. Here is my solution.

Problem 5

Prove that the following problem is NP-complete: Given a weighted directed graph with n vertices and m edges, decide whether the graph has a simple cycle with sum of weights equal to 344. The weight of each edge must be an integer, but can be positive, negative, or zero.

Solution. Here is my solution.