2a) $\frac{dL_c(z)}{dt} = q n_c + k_m(z)[L_b - L_c(z)] + k_r R_s^* n_c - k_f R_s n_c L_c(z) = 0$ ss

$$k_m(z) L_c(z) + k_f R_s n_c L_c(z) = q n_c + k_r R_s^* n_c + k_m(z) L_b$$

$$\boxed{L_c(z) = \frac{q n_c + k_r R_s^* n_c + k_m(z) L_b}{k_m(z) + k_f R_s n_c}}$$

b) transport limited: $k_m \ll 1$

$$L_c(z) \approx \frac{q n_c + k_r R_s^* n_c}{k_f R_s n_c} = \frac{q + k_r R_s^*}{k_f R_s}$$

when there is limited transport, $L_c(z)|_{ss}$ is controlled by binding & unbinding of ligand to surface receptors & production of ligand ⇒ negligible ligand is being transported to surface in comparison to binding & production rates

binding limited: $k_m \gg 1$

$$L_c(z) \approx \frac{k_m(z) L_b}{k_m(z)} = L_b$$

the magnitude of ligand transported from bulk to surface is so large that essentially all of the ligand in the bulk is transported to the surface. Binding & production rates are essentially negligible in comparison to transport rate.

c) $R^*_{total}(z) = R^*_s + R^*_i = \left(\frac{1}{k_e^*} + \frac{1}{k_{deg}}\right)\left(\frac{K_{SS}L_c}{1+K_{SS}L_c}\right)V_s$

where $K_{SS} = \frac{k_e^* k_f}{k_e(k_r + k_e^*)}$

↳ assume $K_{SS}L_c \ll 1$, $L_b = 0$

$$R^*_{tot}(z) \approx \left(\frac{1}{k_e^*} + \frac{1}{k_{deg}}\right) K_{SS}L_cV_s$$

from a) $L_c(z) = \frac{qn_c + k_rR_s^*n_c + \cancel{k_m(z)L_b}^{\to 0}}{k_m(z) + k_fR_sn_c}$

$$\boxed{R^*_{tot}(z) \approx \left(\frac{1}{k_e^*} + \frac{1}{k_{deg}}\right) K_{SS}V_s\left[\frac{qn_c + k_rR_s^*n_c}{k_m(z) + k_fR_sn_c}\right]}$$

d) mitotic activity $= \gamma \cdot R^*_{total}(z)$

$$\frac{k_m(z)}{D_L/z} = \left(\frac{\dot{\gamma}z^2}{D_L}\right)^{1/3}$$

$$k_m(z) = \frac{D_L}{z}\left(\frac{\dot{\gamma}z^2}{D_L}\right)^{1/3}$$