Linear Map

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摘要

This is the note of Linear Map, maded by Len Fu while his learning progress. The main content is from *Linear Algebra Done Right* , 线性代数 北京理工大学出版社. and *Linear Algebra Allenby*

目录

1 Definition and Examples

1.1 Definiton

A mapping T from a vector space V to a vector space W is called a *linear transformation* or a *linear operator* if,

$$T(\alpha v_1 + \beta\ v_2) = \alpha T(v_1) + \beta T(v_2)$$

for all $v_1, v_2 \in V$ and all $\alpha, \beta \in \mathbf{F}$.
Then if T is a linear operator on V if and only T satisfies

$$T(v_1 + v_2) = T(v_1) + T(v_2)$$

and

$$T(\alpha v_1) = \alpha T(v_1)$$

. Thus T is a linear transformation mapping a vector space V to a vector space W if and only if T is a linear operator on V.

1.2 Examples

1.2.1 Linear Operators from R^n to R^m

In general, if A is any $m \times n$ matrix, we can define a linear operator T_A from R^n to R^m

$$T_A(x) = Ax$$

for all $x \in R^n$. The operator T_A is linear, since

$$\begin{aligned} T_A(\alpha x + \beta y) &= A(\alpha x + \beta y) && (1)\\ &= \alpha Ax + \beta Ay && (2)\\ &= \alpha T_A(x) + \beta T_A(y) && (3) \end{aligned}$$

Thus we can think of each $m \times n$ matrix as defining a linear operator from R^n to R^m.

1.2.2 Linear Transformation from V to W

2 The Matrix of a Linear Map

Let $T \in \mathcal{L}(V, W)$. Suppose that $(v_1, ..., v_n)$ is a basis of V and $(w_1, ..., w_m)$ is a basis of W .For each $k = 1, ..., n$, we can write Tv_k uniquely as a linear combination of $w_1, ..., w_m$:

$$Tv_k = a_{1k}w_1 + ... + a_{mk}w_m,$$

where $a_{jk} \in \mathbf{F}\ for\ j\ in\ 1, ..., m.$.
The matrix

$$\begin{bmatrix} a_{11} & ... & a_{1n} \\ . & & . \\ . & & . \\ . & & . \\ a_{m1} & ... & a_{mn} \end{bmatrix}$$

is called the *matrix* of T with respect to the bases $(v_1, ..., v_n)$ and $(w_1, ..., w_m)$. We denote it by

$$\mathcal{M}(T, (v_1, ..., v_n), (w_1, ..., w_m))$$

. or we just write $\mathcal{M}(T)$. The k^{th} column consists of the scalars needed to write Tv_k as a linear combination of $w_1, ..., w_m$.

3 Changing coordinate systems

4 Conclusion

This is the conclusion section. Summarize the findings and state any implications or future work.