As you've solved earlier, that the radial part of the hydrogen atom Hamiltonian

$$H = \frac{1}{2} p_r^2 + \frac{\ell(\ell+1)}{2r^2} - \frac{1}{r}$$

\hbar = m =1

the expectation value for the "moments" with respect to energy eigenstates satisfy

$$8kE\langle r^{k-1}\rangle + (k-1)[k(k-2)-4\ell(\ell+1)]\langle r^{k-3}\rangle + 4(2m-1)\langle r^{k-2}\rangle = 0$$

Using the fact that Hankel matrix of the moments $\langle r^k\rangle$, i.e.

$$\begin{pmatrix} \langle r^0\rangle, & \langle r^1\rangle, & \langle r^2\rangle & \cdots \\ \langle r^1\rangle, & \langle r^2\rangle, & \langle r^3\rangle & \cdots \\ \vdots & \vdots & \vdots & \end{pmatrix}$$

must be positive semidefinite, please derive the allowed region of energy values as we increase the matrix size. Provide analysis with regards to the convergence property for different angular momentum sector ℓ. e.g. why the ground state energy cannot be found as mentioned in the original paper

Can you derive other recurssion relations to impose extra constraint? What are the effect of these new constraint?