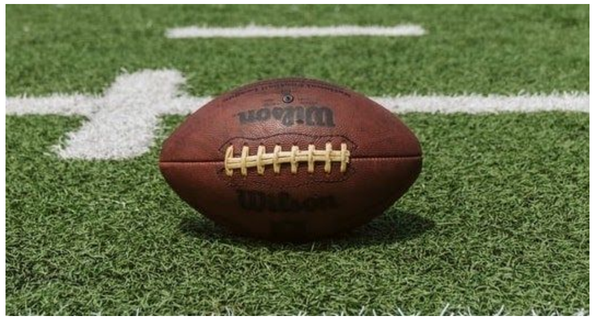


Quarterback Safety in American Football

Modeling the Sport of American Football through the Lens of a Hybrid System

By Len Huang, Megha Jain

TABLE OF CONTENTS

I. AMERICAN FOOTBALL IN A NUTSHELL

American football lends itself to modelling in many ways and has clearly defined rules for what we will refer to as “safety” and “efficiency”. Much work has been done with making models for fútbol / soccer, especially with events like the RoboCup. But we seek to lay the groundwork for how we might approach similar problems of modelling, with American football (to which we refer to as “football”).

Football is played by two teams of eleven players on a rectangular field with goalposts at either end. The offense is the team that has possession of the ball. Their goal is to advance down the field by either running with or passing the ball. The defense is the team without possession of the ball. Their goal is to stop the advance of the offense. Football is played in increments. First, players line up along a line, facing each other. Then, the offense starts an “increment” by handing the ball to the quarterback. Action is continued until either the ball makes its way to the end of the field (or the endzone), or the player with the ball is stopped (often by being tackled to the ground). We will refer to a single increment as a “play”.

For the offensive team, there are three core roles which we will focus on: the offensive linemen (OL), quarterback (QB), and wide receiver (WR). A typical passing play in football involves the QB throwing the ball to the WR as the OL is protecting him from the defense. The QB may need to move around in order to avoid getting hit by the defense. However, the QB must avoid moving too quickly, because if he does, he will no longer be able to accurately pass to the WR. This is the prime challenge we seek to discuss in our model! But more on that later.

For the defensive team, there are two core roles which we will focus on: the defensive linemen (DL) and the linebacker (LB). The goal of the DL is to break past the OL in order to tackle the QB (often referred to as a “sack”) before he is able to pass the ball. The goal of the LB is to prevent the WR from catching the ball and to stop him after he catches it. One rule to note is that the LB is not allowed to touch the WR before he catches the ball, and can only indirectly influence his ability to catch the ball (block it, intercept it, etc). If he does, then it is considered a “pass interference” and counted as a violation against the defense.

II. SUMMARY OF TERMS

Here are some terms that we will be using regularly throughout this proposal. Feel free to look back here if the terms or abbreviations get confusing. We also note some other specifics that may be necessary to know moving forward. Note that some of these positions are more nuanced in actual football, and that we restrict their responsibilities for the sake of a simpler analysis.

1. **Player Roles**
    a. Quarterback (QB): Offensive player responsible for passing the ball to WR’s
    b. Wide Receiver (WR): Offensive player responsible for catching the ball
    c. Offensive Linemen (OL): Offensive player responsible for protecting the QB
    d. Defensive Lineman (DL): Defensive player responsible for attacking the QB
    e. Linebacker (LB): Defensive player responsible for stopping the WR
2. **Logistics**
    a. Football Field: 160ft wide x 300ft long, or 48.8m x 91.44m. The length is often measured in yards (100 yards, where 1 yard = 3ft).
    b. Play Clock: The ball must be passed within 40 seconds
    c. Line of Scrimmage: The length-wise position of the ball at the start of a play
3. **In-Game Actions**
    a. Pass Interference: An illegal move where the LB touches the WR before he catches the ball
    b. Sack: When the DL breaks past the OL and tackles the QB
    c. Touchdown: The Offensive team safely delivers the ball to the “endzone”, or past the 300ft on the field.

III. FOOTBALL THROUGH THE LENS OF HYBRID SYSTEMS

Our goal with this project is to explore how we might view football from the perspective of a hybrid system. A single play in football can be viewed as a collection of subproblems woven together to solve a greater goal of furthering the position of a ball. Some of the subproblems include passing, tackling, and quarterback movement with respect to linemen interactions. To get a better understanding of these subproblems as hybrid systems, we will describe each as a more traditional example of CPS we might see in a textbook.

Passing is very similar to how we might ensure the accuracy of a catapult or missile launching system (perhaps a bit of an extreme comparison). We want to ensure that, given a target, we are able to programmatically determine a path for our projectile to travel along in order to reach the target. This can start by first viewing how we might hit a static target, then a moving target, and finally a moving target as we (the catapult) are moving. Since we are reasoning about football, we are able to restrict the realm of possibility for how our target (WR) and catapult (WR) will move.

Tackling can be reduced to the scenario of two robots following each other. If we consider our players as infinitesimal points, then we simply want to model their intersection as “tackling”. This relates heavily to the “Follow the Leader” lab, where we want to maintain a certain distance from a system in front of us. We will add an additional layer of complexity where we are able to accelerate again after a certain event-triggered control. In other words, once a certain event is triggered, our goal changes from maintaining distance to the leader, and instead intersecting the leader. Within the realm of football, this means that LB is allowed to tackle the WR after he catches the ball.

And finally, we have what our modelling will center around. With passing and tackling in mind, we then seek to ask, how we might be able to model the safety of the quarterback? There are many different things to consider with this. We could say that for the quarterback to be safe, we conservatively deem him to be the “leader” in the Follow the Leader lab. However, this fails since the goal of the follower IS to be “unsafe”. Furthermore, we cannot arbitrarily move backwards both because of the physical limitations of the field and the limitations of how far we can throw to the WR. We also cannot move at arbitrarily quick speeds, since that would impede our ability to pass the ball *(in real football, the QB could choose to run, but this poses significant risk. Introducing this into our model would be introducing another strategy for the QB, as to when he should and should not violate his velocity restrictions, noting that violating it would inhibit his ability to pass).* We must then leverage our ability to move side to side, the 40 second play clock, and the ability to pass to ensure the safety of our QB until the end of the play. There is also another layer of complexity since we now also have teammates that are there to protect us, and it is not wholly our responsibility as the QB to maintain our safety.

We will go on to further define some of the various subtleties and logic needed to describe and model a football play as a hybrid system. But let these comparisons and discussions serve to motivate your understanding of how one might view the game of football from the perspective of a hybrid system.

IV. PROS AND CONS OF dGL AND dL

A sports game seems to lend itself naturally towards a dGL implementation. After all, it is a "game" with a clear notion of players to which we can assign angelic and demonic roles. But there are countless strategies and plays in the game of football, which would be difficult to model. So to view this from a dGL perspective, we would likely need to burn in a set strategy into our players. We'll be doing this by limiting the scope and play of our specific football model. However, a subtle issue arises when using dGL. By burning in a strategy, this may allow an opponent to peek at strategies, giving one or the other an unfair advantage. In real football, there are some contextual clues one can use to guess what your opponent's strategy is. However, it is still much more of a toss up and not very deterministic. While discussing dGL, we will refer to the Offense as the Angel team and the Defense as the Demon team.

In order to avoid this "peeking" problem, it is possible to reduce it to a hybrid systems question to test if a fixed strategy works or not. However, this makes no compromise in the game-like intuition of our model. The ulterior motive of the offense is to protect the quarterback and move the ball forward. This lends itself naturally to a safety condition and efficiency condition, similar to our very first lab, Charging Station. More specifically, our strategy is safe if our quarterback isn't tackled. However, we don't care if he is only safe because we want to win and move the ball forward! It also makes sense to model our QB as the controller, since in the real sport, the QB does not have control over his teammates and what they do, so he must act in a safe way that considers scenarios where the OL may fail to protect him. For now, we will explore a dGL version of our model, and may explore dL in later steps.

V. LITERATURE REVIEW

Modelling football is not the most pressing of issues in our society, so not much research has been explicitly done in this area. However, the nature of our project lends itself very naturally towards a combination of collision-avoidance and distributed systems problems. The ulterior goal is to successfully avoid the collision of our quarterback and another "robot" in a larger system of "robots" (or the 22 total players on the football field). With that in mind, we begin to explore trends within the area of collision avoidance and distributed hybrid systems.

First we set the scene of what we mean by a distributed hybrid system (DHS). Many safety-critical systems in aviation and transportation seek to combine communication, computation, and control [5]. The ideas of computation and control can be unified under "hybrid systems", the ideas of communication and computation can be unified under "distributed systems", and the union of all three can then be referred to as "distributed hybrid systems." The systems include both discrete transitions of system parts that communicate with each other and discrete / continuous dynamics from discrete control decisions and differential equations of movement. A use case that portrays this particularly well is proving the safety of a highway merge; namely, the lack of collision in a system of cars that have both discrete controls and communication with each other. There's not a formal way to reason about these kinds of systems, so Platzer seeks to develop and prove "Quantified Dynamic Logic (QDL)" to formalize and verify the safety of these systems. This may be slightly out of scope for our project, but it does seek to help motivate ideas of how we might reason about the various players in our field as systems that must communicate with each other in order to avoid collision. The example of car collision, however, is a step up in complexity when compared to our model for football. This is because DHS is also able to consider a system whose structure is always changing; namely, a system where cars will appear and disappear as they come and go. We do not have to worry about this since we have a fixed structure of 11 players on either team, but we can definitely benefit from some of the ideas of representing a system with multiple moving parts (literally and figuratively). With a formal QDL in mind, we seek to discuss more about the nature of what a multi agent system or DHS may look like, and then discuss the nature of the collision avoidance problem.

While a distributed hybrid system may be overly complex for our needs, it would still be useful to explore a way to quantify large groups of systems. Especially when discussing Multi-Agent Robotics, it can be helpful to think of these large groups as "collectives" [2]. Depending on the nature of the task, a collective is better than a complicated robot: "Collectives of simple robots may be simpler in terms of individual physical design than a larger, more complex robot, and thus the resulting system can be more economical, more scalable and less susceptible to overall failure" [2]. Another aspect of multi-agent systems is the way they communicate: "the tasks that they perform are typically parallelized with small amounts of coordinating communication at either the start (for truck delivery) or at the end (forestry). In these tasks each element of {r_i} operates independently for the most part, utilizing interagent communication either initially, to parcel up the expected workload in an efficient manner, or penultimately, just before dealing with any work that was not covered during the parallel portion of the processing" [2]. This suggests that the robots are not able to directly communicate with each other while they are performing their tasks, but are able to have moments of communication before and after. This is very similar to how in football, a team will meet together to determine a strategy in which the players follow. But after that each unit within the collective may be up to their own individual task. With the intuition now for how we might represent football as a multi agent system, as well as the logic for reasoning about distributed hybrid systems, we introduce the problem we wish to solve in this context: collision avoidance.

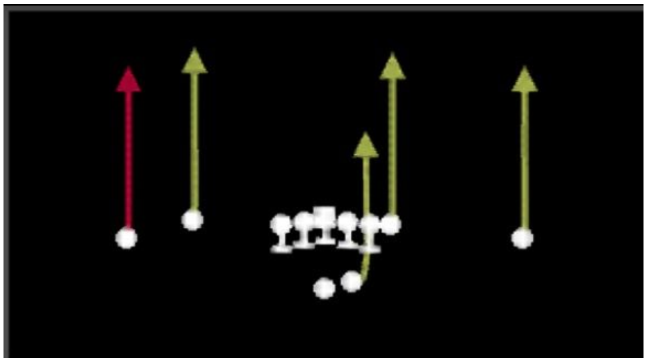
Platzer discusses a case study of aircraft collision avoidance [6]. Namely, how two hybrid systems (airplanes) can communicate with one another and engage in a curved maneuver in order to avoid collision. One idea that we may seek to use from this case study is one of bounded overapproximation. In Section 3.7, when discussing Safe Entry Separation (AC5), they over approximate aircraft distances and speed in order to reduce the polynomial degree and verification complexity of their model. While our model is not as complicated, we borrow this same intuition of overapproximation in order to conservatively guarantee that our Quarterback will be safe. In fact, we use this idea to motivate the generalization of the Offensive and Defensive Lines as literal lines (more on this later).

By looking at distributed hybrid systems and their quantified dynamic logic, and how it might be used to formalize multi-agent systems, in particular one which may focus on collision avoidance and using overapproximations, we begin to piece together various literature that was very important in CPS and Robotics research and use football as a medium to discuss those topics.

VI. INITIAL MODEL

Simplified Football via Hail Mary:

There are various plays in Football which involve certain players running certain routes and such. This can lead to some undesired complexity, so for our model, we will pick a simple football play known as “Hail Mary.”



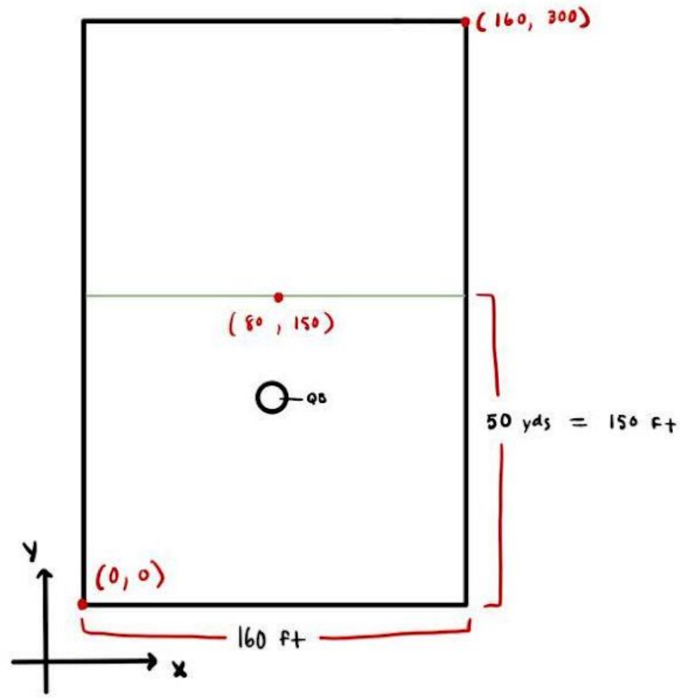
A depiction of a Hail Mary Pass in video game NFL Mobile.

As in the image above, we see that there are 5 Wide Receivers (WR), each with a forward arrow representing the path they will follow. Their goal is to run forward and catch the ball if it is thrown to them while avoiding contact from the Linebacker (LB). The dots with a seemingly inverted-T shaped root to them and are aggregated in the center represent our 5 Offensive Linemen (OL), whose goal is to protect the Quarterback (QB) from the Defensive Line (DL, not in picture). Finally, the last dot towards the bottom that is seemingly being protected, is the QB.

5 Wide Receivers, 5 Offensive Linemen, and 1 QB thus represent the 11 players that make up a football team. However, modeling 22 (considering the other team as well) individual hybrid systems may be beyond the scope of our project. As such, to simplify our model, we will reduce the OL and DL to literal lines, and only use 2 WRs and 1 QB. More modeling decisions will be further elaborated upon in later sections.

Geometry of the Football Field

First, let us understand the geometric representation of the football field. Note that the football field's dimensions are 160 feet by 300 feet. If we consider the football field to be on a coordinate plane, with the units being in feet, we will let the lower left corner be at the point (0, 0), the upper right corner be at (160, 300), and the center be at (80, 150). For generality, we will consider the play to start centered horizontally and vertically on the football field. Therefore the line of scrimmage is the 50-yard line (or y = 150 feet).

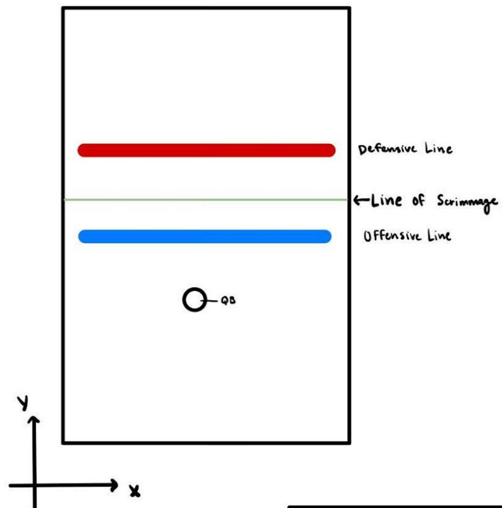


The Players

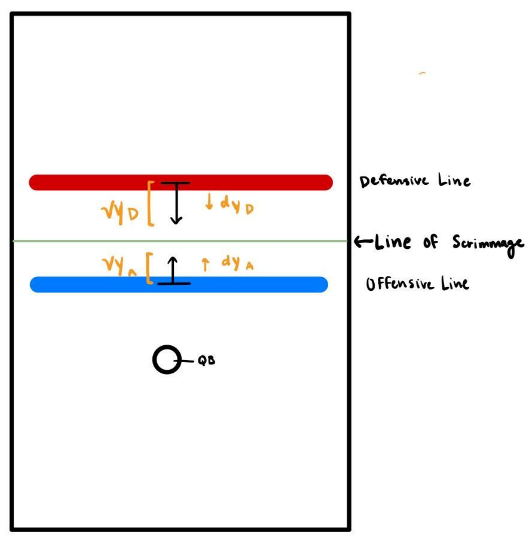
We will break the system down into its smaller subparts, by first examining how the Offensive and Defensive Lines work. To simplify the system, we will consider the Offensive Line and Defensive Line as their own single lines that move as a unit rather than individual players. In real football, the players might struggle to break past one another. For our model, we will simplify this by saying that the Defensive Line slowly approaches the Quarterback.

Defensive and Offensive Lines

In football, there is no fixed rule as to where the Offensive and Defensive Lines need to start exactly. The only constraint is that the Offensive Line starts in front of the Quarterback on one side of the Line of Scrimmage and the Defensive Line starts on the other side of the Line of Scrimmage as visible in the visual to the right. We will be viewing the football field from the perspective of the offense.



Note that the Offensive Line's duty is to protect the Quarterback and the Defensive Line's goal is to tackle the quarterback before he throws the ball. Therefore the Defensive Line travels toward the quarterback, which is the negative direction on the y-axis (dy_D). On the other hand, the Offensive Line travels away from the Quarterback, which means they travel in the positive direction on the y-axis (dy_A). To simplify the model, we consider that both of the lines travel with a constant velocity before they collide. We visualize the velocities and directions of the two lines in the diagram to the right.



The Defensive Line, represented by the red line, is moving with speed vy_D in the direction dy_D, and the Offensive Line (blue) is moving with speed vy_A in direction dy_A. Note that vy_D and vy_A are the magnitudes of the lines' velocities, and are therefore non-negative.

At the start of the play, the two lines will charge towards each other. However, once the two lines collide, they will move as one unit with the velocity being the sum of their initial velocities; this models a perfectly inelastic collision. The NFL Combine is an event where players compete in various athletic tests such as a 40-yard dash, vertical jump and more. We aggregate NFL Combine results for the 40-yard dash to derive an estimate of what each position's average speed might be. From these results, we see that Defensive Linemen are faster than Offensive Linemen across the National Football League; we model this by making the magnitude of the initial velocity of the Defensive Line greater than the magnitude of the velocity of the Offensive Line. Therefore, when the two lines collide, their velocities will counteract one another, effectively dampening the velocity of the Defensive line. One might think of this as two people pushing against one another, but one eventually dominating the other in terms of force, thus pushing them back. Taking into account the force of the Offensive and Defensive lines, we see that they will travel with a velocity that has a magnitude equivalent to half of the difference between the magnitudes of the initial Defensive and Offensive Line's velocities. If we consider that the masses of the Offensive and Defensive Lines are the same, we know that this abides with the Principle of Momentum Conservation because of the following equations.

Let m_A and m_D be the masses of the Offensive and Defensive lines respectively, where $m_A=m_D=m$. Let vy_{Ai} and dy_{Ai} be the initial velocity (magnitude) and direction of the Offensive Line, vy_{Di} and dy_{Di} be the initial velocity (magnitude) and direction of the Defensive Line, and vy_f and dy_f be the final velocity and direction of both lines together after the collision. By the conservation of momentum, we have:

$$m_A \cdot vy_{Ai} \cdot dy_{Ai} + m_D \cdot vy_{Di} \cdot dy_{Ai} = m_A \cdot vy_f \cdot dy_f + m_D \cdot vy_f \cdot dy_f$$

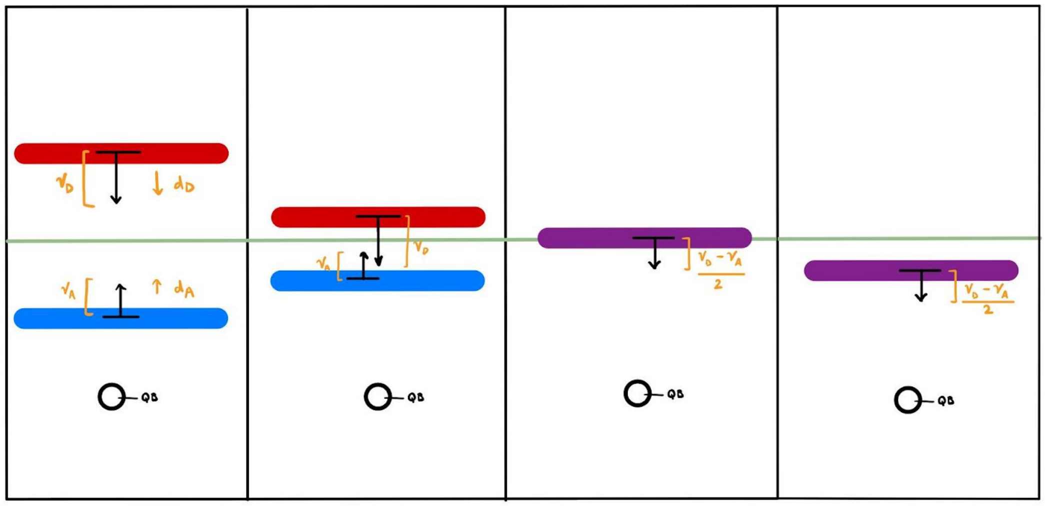
$$m(vy_{Ai} \cdot dy_{Ai} + vy_{Di} \cdot dy_{Di}) = 2 \cdot m \cdot vy_f \cdot dy_f \qquad [\text{since } m = m_A = m_D]$$

$$m(vy_{Ai} - vy_D) = 2 \cdot m \cdot vy_f \cdot dy_f \qquad [\text{since } dy_{Ai} = 1,\ dy_{Di} = -1]$$

$$(vy_{Ai} - vy_{Di}) = 2 \cdot vy_f \cdot dy_f \qquad [\text{eliminate the m}]$$

$$(vy_{Ai} - vy_{Di})/2 = -vy_f$$

Since $vy_{Di} > vy_{Ai}$ (the Defensive Line is faster than the Offensive Line), we know that the left hand side has to be negative as $(vy_{Ai} - vy_{Di})/2 < 0$. Therefore, $vy_f \cdot dy_f < 0$, and vy_f is a magnitude so it cannot be negative. Therefore, $dy_f = -1$, and the direction is the same as the Defensive Line's original direction. We have $(vy_{Ai} - vy_{Di})/2 = -vy_f$. The progression of movement is shown in the following figure.



The Quarterback

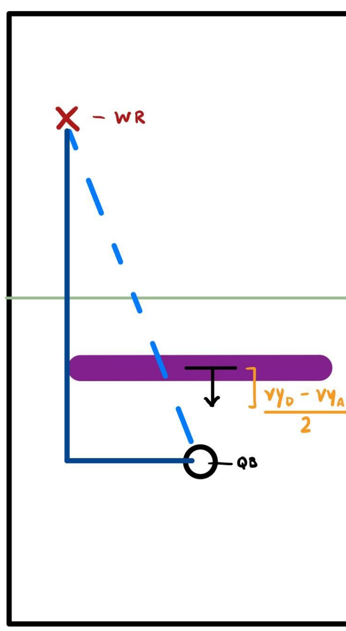
The next component of our system is the Quarterback. The Quarterback is a player that we must protect until he executes a pass. In order for our system to be safe and have a winning strategy, the Quarterback must not be tackled until he executes a pass.. In football, Quarterbacks tend to be smaller than the Offensive and Defensive Linemen. For reference, the average weight of a Defensive Lineman is 310 pounds [4], while the average weight of a Quarterback is 225 pounds [3]. It is quite important that the Quarterback is not tackled by the Defensive Line, so our system is "safe" when the position of the Quarterback is below that of the Defensive Line (and Offensive Line since these move together upon collision).

Our system's winning strategy for Angel, or the offensive team, is when the Quarterback successfully throws the ball to the Wide Receiver. We will introduce the specifics of the Wide Receiver in the next section. However, in reference to the Quarterback, one component of the winning strategy must be that the Quarterback passes the ball (or that the ball is no longer with the Quarterback at the end of the play). The play ends either when the timer runs out, the Quarterback gets tackled, or the Quarterback passes the ball. The second component of the winning strategy is that the Quarterback manages to successfully throw the ball to the Wide Receiver. Realistically, the Quarterback cannot throw the entire length of the football field; an average NFL Quarterback is at most able to throw a ball around 65-70 yards. To simplify our model, we will cap the passing distance at 200 feet.

In modeling the Quarterback, he must be able to move around in all directions and with a maximum velocity of 9 feet per second, which is slightly faster than the average walking speed. We choose this speed because in order to accurately throw, the quarterback cannot move too fast. The Quarterback must be allowed to travel backwards to avoid getting tackled before throwing the ball. He must also be able to move left, right, and forward in order to get as close to the position of the Wide Receiver -- or at least within the range of 200 feet. Combining both the safety condition and the winning strategy introduces a modeling subtlety that makes winning much harder. That is, the Quarterback cannot solely travel backwards to avoid getting tackled by the Defensive Line because that would make remaining within 200 feet of the Wide Receiver impossible. On the other hand the Quarterback's forward movement is bounded by the y-position of the Defensive Line. Since the Wide Receiver and Defensive Lines are continuously moving, the bounds in which the Quarterback must remain in are also continuously changing.

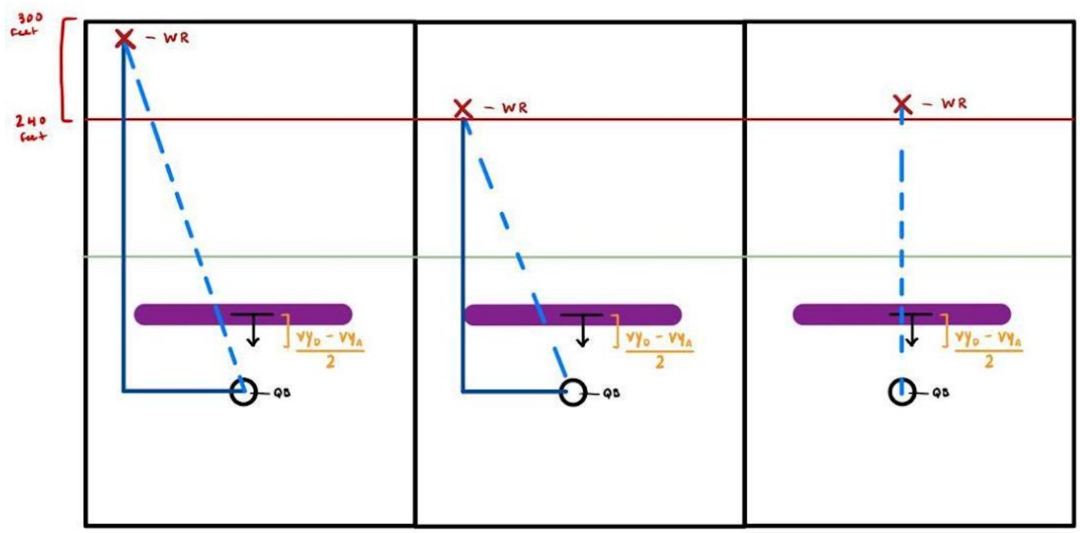
The Wide Receiver and Linebacker

The final part of our hybrid system is the Wide Receiver. We start by modeling it as a stationary target which the Quarterback has to pass to. This model hardcodes the position of the wide receiver at (20, 300), to indicate that catching the ball will result in a touchdown. While our model aims to demonstrate quarterback safety, getting touchdowns is still an overall goal of the game.

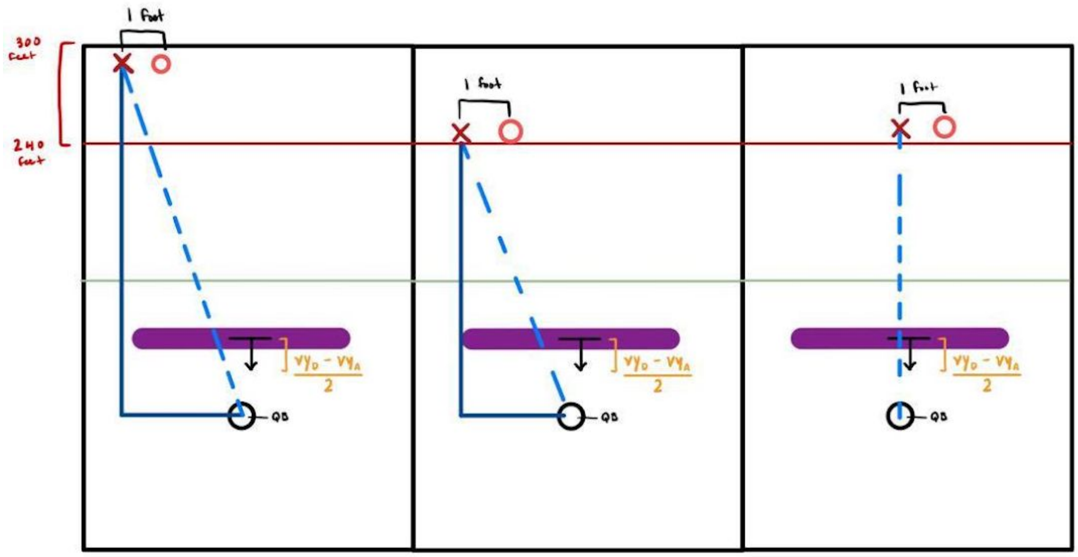


The above figure describes the model above. The same Quarterback, Defensive Line, Offensive Line sub-system is in the lower portion of the diagram. We now add the Wide Receiver (as indicated by the "x") and a dashed blue line to denote the distance between him and the Quarterback. Recall the modeling subtlety that was previously described where this distance between the Wide Receiver and the Quarterback must be within 200 feet.

However, in real football, the Wide Receiver starts behind the line of scrimmage (as do all the offensive players). So it would be helpful to add some sort of motion to the model of the Wide Receiver, in order to get from a starting point to the endzone. For example, if we're starting at the 50 yard line (yA = 150), then we want to model the Wide Receiver moving from (20,150) to (20, 300). We will later introduce movement of the Linebacker who defends the Wide Receiver. Introducing motion for the Wide Receiver consists of enabling movement in the x and y directions to get the ball. In order to make this play realistic, we do not allow our wide receivers to go behind 240 feet (or the 80 yard line). The Wide Receiver may want to move closer to the x-direction of the quarterback in order to ensure that their distance does not exceed 200 feet. The way we model this is by allowing the Wide Receiver to move on a line for a given amount of time; the Quarterback's throw will be triggered when the distance from the Wide Receiver is minimized. If the closest the two will be exceeds 200 feet, they will travel on a new line in order to get closer. The movement of the wide receiver can be seen in the following diagram.



Now that we have a moving target to throw to, we want to model the Linebacker to defend against the Wide Receiver. Modelling the Linebacker shares many similarities with the Follow the Leader Lab. Namely, the Linebacker must remain close without hitting the Wide Receiver, only hitting or tackling him after he catches the ball. We hardcode these notions of the Linebacker being "close" as being within 1 to 3 feet from the Wide Receiver . The Linebacker cannot get closer than 1 foot because that would indicate a pass-interference; however, remaining farther than 3 feet away would make it difficult for the linebacker to tackle the Wide Receiver and end the play. The relationship between the Linebacker and the Wide Receiver can be seen in the following images.



Once the ball is thrown, the Linebacker must close the gap between himself and the wide receiver in order to stop the play. If the Linebacker reaches the wide receivers position at the same time as the ball reaches the receiver, we will consider that an interception.

Proving this model may not be possible because the defense always has a winning strategy if the linebacker can choose its own speed and intercept any ball thrown. As such, we would like to change one small aspect of the model to show that in one case, the offense has a winning strategy, and in the other case, the defense has a winning strategy. Namely, depending on how far apart the Wide Receiver and Linebacker start from one another, either the offense or defense will have a winning strategy.

The offense has a winning strategy if the Linebackers are not able to catch up to the Wide Receiver. In other words, the linebacker will not start directly next to the wide receiver. Instead the linebacker would start a little behind the line of scrimmage. Then when the play starts, the linebacker needs to move much farther to be within 1 to 3 feet of the wide receiver. In that time, the wide receiver could have already received the ball. This reflects a “blitz” defense in football, where the defensive players start closer to the line of scrimmage in order to have more available players for attacking the Quarterback.

The defense has a winning strategy in the cases we described above, with the Linebacker starting within 1 to 3 feet of the Wide Receiver. As described before, the Linebacker can tightly follow the Wide Receiver and eventually tackle him. This reflects a “zone” defense in football, where the defensive players will spread themselves out amongst the field in order to maximize defensive coverage of the field. We will build these two models and prove that in one, the defense has a winning strategy ([ ] - demon’s winning strategy), and in the other the offense (< > - angel’s winning strategy) has a winning strategy.

VII. PROGRESSION

Dampened Defensive Line following the Quarterback

Currently, our model implements the first two subparts we described, which include the Offensive Linemen, Defensive Linemen, and Quarterback. For this stepping stone, we have first simplified the OL-DL collision to include just the Defensive line moving towards the QB with a slower velocity. The velocity of the moving line is equivalent to the speed of the OL + speed of the DL, which ends up being a smaller negative value. This represents the difference in magnitudes between the offensive and defensive lines. The ordinary differential equation for the quarterback is simply movement backwards to avoid getting tackled.

A quick note about where we get our values from. The average OL Speed is 22.5 ft/s, the average DL Speed = 23.7 ft/s. Note that the upper bound on DL Speed is ceil(23.7) = 24. The upper bound on difference = ceil(23.7) - floor(22.5) = 24 - 22 = 2. Recall that the football field has dimensions 300ft x 160ft.

We have proved a much easier model, with the model and proof attached here. The comments next to the variable names describe the purpose of each. (We also attach this model and proof as STAR LAB TRIAL 0.kyx)

```
Definitions
  Real Diff; /* maximum difference in strength of OL and DL */
  Real T; /*Time bound of a play, 40 seconds*/
End.

ProgramVariables
  Real diff; /* difference in the strength of OL and DL */
  Real maxVel; /* maximum realistic velocity */

  Real yA; /*y-position of the offensive line (Angel-team)*/
  Real vyA; /*magnitude of velocity in y-direction of the offensive line*/
  Real dyA; /*direction of velocity in y-direction of the offensive line*/

  Real yD; /*y-position of the defensive line (Demon)*/
  Real vyD; /*magnitude of velocity in y-direction of the defensive line*/
  Real dyD; /*direction of velocity in y-direction of the defensive line*/

  Real yQB; /*y-position of the Quarterback (Angel-team)*/
  Real vyQB; /*magnitude of velocity in y-direction of the Quarter Back*/
  Real dyQB; /*direction of velocity in y-direction of the Quarter Back*/

  Real t; /*running time of the play*/
End.
```

```
Problem
/* Pre-Conditions */
  ( Diff = 2 & T = 40 /* Max Difference + Play Clock, T */
  & 15 < yA & yA < 300 /* Start anywhere after 3 yard line */
  & yD = yA /* OL/DL start as intersecting */
  & yQB = yA - 15 /* QB is 3 yards behind OL */
  & maxVel = 24 /* realistic speed for DL */
  & 0 < diff & diff < Diff /* realistic decrement for OL speed */
  & dyA = 1 & dyD = -1 /* angel moves "up" the field demon moves "down" */
  & vyQB = 9 & dyQB = -1 /* for now, QB moves down. Walking speed ~9ft/s */
  ) -> <
  /* Let defense / demon choose a speed for their DL */
   {
      vyD := *; /* do it non-deterministically */
      ?(0 < vyD & vyD < maxVel); /* ensure realistic velocity */
   }^@

   vyA := vyD - diff; /* On average, OL slower than DL */
   t:= 0;

   /* QB moves backwards, and both OL + DL move at a dampened speed */
   {
      yQB' = dyQB*vyQB, yA' = 0.5*(dyA*vyA + dyD*vyD),
      yD' = 0.5*(dyA*vyA + dyD*vyD), t' = 1 & t <= T
   }
   >
   /*Safety Conditions*/
   ( yQB < yD /* QB not tackled yet */
   & 0 <= yA & yA <= 300 /* angel still on the field */
   & 0 <= yD & yD <= 300 /* demon still on the field */
   & 0 <= yQB & yQB <= 300 /* QB still on the field */
   )
End.

Tactic "STAR LAB 0: Proof"
  unfold ; solve(1.1) ; assignd(1) ; auto
End.
```

Collision of Offensive and Defensive Lines via sequential ODE's

For the next stepping stone of our model, we introduce the concept of two sequential ODEs for the movement of the offensive and defensive lines. The first ODE has a domain constraint of yA < yD, which represents the time when the two lines are still running towards each other. The second ODE has a domain constraint of yA >= yD, which represents the two lines colliding and moving together. When we try to prove this model, the ODE's prove individually; however, upon having the two ODEs together, this model does not prove. This shows us that one of the difficulties we will face in our proofs is having to prove with diamond modalities, as we have much less experience in that realm. (We attach this model as STAR LAB TRIAL 1.kyx).

Add a Stationary Wide Receiver

After successfully proving the two ODE's together, we will introduce a stationary wide receiver as described in the sections above. We will start without introducing a linebacker (defense). To do so, the model will have two more agents: a ball and the wide receiver. We will start by hardcoding the position of the wide receiver as (20, 300). Therefore, the quarterback now has to be within 200 feet of this fixed position. The proof of this should follow easily from the proof of the past stepping stone because the quarterback can immediately throw the ball and the offense would win (which is why the proof of this step is trivial). (We attach the beginning implementation of this model as STAR LAB TRIAL 2.kyx).

Implement two strategies for two Linebacker scenarios

For the following stepping stone, we will finally introduce the linebacker. This is where our model will split into two, a defensive winning strategy and an offensive winning strategy, depending on where the linebacker starts. For the first model, we will allow the linebacker to start within 1 foot of the wide receiver. We will show that the wide receiver intercepts the ball in this situation. For the second model, we will start the linebacker 30 feet behind the line of scrimmage, and we will show that the offense will win. The winning strategy will be the same for the offense as we have described above. In both of these models, we will implement an ODE inspired by "follow the leader" for the quarterback.

Introduce straight line movement for Wide Receiver

With a functioning stationary wide receiver, we then wish to model its movement. We discussed ideas of starting at an ideal position, and moving along lines to minimize the distance between the Wide Receiver and Quarterback. However, this may be a bit difficult to do at first, especially with lateral movement. This is where our choice to model the Hail Mary play comes in handy. In the Hail Mary play, the Wide Receivers only run forward along a straight path. The next step of the proof then would be to include modelling the movement of the Wide Receiver starting from the line of scrimmage and moving forward along a straight path, and to incorporate passing into it.

Include lateral movement for the Wide Receiver

After successfully modelling a straight path play for the Wide Receiver, we can move on to introducing lateral movement. The goal of the Wide Receiver would be to stay close to the quarterback. Therefore, the Wide Receiver will first move in the y-direction to be at 240 feet (which is the lower bound of the wide receiver's position). Then the wide receiver will try to maintain as little of a difference in its x position from the Quarterback's x position, like in Follow the Leader. While this happens, the linebacker will also be following the wide receiver.

At each of these stepping stones, the complexity of the model increases, so the complexity of the proof increases as well.

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