Quarterback Safety in American Football

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Nice to Meet You!

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Agenda

- Intro to American Football
- Motivation
- Approaches to Modelling
- Proofs and Insights

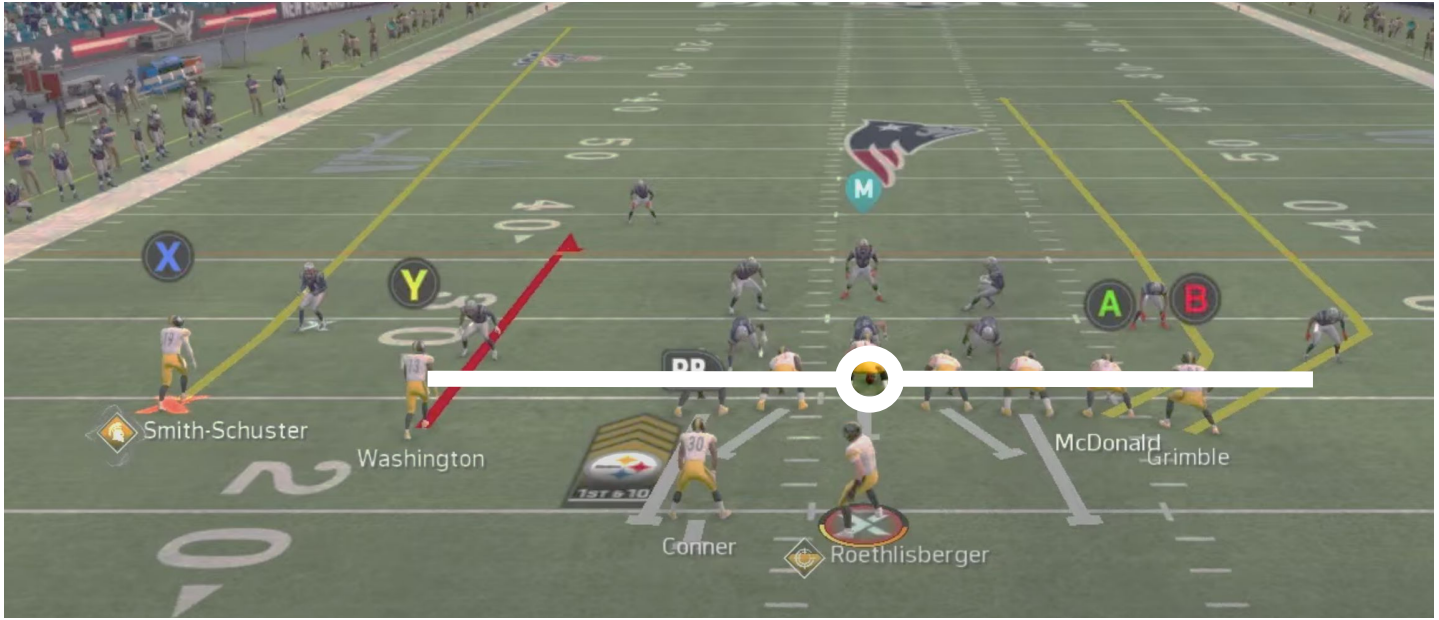
Intro to American Football

- Line of Scrimmage
- Quarterback
- Offensive Linemen
- Defensive Linemen
- Wide Receiver
- Linebacker

Intro to American Football

Game of Advancing the Ball Down the Field. Played in Increments

Intro to American Football (Line of Scrimmage)



Current Lengthwise Position of the Ball

Intro to American Football (Quarterback)

Quarterback: Responsible for Passing the Ball

Intro to American Football (Linemen)

Offensive Line Protects QB, Defensive Line Attacks QB

Intro to American Football (Wide Receivers / Linebackers)

Wide Receivers Run and Catch, Linebackers Prevent This

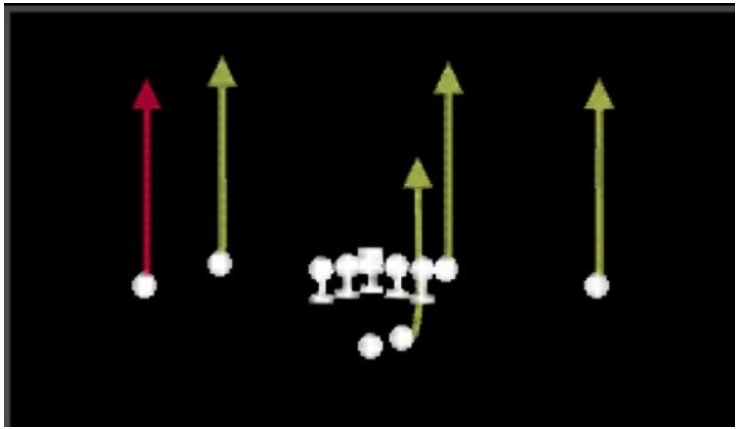
Motivation

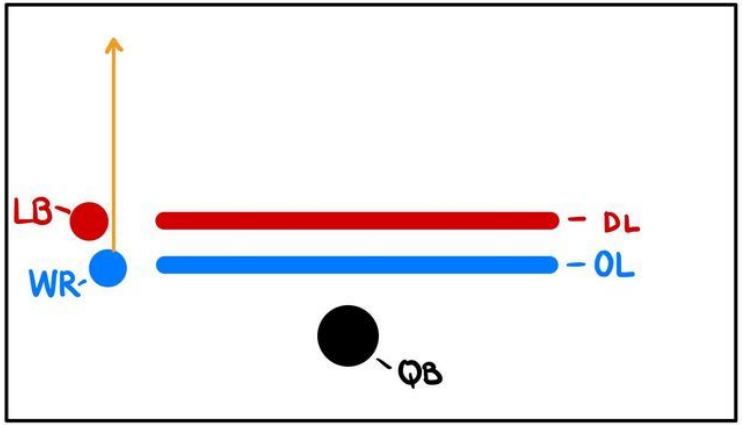
- **Safety** ⇒ QB is not tackled
- **Efficiency** ⇒ Pass is made
- Stepping stone to more multi-agent systems

Approaches to Modelling

- Simplifications
- Modeling Scenarios
  - Linemen Collision (Safety)
  - Passing (Efficiency)
- KeyMaera X Model

Simplifications



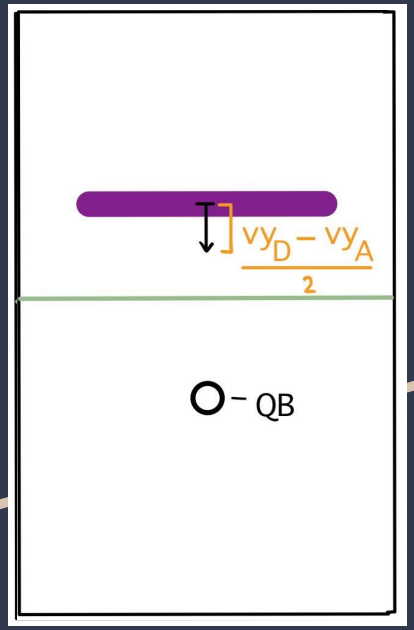


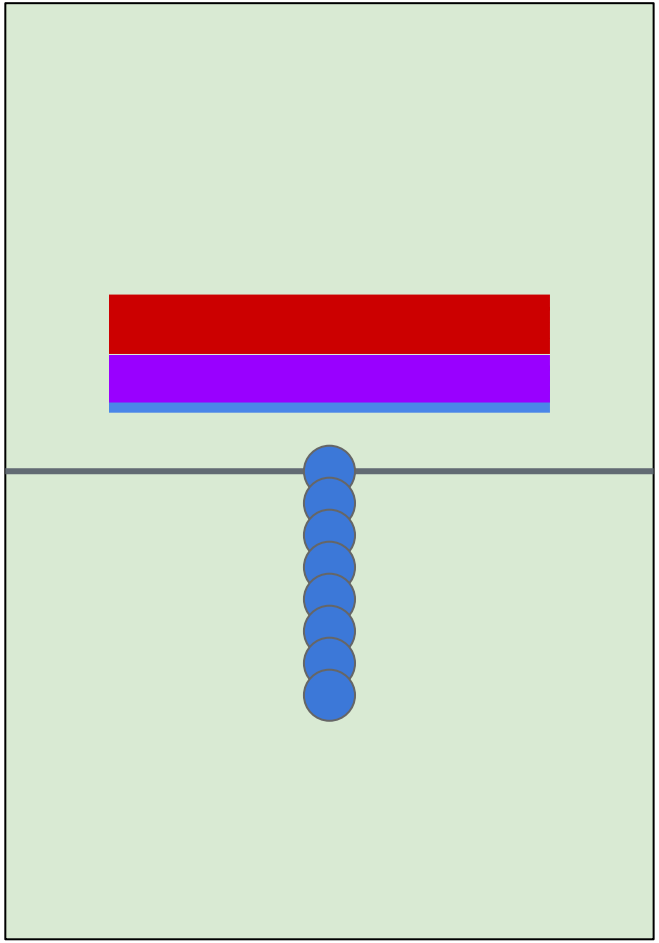
Less Players + One Dimensional

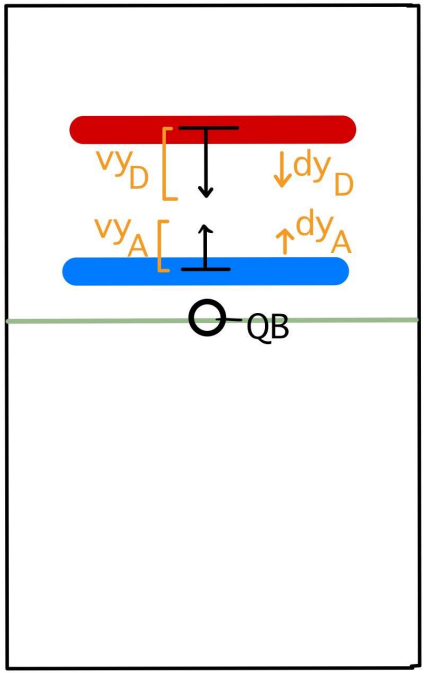
Lineman Collision

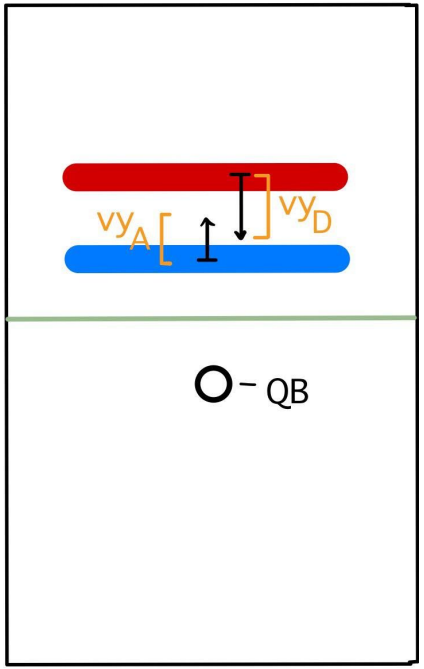
Lineman Collision

(Visualization)

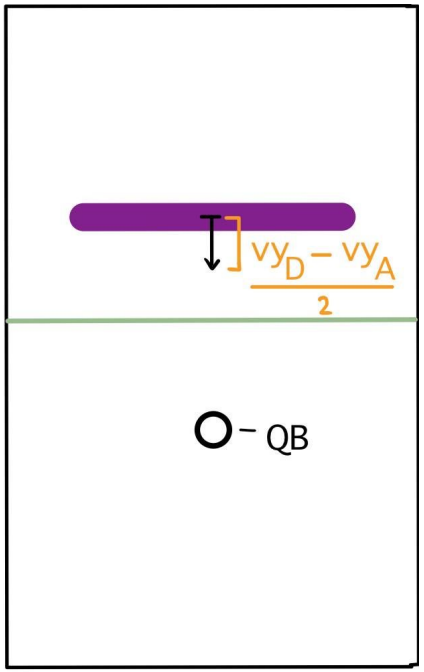




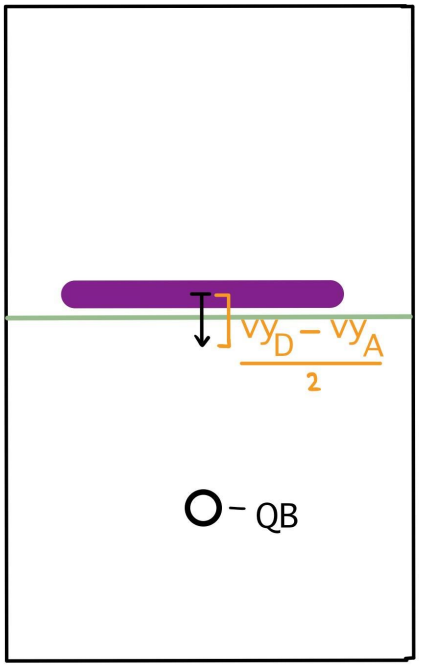




Pre-collision: run towards each other



Perfectly inelastic collision



Post-collision: move with dampened speed towards QB

Lineman Collision **(Movement)**

$$m_A \cdot vy_{Ai} \cdot dy_{Ai} + m_D \cdot vy_{Di} \cdot dy_{Ai} = m_A \cdot vy_f \cdot dy_f + m_D \cdot vy_f \cdot dy_f$$

$$m(vy_{Ai} \cdot dy_{Ai} + vy_{Di} \cdot dy_{Ai}) = 2 \cdot m \cdot vy_f \cdot dy_f \qquad (m = m_A = m_D)$$

$$m(vy_{Ai} - vy_{Di}) = 2 \cdot m \cdot vy_f \cdot dy_f \qquad (vy_{Ai} = 1, vy_{Di} = -1)$$

$$vy_{Ai} - vy_{Di} = 2 \cdot vy_f \cdot dy_f \qquad \text{(eliminate m)}$$

$$\frac{vy_{Ai} - vy_{Di}}{2} = -vy_f$$

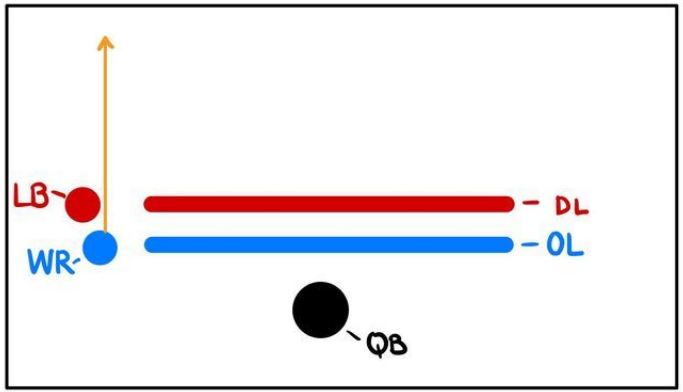
Lineman Collision (Conservation of Momentum)

Passing

Passing (Passing ↔ Open)

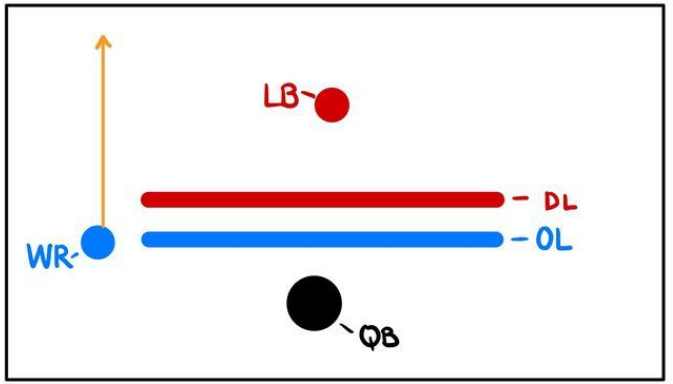
- **Efficiency** ⇒ Pass is made
- Assume Quarterback can **instantaneously** pass as soon as our Wide Receiver is **open**
- **Lemma:** `isOpen(...) == ballPassed(...)`
- Why?
  - Focus on QB Safety
  - Helps us define “pass is made”

Passing (Defensive Considerations)



Man: Wide Receiver must **run past** the Linebacker in order to catch the ball

```
Bool isOpen(WR,LB) ↔ LB < WR
```



Zone: Wide Receiver must **catch before** getting tackled by the Linebacker

```
Bool isOpen(WR,LB) ↔ WR < LB
```

Keymaera X Model

- Variation
- Pre-collision Movement
- Post-collision Movement
- `isOpen() == ballPassed()`
- QB Unhurt + Ball is Passed

```
<
  vyA  := vyD  - diffLine;
  vyLB := vyWR - diffPass;
  t:= 0;

  /* Pre-collision movement */
  { yQB' = dyQB*vyQB,
    yA'  = dyA*vyA,
    yD'  = dyD*vyD,
    yWR' = dyWR*vyWR,
    yLB' = dyLB*vyLB,
    t' = 1
    & yA <= yD /* Pre-collision */
    & t <= T /* Realism */
  }

  /* Keep evolving */
  { yQB' = dyQB*vyQB,
    /* Dampened Movement */
    yD'  = (dyA*vyA + dyD*vyD)/2,
    yWR' = dyWR*vyWR,
    yLB' = dyLB*vyLB,
    t' = 1
  & yA >= yD /* Collided */
  & t <= T /* Realism */
  }

>( yQB < yD /* QB unhurt */
 & isOpen(yWR, yLB, buffer) /* Passed */
 & onField(yQB) & onField(yD)
 & onField(yWR) & onField(yLB)
 & t <= T /* Within 40 second play clock */
 )
```

Proofs and Insights

- Tactic
- Finding the Time

Proof: ✔ All goals closed

Export proof | Browse proof | Redo proof

```
Provable(  ==>  yQB=150&yA=yQB+15&yD=yA+3&yWR=yA&yLB=(yD+300)/2&0 < diffLine&diffLine < 2&0 < diffPass&diffPass < 2&buffer=0&vyQB=4.6
&vyD=23.72&vyWR=26.79-><vyA:=vyD-diffLine;vyLB:=vyWR-diffPass;t:=0;{yQB'=(-1)*vyQB,yA'=1*vyA,yD'=(-1)*vyD,yWR'=1*vyWR,yLB'=(-1)*vyLB,
t'=1&yA<=yD&t<=40}{yQB'=(-1)*vyQB,yD'=(1*vyA+(-1)*vyD)/2,yWR'=1*vyWR,yLB'=(-1)*vyLB,t'=1&yA>=yD&t<=40}>(yQB < yD&yWR+buffer < yLB&(0<
=yQB&yQB<=300)&(0<=yD&yD<=300)&(0<=yWR&yWR<=300)&(0<=yLB&yLB<=300)&t<=40) proved)
```

Tactic to Reproduce the Proof

```
expandAllDefs ; unfold ; assignd(1) ; composed(1) ; solve(1.1) ; solve(1) ; QE
```

$$T_{\text{lc}} = T_{\text{lineman collision}} = \frac{\text{yD - yA}}{\text{dyD * vyD - dyA * vyA}} = \frac{\text{yD - yA}}{\text{2*vyD - diffLine}}$$

$$T_{\text{zone}} = T_{\text{wr open zone}} = \frac{\text{yLB - yWR}}{\text{dyLB * vyLB - dWR * vyWR}} = \frac{\text{yLB - yWR}}{\text{2*vyWR - diffPass}}$$

$$T_{\text{man}} = T_{\text{wr open man}} = \frac{\text{yWR - yLB}}{\text{dWR * vyWR - dyLB * vyLB}} = \frac{\text{yWR - yLB}}{\text{2*vyWR - diffPass}}$$

Questions?