Len Huang lghuang
Megha Jain meghaj2
15-424: LFCPS F20

Modeling American Football with CPS

American football lends itself to modelling in many ways and has clearly defined rules for what we will refer to as “safety” and “efficiency”. Much work has been done with making models for fútbol / soccer, especially with events like the RoboCup. But we seek to lay the groundwork for how we might approach similar problems of modelling, with American football (to which we refer to as “football”).

At its core, football has 11 players on either team and an ulterior motive of moving a ball forward via passing / running. This can be interpreted as a multiple-agent system of angels and demons with probabilistic choices and all sorts of varying complexity. For the sake of our project, we choose to reduce it into a “Follow The Leader” -esque problem. Some terms we’ll be using moving forward:

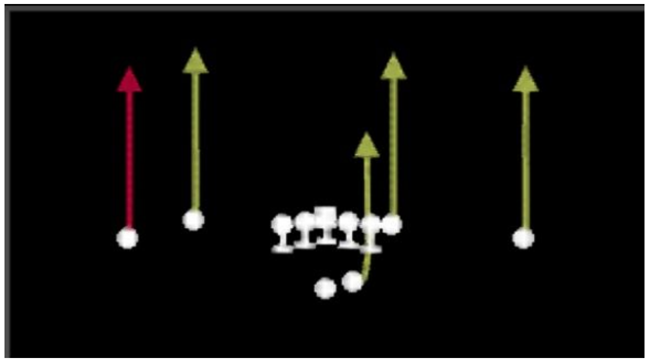
Offensive Players:
- Quarterback (QB): Responsible for passing the ball within 40 seconds
- Offensive Linemen (OL): Responsible for guarding the QB from defenders
- Wide Receiver (WR): Responsible for running forward and catching QB passes

Defensive Players:
- Defensive Linemen (DL): Responsible for attacking the QB
- Linebacker (LB): Prevent WRs from catching *(simplified for model).*

Other Terminologies:
- Passing Play: WRs run and QB tries to pass to them before getting tackled by DL
- Blocked Pass: LB successfully intercepts a pass / interferes with it
- Sack: DL breaks pasts OL and tackles the QB

Note to the football fans: We will use the term “safety” to reference CPS safety, not the position.

Instead of modelling an entire football game, we will model a particular “play” with reduced players. Our offense will consist of 1 QB, 2 WRs, and 2 OLs. The defense will consist of 3 DLs and 2 LBs. Our offense will execute the “Hail Mary” play, where the 2 WRs run in a straight line while the 2 OLs protect our QB. We will let our players be represented by arbitrary points, similar to cars in our labs. Furthermore, we will let the QB be our controller, and our goal is to find a way to keep him safe and efficient. Safety would mean that the QB doesn’t get tackled before he passes / before the 40 second play clock. Efficiency would mean that the QB is able to successfully pass the ball. We will also model our field as a cartesian coordinate plane of size [0, 160] x [0, 360]. We will also model the ball as an arbitrary point, similar to players.



A depiction of a [Hail Mary Pass] in video game NFL Mobile.

Let us move forward with the assumption that players follow their plays exactly. This allows us to assign defined / hard-coded ODEs to each player. We will consider our QB tackled when an enemy player's position equals our QB's. OL/DL usually struggle with one another before breaking past. To model this, we can assign certain thresholds to our linemen. If they are accelerating at each other, and either surpasses a certain velocity threshold, we will consider it "broken through". These are just some preliminary ideas as to how we might model a system of multiple players all with their own unique physics.

Our model involves defensive and offensive players, so eventually we want to define and prove a "winning" strategy, where winning is getting the ball from the QB to the WR. We will mainly draw ideas from papers on modeling multi-agents, allocentric representation, and spacio-temporal analysis. A few papers we are looking at include a case study on aircraft collision maneuvering [4], qualitative motion description regarding RoboSoccer [3] and Intelligent Vehicles [2], as well as Plan Recognition and retrieval for multi-agent systems [1], with the latter being the most relevant for our earlier modeling.

With all these ideas in mind, this is how we might approach the iterative design of our model:

1. **Individual:** 1 OL, 1 DL, 1 QB, Safety of QB

This is a simple scenario of one agent trying to attack us, and one agent trying to protect us.

2. **Doubles:** 2 OL, 2 DL, 1 QB, Safety of QB

Can we extend the above scenario to 2 attackers / defenders on either side?

3. **Triples:** 2 OL, 3 DL, 1 QB, Safety of QB

Let's add an additional attacker to the previous scenario and see if we can still stay safe.

4. **Blitz:** 2 OL, 5 DL, 1 QB, Safety of QB

Let's add two more attackers to represent the full defense. In a blitz, everyone attacks the QB.

5. **Hail Mary Blitz One:** 2 OL, 5 DL, 1 QB, 1 WR, Safety of QB, 1 ball

Now that we're safe even in a blitz, let's become more efficient by adding a wide receiver

6. **Hail Mary Blitz Two:** 2 OL, 5 DL, 1 QB, 2 WR, Safety of QB, 1 ball

We can become even more efficient by adding another wide receiver.

7. **Hail Mary:** 2 OL, 1 QB, 2 WR, Safety of QB, 1 ball

Can we adjust it now so that our QB is safe for some arbitrary defense?

Slowly and slowly, we can build up to our simplified 5v5 model (**Hail Mary)** in this manner, considering new challenges each and every time.

The core correctness of our model lies in these two aspects: the QB remains untouched by the defensive line and the ball reaches a WR. Once the ball reaches the wide receiver or the 40 seconds run out, the play is over and we do not need to worry about the QB remaining untouched because it is expected that movement stops. We intend to prove these properties with a post-condition that states that the position of the ball coincides with the position of one of the WRs. The other necessary logical formula we need is that the positions of any of the defensive linemen should not coincide with the position of the QB.

In order to prove this conjunction, we can utilize differential invariants, but since this is a more complicated system, we will also have to introduce differential cuts and ghosts. The Keymaera proof will follow from this handwritten proof. Our strategy will be to unfold as much as possible and analyze the individual branches for safety and efficiency. Then, we will write in the cuts, loop invariants, and ghosts as necessary.

We expect to face many challenges as multi-agent modeling will become very complicated, and we will need to look at the entire field and set of players as one system. Therefore, we will need to have each controlled player understand what all the other players and the ball are doing around it. Also, if we want to add predictive behavior, that will be a huge challenge which we will only take upon if we successfully complete the first checkpoints. Finally, the proof will be quite challenging due to having multiple agents.

Throughout our project, our deliverables will be updated Keymaera models and proofs for each phase.

References

[1] Huang, Zhanxiang & Yang, Yang & Chen, Xiaoping. (2003). *An approach to plan recognition and retrieval for multi-agent systems.*

[2] Miene, A., Lattner, A. D., Visser, U., & Herzog, O. (n.d.). *Dynamic-preserving qualitative motion description for intelligent vehicles. IEEE Intelligent Vehicles Symposium*, 2004. doi:10.1109/ivs.2004.1336459

[3] Miene, A., Visser, U., & Herzog, O. (2004). *Recognition and Prediction of Motion Situations Based on a Qualitative Motion Description. Lecture Notes in Computer Science, 77–88.* doi:10.1007/978-3-540-25940-4_7

[4] Platzer A., Clarke E.M. (2009) *Formal Verification of Curved Flight Collision Avoidance Maneuvers: A Case Study. In: Cavalcanti A., Dams D.R. (eds) FM 2009: Formal Methods.* FM 2009. Lecture Notes in Computer Science, vol 5850. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-05089-3_35