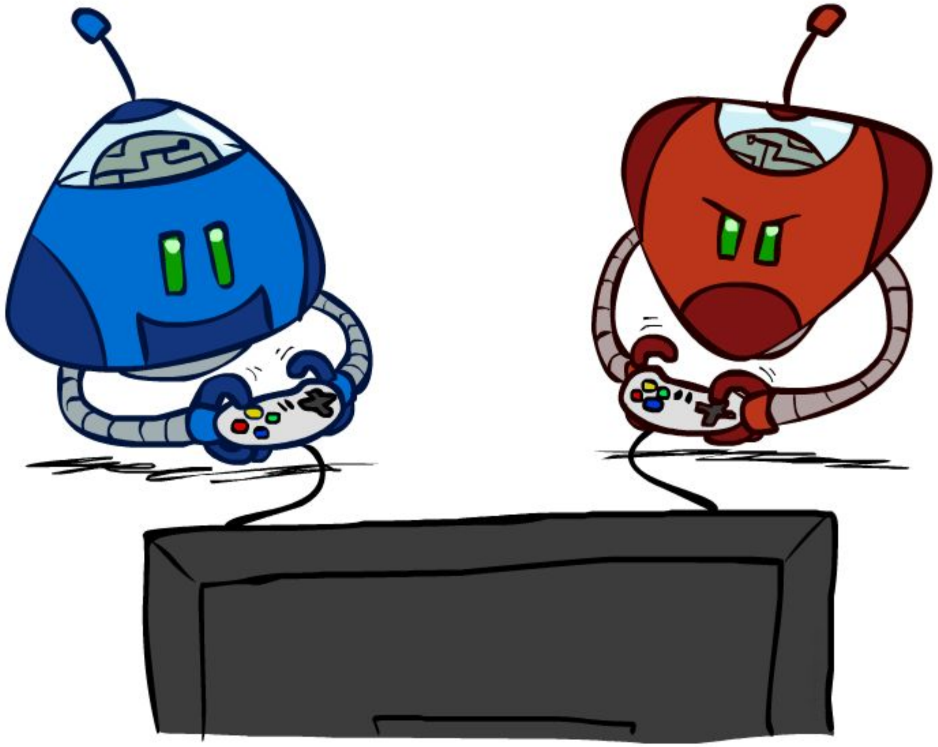
CS 106: Artificial Intelligence

Adversarial Search



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[These slides were adapted from the slides created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. http://ai.berkeley.edu.]

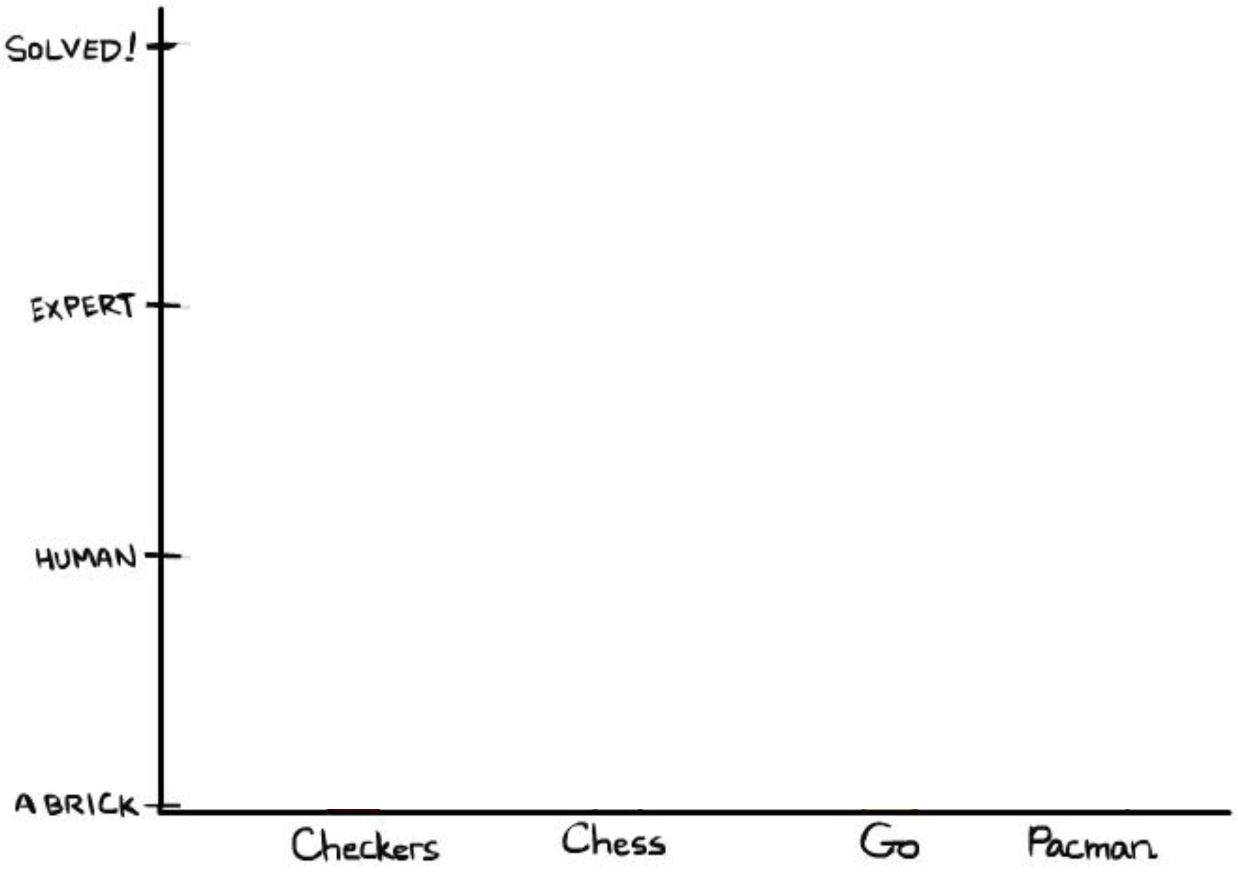
Game Playing State-of-the-Art

- **Checkers:** 1950: First computer player. 1994: First computer champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame. 2007: Checkers solved!

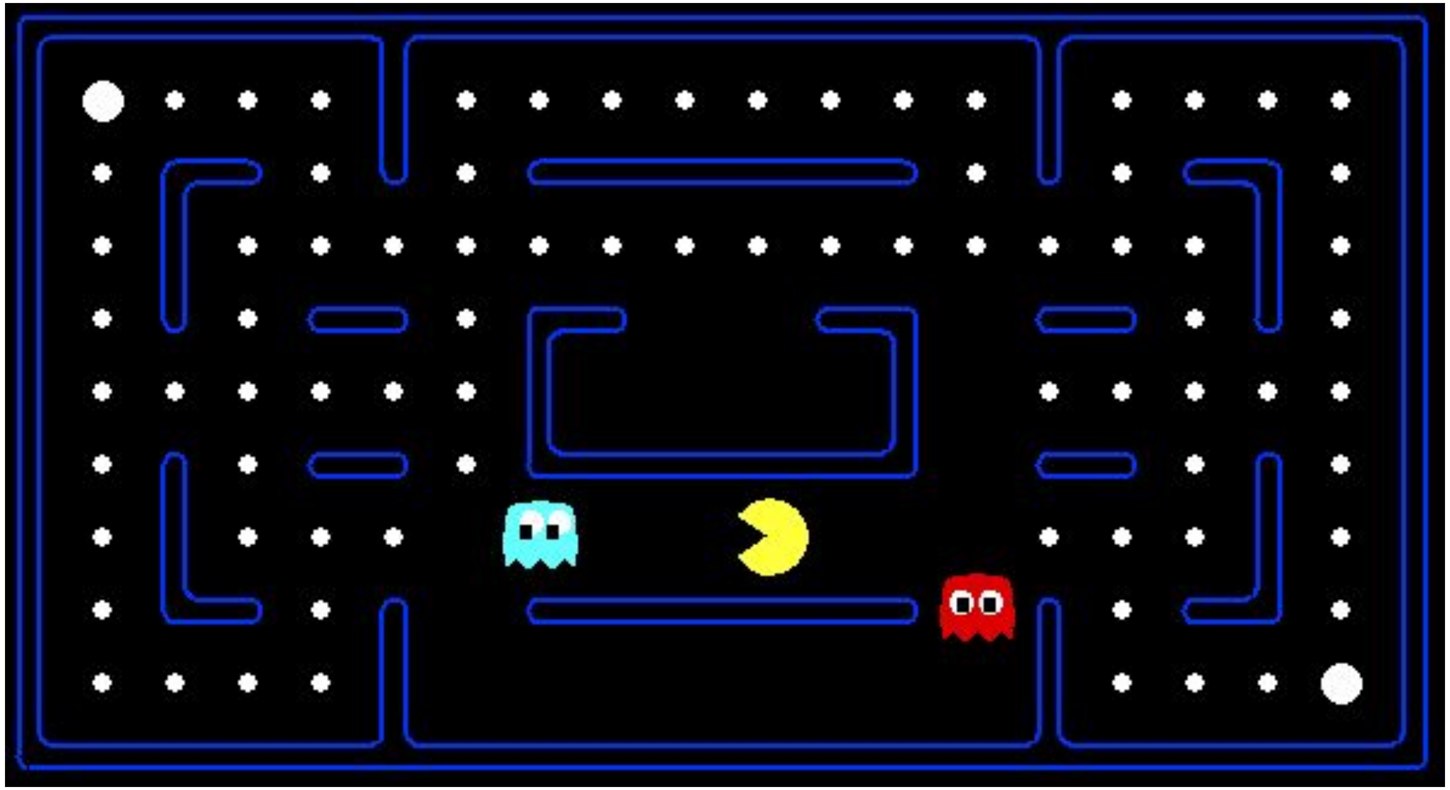
- **Chess:** 1997: Deep Blue defeats human champion Gary Kasparov in a six-game match. Deep Blue examined 200M positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 ply. Current programs are even better, if less historic.

- **Go:** Human champions are now starting to be challenged by machines, though the best humans still beat the best machines. In go, b > 300! Classic programs use pattern knowledge bases, but big recent advances use Monte Carlo (randomized) expansion methods.
- **Go: 2016: Alpha GO defeats human champion.**

- **Pacman**



Behavior from Computation



[Demo: mystery pacman (L6D1)]

Video of Demo Mystery Pacman

Adversarial Games

A simple game

- You choose one of the three bins.
- I choose a number from that bin.
- Your goal is to maximize the chosen number.

A

-$50 $50

B

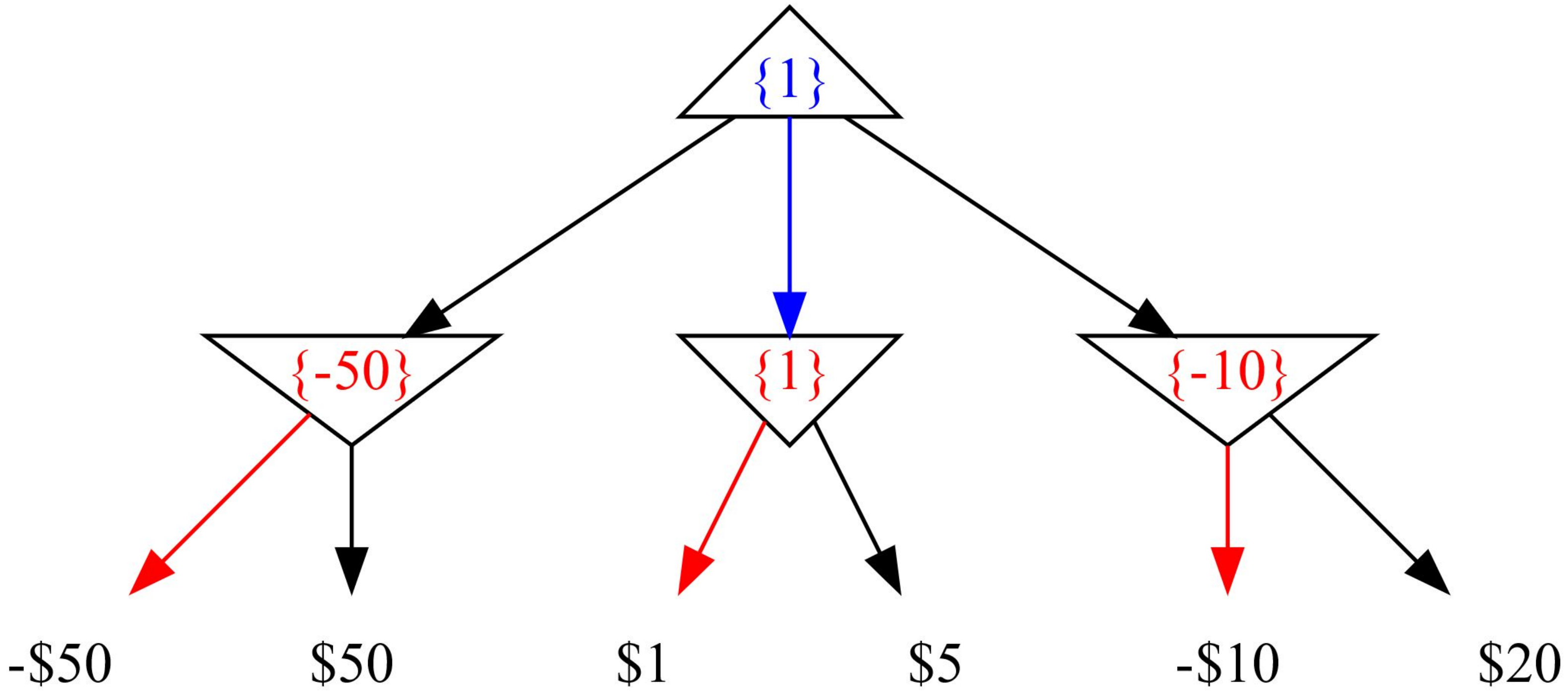
$1 $5

C

-$10 $20

Game tree

- Each node is a decision point for a player.
- Each root-to-leaf path is a possible outcome of the game.
- Your goal is to maximize the chosen number.



Types of Games

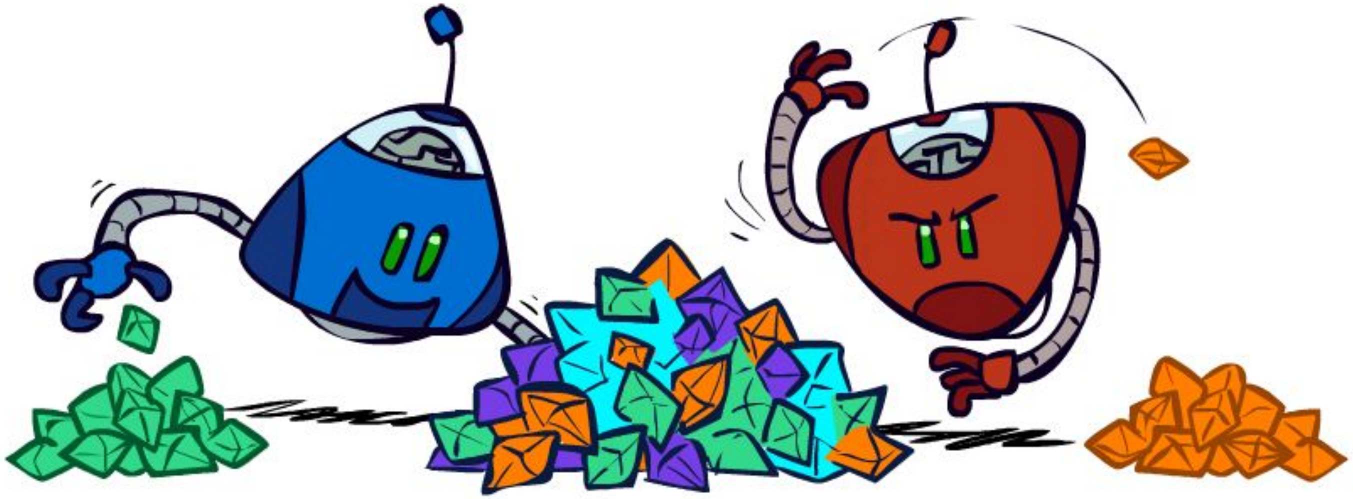
- Many different kinds of games!
- Axes:
  - Deterministic or stochastic?
  - One, two, or more players?
  - Zero sum?
  - Perfect information (can you see the state)?

- Want algorithms for calculating a strategy (policy) which recommends a move from each state

Deterministic Games

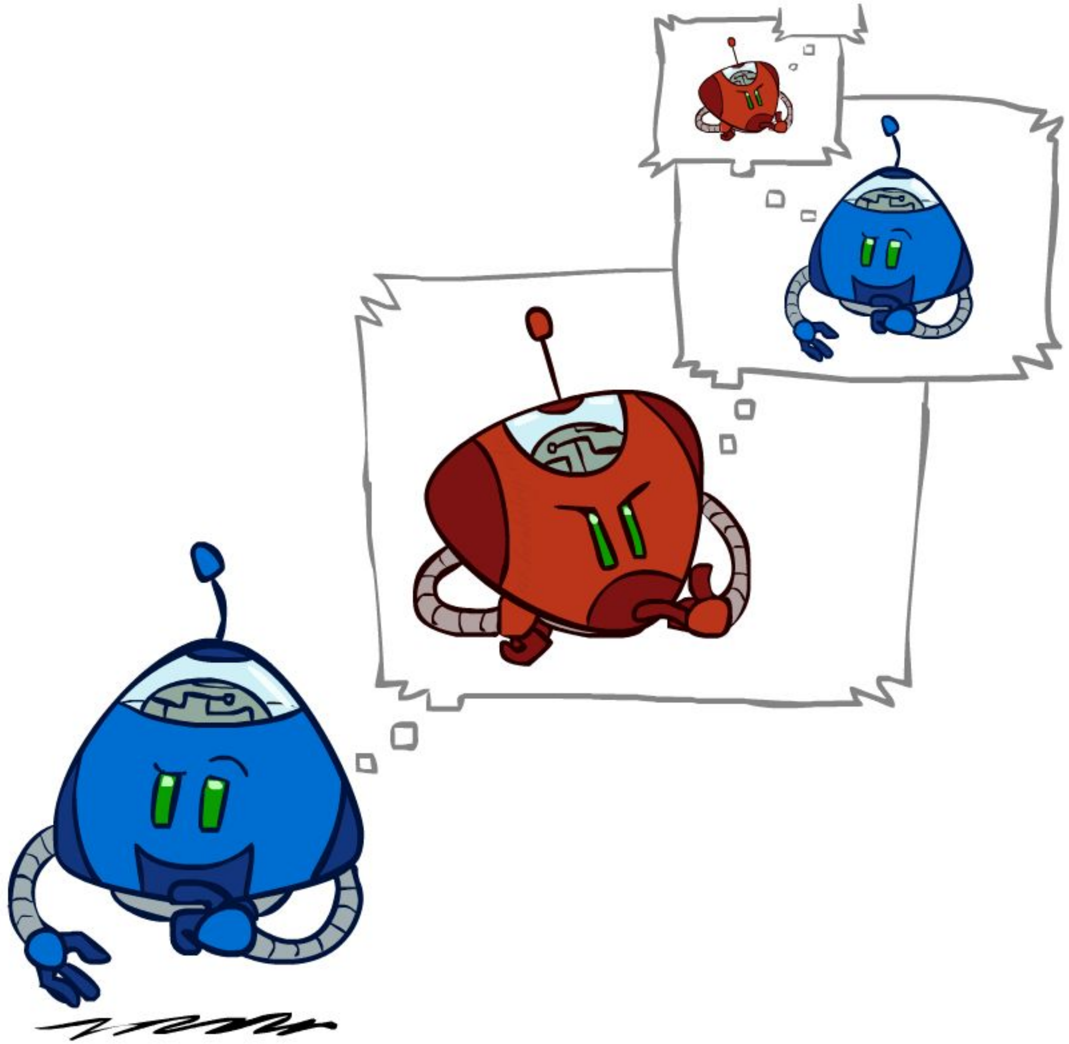
- Many possible formalizations, one is:
  - States S (start at s_0 or s_{start})
  - $\text{Players}(s) \in \text{Players } P = \{1, \dots, N\}$: player who control state s
  - $\text{Actions}(s)$: possible actions from state s (depend on the player)
  - $\text{Succ}(s, a)$: resulting state if choose action a in state s
    Transition Function: $S \times A \rightarrow S$
  - $\text{IsEnd}(s)$: whether s is and end state (game over)
    Terminal Test: $S \rightarrow \{\text{true}, \text{false}\}$
  - $\text{Utility}(s)$: utility for end state s
    Terminal Utilities: $S \times P \rightarrow R$
- Solution for a player is a policy: $S \rightarrow A$

Zero-Sum Games

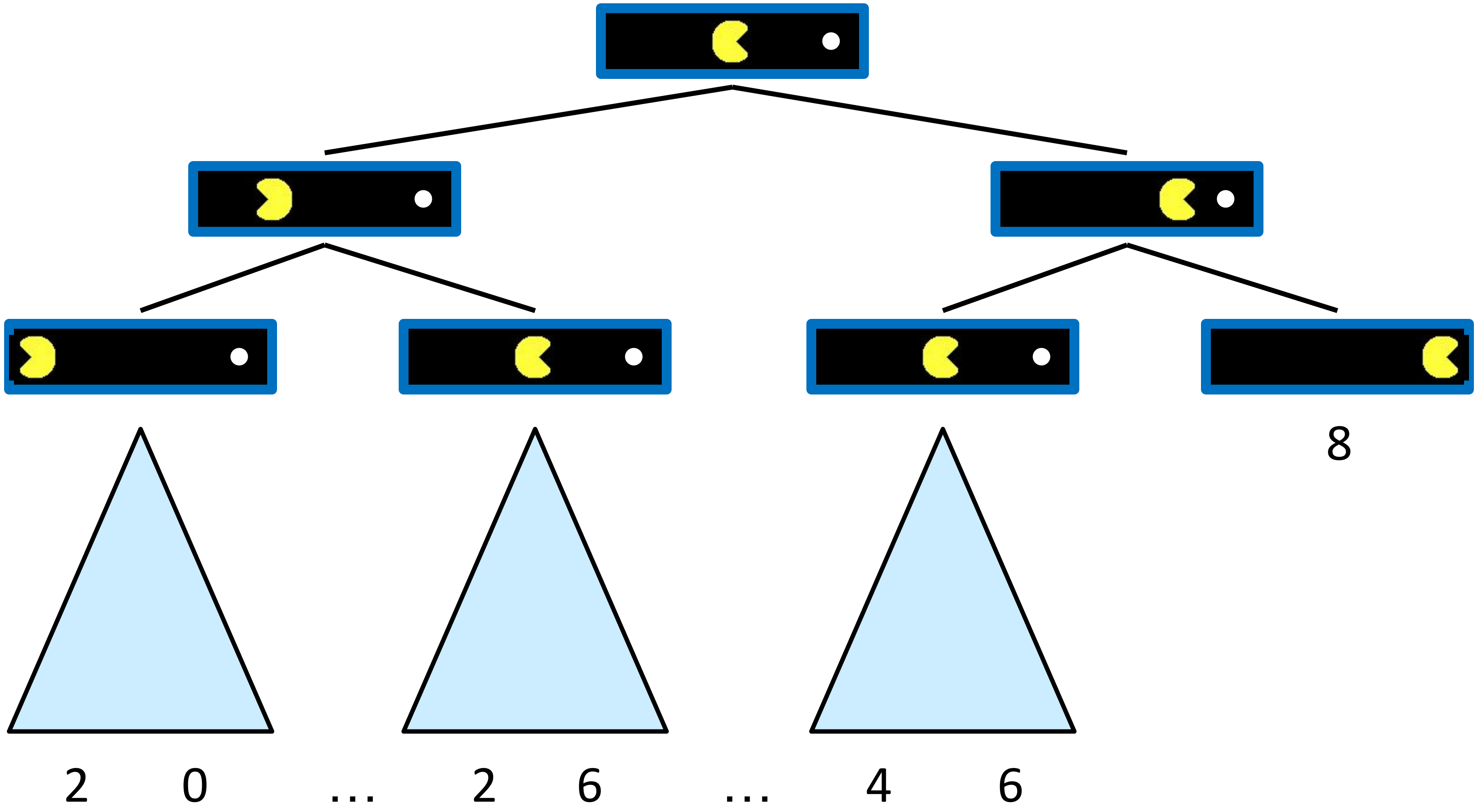


- Zero-Sum Games
  - Agents have opposite utilities (values on outcomes)
  - Lets us think of a single value that one maximizes and the other minimizes
  - Adversarial, pure competition
- General Games
  - Agents have independent utilities (values on outcomes)
  - Cooperation, indifference, competition, and more are all possible

Adversarial Search



Single-Agent Trees

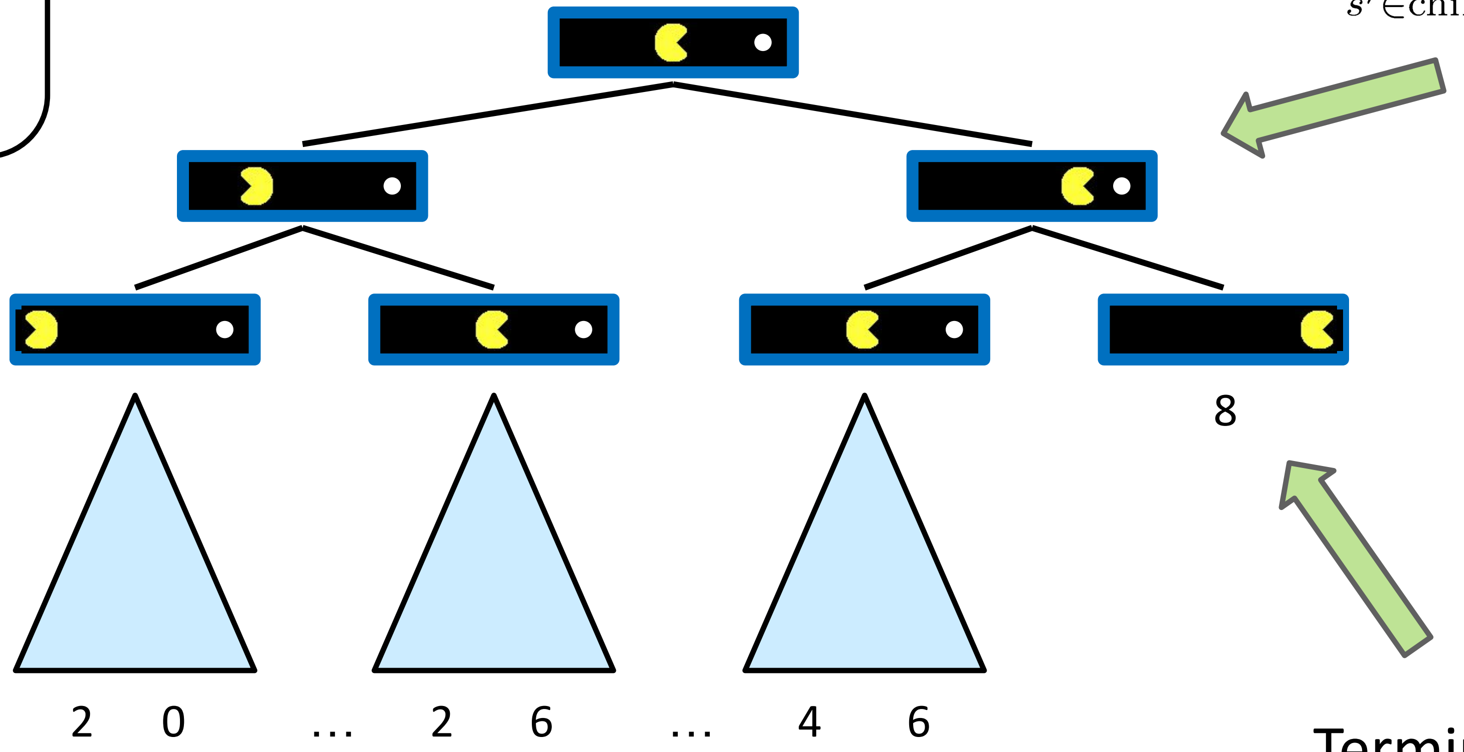


Value of a State

Value of a state:
The best achievable outcome (utility) from that state

Non-Terminal States:

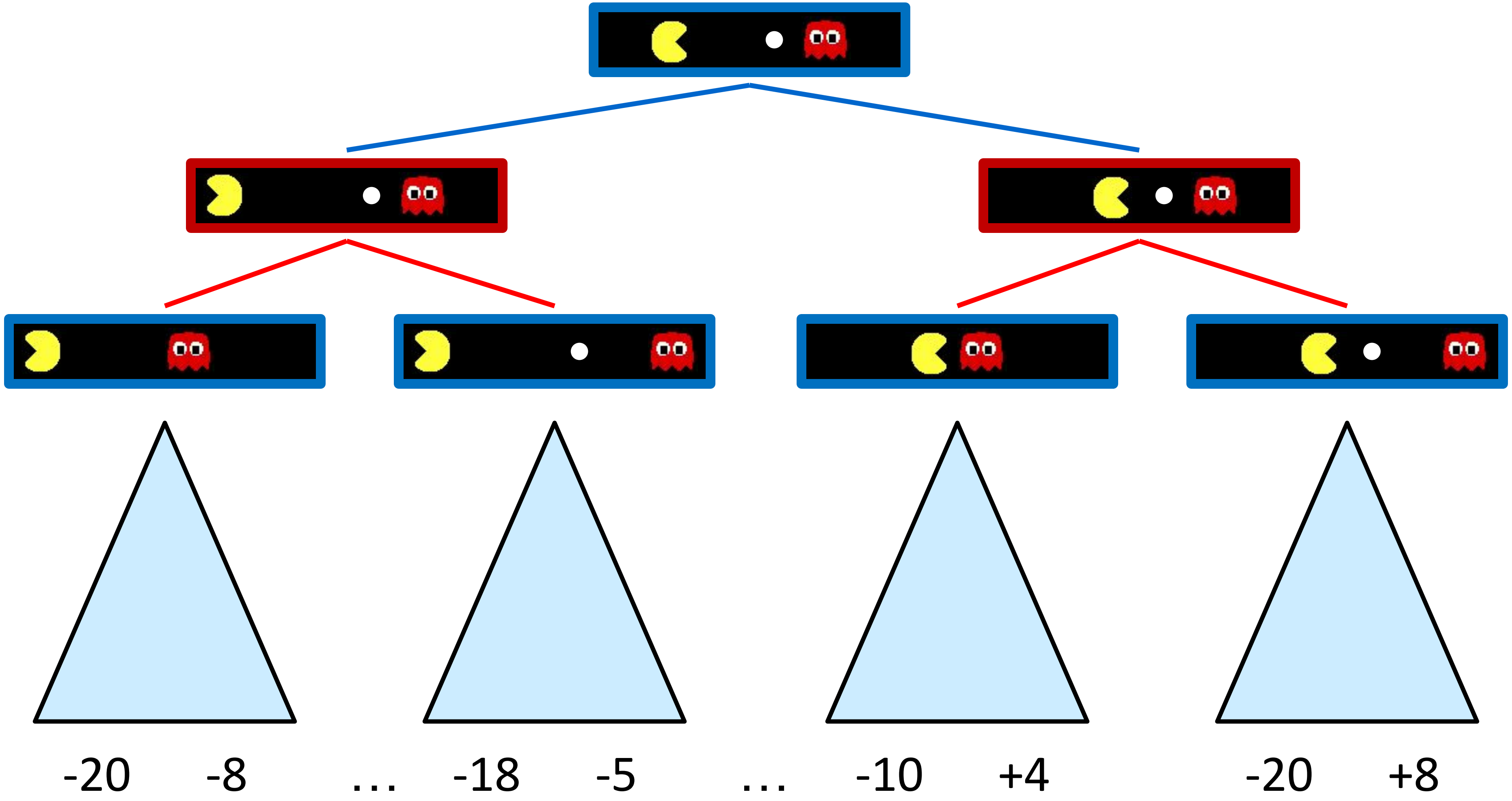
$$V(s) = \max_{s' \in \text{children}(s)} V(s')$$



Terminal States:

$$V(s) = \text{known}$$

Adversarial Game Trees



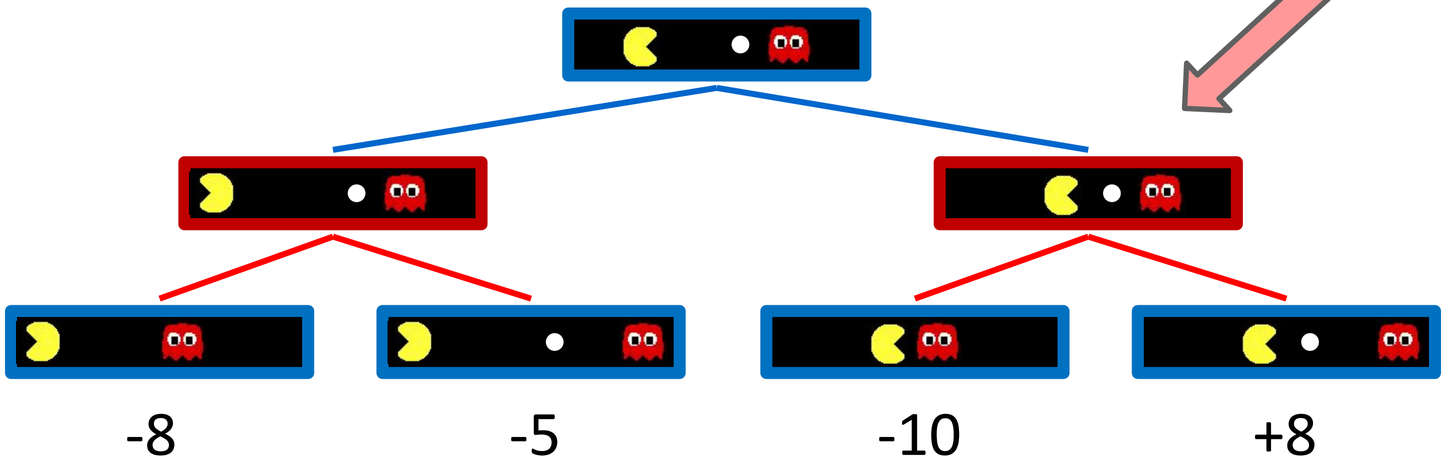
Minimax Values

States Under Agent's Control:

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

States Under Opponent's Control:

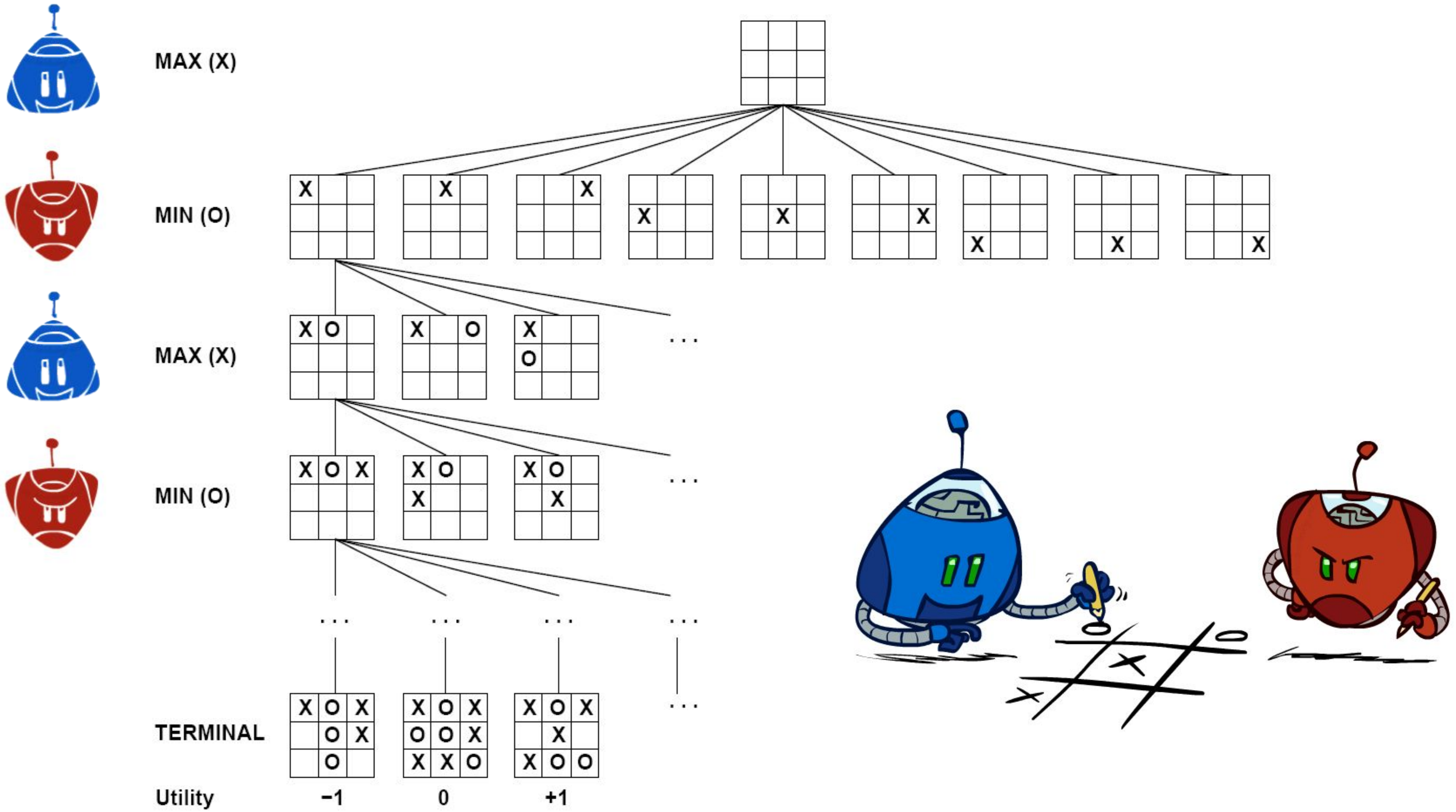
$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

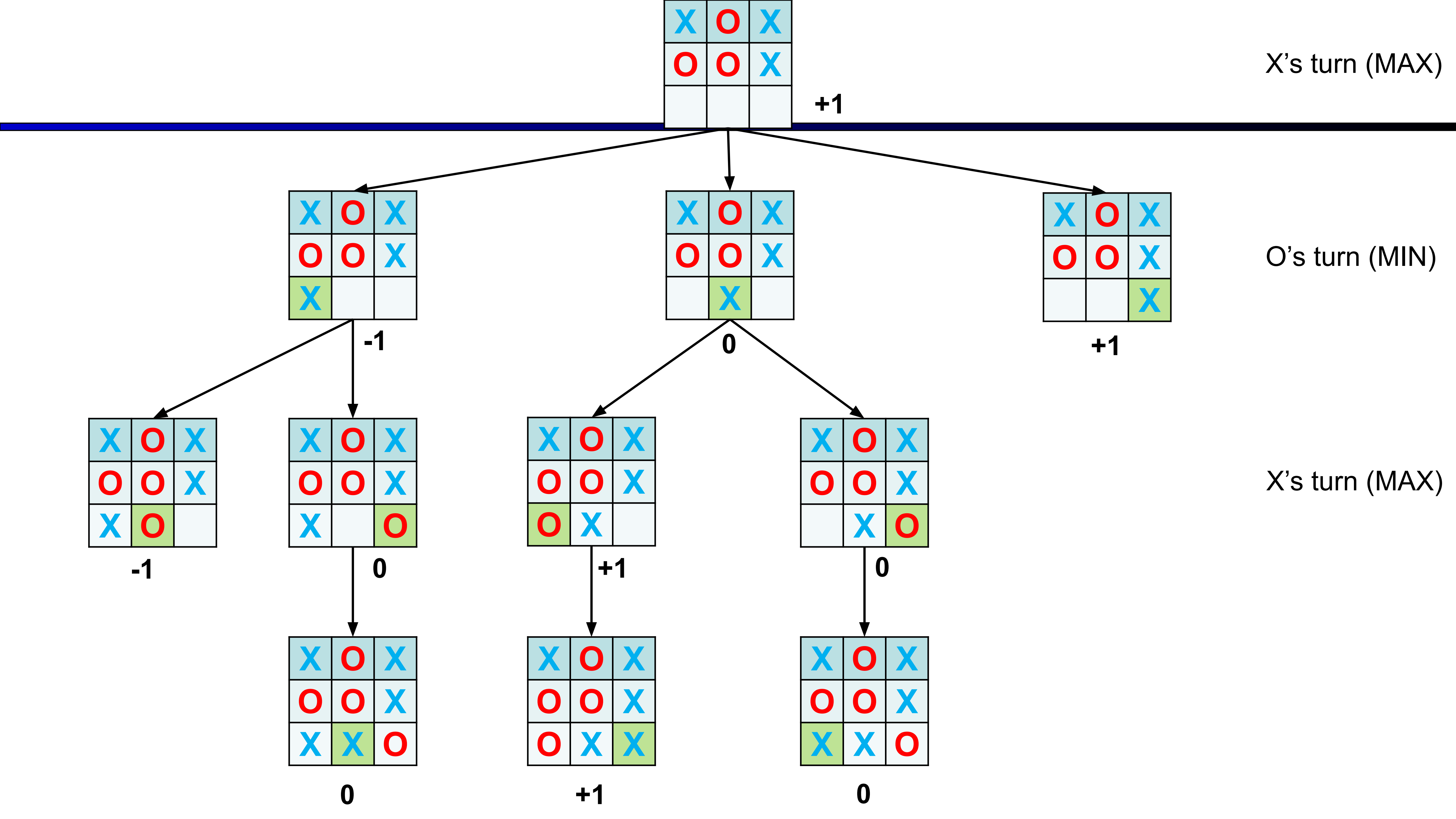


Terminal States:

$$V(s) = \text{known}$$

Tic-Tac-Toe Game Tree



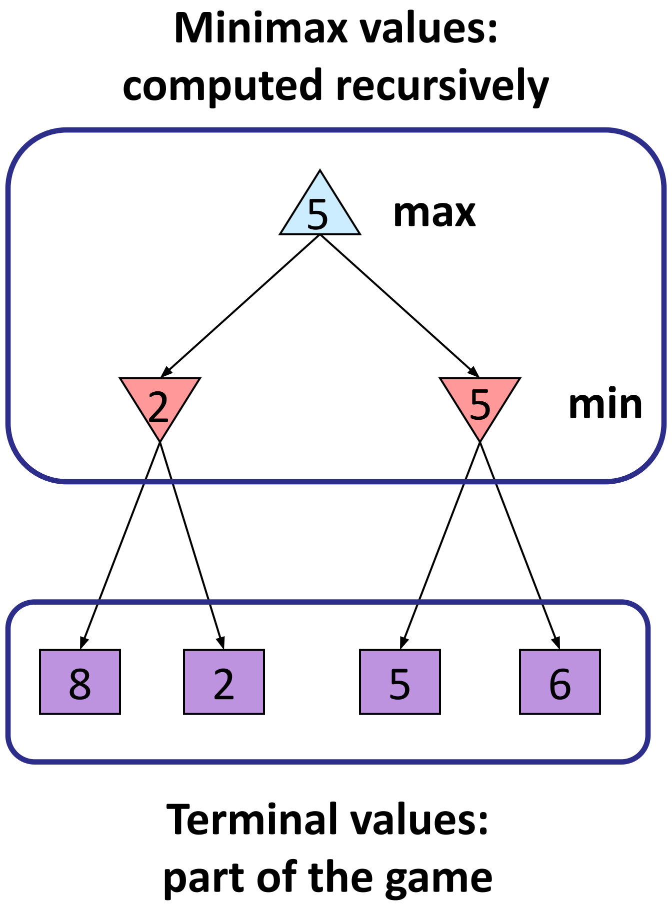


X's turn (MAX)
O's turn (MIN)
X's turn (MAX)
+1
-1
0
+1
-1
0
+1
0
0
+1
0

Adversarial Search (Minimax)

- Deterministic, zero-sum games:
  - Tic-tac-toe, chess, checkers
  - One player maximizes result
  - The other minimizes result

- Minimax search:
  - A state-space search tree
  - Players alternate turns
  - Compute each node's minimax value: the best achievable utility against a rational (optimal) adversary



Minimax Implementation

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, min-value(successor))
    return v
```

```
def min-value(state):
    initialize v = +∞
    for each successor of state:
        v = min(v, max-value(successor))
    return v
```

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

Minimax Implementation (Dispatch)

```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is MIN: return min-value(state)
```

```
def max-value(state):
    initialize v = -∞
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        v = max(v, value(successor))
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```

```
def min-value(state):
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```
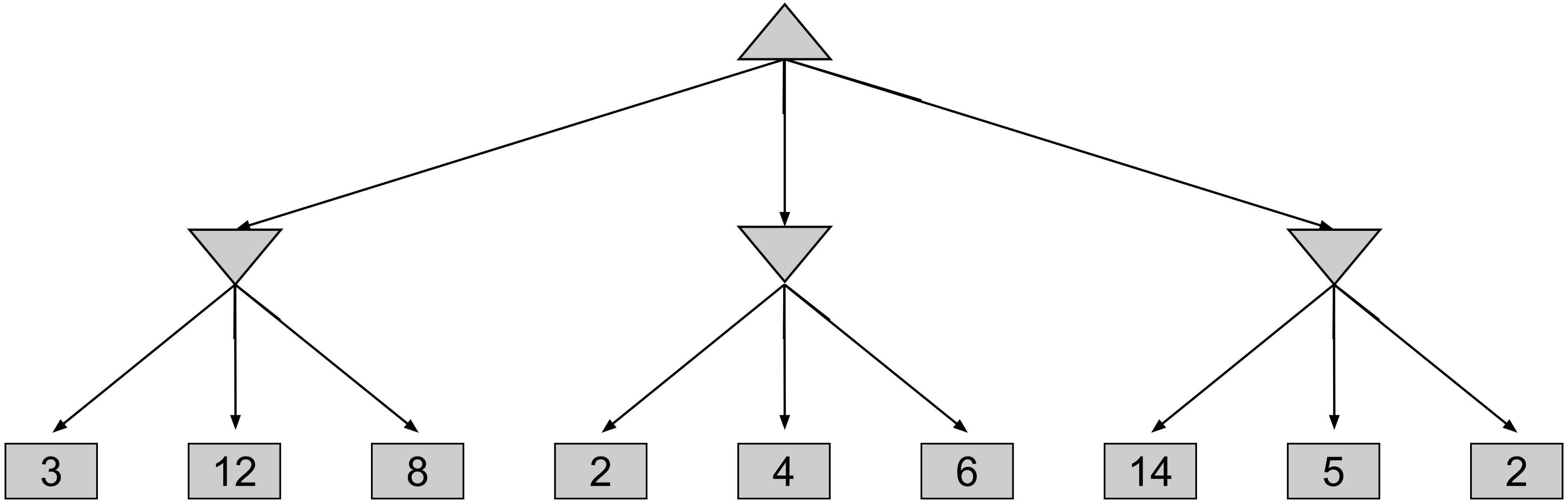
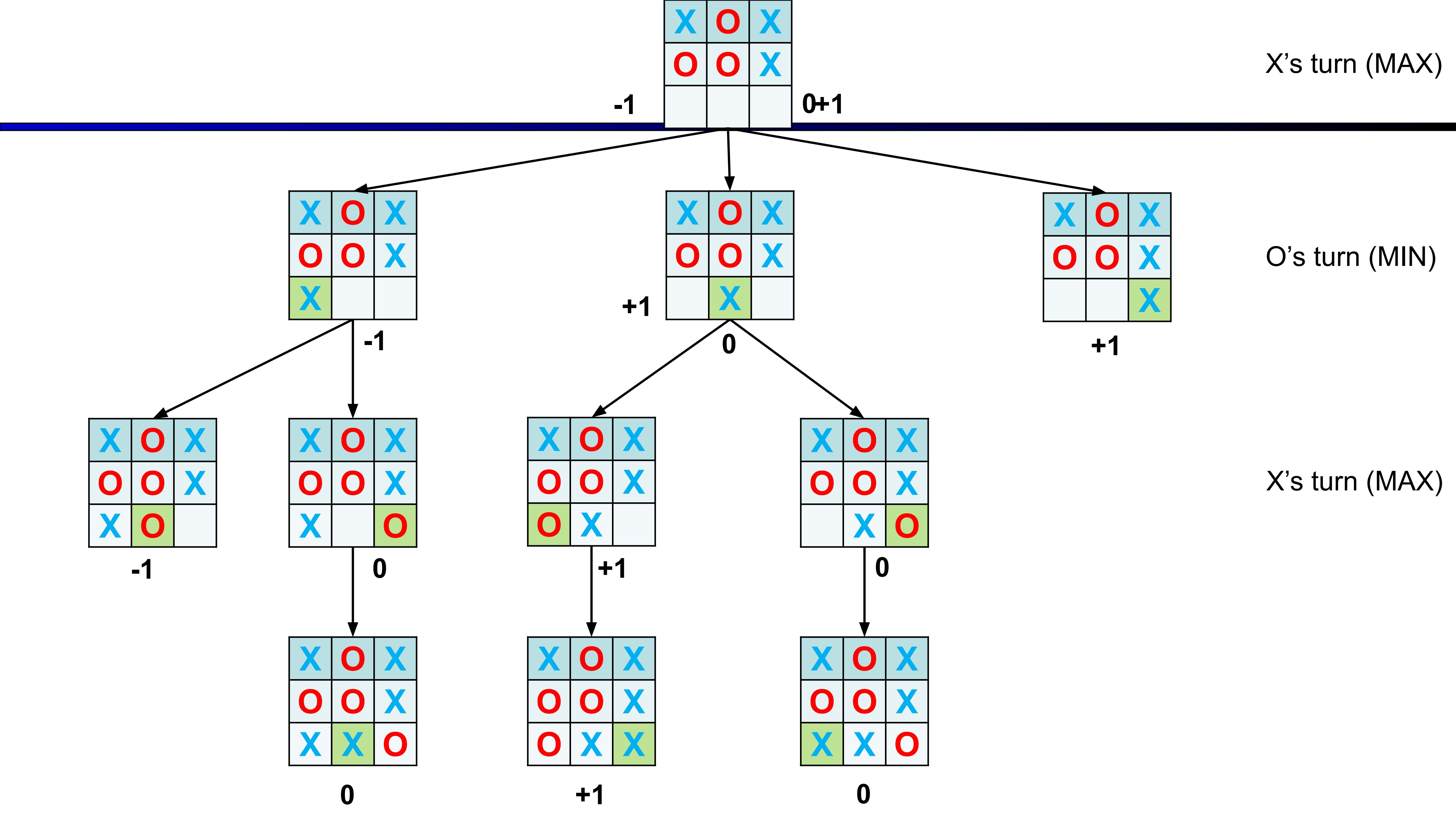

Minimax Example

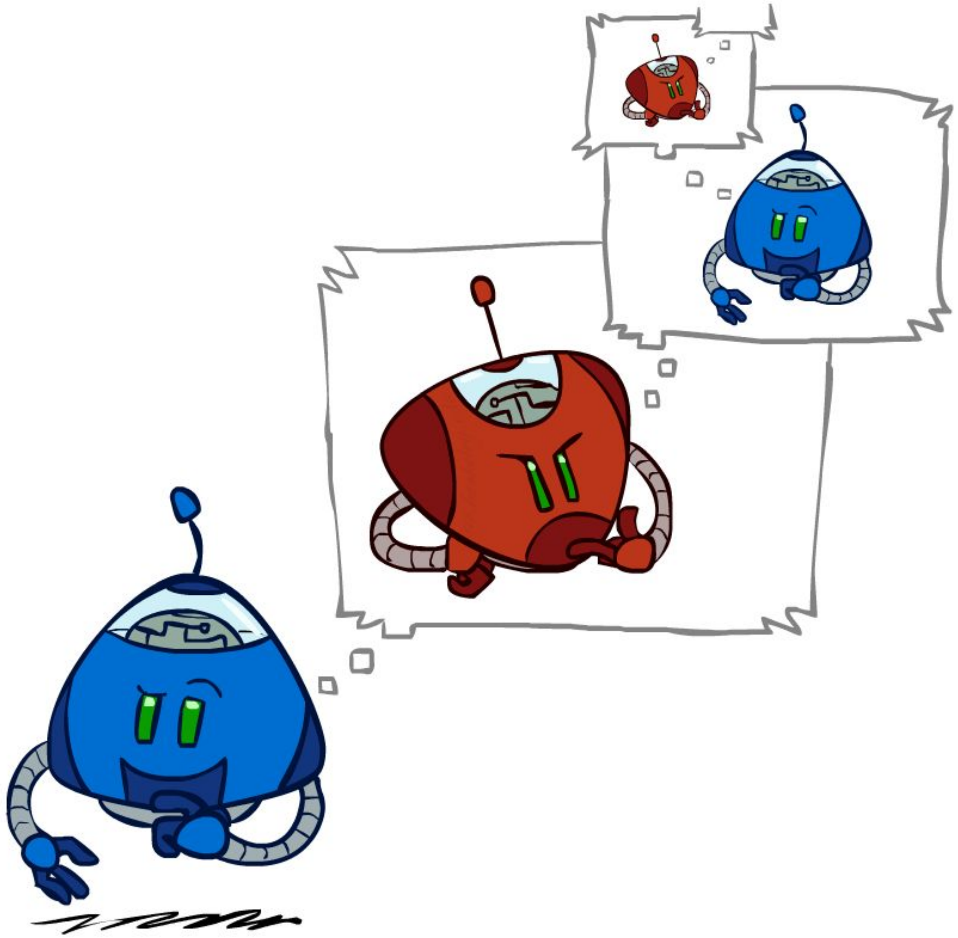



X's turn (MAX)
-1
0+1
O's turn (MIN)
-1
+1
0
+1
X's turn (MAX)
-1
0
+1
0
0
+1
0

Minimax Efficiency

- How efficient is minimax?
  - Just like (exhaustive) DFS
  - Time: $O(b^m)$
  - Space: $O(bm)$

- Example: For chess, $b \approx 35$, $m \approx 100$
  - Exact solution is completely infeasible
  - But, do we need to explore the whole tree?



Minimax Properties

- You choose one of the three bins.
- I toss a fair coin. If head, the bigger number is returned. If tail, the smaller number is returned.
- Your goal is to maximize the chosen number.

A	B	C
-\$50 \$50	\$1 \$5	-\$10 \$20

Minimax Properties

- You choose one of the three bins.
- I toss a fair coin. If head, the bigger number is returned. If tail, the smaller number is returned.
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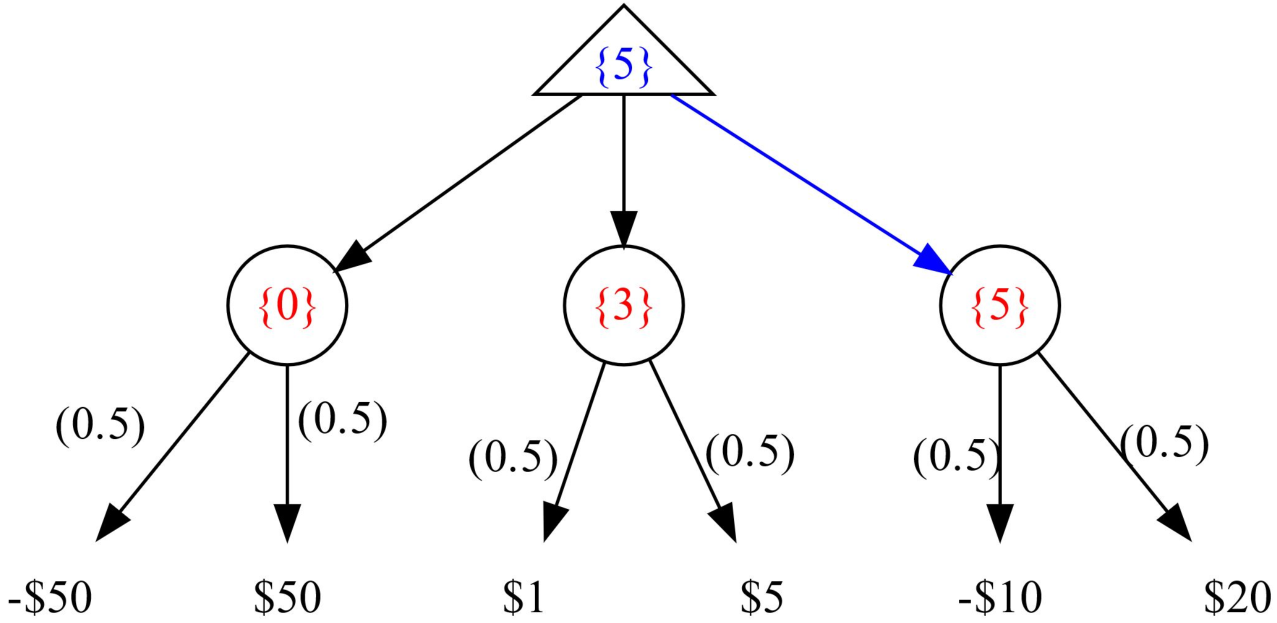
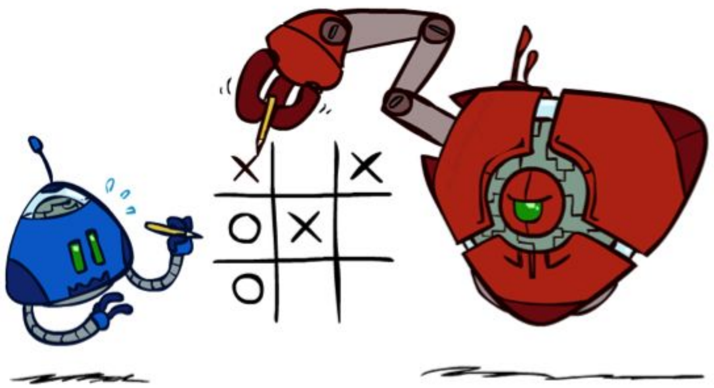
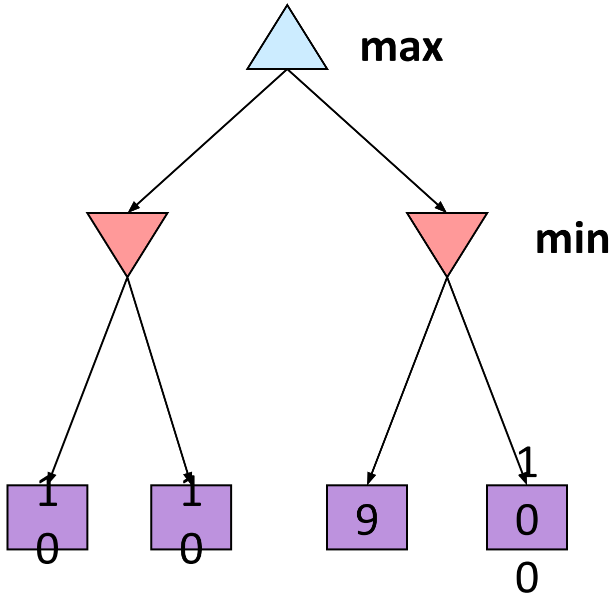
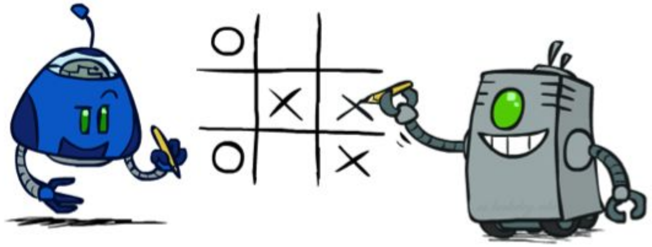


Minimax Properties







Optimal against a perfect player. Otherwise?

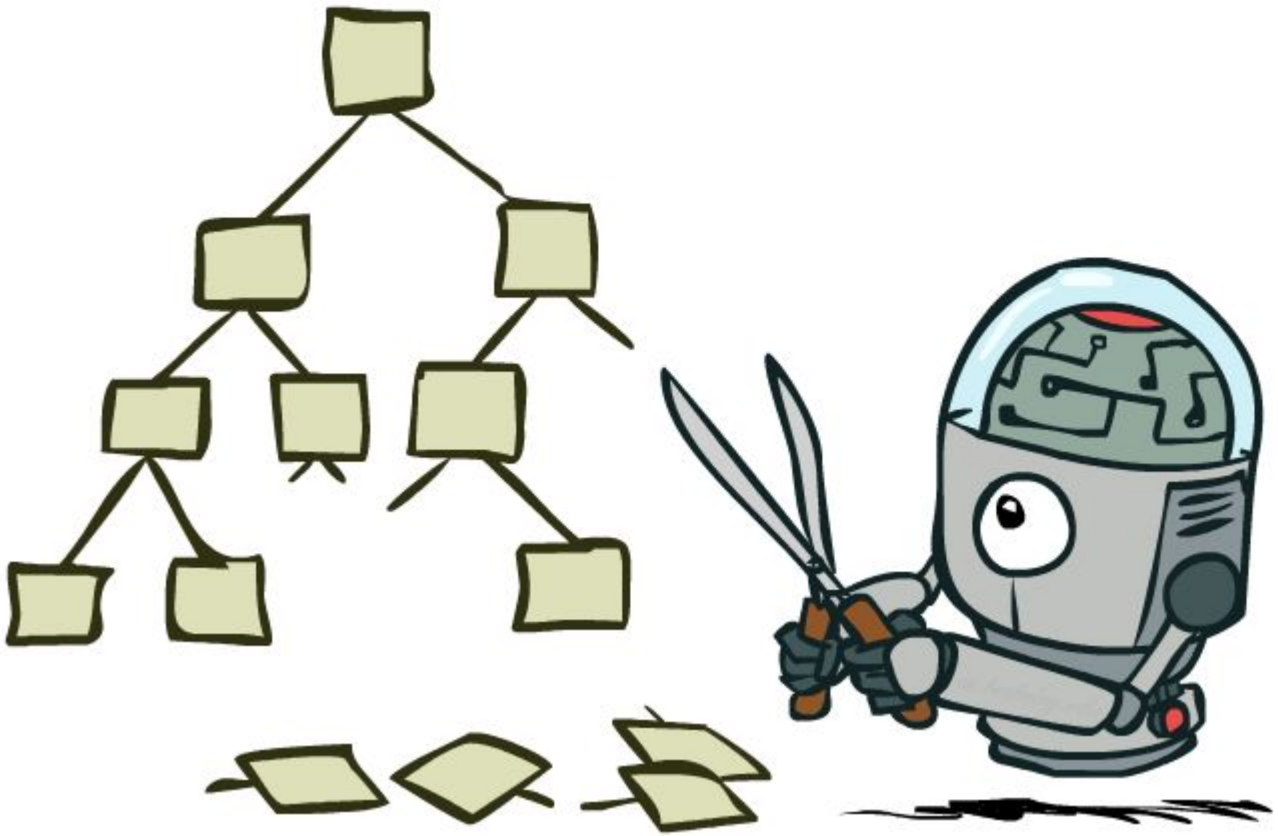
[Demo: min vs exp (L6D2, L6D3)]

Video of Demo Min vs. Exp (Min)

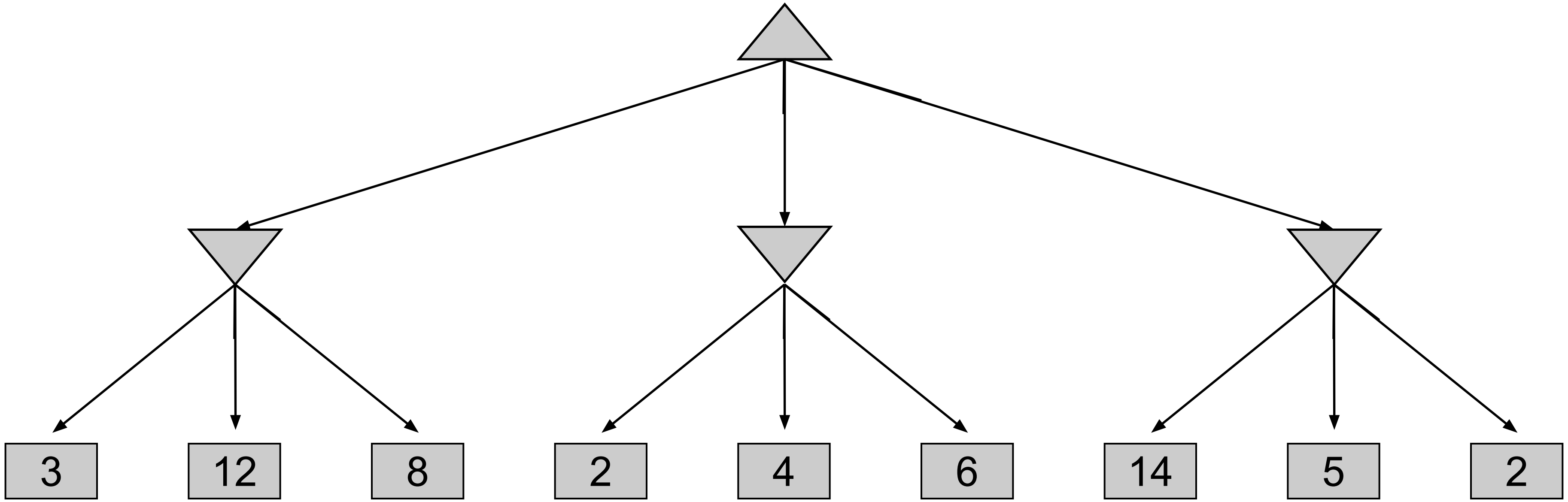
Video of Demo Min vs. Exp (Exp)

Resource Limits

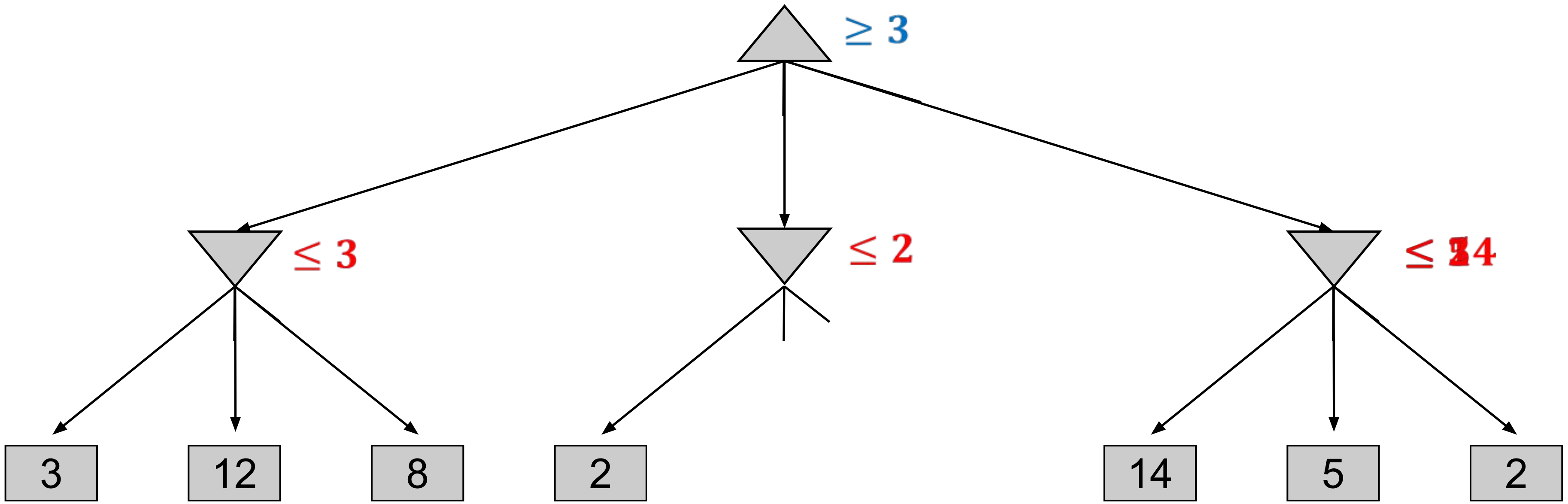
Game Tree Pruning



Minimax Example



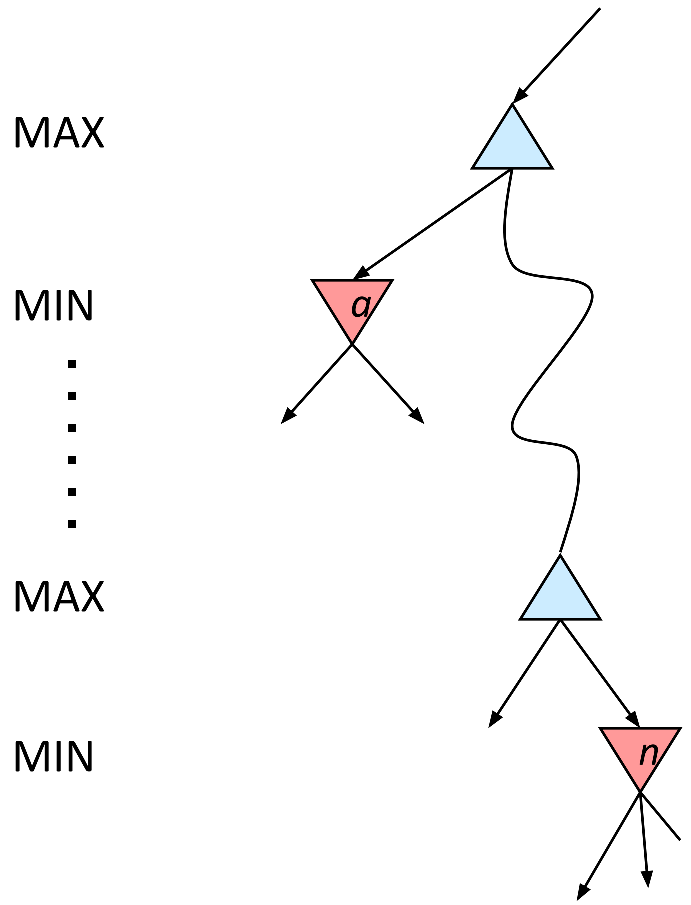
Minimax Pruning



Alpha-Beta Pruning

- General configuration (MIN version)
  - We're computing the MIN-VALUE at some node *n*
  - We're looping over *n*'s children
  - *n*'s estimate of the childrens' min is dropping
  - Who cares about *n*'s value? MAX
  - Let *a* be the best value that MAX can get at any choice point along the current path from the root
  - If *n* becomes worse than *a*, MAX will avoid it, so we can stop considering *n*'s other children (it's already bad enough that it won't be played)

- MAX version is symmetric



Alpha-Beta Implementation

α: MAX's best option on path to root
β: MIN's best option on path to root

```
def max-value(state, α, β):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor, α, β))
        if v ≥ β return v
        α = max(α, v)
    return v
```

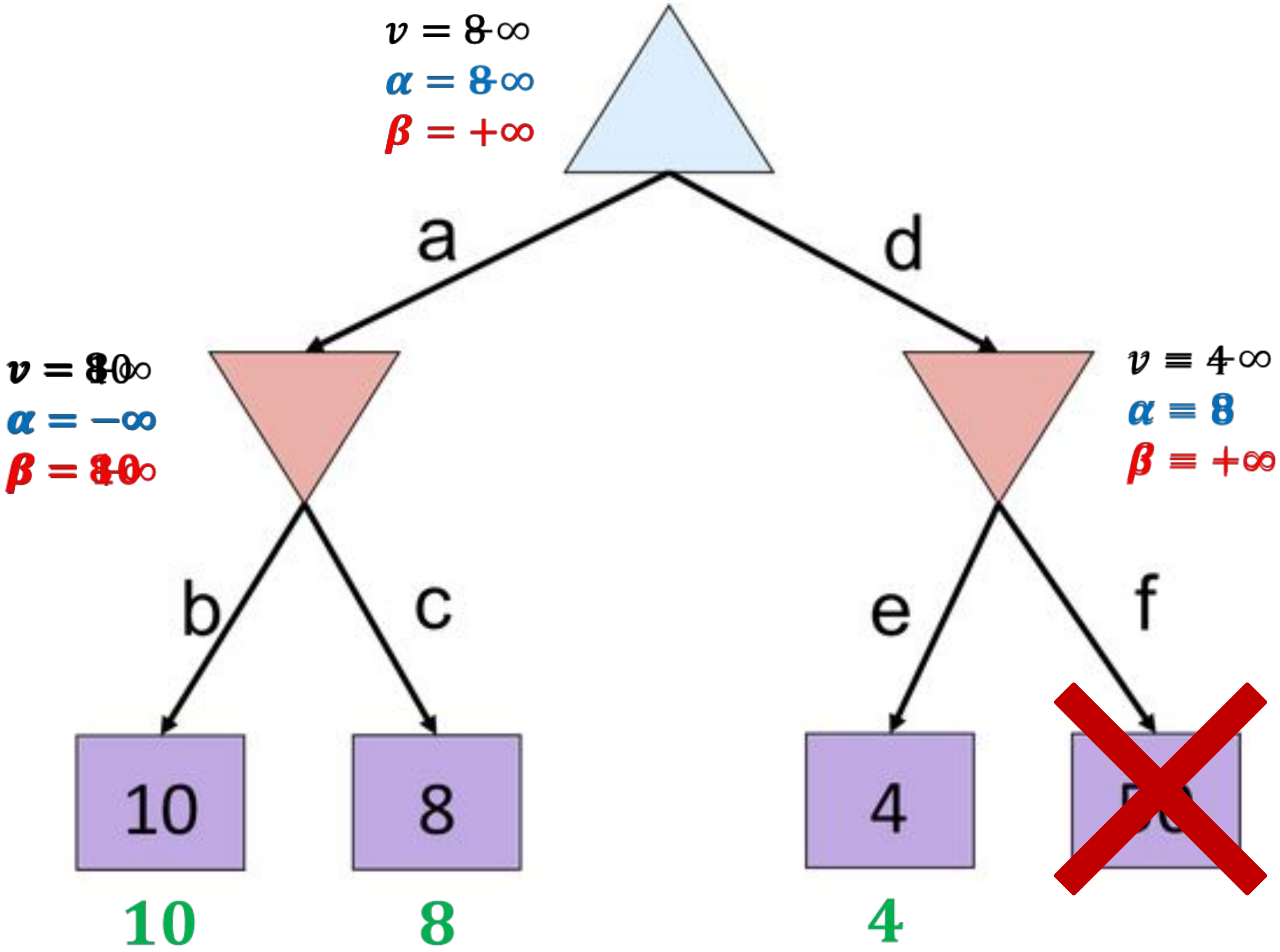
```
def min-value(state , α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α return v
        β = min(β, v)
    return v
```

Alpha-Beta Pruning Example

```
def value(state, α, β):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state, α, β)
    if the next agent is MIN: return min-value(state, α, β)
```

```
def max-value(state, α, β):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor, α, β))
        if v ≥ β return v
        α = max(α, v)
    return v
```

```
def min-value(state , α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α return v
        β = min(β, v)
    return v
```



Alpha-Beta Pruning Example

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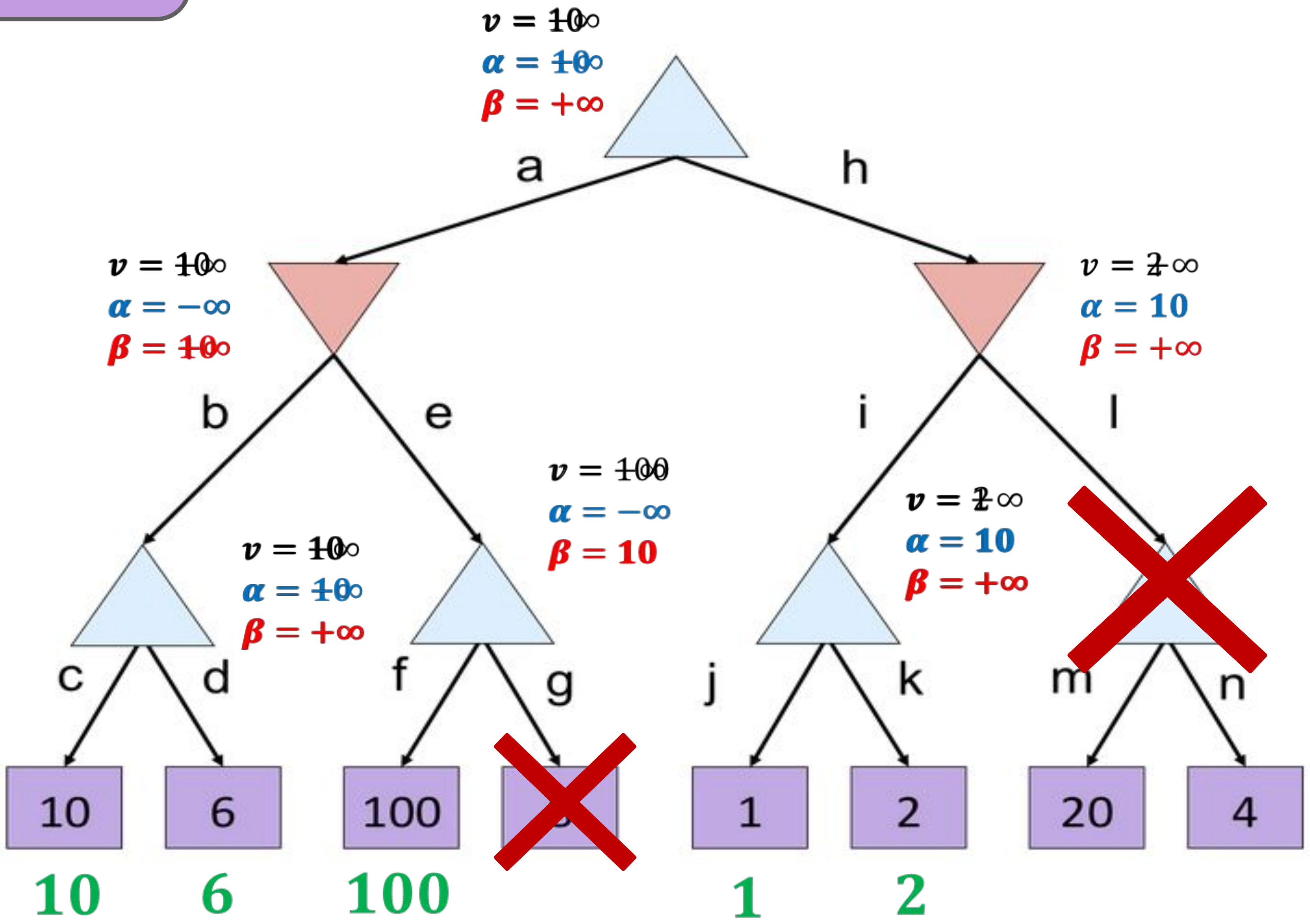
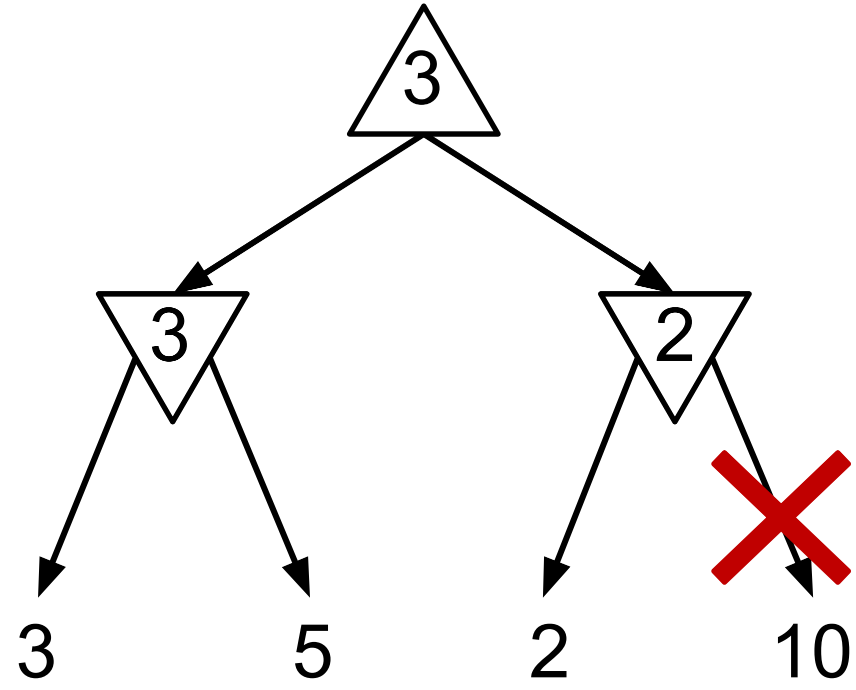


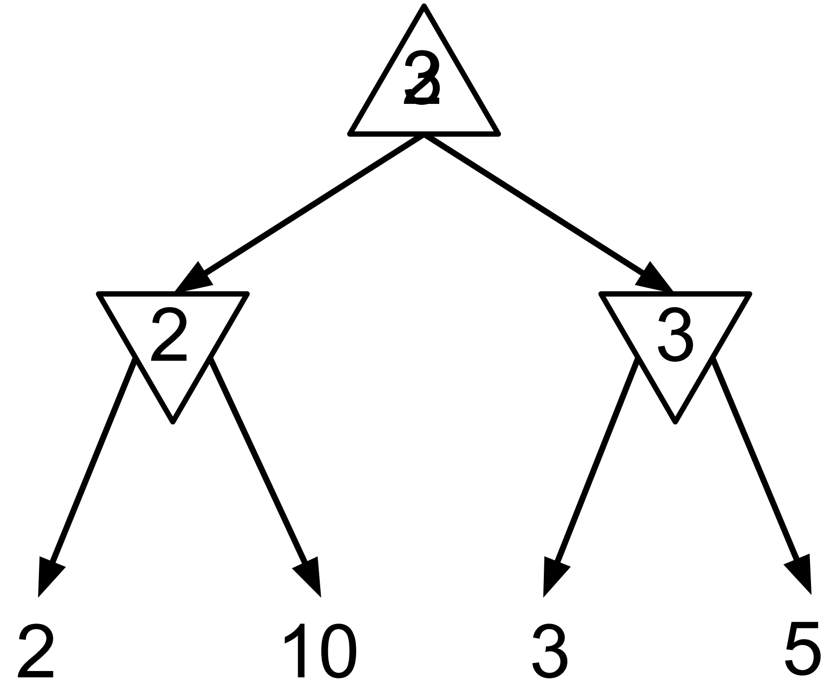
Child Ordering

- Pruning depends on order of actions.
- The 10 node can be pruned.



Child Ordering

- Pruning depends on order of actions.
- The 5 node can't be pruned.



Alpha-Beta Pruning Properties

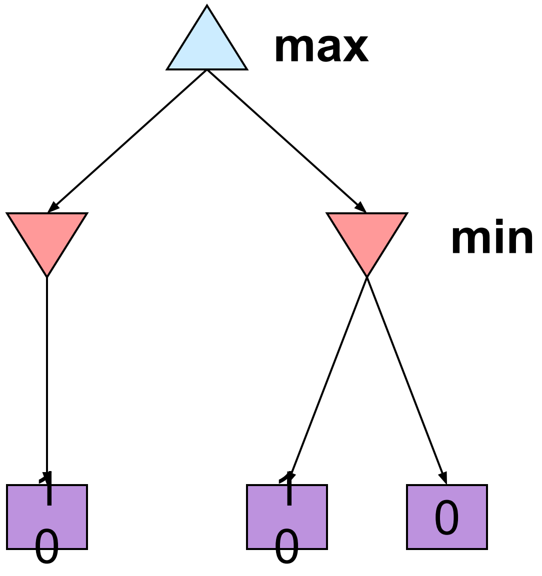
- This pruning has no effect on minimax value computed for the root!

- Values of intermediate nodes might be wrong
  - Important: children of the root may have the wrong value
  - So the most naïve version won't let you do action selection

- Good child ordering improves effectiveness of pruning

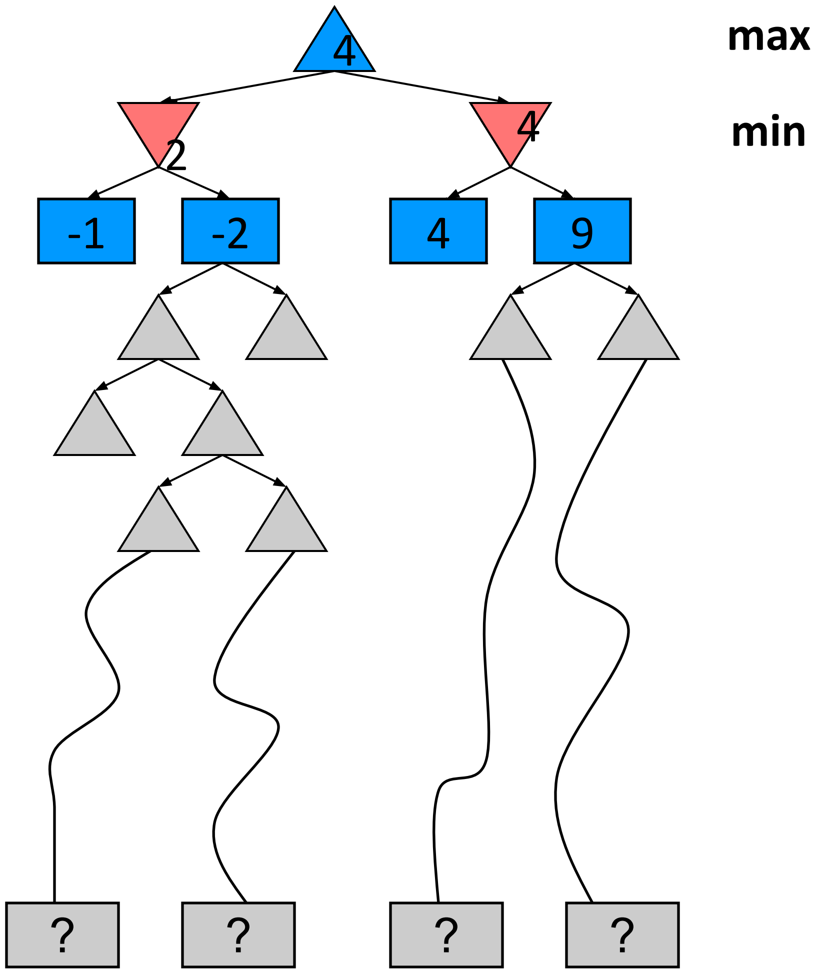
- With "perfect ordering":
  - Time complexity drops to $O(b^{m/2})$
  - Doubles solvable depth!
  - Full search of, e.g. chess, is still hopeless…

- This is a simple example of metareasoning (computing about what to compute)



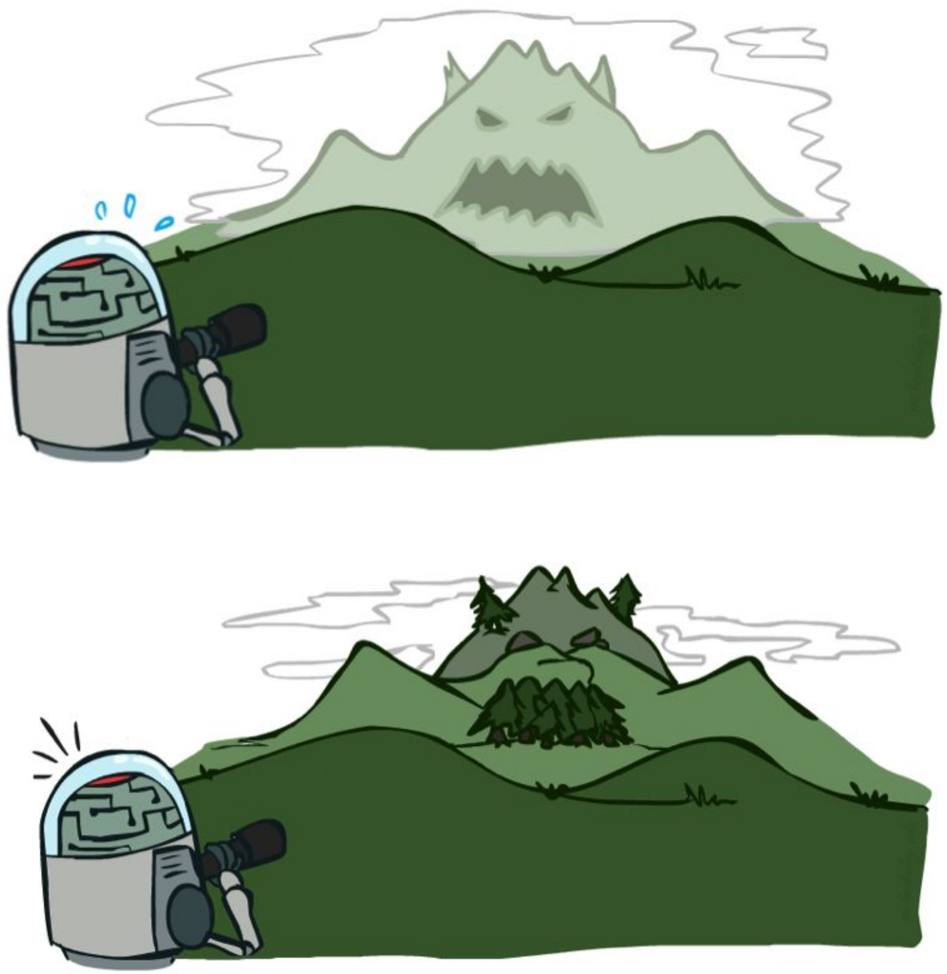
Resource Limits

- Problem: In realistic games, cannot search to leaves!
- Solution: Depth-limited search
  - Instead, search only to a limited depth in the tree
  - Replace terminal utilities with an evaluation function for non-terminal positions
- Example:
  - Suppose we have 100 seconds, can explore 10K nodes / sec
  - So can check 1M nodes per move
  - α-β reaches about depth 8 – decent chess program
- Guarantee of optimal play is gone
- More plies makes a BIG difference
- Use iterative deepening for an anytime algorithm



Depth Matters

- Evaluation functions are always imperfect
- The deeper in the tree the evaluation function is buried, the less the quality of the evaluation function matters
- An important example of the tradeoff between complexity of features and complexity of computation

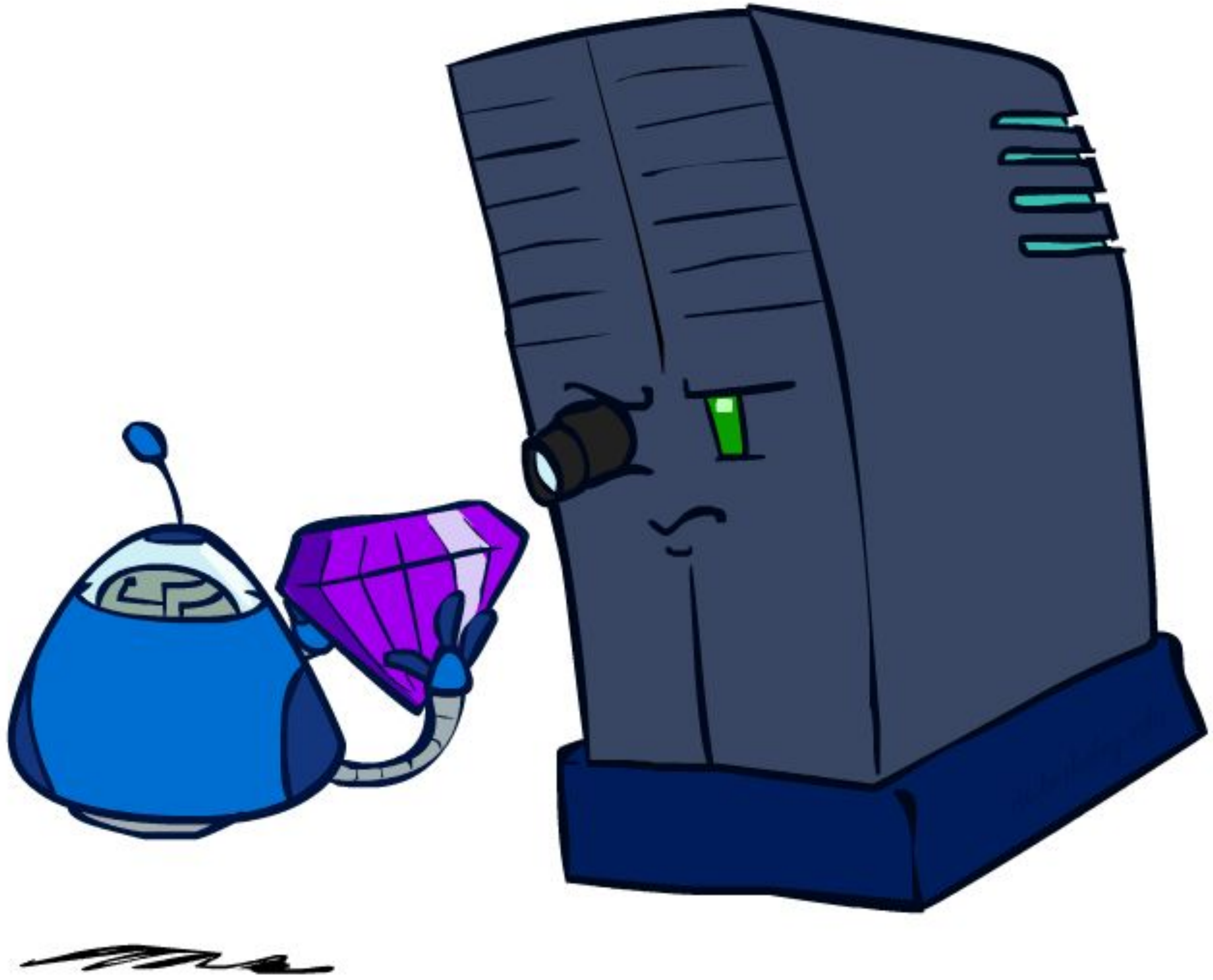


[Demo: depth limited (L6D4, L6D5)]

Video of Demo Limited Depth (2)

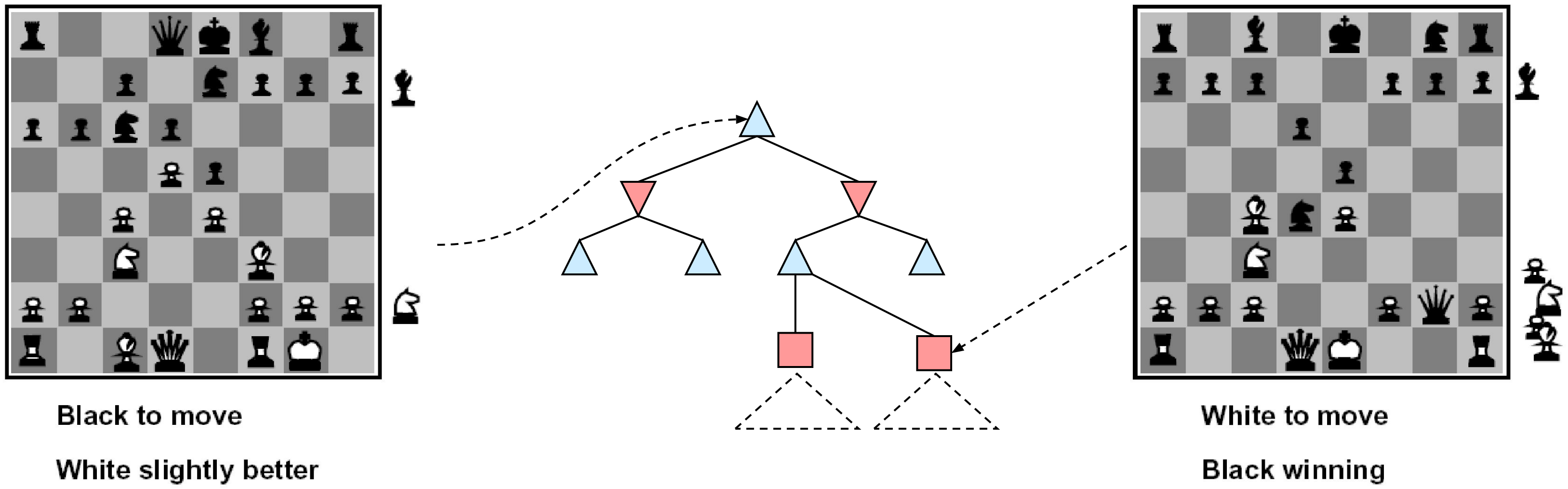
Video of Demo Limited Depth (10)

Evaluation Functions



Evaluation Functions

- Evaluation functions score non-terminals in depth-limited search



- Ideal function: returns the actual minimax value of the position
- In practice: typically weighted linear sum of features:

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

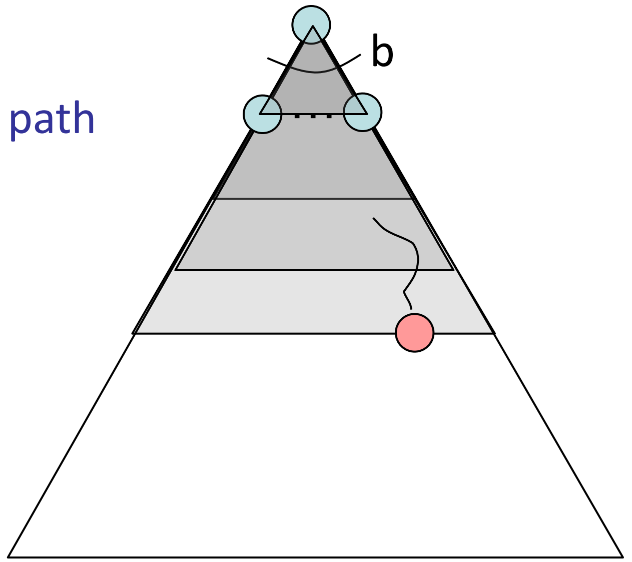
- e.g. $f_1(s)$ = (num white queens – num black queens), etc.

Iterative Deepening

Iterative deepening uses DFS as a subroutine:

1. Do a DFS which only searches for paths of length 1 or less. (DFS gives up on any path of length 2)
2. If “1” failed, do a DFS which only searches paths of length 2 or less.
3. If “2” failed, do a DFS which only searches paths of length 3 or less.

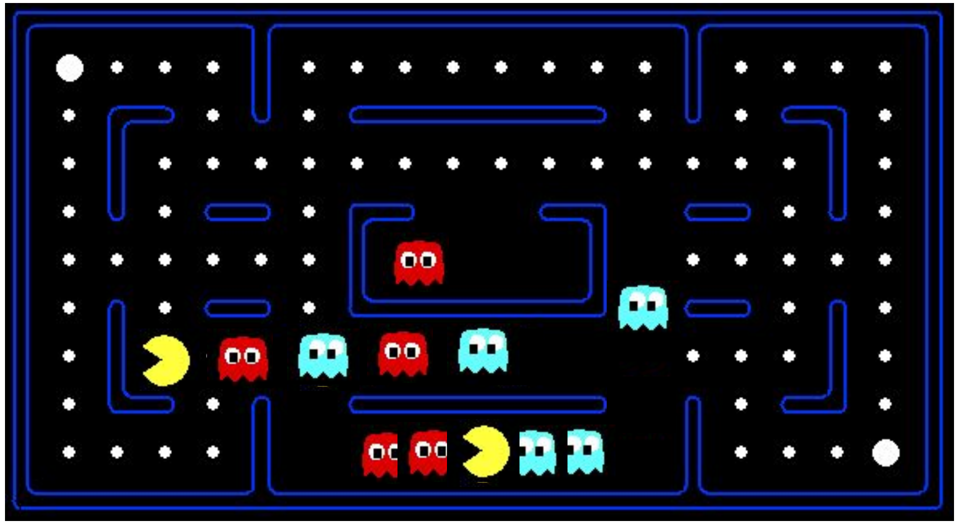
….and so on.



Why do we want to do this for multiplayer games?

Note: wrongness of eval functions matters less and less the deeper the search goes!

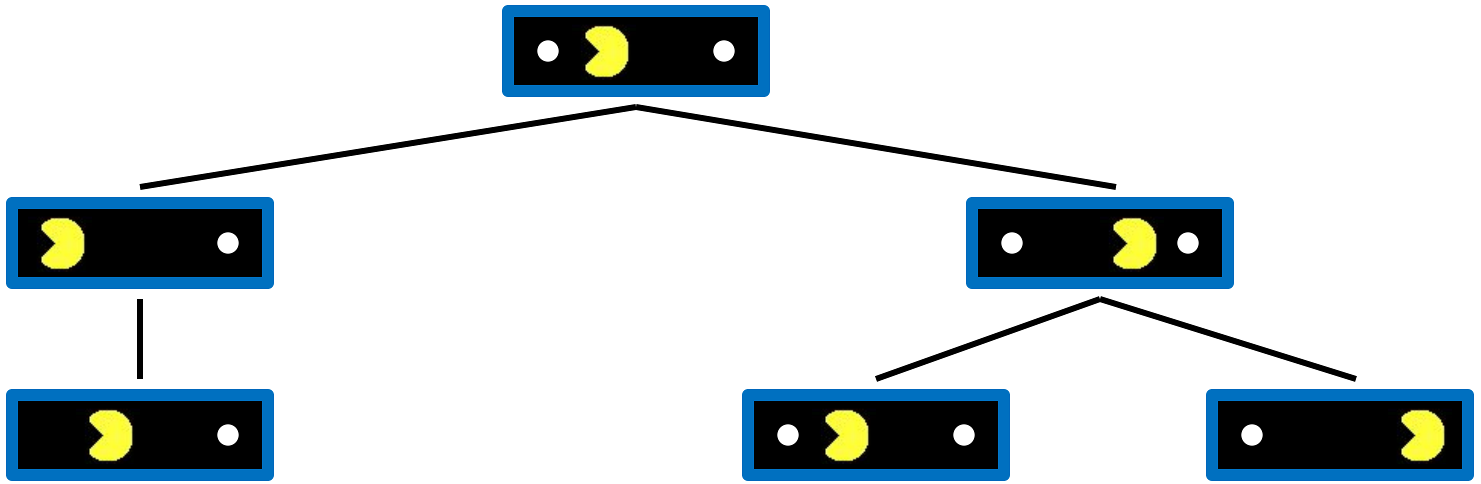
Evaluation for Pacman



[Demo: thrashing d=2, thrashing d=2 (fixed evaluation function), smart ghosts coordinate (L6D6,7,8,10)]

Video of Demo Thrashing (d=2)

Why Pacman Starves



- A danger of replanning agents!
  - He knows his score will go up by eating the dot now (west, east)
  - He knows his score will go up just as much by eating the dot later (east, west)
  - There are no point-scoring opportunities after eating the dot (within the horizon, two here)
  - Therefore, waiting seems just as good as eating: he may go east, then back west in the next round of replanning!

Video of Demo Thrashing -- Fixed (d=2)

Video of Demo Smart Ghosts (Coordination)

Video of Demo Smart Ghosts (Coordination) – Zoomed In

Next Time: Uncertainty!