Q2e_OptimalControl

April 16, 2024

[ ]:
```julia
using JuMP
using Ipopt
using Plots
include("Q2c_VehicleDynamics.jl")

x0 = [-10.0, 0.0, 0.0, 0.0, 0.0, 10.0, 0.0] #TODO Initial Condition
XL = [-40, -20, -3, -pi/5,  -pi/2, 5.0, -pi/12] # States Lower Bound
XU = [300, 20, 3, pi/5, pi/2, 15.0, pi/12] #TODO States Upper Bound
CL = [-2.6, -0.1] #TODO Control Lower Bound
CU = [2.6, 0.1] #TODO Control Upper Bound

model = Model(optimizer_with_attributes(Ipopt.Optimizer)) # Initialize JuMP␣
 ↪model

numStates = 7 #TODO number of states
numControls = 2 #TODO number of control
PredictionHorizon = 8 #TODO Prediction Time
numColPoints = 81 #TODO
Δt = PredictionHorizon/(numColPoints - 1)# Time interval

@variables(model, begin
    # Set xst as a numColPoints x numStates matrix that is between the upper␣
 ↪and lower states bounds
    XL[i]   xst[j in 1:numColPoints, i in 1:numStates]    XU[i]
    #TODO Similarly, set u as a numColPoints x numControls matrix that is
    # between the upper and lower control bounds
    CL[i]   u[j in 1:numColPoints, i in 1:numControls]   CU[i]
end)

# Fix initial conditions
fix(xst[1, 1], x0[1]; force = true) # set the initial condition for x-position␣
 ↪value
#TODO Follow the same way, set the remaining initial conditions,
# set x0[2] to xst[1,2],... and so on.
fix(xst[1,2],x0[2];force = true)
fix(xst[1,3],x0[3];force = true)
fix(xst[1,4],x0[4];force= true)
```

```
fix(xst[1,5],x0[5];force= true)
fix(xst[1,6],x0[6];force= true)
fix(xst[1,7],x0[7];force= true)


# sa means steering angle, sr means steering rate
x = xst[:, 1]; y = xst[:, 2]; v = xst[:, 3]; r = xst[:, 4];   = xst[:, 5];
ux = xst[:, 6]; sa = xst[:, 7];
ax = u[:, 1]; # retract variable
sr = u[:, 2];

#  xst = Matrix{Any}(undef, numColPoints, numStates)
# write the states derivative for all states & controls
 xst = Matrix{Any}(undef, numColPoints, numStates)
for i = 1:1:numColPoints
     xst[i, :] = @expression(model, VehicleDynamics(xst[i, :], u[i, :]))
    #  xst[i, :] = @expression(model, VehicleDynamics(reshape(xst[i, :],(1,7)),
    #   reshape(u[i, :],(1,2))))
end

# add constraint to each state using backward Euler method
for j = 2:numColPoints
    for i = 1:numStates
        @constraint(model, xst[j, i] == xst[j - 1, i] +Δt *  xst[j, i])
    end
end


# TODO write the cost function for each term - Lane change
y_cost = @expression(model, sum((y[j] - 5)^2 * Δt for j= 1:1:numColPoints))␣
 ↪#global y position of C.G Cost
sr_cost = @expression(model, sum((sr[j])^2 * Δt for j= 1:1:numColPoints))
sa_cost = @expression(model, sum((sa[j])^2 * Δt for j= 1:1:numColPoints))
ux_cost = @expression(model, sum((ux[j] - 13)^2 * Δt for j= 1:1:numColPoints))
ax_cost = @expression( model, sum((ax[j])^2 * Δt for j=1:1:numColPoints)) # ax␣
 ↪cost


#TODO define cost weight
# w_y = [1e-5, 1e-4, 1e-3, 1e-2, 1e-1, 1,10];
# w_y = 10
w_y  = 0.05 # change later for 2f
w_sr = 2.0
w_ax = 0.2
w_ux = 0.2
w_sa = 1.0
```

```julia
# Objective: Minimize cost function
@objective(model, Min, w_y * y_cost + w_sr * sr_cost + w_ax * ax_cost + w_ux *␣
↪ux_cost + w_sa * sa_cost) # objective value
optimize!(model) # optimize model
StatesHis = value.(model[:xst]) # retrieve data
if abs(objective_value(model) - 3.65) < 0.1 # check answer
    println("Congrats, your answer is correct")
else
    println("Something went wrong, please try again!")
end

# println("Objective value model = ",objective_value(model))
println("Your y cost is:  ", round(value(y_cost); digits = 3))

#Plot
# plot(StatesHis[:, 1], StatesHis[:, 2], tickfontsize = 10, xlabel = "X (m)",␣
↪ylabel = "Y (m)",guidefont=15) # path plot
# plot(0:Δt:PredictionHorizon, StatesHis[:, 6], tickfontsize = 10, xlabel =␣
↪"time (s)", ylabel = "ux (m/s)",guidefont=15) # Speed plot

display(plot(StatesHis[:, 1], StatesHis[:, 2], tickfontsize = 10, xlabel = "X␣
↪(m)", ylabel = "Y (m)",guidefont=15)) # path plot
display(plot(0:Δt:PredictionHorizon, StatesHis[:, 6], tickfontsize = 10, xlabel␣
↪= "time (s)", ylabel = "ux (m/s)",guidefont=15)) # Speed plot)
```

```
The states derivative is: [9.9 1.496 -1.004 2.587 0.1 1.0 0.1]
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.

Number of nonzeros in equality constraint Jacobian…:     2473
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian…:     3602

Total number of variables…:      722
                     variables with only lower bounds:        0
                variables with lower and upper bounds:      722
                     variables with only upper bounds:        0
Total number of equality constraints…:      560
Total number of inequality constraints…:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  1.1142900e+02 9.50e+00 2.64e-01  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  1.1109759e+02 8.23e+00 6.83e+00  -1.0 3.13e+01    -  1.67e-02 1.33e-01f  1
   2  1.1025958e+02 6.92e+00 5.68e+00  -1.0 2.83e+01    -  6.67e-02 1.60e-01f  1
   3  1.0819221e+02 5.04e+00 4.04e+00  -1.0 2.49e+01    -  9.68e-02 2.71e-01f  1
```

```
   4  1.0469717e+02 3.07e+00 2.31e+00  -1.0 1.94e+01    -  1.38e-01 3.90e-01f  1
   5  9.8916387e+01 1.03e+00 1.23e+00  -1.0 1.29e+01    -  2.15e-01 6.66e-01f  1
   6  9.1320242e+01 2.90e-03 7.91e-01  -1.0 5.54e+00    -  4.76e-01 1.00e+00f  1
   7  7.7137482e+01 1.85e-03 8.01e-01  -1.0 4.29e+00    -  5.18e-01 1.00e+00f  1
   8  3.2102711e+01 2.00e-02 3.97e-01  -1.0 2.02e+01    -  4.57e-01 1.00e+00f  1
   9  1.0894542e+01 1.70e-02 9.82e-02  -1.0 1.50e+01    -  6.62e-01 1.00e+00f  1
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
  10  5.5093386e+00 4.06e-03 6.48e-02  -1.0 7.91e+00    -  1.00e+00 1.00e+00f  1
  11  4.2433091e+00 2.22e-03 1.18e-02  -1.7 3.76e+00    -  1.00e+00 1.00e+00f  1
  12  3.8205562e+00 2.76e-03 8.76e-03  -2.5 1.96e+00    -  9.80e-01 1.00e+00f  1
  13  3.6906527e+00 2.33e-03 3.35e-02  -3.8 5.20e-01    -  8.17e-01 1.00e+00h  1
  14  3.6620614e+00 6.66e-04 5.61e-04  -3.8 1.74e-01    -  1.00e+00 1.00e+00h  1
  15  3.6533282e+00 2.26e-04 2.77e-03  -5.7 7.89e-02    -  8.10e-01 9.78e-01h  1
  16  3.6519431e+00 2.36e-05 4.94e-04  -5.7 2.39e-02    -  9.47e-01 1.00e+00h  1
  17  3.6517338e+00 2.47e-06 5.25e-06  -5.7 8.66e-03    -  1.00e+00 1.00e+00h  1
  18  3.6516747e+00 4.23e-07 1.36e-05  -8.6 2.04e-03    -  9.85e-01 9.76e-01h  1
  19  3.6516712e+00 8.27e-08 3.93e-07  -8.6 1.09e-03    -  1.00e+00 1.00e+00h  1
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  21  3.6516707e+00 4.52e-09 2.21e-08  -8.6 2.59e-04    -  1.00e+00 1.00e+00h  1
  22  3.6516707e+00 8.61e-10 4.21e-09  -8.6 1.13e-04    -  1.00e+00 1.00e+00h  1
  23  3.6516706e+00 1.12e-10 5.46e-10  -9.0 4.07e-05    -  1.00e+00 1.00e+00h  1

Number of Iterations…: 23

                                   (scaled)                 (unscaled)
Objective…:   3.6516706220464390e+00    3.6516706220464390e+00
Dual infeasibility…:   5.4583440954618428e-10    5.4583440954618428e-10
Constraint violation…:   1.1227421770065860e-10    1.1227421770065860e-10
Variable bound violation:   9.1102048405122815e-09    9.1102048405122815e-09
Complementarity…:   2.1361287151070329e-09    2.1361287151070329e-09
Overall NLP error…:   2.1361287151070329e-09    2.1361287151070329e-09


Number of objective function evaluations             = 24
Number of objective gradient evaluations             = 24
Number of equality constraint evaluations            = 24
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 24
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 23
Total seconds in IPOPT                               = 0.051

EXIT: Optimal Solution Found.
Congrats, your answer is correct
Your y cost is:  33.942
This is Ipopt version 3.14.14, running with linear solver MUMPS 5.6.2.
```

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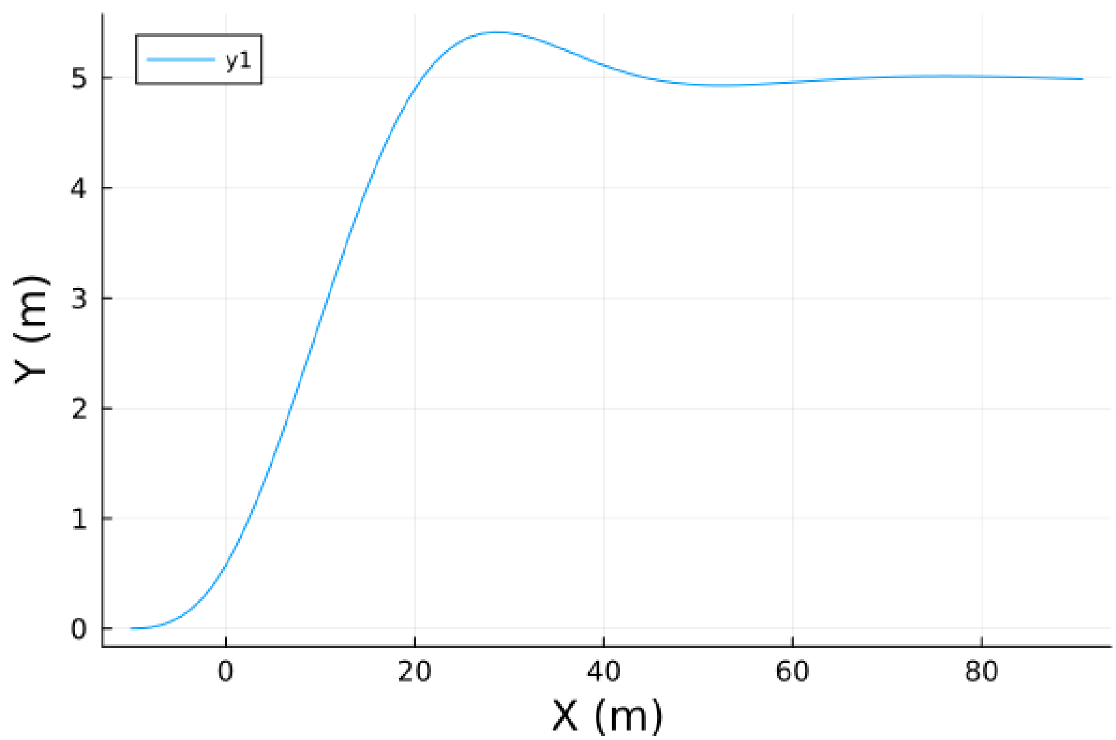

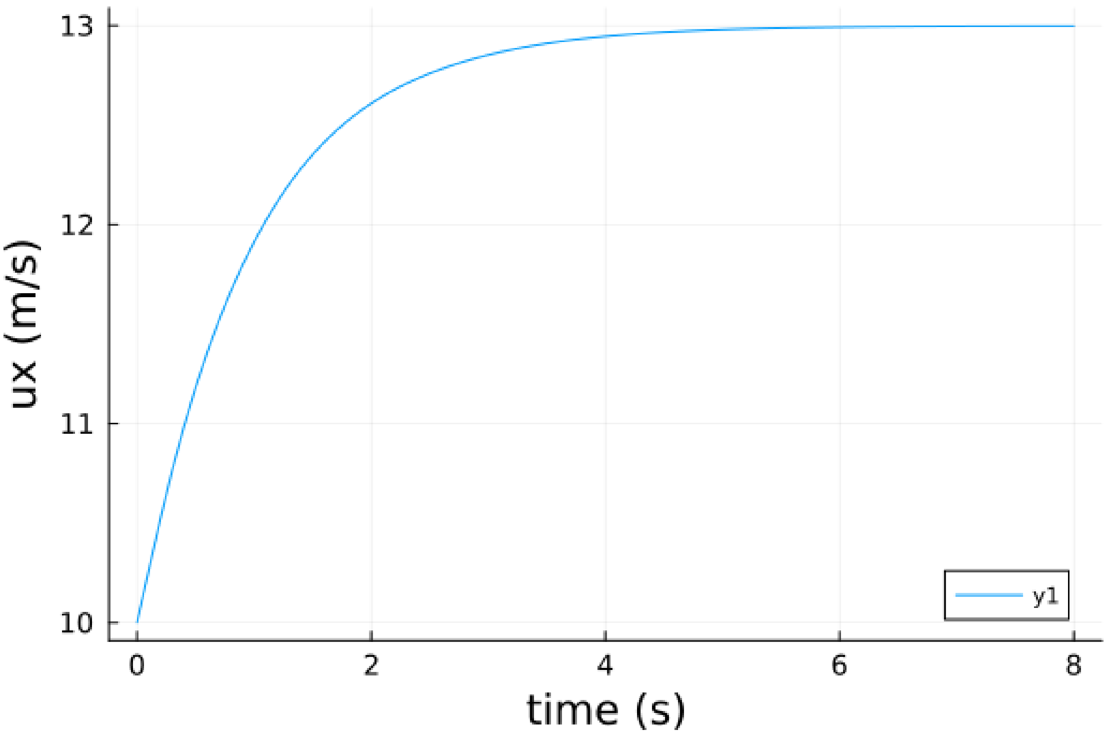
13
12
11
10
0
2
4
6
8
ux (m/s)
time (s)
y1