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Chapter 10

Stochastic Programming Models in Energy

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Abstract

We give the reader a tour of good energy optimization models that explicitly deal with uncertainty. The uncertainty usually stems from unpredictability of demand and/or prices of energy, or from resource availability and prices. Since most energy investments or operations involve irreversible decisions, a stochastic programming approach is meaningful. Many of the models deal with electricity investments and operations, but some oil and gas applications are also presented. We consider both traditional cost minimization models and newer models that reflect industry deregulation processes. The oldest research precedes the development of linear programming, and most models within the market paradigm have not yet found their final form.

Key words: Stochastic programming, energy, regulated markets, deregulation, uncertainty, electricity, natural gas, oil.

1 Introduction

The purpose of this chapter is to discuss the use of stochastic programming in energy models. This is not a well defined topic. Let us therefore start by outlining what this chapter is and what it is not. First, this is not an annotated bibliography. Its purpose is to help the reader see where stochastic programming can be used, and to point to relevant existing literature. We do not attempt to be complete in our references, only to help the reader find good starting points. We shall discuss both existing models and the potential for new arenas.

Then, what shall we understand by the reference to energy models in *stochastic programming*? Generally, stochastic programming refers to a problem class, and not to the choice of solution procedures. Many of the models in this class can be solved both with tools from mathematical programming and as stochastic dynamic programs (SDPs). This book is about stochastic programs solved with tools from mathematical programming. However, the view we have taken in this chapter is that we cannot include or exclude interesting models solely on the basis of what solution method the authors have chosen. Hence, if an existing model represents a stochastic dynamic decision problem which can be formulated as a stochastic program, we include it irrespective of whether it is solved with methodology from mathematical programming or set and solved as an SDP.

Furthermore, to have made the point, this chapter is not about operations research and energy. This ought not to affect our models too much, as we are of the opinion that most real decision are made under uncertainty, but it will affect our referencing to the literature.

As part of the preparation for this chapter we had the privilege of reading a text, which for our field, is very old. Massé (1946) authored two volumes on hydro scheduling. The books are based on work performed before and during World War II. Of course, he does not discuss stochastic programming as such—the term was not invented at the time—but he discusses models and methodology that would fit the premises of this chapter. It is very interesting to see how he walks his readers through some very deep arguments about why deterministic model are not good enough. He points to the fact that looking at a deterministic future is far too optimistic, and that flexibility will be disregarded.

His major point is that hydro scheduling is about releasing water such that the immediate financial gain equals the expected future value of water. The expected future value of water is presented as a function of reservoir level, present inflow (to the extent that there is memory in that process), and time (to represent seasonality). He gives optimality conditions for this case. In fact, he has a long discussion to the effect that all uses of natural resources is a tradeoff between use now and use in a stochastic future. To illustrate the use of statistics about the future, he makes the reference that if you wish to check the probability that you are alive tomorrow, you look at your present health, if you wish to know if you are alive in thirty years, you resort to statistics.

Another fact, dear to all stochastic programmers, is his pointing out that while deterministic multiperiod optimization yields decisions for all periods, a stochastic approach only yields policies or strategies.

A further major issue in the books is the objective function of the optimization. Should we maximize expected profit or expected utility (which he denotes psychological expectation in contrast to mathematical expectation)? He is concerned about some of the well known paradoxes when using expected profit, and he always refers to Borel for these examples. He is also very much concerned about risk, and strongly believes that risk will always be with us.

(He clearly had not thought of hedging in the financial markets.) He comes very close to defining an efficient frontier with expected profit on one axis and the probability of shortage of water on the other. His premise here is that the owner of a reservoir has agreements with some supplier, and that any reasonable agreement will be such that in extremely dry years, the contract cannot be fulfilled.

When decision problems are formulated and solved as deterministic problems, odd and special situations are often automatically excluded from consideration as only the expected values—the normal cases—are considered. Massé has the view that this can be dangerous, as what may appear to be a detail at the time of analysis, may later turn out to have had a major effect on the development: "Le nez de Cléopâtre, s'il eût été plus court, toute la face de la terre aurait été changée."

This chapter has five more sections. Section 2 is on regulated electricity markets. This is clearly the best developed area for use of stochastic programming in energy. Section 3 discusses the much newer area of de/re-regulated electricity markets. We close with two shorter sections on oil and natural gas, and a conclusion.

2 Electricity in regulated markets

This section discusses models for electricity production, thermal and hydro-based, in a setting of a regulated utility. Transmission planning and operations will not be considered. The utility can be either a single producer, or several producers that are perfectly coordinated by choice or by law. Much of the newer literature on electricity production is set in a framework of de(re)regulation and competition. We wait with this subject until the next session.

2.1 Overview of models

Many different models are used in power systems planning. A possible classification divides the different models according to the planning horizon. Long term planning models deal with investments and typically have a 15–20 year horizon. Medium term planning is done over a 1–3 year range, and deal, for example, with reservoir management. Short term planning typically deals with problems with horizons of one week or shorter, such as unit commitment and economic dispatch.

The perspective taken in these models is that of a social planner or an ideal public utility. The industry has traditionally been heavily regulated with considerable central planning. The reason for regulation is that the industry is prune to market failure; use of the transmission grid causes changes in its capacity in other parts of the network (externalities). If demand and supply is not matched at each instant, the whole or large parts of the system breaks down; reliability is a public good. Local utilities have typically had a monopoly within their area, preventing competition.

The demand in these models is mostly portrayed as price-inelastic; a deterministic or stochastic demand is to be met at minimum cost. A standard textbook on electricity operations is Wood and Wollenberg (1996). Some of these models may still have some relevance in a market setting, because the deregulated utility often still has the obligation to serve local demand. Similarly, the utility may have committed to a particular load schedule in the spot market.

2.2 Long term planning

By long term planning we will normally mean planning large investments, be that building thermal units, or constructing hydro reservoirs and turbines. The starting point for such an analysis is always a projection of future load (demand). Let us first briefly see how the load is normally presented in such a setting. The starting point will a load curve for each individual day of the year—possibly split into groups of days with similar patterns. These curved will possess the common pattern of having peaks in the morning and afternoon, reflecting our way of life. The first step is to sort these curves according to decreasing loads, to achieve daily load duration curves. These curves are then added together to form a curve like the one in Fig. 1. To the left is the hour with the highest load during the whole year, and to the right the one with the lowest. For example, h is the number of hours in a year for which the load is at least L (MW).

Normally, this curve is not smooth, but step-wise, as the unit along the horizontal axis is at least one hour. To simplify the presentation, let us assume that there are only two steps, the base load period and the peak load period. What we are about to present is inspired by Murphy et al. (1982), and the first result we are to illustrate comes from that paper. Consider the simplified picture in Fig. 2

The total energy consumption in base load is given by $t_2 L_1$, and in peak load $t_1(L_2 - L_1)$. Assume we have two technologies, for example two different

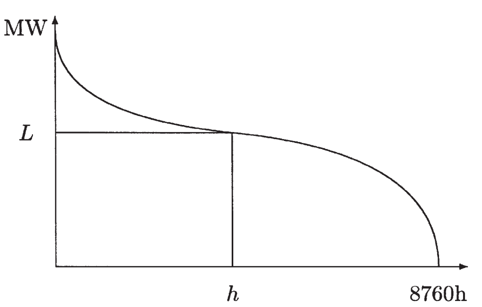

Fig. 1. Load duration curve for one year (8760 h).

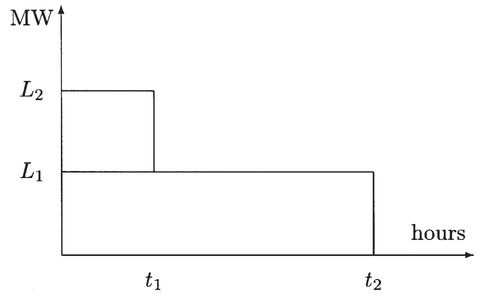

Fig. 2. Simplified load duration curve for illustrative purposes.

types of thermal units, and let us simply denote them '1' and '2'. Let f_1 be the annualized fixed costs per unit of production capacity for technology 1, and let c_1 be the operating costs per MWh, also for technology 1. For technology 2, f_2 and c_2 are similarly defined. Let x_i for $i = 1, 2$ be the installed capacity of technology i. Furthermore, let y_{ib} be the production of base load for technology i and correspondingly y_{ip} for peak load. The (deterministic) problem now becomes:

$$\min \sum_{i=1}^{2} f_i x_i + \sum_{i=1}^{2} c_i (t_2 y_{ib} + t_1 y_{ip}) \tag{2.1}$$

subject to

$$\begin{aligned} y_{1b} + y_{1p} &\le x_1 \\ y_{2b} + y_{2p} &\le x_2 \\ t_2(y_{1b} + y_{2b}) &= d_b (= t_2 L_1) \\ t_1(y_{1p} + y_{2p}) &= d_p (= t_1(L_2 - L_1)) \\ \text{and non-negativity} & \end{aligned} \tag{2.2}$$

where d_b is the load in the base period and d_p the additional demand in the peak period. The first two constraint say that production cannot exceed installed capacity of the two technologies, whereas the last two constraints express that base load and additional peak load must be satisfied.

This model helps us find the optimal investment to meet a known future demand. But, of course, future demand is not known. Hence, as a first step of making the model more realistic, let us assume that there are several possible future load duration curves. Let curve k occur with probability p_k. A straightforward recourse model now becomes.

$$\min \sum_{i=1}^{2} f_i x_i + \sum_{k=1}^{K} p_k \sum_{i=1}^{2} c_i^k (t_2 y_{ib}^k + t_1 y_{ip}^k) \tag{2.3}$$

subject to, for all k

$$\begin{aligned} y_{1b}^k + y_{1p}^k &\leq x_1 \\ y_{2b}^k + y_{2p}^k &\leq x_2 \\ t_2^k(y_{1b}^k + y_{2b}^k) &= d_b^k(= t_2^k L_1^k) \\ t_1^k(y_{1p}^k + y_{2p}^k) &= d_p^k(= t_1^k(L_2^k - L_1^k)) \\ &\text{and non-negativity} \end{aligned} \tag{2.4}$$

where superscript k always refers to load duration curve (scenario) k. Note that the first stage variables x_i do not have a superscript k. Also note that for given values of the first-stage decisions x_i, this problem falls apart and becomes standard transportation network flow problems, one for each load duration curve, see Wallace (1986) for details.

What would we expect to get from (2.3) and (2.4) if we compare its solution to the case where (2.1) and (2.2) are solved with the expected load duration curve, that is, the case where we instead of scenarios use a 'typical' or average year and arrive at a deterministic model? Or even more importantly, how do we expect the solution to the stochastic optimization problem to differ from individual scenario problems? Different technologies for energy production will vary in some aspects. Some will have long lead times, high investments costs and low operating costs, others will have shorter lead times, lower investment costs, but at the price of higher operating costs. In a given scenario, the future demand will be known with certainty. That will tend to produce the use of a technology which perfectly matches the load. This will typically be a technology with high investment costs and low operating costs. The capacity of the unit will perfectly match the needs reflected in the load duration curve (scenario). Over-investments will never take place, and shortages will never occur. Smaller units which are more flexible, but have higher operating costs, will tend to lose out, as their qualities will not be reflected in the model. Why pay for flexibility you do not need? An example of these aspects is shown by Smeers (1990), where he discusses the relationship between coal (expensive but flexible) and nuclear (inexpensive but also inflexible).

An interesting question raised in Murphy et al. (1982) is if there is a version of the deterministic problem, such that if we solve that problem, we obtain the solution to the stochastic problem. From a general point of view, this is something we normally do not find for stochastic programs, but in this case there is a result.

Take the individual load duration curves, and multiply the duration of each block by p_k. In our simplified example, multiply all numbers on the horizontal axis by p_k to obtain $t_1^k p_k$ and $t_2^k p_k$. Notice that a year is 8760 h, thus $\sum t_2^k p_k = 8760$. Create a new load duration curve from these new curves to

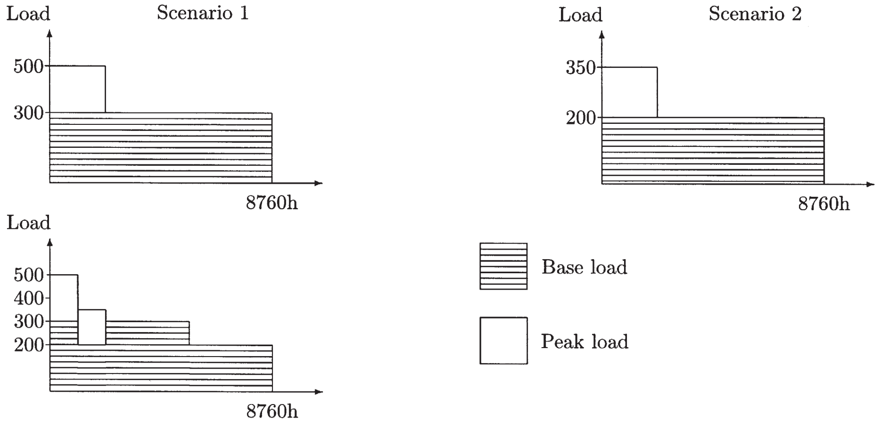

Fig. 3. Horizontal aggregation of two load duration curves.

arrive at a new aggregated curve. This is illustrated in Fig. 3. In the *horizontal* summation we have kept the patterns for peak and base load in the scenarios. This is only to make it easier to see where the columns come from. In reality, in the summation, there are four different load levels, to be treated with four parameters t_i and four load levels L_i. Generally, the number of steps in the sum equals the sum of the number of steps in the individual load duration curves.

The main result here is that if we carry out a deterministic investment analysis using this horizontally aggregated load duration curve, we obtain the same solution as if we had solved the recourse problem above. But the problem is simpler as the number of constraints saying that we cannot use capacity not installed will decrease from k times the number of technologies to just the number of technologies.

But the result is dependent on some assumptions, in particular that the operating costs do not vary with output level, implying that the c_i^k are fixed irrespective of technology i being used for base or peak load (as it is in (2.3) and (2.4)), and irrespective of scenario, so $c_i^k \equiv c_i$. But even so, this is a strong and interesting result for the two-stage case.

Sherali et al. (1982, 1984) discuss the model in greater detail, with an emphasis on cost sharing for the fixed charges f_i. This brings the problem into the realm of peak load pricing, that is, the cost of capacity is always related to those (users or scenarios) that have the maximal use of the capacity.

The load duration curve can also be approximated in a different way. An example is given in Fig. 4. In this case the resulting problem is quadratic. For a discussion, see the very early paper of Louveaux (1980).

Discrete decisions

The basic recourse model above can of course be extended in many directions. First, in many cases the variables x_i can take on only a finite

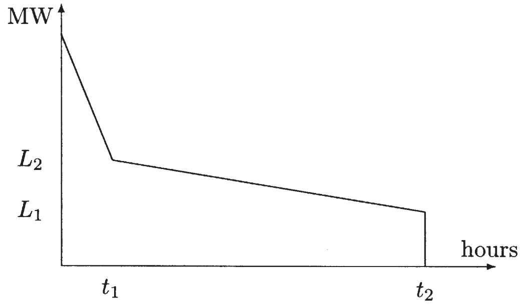

Fig. 4. Discretization of load duration curve resulting in a quadratic objective function.

number of values, which brings us into (stochastic) mixed integer programming. Bienstock and Shapiro (1988) integrates capacity expansion, supply contracts and operations planning in an electricity utility via a two-stage stochastic program with integer variables in both stages.

Multi-stage and lead times

In the same way, there are obvious possibilities in setting up a multi-stage problem, where load duration curves are revealed over time, and investments made stage by stage. Such a setup will show the importance of lead times in construction. The option to wait will favor the technologies with short lead times. This effect is not easy to capture in single- or two-stage models. Hence, this dynamic effect comes in addition to the effect discussed earlier where a stochastic model will be able to favor flexible technologies (that are never optimal in deterministic worlds). A good discussion of this problem can be found in Gardner and Rogers (1999). Flexibility is also discussed by Gardner (1996), where there is a focus on the relationship between flexibility in the financial sense, and the effects of emission control of acid-gas. The paper shows that when acid-gas emission control is added, some of the more flexible technologies lose in the competition. This is particularly true for gas combustion turbines. For further discussions of emission policies, see Manne and Richels (1991, 1995), Manne and Olsen (1996) and Birge and Rosa (1996).

Other contributions in the multi-stage setting are Manne and Richels (1978), Gorenstin et al. (1993), Dantzig and Infanger (1993), Escudero et al. (1995) and Pereira et al. (1995).

Shortage—or lost load

It is traditional in monopoly-based production planning to take it for granted that all demand must be satisfied. Qiu and Girgis (1993) take a different view, and say that since, ultimately, end users must always pay for the investments, they may be better off with a slight probability of an outage. They therefore set up a capacity investment problem where outages are priced rather than forbidden. Taking into account that scenarios (possible load duration curves) will always be somewhat subjectively chosen, and that outages to some extent correspond to worst-case analysis, it may be very good, modeling-wise, to allow for outages at a high cost. There will always be a slight chance that something even worse than the worst scenario of the model could occur, and hence, that an outage could occur even if the model claimed otherwise.

A further discussion of long-term planning can be found in Hobbs (1995).

2.3 *Medium-term planning*

Hydro-thermal scheduling

An important problem in the medium-term scale is hydro-thermal scheduling. For hydro reservoirs, the problem is essentially to strike a balance between immediate and future (opportunity) costs of using the water. Stochastic optimization models for this problem are in daily use in hydro-dominated systems.

The following section presents the production scheduling problem. There are T time periods, or *stages*, as illustrated in Fig. 5.

Periods are time intervals between stages, which are discrete points in time. The first period is deterministic. To simplify exposition, the problem is formulated for a producer with only one reservoir.

The producer is operating an ongoing business with an infinite future. We would like to avoid end effects, which are distortions in the model decisions due to the fact that the model has a finite horizon, whereas the real business problem has an infinite horizon. For example, if in the model the value of the reservoir at the end of the model horizon is too low, say equal to zero, then the end effect would be that too much water is sold in the last stage. There are several alternatives for this problem. One is choosing the date of stage T such that it makes sense to constrain the reservoir to be either empty or full at that date, i.e., in the spring before snowmelt, or in the fall before winter sets in. Another alternative requires estimating the end-of-horizon value of water in the reservoirs from a more aggregate model with a longer time span. Third, one can choose the time horizon of the model to be long enough to make the first stage decisions unaffected by the choice of horizon reservoir value function. See Grinold (1980) for an approach to dealing with end effects in general energy models. Grinold's observation is that the dual variables at the end of the horizon should be in a steady state, and suggests introducing dual constraints (i.e., primal variables) to ensure this. Another way of achieving steady state dual variables in the reservoir management problem is described by Lindqvist (1962). It involves starting with a guess for the dual variables, which in this case equals the incremental values of stored water in the

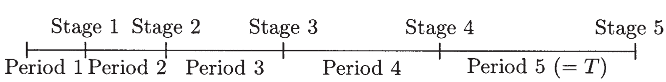

Fig. 5. Time scale example.

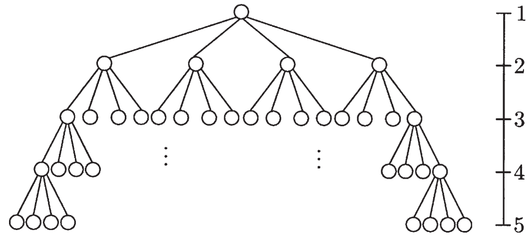

Fig. 6. Event tree and time scale example for $T = 5$. The nodes represent decisions, while the arcs represent realizations of the uncertain variables.

(equivalent) reservoir, and iterating over the last year of the planning horizon replacing the guesses with the water values obtained for one year before the end of the horizon.

The stochastic variables are inflow, and demand δ. Scenarios are possible histories of realizations of the stochastic variables up to the end of the horizon. The event tree in Fig. 6 shows how the uncertainty unfolds over time. A scenario in the event tree is a path from the root node to a leaf node. Each node n represents a *decision point*, or equivalently a *state*, corresponding to a realization of the random variables up to the stage of state n, denoted $t(n)$. The root state is $n = 1$, and scenarios are uniquely identified by states at the last stage, belonging to the set $\mathcal{S}$. The set of all states is denoted $\mathcal{N}$. The states have unconditional probabilities P_n, satisfying $\forall\ t \sum_{n|t(n)=t} P_n = 1$. Every state except the root has a parent state $a(n)$. Let stage t decisions (for period t) be made *after* learning the realization of the stochastic variables for that period.

The inflow process is multidimensional and has strong seasonal components. The main bulk of inflow to reservoirs in North America and northern Europe comes during spring, whereas in winter the precipitation accumulates as snow. Forecasting the inflows and capturing the structure of the processes and their degree of predictability is of vital importance to hydro scheduling models. This issue is discussed by Tejada-Guibert et al. (1995).

The decision variables are reservoir discharge, v_n, spill, r_n, and reservoir level l_n. Each variable in the problem is indexed by the state to which it belongs. Power generation is generally a nonlinear function of the height of the water in the reservoir and the discharge, and could be non-convex. In our exposition we disregard head variation effects, and assume that generation is proportional to flow through the station, ρv_n, where ρ is the constant hydro-plant efficiency. In practice however, head variation effects can be significant, particularly when balancing reservoirs with different characteristics. If a downstream reservoir has little storage capacity, then keeping its head up in order to maximize efficiency may lead to increased risk of spilling water if inflow increases too rapidly. This area needs further research.

Let $V_L(l_n)$ be the value of the reservoir at the end of the horizon as a function of the reservoir level. This function must be specified to avoid end effects. If a long term scheduling model is available, V_L may be extracted from this model, e.g., in the form of incremental value of stored water in reservoirs.

It is assumed that there is no direct variable cost of hydro production. Thermal generation is represented by p_{in}, for energy generated by unit $i \in \mathcal{I}$ in state n. This incurs variable cost $FC_i(p_{in})$, a convex function of p_{in}. It consists mainly of fuel costs, and is usually modeled as linear, piecewise linear or quadratic. Nonconvex cases are plausible, for example in the case of a unit that can be fed multiple types of fuels. With a typical minimum time resolution of one week it is reasonable not to include startup costs for thermal generation. Let γ be a discount interest rate, N_t is the number of years (in fractions of years) to period t,

$$\min_{p,v} \sum_{n \in \mathcal{N}} \left[P_n (1+\gamma)^{-N_{t(n)}} \sum_{i \in \mathcal{I}} FC_i(p_{in}) \right] - \sum_{s \in \mathcal{S}} P_s (1+\gamma)^{-N_T} V_L(l_s) \tag{2.5}$$

The hydro reservoir balance is

$$l_n - l_{a(n)} + v_n + r_n = \nu_n, \tag{2.6}$$

The demand constraint reads

$$\rho v_n + \sum_{i \in \mathcal{I}} p_{in} \geq \delta_n \tag{2.7}$$

Time-dependent upper and lower limits on release and reservoir level are imposed using

$$\underline{v}_{t(n)} \leq v_n \leq \overline{v}_{t(n)}, \tag{2.8}$$

$$\underline{l}_{t(n)} \leq l_n \leq \overline{l}_{t(n)}, \tag{2.9}$$

$$r_n \geq 0, \tag{2.10}$$

for $n \in \mathcal{N}$ and with initial reservoir level given.

Load curtailment is sometimes modeled as an extra thermal unit having a marginal cost equal to an estimate of the marginal cost of unserved energy.

In many systems the transmission system limits the opportunities for hydro-thermal scheduling. There will be cases when transferring more electric energy from one node of the system to another will not be possible. Electricity flow in transmission networks is governed by Kirchoff's laws and is limited by line capacities. These physical phenomena must be taken into account when including transmission constraints in the scheduling problem. Accurate mathematical representations of these features typically involve nonlinear and nonconvex equations with phase angles, voltages and power flows.

For hydro-thermal scheduling, the network constraints are usually linearized, however, into linearized (DC) power flow (Wood and Wollenberg, 1996). This problem has been modeled by Gorenstin et al. (1992).

Some schedulers feel more comfortable using deterministic models for this problem. An important question in this context is what is the value of stochastic optimization? Starting with Massé (1946), many researchers argue that the stochastic aspects of the problem are important, and their neglect should result in some loss. Deterministic solutions will underestimate the true costs and the risk of spilling water, and deterministic models will not see any value in waiting with releasing water in order to learn more about future demand and/or inflow. The degree of cost underestimation depends on the problem, e.g., Tejada-Guibert et al. (1995) show that it depends, for a given system, on the demand and the severity of penalties on shortages. Philbrick and Kitanidis (1999) show that the performance of deterministic solutions is particularly poor for reservoir systems with limited storage capacity.

The typical horizon for hydro scheduling is a few months to a few years. A typical length of the first time step ranges from one week to a month. The hydro scheduling model gives signals to hydro unit commitment via marginal values of stored water in the reservoirs and/or via total generation during the first week.

2.4 Short term planning

Hydro unit commitment

In the hydro unit commitment problem the scheduler must determine what turbine units to run in each time step (hourly or shorter) the next day or week, and at which output level the running units should generate. Generating stations may have several turbines each and may be coupled by their location along the same river system. Turbines incur startup costs and the generation of each station varies nonlinearly with the volume and with the net head[1] of the hydro discharge. This problem can be formulated as a large mixed-integer nonlinear programming model with an objective of minimizing cost subject to meeting a given demand. The cost is measured in terms of the volume of water used or as the opportunity cost associated with that volume, i.e., using the marginal values of stored water in each reservoir coming from medium-term generation planning models. Stochasticity in such models may reside in load, inflow, unit availability or cost. Of these, load is considered most important, since it is temperature dependent, and temperature cannot be predicted with a precision better than a few degrees even just a few hours in advance. Other factors such as hydro inflow are accurately predictable on such short time scales. The stochastic hydro unit commitment problem has been studied by Philpott et al. (2000).

[1] The net head is the difference between the height of the water immediately before and immediately after the power station.

If the net head varies significantly on short term with upper reservoir storage level, which is usually the case for small upper reservoirs and/or small maximum head, there may be significant gains from letting reservoir levels cycle between high and low levels during the course of the day or week. Such problems call for global optimization techniques, such as in Feltenmark and Lindberg (1997). Other complicating issues are time delays from flows leaving a station to arriving downstream, a delay that may depend on the flow rate. Further, the generation function is not always concave in discharge, making the standard approach of replacing it with a piecewise linear function problematic. Discharge ramping and reservoir level constraints due to navigational, environmental and recreational requirements add to the difficulty. To avoid end effects, horizon unit states must be constrained or valued. In the case of time delays, in-transition flows at the horizon must be dealt with similarly.

Thermal unit commitment

In contrast to hydro unit commitment, where there are power stations situated along rivers, the problem here is characterized by higher startup costs and restrictions preventing thermal stress. When starting up a coal fired unit there is a time delay before the unit is available for generation. The task is to find a cost-minimal schedule, and a production level, for each generating unit over time. The problem is to decide which units will be on/running, and how much units that are on (committed) will produce. As mentioned above, the load that is to be met and the availability of the generating units are uncertain parameters affecting the problem.

This has been modeled as a mixed integer stochastic program and has been explored by Takriti et al. (1996), Carpentier et al. (1996), Dentcheva and Römisch (1998) and Caroe and Schultz (1998). See also Gröwe-Kuska et al. (2002), Nowak and Römisch (2000) and Gollmer et al. (2000).

Having $|\mathcal{I}|$ thermal and $|\mathcal{J}|$ hydro plants, the objective is to minimize the operating costs. Let u_{in} be unit states, i.e., a binary decision variable that represents whether thermal unit i is running ($u_{in} = 1$) or not ($u_{in} = 0$) in state n. Operating costs consist of fuel cost $FC_i(p_{in}, u_{in})$ and startup costs $SC_{in}(u_i)$ for thermal units. Startup costs may depend on the amount of time the unit was down before startup. Thus u_i is a vector consisting of u_{in} and unit states u_i for one or several predecessor states of n. Hydro plants only contribute to the objective function with the water value $V_S(l_s)$ at the end of the time horizon T, where the vector $l_s = [\{l_{js}\}_{j \in \mathcal{J}}]$. This water value function approximates the objective value for the problem (2.5)–(2.10) with T going to infinity. The objective function reads:

$$\min_{u,p,v} \sum_{n \in \mathcal{N}} \sum_{i \in \mathcal{I}} FC_i(p_{in}, u_{in}) + SC_{in}(u_i) - \sum_{s \in \mathcal{S}} V_S(l_s). \tag{2.11}$$

The local demand δ_n has to be satisfied in all states:

$$\forall\, n \in \mathcal{N}: \sum_{i\in\mathcal{I}} p_{in} + \sum_{j\in\mathcal{J}} \rho_j v_{jn} \geq \delta_n, \tag{2.12}$$

where $\rho_j v_{jn}$ is the generation of hydro unit j. Additional constraints follow (index i omitted since all constraints are for each unit i): The output of a unit should be zero if the unit is offline; otherwise it should be between a lower and an upper bound ($\underline{p}$ and $\overline{p}$):

$$\forall\, n \in \mathcal{N}: u_n\underline{p} \leq p_n \leq u_n\overline{p}. \tag{2.13}$$

Further single-unit constraints are minimum up- and down-times and additional must-on/off constraints. Minimum up- and down-time constraints are imposed to prevent thermal stress and high maintenance costs due to excessive unit cycling. Denoting by $\underline{\tau}$ the minimum down-time of the unit, the corresponding constraints are described by the inequalities (temporarily switching to time subscripts instead of states):

$$u_{t+\tau} + u_{t-1} - u_t \leq 1, \tag{2.14}$$

for all $\tau = 1, \ldots, \min\{T - t, \underline{\tau} - 1\}$. Analogous constraints can be formulated for describing minimum up-time restrictions, see e.g., Gröwe-Kuska et al. (2002).

A reserve margin $r_t \geq 0$ is often imposed via reserve constraints

$$\forall\, n \in \mathcal{N}: \sum_{i\in\mathcal{I}} (u_{in}\overline{p}_i - p_{in}) \geq r_{t(n)} \tag{2.15}$$

to ensure that the model recommends an on-line capacity that exceeds the predicted load, giving a 'spinning reserve'. This is used, particularly in deterministic models, to avoid an energy imbalance resulting from the unexpected failure of a generating unit or an unexpected increase in load, which may cause very costly brownouts or blackouts. Carpentier et al. (1996) discuss the relationship between spinning reserve in a deterministic model of the problem compared to a stochastic model without spinning reserve, and uses rolling horizon optimization to arrive at an optimal level of reserve margin. The models consider uncertainty in generator availability.

Deterministic approaches

Unit commitment is usually solved as a deterministic large scale mixed integer program (Sheble and Fahd, 1994). It is therefore interesting to learn about qualitative differences between stochastic programming solutions of the unit commitment problem and deterministic solutions. A priori we can state that deterministic solutions will be characterized by extensive use of large plants with high start-up costs, with relatively few starts. SP solutions on the other hand, will typically use smaller units and will involve more startups of flexible but possibly high-marginal cost plants such as gas fired units. Deterministic models will know exactly how much power is needed at any time and can thus plan to run low fuel cost plants at high output for long periods of time. The gains that the model sees from such scheduling will outweigh the high startup costs that typically come with such plants.

In deterministic models, a common approach to the uncertainty regarding generator failure is to "derate" the units' maximum generation rate according to the probability of availability. However, this will underestimate the expected operations cost and the probability of load being larger than peaking capacity. Recognizing such operations cost underestimation, a class of models known as production costing models have been developed (Wood and Wollenberg, 1996). The purpose of these models is a more accurate estimation of production costs by simulating and/or optimizing the dispatch of generation under uncertainty of load and generator outages. Production costing models are used both in long-term and operations planning. These models are conceptually not much different from SP-based generation planning models, in fact, good SP models lessen the need for separate production costing and reliability models. SP contributions in this class have been made by e.g., Bloom (1983), Pereira et al. (1987, 1992) and Hobbs and Ji (1999). See also the review by Hobbs (1995).

Economic dispatch and optimal power flow

Optimal dispatch of power under uncertainty has been considered by e.g., Bunn and Paschentis (1986), Gröwe and Römisch (1992) and Gröwe et al. (1995). The models have a short time horizon, usually a day, with hourly or finer resolution. The unit commitment schedule (the unit states u_{in}) is regarded as given, and the problem is to determine a generation schedule (in the p_{in} variables) that minimizes operating costs ($FC_i(p_{in}, u_{in})$) and satisfies the demand.

Since this problem is near real-time operations, it is meaningful to include transmission issues. This is considered by Pereira et al. (1987), who solve a two-stage optimal power flow problem.

2.5 Solution methods and computations

We focus on the modeling process and not the solution methods. However, most of the SP energy papers focus on the solution method used to solve the model, not the modeling process itself. Still, there is a relationship between research on solution methods and model development, because models tend to be developed only if there is hope of solving the model. Thus, as new solution algorithms are published, new models are reported solved using twists of the state of the art algorithms. Thus solution methods are discussed briefly in this subsection.

For hydro planning problems, stochastic dynamic programming has been used for a long time; an early reference is Massé (1946). For surveys see Yakowitz (1982), Yeh (1985) and Stedinger (1998). These methods have also been used in unit commitment and expansion planning. However, a well known problem with these methods is the curse of dimensionality. To use them, it has been necessary to aggregate and/or decompose the problems before solving them. An example of this is the aggregation of several hydro reservoirs and connected power stations into a single equivalent reservoir/power station pair. Relatively good heuristics have been developed for supporting the aggregation/de-aggregation approximation process. Important applications are presented by Terry et al. (1986) and Gjelsvik et al. (1992). A somewhat different approach for the multireservoir problem, using decomposition, is presented by Turgeon (1980) and Sherkat et al. (1985). Still, methods that could handle multidimensional problems having many state variables, were in demand. In the late 1970s, authors at Stanford University (Birge, 1985) began experimenting with nested Benders' decomposition, and in electricity models this was used and refined by Pereira and Pinto (1985), Jacobs et al. (1995) and Morton (1996).

This method was able to solve multidimensional state type problems, but was unable to match SDPs time decomposition abilities with respect to solving stochastic programs having many stages. Nested Benders' decomposition works on a scenario tree whose number of nodes explodes with the number of stages, and the size of the problem to be solved is proportional to the number of such nodes. For a comparison of the main algorithms on reservoir management problems see Archibald et al. (1999).

With this background, the algorithm of Pereira and Pinto (1991) created a lot of interest in the energy optimization community. Termed stochastic dual dynamic programming (SDDP), it effectively combines the state-time decomposition features of dynamic programming and the benefits of nested Benders' decomposition. It represents a very important expansion of nested Benders' decomposition using two important concepts of cut sharing and sampling. Commercial software based on this algorithm is in widespread use.[2]

In a deregulated setting, spot market prices become important as input to power scheduling models. Assuming the price-taker case, prices are exogenous and can be treated by ordinary linear stochastic programming.[3] Prices are autocorrelated, so the current price carries information about the likely future outcomes of price. Thus it must be treated as a state variable, which posts a problem in SDDP, because the future cost function is no longer convex in all state variables. (As is well known and probably shown in earlier chapters in

[2] The Stanford group under G. B. Dantzig worked out a similar decomposition/sampling algorithm based on importance sampling approximately at the same time (Dantzig, 1989).

[3] Ordinary is meant in contrast to game-theoretic approaches.

this volume, the recourse cost function, or future cost function, is concave in changes to objective function coefficients and convex in changes to right hand side coefficients.) Thus the future cost function can no longer be supported by cuts. This issue is discussed by Gjelsvik and Wallace (1996), who introduce an algorithm that can handle stochastic prices by not sharing cuts across price states. During the course of the algorithm the future cost function (a function of all state variables) is built for each price state at each stage. Pereira et al. (2000) approach the issue of stochastic prices causing nonconvex recourse functions by using a cut classification scheme.

Stochastic unit commitment problems are not yet in daily use, as far as we know, and for algorithmic work on these stochastic integer problems we refer to Gröwe-Kuska et al. (2002) and to other chapters in this volume.

3 Electricity in deregulated markets

This section discusses issues related to electricity production under market conditions. Researchers have studied hydrothermal scheduling, risk management, unit commitment and bidding problems in deregulated market settings. Assumptions on market form, institutional and market design and existence of derivative markets vary.

At the time of writing, electricity markets are still in transition from the old regulated regime, motivating the development of hybrid models where there is both a demand constraint and a wholesale market. The local load is to be met at each instant, but the producer can choose to serve this load by his own production capacity or by buying capacity in the market. The producer may also produce more than the local load, selling the surplus in the market. See f.ex. Takriti et al. (2000) and Gröwe-Kuska et al. (2002). A hybrid approach may also be motivated by "imperfections" causing constraints on how much the company can buy or sell in the wholesale market, or by a significant difference between the price of buying electricity in the spot market and the price of selling to the spot market.

3.1 System-wide models

Some models try to capture aspects of the whole electricity system, having the power price as endogenous variable, i.e., as a result of matching supply and demand. Examples of such models are MARKAL, MPS, which focuses on markets with a large share of hydro power, and BALMOREL. Stochastic programming efforts related to these models are reported by Fragniere and Haurie (1996), Botnen et al. (1992) and Hindsberger (2003, papers F and G). These models serve the needs of utility planners and policy makers in that they can derive scenarios of market prices of electricity. The major advantage of such models is that they capture the specific aspects of electricity and that the scenarios generated are consistent with the assumptions underlying analyses regarding e.g., future system-wide capacity and emission allowance policies. These models are all developed for regulated markets. However, they have become very popular for generating price scenarios in deregulated markets. The reason is that in perfect markets, price will equal (long term) marginal cost. A regulated market, as described in Section 2 of this chapter, is normally based on a policy of efficiency and cost minimization so as to achieve exactly the same result—price equal to long term marginal cost. Care must be taken, however, so that the price-scenario generation does not take the form of pure scenario analysis, that is, a large number of "What-if"-questions on the external events. That would result in prices of electricity being too low, as each path of investments would be done under full knowledge of the future, underestimating the need to invest in (expensive) flexibility. The modern versions of MARKAL, like the one referenced above and Kanudia and Loulou (1998), take this into account.

It is also important to remember the setting here. These models can be used to generate price-scenarios (consistent with external events) for a *small* market participant who does not herself affect the market. A policy-maker can view the price scenarios as the *result* of her actions, but cannot use them to make other policy decisions. That would create a logical loop.

These models could of course also have been discussed in the previous section on regulated markets, as some of them represent long term stochastic investment models in the light of random demand and emission policies.

3.2 The role of futures markets

The electricity markets are developing into regional commodity markets. This can be seen in the contract market where there is decreasing use of complex physical sales contracts and increasing use of standardized financial contracts. As these derivative markets mature, they will serve an important role in risk sharing and in giving economic signals to investment and operations planning.

A common commodity contract is a forward contract, which entitles the buyer of a contract the difference between the spot price and the agreed contract price in the settlement period of the contract. In some markets these contracts are known as contracts for differences (CfDs).

If the commodity can be stored, such as coal, oil and gas, the contract price will be closely related to storage costs and interest rates, due to the arbitrage opportunities that would otherwise be present. If futures prices are higher than current spot prices compounded at the risk free rate plus storage costs, an arbitrageur can buy a unit of the good (at price S_0), finance this with a bank loan, and short sell a futures contract. The storage costs may include opportunity costs associated with the operational benefits of having the commodity immediately available in storage (convenience yield). At the time of maturity, the arbitrageur sells the good (at S_T), pays back the loan

($-e^{rT}S_0$), pays storage costs (C) and settles the futures contract ($F_T - S_T$). The safe future value of this project is

$$FV = S_T - S_0\, e^{rT} - C + F_T - S_T = F_T - S_0 e^{rT} - C.$$

If this value is positive, the futures price F_T is too high compared to the current spot price since the arbitrageur actually can make money on this deal. Clearly, this value must be nonpositive in a reasonable model of price dynamics. Similarly, a speculator holding the commodity in stock may arbitrage on temporarily reducing his storage by selling a unit of the commodity and buying a futures contract. This means that the above future value must be nonnegative, leading to $F_T = S_0\, e^{rT} + C$ for commodities that are stored.

Electricity can to a certain extent be stored as potential energy in reservoirs. Hydroelectric producers are thus in a position to arbitrage between the spot and futures markets using their reservoirs, hydro stations and possibly pumps. If aggregate reservoir capacity is large then such behaviour can be expected to influence the pricing of electricity futures relative to spot. In many power systems, however, the aggregate reservoir capacity is low compared to aggregate system capacity, and the price will be determined by short term equilibrium of supply and demand for the contract. Supply and demand are driven by hedging and speculation, where selling hedgers are the producers and buying hedgers are power marketers and large industry. Speculators take positions on either side depending on their capacity and willingness to take risks and their expectations on the future spot price or the future movement of the contract price.

Regardless of its determination, the contract price represents the current market value of future delivery of the commodity. This is obviously important for investment and operational planning. If an electricity company is considering an investment that will give a certain production capacity in a future time period, the current value of the revenues coming from the use of that capacity is given by multiplying the capacity with the forward price for settlement in the same future period, and discounting to present using the risk free rate of interest. This is a simple valuation procedure that will value production resources in a way that is consistent with how the market prices contracts. Rational decisions based on such valuation will contribute to maximizing the market value of the firm owning the production assets.

Valuation of future production is needed in stochastic programming models in energy. It is what many such SP models are about. These models are based on describing the uncertainty in the form of scenarios of the spot price of the commodity. However, basing the scenarios on forecasts of spot prices will not give a valuation that is consistent with the market. Price scenarios need to be adjusted for risk in order to give consistent valuation;

they must be adjusted so that the values of derivatives as calculated in the scenario tree are the same as can be observed in the market for futures, options and other contracts.[4] Once this adjustment is made, the appropriate discount interest rate to use is the risk free one, and the SP is now in the position to value the decision flexibility using a price of risk that is consistent with the market.

Adjusting for risk is necessary because expected spot prices in future periods are generally different from forward prices for the same future periods. This in turn is due to the limited capacity or willingness of speculators to trade on the mentioned difference, called the *risk premium*.

This quantity, defined more formally as $E[S_T] - F_T$ where $E[S_T]$ is the expected spot price at future time T, and F_T is the current forward price for delivery at time T.[5]

Note that if one makes optimal decisions based on price forecasts (i.e., on $E[S_T]$), the expected profit is maximized. If one makes decisions based on risk adjusted prices (i.e., on F_T), the value of cash flows is maximized. Thus, one cannot have profit maximization and shareholder value maximization at the same time. Only the latter will maximize the value of the firm.

When constructing scenario trees (or more generally, when modeling the stochastic processes involved) we must therefore make sure that the path of the expected spot price in the tree matches that of the term structure of futures prices, and that the path of the standard deviation of price returns in the tree matches the term structure of volatility (which has to be estimated, see Hull (2000)). One should possibly also match higher moments and dynamic properties of commodity prices such as mean reversion. An approach for scenario generation based on matching such statistical properties is described by Høyland and Wallace (2001). Alternatively, one may prefer modeling the stochastic processes as stochastic integrals, i.e., a parametric approach. This would have the advantage of capturing the theoretical developments in the financial commodity pricing literature, as in e.g., Schwartz (1997) and Lucia and Schwartz (2000). In this case, the scenario trees can be built using the approach of Pflug (2001) or by discretizing the continuous stochastic processes directly as in Hull (2000).

Example 1. Let us give a very simple example of how market data can be used to extract useful information for a stochastic program, and in particular to obtain an understanding of the world in which stochastic programs operate.

Assume we are facing an uncertain future price for electricity. Assume that presently, 1 MW delivered in the next period costs 100. In the next period, we know (e.g., by estimation), that the price for immediate delivery will

[4] At least approximately the same. Current market prices will change in the future. See Hull (2000, Chap. 18.6).

[5] This is positive for most commodities most of the time (Hull, 2000).

increase to 125 or decrease to 70. Each of these cases occurs with a true probability of 50%. In the next period, our production will be worth 2000 if prices go up, and 1500 if they go down. Hence, the expected value of our production, using the true probabilities, is 1750. However, we should note that $100 \neq 0.5 \times 125 + 0.5 \times 70$. Let us disregard discounting, and assume that there is a risk free asset that costs 100 now, and pays 100 in any state in the next period. We then have two instruments (price of electricity delivered in the next period and a risk free asset) in a world with two states, and we can set up the equations for the state prices π_1 and π_2.

$$
\begin{aligned}
100 &= 125\pi_1 + 70\pi_2 \\
100 &= 100\pi_1 + 100\pi_2
\end{aligned}
$$

which yields $\pi_1 = 0.5455$ and $\pi_2 = 0.4545$. Hence, the market value of our production equals

$$0.5455 \times 2000 + 0.4545 \times 1500 = 1773,$$

above its expected value using true probabilities. Someone understanding markets better could obtain arbitrage by for example buying our production for above its expected value (according to the stochastic program), say for 1751, which should make us happy, and then sell it in the forward market for its true market value, 1773, to obtain a risk free profit (arbitrage) of 22.

Again, the purpose of this example is to observe a fact about market values. To maximize the market value of our electricity production we need to use risk adjusted probabilities, and not the physically correct ones. Of course, often we do not know the true ones either, but that is not the point here—the point is that the relevant probabilities to look for are the risk adjusted ones. And they are to be found in the market prices of contracts, not in historical spot price data. This also means that if we use the true probabilities in a stochastic program, we shall not be maximizing market value, and hence, we open up our business for speculation based on the true values of risk.

As a theoretical digression, note that transforming a scenario tree, or a stochastic process with an associated probability measure $\mathcal{P}$, for the true or forecasted spot price, to a scenario tree matching the term structure of futures prices and volatility, is equivalent to changing the probability measure into an equivalent martingale measure $\mathcal{Q}$. The existence and uniqueness of such a probability measure can be analyzed via stochastic programming by setting up the problem of hedging a general contingent claim (contract) in the original ($\mathcal{P}$-measure) scenario tree. This has been done by e.g., Ross (1977), Kreps (1979), Naik (1995) and King (2002).

3.3 Energy bidding

The bidding problem can be viewed as a short term optimization problem in which the market participant offers to buy or sell capacity to the market in the form of price-quantity pairs for given time intervals that typically are 30 min or one hour long. A market operator collects such bids and calculates clearing prices and quantities for each node or zone in the network, resulting in a dispatch for the system. The price clearing process aims at maximizing the sum of consumer and producer surplus as implied by the bids, subject to transmission constraints, reserve constraints and possibly other technical constraints. In sending their bids, individual market participants try to maximize profits resulting from the dispatch (thus buyers minimize the cost of buying electricity).

The exact setup regarding market structure and market rules differs from market to market. The Electricity Pool of England and Wales was the first to be established, in 1988, and has served as a model for much of the restructuring worldwide, e.g., in Australia, New Zealand and parts of Latin America and North America. These countries use a centralized dispatch and pricing mechanism, called an electricity pool. The second country to deregulate was Norway in 1991. The Norwegian electricity trade is much more decentralized and its structure has been adopted by the other Nordic countries and in some aspects by California. Participation in the organized markets is voluntary and there is demand-side bidding. This is called a bilateral market.

Some markets have only a few or even only one round of bidding, and after these rounds the generator is assigned a generation schedule for the near future (e.g., for the next 12–36 h). In this situation, after the market operator has announced the dispatch, the traditional unit commitment models that include a demand constraint become relevant again. Furthermore, the bidding problem, i.e., determining optimal bids to send to the market operator, becomes a nontrivial task that can be supported by optimization models. Nowak et al. (2000) study this problem and present an integrated stochastic unit commitment and bidding model.

Neame et al. (1999) consider the bidding problem for a price taker in an electricity pool type market. The bids are required to be in the form of a piecewise constant increasing supply curve, i.e., a set of price-quantity pairs. If the bids could be in any form, price-taking generators maximize their profit by bidding according to the marginal cost of generation. However, since marginal cost is not generally a piecewise constant curve having a finite number of price-quantity pairs, the generator needs to optimize his bid curve. The authors study this problem and finds, among other things, that it is nonconvex. For special cases dynamic programming algorithms for computing the globally optimal bid are presented.

Anderson and Philpott (2002a) consider bidding problems in day-ahead markets for producers having market power under varying assumptions on the allowed smoothness of the bids, and on whether there is uncertainty in demand only or also in the supply functions offered by competing generators. An important vehicle in the analysis is the "market-distribution function" Ψ encapsulating uncertainty in demand and competitor behavior. Let $\Psi(q, p)$ be the probability of not being fully dispatched by the market operator if quantity q is offered at price P. The generator is said to be fully dispatched if the whole offer was knocked down in the auction, i.e., the market operator declares it will use all of the quantity q offered. The other cases are those of not being dispatched, and of being partially dispatched if a fraction of the quantity is cleared. The problem is to find a supply curve $s = ((q(a),\ p(a), 0 \leq a \leq A)$ where $q(a)$ is the quantity the generator is willing to supply at a corresponding price $p(a)$. This curve is assumed to be continuous with $q(\cdot)$ and $p(\cdot)$ non-decreasing in the parameter a. If $C(q)$ is the cost associated with generation of q, the payoff resulting from a dispatch (q, p) is $R(q, p) = qp - C(q)$. If the generator has sold a quantity Q at price f via physical or financial contracts for delivery in the period in question, the payoff is $R(q, p) = qp - C(q) + Q(f - p)$. With a continuous market distribution function the expected payoff becomes a line integral given by

$$V(s) = \int_s R(q, p)\, \mathrm{d}\Psi(q, p),$$

see Anderson and Philpott (2002b). With certain (nonrestrictive) differentiability and monotonicity properties for Ψ, the problem can be formulated with respect to the parameter a as follows:

$$\begin{aligned}
\max \quad & \int_0^A R(q, p)\left[\frac{\frac{\partial\Psi}{\partial q}(q, p)q'(a) + \partial\Psi}{\partial p(q, p)p'(a)}\right]\mathrm{d}a \\
\text{s.t.} \quad & 0 \leq q(a) \leq q_M \\
& 0 \leq p(a) \leq p_M \\
& q'(a) \geq 0 \\
& p'(a) \geq 0
\end{aligned}$$

where q_M is the maximum capacity of the generator and p_M is some upper bound on price. This is a nonlinear optimal control problem, and the authors analyze its properties and various extensions, for example to the case where the generator is required to submit piecewise constant bid curves instead of smooth continuous curves.

3.4 Scheduling in a market

We first assume that this generation utility is not large enough to be able to influence electricity prices by changing the amount of generation capacity offered to the market. The market is liberalized, but not necessarily perfectly competitive.

We discuss how different classical power generation planning problems change in the face of liberalization.

Significant changes are necessary in traditional long and mid-term power scheduling, unit commitment and economic dispatch. Under the price taker assumption and that the utility does not have to worry about the transmission constraints, either because transmission is not the utility's responsibility or because there is sufficient capacity in the transmission grid, these changes affect both planning objectives and constraints.

First, the objective of the planning models should now be to maximize utility profits instead of minimizing overall system costs. The revenues are (hopefully) greater than the generation costs. From an optimization point of view, this may not amount to more than multiplying the objective function by -1 and maximize instead of minimize, but for the management focus the change is more profound.

Second, the demand constraint in these models becomes superfluous (except possibly in the very short run). Since utilities no longer have an obligation to serve demand by using only own generation resources, they now can use the spot and contract markets (i.e., other companies' resources) to meet customer obligations.

Third, reserve constraints, as used in unit commitment, also become unimportant for the utility. This happens because spinning reserve and other ancillary services become the responsibility of the system operator rather than the utilities collectively, or because well-functioning markets for different levels of reserve develop.

To see why the demand constraint becomes superfluous, consider the following problem, where e_t is the net sale (selling minus buying) in the spot market and π_t is the spot market price:

$$\min_{u,p,v} \sum_{n\in\mathcal{N}} P_n \left\{ (-\pi_n e_n) + \sum_{i\in\mathcal{I}} FC_i(p_{in}, u_{in}) + SC_{in}(u_i) \right\} - \sum_{s\in\mathcal{S}} P_s V(l_s) \tag{3.1}$$

$$\text{s.t. } \forall\, n \in \mathcal{N}: \sum_{i\in\mathcal{I}} p_{in} + \sum_{j\in\mathcal{J}} v_{jn} - e_n \geq \delta_n. \tag{3.2}$$

This formulation assumes that the cost of buying is the same as the income of selling the same energy volume. With a significant difference between purchase price and sale price, the argument becomes invalid. Due to the presence of operating ranges $[\underline{p}_i, \overline{p}_i]$ for each unit, the demand constraint (3.2) may not be satisfied as an equality in an optimal solution, as one might expect from cost minimization. However, this rarely occurs in practice and we ignore this possibility. If there are no binding constraints on it, the net sale variable e_n and (3.2) can be substituted out to give the following model:

$$\begin{aligned}\min_{u,p,v} \sum_{n\in\mathcal{N}} P_n \sum_{i\in\mathcal{I}} FC_i(p_{in}, u_{in}) + SC_{in}(u_i) - \sum_{s\in\mathcal{S}} P_s V(l_s) \\ - \pi_n\left(\sum_{i\in\mathcal{I}} p_{in} + \sum_{j\in\mathcal{J}} v_{jn} - \delta_n\right)\end{aligned} \tag{3.3}$$

This is a model that is decomposable; one can solve for each thermal unit and for each group of hydrologically coupled hydro units independently.

The total implication for the models is that all or most constraints coupling the different generating units should be removed as the deregulation process is moving forward. The management is left with a set of decoupled subproblems for power operation planning, one for each unit or plant, instead of one big problem with the plants depending on each other to cover demand and spinning reserve.

In a liberalized market, the following tasks are most important for a generation utility: Risk management, hydro scheduling, unit commitment and bidding in the organized markets. In addition, short- and long term market analyses are important. Forecasting the future development of prices and other uncertain factors from now to several months or years into the future is important for trading and risk management. Short term forecasting of prices, loads and inflows is important for short term operational planning.

Hydro scheduling

Next, we present hydro scheduling. Gjelsvik and Wallace (1996), Fosso et al. (1999), Pereira et al. (2000) study hydro scheduling assuming perfect competition. For simplicity, we show a model with a single reservoir. For cascaded reservoirs, a multi-reservoir formulation is warranted. Since hydro plants are independent of each other under our assumptions, we omit the index j.

The release decisions for period t are taken after learning the realization of the stochastic variables for that period.

Decision variables and parameters for hydro scheduling are measured in energy units. The problem can be formulated as:

$$\max \sum_{n\in\mathcal{N}} ((1+\gamma)^{-N_{t(n)}} P_n \pi_n v_n) + (1+\gamma)^{-N_T} \sum_{s\in\mathcal{S}} P_s V(l_s) \tag{3.4}$$

$$\text{s.t.} \quad \forall\, n \in \mathcal{N}: l_n - l_{a(n)} + v_n + r_n =_n , \tag{3.5}$$

$$\forall\, n \in \mathcal{N}: \underline{l}_{t(n)} \leq l_n \leq \bar{l}_{t(n)}, \tag{3.6}$$

$$\forall\, n \in : \underline{v}_{t(n)} \leq v_n \leq \bar{v}_{t(n)}, \tag{3.7}$$

where $\underline{l}_{t(n)}$, $\bar{l}_{t(n)}$, $\underline{v}_{t(n)}$ and $\bar{v}_{t(n)}$ are lower and upper bounding parameters for reservoir level and discharge, and spill $r_n \geq 0$. Equation (3.5) is the energy balance in the reservoir, and (3.6) and (3.7) impose lower and upper bounds on reservoir level and discharge.

Using deterministic models for hydro scheduling in a market setting will lead to operating policies that essentially allocates the water to the periods with the highest prices. As in the case without markets, the spilling risk will be underestimated and true profit will be overestimated. There will be no extra release in the fall in case of extra inflow at near maximum reservoir levels, and no holding back water before the spring flood in case snow melting starts late and prices skyrocket.

Market power

Operations scheduling in deregulated markets when the operator has market power is discussed by Scott and Read (1996). Their focus is on imperfect competition due to the low number of suppliers in New Zealand. In particular, they develop a hydro scheduling model for a Cournot-type producer having the contract position as exogenously given. A multistage stochastic programming algorithm is developed to solve the optimization problem with a Cournot market equilibrium superimposed on it at each stage. A similar study by Kelman et al. (2001) reach the same conclusions as Scott and Read, namely that the more contracts the strategic generators have sold, the less incentive they have to withhold capacity in order to increase prices. A major limitation in these analyses is that buying and selling of contracts is in reality determined simultaneously with production. The players are also limited in the degree to which they can dynamically anticipate and react to opponents' strategies.

Unit commitment

Thermal unit commitment for price takers can be formulated as follows (index i omitted):

$$\max_{u,p} \sum_{n \in \mathcal{N}} (P_n \pi_n p_n - FC_n(p_n, u_n) - SC_n(u)) \tag{3.8}$$

$$\forall\, n \in \mathcal{N}: u_n \underline{p} \leq p_n \leq u_n \bar{p}. \tag{3.9}$$

Further, single-unit constraints are minimum up- and down-times and additional must-on/off constraints as explained in Section 2.

The decomposition that the liberalization induces should have profound implications for the organization of the utilities: Now each plant manager can be given responsibility for operating as she thinks is best. She can and should be supported by planning models that now sensibly only includes *local* generating units. Tseng and Barz (2002) consider such stochastic single-unit commitment problems.

In summary, we propose that generation utilities comprehensively revise their generation planning models. New models should include (stochastic) prices instead of using demand constraints and spinning reserve constraints. The problems become much easier to solve, thanks to the decoupling effects of the new markets.

3.5 Risk management

Basic financial theory implies that it is not necessary to hedge at the corporate level, since investors can do that on their own account. In practice, however, there are "market imperfections" that make the case for risk management, for example the fact that it is cheaper for a firm to operate in the power derivatives markets than for individual owners, due to the economy of scale in the risk management function.

Example 2. Let us illustrate the use of financial instruments on risk management to see, in a very simple world, how the instruments can change the risk picture. In Fig. 7, the first figure shows the distribution of profits from one unit of production without any financial contracts. Assume next that we sell 50% of our production in the forward market at the expected price of 100. That results in a new distribution of profits, given in the right-hand part of Fig. 7. The risk has clearly decreased (even if we are not very specific about what we mean by risk).

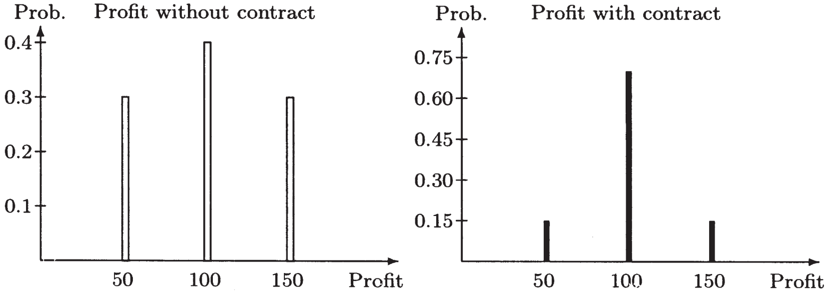

Fig. 7. Distribution of profit without and with a forward contract. The horizontal axis shows profit for one unit of production, the vertical axis probabilities.

Table 3.1
Production, prices and probabilities with and without a forward contract

Production	50	150	50	150
Price	10	10	20	20
Probability	10%	40%	40%	10%
Profit without contract	500	1500	1000	3000
Profit with contract	1000	2000	500	2500

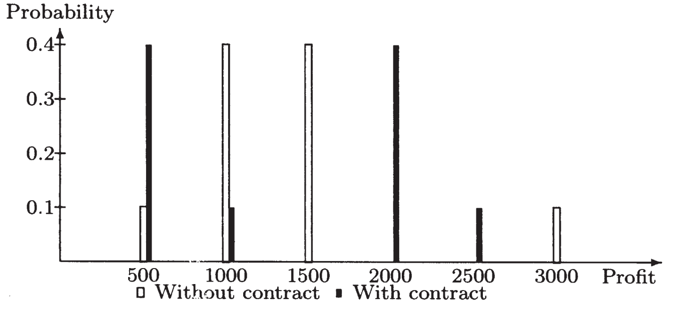

Fig. 8. Distribution of profit without and with a forward contract. The horizontal axis shows total profit, the vertical axis probabilities.

This example indicates that trading in the forward market will reduce the risk. But this may not be the case. Assume that we are facing uncertain production and uncertain prices, as outlined in Table 3.1. The most likely situations (each having 40% probability) is low production and high prices (low inflow) or high production and low prices (high inflow). But there are two other cases, representing the possibility that while we have high inflow to our reservoirs, the general picture is the reverse. Hence, there is a chance of seeing low prices and low production at the same time. The same goes for high production and high prices.

Consider the illustration in Fig. 8. The white set of columns shows the profit without any financial contracts. Assume next that we sell 100 units of production (the expected production) in the forward market at a price of 15 (the expected price). The last row in Table 3.1 shows the resulting profit. Each number is the sum of the income from the forward contract (1500) and sales or purchases in the “spot” market for what is left or what is missing relative to our forward contract. The result is the distribution in black in Fig. 8. We see that the variance has increased, and that most measures of risk will show the same. In this case, selling the expected production at expected prices increased the risk.

The purpose of these two examples is simply to illustrate the use of financial instruments for risk management, and a warning that using these markets to fix income in the future will not automatically mean reduced risks.

Mo et al. (2001) and Fleten et al. (2002) suggest that production planning and contract risk management should be integrated in order to maximize expected profit at some acceptable level of risk. However, in some circumstances (no production uncertainty or basis risk) production planning can be done independently from hedging (separation). So then it is possible to have a relatively decentralized organization, with local plant managers having much responsibility, and a centralized treasury department in charge of overall risk management. The main tasks of such a department are to speculate and hedge using derivatives in order to satisfy the goals of owners and top management regarding expected profit and risk. Of course, this requires that the relevant attitude toward risk must be expressed.

The requirements needed to invoke the separation theorem are not likely to be met 100% in practice. However, the benefits of a decoupled set of models and corresponding decentralized organizational units will probably outweigh the small theoretical gain from integrating production planning and trading.

Another argument for separating risk management is as follows: From financial theory we know that the market value of any financial contract is zero at the time it is entered into. This also holds for electricity contracts that are fairly priced, and consequently, buying a new contract will not change the market value of the electricity portfolio in question. In particular, buying and selling a range of contracts that jointly are selected in order to minimize the risk of a given electricity portfolio, will not alter the market value of that portfolio. However, operational decisions *do* change the market value of the electricity portfolio, and so generation should be allocated in order to achieve maximal market value. Any production decision that deviates from the value-maximal strategy will erode value for the owners of the generation utility. Consequently, a natural setup for the coordination of generation planning and risk management is to first schedule generation so that market value is maximized. Second, given this optimal strategy, find a set of contracts (or a trading strategy) that will minimize the risk of the total portfolio.

We model the risk management problem as follows: Given calculated (optimal) profit from hydro and thermal generation in each state in the scenario tree, summed over all plants, dynamically trade in futures and options in order to minimize some risk measure.

Let Π_t be the stochastic profit estimated from all generation activities in period t. This information must be extracted from the optimal objective function value of hydro and thermal sub-models (3.4). The scenario tree used for these sub-models is assumed to be identical to the one used for risk management.

Trading in forward contracts is modeled by the variables f_{kn}, g_{kn} and h_{kn}. Let f_{kn}, $k = 2, \ldots, T$, $n \in \{\mathcal{N} : t(n) < k\}$ be the position, measured in energy units, in state n for a contract with delivery in period k. Negative f_{kn} represent a short position in product k. Buying and selling forward contracts are represented by g_{kn} and h_{kn} (both nonnegative). Contract prices are denoted φ_{kn}, and markets are infinitely liquid and perfectly competitive.

The position accumulated in state n is

$$f_{kn} = f_{k,a(n)} + g_{kn} - h_{kn}, \tag{3.10}$$

with the initial forward position given. Contract variables and rebalancing constraints (3.10) are only defined for relevant states satisfying $t(n) < k$.

Rebalancing decisions are made in each state n, after the realizations of the random electricity prices for period $t(n)$ are known. Transaction costs are proportional to the trade volume and is T_F per unit energy bought or sold.

European-type option contracts can also be included. To conserve space, the involvement of options in rebalancing, profit measurement and objective is not shown (see Fleten (2000) for models including options).

Modeling of risk depends on the attitude toward risk in the generation utility. A simple approach that leads to a piecewise linear model is to minimize expected shortfall (Kusy and Ziemba, 1986). Shortfall is defined as profit underperformance relative to some preset profit targets at various periods. Let Π_n^{tot} be the profit to be measured. The exact definition of this profit depends on how the generation utility defines risk. A possible definition is:

$$\begin{aligned} \forall\, n \in \mathcal{N}: \Pi_n^{\text{tot}} &= \Pi_n + \pi_n f_{t,a(n)} \\ &+ \sum_{k<t(n)} [(\varphi_{tn} - T_F)h_{kn} - (\varphi_{kn} + T_F)g_{kn}], \end{aligned} \tag{3.11}$$

where $f_{t,a(n)}$ is the forward position in the product that has delivery in period t, during the actual delivery period.

Let C_{mt} be the marginal shortfall cost in segment (piece) m and let s_{nm} be shortfall. The following constraint defines shortfall variables:

$$\Pi_t^{\text{tot}} + \sum_m s_{mn} \geq B_t, \tag{3.12}$$

for all states n for which there is a profit target B_t.

Let W be a weight parameter. In order to avoid incurring excess transaction costs, the objective function maximizes expected profit minus the weight times expected shortfall:

$$\max_{f,g,h} \sum_{n\in\mathcal{N}} P_n(1+r)^{-N_{t(n)}} \left[\Pi_n^{\text{tot}} - W \sum_m C_{mt} s_{mn} \right]. \tag{3.13}$$

This model does not treat physical and financial forward-type contracts differently. The reason is that with the assumptions we have made, a financial contract is a perfect substitute for a physical contract. It generates the exact same cash flow. Some generation utilities in newly liberalized markets have physical bilateral sales contracts that have a minimum energy volume that is very large and whose tariff structure is complex. The market for such wholesale consumption contracts will quickly become competitive, since small power marketers can sell such contracts and cover the liability in the spot and financial markets. The integrality of these contracts (large minimum volume) will not be a problem either, since one can always add or delete energy volume by buying or selling additional (physical or financial) contracts. The decision support tool needed for such bilateral sales contracts is thus not only a portfolio optimization model, but also a good model for pricing the specialties (e.g., embedded physical load risk) of the individual contracts. See e.g., Thompson (1995) for such an approach applied to take-or-pay contracts.

3.6 Capacity expansion

In a deregulated and well-functioning market, capacity expansion decisions should be analyzed in view of their profit and market value adding potential, and not their ability to serve growing demand at minimum cost. As such, future electricity prices, as opposed to demand, is the central object of analysis. A lot of work remains to be done on this arena, but as a starting point the readers are referred to Deng and Oren (2001), who analyze an investment in a gas-fired power plant using a stochastic dynamic programming model that includes startup costs, operating-dependent efficiency and ramping constraints.

4 Oil

4.1 Optimal field development

Haugland et al. (1988) discuss an optimization model for an oil field based on a two-dimensional reservoir model of the same type that is used in reservoir simulations (but of course much simpler). The goal is to determine platform capacity, the number of wells (and their placement and timing), plus the production profile of each well. This way of using the reservoir simulation equations within an optimization model provides a setting that spans two different fields of research. This is useful both for quality and acceptance.

But since the model is deterministic, all aspects of flexibility are gone, including the postponement of decisions. Jonsbråten (1998a) adds one type of stochasticity to these models by assuming that future oil prices are random. He describes them using scenarios. He then solves the resulting stochastic mixed integer program with scenario aggregation on the continuous variables and a heuristic for finding feasible integer solutions. He observes what is expected, namely that as soon as stochasticity is introduced, timing of decisions, in order to take into account accumulation of information, becomes important.

It is clearly possible to expand this type of models to include other types of randomness. However, we should be aware that gathering of information about the reservoir (over time) will depend on the actual decisions made. Stochastic programming for such cases is barely developed. An initial discussion can be found in Jonsbråten's doctoral thesis (Jonsbråten, 1998b).

4.2 Scheduling arrivals of tankers at a refinery

This problem originates from Bjørstad et al. (1991), and is interesting as its randomness is different from what we have seen elsewhere. A refinery is about to receive a large ship for loading of gasoline for export. For simplicity, we shall assume that gasoline is characterized by two qualities, namely sulphur content (the lower, the better) and octane number (the higher, the better). In reality, there are many other properties, but this is enough for our example. For the arriving ship, it is known how much gasoline it needs, and there are given a minimal value for octane number and a maximal value for sulphur content. At a refinery, gasoline is not stored as final products, but rather as intermediate components, such as propane, butane etc. These are the results of the refining process, and are stored in tanks (with limited capacity). To fulfill an export order, one mixes components from the different tanks, to achieve a product with the desired properties. It is not always possible to achieve exactly the boundary values of the qualities, and in such a case it is a goal to give away as little extra quality as possible. Clearly, if one gives away very little extra quality in one shipment, one may be left in a situation in terms of stored components, such that for later shipments the quality giveaway is very high. Hence, one needs to have a somewhat long view on the production.

For many refineries, the arrival time of the exporting ships is uncertain. This is caused mainly by bad weather, but other reasons are of course possible as well. Hence, although both production of components, and requirements (quantities and qualities) relating to arriving vessels may be known for some periods into the future, the very fact that their arrival times are unknown will cause some concern. Problems may occur both with respect to production (full tanks because no ship arrived), and the mixing of gasoline for a specific ship, since production continuously change the qualities of the contents in the tanks. Even more severe effects occur if ships arrive in an unexpected order.

Assume we look four periods into the future, and that we know that during those periods three ships will arrive. The model is run when a ship arrives, so ship 1 is known to arrive in the first period. Figure 9 shows the six possible arrival sequences, with given estimated probabilities.

The scenario representing what we expect to happen is scenario 2, where ship A arrives in period 2, and ship B in period 4. This has a probability of 60%.

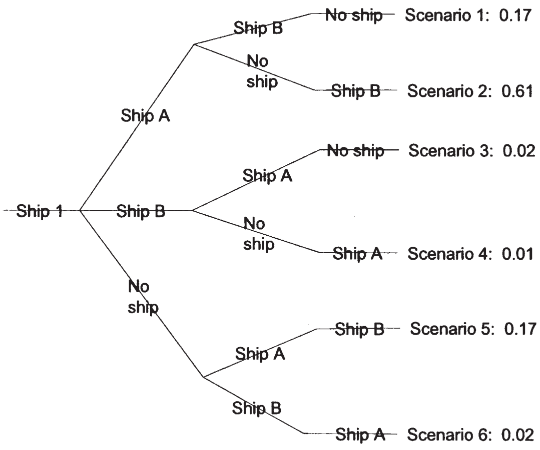

Fig. 9. The possible arriving sequence for the three ships over four periods, with given probabilities.

This *sequence* is almost certain, as it is only 5% chance that B arrives before A. So in a deterministic model, we would clearly use scenario 2.

But let us describe the problem in some detail. When a continous model is made discrete, as it is here, there is always the need to make assumptions about the order in which things happen. The assumptions here are:

- No two ships arrive in the same period.
- Gasoline for export is subtracted from the tanks *before* the production of the period is added.
- The periods are long enough to finish loading a ship.

In this example, it turns out that if we solve the problem corresponding to the most likely scenario, we may end up with all sorts of problems later on. For some scenarios we shall experience serious problems with high giveaways, tanks that fill up (resulting in stopped production) and orders that cannot be fulfilled. The optimal solution to the stochastic program takes the future appropriately into account and avoids these problems. In this case, the optimal solution is a scenario solution. But as always, this can be determined only by solving the stochastic program.

4.3 Refinery planning

In addition to the problem outlined above about the arrival of tankers to a refinery, there will always be interesting problems related to the refining process itself. Examples of short-term decisions are what qualities to produce and what tanks to use, medium-term decisions concern which crude oils to buy when, and of course there are long term investment problems. An example of a model in this area is Escudero et al. (1999).

5 Gas

5.1 Scheduling of gas fields

Haugen (1993) discusses the following question from the North Sea. Gas was at the time mostly sold on long term contracts. The income of the producer depended to a large extent on his ability to meet the contracted volumes. The market was connected to the offshore gas fields by pipelines. Some of the fields and pipelines already existed, but new ones had to be developed to satisfy the demands (i.e., the contracts). A stochastic dynamic programming model was set up to decide which fields should be developed when, and which pipelines should be constructed when. Although, as the author points out, many aspects of such a problem are random, this paper focuses on resource uncertainty. The uncertainty is described by defining a production profile, consistent with how that is normally done in the industry, and then letting the time at peak production be stochastic. The size of the peak production is a design variable (production capacity of the platform), while the time spent there is a function of field properties, and hence, random. Some small examples are given. The main result, apart from the model itself, is the fact that the author is not able to extract simple decision rules. This is not a negative result, but shows that the problem is inherently difficult, and that care must be taken (in the real world) when arguments are made on how to develop such fields and infrastructure. Simple arguments are very likely to be false.

5.2 Use of gas storage

In light of the nature of stochastic programs, storage will always be important. Storage of gas will be a way to solve many different problems of flexibility. To mention but a few:

- A gas producer has an obligation to deliver certain amounts of gas at certain points in the network at certain times. He is aware that at times there are interruptions in his production or transportation systems. By having storage facilities near the delivery points, he can reduce the chance of failing to deliver.
- At certain points in the network spot markets for gas exist. For some producers it is hard to take part in such markets, as it may take them several days from a decision about increased production is made until the gas actually reaches the point of spot delivery. A storage facility near the spot market will make it possible to take part in a potentially profitable spot market.
- A local distribution company may have as its sole goal to supply its customers according to their (random) demand at lowest possible cost. In this case storage can both help buy gas at times when it

is cheap, as well as supply gas in periods of high demand (typically cold periods) where there may be problems of delivery (in addition to high costs). The problems may be caused both by lack of available gas (limited production capacity) and lack of transportation capacity.
- Utilities producing electricity from gas will have very similar problems as above. They can save money as well as secure supply of gas by having a storage facility.

The storage facility will create both strategic and operational decisions. The strategic decisions, which normally are the ones interesting from a stochastic programming point of view, can be such as:

- Building a storage facility—in many ways a classical facility location problem.
- Renting (part of) a storage facility. If there are several possibilities, this is also a kind of facility location problem.
- Investing in equipment determining the speed by which gas can be put into and removed from storage—by some called deliverability.

There are also more indirect strategic decisions, such as changing the production capacity of a gas field (changing the number of wells, for example) to take into account the value of being able to add gas to the storage at an increased rate also in periods of high production with direct delivery.

These strategic decisions can show up in many stages of a model. For example, rental of storage capacity can be updated at times, new contracts can be entered into, old ones continued or dropped. This way, storage rental turns into a portfolio problem, where characteristics are geography, deliverability and size. As always, we should expect that the more flexible is a certain storage, the more it costs to build or rent.

Operational decisions are more obvious. In combination with purchases, production or delivery, whichever is the relevant trade, we must optimally use the storage facilities to maximize profit or minimize costs, whatever is the objective.

Useful references here are Butler and Dyer (1999), Bopp et al. (1996) and Takriti et al. (2001).

5.3 Portfolio management of gas contracts

Whether we are selling or buying natural gas, the chance is that we need to buy or sell the gas on contracts of different types. These may vary in price and duration. The price difference may stem from differences in forward prices, such that gas on a one-year contract may cost more or less than gas on a two-year contact. But the differences may also stem from how the gas price depends on other entities, such as oil price, or by special rules on renegotiations of contract details.

In such a picture, we are faced with a portfolio management problem. The goal may be to buy or sell gas so as to obtain an optimal tradeoff between expected profit and some measure of risk. Haurie et al. (1992) discuss this problem for a Canadian producer. The different gas contracts have different time spans and different rules for how prices are set. They have many different models. The first is in the spirit of the Markowitz' mean-variance model, the last is a stochastic program with recourse. Risk is measured in terms of the variance of profits.

6 Conclusion

The purpose of this chapter has been to give an introduction to the use of stochastic programming in energy. Based on the available literature, the focus has naturally been on electricity production, but we have tried to provide some pointers also for natural gas (particularly the treatment of contracts) and oil. The purpose has not been to have a full overview over the literature, but to provide the reader with pointers to interesting problems and starting points for reading.

Stochastic programming used in regulated markets, that is, in monopolies, is a well-established activity. The first articles go far back, and the literature is enormous. Articles typically mix discussions of models and methods, and very often the chosen methodology is stochastic dynamic programming (SDP). We have chosen to base our presentation on models rather than methods, so as to avoid a split of papers into two arbitrary piles; those that use stochastic programming (as understood in this handbook) and those that use SDP. For regulated markets, as that is such a well established field, and since methods and models are almost always mixed, we have chosen to discuss also methodology in that section. For deregulated markets, on the other hand, we have chosen to focus very little on methodology, simply assuming that the reader will use the rest of this handbook to look for appropriate methodology. Instead, we have tried to focus on what the new markets may bring us, and tried to point to relevant theory outside stochastic programming, in particular market theory and options theory. The deregulated markets have not found their final forms, so it is impossible to provide the reader with clear-cut descriptions of where we will end up. Hence, our goal has been to assist and present ideas.

Many problems in resource management concern situations where our decisions will change the (conditional) probability distributions. Drilling exploration wells in an oil or gas field is a good example. As stochastic programming, as it stands today, cannot treat this case in any good way, we have chosen to let those problems rest, and mostly focused on problems where the uncertainty is external to the model at hand.

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