Friction and adhesion between corn stover particles

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Abstract

Corn stover interactions involving cob fractions were found to contain the largest peaks and variance in friction coefficients (Table 3). Cob particles with woody-ring sub-fractions on the contacting surface were observed to be significantly harder and ’claw-like’ in nature. Whereas the pith region of the cob was very similar to the exposed pith in stalk particles. Particles of stalk and husk both showed moderate friction coefficients.

From the observation that particles of corn cob show a larger standard deviation in the determined coefficient of friction, it can be hypothesized that particles of corn cob will have a high contribution to an erratic feedstock flow behavior. It can also be hypothesized that the cob will generate high frictional forces considering the high cohesive forces and moderate friction coefficients observed in a corn cob. The experimental data suggest that high-stress feedstock handling applications may be more stable with the removal of corn cob.

The anatomical fractions discussed in this paper were categorized using macro-observations for the purpose of potential mechanical separation. Under further review, it appears each corn stover fraction contained an associated set of sub-fractions that could be found in at least one other fraction. Sub-fractions could be generally characterized as woody, pithy, or leafy. Additional steps in comminuting particles to expose ’sub-fractions’ may allow fluid separation techniques to create more uniform and, therefore, desirable products. It is recommended to repeat this experiment using two additional comminution methods (i.e., knife mill and hammer mill) practiced in the industry to study varying effects of size reduction equipment on friction coefficients.

# Introduction

Department of Energy’s Bioenergy Technology Office (BETO) considers corn stover a key candidate as a biomass source for its abundance and current use in integrated biorefineries (IBR) (Doyle 2014; USDOE 2011; Langholtz, Stokes, and Eaton 2016; Perlack et al. 2011). Corn stover refers to the remaining corn plant fractions (i.e., stalk, husk, and cob) after the lower stalk is cut away and the grain has been harvested. While many want to bring the advantages of new biofuel technologies to industrial production, there is a significant financial risk due to frequent misrepresentation of physical and chemical variabilities in effective lignocellulosic biomass handling and feeding (Crawford et al. 2016; Ray et al. 2020; Ray et al. 2017). In the most recent 2016 evaluation, total biofuel production only reached 7% of the expected 58 billion gallons per year design capacity (Westover and Hartley 2018). The challenges in the handling of biomass are one of the crucial impediments to adopting biomass as feedstocks for biofuel production.

BETO (2016) reports that flowability challenges are mainly associated with biomass density, particle size distribution, moisture content, angle of repose, shear stress, bridging tendency, cohesive strength, and friction of equipment surfaces (Westover and Hartley 2018). The variations in biomass factors such as shape, size, moisture content, and low bulk density are considered to make it challenging to handle and transport in its original form (Kaliyan et al. 2009).

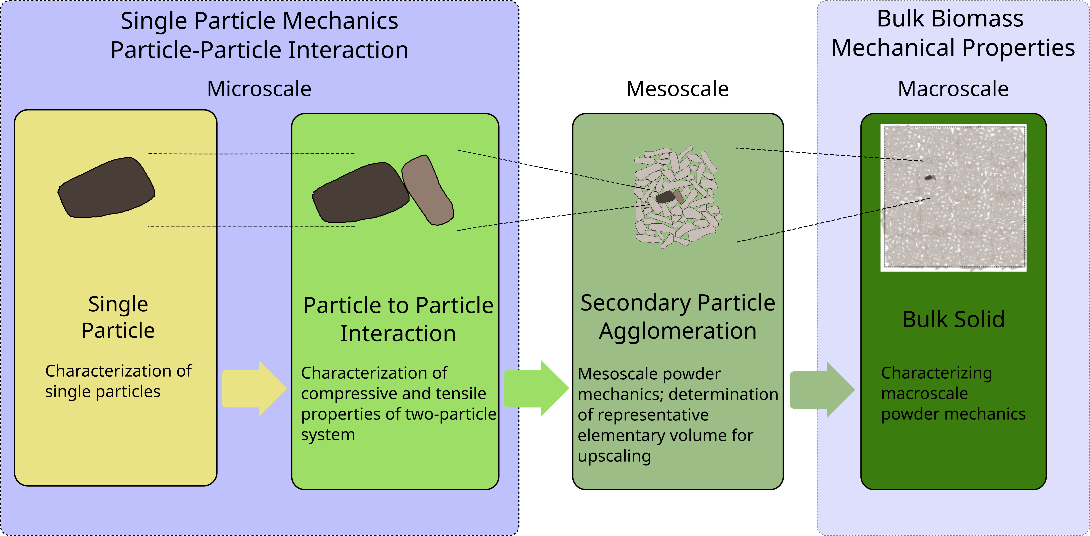
The root cause of such variations in bulk behavior of milled biomass originates from the multiscale-nature as depicted in the Figure [1](#fig:org703b856). This means in order to develop a first-principle based understanding of bulk biomass behavior, quantitative knowledge of interactions between particles is necessary. ~~With such knowledge, one can investigate and develop a predictive model of the mechanical behavior of bulk biomass.~~ One of earlier mention on the friction coefficient between particle and bulk behavior was from Raynolds (1885), where he mentioned that “we cannot assume that tan*φ* has any relation to the actual friction between the molecules,” where *φ* is the angle of repose. In later development of the bulk mechanism of particulate system, this concept was further developed into the Mohr-Coulomb’s law, where the two material properties are named as ‘coefficient of cohesion’ and ‘internal angle of friction.’ While it is widely accepted that the angle of repose coincides with the angle of repose, it is difficult to find an experimental study corroborating this concept. One clear concept is that mechanical interactions between particles will give rise to the frictional resistance to the failure of bulk materials. This study aims to taking a step toward to bridging this knowledge gap between bulk mechanical behavior and inter-particle mechanical behavior at the underlying scale.

Figure Schematic of the emerging bulk mechanical properties from the mechanical interactions between biomass particles to the bulk mechanical behavior of milled biomass

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* Janssen’s equation (Janssen 1895) and stresses on silo wall(Schulze and Schwedes 1994)
* Still not quantified Reynolds’ question on the relationship between the angle of repose and friction between particles (Reynolds 1885)
* Recent studies with DEM still relying on fitted parameters that are not determined with particles (Xia et al. 2023; F. Chen et al. 2023, 2022)

Understanding interparticle mechanics associated with each corn stover fraction will provide valuable information for understanding the overall poor flow behavior of milled corn stover. More specifically, such quantitative information will help identify certain fractions that may be significantly responsible for the poor flowability behavior of comingled corn stover fractions.

There exists a ASTM standards concerning the measuring friction coefficient including ASTM G115-10 (2018). However, listed methods in ASTM G115-10 (2018) assumes a specific geometry, e.g., flat surface or sphere, and not feasible to implement for testing milled biomass particles, which are much smaller in size and irregular in shape. Also test protocols reflect repeated sliding in cyclic motion or rotation, which is implemented in a typical tribometer. This also reflects specific industrial applications such as ball bearing or sliding frames, which are not applicable to the interactions of biomass particles relevant to the bulk biomass flow.

* In other words, the current friction measurements focus on interactions between sliding surfaces, e.g., bearing. This focus is also reflected in the DEM framework, requiring sliding and rolling friction coefficient. From the perspective of the handling bulk biomass, or any other cohesive particulate materials, the friction coefficient of incipient relative motion between particles is of interest.

Therefore, we developed an in-house instrument and protocol that can determine mechanical interaction between biomass particles following the Derjaguin’s adhesive friction model (Gao et al. 2004; Derjaguin 1934; Derjaguin, Muller, and Toporov 1975).

This study aims to gain quantitative knowledge of the interparticle mechanics of fractionated corn stover through the development of an interparticle mechanics tester capable of accommodating biomass particles and a test protocol to determine friction and adhesion properties. With the novel data on interparticle mechanical properties of corn stover particles from different anatomical origins, this study aims to examine the significance of the difference in friction coefficients between corn stover particles from different anatomical fractions. The knowledge gained by this study will contribute to reducing operational challenges by removing fractions with high friction and traction adhesion contributions.

## Other notable quotes

page 121: Variability of biomass exceeds handling machinery design specification: This paper also mentions the start-up or continuous running efficiency

# Materials and Methods

This section details the method used to determine friction coefficients between anatomical fractions of corn stover.



Figure Sample images of individual corn stover fractions used in the present study. 4mm cob Crumbles® (top left), 4mm husk Crumbles® (top right), 4mm stalk Crumbles® (bottom left), and 4mm leaf Crumbles® (bottom right)

## Material Preprocessing History

A square bale of field-dried corn stover from Antares Corn Stover (Hardin, IO) was processed by Forest Concepts, LLC (Auburn, WA). The bale was hand separated and placed into labeled bins designated for each anatomical fraction. Individual fractions were then comminuted to Crumbles® using Forest Concepts’ Crumbler® Rotary Shear technology and conveyed into a final screening process. The screening was performed using a 3-deck orbital screen to filter out fractions with geometric mean diameter outside the 4mm target.

The physical properties of particles of different tissue types of corn stover indicate that the crumbled particles are in consistent size (Table [1](#tab:org9975225)). However, the bulk density values are significantly different from each other (Kruskal-Wallis test, p = 5e-13 < 0.05).

Table Physical properties of corn stover particles of different tissue type (Forest Concepts, LLC analytics laboratory in Auburn, WA) following (ANSI/ASAE S424.1 1992).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Anatomical Fraction | Moisture Content | Geometric Mean  Particle Size D50 | Standard Deviation of Particle Size | Loose Bulk Density  (oven-dried) | Tapped Bulk Density  (oven-dried) |
|  | (% w.b.) | (mm) | (mm) | (kg/m3) | (kg/m3) |
| Cob | 9.7 | 6.6 | 1.6 | 179 | 217 |
| Husk | 9.7 | 4.6 | 1.6 | 48 | 66 |
| Leaf |  |  |  |  |  |
| Stalk | 9.7 | 4.5 | 1.8 | 76 | 98 |

## Inter-particle Mechanics (IPM) Tester

The existing tribometer, similar to the instrument used by Chen et al.(Chen, Wassgren, and Ambrose 2020), can be used in determining friction coefficient. However, an additional attachment is needed because of the dimension of typical milled biomass particles. In addition, the cyclic reciprocating movement does not simulate the frictional interaction between biomass particles during handling. Furthermore, this arrangement is not adequate to determine adhesive friction following Derjaguin’s law (Derjaguin 1934; Derjaguin and Toporov 1994).

The development of a new Inter-particle Mechanics (IPM) tester was needed (Figure 3) to measure the traction force between a stationary and moving particle under a varying magnitude of normal load. This arrangement allows for one test run to produce multiple normal force and lateral force measurements and alleviates the requirement to conduct separate tests with different normal forces in a conventional friction tester (Pitenis, Dowson, and Gregory Sawyer 2014; ASTM G115-10 2018). The development of the new design was essential for simulating any possible orientation of two interacting particles. This design allows friction coefficients to be determined over multiple untested regions without resampling surfaces or replacing test particles between each repetition. With this design, it is also possible to prepare and condition multiple test samples, e.g., conduct experiments in an environment-controlled chamber at a different environmental relative humidity.

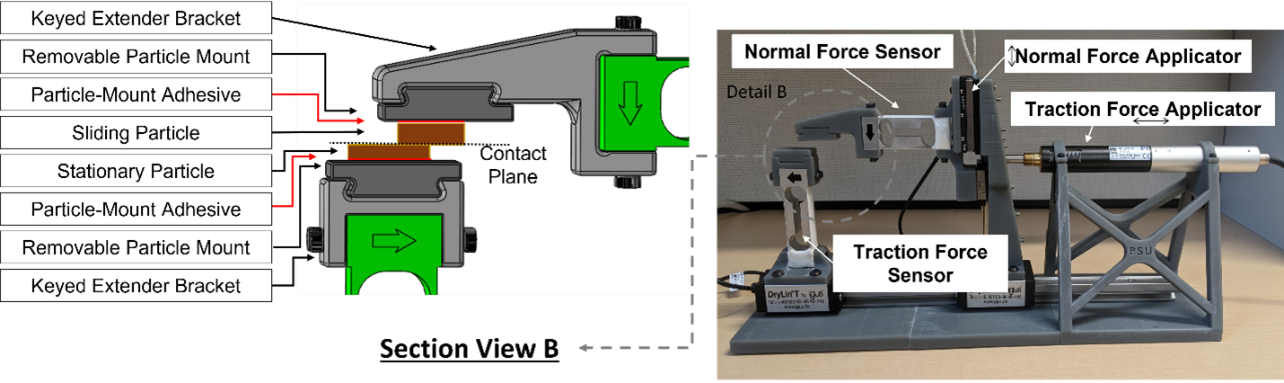


Figure Inter-particle mechanics tester (right) with a detailed view of sample mounting (left)

## Specifications and Operational Parameters of IPMT

An IPM tester was developed to perform friction tests on fully supported particles, unsupported (over-hanging), or combinations of both. Sample holders and component position/capacity are easily adjustable to handle particles of any size and shape. Considering the need to characterize the frictional contact with the perspective of storage and incipient flow, IPMT was designed to have a minimum incremental push and pull motion of 0.05 μm, and a typical push and pull velocity is set to 1.2 mm/s. To accommodate a typical biomass particle dimension, the maximum push and pull travel ranges up to 25 mm to accommodate a typical biomass particle size. For vertical direction movement, the normal force actuator's maximum displacement is 14mm to accommodate a typical biomass particle size. The resolution of the actuator is 6 nm. Combined vertical and lateral displacement actuators make it possible to control the initial particle contact location if there exists a specific region of interest. The maximum forces of vertical and normal force censors are 20.02 N and 2.94 N, respectively, whereas the accuracy of both force sensors is ±0.02% of the full scale. Sampling rates of displacement and force sensors are equal to or higher than 1 kHz. Normal and traction force data were collected at 80Hz, using National Instruments™ LabVIEW (Version 2022, Austin TX) and two 24-bit Di-1000U signal digitizers (Loadstar Sensors - Freemont, CA).

## Experimental Design and Setup

An experimental procedure was developed to perform friction tests with minimal experimental error. A sample size of 15 was selected for each possible combination of anatomical fractions to ensure the quality of a subsequent analysis of variance (Kruskal-Wallis test with ) that examined the magnitude and source of variance. The Wilcoxon signed-rank (signed-rank) test was conducted to test the null hypotheses, i.e., friction coefficients and adhesion forces between corn stover particles from different anatomical fractions are not statistically different, without assuming a statistical distribution of determined friction coefficient and adhesion forces.

To prepare an inter-particle mechanics test, approximately 50 g samples were collected from each anatomical fraction of milled corn stover. Each sample was held in a controlled environment for 72 hours to allow for particles to reach an equilibrium moisture content at approximately 23 °C and 40 % RH. The resulting moisture contents of cob, husk, leaf, and stalk are 8.9 ± 1.0 %, 7.0 ± 1.4 %, 7.0 ± 1.2 %, and 8.9 ± 1.12 %, respectively (values after ± represent the standard error with n = 11).

Once particle moisture was equalized, particles were randomly selected and adhered to removable particle mounts (Figure 4) using Permatex *Ultra Bond™ Super Glue*. Conservative volumes of adhesives were applied to minimize the absorption into particles and ensure proper curing. From the visual inspection, it appeared that no glue simps into the particle samples, which is attributed to the high viscosity of the chosen glue and quick initial curing time. A curing period of longer than 24 hours was established for experimental consistency. The curing environment is identical to the moisture-equilibrating environment. Finally, the grain orientations were randomly chosen with every particle.

* 

Figure Photograph of corn stover particles adhered to plastic particle mount for conditioning and testing. Samples shown in the picture are 4mm husk (top) and cob (bottom) fractions, respectively.

Once the adhesive had cured, particle pairs were randomly selected for testing. Table 2 lists the combinations of milled corn particles of different tissue types and conducted a number of repetitions for statistical analysis with an appropriate level of confidence.

Table Experimental plan for the required number of corn fraction test pairs and associated repetitions

|  |  |  |
| --- | --- | --- |
| Base Particle | Interacting Particle | Minimum Repetitions |
| Cob | Cob | 15 |
| Cob | Husk | 15 |
| Cob | Leaf | 15 |
| Cob | Stalk | 15 |
| Husk | Husk | 15 |
| Husk | Leaf | 15 |
| Husk | Stalk | 15 |
| Leaf | Leaf | 15 |
| Leaf | Stalk | 15 |
| Stalk | Stalk | 15 |

An inter-particle mechanics test started with establishing contact between particle samples indicated by a positive normal force reading typically ranging between 3 N and 2 N. Then, lateral sliding was initiated at a speed of 0.5 mm/min. The normal and lateral forces were tracked and recorded simultaneously until the contact between two particles was lost, which resulted in a sudden change in force measurement. The middle two quartiles were selected and extracted as steady-state data for further analysis and determination of friction coefficient and adhesion force.

To determine the cohesive frictional contact, a modified Amonton’s equation by Derjaguin (Derjaguin 1934; Gao et al. 2004; Derjaguin, Muller, and Toporov 1975)

Where is tractional force in the lateral direction, is the adhesion force, is the coefficient of friction, and is the normal force representing the contact force. From this linear relationship, the coefficient of friction (*μ*) was determined from the slope of the best-fit line, and the adhesion force () was determined from the y-axis, i.e., the traction force axis, the intercept of the best-fit line. Therefore, linear regression analyses were conducted to determine the coefficient of friction (*μ*) and adhesion force () from experimental data.

# Results and Discussion

## Validation of the IPM

To validate the developed Inter-Particle Mechanics tester, a polypropylene sheet was used as a reference material. A polypropylene sheet with 1/16 inch thickness was obtained from McMaster-Carr, which lists the friction coefficient to range from 0.25 to 0.28 based on the specification of the manufacturer. 20 samples were prepared and subjected to the IPM test. The obtained 20 test results of frictional interaction between two polypropylene sheet samples are shown in Figure 5, which clearly shows a linear relationship between normal force and traction force with detectable traction force.

Using Derjaguin’s adhesive contact model, the coefficient of friction and adhesion force were determined from the slope and intercept of the obtained linear relationship through the linear regression analysis. The summary of the determined friction coefficient () and adhesion force () are listed in Table 3 and shown in Figure 6. The specification listed in by the vendor (McMaster-Carr) ranges between 0.25 and 0.28. The determined friction coefficient is smaller than the listed range. This is an expected trend because the friction coefficient is reported to be smaller with the smaller normal force and initial sliding friction segment for polypropylene (Sędłak et al. 2017). This observation supports the accuracy of the Interparticle Mechanics Tester in determining frictional characteristics.

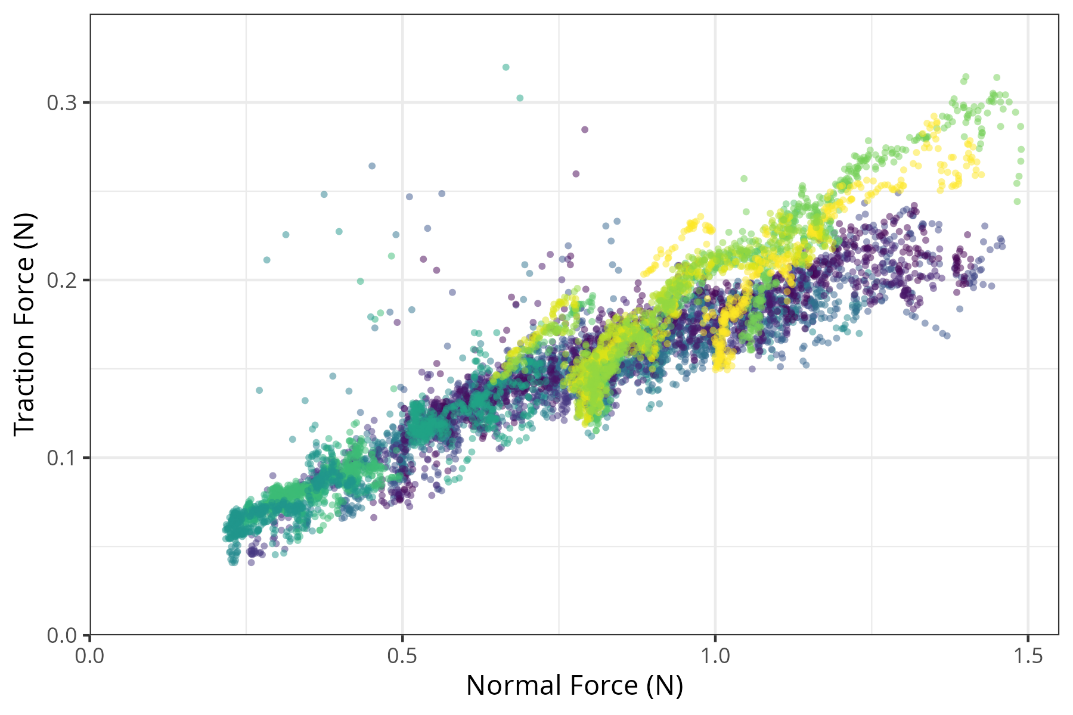
Figure Validation test results of polypropylene

Table Summary statistics of Polypropylene friction coefficient and adhesion force determined using the Interparticle Mechanics Tester

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Interparticle Mechanics  Properties | Sample Size | Min | Q1 | Median | Mean | Q3 | Max | Interquartile Range |
| friction coefficient  () | 20 | 0.10 | 0.12 | 0.14 | 0.16 | 0.20 | 0.26 | 0.07 |
| adhesion force  (, N) | 20 | -0.07 | 0.00 | 0.04 | 0.02 | 0.05 | 0.08 | 0.05 |

A graph of a graph of a graph of a graph of a graph of a graph of a graph of a graph of a graph of a graph of a graph of a graph of a graph of

Description automatically generatedFigure Interparticle Mechanics Test results of polypropylene sheet samples. Each color represents a different run of the experiments (n=20)

## Friction Coefficient and Adhesion Force of Corn stover particles from different types

To examine the variability in frictional contact mechanics between corn stover particles from different tissue types, corn stover samples were manually fractionated into the cob, husk, leaf, and stalk, then Crumbled® using a 4 mm screen by Forest Concepts, LLC. Twenty particle sample pairs for each tissue type were prepared, and the interparticle mechanics tests were conducted. A typical experimental data and regression analysis result is shown in Figure 7.

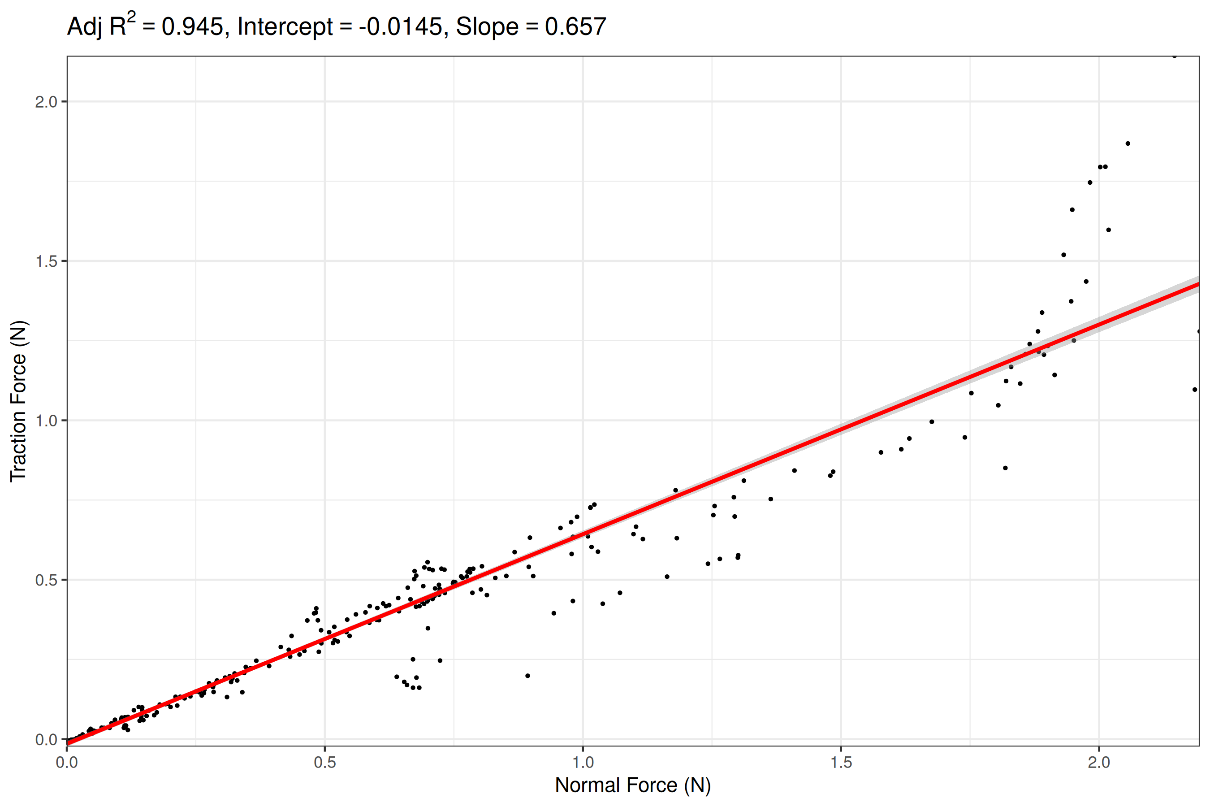


Figure Typical corn stover particle result (cob particle vs. stalk particle shown)

Corn stover interactions involving cob fractions were found to contain the largest peaks and variance in friction coefficients (Table [4](#tab:org6c74d9c)). Cob particles with woody-ring sub-fractions on the contacting surface were observed to be significantly harder and ’claw-like’ in nature. Whereas the pith region of the cob was very similar to the exposed pith in stalk particles. Particles of stalk and husk both showed moderate friction coefficients.

Table Determined Coefficient of friction and adhesion force values of corn stover particles from different tissue types

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Paired Fractions | Average Friction Coefficient | Standard Deviation |  | Mean Adhesion Force |
| cob-cob | 0.86 | 0.26 |  |  |
| husk-husk | 0.23 | 0.06 |  |  |
| leaf-leaf | 0.28 | 0.09 |  |  |
| stalk-stalk | 0.28 | 0.09 |  |  |

Table Analysis of variation (Kruskal-Wallis test) in the coefficient of friction and adhesion force between corn stover particles

| Particle Pair | Comparison Pair | Sum<Cr? | Result for Coefficient of Friction | Result for Adhesion Force |
| --- | --- | --- | --- | --- |
| cob-cob | husk-husk |  |  |  |
|  | leaf-leaf |  |  |  |
|  | stalk-stalk |  |  |  |
| husk-husk | leaf-leaf |  |  |  |
|  | stalk-stalk |  |  |  |
| leaf-leaf | stalk-stalk |  |  |  |

Kruskal-Wallis test (, Table 5) showed the key source of variance in friction coefficients was found to be between different combinations of fractions. Leading to further investigation of key sources of variances. Without an understanding of the population distribution, a Wilcoxon signed-rank test was conducted to determine the significance across the variance with each corn stover fraction combination. As shown in Table [5](#tab:orgefb11ac), the results suggest that cob fractions had a significantly different friction coefficient.

TODO and Discussion Points

* Add result of the adhesion force
* Populate the Kruskal-Wallis test results for both friction coefficient and adhesion force
* Discuss that mu and c are significantly different between fractions: Dunn test
* How does this relate to the Mohr-Coulomb model parameters
* How does this relate to Reynolds dilatancy

Dilatancy, or the volume change in granular materials under shear deformation, is a critical phenomenon in understanding the mechanical behavior of bulk materials. This concept was first scientifically described by Osborne Reynolds in 1885/1886 and is often referred to as Reynolds dilatancy. It explains how dense granular materials expand (dilate) when sheared due to the interlocking of grains, while loose granular materials may compact instead(Sakaie et al. 2008).

**Constitutive Models and Dilatancy Prediction**

Constitutive models are mathematical frameworks used to describe material behavior under different loading conditions. Several constitutive models explicitly incorporate or predict dilatancy:

Mohr-Coulomb Model:

This widely used elasto-plastic model accounts for shear strength and includes a dilation angle (ψ) to describe the relationship between plastic volume change and plastic shear strain. It is particularly effective for simulating granular materials like soils and sands under shear.

Modified Cam-Clay Model:

A critical-state-based model that incorporates volumetric changes due to shear and compression. It is commonly applied to clays but can also model granular materials where volume change influences resistance to shear (DeSimone and Tamagnini 2005; Maranha and Maranha das Neves 2009).

Hypoplasticity Models:

These advanced models, such as Sand Hypoplasticity (von Wolffersdorff, 1996), are incrementally non-linear and include dilatancy as a function of stress state and void ratio. They are particularly suited for granular soils under complex loading paths.

Critical-State Models:

Critical-state soil mechanics frameworks inherently predict dilatancy behavior by defining a critical void ratio at which no volume change occurs during shear. These models are fundamental for understanding transitions between dilative and contractive states in granular assemblies (Yin et al. 2017; Massoudi 2023).

Double-Yield Surface Models:

These models, often used for coarse granular materials, include separate yield surfaces for shear sliding and compression, allowing them to capture dilative behaviors during phase transformations (Yin et al. 2017).

Non-Local Constitutive Models:

Recent developments in non-local models incorporate mesoscopic stress states to predict deformation rates and density changes, explicitly addressing dilatancy in slow granular flows (Dsouza and Nott 2020).

SANISAND Model:

An advanced elasto-plastic model that integrates fabric-dilatancy effects, making it effective for simulating sand behavior under cyclic loading.

Reynolds Dilatancy

Reynolds' work highlighted the fundamental mechanism of dilatancy in dense granular materials: grains interlock and expand when sheared due to geometric constraints. This phenomenon is central to many constitutive models, particularly those based on critical-state mechanics or incorporating dilation angles (e.g., Mohr-Coulomb). Reynolds dilatancy has been experimentally validated using techniques like Magnetic Resonance Imaging (MRI) to observe density changes during shear(Sakaie et al. 2008).

Summary of Major Constitutive Models Addressing Dilatancy

Model Key Features Related to Dilatancy

Mohr-Coulomb Dilation angle (ψ) defines plastic volume change under shear

Modified Cam-Clay Critical-state framework with volumetric hardening/softening

Hypoplasticity Non-linear stress-strain relations with void ratio-dependent dilatancy

SANISAND Fabric-dilatancy coupling for sands

Double-Yield Surface Captures phase transformation-induced dilation

Non-Local Models Stress-dependent density changes at mesoscopic scales

These models provide robust tools for predicting and analyzing the mechanical behavior of granular materials, including their dilative or contractive responses under various loading conditions.

* Implication of DEM contact model, especially with the contact model choices
* Derjaguin’s linear model and other polynomial models reflecting the observation of Sędłak (Sędłak et al. 2017) hinting that the lower friction coefficient in the smaller normal force

# Conclusions

From the observation that particles of corn cob show a larger standard deviation in the determined coefficient of friction, it can be hypothesized that particles of corn cob will have a high contribution to an erratic feedstock flow behavior. It can also be hypothesized that the cob will generate high frictional forces, considering the high cohesive forces and moderate friction coefficients observed in a corn cob. The experimental data suggest that high-stress feedstock handling applications may be more stable with the removal of corn cob.

It is planned to repeat tests with variations in moisture content. High adhesive forces in cob particles containing ’woody-ring’ subfractions may be reduced as moisture levels increase towards the fiber saturation point, but further studies are required to understand any adverse effects on friction coefficients.

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## Future studies

Using DEM in biomass flow studies

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