Reading Week Assignment CS1003
Student Name : Caroline Liu

1(a)
i

| | | B |
|---|---|---|
| A | 0 | 1 |
| | 1 | 1 |

$A \cup B$

$A \bar{\cup} (A \bar{\cup} B)$
$= A \cup (\overline{A \cup B})$
$= \bar{A} \cap (A \cup B)$

| | | B |
|---|---|---|
| A | 0 | 1 |
| | 0 | 0 |

$A \bar{\cup} (A \bar{\cup} B)$

hence, $A \cup B \neq A \bar{\cup} (A \bar{\cup} B)$

ii

| | | B |
|---|---|---|
| A | 1 | 1 |
| | 1 | 0 |

$\overline{A \cap B}$

$\bar{A} \bar{\cup} (B \bar{\cup} A)$
$= \bar{A} \cup (\overline{B \cup A})$
$= A \cap (B \cup A)$

| | | B |
|---|---|---|
| A | 0 | 0 |
| | 1 | 1 |

$\bar{A} \bar{\cup} (B \bar{\cup} A)$

hence, $\overline{A \cap B} \neq \bar{A} \bar{\cup} (B \bar{\cup} A)$

iii

$A \bar{\cup} (B \bar{\cup} C)$
$= A \cup (\overline{B \cup C})$
$= \bar{A} \cap (B \cup C)$

| | | | B | |
|---|---|---|---|---|
| A | 0 | 1 | 1 | 1 |
| | 0 | 0 | 0 | 0 |
| | | C | | |

$A \bar{\cup} (B \bar{\cup} C)$

$(A \bar{\cup} B) \bar{\cup} C$
$= (\overline{A \cup B}) \cup C$
$= (A \cup B) \cap \bar{C}$

| | | | | B |
|---|---|---|---|---|
| A | 0 | 0 | 0 | 1 |
| | 1 | 0 | 0 | 1 |
| | | C | | |

$(A \bar{\cup} B) \bar{\cup} C$

hence $A \bar{\cup} (B \bar{\cup} C) \neq (A \bar{\cup} B) \bar{\cup} C$

(b) 30 Students from A, B and C, hence $A + B + C = 30$

| | | | B | |
|---|---|---|---|---|
| A | 0 | 6 | 2 | 3 |
| | 2 | 2 | 5 | 10 |
| | | C | | |

i 10 people belong to the Archaeological and Botanical but not the Choral.
ii $2 + 2 + 5 + 10 = 19$
Ans: 19 people belong to the Archaeological society.

2(a) i $(p \rightarrow q) \rightarrow p$

| (p | $\longrightarrow$ | q) | $\longrightarrow$ | p |
|---|---|---|---|---|
| $\perp$ | T | $\perp$ | $\perp$ | $\perp$ |
| $\perp$ | T | T | $\perp$ | $\perp$ |
| T | $\perp$ | $\perp$ | T | T |
| T | T | T | T | T |

hence, $(p \rightarrow q) \rightarrow p$ is a contradiction, not a tautology.

ii

| $(p$ | $\longrightarrow$ | $q$ | $\wedge$ | $r)$ | $\longrightarrow$ | $(p$ | $\longrightarrow$ | $q)$ | $\wedge$ | $(p$ | $\longrightarrow$ | $r)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\perp$ | T | $\perp$ | $\perp$ | $\perp$ | T | $\perp$ | T | $\perp$ | T | $\perp$ | T | $\perp$ |
| $\perp$ | T | $\perp$ | $\perp$ | T | T | $\perp$ | T | $\perp$ | T | $\perp$ | T | T |
| $\perp$ | T | T | $\perp$ | $\perp$ | T | $\perp$ | T | T | T | $\perp$ | T | $\perp$ |
| $\perp$ | T | T | T | T | T | $\perp$ | T | T | T | $\perp$ | T | T |
| T | $\perp$ | $\perp$ | $\perp$ | $\perp$ | T | T | $\perp$ | $\perp$ | $\perp$ | T | $\perp$ | $\perp$ |
| T | $\perp$ | $\perp$ | $\perp$ | T | T | T | $\perp$ | $\perp$ | $\perp$ | T | T | T |
| T | $\perp$ | T | $\perp$ | $\perp$ | T | T | T | T | $\perp$ | T | $\perp$ | $\perp$ |
| T | T | T | T | T | T | T | T | T | T | T | T | T |

hence, $(p \to q \wedge r) \to (p \to q) \wedge (p \to r)$ is a tautology.

(b)

| | | |
|---|---|---|
| 1. | $p \vee (C \to \neg S)$ | premise |
| 2. | $p \to \neg C$ | premise |
| 3 | $C$ | premise |
| 4. | $\neg(S \to p)$ | negate conclusion |
| 5 | $\neg p$ | $\beta(2,3)$ |
| 6 | $S$ | $\alpha(4)$ |
| 7 | $C \to \neg S$ | $\beta(1,5)$ |
| 8 | $\neg S$ | $\beta(3,7)$ |
| 9 | $\perp$ | $\times(6,8)$ |

hence, the argument is valid

3(a)

| | | |
|---|---|---|
| 1. | $TB \to (BL \to MA)$ | premise |
| 2. | $(MA \wedge FD) \to \neg GH$ | premise |
| 3 | $\neg GJ \to (FD \wedge GH)$ | premise |
| 4 | $\neg(GH \to (FD \vee GJ))$ | negate conclusion |
| 5 | $GH$ | $\alpha(4)$ |
| 6 | $\neg(FD \vee GJ)$ | $\alpha(4)$ |
| 7 | $\neg FD$ | $\alpha(6)$ |
| 8 | $\neg GJ$ | $\alpha(6)$ |
| 9 | $FD \wedge GH$ | $\beta(3,8)$ |
| 10 | $FD$ | $\beta(8,9)$ |
| 11 | $\perp$ | $\times(7,10)$ |

hence, the conjecture $GH \to FD \vee GJ$ can be inferred from the premises.

(b)

| | | |
|---|---|---|
| 1. | $TB \rightarrow (BL \rightarrow MA)$ | premise |
| 2. | $(MA \wedge FD) \rightarrow \neg GH$ | premise |
| 3. | $\neg GJ \rightarrow (FD \wedge GH)$ | premise |
| 4 | $\neg((TB \wedge \neg GH) \rightarrow (\neg MA \rightarrow GJ))$ | negate conclusion |
| 5 | $TB \wedge \neg GH$ | $\alpha(4)$ |
| 6 | $\neg(\neg MA \rightarrow GJ)$ | $\alpha(4)$ |
| 7 | $\neg GJ$ | $\alpha(6)$ |
| 8 | $\neg GH$ | $\alpha(5)$ |
| 9 | $FD \wedge GH$ | $\beta(3,7)$ |
| 10 | $GH$ | $\alpha(9)$ |
| 11. | $\perp$ | $X(8,10)$ |