Name: Caroline Liu
Module: CS1003

Q1. $A = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$

(i) $\det A = (3)(2) - (1)(4) = 6 - 4 = 2$

$\det B = (1)(3) - (4)(2) = 3 - 8 = -5$

(ii) $B^T A^T = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$

$= \begin{pmatrix} 5 & 8 \\ 15 & 22 \end{pmatrix}$

$(AB)^T = \left(\begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}\right)^T$

$= \begin{pmatrix} 5 & 15 \\ 8 & 22 \end{pmatrix}^T$

$= \begin{pmatrix} 5 & 8 \\ 15 & 22 \end{pmatrix} = B^T A^T$

$\Rightarrow (AB)^T = B^T A^T$

Q2 $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 5 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$

(i) $AB = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 5 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$

$= \begin{pmatrix} 9 & 13 & 11 \\ 9 & 17 & 13 \\ 6 & 10 & 8 \end{pmatrix}$

$BA = \begin{pmatrix} 2 & 5 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{pmatrix}$

$= \begin{pmatrix} 15 & 12 & 27 \\ 5 & 6 & 11 \\ 7 & 6 & 13 \end{pmatrix}$

(ii) No, $AB \neq BA$.

Reason: the multiplication of matrices is not commutative.

Q3 $\begin{array}{c} \begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & 3 & 1 & 3 \\ -1 & 1 & 2 & 6 \end{pmatrix} \end{array} \begin{array}{l} R1 \\ R2 \\ R3 \end{array}$

$\xrightarrow[R3+R1]{R2-2R1} \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & 3 & 8 \end{pmatrix}$

$\xrightarrow[R3-2R2]{R1-R2} \begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 5 & 10 \end{pmatrix}$

$\xrightarrow{\frac{1}{5}R3} \begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$

$\xrightarrow[R2+R3]{R1-2R3} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$

$\Rightarrow$ $x = -1$
$y = 1$
$z = 2$