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Module: CS1003

Q1.

$$x + y + z = 2$$
$$2x + y = 3$$
$$x - y - 3z = 0$$

Using Gaussian Elimination :

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & 0 & 3 \\ 1 & -1 & -3 & 0 \end{pmatrix} \begin{matrix} R1 \\ R2 \\ R3 \end{matrix}$$

$$\xrightarrow[R3 \to R3 - R1]{R2 \to R2 - 2R1} \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & -1 & -2 & -1 \\ 0 & -2 & -4 & -2 \end{pmatrix}$$

$$\xrightarrow[R3 \to R3 + 2R2]{R2 \to R2 \times (-1),\ R1 \to R1 - R2} \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\because$ R3 : $0x + 0y + 0z = 0$ i.e. $0 = 0$

$\therefore$ linear system corresponding to matrix:

$$x - z = 1$$
$$y + 2z = 1$$

let $z = t$, $t \in R$

hence : $x = 1 + t$

$y = 1 - 2t$

$$\boxed{\begin{array}{l} \text{Solutions :} \\ x = 1 + t \\ y = 1 - 2t \\ z = t \\ \text{for all } t \in R \end{array}}$$

Q2.

$$\begin{pmatrix} 2 & 4 & 1 \\ 3 & 3 & 2 \\ 4 & 1 & 4 \end{pmatrix}$$

Inverse :

$$\begin{array}{c} \left(\begin{array}{ccc|ccc} 2 & 4 & 1 & 1 & 0 & 0 \\ 3 & 3 & 2 & 0 & 1 & 0 \\ 4 & 1 & 4 & 0 & 0 & 1 \end{array}\right) \end{array} \begin{array}{l} R1 \\ R2 \\ R3 \end{array}$$

$$\begin{array}{l} R1 \to R1 \times \frac{1}{2} \\ R2 \to R2 - 3R1 \\ \xrightarrow{\hspace{3em}} \\ R3 \to R3 - 4R1 \end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & -3 & \frac{1}{2} & -\frac{3}{2} & 1 & 0 \\ 0 & -7 & 2 & -2 & 0 & 1 \end{array}\right)$$

$$\begin{array}{l} R2 \to R2 \times (-\frac{1}{3}) \\ R1 \to R1 - 2R2 \\ \xrightarrow{\hspace{3em}} \\ R3 \to R3 + 7R2 \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{6} & -\frac{1}{2} & \frac{2}{3} & 0 \\ 0 & 1 & -\frac{1}{6} & \frac{1}{2} & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{5}{6} & \frac{3}{2} & -\frac{7}{3} & 1 \end{array}\right)$$

$$\begin{array}{l} R3 \to R3 \times \frac{6}{5} \\ R2 \to R2 + \frac{1}{6}R3 \\ \xrightarrow{\hspace{3em}} \\ R1 \to R1 - \frac{5}{6}R3 \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & \frac{4}{5} & -\frac{4}{5} & \frac{1}{5} \\ 0 & 0 & 1 & \frac{9}{5} & -\frac{14}{5} & \frac{6}{5} \end{array}\right)$$

$$\boxed{\text{hence, inverse matrix} = \begin{pmatrix} -2 & 3 & -1 \\ \frac{4}{5} & -\frac{4}{5} & \frac{1}{5} \\ \frac{9}{5} & -\frac{14}{5} & \frac{6}{5} \end{pmatrix}}$$

Q3.

$$3x - y = 5$$
$$2x + 3y = 1$$

$$\det(D) = \begin{vmatrix} 3 & -1 \\ 2 & 3 \end{vmatrix}$$

$$\det(D) = 9 + 2 = 11$$

Using Cramer's rule:

$$x = \frac{\begin{vmatrix} 5 & -1 \\ 1 & 3 \end{vmatrix}}{11} = \frac{15+1}{11} = \frac{16}{11}$$

$$y = \frac{\begin{vmatrix} 3 & 5 \\ 2 & 1 \end{vmatrix}}{11} = \frac{3-10}{11} = -\frac{7}{11}$$

Solutions to linear system:

$$(x, y) = \left(\frac{16}{11}, -\frac{7}{11}\right)$$