# Weekly Assignment 4

Caroline Liu
STU3009 - Statistical Methods for Computer Science
Trinity College Dublin

February 18, 2020

## Declaration concerning plagiarism

I have read and I understand the plagiarism provisions in the General Regulations of the University Calendar for the current year, found at https://www.tcd.ie/calendar/

I have completed the Online Tutorial in avoiding plagiarism 'Ready, Steady, Write', located at http://tcd-ie.libguides.com/plagiarism/ready-steady-write

Student Name - Caroline Liu
Student Number - 16327123

# Question 1

**Consider an experiment where we roll two 6-sided dice. Let random variable $Y$ be the sum of the values rolled. The sample space is $(1,1),(1,2),(1,3),...,(6,6)$ and recall that a random event is s subset of the sample space.**

## Question 1(a)

**What random event corresponds to $Y = 2$ ?**

**Solution -** $(1,1)$.

## Question 1(b)

**What random event corresponds to $Y = 3$ ?**

**Solution -** $(1,2),(2,1)$.

## Question 1(c)

**What random event corresponds to $Y = 4$ ?**

**Solution -** $(1,3),(2,2),(3,1)$.

## Question 1(d)

**Now let $X$ be the indicator random variable associated with the event $(1,1),(2,2),(3,3)$. What's the probability that $X = 1$ ?**

**Solution -**

$$0.0833$$

**Justification -**

- The sample space of the experiment consist of $6^2 = 36$ possible events.
- There are 3 elements associated with the random variable $X$, namely the three, $(1,1),(2,2),(3,3)$.
- Therefore the probability that $X = 1$ is $P(X = 1) = \frac{3}{36} \approx 0.0833$

# Question 2

**Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 3 times.**

## Question 2(a)

**What are the possible values of X ?**

**Solution -**

$$X = \{-3, -1, 1, 3\}$$

**Justification-**

- In the following, let $H$ represent heads and let $T$ represent tails.
- When a coin is tossed 3 times, the possible outcomes are sample space,

  $S = \{(T, T, T), (T, T, H), (T, H, T), (H, T, T), (T, H, H), (H, T, H), (H, H, T), (H, H, H)\}$.
- 
  - 0 heads and 3 tails, difference $= 0 - 3 = -3$
  - 1 head and 2 tails, difference $= 1 - 2 = -1$
  - 2 heads and 1 tail, difference $= 2 - 1 = 1$
  - 3 heads and 0 heads, difference $= 3 - 0 = 3$
- Hence the possible values of $X$ can be $-3, -1, 1, 3$, therefore $X = \{-3, -1, 1, 3\}$

## Question 2(b)

**What is $P(X = -3)$ ?**

**Solution -**

$$0.125$$

**Justification -**

- The total number of possible outcomes when a coin is tossed 3 times is $2^3 = 8$ outcomes.
- There is only one event in which $X = -3$, namely when there are 3 tails and 0 heads.
- Therefore $P(X = -3) = \frac{1}{8} = 0.125$

## Question 2(c)

**What is $P(X = -1)$ ?**

**Solution -**

$$0.375$$

**Justification -**

- There are $2^3 = 8$ possible outcomes.
- For $X = -1 = 1 - 2$, there are 2 heads and 1 tail. There are 3 possible outcomes for this result.
- Therefore, $P(X = -1) = \frac{3}{8} = 0.375$

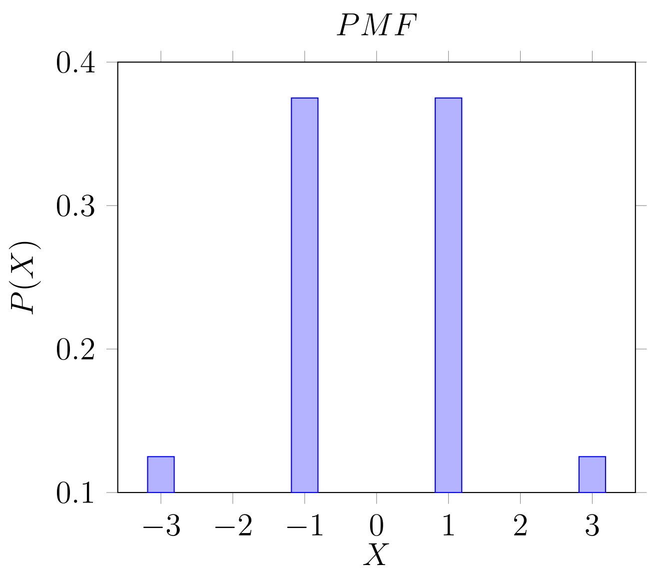
## Question 2(d)

**If the coin is assumed fair, calculate the PMF and CDF of $X$ and plot a sketch of both.**

**Solution -**

**Justification -**

- PMF
  - $X$ takes the values within $\{-3, -1, 1, 3\}$.
  - $P(X = -3) = Prob(3 \text{ tails}) = \binom{3}{0} \times (\frac{1}{2})^0 \times (\frac{1}{2})^3 = \frac{1}{8}$
  - $P(X = -1) = Prob(2 \text{ tails, } 1 \text{ head}) = \binom{3}{1} \times \frac{1}{2} \times (\frac{1}{2})^2 = \frac{3}{8}$
  - $P(X = 1) = Prob(1 \text{ tail, } 2 \text{ heads}) = \binom{3}{3} \times (\frac{1}{2})^2 \times \frac{1}{2} = \frac{3}{8}$
  - $P(X = 3) = Prob(3 \text{ heads}) \binom{3}{3} \times (\frac{1}{2})^3 \times (\frac{1}{2})^0 = \frac{1}{8}$




- CDF
  - Assumed that here, $x$ takes the values in $\{-3, -1, 1, 3\}$, hence $X$ lies within $x = \{-3, -1, 1, 3\}$.
  - If $x < -3$, then
  $$F(X) = P(X \leq x) = 0, for\ x < -3$$
  .
  - Next if $x = 3$,
  $$F(x) = P(X \leq x) = 1, for\ x = 3$$

– Next if $-3 \leq x < -1$,

$$F(x) = P(X \leq x) = P(X = -3) = \frac{1}{8}, for \ -3 \leq 0 < -1$$

– Next if $-1 \leq x < 1$,

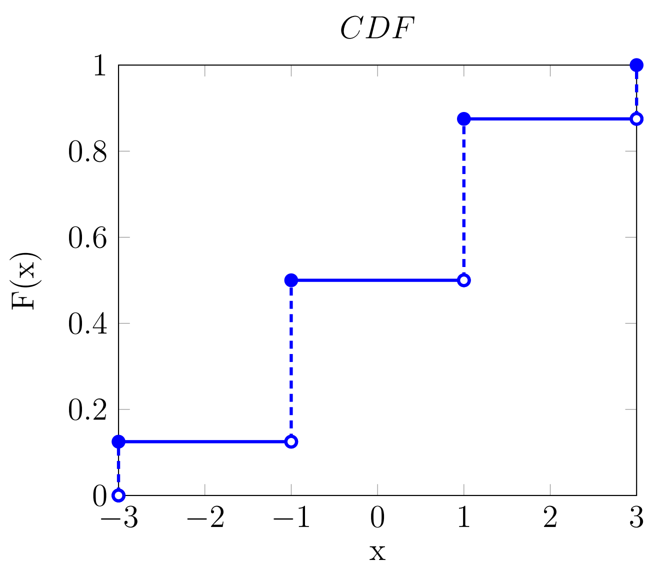
$$F(x) = P(X \leq x) = P(X = -3) + P(X = -1) = \frac{1}{8} + \frac{3}{8} = \frac{1}{2}, for \ -1 \leq x < 1$$

– Finally if $1 \leq x < 3$,

$$F(x) = P(X \leq x) = P(X = -3)+P(X = -1)+P(X = 1) = \frac{1}{8}+\frac{3}{8}+\frac{3}{8} = \frac{7}{8}, for\ 1 \leq x < 3$$

– Thus, to summarise, we have

$$F(x) = \begin{cases} 0 & \text{for } x < -3 \\ \frac{1}{8} & \text{for } -3 \leq x < -1 \\ \frac{1}{2} & \text{for } -1 \leq x < 1 \\ \frac{7}{8} & \text{for } 1 \leq x < 3 \\ 1 & \text{for } x = 3 \end{cases}$$



# Question 3

**Four 6-sided dice are rolled. The dice are fair, so each one has equal probability of producing a value in 1,2,3,4,5,6. Let X =the minimum of the four values rolled. (It is fine if more than one of the dice has the minimal value.)**

## Question 3(a)

**What is $P(X \geq 1)$ ?**

**Solution -**
**Justification -**

- $X \geq 1$ means the events where the minimum of the four values rolled are greater than 1.
- Since the minimum value on a die is 1, this would always be true.
- Hence, the probability $P(X \geq 1) = 1$.

## Question 3(b)

**What is $P(X \geq 2)$ ?**

**Solution -**

$$0.4823$$

**Justification -**

- Assuming he dice are fair.
- $P(X \geq 2) = 1 - P(X < 2)$
- Therefore $P(X < 2) =$ all the events where the minimum value rolled is 1.

$$P(X < 2) = \binom{4}{4} \times (\frac{1}{6})^4 \times (\frac{5}{6})^0 + \binom{4}{3} \times (\frac{1}{6})^3 \times (\frac{5}{6}) + \binom{4}{2} \times (\frac{1}{6})^2 \times (\frac{1}{6})^2 + \binom{4}{1} \times (\frac{1}{6}) \times (\frac{1}{6})^3$$

$$P(X < 2) = \frac{671}{1296}$$

- Therefore $P(X \geq 2) = 1 - P(X < 2) = 1 - \frac{671}{1296} = \frac{625}{1296} \approx 0.4823$

## Question 3(c)

**What is the CDF of $X$ *i.e.* $P(X \leq k)$ for all values of $k$ ?**

**Solution -**

- Assuming the four dice are fair. results only have an affect when $k = \{1, 2, 3, 4, 5, 6\}$
- If $k < 1$, $P(X < 1) = 0$
- If $1 \leq k < 2$, the solution set is $\{1\}$

$$P(X \leq 1) = (\frac{1}{6})^4 = \frac{1}{1296}$$

- If $2 \leq k < 3$, the solution set is $\{1, 2\}$

$$P(X \leq 2) = (\frac{2}{6})^4 = \frac{1}{81}$$

- If $3 \leq k < 4$, the solution set is $\{1, 2, 3\}$

$$P(X \leq 3) = (\frac{3}{6})^2 = \frac{1}{16}$$

- If $4 \leq k < 5$, the solution set is $\{1, 2, 3, 4\}$

$$P(X \leq 4) = (\frac{4}{6})^4 = \frac{16}{81}$$

- If $5 \leq k < 6$, the solution set is $\{1, 2, 3, 4, 5\}$

$$P(X \leq 5) = (\frac{5}{6})^4 = \frac{625}{1296}$$

- If $k = 6$, the solution set is $\{1, 2, 3, 4, 5, 6\}$

$$P(X = 6) = 1$$

- Thus, to summarise, we have the CDF -

$$F(k) = \begin{cases} 0 & \text{for } k < 1 \\ \frac{1}{1296} & \text{for } 1 \leq k < 2 \\ \frac{1}{81} & \text{for } 2 \leq k < 3 \\ \frac{1}{16} & \text{for } 3 \leq k < 4 \\ \frac{16}{81} & \text{for } 4 \leq k < 5 \\ \frac{625}{1296} & \text{for } 5 \leq k < 6 \\ 1 & \text{for } x = 6 \end{cases}$$