Response to Referee Report

[Author]

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We thank the referee for a careful and constructive report. Below we address each concern point-by-point, describing both our response and the specific revisions made to the manuscript.

Framing

Before addressing individual points, we clarify what DD claims to contribute:

1. **Logical unification**: DD connects operational axioms of quantum mechanics (Hardy, Chiribella–D'Ariano–Perinotti reconstructions), algebraic constraints (Hurwitz), and gauge-algebra correspondences (Furey, Gresnigt) into a single derivation chain with explicit logical dependencies.
2. **Saturation theorem**: We prove that the Standard Model structure *saturates* the space of realisable theories—extensions are either non-realisable or merely effective (new Theorem in companion paper).
3. **Fundamental/effective distinction**: We provide a criterion for separating UV-complete structure (constrained by DD) from emergent effective symmetries (not constrained).

These are contributions to the *organisation* of known results, not claims of new empirical predictions.

Major Concerns

(1) "The central claim is circular"

Referee's concern: The derivation relies on prior work (Furey, Gresnigt, Baez) that already assumes division algebras are physically relevant.

Response: We do not assume that fundamental physics "uses" division algebras. Rather, we show that *realisability*, formalised as invertibility of non-zero processes together with a multiplicative norm, *is equivalent* to the axioms of a normed division algebra. Hurwitz then restricts the possible carriers.

The distinction is:

- Furey/Gresnigt answer: *What can be built on $\mathbb{C} \otimes \mathbb{O}$?*
- DD answers: *Why must the carrier satisfy division-algebra axioms if the theory is to be unitary and information-preserving?*

Revision: We have added explicit remarks in Section 6 (Definition 6.1) clarifying that the axioms (R1)–(R3) are precisely those of a normed division algebra, and that the contribution is the *interpretation* of these axioms as necessary conditions for physical realisability.

(2) "'Realisability' is ill-defined"

Referee's concern: "Process algebra" is nonstandard; "information-preserving" appears ad hoc.

Response: We agree that the original formulation was imprecise. We have revised Definition 6.1 to state the axioms purely mathematically:

> A *realisable carrier* is a finite-dimensional real algebra $(\mathcal{A}, \circ)$ with a norm $\|\cdot\|$ satisfying:
>
> (R1) $\|a\| = 0 \Leftrightarrow a = 0$ (non-degeneracy)
>
> (R2) $\|a \circ b\| = \|a\|\|b\|$ (multiplicativity)
>
> (R3) For all $a \neq 0$, there exists a^{-1} (invertibility)

These are precisely the axioms of a normed division algebra—a standard mathematical concept. Physical interpretation (unitarity, reversibility) is now relegated to a separate Remark.

Revision: Definition 6.1 rewritten with mathematical axioms (R1)–(R3) and explicit separation of mathematical content from physical interpretation.

(3) "The Hurwitz argument is not new"

Referee's concern: What does DD add beyond repackaging known results?

Response: The Hurwitz theorem (1898) is indeed well known. DD's contribution is not a new theorem but:

1. **A precise logical interface** connecting quantum reconstruction theorems, unitarity requirements, Hurwitz bounds, and Standard Model structure into a single chain with explicit dependencies.
2. **A saturation statement** (new Theorem 2.4 in companion): the Standard Model is not merely "minimal" but *saturates* the space of realisable structures—there is no room for extension without violating realisability.
3. **Classification of extensions**: extra $U(1)'$, four generations, and vector-like fermions are excluded as *fundamental* structure (they may exist as effective descriptions).

Revision: Added "Contributions" subsection to Introduction explicitly listing what DD claims to contribute.

(4) "The three-generation argument is weak / numerology"

Referee's concern: The connection between sedenion subalgebras and fermion generations is asserted, not derived.

Response: We accept this is the most delicate point. We have made two changes:

1. **Scope clarification**: Theorem 6.5 now explicitly states it holds "within the Gresnigt realisation class." This is not a limitation but a precise statement: the result is conditional on a specific algebraic framework that currently provides the most developed connection between division algebras and fermion generations.
2. **Interpretation Lemma**: We clarify why three octonionic subalgebras must be interpreted as generations: they carry identical gauge representations, introduce no new quantum numbers, and are permuted by S_3. In Standard Model terminology, this is precisely the definition of "generation."

We do not infer "3" from coincidence; we identify an automorphism-permuted replication of an identical representation sector with no new charges.

Revision: Theorem 6.5 reformulated with explicit scope qualifier; attribution to Gresnigt (2023) made explicit throughout.

(5) "Mass hierarchy section overreaches"

Referee's concern: The "Monotonicity Theorem" assumes "depth-dimension monotonicity" without proof.

Response: Agreed. We have reclassified this result as a *Proposition* with explicit conditional structure:

> **Proposition** (Conditional Monotonicity): Within any framework satisfying (1) masses arise from RG-renormalised operators, and (2) depth-dimension monotonicity holds, the mass ordering is monotone with depth.

This does not predict mass values; it predicts a robust ordering structure conditional on stated assumptions.

Revision: "Theorem" changed to "Proposition" in companion paper; conditional structure made explicit.

(6) "Counterexamples are unconvincing"

Referee's concern: Has the author verified that $U(1)'$ extensions genuinely violate realisability?

Response: The argument proceeds via the Saturation Theorem (new):

1. The gauge structure $SU(3) \times SU(2) \times U(1)$ exhausts the automorphism content of $\mathbb{C} \otimes \mathbb{O}$.
2. An additional $U(1)'$ requires either a second independent phase rotation (impossible: single $U(1)$ saturates abelian content) or extension beyond octonions (requires sedenions or higher $\Rightarrow$ zero divisors $\Rightarrow$ violates realisability).

Effective $U(1)_{B-L}$ is *not* excluded; DD constrains *fundamental* gauge structure, not emergent effective symmetries.

Revision: Added Section 7.4 "Fundamental vs. Effective Structure" with explicit table distinguishing fundamental from effective.

(7) "Philosophical claims are overblown"

Referee's concern: Statements like "Existence is the closure of distinctions" are rhetoric, not philosophy.

Response: We have moved interpretive statements to a separate one-page note ("Realisability Axiom") intended for philosophical context. The main paper now focuses on mathematical and physical content.

Revision: Philosophical framing removed from main paper; concentrated in separate axiom note marked as "Interpretive."

Specific Questions

Q1: Can you state realisability purely mathematically?

Yes. See revised Definition 6.1 with axioms (R1)–(R3). This is precisely a normed division algebra.

Q2: What prediction differs from standard model-building?

DD does not predict new particles. It predicts *structural impossibility* of certain extension classes as fundamental UV-complete structure while preserving realisability axioms. Specifically: no fourth generation, no fundamental extra $U(1)$, no fundamental vector-like fermions.

Q3: If sedenions have zero divisors, why are they relevant?

Sedenions are not the carrier of fundamental processes. They serve as an *analytical tool*: the attempt to extend beyond Hurwitz fails to produce a new division structure but leaves a "residue" of exactly three octonionic subalgebras permuted by S_3. Sedenions mark the boundary, not the foundation.

Q4: Exhibit anomaly-free, consistent structure violating DD

Any structure violating (R1)–(R3) violates DD by construction. The question is whether there exist anomaly-free, perturbatively consistent theories that also violate invertibility or norm multiplicativity. Such theories would describe non-unitary or irreversible processes at the fundamental level. This is the content of the realisability constraint: we exclude such theories *by axiom*, not by calculation.

Summary of Revisions

1. Definition 6.1 rewritten with pure mathematical axioms (R1)–(R3)
2. Theorem 6.5 scoped to "Gresnigt realisation class"
3. Mass-depth result reclassified as Proposition (conditional)
4. Saturation Theorem added (companion §2.4)
5. Fundamental/Effective distinction added (companion §7.4)
6. Contributions block added to Introduction
7. Philosophical content moved to separate axiom note

We believe these revisions address all substantive concerns while preserving the core contribution: DD as a constraint framework that organises known results into a unified derivation chain with explicit logical dependencies.