MATH 3100 FALL 2020. LECTURE SUMMARIES

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1. 8/25

Section 1.1 in the textbook.

(1) Sample space — long abstract definition which encompasses all possible mathematical models of randomness we are going to see in the course
(2) Examples of sample spaces — coin tossing, dice rolling.
(3) We are discussing finite sample spaces so far. Out of finite sample spaces, a special case is formed by *finite sample spaces with equally likely outcomes*. In them, we have $\mathsf{P}(\omega) = \dfrac{1}{\#\Omega}$ for all $\omega \in \Omega$, and $\mathsf{P}(A) = \dfrac{\#A}{\#\Omega}$ for all events A.
(4) Repeated experiments, sample space $\Omega^n = \Omega \times \ldots \times \Omega$ (Cartesian power), where
$$\Omega^n = \{(a_1, \ldots, a_n) \colon a_i \in \Omega\}$$
is the space of ordered n-tuples of elements from Ω. The sample space Ω^n models the experiment corresponding to Ω, repeated (independently) n times.
(5) Finer point. In uncountable sample spaces, usually it is not possible to define P consistently for all subsets. Therefore, we need to restrict the definition of event to "good" subsets of Ω.

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Section 1.2 in the textbook.

(1) Random sampling. We stay in the scenario with finite sample spaces, equally likely outcomes.
(2) We discuss three main sampling schemes of k objects out of n objects.
(3) If we sample with replacement and order matters, then $\#\Omega = n^k$
(4) If we sample without replacement and order matters, then
$$\#\Omega = n(n-1)\ldots(n-k+1) = \frac{n!}{(n-k)!}.$$
If $k = n$, we talk about random permutations of n objects.
(5) If we sample without replacement and order does not matter, then
$$\#\Omega = \frac{n(n-1)\ldots(n-k+1)}{k!} = \frac{n!}{(n-k)!\,k!} = \binom{n}{k}.$$
(6) Hypergeometric distribution. Imagine we have n objects separated into a number of types $1, \ldots, L$, and there are m_j objects of type j. So that $m_1 + \ldots + m_L = n$. Sample k objects at random from n. The probability that in this sample there are p_j objects of type j is equal

to

$$\frac{\binom{m_1}{p_1}\cdots\binom{p_L}{m_L}}{\binom{n}{k}},$$

where $p_1 + \ldots + p_L = k$.

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