

MATH 3100 FALL 2020. LECTURE SUMMARIES

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1. 8/25

Section 1.1 in the textbook.

- (1) Sample space — long abstract definition which encompasses all possible mathematical models of randomness we are going to see in the course
- (2) Examples of sample spaces — coin tossing, dice rolling.
- (3) We are discussing finite sample spaces so far. Out of finite sample spaces, a special case is formed by *finite sample spaces with equally likely outcomes*. In them, we have $P(\omega) = \frac{1}{\#\Omega}$ for all $\omega \in \Omega$, and $P(A) = \frac{\#A}{\#\Omega}$ for all events A .
- (4) Repeated experiments, sample space $\Omega^n = \Omega \times \dots \times \Omega$ (Cartesian power), where

$$\Omega^n = \{(a_1, \dots, a_n) : a_i \in \Omega\}$$

is the space of ordered n -tuples of elements from Ω . The sample space Ω^n models the experiment corresponding to Ω , repeated (independently) n times.

- (5) Finer point. In uncountable sample spaces, usually it is not possible to define P consistently for all subsets. Therefore, we need to restrict the definition of event to “good” subsets of Ω .

2. 8/27

Section 1.2 in the textbook.

- (1) Random sampling. We stay in the scenario with finite sample spaces, equally likely outcomes.
- (2) We discuss three main sampling schemes of k objects out of n objects.
- (3) If we sample with replacement and order matters, then $\#\Omega = n^k$
- (4) If we sample without replacement and order matters, then

$$\#\Omega = n(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!}.$$

If $k = n$, we talk about random permutations of n objects.

- (5) If we sample without replacement and order does not matter, then

$$\#\Omega = \frac{n(n-1)\dots(n-k+1)}{k!} = \frac{n!}{(n-k)!k!} = \binom{n}{k}.$$

- (6) Hypergeometric distribution. Imagine we have n objects separated into a number of types $1, \dots, L$, and there are m_j objects of type j . So that $m_1 + \dots + m_L = n$. Sample k objects at random from n . The probability that in this sample there are p_j objects of type j is equal

to

$$\frac{\binom{m_1}{p_1} \cdots \binom{p_L}{m_L}}{\binom{n}{k}},$$

where $p_1 + \dots + p_L = k$.

3. 9/1

Sections 1.3 and 1.4 in the textbook.

- (1) Geometric distribution $P(k) = p^{k-1}(1-p)$, $k = 1, 2, \dots$. This is an example of an infinite Ω . Here Ω is countable. Countable and finite sample spaces have a special unifying name, “discrete sample spaces”.
- (2) Geometric series, its sum = $\frac{\text{first term}}{1 - \text{ratio}}$.
- (3) Continuous uniform distribution on $[0, 1]$ — another example of an infinite Ω . This Ω is uncountable.
- (4) $P(A)$ behaves like area of the event, both in continuous uniform case and in general (in some sense).
- (5) Operations on events and their probabilities: decomposition, complement, monotonicity, inclusion-exclusion.

4. 9/3

Sections 1.4 and 1.5 in the textbook.

- (1) Operations on events and their probabilities, and corresponding examples: decomposition, complements, monotonicity, inclusion-exclusion.
- (2) For monotonicity, a proof that we will see T with probability 1, after repeatedly tossing a fair coin.
- (3) For inclusion-exclusion, discussed a hard problem of computing the probability that no one has their own hat, if the hats are randomly permuted.
- (4) Random variable is a function on the sample space. This is the second fundamental definition of the course.
- (5) Discussed the definition, examples of random variables on discrete and continuous sample spaces.
- (6) Probability mass function (for discrete random variables). Probability distribution.

5. 9/8

Conditional probability and Bayes’ rule (Sections 2.1 and 2.2 d)

- (1) Definition of conditional probability $P(A | B)$
- (2) Multiplication rule $P(AB) = P(B)P(A | B)$.
- (3) Law of total probability $P(A) = \sum_{k=1}^N P(A | B_k)P(B_k)$, where $\Omega = \bigcup_{k=1}^N B_k$ is a partition of the sample space.
- (4) Bayes’s formula

$$P(B | A) = \frac{P(A | B)P(B)}{P(A)} = \frac{P(A | B)P(B)}{P(A | B)P(B) + P(A | B^c)P(B^c)}.$$

6. 9/10

- (1) Some hints on the most challenging problems from Problem Set 3.
- (2)