MATH 3100 FALL 2020. LECTURE SUMMARIES

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1. 8/25

Section 1.1 in the textbook.

(1) Sample space — long abstract definition which encompasses all possible mathematical models of randomness we are going to see in the course
(2) Examples of sample spaces — coin tossing, dice rolling.
(3) We are discussing finite sample spaces so far. Out of finite sample spaces, a special case is formed by *finite sample spaces with equally likely outcomes*. In them, we have $\mathsf{P}(\omega) = \dfrac{1}{\#\Omega}$ for all $\omega \in \Omega$, and $\mathsf{P}(A) = \dfrac{\#A}{\#\Omega}$ for all events A.
(4) Repeated experiments, sample space $\Omega^n = \Omega \times \ldots \times \Omega$ (Cartesian power), where

$$\Omega^n = \{(a_1, \ldots, a_n) \colon a_i \in \Omega\}$$

is the space of ordered n-tuples of elements from Ω. The sample space Ω^n models the experiment corresponding to Ω, repeated (independently) n times.
(5) Finer point. In uncountable sample spaces, usually it is not possible to define P consistently for all subsets. Therefore, we need to restrict the definition of event to "good" subsets of Ω.

2. 8/27

Section 1.2 in the textbook.

(1) Random sampling. We stay in the scenario with finite sample spaces, equally likely outcomes.
(2) We discuss three main sampling schemes of k objects out of n objects.
(3) If we sample with replacement and order matters, then $\#\Omega = n^k$
(4) If we sample without replacement and order matters, then

$$\#\Omega = n(n-1)\ldots(n-k+1) = \frac{n!}{(n-k)!}.$$

If $k = n$, we talk about random permutations of n objects.
(5) If we sample without replacement and order does not matter, then

$$\#\Omega = \frac{n(n-1)\ldots(n-k+1)}{k!} = \frac{n!}{(n-k)!\,k!} = \binom{n}{k}.$$

(6) Hypergeometric distribution. Imagine we have n objects separated into a number of types $1, \ldots, L$, and there are m_j objects of type j. So that $m_1 + \ldots + m_L = n$. Sample k objects at random from n. The probability that in this sample there are p_j objects of type j is equal

to

$$\frac{\binom{m_1}{p_1}\cdots\binom{p_L}{m_L}}{\binom{n}{k}},$$

where $p_1 + \ldots + p_L = k$.

3. 9/1

Sections 1.3 and 1.4 in the textbook.

(1) Geometric distribution $\mathsf{P}(k) = p^{k-1}(1-p)$, $k = 1, 2, \ldots$. This is an example of an infinite Ω. Here Ω is countable. Countable and finite sample spaces have a special unifying name, "discrete sample spaces".
(2) Geometric series, its sum $= \dfrac{\text{first term}}{1 - \text{ratio}}$.
(3) Continuous uniform distribution on $[0, 1]$ — another example of an infinite Ω. This Ω is uncountable.
(4) $\mathsf{P}(A)$ behaves like area of the event, both in continuous uniform case and in general (in some sense).
(5) Operations on events and their probabilities: decomposition, complement, monotonicity, inclusion-exclusion.

4. 9/3

Sections 1.4 and 1.5 in the textbook.

(1) Operations on events and their probabilities, and corresponding examples: decomposition, complements, monotonicity, inclusion-exclusion.
(2) For monotonicity, a proof that we will see T with probability 1, after repeatedly tossing a fair coin.
(3) For inclusion-exclusion, discussed a hard problem of computing the probability that no one has their own hat, if the hats are randomly permuted.
(4) Random variable is a function on the sample space. This is the second fundamental definition of the course.
(5) Discussed the definition, examples of random variables on discrete and continuous sample spaces.
(6) Probability mass function (for discrete random variables). Probability distribution.