

MATH 3100 FALL 2020. LECTURE SUMMARIES

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1. 8/25

Section 1.1 in the textbook.

- (1) Sample space — long abstract definition which encompasses all possible mathematical models of randomness we are going to see in the course
- (2) Examples of sample spaces — coin tossing, dice rolling.
- (3) We are discussing finite sample spaces so far. Out of finite sample spaces, a special case is formed by *finite sample spaces with equally likely outcomes*. In them, we have $P(\omega) = \frac{1}{\#\Omega}$ for all $\omega \in \Omega$, and $P(A) = \frac{\#A}{\#\Omega}$ for all events A .
- (4) Repeated experiments, sample space $\Omega^n = \Omega \times \dots \times \Omega$ (Cartesian power), where

$$\Omega^n = \{(a_1, \dots, a_n) : a_i \in \Omega\}$$

is the space of ordered n -tuples of elements from Ω . The sample space Ω^n models the experiment corresponding to Ω , repeated (independently) n times.

- (5) Finer point. In uncountable sample spaces, usually it is not possible to define P consistently for all subsets. Therefore, we need to restrict the definition of event to “good” subsets of Ω .

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Section 1.2 in the textbook.

- (1) Random sampling. We stay in the scenario with finite sample spaces, equally likely outcomes.
- (2) We discuss three main sampling schemes of k objects out of n objects.
- (3) If we sample with replacement and order matters, then $\#\Omega = n^k$
- (4) If we sample without replacement and order matters, then

$$\#\Omega = n(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!}.$$

If $k = n$, we talk about random permutations of n objects.

- (5) If we sample without replacement and order does not matter, then

$$\#\Omega = \frac{n(n-1)\dots(n-k+1)}{k!} = \frac{n!}{(n-k)!k!} = \binom{n}{k}.$$

- (6) Hypergeometric distribution. Imagine we have n objects separated into a number of types $1, \dots, L$, and there are m_j objects of type j . So that $m_1 + \dots + m_L = n$. Sample k objects at random from n . The probability that in this sample there are p_j objects of type j is equal

to

$$\frac{\binom{m_1}{p_1} \cdots \binom{p_L}{m_L}}{\binom{n}{k}},$$

where $p_1 + \dots + p_L = k$.

3. 9/1

Sections 1.3 and 1.4 in the textbook.

- (1) Geometric distribution $P(k) = p^{k-1}(1-p)$, $k = 1, 2, \dots$. This is an example of an infinite Ω . Here Ω is countable. Countable and finite sample spaces have a special unifying name, “discrete sample spaces”.
- (2) Geometric series, its sum = $\frac{\text{first term}}{1 - \text{ratio}}$.
- (3) Continuous uniform distribution on $[0, 1]$ — another example of an infinite Ω . This Ω is uncountable.
- (4) $P(A)$ behaves like area of the event, both in continuous uniform case and in general (in some sense).
- (5) Operations on events and their probabilities: decomposition, complement, monotonicity, inclusion-exclusion.