

# MATH 3100 SPRING 2022. LECTURE SUMMARIES

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## 1. LECTURE 1. JANUARY 19

Introductory lecture, just saying hello to the students.

## 2. LECTURE 2. NO DEADLINE

Section 1.1

A recording of an in-class piece, explaining the definition of the probability space  $\Omega$ , events  $\mathcal{F}$ , and probability measure  $P(A)$ .

## 3. LECTURE 3. JANUARY 21

Sections 1.1–1.2.

- Recalling  $(\Omega, \mathcal{F}, P)$ .
- Finite sample spaces, when it is enough to specify  $P(\{\omega\})$  for each singleton  $\omega \in \Omega$ . We only need to have  $P(\{\omega\}) \geq 0$  and  $\sum_{\omega \in \Omega} P(\{\omega\}) = 1$
- Biased coin, 2 flips — example
- Equally likely outcomes,  $P(A) = \frac{\#A}{\#\Omega}$ .
- Example with urns.

## 4. LECTURE 4. JANUARY 24

Section 1.2. Discussing 3 settings, when we sample  $k$  times from  $n$  objects. This is an instance of equally likely outcomes. Recall the factorial:

$$\begin{cases} n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n, & n \geq 1 \\ 0! = 1. \end{cases}$$

So,  $0! = 1$ ,  $1! = 1$ ,  $2! = 2$ ,  $3! = 6$ ,  $4! = 24$ , and so on.

- Sampling with replacement, order matters:  $\#\Omega_k = n^k$ .
- Sampling without replacement, order matters:  $\#\Omega_k = n(n-1) \dots (n-k+1) = \frac{n!}{(n-k)!}$ .
- Sampling without replacement, order does not matter:

$$\#\Omega_k = \frac{n(n-1) \dots (n-k+1)}{k!} = \frac{n!}{k! (n-k)!} = \binom{n}{k}.$$

The quantity  $\binom{n}{k}$  has a special name, “binomial coefficient”.

## 5. LECTURE 5. JANUARY 26

## Section 1.3.

- Probability space for rolling a die until you see a “6”
- How to sum the geometric progression
- Continuous sample spaces,  $[0, 1]$  with length measure
- How to draw a random line in the plane (one example)

## 6. LECTURE 6. JANUARY 28

## Section 1.4. Rules of probability (theory only).

- Venn diagrams. Probability “behaves like the area of a Venn diagram”
- Decomposing an event
- Complements, also  $P(A) = P(AB) + P(AB^c)$
- Monotonicity of probability
- Inclusion-exclusion

## 7. LECTURE 7. JANUARY 31

Section 1.4. Examples on rules of probability — inclusion/exclusion principle; helpful passing to the complement event  $A^c$ .

A first look on random variables: if we flip a fair coin 5 times, and let  $Y$  be the number of Heads, what is the probability distribution of  $Y$ ? This is described in terms of the probability mass function, which in this case takes the form

$$P(Y = 0) = \frac{1}{32}, \quad P(Y = 1) = \frac{5}{32}, \quad P(Y = 2) = \frac{10}{32}, \quad P(Y = 3) = \frac{10}{32}, \quad P(Y = 4) = \frac{5}{32}, \quad P(Y = 5) = \frac{1}{32}.$$

## 8. LECTURE 8. FEB 2

## Section 1.5. Random variables.

- Definition of a random variable
- Splitting of  $\Omega$  into level sets
- Discrete, continuous, and other random variables
- Examples of discrete random variables
- Probability mass function (pmf)
- Distribution of a random variable, example of two random variables with the same distribution which are not the same.

## 9. LECTURE 9. FEBRUARY 4

## Sections 2.1, 2.2. Conditional probabilities, Bayes rule.

- Conditional probability — motivating example
- Definition of  $P(A | B)$
- Chain product of conditional probabilities
- Total probability formula (averaged conditional probabilities)
- Bayes rule
- An example, testing for rare diseases

## 10. LECTURE 10. FEBRUARY 7

## Section 2.3. Independence.

- Recall Bayes rule. One more example.
- Independence of two events.
- Independence of  $A, B$  is equivalent to the independence of  $A^c, B$  and of  $A^c, B^c$ .
- Examples 2.18, 2.21
- Independence of multiple events means we need lots of product rules.
- Independence of random variables. Coin flips. Digits in uniformly random  $\omega \in [0, 1]$ .