

MATH 3100 SPRING 2022. LECTURE SUMMARIES

LEONID PETROV

1. LECTURE 1. JANUARY 19

Introductory lecture, just saying hello to the students.

2. LECTURE 2. NO DEADLINE

Section 1.1

A recording of an in-class piece, explaining the definition of the probability space Ω , events \mathcal{F} , and probability measure $P(A)$.

3. LECTURE 3. JANUARY 21

Sections 1.1–1.2.

- Recalling (Ω, \mathcal{F}, P) .
- Finite sample spaces, when it is enough to specify $P(\{\omega\})$ for each singleton $\omega \in \Omega$. We only need to have $P(\{\omega\}) \geq 0$ and $\sum_{\omega \in \Omega} P(\{\omega\}) = 1$
- Biased coin, 2 flips — example
- Equally likely outcomes, $P(A) = \frac{\#A}{\#\Omega}$.
- Example with urns.

4. LECTURE 4. JANUARY 24

Section 1.2. Discussing 3 settings, when we sample k times from n objects. This is an instance of equally likely outcomes. Recall the factorial:

$$\begin{cases} n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n, & n \geq 1 \\ 0! = 1. \end{cases}$$

So, $0! = 1$, $1! = 1$, $2! = 2$, $3! = 6$, $4! = 24$, and so on.

- Sampling with replacement, order matters: $\#\Omega_k = n^k$.
- Sampling without replacement, order matters: $\#\Omega_k = n(n-1) \dots (n-k+1) = \frac{n!}{(n-k)!}$.
- Sampling without replacement, order does not matter:

$$\#\Omega_k = \frac{n(n-1) \dots (n-k+1)}{k!} = \frac{n!}{k! (n-k)!} = \binom{n}{k}.$$

The quantity $\binom{n}{k}$ has a special name, “binomial coefficient”.

5. LECTURE 5. JANUARY 26

Section 1.3.

- Probability space for rolling a die until you see a “6”
- How to sum the geometric progression
- Continuous sample spaces, $[0, 1]$ with length measure
- How to draw a random line in the plane (one example)

6. LECTURE 6. JANUARY 28

Section 1.4. Rules of probability.

- Venn diagrams. Probability “behaves like the area of a Venn diagram”
- Decomposing an event
- Complements, also $P(A) = P(AB) + P(AB^c)$
- Monotonicity of probability
- Inclusion-exclusion