MATH 3100 SPRING 2022. LECTURE SUMMARIES

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1. Lecture 1. January 19

Introductory lecture, just saying hello to the students.

2. Lecture 2. No deadline

Section 1.1

A recording of an in-class piece, explaining the definition of the probability space Ω, events $\mathcal{F}$, and probability measure $P(A)$.

3. Lecture 3. January 21

Sections 1.1–1.2.

- Recalling $(\Omega, \mathcal{F}, P)$.
- Finite sample spaces, when it is enough to specify $P(\{\omega\})$ for each singleton $\omega \in \Omega$. We only need to have $P(\{\omega\}) \geq 0$ and $\sum_{\omega\in\Omega} P\left(\{\omega\}\right) = 1$
- Biased coin, 2 flips — example
- Equally likely outcomes, $P(A) = \frac{\#A}{\#\Omega}$.
- Example with urns.

4. Lecture 4. January 24

Section 1.2. Discussing 3 settings, when we sample k times from n objects. This is an instance of equally likely outcomes. Recall the factorial:

$$\begin{cases} n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n, & n \geq 1 \\ 0! = 1. \end{cases}$$

So, $0! = 1$, $1! = 1$, $2! = 2$, $3! = 6$, $4! = 24$, and so on.

- Sampling with replacement, order matters: $\#\Omega_k = n^k$.
- Sampling without replacement, order matters: $\#\Omega_k = n(n-1)\ldots(n-k+1) = \dfrac{n!}{(n-k)!}$.
- Sampling without replacement, order does not matter:

$$\#\Omega_k = \frac{n(n-1)\ldots(n-k+1)}{k!} = \frac{n!}{k!\,(n-k!)} = \binom{n}{k}.$$

The quantity $\binom{n}{k}$ has a special name, "binomial coefficient".

5. Lecture 5. January 26

Section 1.3.

- Probability space for rolling a die until you see a “6”
- How to sum the geometric progression
- Continuous sample spaces, $[0, 1]$ with length measure
- How to draw a random line in the plane (one example)

6. Lecture 6. January 28

Section 1.4. Rules of probability (theory only).

- Venn diagrams. Probability “behaves like the area of a Venn diagram”
- Decomposing an event
- Complements, also $P(A) = P(AB) + P(AB^c)$
- Monotonicity of probability
- Inclusion-exclusion

7. Lecture 6. January 31

Section 1.4. Examples on rules of probability — inclusion/exclusion principle; helpful passing to the complement event A^c.

A first look on random variables: if we flip a fair coin 5 times, and let Y be the number of Heads, what is the probability distribution of Y? This is described in terms of the probability mass function, which in this case takes the form

$$P(Y = 0) = \frac{1}{32}, \quad P(Y = 1) = \frac{5}{32}, \quad P(Y = 2) = \frac{10}{32}, \quad P(Y = 3) = \frac{10}{32}, \quad P(Y = 4) = \frac{5}{32}, \quad P(Y = 5) = \frac{1}{32}.$$