MATH 3100 SPRING 2022. LECTURE SUMMARIES

LEONID PETROV

1. Lecture 1. January 19

Introductory lecture, just saying hello to the students.

2. Lecture 2. No deadline

Section 1.1

A recording of an in-class piece, explaining the definition of the probability space Ω, events $\mathcal{F}$, and probability measure $P(A)$.

3. Lecture 3. January 21

Sections 1.1–1.2.

- Recalling $(\Omega, \mathcal{F}, P)$.
- Finite sample spaces, when it is enough to specify $P(\{\omega\})$ for each singleton $\omega \in \Omega$. We only need to have $P(\{\omega\}) \geq 0$ and $\sum_{\omega \in \Omega} P(\{\omega\}) = 1$
- Biased coin, 2 flips — example
- Equally likely outcomes, $P(A) = \frac{\#A}{\#\Omega}$.
- Example with urns.

4. Lecture 4. January 24

Section 1.2. Discussing 3 settings, when we sample k times from n objects. This is an instance of equally likely outcomes. Recall the factorial:

$$\begin{cases} n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n, & n \geq 1 \\ 0! = 1. \end{cases}$$

So, $0! = 1$, $1! = 1$, $2! = 2$, $3! = 6$, $4! = 24$, and so on.

- Sampling with replacement, order matters: $\#\Omega_k = n^k$.
- Sampling without replacement, order matters: $\#\Omega_k = n(n-1)\ldots(n-k+1) = \dfrac{n!}{(n-k)!}$.
- Sampling without replacement, order does not matter:

$$\#\Omega_k = \frac{n(n-1)\ldots(n-k+1)}{k!} = \frac{n!}{k!\,(n-k!)} = \binom{n}{k}.$$

The quantity $\binom{n}{k}$ has a special name, "binomial coefficient".

5. Lecture 5. January 26

Section 1.3.

- Probability space for rolling a die until you see a "6"
- How to sum the geometric progression
- Continuous sample spaces, $[0, 1]$ with length measure
- How to draw a random line in the plane (one example)

6. Lecture 6. January 28

Section 1.4. Rules of probability (theory only).

- Venn diagrams. Probability "behaves like the area of a Venn diagram"
- Decomposing an event
- Complements, also $P(A) = P(AB) + P(AB^c)$
- Monotonicity of probability
- Inclusion-exclusion

7. Lecture 7. January 31

Section 1.4. Examples on rules of probability — inclusion/exclusion principle; helpful passing to the complement event A^c.

A first look on random variables: if we flip a fair coin 5 times, and let Y be the number of Heads, what is the probability distribution of Y? This is described in terms of the probability mass function, which in this case takes the form

$$P(Y=0)=\frac{1}{32},\quad P(Y=1)=\frac{5}{32},\quad P(Y=2)=\frac{10}{32},\quad P(Y=3)=\frac{10}{32},\quad P(Y=4)=\frac{5}{32},\quad P(Y=5)=\frac{1}{32}.$$

8. Lecture 8. Feb 2

Section 1.5. Random variables.

- Definition of a random variable
- Splitting of Ω into level sets
- Discrete, continuous, and other random variables
- Examples of discrete random variables
- Probability mass function (pmf)
- Distribution of a random variable, example of two random variables with the same distribution which are not the same.

9. Lecture 9. February 4

Sections 2.1, 2.2. Conditional probabilities, Bayes rule.

- Conditional probability — motivating example
- Definition of $P(A \mid B)$
- Chain product of conditional probabilities
- Total probability formula (averaged conditional probabilities)
- Bayes rule
- An example, testing for rare diseases

10. Lecture 10. February 7

Section 2.3. Independence.

- Recall Bayes rule. One more example.
- Independence of two events.
- Independence of A, B is equivalent to the independence of A^c, B and of A^c, B^c.
- Examples 2.18, 2.21
- Independence of multiple events means we need lots of product rules.
- Independence of random variables. Coin flips. Digits in uniformly random $\omega \in [0, 1]$.

11. Lecture 11. February 11

Sections 3.1, 3.2.

- Discrete random variables. They are determined through pmf. Pmf properties (2) - non-negativity, sum to one. Probability through pmf as a sum.
- Continuous random variables, in the same fashion. Probability density function. Properties of pdf (2); Probability through integrals.
- Cumulative distribution function (cdf).
  - definition
  - how it looks for continuous r.v.; connection to pdf
  - how it looks for discrete r.v.

  Some examples.

12. Lecture 11.1. February 11

This is an example of finding cdfs of random variables (X, Y) which are coordinates of a uniformly random point thrown into some figure in the plane.

13. Lecture 12. February 14

Section 3.3. Expectation.

- Expectation - discrete rv
- Expectation - continuous rv
- Example - Bernoulli; binomial without proof
- Example - geometric
- Derivative technique
- Use derivatives to get expectation of geometric
- Application to binomial distribution.
- One more example of the computation of expectation for a continuous random variable.