MATH 3100: INTRODUCTION TO PROBABILITY

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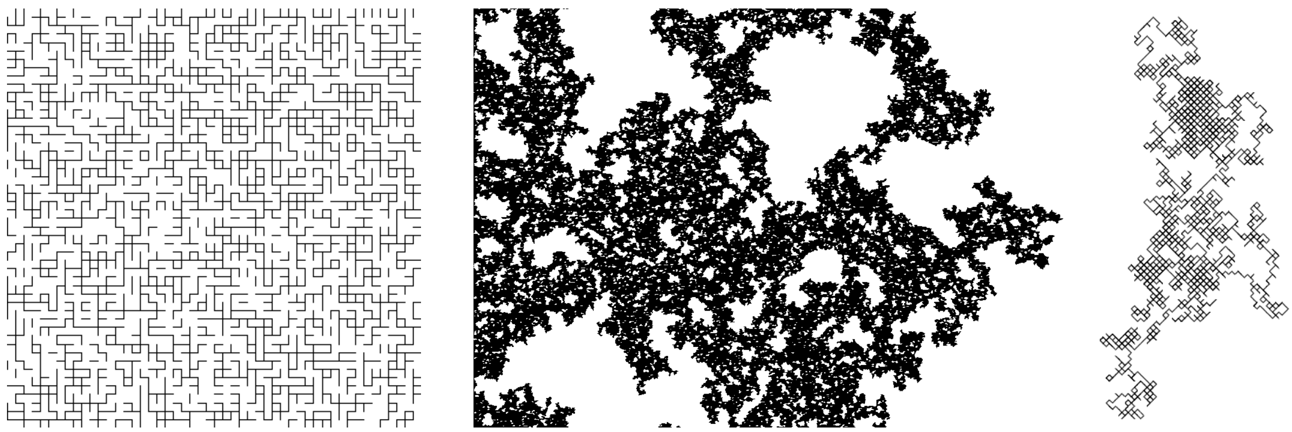
1. *A mathematical study of randomness*

How random is everything around us, and what chance do we have of understanding it? What to do when you're not certain, and how to do it right? How many falling stars will you see as you walk outside one beautiful night?

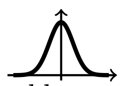
Probability theory is a mathematical study of uncertainty. It is a rigorous foundation of statistics — and many areas of human knowledge operate in a language of statistics nowadays (yes, and robots use it, too!). The course introduces fundamental concepts, ideas, and techniques of probability theory. It will provide you with foundational mathematical knowledge needed to address the questions above, and will help you develop intuition about randomness.



Examples of random structures: bond percolation `close-up` (left), at a `larger scale` (center), and a `random walk` (see also a `simulation` of a random walk). Note: this PDF has green clickable links, like in the previous sentence.

What you will get from this course.

1. Mastery of basic probability concepts:
   (a) What is a probability space and how to translate commonly-sounding problems into this language;
   (b) How to count (in an advanced way) to compute probabilities;
   (c) What is a random variable, a probability distribution, and what are their main quantitative properties;
   (d) How commonly encountered probability distributions (binomial, Poisson, exponential, Gaussian) look like and behave, what are their properties, and in which situations they typically arise.

2. How large random systems behave, and what the bell-shaped curve  has to do with this.

3. How to describe and quantify mutual dependence of random events, and how to use such a description to infer properties of "hidden" random events.

Date: Sunday 11th December, 2016, 01:18.
An up to date syllabus is always on `GitHub` at `https://github.com/lenis2000/Syllabi/blob/master/Syllabus_3100_s17.pdf`. For direct PDF download use `this link`. LaTeX source with *changes* to the syllabus is `here` (click "History").

4. How to apply probability theory to model real-life processes like queues (consisting of people or requests at an internet server).
5. How to collaborate on solving probability problems in pairs, small groups, and online, and present solutions clearly and efficiently.
6. How to design probability problems (for example, for the final exam), and evaluate problems presented by others.
7. In what ways probability theory is connected to science, engineering, and other branches of knowledge.

Prerequisite. You should have taken at least one semester of calculus, because the study of random variables often requires single and double integrals and infinite series.

What this course is and what it is not. This course in probability *theory* belongs to pure mathematics, with rigorous definitions, calculations, and proofs. However, the objects which we study are motivated by real-life applications, and so pure mathematical arguments often appeal to our common sense understanding of these objects. There will be many opportunities to explore (and discover new) connections of the theory studied in the course with real world.

Also, this course does not discuss *applications to statistics*. Probability theory focuses on developing mathematical side, and statistics applies these mathematical theories to real data (coming from observations). In this course we will not discuss how to analyze data coming from observations — there are courses in statistics for that.

2. *Necessary information*

2.1. Meeting times.

Class times	MoWe, 2:00PM – 3:15PM Monroe Hall 124
Final exam	Mo, May 8, 2:00PM – 5:00PM Monroe Hall 124

Instructor: Leonid Petrov
Email: We use `Slack` instead, see Section 4
Office: 209 Kerchof Hall
Office hours: (TBA), or by appointment (you can make as many as you want)

2.2. **About the instructor.** I am an assistant professor in the Department of Mathematics at UVA, and I've been here since 2014. My research area is probability theory (very appropriate for this course!). More precisely, I am using exact formulas to study large random systems. I also like computer simulations of random systems, some examples are `at this link` or at my office door. I'll be happy to tell you more if you're interested.

2.3. **Textbook.** "Probability" by Jim Pitman. See also Section 5 below for discussion and more helpful resources.

3. *Assessing your learning*

(first: communicate things clearly; what probability theory is and what it is not; etc)

3.1. **Course engagement (12%).** There will be short quizzes, group brainstorming sessions, and other activities during the class (in particular, using `Slack`), and actively engaging in them is important to enhance the grasp on the foundations of probability theory and problem-solving skills. The grade for this part will come from:

- Quizzes.
- Attendance, which may sometimes be taken.
- Ungraded homework. All homework will be collected and some of it will be graded, see below. If a homework is ungraded, collecting it will help me identify and address troubles with the material. Ungraded homework which is turned in will generally receive a "check" mark. Rarely a "check−" will be assigned if less than half of homework problems was attempted, or something similar.

- In-class work sometimes it will be collected and assessed, too.
- Participation in polls, weekly check-ins, and discussions (in particular, answering other student's questions) in `Slack`.

3.2. **Homework (20%).** Learning mathematics means *doing* mathematics. In this course, this amounts to solving problems. Homework assignments will help you achieve mastery of knowledge and skills pertaining to the course, and prepare for in-class work. You are encouraged to work together on homework assignments (also can do it online via `Slack`, which allows private groups of up to 9 people). Teams of two work very well. Most mathematicians work in pairs to take advantage of the challenge-defend discussions that help us understand things better. However, each student needs to submit her/his own homework assignments, and should work individually when writing them up to demonstrate the understanding of the material. The homework assignments are due approximately once a week. Some will be graded and some not — this will be announced in advance.

3.3. **Projects (13%).** There will be 2 longer projects in the course. The first project is a group assignment in the middle of the semester that will help you apply your knowledge and skills to other areas and/or to real-world situations, and will help you enhance your collaborative and presentation skills. You can choose to either make a computer simulation of an interesting random system and then experimentally describe its behavior; or to come up with a probabilistic model of a real-world phenomenon, and use the model to quantitatively understand it. Possible group projects topics are available at `https://TBA...`. In the second assignment, you will get to compose a good problem for the final exam (which even has a chance to end up in the actual exam!). This will put you in the shoes of the instructor, and will let you look at the course material at a different angle.

3.4. **Tests (15%+15%+25%, total: 55%).** There will be 2 tests during the semester, and a final exam (the second test and the exam are cumulative). They present an ultimate opportunity to demonstrate your knowledge and problem-solving abilities. Tests and the exam will usually consist of problems similar to the ones from the textbook and lectures, and occasionally there can be conceptual theoretic questions. A two-sided letter size formula sheet, hand-written by yourself, will be allowed on each test and the exam. Preparing this formula sheet will help you review the material, and paint a systematic picture in your head. I encourage you to collaborate on test preparation, but needless to say that during tests and exams each student must work individually.

4. *Communication*

My email is petrov@virginia.edu, but for the communication we will use `Slack` — an industrial standard of work chats, and it has a web version and apps for all platforms. The course group is at `https://TBA.slack.com`, and you'll get an invitation to join by e-mail. Please let me know if you have issues with access. It is also expected that you will bring a device with `Slack` app (e.g., a smartphone) to each class, and also check-in with me every week (by 10am on Monday).

5. *How to be successful in the course*

textbook

- You don't have to bring the textbook to all classes, although this may be helpful sometimes.
- I strongly encourage you to read the textbook. It includes many examples and extra exercises which augment the concepts discussed in class.
- The textbook contains much more material than will be covered in classes, so it makes sense to come to all classes and see which parts of the textbook are covered and which are omitted.
- Class notes which I prepare for each lecture will be made available on `Slack`.

other references / other textbooks

khan academy, wikipedia, other places containing basic stuff on probability theory

— office hours, availability in `Slack`, your fellow students, etc.

The Math Department Tutoring Center is available for helping students in this course: see `http://people.virginia.edu/~psb7p/MTCsch.html` for more information and schedule.

6. *Approximate course schedule*

Add/drop information: http://www.virginia.edu/registrar/reginst1158.html#Deadlines

NOTE: Please don't make travel plans that conflict with tests or final exam

Week	**Topic**	**Notes**	**Homework**	**Sections**	**Slack direct message check-in (by 10 am on Monday each week)**	**Due items for the projects**
1: 1/18	What is probability theory	1/18: Introduction		TBA		
2: 1/23. 1/25	Advanced counting. Conditional probabilities			TBA	Individual: send me a test direct message answering why you are taking the course - to make sure it works	
3: 1/30. 2/1	Repeated trials and Gaussian approximation	2/1: group forming		TBA	Individual: briefly describe a real-world problem (preferably in one of your other courses) that you think can be accessed via probability theory	Wed 2/1 class time: form groups of up to 6 people and let me know. It will be reflected in slack
4: 2/6. 2/8	Random variables	2/8: real-world problem discussions	By 2/6 class time: read this link to see how random variables look like	TBA	Group: send me a test message in the private group channel to make sure it works. Also mention which real-world problem you are thinking about.	Wed 2/8 class time: choose a real-world problem that you would like to turn into a probability problem
5: 2/13. 2/15	Expectation and conditional expectation	2/15: probability problems discussions		TBA	Group: are you doing OK with formulating your probability problem? If you are not sure talk to me during office hours - or in Slack.	Mon 2/13 class time: Choose track: analysis or simulation. Formulate a (draft) probability problem / simulation model associated with your real-world setting
	Test 1: 2/20			list all sections		
6: 2/20. 2/22	Poisson processes			TBA	Individual: are there any last-minute questions before the test?	
7: 2/27. 3/1	Poisson processes. Exponential and gamma distributions	3/1: project discussion before the break		TBA	Group: send me a polished version of the probability problem / simulation model you are working on	Wed 3/1 class time: present a draft solution of your probability problem / draft simulation of your model (at least in the simplest case)
8: 3/13. 3/15	Poisson processes. Exponential and gamma distributions			TBA	Individual: how was your break? // Group: is everything OK with the project? Need any help?	Over the break and this week: continue working on the group project

9: 3/20. 3/22	Continuous distributions. General Gaussian approximation	3/22: full class: group presentations		TBA	Group: are you OK with your reports? who is going to present? Need any help?	Mon 3/20 class time: group reports due; Wed 3/22: group presentations
10: 3/27. 3/29	Continuous distributions			TBA	Individual: any feedback on the group project? Are you doing OK in the course so far?	
	Test 2: 4/3			list all sections		
11: 4/3. 4/5	Joint and conditional distributions			TBA	Individual: are there any last-minute questions before the test?	Sun 4/8 by 11:59pm: select topic at https://TBA on which you will compose your problem
12: 4/10. 4/12	Joint and conditional distributions			TBA	Individual: which textbook problem did you choose? Are you doing OK with the evaluation of this textbook problem?	Wed 4/12 class time: textbook problem evaluation due
13: 4/17. 4/19	Joint and conditional distributions	Some of the final exam review problems will be posted by 4/23		TBA	Individual: are you doing OK with composing your problem?	Wed 4/19 class time: your problem due
14: 4/24. 4/26	Bivariate normal distributions. Applications to statistics			TBA	Individual: are you doing OK with peer evaluation of a problem you were given?	Sun 4/29 by 11:59pm: peer evaluation due
15: 5/1	Discussion of peer evaluation and final exam review	5/1 full class: discussion and review		TBA	Individual: which topic you found the clearest in the course? which was the muddiest?	
	Final exam: TBA			list all sections	You can ask questions before the final exam in the #general channel at any time	

7. *Policies*

7.1. `Slack`**.** Although `Slack` is a chatting app, it should be used professionally, especially in public discussions. The app also supports private direct messages and I encourage to use them to collaborate on homework problems and projects, please note that in principle the admin (i.e., myself) can obtain access to **all** direct messages between members of the team. The procedure would involve sending a paper request via the usual mail, and everyone will be notified if the direct messages are accessed — so this can happen only in extreme circumstances.

7.2. **Independent work, honor code.** You are required to work independently on the quizzes. So when working together with others, make sure you are preparing yourself to take the quiz independently. The honor code is taken seriously. Any honor code violations pertaining to the quizzes will be automatically referred to the Honor Committee.

7.3. **Special needs accommodations.** All students with special needs requiring accommodations should present the appropriate paperwork from the Student Disability Access Center (SDAC). It is the student's responsibility to present this paperwork in a timely fashion and follow up with the instructor about the accommodations being offered. Accommodations for test-taking (e.g., extended time) should be arranged at least 5 business days before an exam.