MATH 3100: INTRODUCTION TO PROBABILITY

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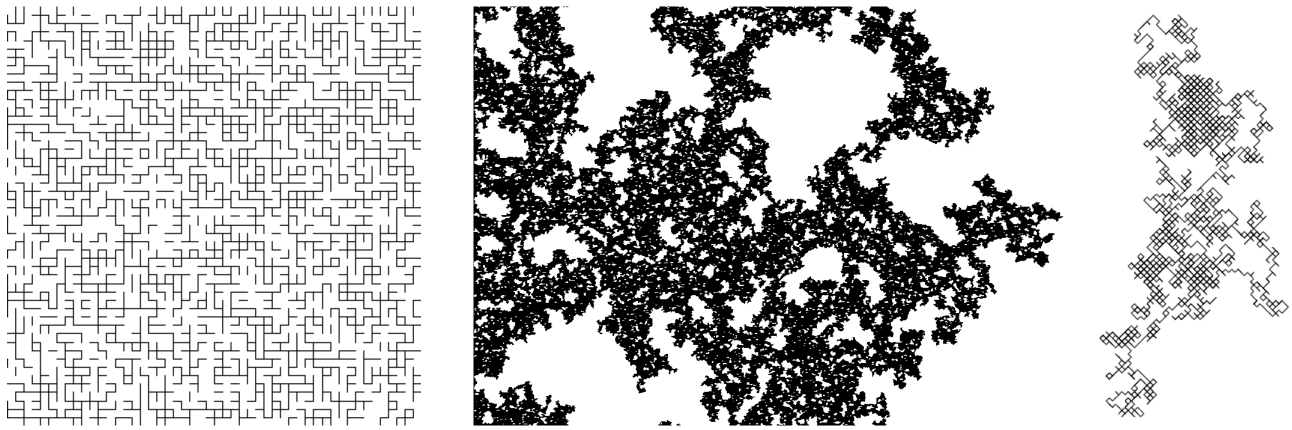
1. *A mathematical study of randomness*

How random is everything around us, and what chance do we have of understanding it? What to do when you're not certain, and how to do it right? How many falling stars will you see as you walk outside one beautiful night?

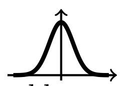
Probability theory is a mathematical study of uncertainty. It is a rigorous foundation of statistics — and many areas of human knowledge operate in a language of statistics nowadays (yes, and robots use it, too!). The course introduces fundamental concepts, ideas, and techniques of probability theory. It will provide you with foundational mathematical knowledge needed to address the questions above, and will help you develop intuition about randomness.



Examples of random structures: bond percolation `close-up` (left), at a `larger scale` (center), and a `random walk` (see also a `simulation` of a random walk). Note: this PDF has green clickable links, like in the previous sentence.

What you will get from this course.

1. Mastery of basic probability concepts:
   (a) What is a probability space and how to translate commonly-sounding problems into this language;
   (b) How to count (in an advanced way) to compute probabilities;
   (c) What is a random variable, a probability distribution, and what are their main quantitative properties;
   (d) How commonly encountered probability distributions (binomial, Poisson, exponential, Gaussian) look like and behave, what are their properties, and in which situations they typically arise.

2. How large random systems behave, and what the bell-shaped curve  has to do with this.

3. How to describe and quantify mutual dependence of random events, and how to use such a description to infer properties of "hidden" random events.

Date: Sunday 11$^{\text{th}}$ December, 2016, 03:12.

An up to date syllabus is always on `GitHub` at `https://github.com/lenis2000/Syllabi/blob/master/Syllabus_3100_s17.pdf`. For direct PDF download use `this link`. LaTeX source with *changes* to the syllabus is `here` (click "History").

4. How to apply probability theory to model real-life processes like queues (consisting of people or requests at an internet server).
5. How to collaborate on solving probability problems in pairs, small groups, and online, and present solutions clearly and efficiently.
6. How to design probability problems (for example, for the final exam), and evaluate problems presented by others.
7. In what ways probability theory is connected to science, engineering, and other branches of knowledge.

Prerequisite. You should have taken at least one semester of calculus, because the study of random variables often requires single and double integrals and infinite series.

What this course is and what it is not. This course in probability *theory* belongs to pure mathematics, with rigorous definitions, calculations, and proofs. However, the objects which we study are motivated by real-life applications, and so pure mathematical arguments often appeal to our common sense understanding of these objects. There will be opportunities to explore (and discover new) connections of the theory studied in the course with real world.

Also, this course does not thoroughly discuss *applications to statistics* (except maybe in the group project, see Section 3.3). Probability theory focuses on developing the mathematical side, and statistics applies these mathematical theories to real data (coming from observations). In this course we will not discuss how to analyze data coming from observations — there are courses in statistics for that.

2. *Necessary information*

2.1. Meeting times.

Class times	MoWe, 2:00PM – 3:15PM Monroe Hall 124
Final exam	Mo, May 8, 2:00PM – 5:00PM Monroe Hall 124

Instructor: Leonid Petrov
Email: We use `Slack` instead, see Section 4
Office: 209 Kerchof Hall
Office hours: (TBA), or by appointment (you can make as many as you want)

2.2. About the instructor.

I am an assistant professor in the Department of Mathematics at UVA, and I've been here since 2014. My research area is probability theory (very appropriate for this course!). More precisely, I am using exact formulas to study large random systems. I also like computer simulations of random systems, some examples are `at this link` or at my office door. I'll be happy to tell you more if you're interested.

I also co-organize the UVA Math Club (`http://pi.math.virginia.edu/mathclub/`), let me know if you'd like to be added to the Math Club mailing list.

2.3. Textbook.

"Probability" by Jim Pitman. See also Section 5 below for discussion of how we'll use the textbook, and for other helpful resources.

3. *Assessing your learning*

Learning mathematics means *doing* mathematics: during class meetings, on your own, and in groups. In this course, doing mathematics mainly amounts to solving problems. There are five aspects which are assessed in this course:

3.1. Homework (20%).

- Weekly homework will consist of textbook and other problems aligned with lectures, to help you practice new concepts and techniques. The homeworks are usually due on Mondays, and will be assigned at least a week before the due date.
- You are encouraged to work together on homework assignments (also can do it online via `Slack`, which allows private groups of up to 9 people), to take advantage of the challenge-defend discussions that help understand things better. However, each student needs to submit her/his own homework

assignments, and should work individually when writing them up to demonstrate the understanding of the material.

- The homeworks are graded "coarsely", that is, each homework will be assigned one of four grades:

Grade	VG (very good)	G (good)	OK	N
%	100%	90%	75%	0%

It is expected that most of the students will get VG or G grades. It is important that you submit your homework on time, otherwise you'll get "N" for it. Also, the "N" grade can be assigned if you submit the homework with less than 3/4 of the problems attempted.

- The homeworks *must be submitted only on Collab* (i.e., hard copies are not accepted). Take pictures or scan your work, make sure it's readable, put it into a *single PDF file with correct orientation*, and upload it before the deadline.

  Submitting work like this has many benefits: (1) you retain a paper copy to prepare for tests; (2) your submitted work is never misplaced or lost, and there is a digital trail; (3) the grading will be much faster and will allow me to immediately incorporate my impressions of the homeworks into the class discussions. Moreover, (4) knowing how to scan and make a single PDF file is a valuable technical skill for the future: ask me and I can help you learn it.

3.2. **Course engagement / quizzes (8±4%).** This includes short pop quizzes during lectures (at random days), and activity in `Slack` (including answering other student's questions).

3.3. **Group project (12±4%).** [1]There will be a group project on applications of probability theory. Its purpose is to put you in a collaborative environment, and help you explore how probability theory can be applied to real-world situations. In fact, nowadays applications of probability theory (along with statistics, and sometimes under the names of modeling or operations research) span almost every area of human knowledge and activity, so there is a lot to choose from.

There will be about 10 groups. There are two tracks of this project for each group to choose from: you can either make a computer simulation of an interesting random system and then experimentally describe its behavior; or to come up with a probabilistic model of a real-world phenomenon, and use the model to quantitatively understand it. In the end each group will produce a brief report which will be peer-reviewed by other groups, and will give a short presentation on the project.

3.4. **Midterm test (25%).** The midterm test during lecture on (TBA) will have similar taste as homeworks, and will test basic knowledge of the material.

A two-sided letter size formula sheet, hand-written by yourself, will be allowed on the midterm test and the final exam. Preparing this formula sheet will help you review the material, and paint a systematic picture in your head. Formula sheets cannot contain any photocopied or printed material — do everything by hand (of course, you can include any theorems, formulas, pictures, examples, etc). The use of calculators (but not in mobile phones) is allowed on midterm test and the final exam.

I encourage you to collaborate on test preparation, but needless to say that during the test and the exam each student must work individually.

3.5. **Final exam (35%).** The final exam will be cumulative, but will put more focus on topics covered after the midterm.

A note on letter grades. The exact percentage levels by which course percent grade is converted to letter grade will be determined after the final exam. These levels will be *no worse* than the standard Collab ones. This means that having grade $\geq 90\%$ guarantees that you will get at least $A-$, and so on. For reference, here is the standard Collab scale:

Grade	$A+$	A	$A-$	$B+$	B	$B-$	$C+$	C	$C-$	$D+$	D	$D-$
Minimum %	100	95	90	87	83	80	77	73	70	67	63	60

[1]The course engagement and group project together add up to exactly 20%, but I am not yet sure how these 20% will be distributed; in particular, this depends on the number of pop quizzes we end up having.

4. *Communication*

4.1. **Slack.** My email is petrov@virginia.edu, but for the communication we will use `Slack` — an industrial standard of work messengers, with a web version and apps for all platforms. This will make me more accessible if you have questions, and also will let you answer questions of your fellow students. There will be course content available *only* on `Slack` — my lecture notes which I prepare for each class.

The course "team" is at `https://3100-s17-uva-petrov.slack.com`. You'll get an invitation to join by e-mail, and it is expected that you register. Please let me know if you have issues with access. It is also expected that you will check announcements, and will participate in (or at least read) the discussions of the course material.

Some things to note:

- You need an email address to use `Slack`, and it may be visible to other students. Normally it's your UVA email address (I'll send an invite there), but if you are not comfortable sharing your UVA email, let me know and I'll be happy to send invite to another address. And you can later change the email address yourself in the settings.
- There are private messages where you can ask me questions one-on-one. You can also create private groups with up to 9 people, which is good for homework collaboration (but read Section 5.2 on collaboration).
  Private channels (containing myself) invisible to other students will be created for each group for discussions of the group project.
- Public messaging is separated into channels (#general for class-wide questions/answers where you are welcome to post; #announcements and #class-notes where I will post relevant information; special channels for midterm and final preparation, etc.).
- **Privacy**: Although `Slack` is a messaging app, it should be used professionally, especially in public discussions. The app supports private direct and group messages. But please note that in principle the admin (i.e., myself) can obtain access to **all** direct messages between members of the team. The procedure would involve sending a paper request via the usual mail, and everyone will be notified if the direct messages are accessed — so this can happen only in extreme circumstances.

4.2. **Collab.** Solutions to homeworks / quizzes / midterms will be posted to both `Slack` and Collab. Grades will be posted to Collab as usual. Homework assignments will be posted to Collab and will be also announced in `Slack`'s #announcements channel. Your homework solutions can be submitted only through Collab, and will be graded there.

If you have anonymous comments on anything related to the course, you can make them via Collab.

5. *How to succeed in the course*

5.1. **Textbook and other resources.**

(TODO from hear below)
textbook

- You don't have to bring the textbook to all classes, although this may be helpful sometimes.
- I strongly encourage you to read the textbook. It includes many examples and extra exercises which augment the concepts discussed in class.
- The textbook contains much more material than will be covered in classes, so it makes sense to come to all classes and see which parts of the textbook are covered and which are omitted.
- Class notes which I prepare for each lecture will be made available on `Slack`.

other references / other textbooks
khan academy, wikipedia, other places containing basic stuff on probability theory
— office hours, availability in `Slack`, your fellow students, etc.

The Math Department Tutoring Center is available for helping students in this course: see `http://people.virginia.edu/~psb7p/MTCsch.html` for more information and schedule.

5.2. **Collaboration and honor code.**

6. *Approximate course schedule*

Add/drop information: http://www.virginia.edu/registrar/reginst1158.html#Deadlines

NOTE: Please don't make travel plans that conflict with tests or final exam

Week	Topic	Notes	Homework	Sections	Slack direct message check-in (by 10 am on Monday each week)	Due items for the projects
1: 1/18	What is probability theory	1/18: Introduction		TBA		
2: 1/23. 1/25	Advanced counting. Conditional probabilities			TBA	Individual: send me a test direct message answering why you are taking the course - to make sure it works	
3: 1/30. 2/1	Repeated trials and Gaussian approximation	2/1: group forming		TBA	Individual: briefly describe a real-world problem (preferably in one of your other courses) that you think can be accessed via probability theory	Wed 2/1 class time: form groups of up to 6 people and let me know. It will be reflected in slack
4: 2/6. 2/8	Random variables	2/8: real-world problem discussions	By 2/6 class time: read this link to see how random variables look like	TBA	Group: send me a test message in the private group channel to make sure it works. Also mention which real-world problem you are thinking about.	Wed 2/8 class time: choose a real-world problem that you would like to turn into a probability problem
5: 2/13. 2/15	Expectation and conditional expectation	2/15: probability problems discussions		TBA	Group: are you doing OK with formulating your probability problem? If you are not sure talk to me during office hours - or in Slack.	Mon 2/13 class time: Choose track: analysis or simulation. Formulate a (draft) probability problem / simulation model associated with your real-world setting
	Test 1: 2/20			list all sections		
6: 2/20. 2/22	Poisson processes			TBA	Individual: are there any last-minute questions before the test?	
7: 2/27. 3/1	Poisson processes. Exponential and gamma distributions	3/1: project discussion before the break		TBA	Group: send me a polished version of the probability problem / simulation model you are working on	Wed 3/1 class time: present a draft solution of your probability problem / draft simulation of your model (at least in the simplest case)
8: 3/13. 3/15	Poisson processes. Exponential and gamma distributions			TBA	Individual: how was your break? // Group: is everything OK with the project? Need any help?	Over the break and this week: continue working on the group project

9: 3/20. 3/22	Continuous distributions. General Gaussian approximation	3/22: full class: group presentations		TBA	Group: are you OK with your reports? who is going to present? Need any help?	Mon 3/20 class time: group reports due; Wed 3/22: group presentations
10: 3/27. 3/29	Continuous distributions			TBA	Individual: any feedback on the group project? Are you doing OK in the course so far?	
	Test 2: 4/3			list all sections		
11: 4/3. 4/5	Joint and conditional distributions			TBA	Individual: are there any last-minute questions before the test?	Sun 4/8 by 11:59pm: select topic at https://TBA on which you will compose your problem
12: 4/10. 4/12	Joint and conditional distributions			TBA	Individual: which textbook problem did you choose? Are you doing OK with the evaluation of this textbook problem?	Wed 4/12 class time: textbook problem evaluation due
13: 4/17. 4/19	Joint and conditional distributions	Some of the final exam review problems will be posted by 4/23		TBA	Individual: are you doing OK with composing your problem?	Wed 4/19 class time: your problem due
14: 4/24. 4/26	Bivariate normal distributions. Applications to statistics			TBA	Individual: are you doing OK with peer evaluation of a problem you were given?	Sun 4/29 by 11:59pm: peer evaluation due
15: 5/1	Discussion of peer evaluation and final exam review	5/1 full class: discussion and review		TBA	Individual: which topic you found the clearest in the course? which was the muddiest?	
	Final exam: TBA			list all sections	You can ask questions before the final exam in the #general channel at any time	

7. *Policies*

7.1. **`Slack`.** Although `Slack` is a chatting app, it should be used professionally, especially in public discussions. The app also supports private direct messages and I encourage to use them to collaborate on homework problems and projects, please note that in principle the admin (i.e., myself) can obtain access to **all** direct messages between members of the team. The procedure would involve sending a paper request via the usual mail, and everyone will be notified if the direct messages are accessed — so this can happen only in extreme circumstances.

7.2. **Independent work, honor code.** You are required to work independently on the quizzes. So when working together with others, make sure you are preparing yourself to take the quiz independently. The honor code is taken seriously. Any honor code violations pertaining to the quizzes will be automatically referred to the Honor Committee.

7.3. **Special needs accommodations.** All students with special needs requiring accommodations should present the appropriate paperwork from the Student Disability Access Center (SDAC). It is the student's responsibility to present this paperwork in a timely fashion and follow up with the instructor about the accommodations being offered. Accommodations for test-taking (e.g., extended time) should be arranged at least 5 business days before an exam.