MATH 3340: COMPLEX VARIABLES WITH APPLICATIONS

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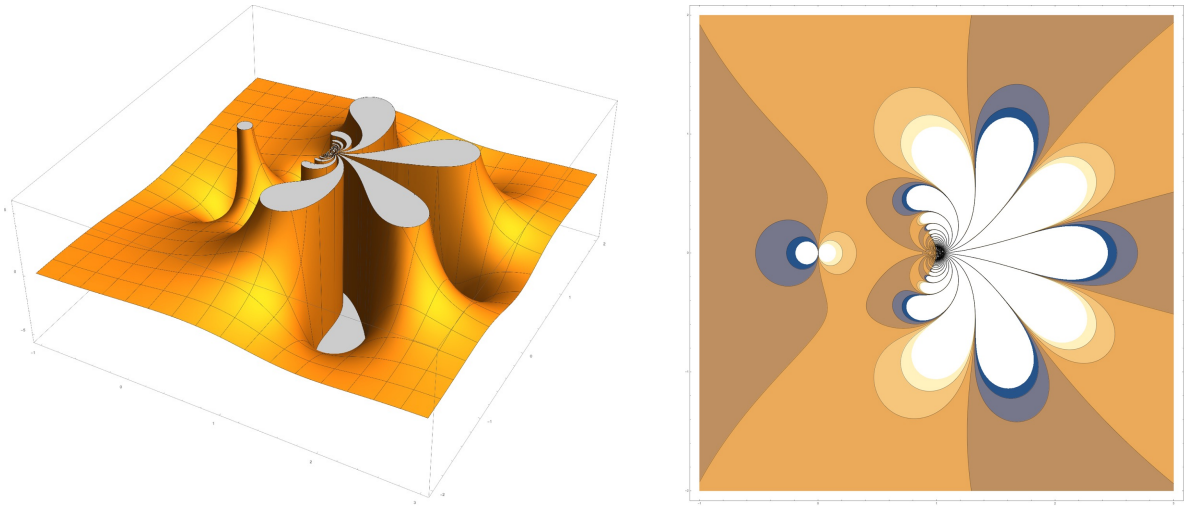
Important first note: The enrollment deadline is February 1, after five classes have already happened. Before this deadline, two quizzes and two homework assignments are due, which are a substantial part of the overall course grade. If you are considering taking my class and plan to enroll late, please come to the first classes, even if not officially registered, and submit the coursework — otherwise, you will miss the credit for these first assignments.



Real part of a particularly complex complex function, and its contour plot on the right.

1. *Complex variables*

The course is centered around the theory of functions of a single complex variable and its applications. Complex analysis is a central part of Mathematics. Many concepts work easier and much more naturally in the complex setup:

- If a function $f(z)$ of the complex variable z has one derivative at a point z_0, then it has infinitely many derivatives, and possesses a power series (Taylor) expansion at z_0, which converges to our function. Compare this with the "bad" behavior of the function $f(x) = e^{-1/x}$ for $x > 0$ (and $f(x) = 0$ for $x \le 0$) of the real variable x, which has infinitely many derivatives but whose Taylor series at 0 is identically zero.
- Any algebraic equation, even $x^8 + 1 = 0$, has a solution over the complex numbers (even if no real solutions). In fact, the equation $x^8 + 1 = 0$ has 8 different solutions and they all can be illustrated by the vertices of a perfect octagon in the complex plane.

Date: Compiled on Thursday 12th January, 2023, 07:01.
An up to date syllabus is always on `GitHub` at https://github.com/lenis2000/Syllabi/blob/master/Syllabus_3340_s23.pdf. For direct PDF download use this link. LaTeX source with *changes* to the syllabus is here (click "History").
Note that this PDF has green clickable links.

After taking this course, you will be able to solve problems and understand the basics of complex numbers, analytic functions, complex integration, Cauchy formulas, power series, residues, and conformal mappings. Moreover, you will learn how to apply these tools to other parts of mathematics and some physical models.

Prerequisites. Good command of single and multivariable calculus at the level of UVA MATH courses 1310, 1320, and 2310.

2. *Necessary information*

Class times: MoWe 2:00 PM – 3:15 PM in Monroe 111.
The general structure of each week is:

- On Mondays, we have lectures and quizzes. The lectures are recorded and posted to canvas. After the lecture, I may upload an additional recorded piece to canvas if we need the material.
- On Wednesdays, we work together in groups to solve homework problems. Homework is due the next Tuesday night.

Exams:

- **Midterm 1:** Monday, February 13, class time (2:00 PM – 3:15 PM in Monroe 111)
- **Midterm 2:** Wednesday, April 5, class time (2:00 PM – 3:15 PM in Monroe 111).
- **Final exam:** Thursday, May 4, 2:00 PM – 5:00 PM. Monroe 111.

Please do not make travel plans that conflict with the midterms or the final exam. If you have an unavoidable conflict, please notify me as soon as possible.

Instructor: Leonid Petrov

Email: lenia.petrov@gmail.com (preferred)

Office: 209 Kerchof Hall

Office hours: TBA
You are welcome to make an appointment and meet outside the usual office hours. For this, please use the online tool located at https://lpetrov.cc/teaching/. (I am automatically available during office hours — and you cannot schedule appointments online for those times.) You can make as many extra appointments as you want.

Course webpage: We will use the Canvas course page for homework submissions, course materials, recorded lectures, and communication. We will use Canvas discussions as a public space to ask and answer questions. *Please keep the Canvas notifications on for announcements.* The Canvas mobile App is also useful for quick course communication.

3. *Course materials*

The textbook is "*Fundamentals of Complex Analysis*" (3rd edition) by Saff and Snider, Pearson, ISBN-10: 0139078746. We will discuss material from Chapters 1–6, and selected topics from Chapters 7–8.

In addition, there is a number of online resources which may help you while doing the homework: Khan Academy, Wikipedia, YouTube, and many other places contain lots of basic material on complex analysis. For example, check out this video by 3blue1brown: https://www.youtube.com/watch?v=5PcpBw5Hbwo. Google Search in general is also a valuable resource.

4. *Assessing your learning*

Learning mathematics means *doing* mathematics: during class meetings, on your own, and in groups. In this course, doing mathematics mainly amounts to solving problems. Below are the concrete aspects which are assessed in this course:

4.1. **Homework.** Weekly homework will consist of problems aligned with lectures and quizzes, to help you practice and enrich the material presented in class. Putting an adequate effort into solving the homework problems and communicating your solutions clearly is of paramount importance for your learning. The homeworks are due **in class** on the specified date, and will be assigned at least a week before the due date. Please **put your problems in order**, indicating clearly which problems you're skipping — this will greatly help with the grading.

Homework solutions are posted soon after the homework deadline, so late work cannot be accepted. The lowest homework grade will be dropped.

The homeworks are graded "coarsely", that is, each homework will be assigned one of four grades:

Grade	VG (very good)	G (good)	OK	N
	All problems solved correctly with minor issues like arithmetic mistakes, and solutions explained in full detail	Most problems solved correctly, and solutions explained in reasonable (close to full) detail	More than 3/4 of problems attempted, many solutions are incorrect, incomplete, or not explained in detail, but the work displays adequate understanding of most of the material	Work not submitted on time, or less than 3/4 of problems attempted, or most solutions are incomplete, or work clearly displays lack of understanding of most of the material
%	100%	90%	75%	0%

It is expected that most students who put reasonable effort into the homework will get VG or G grades.

Note on collaboration on homework assignments. Group work on homework problems is allowed and encouraged. Discussions are in general very helpful and inspiring when learning mathematics. Nevertheless, before talking to others, get well started on the problems, and contribute your fair share to the process.

When completing the written homework assignments, everyone must write up his or her own solutions in their own words. It is very important that you truly understand the homework solutions you hand in, otherwise you may be unpleasantly surprised by your in-class test results.

Needless to say that when working on in-class assignments (quizzes, tests) you are required to work alone.

4.2. **Quizzes.** There will be short quizzes (10-15 minutes) during the classes at random days. They will test the previous week's material and/or recent homework topics. Quizzes are not announced in advance, and there can be two quizzes on a given week.

You should view quizzes as testing your "work in progress", which will allow me to adjust the pace of the course. For this reason, the overall quiz grade is included in the same "bucket" with class participation and office hours discussion, see below.

4.3. **Midterm tests and the final exam.** The midterms and the final exam will feature problems modeled after homework. The final exam is comprehensive, with a focus on the last part after the second midterm.

The exams will be aimed at checking not so much memorization and routine computational skills, but rather understanding of fundamental concepts and principles and the ability to apply

the material learned to solving various problems, including those a student might have never seen before. A missed exam gives a score of zero, unless a student has contacted the instructor a week in advance and agreed upon a procedure to make it up. Under the rules of the College, early examinations are not permitted.

4.4. **How to succeed in the course.** The best way to learn in the course is to come to all lectures, take good notes (some notes may be provided), ask many questions, do all the homework problems, and express your solutions clearly. This will prepare you well for quizzes, midterms, and the final exam.

Mathematical questions are appreciated and encouraged any time during the class. Please use the office hours as much as possible for additional clarifications and occasional homework help. Remember that I am available outside of office hours by appointment which you can book at https://lpetrov.cc/teaching/

4.5. **Grade distribution.** Your grade will consist of:

- Homework — 20%, lowest homework dropped
- Quizzes, class participation, office hours discussion — 15%, one or two lowest quizzes dropped
- Midterms — 15% each
- Final exam — 35%

The score above 90% is usually enough for an A. The score below 50% usually means failing. Other factors such as in-class participation and improvement over time may impact positively your final grade. Excessive absence may lower the final grade.

5. *Course schedule (updated as we go)*

All sections are from the main textbook (see Section 3).

[week 1] 8/23[Q], 8/25. Introduction. Sample space, axioms of probability, random sampling, review of counting, infinitely many outcomes, geometric series. Hypergeometric sampling. (1.1-1.3).

[week 2] 8/30, 9/1[Q]. Problem set 1 due on Tuesday. Infinitely many outcomes. Geometric series. Rules of probability, Venn diagrams. Random variables (first look). (1.3-1.5).

[week 3] 9/6, 9/8[Q]. Problem set 2 due on Tuesday. Conditional probability, Bayes' formula, independence. (2.1-2.3).

[week 4] 9/13, 9/15[Q]. Problem set 3 due on Tuesday. Independent trials, birthday problem, conditional independence, probability distribution of a random variable (2.4-2.5, 3.1).

[week 5] 9/20[Q], 9/22. Problem set 4 due on Tuesday. Probability distribution of a random variable, cumulative distribution function (3.1-3.2).

[week 6] 9/27, 9/29. **Midterm 1, September 27.** No problem set due, practice problems posted a week before the midterm. Cumulative distribution function, expectation. (3.3-3.4).

[week 7] 10/6[Q]. Shorter problem set 5 due on Thursday. Expectation and variance. (3.3-3.4).

[week 8] 10/11, 10/13[Q]. No problem set due on Tuesday. Gaussian distribution, normal approximation, law of large numbers, applications of normal approximation (3.5, 4.1-4.3).

[week 9] 10/18, 10/20[Q]. Problem set 6/7 (combined) due on Tuesday. Poisson approximation, exponential distribution, Poisson process (4.4-4.6).

[week 10] 10/25, 10/27. **Midterm 2, October 27.** No problem set due. Poisson process, gamma distribution (4.6).

[week 11] 11/1, 11/3[Q]. Shorter problem set 8 due on Thursday. Joint distributions (6.1-6.3). Sums of independent random variables and related topics (survey of selected material of chapters 7 and 8).

[week 12] 11/10[Q]. Problem set 9 due on Thursday.

[week 13] 11/15(class cancelled), 11/17. Problem set 10 due on Tuesday. Law of large numbers, central limit theorem (`9.1-9.3`).
[week 14] 11/22. Conditional distributions (`10.1-10.3`).
[week 15] 11/29, 12/1[Q]. Problem set 11 due on Tuesday. Conditional distributions (`10.1-10.3`).
[week 16] 12/6. Shorter problem set 12 due on Tuesday. Conditional distributions (`10.1-10.3`), final exam practice problems.

[Q] means quiz, [M] means midterm.

6. *Policies*

6.1. **Late/make up work.** Each homework assignment will have a due date and time by which it must be submitted to Canvas. After the 1-hour grace period, late assignments are not accepted. There will also be no make-up for the quizzes or midterm tests. However, if you have special needs, emergencies, or unavoidable conflicts, please let me know as soon as possible so we can arrange a workaround for the midterms and the final exam.

6.2. **Special needs.** All students with special needs requiring accommodations should present the appropriate paperwork from the Student Disability Access Center (SDAC). The student must present this paperwork in a timely fashion and follow up with the instructor about the accommodations being offered. Accommodations for midterms or final exams (e.g., extended time) should be arranged (booked with SDAC) at least five days before an exam.

6.3. **Honor Code.** The University of Virginia Honor Code applies to this class and is taken seriously (in particular, see Section 4.1 on homework collaboration). Any honor code violations, especially in written tests (midterms and the final exam), will be referred to the Honor Committee.