II.] Suren Kyuryumyan (6)

Orthogonal polynomials &
— the Hermite kernel

<u>Thm.</u> (Existence of orth. poly's) $\quad \zeta_0 = 1$

$\exists! \; P_n(x), \qquad \int P_m(x) P_n(x) \, d\mu(x) = \zeta_n \delta_{mn}$

Projection operator (kernel)

$f \in L_2(\mu, \mathbb{R})$

$\inf \left\{ \int \left| f(x) - \sum_{j=0}^{n} f_j \, P_j(x) \right|^2 d\mu(x), \; f_j \in \mathbb{R} \right\}$

obtained at $\quad f_j = \int \frac{f(x) \, P_j(x)}{\zeta_j} d\mu(x)$

$x \, P_n = P_{n+1} + \alpha_n P_n + \beta_n P_{n-1}$

$w(x) = e^{-x^2/2}$

$P_0 = 1 \qquad P_1 = x - \alpha_0$

Hermite kernel

$$K_n(x,y) = \sqrt{w(x)w(y)}\ \frac{1}{h_{n-1}}\ \frac{P_n(x)P_{n-1}(y) - P_{n-1}(x)P_n(y)}{x-y}$$

$$h_n = \int_{\mathbb{R}} P_n^2(x)\, w(x)\, dx$$

$$h_n = n! \cdot \sqrt{2\pi}$$

$$\frac{1}{\sqrt{n}} K_n\left(\alpha\sqrt{n} + \frac{x}{\sqrt{n}},\ \alpha\sqrt{n} + \frac{y}{\sqrt{n}}\right) \xrightarrow[n\to\infty]{} \frac{\sin(x-y)}{x-y} \cdot \mathrm{const.}$$

Asymptotic formula for Hermite

$$N = 2n+1$$

$$\lambda_n^{-1} e^{-x^2/2} P_n(x) = \cos\left(N^{1/2} x - \frac{\pi n}{2}\right) + \dots$$

(a whole asymptotic decomposition)

$$+ \dots + O(n^{-1})$$

$$\lambda_n = \frac{\Gamma(n+1)}{\Gamma(\frac{n}{2}+1)}$$

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first component has

$$\cos\left(N^{1/2} x - \frac{\pi n}{2}\right)$$

as the main term.