AIM Workshop:
All roads to the KPZ universality class

Organizers:

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Reading list

Foundations

- [KPZ86] The seminal paper introducing the KPZ equation, which describes the evolution of a growing interface in terms of a stochastic PDE.
- [Cor12] A comprehensive survey explaining key breakthroughs leading to the exact one-point distribution for the KPZ equation with narrow-wedge initial data.
- [Hai14] A groundbreaking work that rigorously constructs a solution to the KPZ stochastic PDE using the theory of regularity structures, resolving longstanding analytical challenges.
- [ACQ11] A seminal paper that computes the one-point distribution of the KPZ equation, linking KPZ fluctuations to Tracy–Widom statistics through integrable probability methods.

Recent advances

- [CH14], [CH16] Develop the Brownian Gibbs property for the Airy and KPZ line ensembles.
- [MQR21] Constructs the KPZ fixed point—the universal Markov process limit of KPZ-class models—with explicit Fredholm determinant formulas for multi-point distributions.
- [QR22] Develops the KP equation for the KPZ fixed point distributions. This and the previous paper are summarized in [Rem22].
- [DOV22] Establishes the directed landscape as the scaling limit of last-passage percolation models, serving as the universal random geometry underpinning KPZ growth.
- [BSS24] Introduces the stationary horizon as the unique invariant coupling for the KPZ fixed point, linking it to the structure of semi-infinite geodesics in the directed landscape.
- [BCY24] Develop a framework of two-layered Gibbs measures to describe stationary measures for geometric LPP and log-gamma polymer models, providing a unified description of the open KPZ stationary measure.

Cutting edge work by participants (in no particular order)

- [BB24] ASEP via Mallows coloring
- [BC24] Mallows Product Measure

- [AB24] Colored Line Ensembles for Stochastic Vertex Models
- [ACH24b] Scaling limit of the colored ASEP and stochastic six-vertex models
- [ACH24a] KPZ fixed point convergence of the ASEP and stochastic six-vertex models
- [LS23] Contour integral formulas for PushASEP on the ring
- [MR23] Exact solution of TASEP and variants with inhomogeneous speeds and memory lengths
- [ANP23] Colored Interacting Particle Systems on the Ring: Stationary Measures from Yang–Baxter Equation
- [Zha23] Shift-invariance of the colored TASEP and finishing times of the oriented swap process
- [KZ24] Asymptotics of dynamic ASEP using duality
- [FKZ24] Orthogonal polynomial duality and unitary symmetries of multi-species ASEP(q, θ) and higher-spin vertex models via $*$-bialgebra structure of higher rank quantum groups
- [AH23] Strong Characterization for the Airy Line Ensemble
- [DLM23] Large deviations for the q-deformed polynuclear growth
- [CHHM23] Exceptional times when the KPZ fixed point violates Johansson's conjecture on maximizer uniqueness
- [BL24] Pinched-up periodic KPZ fixed point
- [LZ25] An upper tail field of the KPZ fixed point
- [TTB+24] Partial yet definite emergence of the Kardar-Parisi-Zhang class in isotropic spin chains
- [JRAS22] Ergodicity and synchronization of the KPZ equation
- [GRASS23] Jointly invariant measures for the KPZ equation
- [She23] Independence property of the Busemann function in exactly solvable KPZ models
- [BS23] Time correlations in KPZ models with diffusive initial conditions
- [BLS22] Limiting one-point distribution of periodic TASEP
- [BPS23] Differential equations for the KPZ and periodic KPZ fixed points
- [DS24] Viscous shock fluctuations in KPZ
- [Tan24] An invariance principle of 1D KPZ with Robin boundary conditions
- [Tsa22] Exact lower-tail large deviations of the KPZ equation
- [LT21] Short time large deviations of the KPZ equation
- [WY24] From asymmetric simple exclusion processes with open boundaries to stationary measures of open KPZ fixed point: the shock region
- [Bus23] Non-existence of three non-coalescing infinite geodesics with the same direction in the directed landscape
- [DZ24] Characterization of the directed landscape from the KPZ fixed point
- [HP24] The Directed Landscape is a Black Noise
- [Wu23] The KPZ equation and the directed landscape

- [MPPY24] Grothendieck Shenanigans: Permutons from pipe dreams via integrable probability
- [PS24] Rewriting History in Integrable Stochastic Particle Systems
- [DK24] Global asymptotics for β-Krawtchouk corners processes via multi-level loop equations
- [CM24] The Symplectic Schur Process
- [IMS22] Solvable models in the KPZ class: approach through periodic and free boundary Schur measures
- [IMS23] New approach to KPZ models through free fermions at positive temperature
- [DFV24] Arctic curves of the T-system with slanted initial data

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