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The Bloomberg Corporate Default Risk Model (DRSK) for Private Firms

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Abstract

The DRSK private firm model produces estimates of real-world default probabilities (DPs) for private companies. The product covers all firms for which the requisite data is available, providing point in time DP term structures for about 500,000 private firms globally.

This year, we are recalibrating the model to account for changes made in the DRSK public firm model. The recalibration includes various enhancements to bring the private model more in alignment with the public model. The new model largely improves accuracy ratios and R-squared values. We describe the new model, analyze its performance and compare it to the previous model.

Keywords. DRSK, private firms, real-world default probability, distance to default, credit risk, logistic regression.

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1 Introduction

Corporate credit risk is the risk that a firm fails to satisfy its financial obligations. This typically first occurs when a firm fails to make a scheduled bond or loan payment, or is unable to meet a margin call. Such failures are known as defaults. They often lead to bankruptcy proceedings where the firm is restructured, acquired or liquidated, and creditors incur losses. As such, it is important for investors to measure and manage corporate credit risk and anticipate default and bankruptcy events.

Most of the research into credit risk modeling focuses on publicly traded firms. As discussed in Bondioli et al. [Bon+21], in-depth financial analysis, statistical models, structural models, reduced form models, and various combinations thereof all appear in the literature. Of these approaches, the financial analysis approach of the nationally recognized statistical rating agencies (NRSROs) such as Moody's, S&P and Fitch avoid the need for stock prices and is thus applicable to private firms. However, the NRSRO approach only yields a stack ranking of firms, not direct estimates of default rates, and NRSRO ratings are only available for a small segment of the market.

The literature on modeling credit risk for private firms is less extensive. It focuses primarily on statistical models and the selection of appropriate factors. Altman [Alt00] extends his eponymous Z-score to private firms by replacing the market capitalization with the book value of the equity. Altman and Sabato [AS07] improve on this by updating the factors used and replacing the multivariate discriminant analysis with a logistic regression. Falkensten, Boral, and Carty [FBC00] use a probit regression on transformed financial ratios and firm size to estimate private firm default probabilities (DPs). Zhou et al. [Zho+05] develop a model for North American privately held firms using maximum likelihood with l_1-regularization applied to various financial ratios and economic indicators.

The DRSK private model follows the same general structure. Like in the DRSK public model [Bon+21], we use a logistic regression with factors capturing market conditions and financial ratios. Our innovation is to include a sector and region specific factor that is specifically constructed to capture credit risk. The factor we add is the average distance to default (DD) of public firms in each sector and region. This enables the model to reflect the current level of market credit risk to which each firm is exposed.

For this model update, we recalibrate the model to adjust for the updates made to the public firm model. We also make various adjustments to bring the private model into closer alignment with the public model, and enlarge our data calibration set. The model now produces DPs for almost 500,000 firms.

The new model outperforms the previous model. Both adjusted pseudo R-squared values and accuracy ratios have improved, with the North American bank sub-model achieving an accuracy ratio (AR) of 98%. And the model's predictive power has improved, with average default rates rising substantially as default events are approached.

The remainder of this document is organized as follows. Section 2 reviews the process of default and bankruptcy and specifies the types of credit events that the model is based on. Section 3 describes the composition of the model's data set. The model is described in Section 4. Section 5 explores the performance of the risk factors that the model uses. The model's performance is analyzed in Section 6. Section 7 compares the updated model to the previous model. Section 8 summarizes the results.

2 Default definition

The DRSK private model estimates the probability of a credit event occurring within a particular time frame. A credit event in the private model is defined in the same way as in the public model, namely the first occurrence of a default, bankruptcy, government intervention, or distressed exchange for a given firm, as detailed in Bondioli et al. [Bon+21]. For simplicity of exposition, we will refer to credit events simply as defaults, with the understanding that default events, default rates, etc., are referring to all credit events as defined above.

Like in the public model, the private model uses Bloomberg's corporate action database (CACS), and our hand collected set of distressed exchanges and government interventions.

An additional source of government interventions is the Federal Deposit Insurance Corporation (FDIC) takeover list. This is a list of failing banks that the FDIC has intervened with to directly resolve their difficulties. Details of how the FDIC resolution process works is given in *Resolutions Handbook* [Fed19]. We augment our list of government interventions with the FDIC's takeover list. This helps to make the North American bank credit event data substantially more complete than for other regions and sectors.

3 Data set

The private model data set consists of monthly observations of private firms from February 1998 to March 2020. This includes almost 500,000 firms with sufficient data for calculating DPs, and over 4,000 credit events.

3.1 Market composition

Tables 1 and 2 show the breakdown of the private model data set by region and sector, respectively. The bulk of firms are domiciled in Western Europe (61%), North America (21%), and Asia (12%). This is in contrast with the public model data set, where most firms are domiciled in Asia (45%), followed by North America (29%) and Western Europe (16%). We attribute the differences more to variance in the availability of data for private firms than to differences in market composition between public firms and private firms.

Table 1: Private model data set by region. We give the number and percentage of firms in each region, the number and percentage of defaults in each region, and each region's default rate.

	Count		Percentage		
Region	Firms	Defaults	Firms	Defaults	Default rate
Africa	1,868	8	0.4%	0.2%	0.4%
Asia	57,145	446	11.6%	10.4%	0.8%
Eastern Europe	24,557	70	5.0%	1.6%	0.3%
Latin America	5,751	67	1.2%	1.6%	1.2%
Middle East	2,595	21	0.5%	0.5%	0.8%
North America	101,615	2,522	20.6%	59.0%	2.5%
Western Europe	298,871	1,144	60.7%	26.7%	0.4%
Total	492,402	4,278	100.0%	100.0%	0.9%

Table 2: Private model data set by sector. We give the number and percentage of firms in each sector, the number and percentage of defaults in each sector, and each sector's default rate.

	Count		Percentage		
Sector	Firms	Defaults	Firms	Defaults	Default rate
Communications	14,344	329	2.9%	7.7%	2.3%
Consumer Discretionary	152,171	1,023	30.9%	23.9%	0.7%
Consumer Staples	48,508	166	9.9%	3.9%	0.3%
Energy	11,913	321	2.4%	7.5%	2.7%
Financials	58,320	888	11.8%	20.8%	1.5%
Health Care	27,991	260	5.7%	6.1%	0.9%
Industrials	110,094	636	22.4%	14.9%	0.6%
Materials	36,884	254	7.5%	5.9%	0.7%
Technology	22,806	356	4.6%	8.3%	1.6%
Utilities	9,371	45	1.9%	1.1%	0.5%
Total	492,402	4,278	100.0%	100.0%	0.9%

Firms in our sample fall into ten different sectors[1], with consumer discretionary having the largest representation (31%), followed by industrials (22%) and financials (12%). The remaining sectors range from 2% (utilities) to 10% (consumer staples). This is similar to the public model data set breakdown, although in that data set, consumer discretionary and industrials account for smaller percentages of firms (18% and 13%, respectively).

3.2 Default rates

Observed default rates for private firms differ substantially between regions and sectors. On a regional basis, they range from 0.3% for Eastern Europe, to 2.5% for North America. While the public firm default rates for Africa and the Middle East are similar, the default rates for the remaining regions are 2 to 10 times higher. Sector default rates are similarly lower in the private model data set.

As in the public model, regional differences can be attributed in part to differences in countries' bankruptcy laws. But these differences are exacerbated by other factors. These other factors also account for the low rate of reported defaults for private firms.

For firms with public debt outstanding (bonds, syndicated loans, etc.), default events are a matter of public record. But private firms tend not to finance through public debt offerings. They often finance through private loans and lines of credit. Public information is available if the loans are syndicated, but more often they are not. Insolvency resolution then becomes a private matter between the firm and its creditors, rather than a matter of public record. Under-reporting of private firm default events is corroborated by the above data.

3.3 Calibration data set

Coping with missing default records is challenging. One approach is to augment the data with a more reliable data set. Since default events for public firms with public debt is generally a matter of public record, the data for such firms is more representative of actual default rates. So we include the public firms in our calibration data set. The only exception is for the bank sub-model, where defaults are more generally available, and our data set is augmented by the FDIC failed bank list.

Since financials tend to be more accurate and more readily available for larger firms, we also only include firms in the calibration set that are sufficiently large. Large regions with poor data can also skew results, so we adjust the firm size cutoff to de-emphasize such regions.

As a final step, like in the public model, we also over-sample credit events. To enforce consistency between the public and private models, we over-sample defaulting firms so that the public and private models yield the same average default rate on the public calibration set. We iterate to solve for the over-sampling weight that yields this match.

[1] The sectors referred to here are the top level sectors in the Bloomberg Industry Classification System (BICS).

4 DRSK Model

The core of the DRSK private model is a logistic regression model that estimates the probability of a credit event occurring within 1-year based on observations of various financial ratios and market measures. The core model is augmented to construct a term structure of default probabilities. Credit grades are assigned based on the 1-year DPs.

To capture differences in credit risk between regions and sectors, we divide the data into two regions (North American and global) and three sectors (banks, non-bank financials and non-financials), and model each pair separately, yielding six separate sub-models:

1. Banks in North America (FinBank_NA)
2. Banks not in North America (FinBank_GLB)
3. Non-bank financials in North America (FinOth_NA)
4. Non-bank financials not in North America (FinOth_GLB)
5. Non-financials in North America (NonFin_NA)
6. Non-financials not in North America (NonFin_GLB)

Each sub-model is separately calibrated and analyzed. North American models are calibrated to North American firms. Global models are calibrated to North American and Western European firms. North American firms are included to help to make up for under-reporting of defaults in Western Europe. Other regions are excluded from the calibration sets because, as noted in Section 3.1, under-reporting is too severe to include them.

4.1 Model structure

In the DRSK public model, the main factor is the underlying calculated distance to default (DD) for each individual firm. For the private firm model, market capitalization is unavailable, so such a DD calculation cannot be performed. To compensate for this lack of data, various other factors are introduced, as listed in Table 3.

To capture the state of the market and its impact on default rates, we include DD averages, averaged by sector and region. This enables the model to respond to market conditions that are not reflected in the latest financial reports.

To characterize the idiosyncratic state and structure of each firm, we use additional regression factors based on financial ratios. For all sectors, the factors include return on assets (ROA) and leverage (LVG). For banks, we include the non-performing loans ratio (NPL) and the tier 1 capital ratio ($T1CR$). Other sectors use the negative book equity indicator (NEQ), and non-financials also use the liability to sales ratio (LTS).

This yields the following general structural form:

$$DP(F) = p(B(F)) = \frac{1}{1 + \exp(-f(B(F)))} \tag{4.1}$$

$$f(B(F)) = \beta_0 + \sum_i \beta_i B_i(F) \tag{4.2}$$

where the sum is over the factors used for the firm's sector, $B_i(F)$ is firm F's ith factor, and β_i is its corresponding coefficient.

Table 3: Factors for each sub-model

Type of firm	Model factor	Notation
Banks	Region and sector average DD	DD
	Return on assets	ROA
	Leverage	LVG
	Non-performing loans	NPL
	Tier 1 capital ratio	$T1CR$
Non-bank financials	Region and sector average DD	DD
	Return on assets	ROA
	Leverage	LVG
	Negative book-equity indicator	NEQ
Non-financials	Region and sector average DD	DD
	Return on assets	ROA
	Leverage	LVG
	Negative book-equity indicator	NEQ
	Liability to sales	LTS

4.2 DP term structure

The core logistic regression model in Equation 4.2 yields an estimate of the 1-year cumulative DP. As the number of private defaults remains small relative to the full universe of private firms, statistically constructing a term structure by regression (like we do for the public model) is inadvisable.

To construct a DP term structure for private firms, we instead rely on information extracted from the term structure of publicly traded firms, which is more transparent and relatively more complete. We essentially lift the average term structure for public firms with the similar 1-year DPs. To account for the differences in term structure between financial firms and non-financial firms, we do this separately for each.

4.3 DP mapping to credit grades

Credit grades for the private model are calculated the same way as in the public model, namely by DP bands with smoothed transitions. See Bondioli et al. [Bon+21] for the details.

5 Factor analysis

In this section we detail each model factor and discuss its ability to estimate credit risk. For each factor, we look at its performance in two different ways – on a distribution basis and on a realized default rate basis.

For the distributional analysis, we split the calibration set into two subsets – the observations within one year of a credit event (the defaulting firm subset), and the remaining observations (the non-defaulting firm subset). The distributions are illustrated using box plots.[2] We look for differences in the distributions of the factors on the two subsets. Ideally, the means will differ substantially, and the distributions themselves will show significant separation.

For the realized default rate analysis, we sort the observations on the factor and calculate a rolling average of the realized default rate over the samples. The more monotonic the graph is, the better the factor is at estimating default rates. These analyses are run separately for the calibration set for each sub-model.

As is illustrated in the following sections, while all of the factors have different explanatory power, they are all significant predictors of default.

5.1 Distance to default

Financial statements are released at most quarterly and only slowly respond to market stress. So, a default model that only depends on financial statements will not be responsive to the current state of the market and its corresponding level of credit risk. The DRSK public firm model overcomes this limitation by using the distance to default for each firm [Bon+21]. The DD combines the current market view of the firm with its capital structure, creating an effective measure of credit risk. But DD calculation requires knowing the firm's market capitalization, which is unavailable for private firms.

To construct a private firm default model that responds to market conditions, a factor is needed that captures the current level of credit risk in the market. Since the DD is so effective in capturing credit risk in the public firm model, we use it as our measure of general market credit risk. We

[2]The box plot displays the distribution of the data based on five key statistics - minimum, first quartile (Q1), median (Q2), third quartile (Q3) and maximum. The minimum and the maximum values are represented by $(Q1 - 1.5 * IQR)$ and $(Q3 + 1.5 * IQR)$, respectively, where $IQR \equiv Q3 - Q1$, denotes the interquartile range. Data points outside the minimum and maximum values are considered outliers.

classify public firms by region and sector, and use their average DD as a measure of the general market credit risk for firms in that region and sector. The regions are as defined in Section 3. The sectors are based on the BICS classification, levels 1 and 2. Throughout the remainder of this document, we refer to this factor as the sector DD, or just the DD.

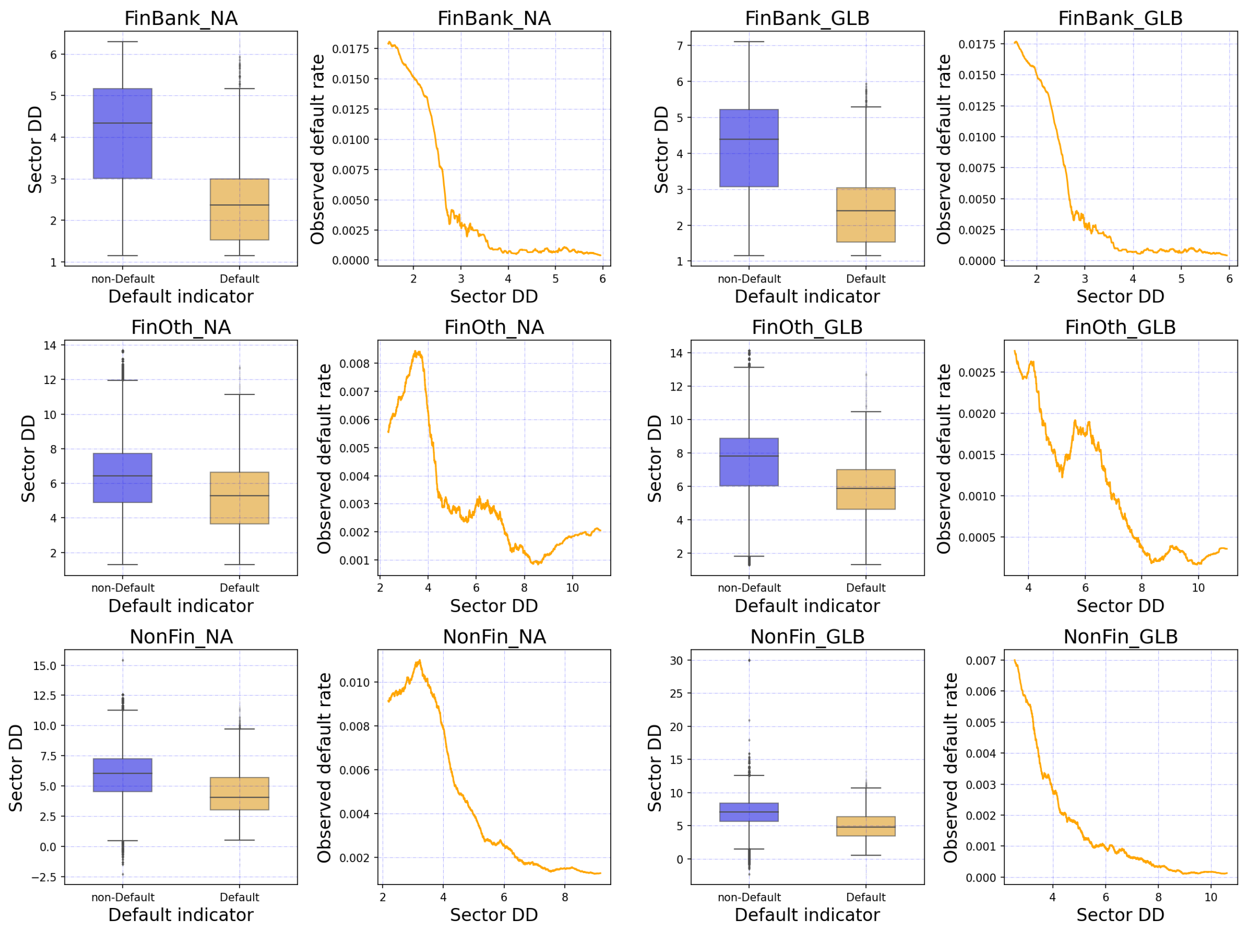

Figure 1: The DD distribution for defaulting and non-defaulting firms is shown, along with the realized default rate as a function of the DD. The DD values are inversely related to the observed default rates.

Since the DD factor in the private model is the average of public firm DDs for firms within a given region and sector, the factor should be seen as a systematic risk factor. This is in contrast to the public firm model, where the DD factor also captures idiosyncratic risk. In both models, the DD factor is expected to be inversely related to default rates. Similarly, drops in the DD factor are expected to coincide with default rate increases. The latter is expected to occur when a sector or region is under economic contraction.

Figure 1 shows that sector DDs have the expected behavior. As shown in the box plots, firms that default within a year have smaller DDs compared to non-defaulted firms. The default rate curves show that average realized default rates are high for low DDs and rapidly fall off as DDs rise. For

FinOth_NA and NonFin_NA, the graph exhibits a hump shape for low DDs which is related to the lack of observed defaults in the private firm data set.

5.2 Return on assets

The return on assets (ROA) is the ratio of the firm's net income to its total assets. This accounting ratio is a useful indicator of the firm's profitability. A negative ROA indicates that the firm is operating at a loss and is more likely to default. A positive ROA indicates that the firm is successful at generating revenues and profits on its investments and is therefore less likely to default.

Figure 2 compares the ROA to realized default rates. Firms with positive ROA values rarely default, and default rates increase as ROA values become more negative. Defaulted firms have a lower ROA median and a wider ROA range. This effect is more pronounced for banks, which justifies splitting financial sector into banks and other financials.

5.3 Leverage

Capital is the buffer that a firm can use to cushion losses on its assets while still being able to meet its liabilities. The more capital a firm has in relation to its assets, the more it can cover losses and the safer the firm is. The leverage ratio (LVG) factor captures this notion.

Leverage ratios are such an important measure of credit risk that banking regulations require limiting them. The Federal Reserve has mandated leverage ratio limits since the 1980s, and Basel has more recently instituted them as well [BS19].

Leverage ratios can be calculated in a variety of ways. We use tangible equity to tangible assets for financial firms, and total assets to total liabilities for non-financial firms. Note that since the equity component is in the numerator, the LVG factor decreases with increased leverage.

As can be seen in Figure 3, default rates do decrease as the LVG factor increases. This relationship is strongest for banks, where the distribution of the LVG factor for defaulting firms is also much lower than for non-defaulting firms. The relationship is weaker for non-financials, where there is a significant, but lesser distributional difference, the default rates do not rise as much as leverage decreases, and default rates are not monotonic as a function of leverage. The relationship is weakest for non-bank financials, where the distributions are closer, and the drop in default rates is less pronounced as the LVG factor increases. None the less, in general, the factor is still useful in the model.

5.4 Nonperforming loans ratio

Banks have a more complex liability structure than other institutions. In particular, their business centers around lending, so it makes sense to include some measure of the losses in the bank's loan

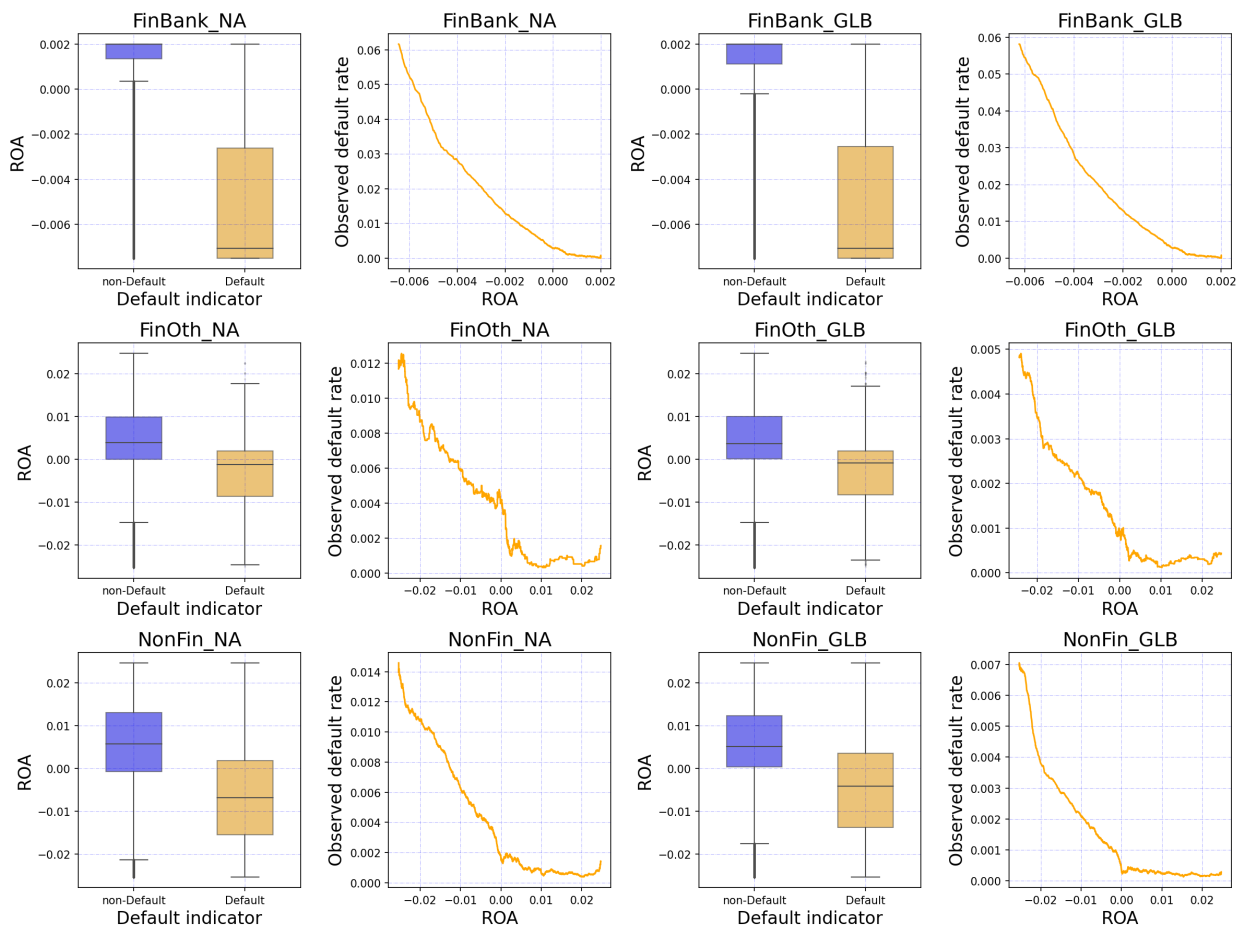

Figure 2: The ROA distribution for defaulting and non-defaulting firms is shown, along with the realized default rate as a function of the ROA. Default rates increase as ROA becomes more negative.

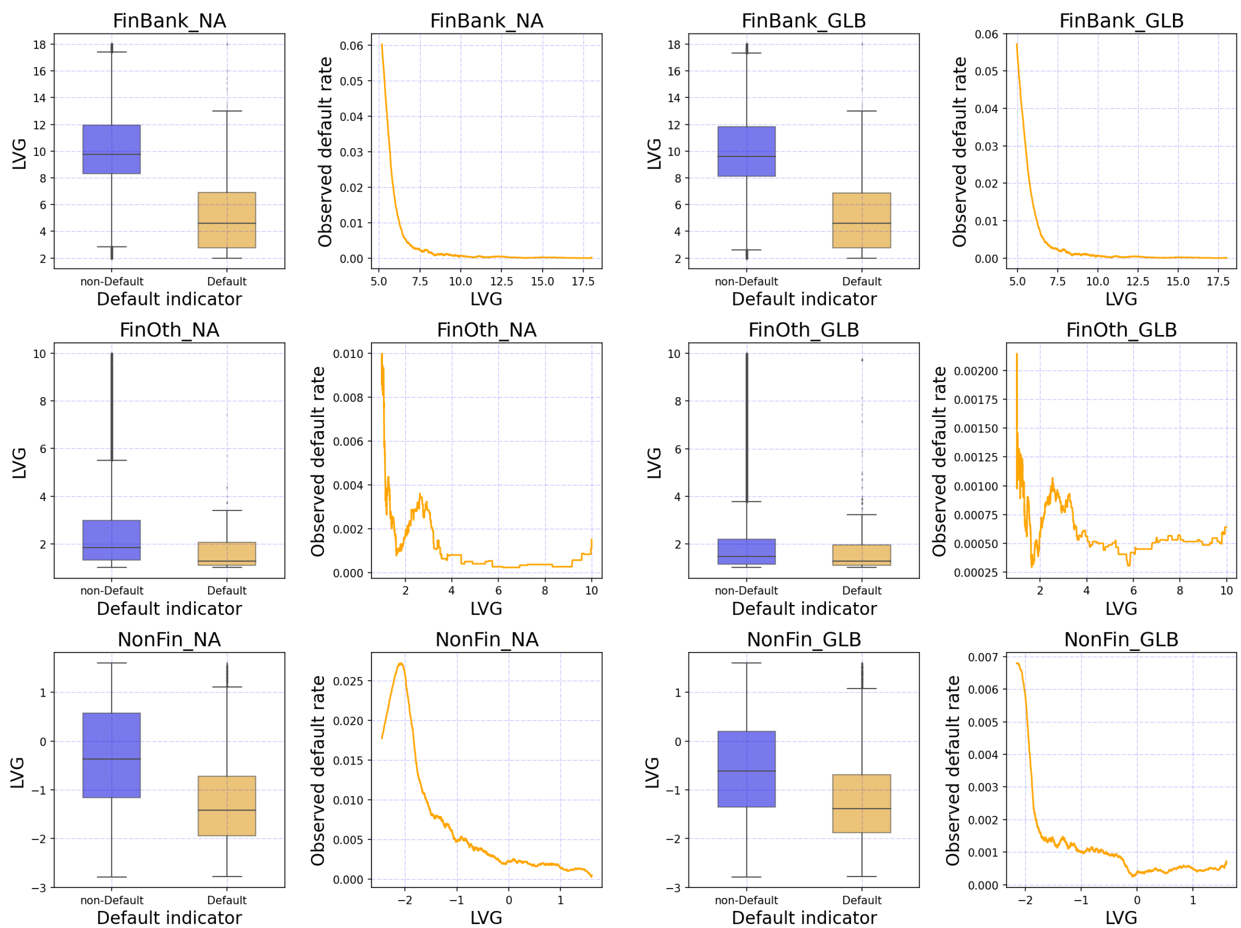

Figure 3: The LVG distribution for defaulting and non-defaulting firms is shown, along with the realized default rate as a function of the LVG (expressed in percentage). Default rates decrease as LVG increases.

book. Towards this end, we include the non-performing loans ratio (NPL) as a factor. The NPL is the ratio of the notional of non-performing loans minus the loan loss reserves to the total assets. Firms with high NPL are more likely to be higher credit risks.

The box plots in Figure 4 show that the NPL is a strong indicator of default for banks. The NPLs are substantially higher for defaulted firms, with about 75% of NPLs above 105, whereas for non-defaulted firms, 95% of NPLs are below 40. And default rates increase substantially and monotonically as NPLs rise.

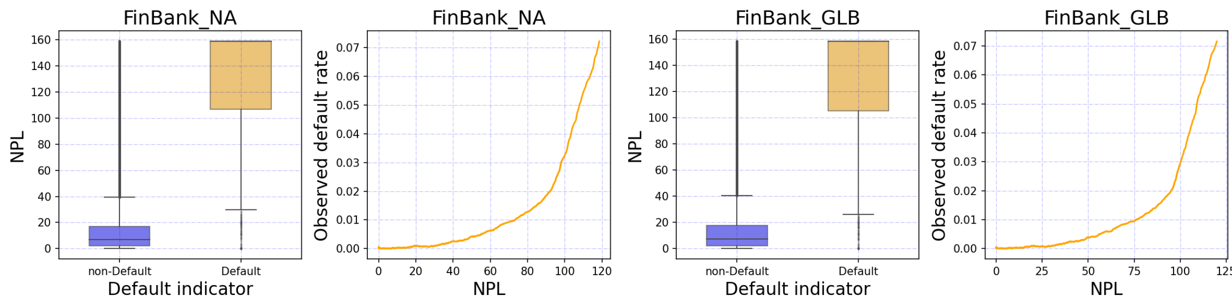

Figure 4: The NPL distribution for defaulting and non-defaulting firms is shown, along with the realized default rate as a function of the NPL. NPLs are substantially higher for defaulted firms.

5.5 Tier 1 capital ratio

The Tier 1 capital ratio ($T1CR$), first introduced in 1988, is a key regulatory ratio in the Basel I Accords [BS88]. The Accords are designed to ensure that banks have enough capital to meet their obligations and manage unexpected losses. The $T1CR$ is measured as the ratio of total Tier 1 capital to total risk-weighted assets. The lower the $T1CR$ the closer a bank is to regulatory constraints and the greater the need to lower the amount of the risk-weighted assets to improve the ratio. According to Basel III, banks should maintain a $T1CR$ of at least 10.5% of their risk-weighted assets [BS20].

The box plots in Figure 5 indicate that defaulted firms have a median $T1CR$ around 6%, well below the regulatory ratio. On the other hand, the median $T1CR$ for non-defaulted firms is around 14%. The figure also indicates that observed default rates are very high for low levels of $T1RC$ and decrease sharply as this ratio rises.

5.6 Liability to sales ratio

A firm without sales is unlikely to survive, so it is important to include a measure of sales in a credit risk model. Total sales cannot be used, as it would predominantly capture the size of a firm, and not the strength of its sales relative to its costs. One common way to measure sales strength is via the asset turnover ratio, which expresses sales as a percentage of total assets. We found

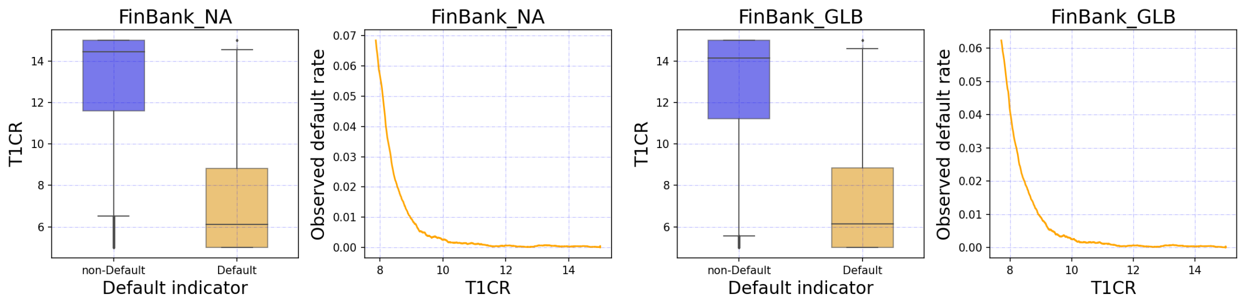

Figure 5: The $T1CR$ distribution for defaulting and non-defaulting firms is shown, along with the realized default rate as a function of the $T1CR$ (expressed in percentage). Default rates are extremely high for low levels of $T1CR$.

that this ratio is too correlated to other factors to be included in the model. Instead we found the liability to sales ratio (LTS) to be an effective addition for capturing sales strength.

Figure 6 shows that LTS are higher for defaulting firms and lower for non-defaulting firms. Similarly, default rates are very low for low LTS and they rise sharply as LTS rises.

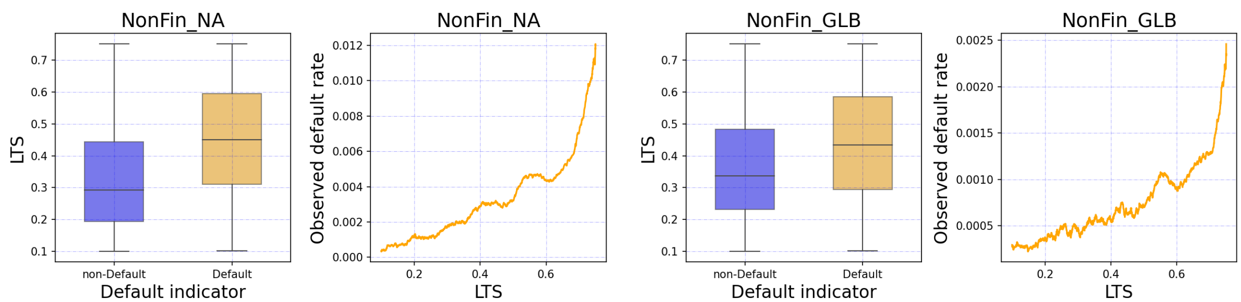

Figure 6: The LTS distribution for defaulting and non-defaulting firms is shown, along with the realized default rate as a function of the LTS. Default rates rise as LTSs rise.

5.7 Negative book-equity indicator

Book-equity is defined as a firm's total assets minus its total liabilities. A negative book-equity indicates that the firm's value of assets is below the value of its liabilities and, thus, the firm is at a substantial risk of default. Book equity cannot be directly used as a factor because it is an absolute measure and may penalize firms with small book equity, such as growth firms. So it is not surprising that we found that book equity itself is not a strong indicator of credit risk. We found, however, that firms with negative book-equity are substantially riskier than the ones with positive book-equity. Therefore, we use the negative book-equity indicator (NEQ) as an additional risk factor for non-banks. This factor is not relevant for banks because their regulatory capital requirements preclude negative book-equity. Table 4 illustrates the average realized default rates for NEQ values.

Table 4: Realized default rate (%) by NEQ. Default rates are higher for firms with negative book-equity ratios.

Region	Sector	$NEQ = 0$	$NEQ = 1$
North America	Non-bank financials	0.17%	1.90%
	Non-financials	0.21%	2.66%
Global	Non-bank financials	0.07%	0.47%
	Non-financials	0.07%	0.90%

The table clearly shows that for each model, firms with negative equity have substantially higher default rates than the remaining firms.

6 Model performance

A variety of tests are typically used for validating credit risk models [SK07; MKL09]. Standard statistical tests are often employed, including goodness-of-fit, coefficients significance and pseudo R-squared. For logistic regressions, and for credit risk modeling in particular, it is common to use receiver operating characteristics (ROC) curves and accuracy ratios (ARs). It is also important to confirm parameter stability. As a result, we review a variety of analyses to analyzed model performance:

1. The overall model fit
2. The DP distribution
3. The comparison of model DP to realized default rates
4. The DP predictive power
5. The model coefficient stability and behavior

We present the overall model fit analysis in Section 6.1, where each sub-model's adjusted pseudo R-squared, the ROC curves and the ARs are given. The ROC curves illustrate the model's ability to distinguish between defaulting firms and non-defaulting firms. We look at the ARs both on an overall basis as well as on a walk-forward basis, the latter comparing the in-sample AR each year to the out-of-sample AR calculated the following year.

To confirm that the produced model DPs follow expectations, we look at the distribution over time of the 21 DRSK credit grades. This analysis is in Section 6.2.

Another check is to compare realized and model estimated default rates. Given that the model is regression based, overall averages will match on the calibration set. Instead, we proceed as in the factor analysis. We sort the observations by model default rate and compare the rolling average of the model default rate to that of the realized defaults (Section 6.3). To confirm alignment between the public firm and private firm models, we also compare the two models' average default rates over time.

The model's predictive power is analyzed by exploring the average model default rate as a function of time-to-default for each sub-model. This analysis is in Section 6.4.

Section 6.5 analyzes the behavior and stability of the model coefficients. Using a time based k-fold cross validation test, we confirm that the model is stable and is not over-fitting the data.

6.1 Overall model fit

Table 5 gives each sub-model's McFadden's adjusted pseudo R-squared [McF74]. The logistic regression accounts for 28% (FinOth_GLB) to 60% (FinBank_NA) of the total variation. This is competitive with, and sometimes better than the public model, which has an adjusted pseudo R-squared range from 34% (global financials and non-financials) to 41% (North American financials).

Table 5: Adjusted pseudo R-squared

Region	Sector	Adjusted Pseudo R-squared
North America	Banks	60%
	Non-bank financials	37%
	Non-financials	44%
Global	Banks	56%
	Non-bank financials	28%
	Non-financials	49%

Figure 7 plots the ROC curves, illustrating each sub-model's ability to distinguish between firms defaulting within a year and the remaining firms. The FinBank_NA and FinBank_GLB sub-models perform best, with an AR of 98%. The other models group together, with ARs ranging from 79% (FinOth_NA) to 86% (NonFin_GLB). The high ARs and steepness of the curves indicate that the model's ability to distinguish between defaulting and non-defaulting firms high and on par with the public firm model, which has ARs ranging from 92% for non-financials to 97% for banks.

We also perform a walk-forward validation test to assess the model's stability and out-of-sample performance. For each year from 2009 to 2018, we calibrate the model on data through the end of that year, and calculate ARs on the data for the following year (Figure 8). The in-sample ARs are stable and remain high, ranging from 75% (FinBank_GLB) to 98% (FinBank_NA). The out-of-sample ARs exhibit more volatility, but still show good performance, ranging from 60% to 98%.

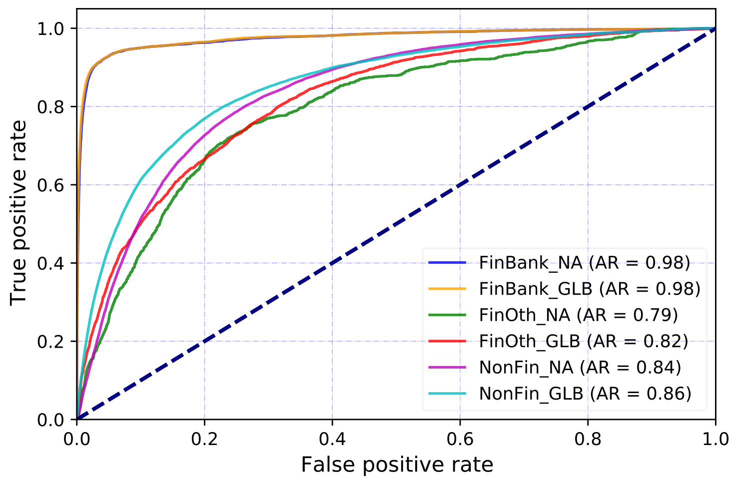

Figure 7: ROC curves. All models exhibit good performance.

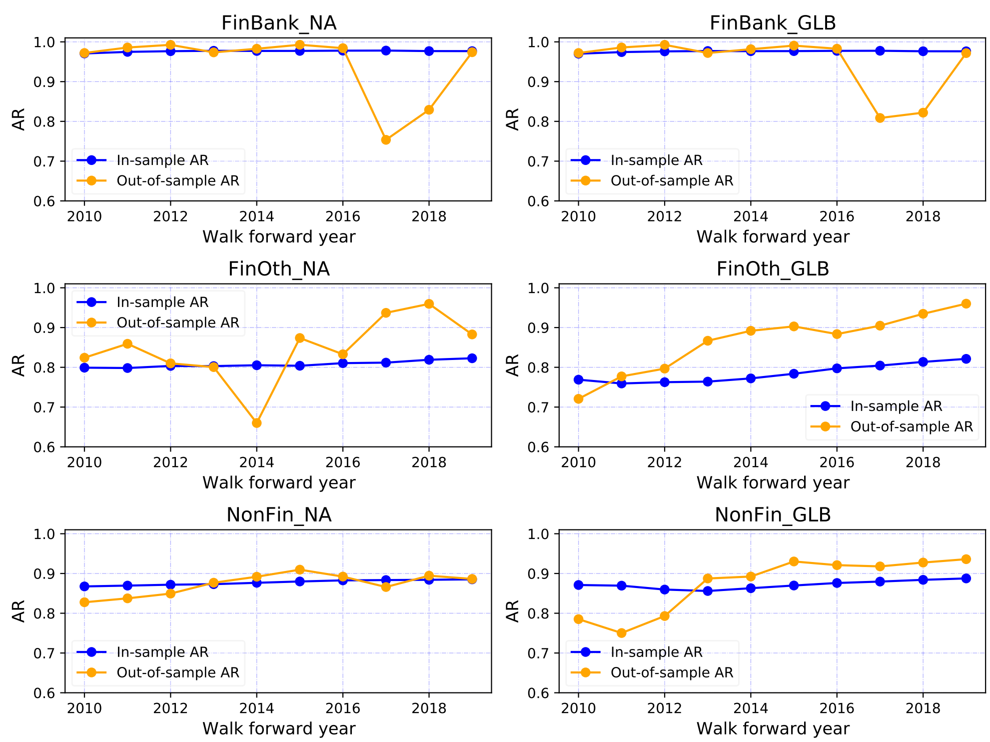

Figure 8: AR walk-forward test. The out-of-sample AR values range between 65% and 98%, suggesting that DRSK predictive ability remains high across different regions and sectors.

6.2 DP distribution

Figure 9 illustrates the distribution of DPs by displaying how the percentage of firms in each DRSK credit grade varies over time for the private firms covered by each model (not for the calibration set of each model). The downward shift of the credit grade distribution around 2000 and 2008 shows that the model captures the increases in credit risk that occur during times of economic contraction.

Except for the FinBank_GLB model, the private model credit grade distributions are more concentrated around the central credit grades than the public model distributions. This is expected, given that the private model uses sector average DDs instead of firm level DDs. Unlike the other models, the FinBank_GLB model puts a substantial percentage of firms in the IG1 range, except for during the 2008 financial crisis. This is probably a consequence of the low number of bank default records outside of North America.

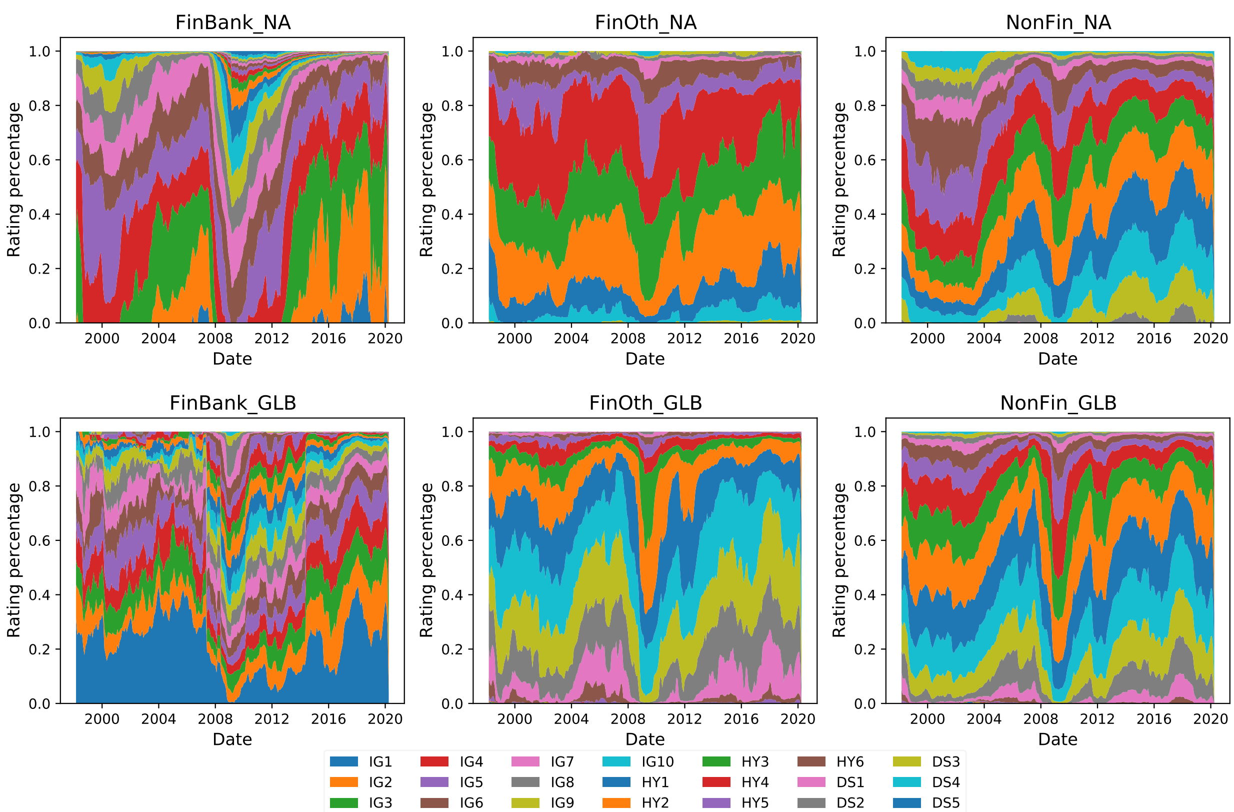

Figure 9: DP distribution by credit grade. Model credit grades are very sensitive to economic conditions.

6.3 Realized default rate comparisons

The comparison of estimated default rates to realized default rates is given in Figure 10. It shows that estimated DPs are largely in alignment with realized default rates. Realized and estimated default rates are closest for the bank sub-models, followed by the non-financials models. The relationship between the two is weaker for the non-bank financials, but the two are still in alignment. This is partly due to the smaller size of this cohort, leading to more idiosyncratic default observations.

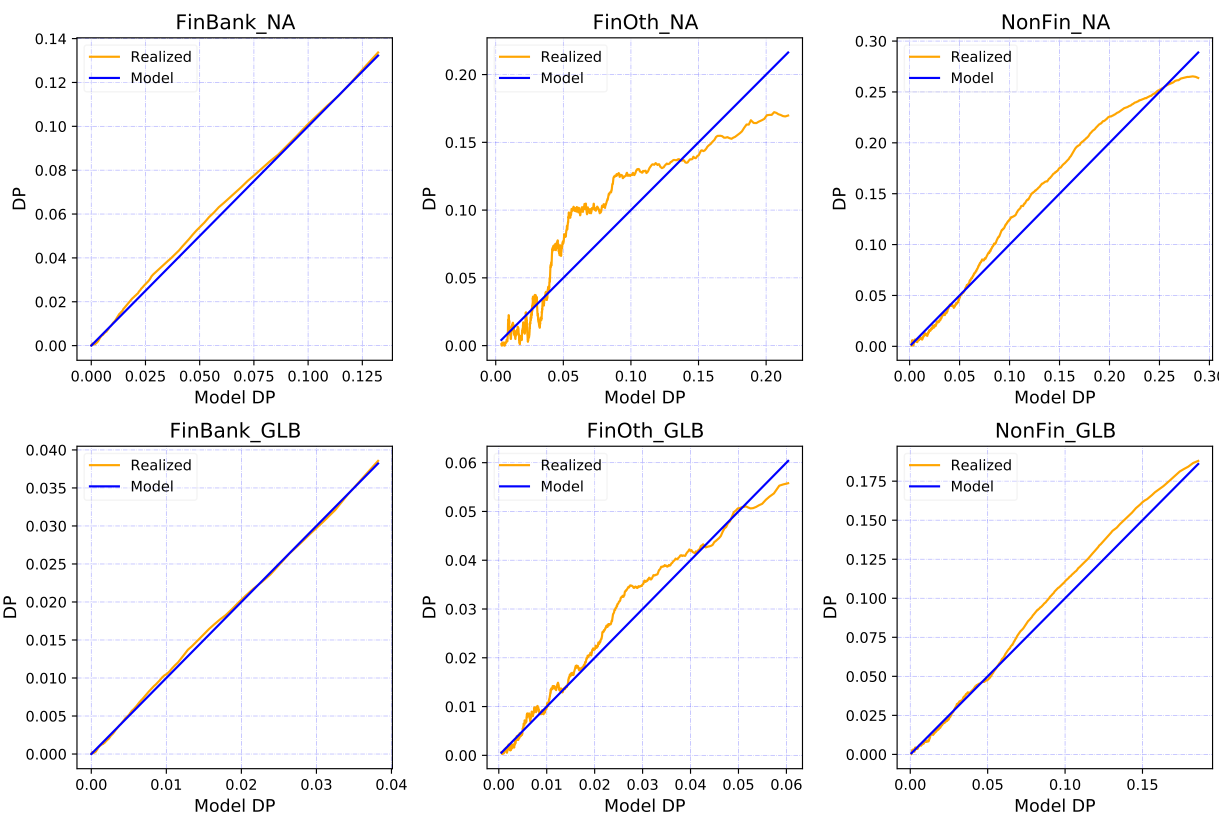

Figure 10: DP vs. realized default rate. Default rates and model DPs are generally in alignment.

In addition, we compare the public and private model average DPs over time on the public firms (Figure 11. The figures show that the two models are generally in alignment as well. The largest differences are in the FinBank_GLB model, where, in 2009, the private model gives an average default rate of about 1.1%, whereas the public model is at 0.6%.

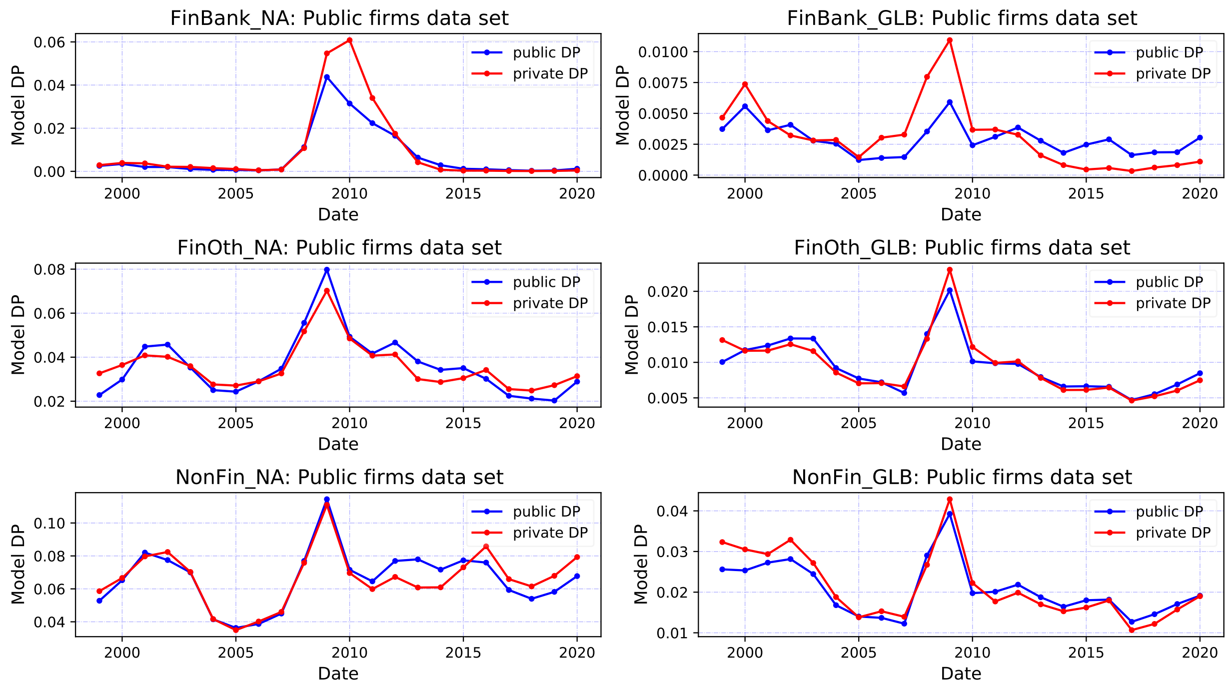

Figure 11: Private vs public DP time series. The average model DP for public firms and private firms are generally in good alignment.

6.4 Predictive power

We test each sub-model's ability to predict defaults by looking at the sub-model's average default probability as a function of the number of months-to-default. A model that has good predictive power should show an increasing average default probability as the time-to-default decreases.

Figure 12 shows that DPs of defaulted firms tend to increase as the time-to-default decreases. The FinBank_NA sub-model exhibits the best performance, with average DPs rising from 0.5% to 54%. Similarly, the FinBank_NA model has DPs rising from 0.2% to 31%. The non-financial models also exhibit a strong trend, although average default rates prior to default only rise to about 15–20%. The non-bank financials sub-models exhibit the same trend, but to a lesser extent, with default rates only rising to about 4% for the FinOth_GLB sub-model and to 11% for the FinOth_NA sub-model.

6.5 Coefficient behavior

In the walk-forward coefficient test, for each year from 2010 to 2019, we recalibrate the model using data up to that year, and analyze the resultant model coefficient time series. This causes the selection of firms in each regression to change. New firms will be added, defaulted firms excluded, etc. Also changes in regulations might have an impact. So, some variation in the coefficients over

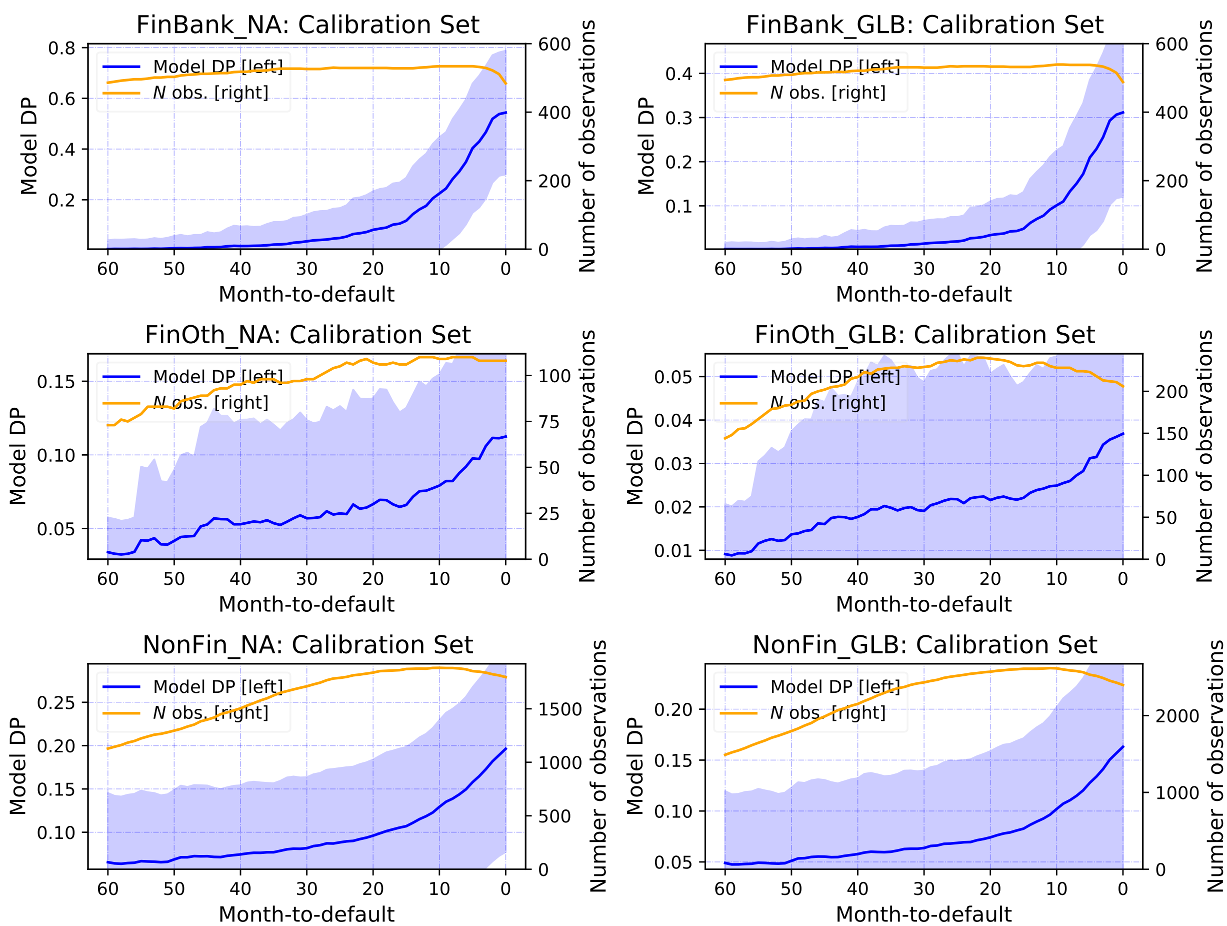

Figure 12: DP versus month-to-default. The shaded area represents ± the standard deviation of DP. Model DPs have a strong predictive ability for a horizon of up to five years.

time is expected. But we would like to see that the coefficients are still relatively stable, with high Z-values[3].

The Z-values in the walk-forward test have high magnitude and the coefficient standard deviations are low, so we can conclude that the coefficients have high significance. The average year over year change of the coefficients themselves is around 8% for the last 10 years, so on average, the model is stable and capturing intrinsic market behavior. The individual coefficients themselves exhibit greater variance, but they stabilize in 2016, remaining within 10% of each other from there on.

7 Comparison to the previous model

Since the private model uses the public model's sector DD as an input, the update of the public model necessitated recalibrating the private model. Recalibrating the model subsequently necessitated additional changes and adjustments:

- The calibration data set was updated to include newer data and more recent defaults.
- The sector classification scheme was updated from BICS version 1 to BICS version 2.
- The model was calibrated to match the public model on the public calibration set. The previous model used a different methodology for over-sampling defaulting firms.
- Where applicable, fallbacks are used for financial data fields when primary fields are unavailable.

The new model exhibits improved performance, especially for the FinBank_GLB sub-model. As can be seen in Table 6, the new model largely outperforms the previous model across regions and sectors. The new sub-models have higher R-squared values, with FinBank_NA showing the biggest improvement, rising from 49% to 60%. Aside from a slight decrease in the AR for the FinOth_NA and NonFin_NA sub-models, the new models also have higher ARs.

The previous model and the new model are mostly in alignment over time (Figure 13). The biggest change is that the new FinOth_NA and NonFin_NA sub-models give higher DPs on average.

We also compare the DP distributions of the two models. Given the number of changes made, this cannot be done over the entire history. Instead, it has to be focused on a particular month.

Figure 14 shows the distributional changes for March 2009, which is in the middle of the 2008 financial crisis. The previous FinBank_NA sub-model assigned an IG1 credit grade to a large percentage of the population. The new sub-model assigns them to the IG6 range. Overall, the non-bank financials and non-financials sub-models rank firms as riskier than the previous model and FinBank_GLB remains about the same.

Figure 15 repeats this analysis for March 2019, a time of less financial market stress than March

[3] The Z-value of a coefficient is the ratio of the coefficient to its standard deviation. The larger the Z-value is in absolute magnitude, the more certainty there is that the coefficient is meaningful.

2009. The new model still generally produces higher DPs than the previous model, but the differences are much smaller. In this time period, the new model ranks North American financials as significantly riskier than in the previous model, and non-financials as somewhat riskier. FinOth_GLB is now about the same. FinBank_GLB has gotten a little safer, although investment grade North American banks are somewhat riskier in the new model than they were in the previous model.

Overall, the new model is more sensitive to economic conditions than the previous model.

Table 6: Adjusted pseudo R-squared and AR - previous vs. new. The new model largely outperforms the previous model.

Region	Sector	Adjusted pseudo R-squared		AR	
		Current	New	Current	New
North America	Banks	49%	60%	97%	98%
	Non-bank financials	21%	37%	72%	71%
	Non-financials	23%	44%	86%	83%
Global	Banks	55%	56%	61%	75%
	Non-bank financials	20%	28%	78%	78%
	Non-financials	42%	49%	77%	77%

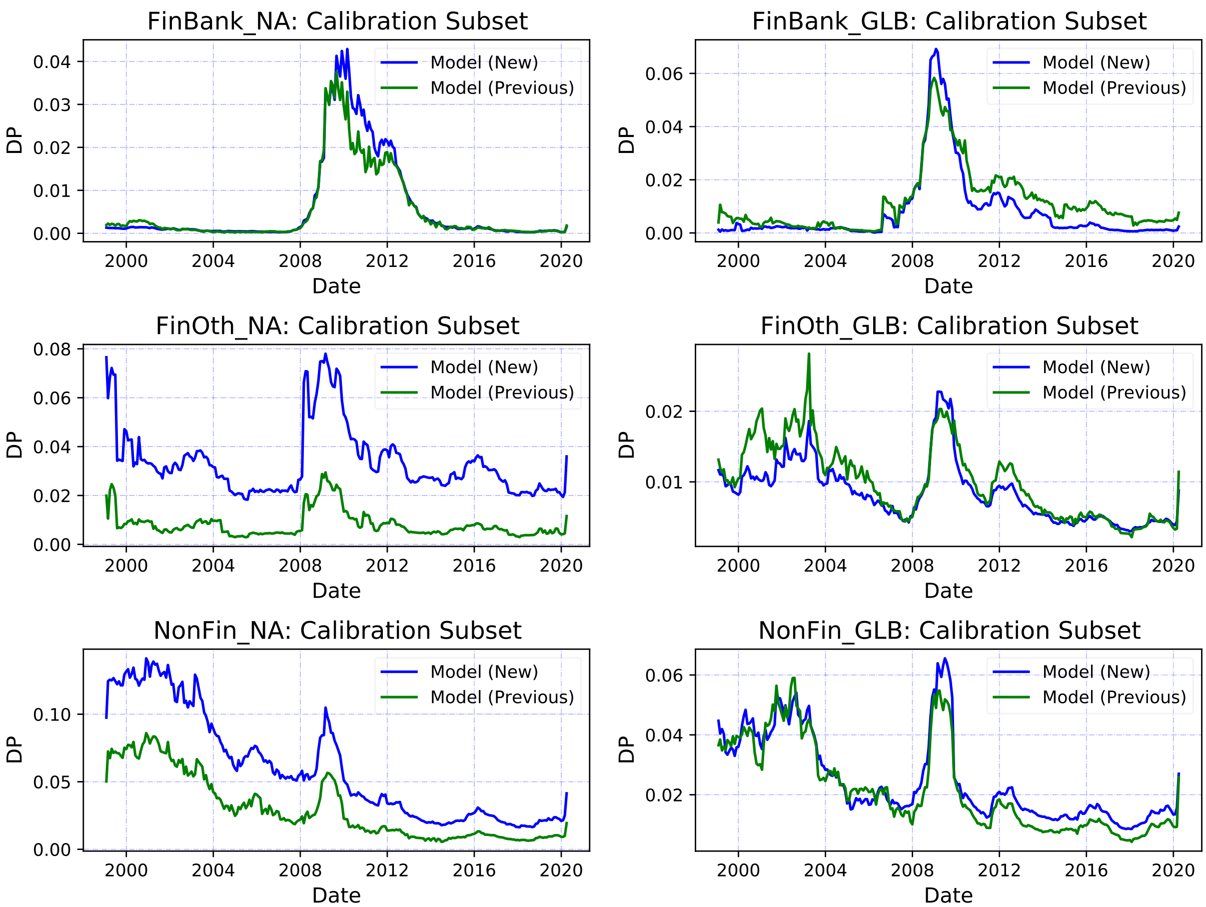

Figure 13: DP time-series - previous vs new model. The two models show similar behavior over time. The average DPs are close for all sub-models, except for FinOth_NA and NonFin_NA where the new DPs are higher.

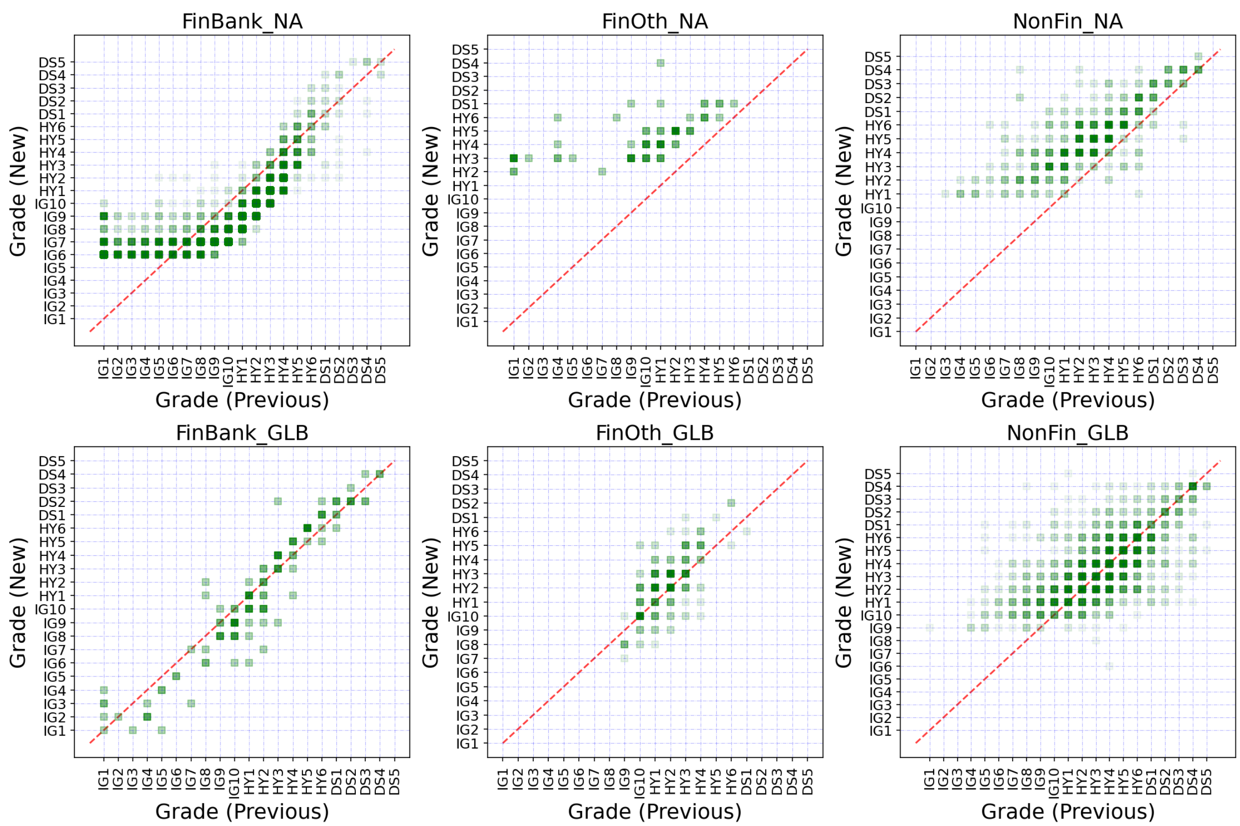

Figure 14: DP scatter plot - previous vs. new model (March 2009). During the financial crisis of March 2009, the new model tends to give firms higher DPs than the previous model.

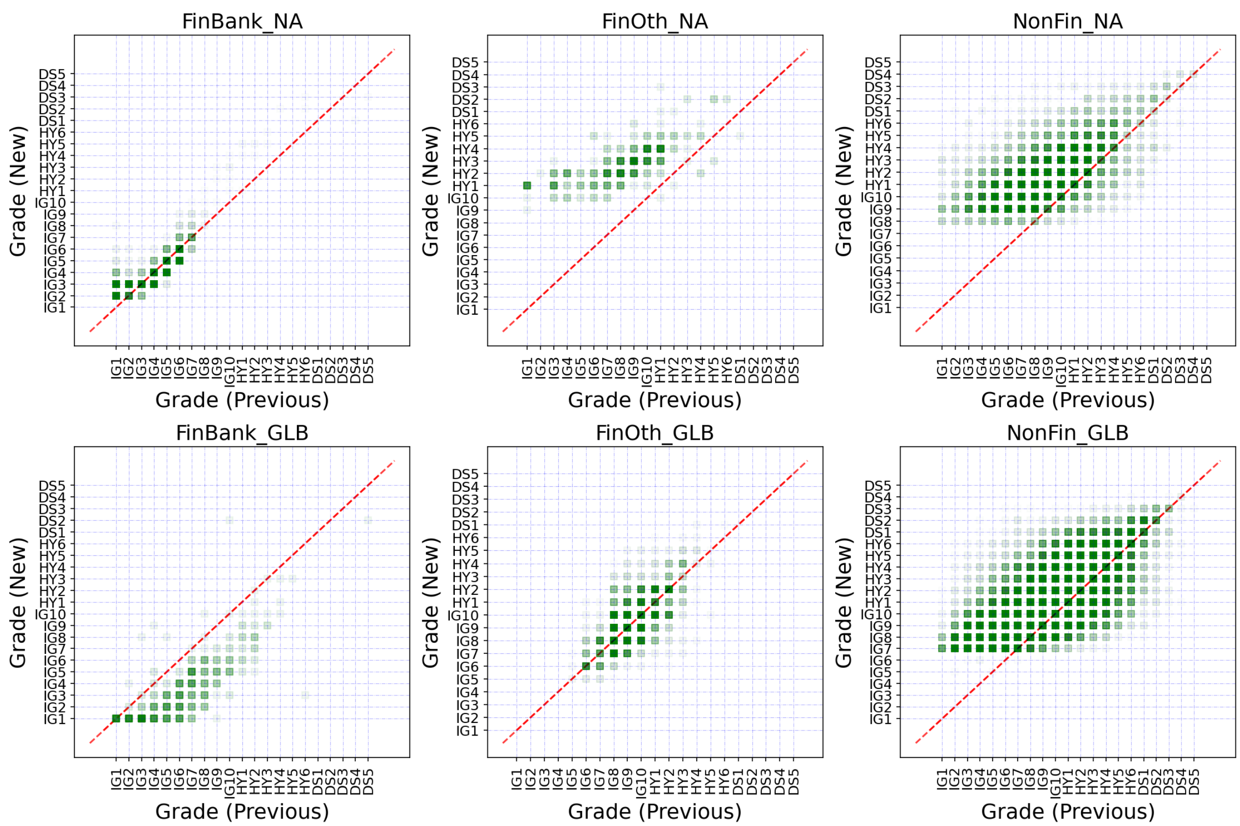

Figure 15: DP scatter plot - previous vs. new model (March 2019). Comparing to March 2009, the new model's DPs are still higher than the old model's but they have dropped closer.

8 Summary

The new DRSK model for private firms was recalibrated this year to account for the update of the new DRSK public firm model. The process included updating the calibration data set and methodology, and moving to the second version of BICS. We reviewed the model structure and its performance and compared it to the previous model.

We showed that the new model performs well via a variety of tests. Adjusted pseudo R-squared values have improved, as have most ARs. The model's predictive power is good in general, and is relatively strong for some sub-models. The estimated model DPs are more aligned with historical default rates and more consistent with public firm DPs.

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