Spatial Redistribution of Carbon Taxes

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Abstract

Policies to slow down climate change are high on the policy agenda and their distributional consequences are actively debated. This paper makes two contributions to the discussion. First, it empirically identifies the spatial dimension between rural and urban households as important for the distributional consequences of carbon taxes, as annual carbon footprints of German households in rural areas are 2.2 tons higher than those of urban households, around 12 percent of an average household's carbon footprint. Second, I build a quantitative spatial general equilibrium model to evaluate different policies of recycling carbon tax revenues on the spatial redistribution and their political support along the transition towards clean technologies. I find that spending carbon tax revenues on lump-sum transfers redistributes from rural to urban households. For a carbon tax of 300 Euros per ton, the difference in the net present value of net transfers amounts to 8,000 Euros. Place-based transfers avoid this spatial redistribution without reducing the speed of transitioning to clean technologies. This has important implications for the political support of these policies, as place-based transfers allow to set higher carbon taxes under the constraint that the policy is beneficial for a majority of households in both regions.

Keywords: Climate change, Inequality, Tax and Transfer policies, Spatial Economics

JEL: E21, H23

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1 Introduction

Climate policies and their distributional consequences are high on the political agenda of governments around the world. In particular, carbon taxes are considered a key policy instrument to reduce carbon emissions. Understanding their effectiveness and distributional consequences is crucial not only for efficiency but also for their political support. One dimension of redistribution that has garnered significant attention in the public debate but has been widely neglected by quantitative macroeconomists, is the spatial dimension between rural and urban households. Do carbon taxes redistribute across space between rural and urban regions? And if so, by how much and does it change over the transition period towards clean technologies? How much do these results depend on the way carbon tax revenues are recycled back to households and what are the implications for the political support of these policies?

This paper provides answers to these questions in a two-step approach. First, I empirically document the heterogeneity of energy consumption and consequently of carbon footprints in the German household sector in 2018. I focus on the dimensions of income and space and provide evidence for substantial heterogeneity in carbon footprints. Second, I build a dynamic general equilibrium, heterogeneous agent model with two regions and a transition towards clean technologies. I investigate the effects of a carbon tax on the distributional consequences across income groups and regions. Along these dimensions, I compare different policies of recycling carbon tax revenues back to households and study the implications on their political support in the overall population and within regions.

There are three main findings. First, in the empirical analysis, I show that annual carbon footprints of rural households are 2.2 tons higher than those of urban households, around 12 percent of an average household's carbon footprint. This difference is entirely driven by emissions from gasoline and heating energy and is robust along the income distribution. Second, as a consequence, spending carbon tax revenues on lump-sum transfers redistributes from rural to urban regions. Based on the quantitative model, the difference in the net present value of net transfers from a carbon tax of 300 Euros per ton is 8,000 Euro, around 25 percent of an average household's annual net income. Third, place-based transfers avoid this spatial redistribution without decreasing the speed of transitioning to clean technologies. Hence, they allow to set a higher carbon tax under the constraint that a majority of households in both regions benefits from the tax.

For the empirical analysis, I combine rich consumption data from the German Income and Consumption Survey (Einkommens- und Verbrauchsstichprobe, EVS) with the EXIOBASE dataset, which provides information on the amount of carbon emissions produced by different consumption goods. The consumption data reveals that rural households spend 70 percent more on gasoline, 25 percent more on heating energy and are more likely to use heating technologies that generate more carbon emissions, particularly oil heating systems. These differences are computed controlling for multiple household characteristics, like net household income and age, and are significant. For computing carbon emissions at the household level, I follow the literature and distinguish between direct and indirect emissions, where the former refer to emissions generated by consumption itself, like driving a car or heating a flat, and the later refer to emissions generated by producing and transporting goods (Hardadi et al., 2021). Consistent with previous findings from the literature, household carbon emissions increase with income, nearly doubling from the lower to the upper third of the income distribution. When looking at the spatial heterogeneity, I do not find any significant differences for indirect emissions. For direct emissions, however, rural households emit around 2.2 tons of carbon emissions more each year, around 36 and 12 percent of an average household's direct and total carbon footprint, respectively. This difference is also computed controlling for household characteristics and is significant. Further, this difference is generated by emissions from gasoline and heating energy to similar shares and constant along the income dimension.

Guided by this empirical evidence, I build a quantitative general equilibrium model with two regions and two types of housing and car technologies. The two regions differ, among others, in their household-specific amenities and energy consumption requirements and are called *rural* and *urban*. The two housing and car technologies differ in whether they emit carbon emissions and how efficiently they can transform raw energy into temperature and vehicle miles traveled and are called *dirty* and *clean*.[1] As the majority of German households are tenants and as tenants are more mobile and thus drive housing prices across space, I model households as tenants. They can decide in which region to live and which technology to use. Next to these discrete decisions, they decide upon the continuous levels of their housing size, heating energy, car energy and non-housing, non-energy consumption. Besides the household sector, there is a firm sector consisting of three firms. First, there is a construction firm which builds dirty and clean housing and transforms dirty into clean housing. Second, there is a competitive renting firm, which buys housing from the construction firm and rents it out to households. Last, there is a competitive production firm which produces cars, energy and the non-housing, non-energy good. Finally, there is a government, which in the baseline only extracts the construction firms' profits and redistributes it back to households.

I calibrate this model to the German economy in 2018 and start out in an initial stationary equilibrium without clean technologies.[2] I introduce clean technologies exogenously in 2019, which starts a transition towards them, as, consistent with empirical estimates, they transform raw energy into temperature and vehicles miles traveled more effectively. I compare a transition without any policy intervention with a transition with a carbon tax of 300 Euros per ton of carbon emissions on energy consumption. This level is well in line with estimates about future carbon taxes within the Emission Trading System 2 (ETS2) proposed by the European Union and explicitly targeted at emissions from heating and car energy (Kalkuhl et al., 2023). I compare three ways of recycling back the carbon tax revenues. First, I will look at lump-sum transfers which have been shown to have important implications for redistribution along the income dimension as they revert the regressive redistribution of carbon taxes without transfers into progressive redistribution (Douenne, 2020). Second, I zoom into the spatial dimension and look at place-based transfers which are set to prevent any redistribution across regions such that all revenues raised within a region are rebated in the same region. Last, I introduce subsidies on housing renovations which increases the speed of transitioning to clean houses.

I start by evaluating the long-run consequences of these policies by comparing their stationary equilibria. I find that, without any policy intervention, the share of clean cars and clean houses rises to 84 and 92 percent, respectively. As clean technologies are more effective in transforming raw energy into temperature and vehicle miles traveled, energy consumption levels in the final stationary equilibrium

[1] For housing, one can think of badly insulated houses with oil heating systems and very well insulated houses with heat pumps. For cars, one can think of traditional gasoline or diesel-powered cars and electric cars.

[2] Based on data from the German Federal Motor Transport Authority (*Kraftfahrtsbundesamt*) and the German Federal Association of the Energy and Water Industry (*Bundesverband der Energie- und Wasserwirtschaft*) the shares of electric cars and heat pumps relative to all cars and heating systems were only 0.1 and 2.2 percent, respectively, which justifies this assumption.

fall relative to the initial stationary equilibrium without clean technologies by 60 and 9 percent for heating and car energy and household's carbon emissions fall by 82 percent. Since, heating energy and housing consumption are complements, a decrease in the effective price for heating energy, caused by a more efficient heating technology, increases housing demand. As the housing construction technology exhibits decreasing returns to scale, this leads to an increase in housing prices by around 6 percent. In the rural area, this increase is around 0.5 percentage points higher than in the city because of net migration from the urban region. Rural households have higher energy consumption levels, which in the final stationary equilibrium become cheaper, creating an incentive to move to the rural region. As the clean technologies are more efficient, household's welfare, measured as the consumption equivalent variation (CEV) in terms of the non-housing, non-energy good, increases by 2.87 percent relative to the initial stationary equilibrium without clean technologies. When introducing the carbon tax and reimbursing households via lump-sum transfers, the share of clean technologies in the final stationary equilibrium increases to 99 and 98 percent for cars and houses. Thus, energy consumption falls by 63 and 12 percent, carbon footprints fall by 99 percent and housing prices increase by 6 percent. As the share of households with the clean, more efficient technologies increases, the positive welfare effects rise slightly by 0.15 percentage points. For these long-run outcomes, the results hardly change depending on what the carbon tax revenues are used for as carbon footprints and thus carbon tax revenues are very small.

Next, I evaluate the different policy scenarios along the transition between the stationary equilibria. The speed of transitioning to clean technologies strongly varies between the policy scenarios. Without any policy intervention, the shares of households with clean houses and cars increases to 26 and 31 percent by 2040 and to 65 and 74 percent in 2060, before they converge to their stationary equilibria by around 2100. When introducing carbon taxes, this transition happens faster. The share of clean cars increases to 43 and 90 percent in 2040 and 2060 and does not depend on the way carbon tax revenues are reimbursed. When spending the carbon tax revenues on lump-sum or place-based transfers, the share of households with clean houses increases to 38 and 72 in 2040 and 2060. When spending the revenues on subsidies on housing renovations, the transition is further accelerated such that the share increases to 45 and 78 percent in 2040 and 2060. These differences in the speed

The speed of this transition has important implications for the level of spatial redistribution over time. Spending the tax revenues from a carbon tax of 300 Euros per ton on lump-sum transfers redistributes around 300 Euros from rural to urban households annually for the first years of the transition. As more households adopt clean technologies over time, carbon footprints and hence the level of redistribution fall and converge to a situation without spatial redistribution by around 2060. The difference in the net present value of net transfers is around 8,000 Euros, corresponding to 25 percent of an average household's annual net income. For the case of place-based transfers, there is no spatial redistribution by construction and when spending the carbon tax revenues on subsidies for renovations, the difference in tax burdens is similar to the one for lump-sum transfers. Thus, when spending the carbon tax revenues on lump-sum transfers or housing renovations, we observe net migration to the city for the first years of the transition, which peaks around the year 2040. At that point, the share of the urban population will rise by 1.3 percentage points, corresponding to around one million households. Towards the end of the transition, there is net-migration to the rural regions as first, the level of spatial redistribution falls and second, energy efficient technologies, in particular in the rural regions, become more important.

The general-equilibrium effects of carbon taxes on housing prices are sizeable. Without policy intervention the price premia for clean houses in rural and urban regions relative to the one for dirty houses in the initial stationary equilibrium, jump to 13 and 11 percent upon introducing the clean technologies, where the price for dirty houses remains unchanged. As the construction firm builds new clean houses and renovates dirty into clean houses, the prices for clean and dirty houses converge. When spending the carbon tax revenues on lump-sum transfers, the initial rise in the price premium for clean houses is slightly higher, around 14 and 12 percent in rural and urban regions. But now also the prices for dirty houses drop relative to the initial steady state by 11 and 7 percent for rural and urban regions. This drop results from a decrease in housing demand as heating energy, a complementary good to the housing size, becomes more expensive due to the carbon tax. As rural households consume more heating energy and use heating technologies which pollute more, this effect is stronger for them. When spending the carbon tax revenues on place-based transfers, the rural housing price increases by around 2 percentage points whereas the urban price decreases by the same amount as this policy avoids spatial redistribution and thus net migration from rural to urban regions. Last, spending the tax revenues on subsidies for renovations decreases the clean housing price due to a higher supply by around 4 percentage points.

In a next step, I evaluate these policy scenarios based on their political support. I start by comparing them regarding their monetary consequences for households in terms of the net present value of net transfers, i.e. what households receive as transfers, including transfers by the government from higher firm profits, minus what they pay as carbon taxes. When spending the carbon tax revenues on lump-sum transfers, around 48 and 74 percent of rural and urban households have positive net present values, meaning they benefit.[3] For the cases of place-based transfers and subsidies these shares are 61 and 0 percent in both regions. For the later this is by construction, as no transfers are paid and net transfers are thus necessarily negative. These share are close to constant when introducing a low- and a high-carbon tax scenario of 100 and 500 Euros per ton as they are determined by the share of households emitting less than the average carbon footprint in the overall population or within regions. Next, I evaluate these policies based on their welfare consequences for households. As the carbon tax distorts household's consumption decisions, the share of households who benefit in welfare terms is lower than the one based on monetary terms. With lump-sum transfers these shares are 24 and 63 percent, for place-based transfers 36 and 59 percent and with subsidies for housing renovations 6 and 9 percent for rural and urban households. In a last step, I try to incorporate the positive externalities from reducing carbon emissions on household's welfare in a reduced form. For doing so I make two assumptions. First, I assume the social costs of carbon to be 500 Euros per ton, which is at the higher end of what people traditionally use but within the range of recent estimates (Bilal and Känzig, 2024; Rennert et al., 2022). Second, I assume that this tax is set for the European Union and that households within this union do not care about households from other regions. As these are two ad-hoc assumptions, the following results become more qualitative than quantitative. Including the benefits of reducing carbon emissions into the analysis increases the share of households benefiting from carbon taxes to 29 and 67 percent for lump-sum transfers, 40 and 62 percent for place-based transfers and 10 and 13 percent

[3]Households are classified into *rural* and *urban* based on where they live in the initial steady state.

for subsidies on renovations for rural and urban households, respectively. Varying the level of the carbon tax shows that higher carbon taxes are politically feasible under the constraint that a majority of households in both regions benefit if carbon tax revenues are spend on place-based transfers.

This paper relates to two strands of literature. First, it contributes to the empirical literature documenting a high heterogeneity of carbon emissions in the household sector. The focus of this literature has been to study the heterogeneity along the income dimension. A key finding in this literature is that carbon emissions increase with income, which has been documented for Germany (Hardadi et al., 2021; Miehe et al., 2016) as well as for other European (Duarte et al., 2012; Isaksen and Narbel, 2017; Kerkhof et al., 2008) and non-European countries (Perobelli et al., 2015; Wiedenhofer et al., 2017). Recently, horizontal heterogeneities of carbon footprints within income groups caught more attention, indicating that households in rural areas have larger carbon footprints than those in urban areas (Douenne, 2020; Gill and Moeller, 2018; Tomás et al., 2020). I am contributing to this literature by quantifying the heterogeneity for Germany using the most recent available data and identifying a key role for emissions from car and heating energy. As car and heating technologies are changing rapidly and vary substantially across countries (Rosenow et al., 2022), studying specific countries with up-to-date data is key for analyzing the heterogeneous burden of carbon taxes.

Second, this paper relates to the literature of studying the distributional consequences of climate policies. This literature has so far focused on redistribution along the income distribution and between different generations. Känzig (2021) shows that the poor bear higher economic costs from carbon taxes as their energy consumption share is higher and, importantly, their income falls more through general equilibrium effects in the labor market. Similarly, Kuhn and Schlattmann (2024) find a policy trade-off between carbon emission reduction and redistribution as policies which maximize the reduction in carbon emission redistribute substantially from poor to rich households. Fried et al. (2018) evaluate the distributional effects of a carbon tax on households living in a current and a future steady state in a general equilibrium life-cycle model calibrated to the U.S. economy. They find that households in the current steady state prefer uniform, lump-sum rebates, while households in the future steady state prefer reducing existing distortionary taxes. Relatedly, Kotlikoff et al. (2021) compute the optimal carbon tax path in an overlapping generations model and find that it raises all generations' welfare by almost 5 percent but requires major intergenerational transfers. Douenne et al. (2023) study the optimal fiscal policy to address climate change and inequality jointly and find that the revenues from carbon taxes are optimally split between reducing tax distortions and increasing transfers equally. I contribute to this literature by studying the distributional consequences of carbon taxes along the unexplored but oftentimes discussed spatial dimension between rural and urban households.

The remainder of this paper is structured as follows. Section 2 introduces the data employed and presents the empirical results. Section 3 presents the model and explains the calibration strategy. Finally, Section 4 introduces the policy experiments and presents the results, before Section 5 concludes.

2 Empirical evidence

This section first introduces the datasets used in this paper before empirically documenting the heterogeneity in energy consumption pattern for households living in rural and urban regions. Lastly, I translate these consumption patterns into carbon footprints to document the level of spatial heterogeneity along this dimension.

2.1 Data

The empirical analysis is based on two datasets. First, I employ the German Income and Consumption Survey (*Einkommens- und Verbrauchsstichprobe*, EVS), which provides repeated cross sections on consumption expenditures of households similar to *Consumer and Expenditure Survey* (CEX) in the U.S. The EVS provides detailed information on around 43,000 households, which corresponds to 0.1 percent of the German households, and sample weights allow to construct representative statistics for the entire German population. It is collected every 5 years and is considered to be of excellent quality as it is also used for computing the consumption basket for the German CPI. I pool together the two most recent versions from 2013 and 2018. Second, I use the EXIOBASE v3.6 dataset in order to quantify the carbon emissions generated by different consumption goods. This dataset is compiled from multi-regional input-output tables and differentiates between 44 countries and five rest of the world regions, 163 industries, and 200 products.[4] I bridge the two datasets based on the bridging strategy developed in (Hardadi et al., 2021). For calibrating the model, I further utilize data from the German Socioeconomic Panel (SOEP).[5]

2.2 Spatial heterogeneity in energy consumption

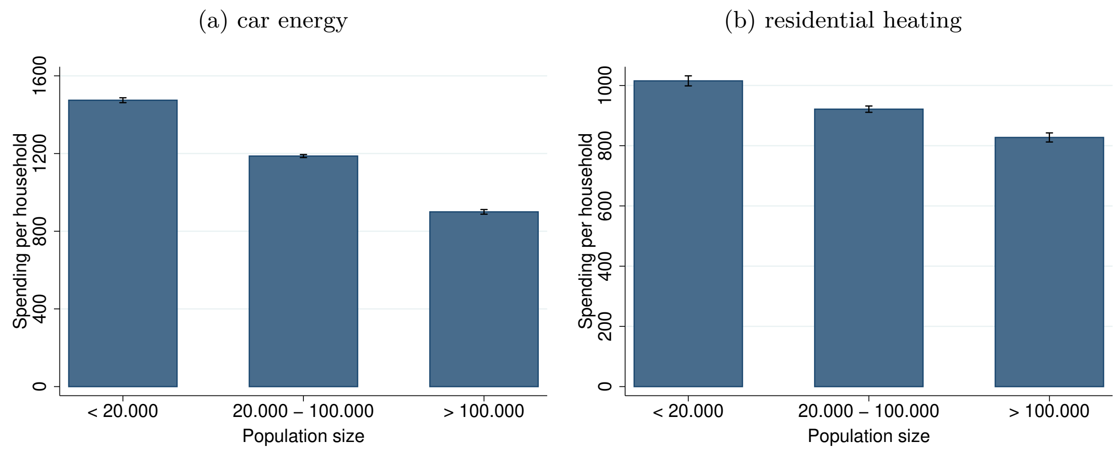
For analyzing the heterogeneity in energy consumption patterns for households living in rural and urban regions, I focus on car energy and residential heating consumption. These two consumption goods are not only considered to be of key importance for carbon footprints, as they make up around a third of total household carbon emissions (Hardadi et al., 2021), but they are also key for understanding the heterogeneity in carbon footprints across space, as I will document. I distinguish between three levels of city sizes, small villages of less than 20,000 inhabitants, small cities with population sizes between 20,000 and 100,000 and large cities with more than 100,000 inhabitants. This definition leaves around one third of the total population in each city size category. In order to identify the differences in energy consumption for comparable households living in cities of different sizes, I regress the annual expenditures per household for car energy and residential heating not only on the city size category but also on the age of the main earner, the households net disposable income, the households size and the survey year. For residential heating expenditures, I further control for the heating technology to isolate the level of expenditures from the heating technology.

Figure 1 shows the resulting average expenditures for comparable households depending on where they live. Thus, Figure 1a indicates that an average household with respect to age, net household income, household size and the survey year who lives in a village of less than 20,000 inhabitants spends around 70 percent more on car energy compared to a households with the same average characteristics who lives in a larger city of more than 100,000 inhabitants. Similarly, Figure 1b shows that households in rural areas spend around 25 percent more on residential heating than comparable households

[4]For more information see Stadler et al. (2018).
[5]For more information see Goebel et al. (2019).

in urban regions. These differences are highly significant as indicated by the black confidence intervals.

Figure 1: Energy expenditures across space

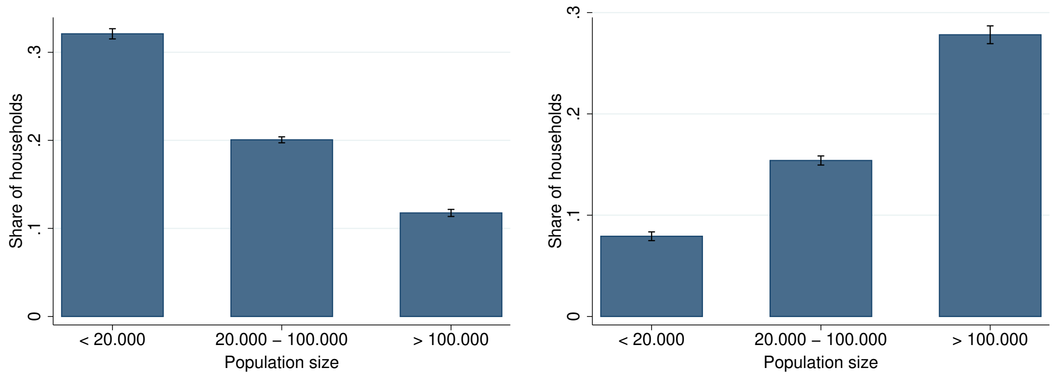


Notes: Panel (a) shows the average annual expenditures per household in Euro for car energy for households living in cities of different population sizes. These averages are computed controlling for the age of the main earner, households net disposable income, the household size, and the survey year. Panel (b) shows the average annual expenditures per household in Euro for residential heating for households living in cities of different population sizes. Besides the age of the main earner, households net disposable income, the household size, and the survey year, these averages are computed controlling also for the heating technology.

Besides the level of energy consumption, households in rural and urban regions do also differ in the way in which they heat. While in both regions around half of the population heats with natural gas, Figure 2 shows that the share of households heating with oil and district heating differs substantially. Figure 2a indicates that more than 30 percent of households in rural areas are using oil heating systems, whereas only around 11 percent in urban areas use this technology. The reverse picture, we observe in Figure 2b depicting the share of households using district heating. While close to 30 percent of households in the city use district heating, only around 7 percent of households in rural areas do so. These shares are again computed for an average household in terms of the main earner's age, household net disposable income, the households size and survey year. Importantly, heating with oil generates substantially more carbon emission compared to district heating. According to the most recent estimates from the International Institute for Sustainability Analysis and Strategy based on the Global Emission Model for Integrated Systems (GEMIS) Version 5.0, producing one kWh of oil and district heating generated 315 and 237 grams of carbon emission in Germany in 2015, respectively. Given that the share of renewable energy increases for district heating while the level of carbon emission emitted by burning oil stays constant, this difference can be considered a lower bound along the transition period towards clean technologies.

Figure 2: Heating technologies across space

(a) oil heating (b) district heating



Notes: Panel (a) shows the share of households heating with oil for different in cities sizes. These averages are computed controlling for the age of the main earner, households net disposable income, the household size, and the survey year. Similarly, Panel (b) shows the share of households using district heating for different in cities sizes. Again, I control for the age of the main earner, households net disposable income, the household size, and the survey year.

2.3 Spatial heterogeneity in carbon footprints

As a last step in the empirical analysis, I compute the level of carbon emissions emitted per household and year. For doing so, I follow the literature and distinguish between direct and indirect emissions (Hardadi et al., 2021). The former refer to emissions directly generated by household's consumption, like driving a car or heating an apartment, and account for around 30 percent of total household emissions. The later refer to emissions generated by producing and transporting goods and services, like transporting a banana from South America to Europe. Emissions generated by public transportation, like from bus rides, are also included in indirect emissions.

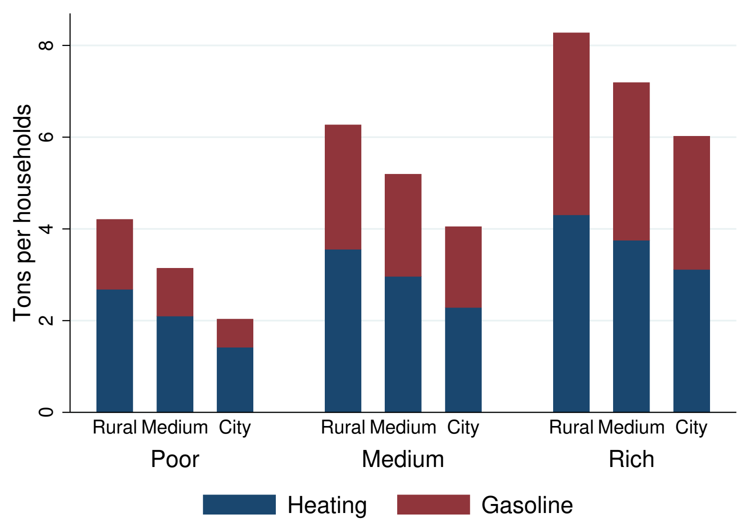
For computing direct emissions, I divide household's expenditures for each of the different items, like gasoline or oil, by the respective yearly average price to get the quantities.[6] In a second step, I take estimates from the natural science literature on the level of emissions generated by consuming these quantities to compute the level of direct emissions at the household level.[7] For indirect emissions, I follow Hardadi et al. (2021) and impute carbon emissions from consumption by linking the consumption categories of the EVS to those of the EXIOBASE. My imputation differs from Hardadi et al. (2021) in two dimensions. First, while they estimate carbon footprints for an average household and for eleven income groups, I impute carbon emissions of consumption at the household level. This is crucial for my empirical analysis as it allows me to differentiate between carbon footprints of households in rural and urban regions. Second, they correct for expenditure underreporting in the EVS data. I also compute results corrected for expenditure underreporting as robustness but find differences to be negligible for my analysis. Therefore, I abstain from this adjustment.

[6] I assume identical prices for households living in rural and urban areas. This assumption is based on a recent study by the German Federal Cartel Office which finds no significant prices differences between urban and rural regions (Bundeskartellamt, 2020).

[7] An alternative would be to take estimates for the sum of all direct emissions generated by the household sector in Germany and distribute them to households based on their consumption expenditures. Both approaches yield very similar results.

Figure 3 depicts household's average annual direct carbon emissions in tons per year along the income dimension and for different city sizes. Households are first grouped into three equally-sized categories based on their net disposable household income before being sorted into the three city size categories within the income groups. First, note that I corroborate the well documented finding in the literature that carbon emissions increase along the income dimension (Duarte et al., 2012; Hardadi et al., 2021; Miehe et al., 2016; Wiedenhofer et al., 2017). But more importantly, I show that there is substantial heterogeneity in carbon footprints within income groups, where the city size is key for explaining this heterogeneity. Households in rural areas generate around 2.2 tons more carbon emissions compared to households in urban areas. This difference corresponds to around 12 percent of total carbon emissions of an average household in Germany and stems from emissions generated by residential heating and car energy in equal parts. Interestingly, this difference in carbon footprints is constant along the income dimension which is remarkable given the substantial increase in emissions with income.

Figure 3: Direct carbon footprints across space and income



Notes: This figure shows the average carbon footprints for households of different income and city size groups. First, I divide the total sample of households into three equally-sizes groups according to their net disposable household income. Second, I group households within these income groups based on whether they live in small villages of less than 20,000 inhabitants, in small cities with population sizes between 20,000 and 100,000, or in large cities with more than 100,000 inhabitants.

For the indirect emissions, I again corroborate the finding in the literature that carbon emissions increase along the income dimension as Figure A1 in Appendix A shows. Differently to the direct emissions, for the indirect emissions there is hardly any variation across space. Hence, for studying the distributional consequences across space, I will focus on the direct emissions stemming from car energy and residential heating.

3 Model

This section develops a quantitative general equilibrium model with two regions and two types of technologies for cars and housing units, one is clean and does not pollute carbon emissions, the other one is dirty and does. Besides a household sector, there is a firm sector constructing and renovating housing, renting out housing and producing final consumption goods. Finally, there is a government which, in the baseline, only extracts the firms' profits and redistribute it to households.

3.1 Household sector

The economy is populated by a continuum of measure one of infinitely lived tenants. I focus on tenants as they make up the majority of German households and as they are more mobile and hence drive prices in housing markets across space. Households can migrate between a rural region, denoted by r, and an urban region, denoted by u. In each region, there is a housing stock for clean and dirty housing, denoted $H^{r,cl}$, $H^{r,di}$ and $H^{u,cl}$, $H^{u,di}$, respectively. Time is discrete.

3.1.1 Preferences

Household's per period utility follows a CRRA specification over a Cobb-Douglas aggregate of non-housing and housing consumption. In case the household lives in the urban region, indicated by $l = 1$, he additionally receives household-specific city amenities κ. These amenities are constant over time and can be interpreted as the region in which the household grew up or in which his family and friends live. Both, housing and non-housing are modeled as composite goods. Housing consists of the housing size h and effective heating energy $(1 + \phi^{h,j})e^h$, while non-housing consumption consists of effective car energy consumption $\left(1 + \phi^{c,j}\right)(e^c - \underline{e^{c,l}})$ and non-housing, non-car energy consumption x. The parameters $\phi^{c,j}$ and $\phi^{h,j}$ describe the efficiency of transforming raw heating and car energy into effective heating and car energy, meaning into temperature and vehicles miles traveled. Note, that these production efficiencies depend on whether the housing stock or the car is dirty, denoted $j = di$ or clean, denoted $j = cl$.[8] To capture the empirical observation that households in rural regions consume more car energy, there is a location-specific subsistence level of car energy consumption $\underline{e^{c,l}}$. This level can be thought of as car energy for commuting from which households do not derive any utility.

Hence, the per-period utility function of households reads[9]

$$
\begin{aligned}
u(x, h, e^h, e^c, l) &= \frac{1}{1-\sigma}\left(\tilde{x}^{\gamma}\tilde{h}^{1-\gamma}\right)^{1-\sigma} + l\kappa \\
\tilde{x} &= \left(\mu_x x^{\frac{\nu_x - 1}{\nu_x}} + (1-\mu_x)\left[\left(1+\phi^{c,j}\right)\left(e^c - \underline{e^{c,l}}\right)\right]^{\frac{\nu_x - 1}{\nu_x}}\right)^{\frac{\nu_x}{\nu_x - 1}} \\
\tilde{h} &= \left(\mu_h h^{\frac{\nu_h - 1}{\nu_h}} + (1-\mu_h)\left[(1+\phi^{h,j})e^h\right]^{\frac{\nu_h - 1}{\nu_h}}\right)^{\frac{\nu_h}{\nu_h - 1}}
\end{aligned}
$$

[8] I will calibrate these parameters according to empirical estimates in the literature as described in more detail in Section 3.4.5.

[9] For readability, I am omitting subscripts for the year of the transition. Once I turn to the recursive formulation of the dynamic household problem, I will introduce them.

where γ, μ_x, and μ_h measure the relative taste for non-housing consumption, non-housing non-car energy consumption, and the housing size, respectively. The parameters ν_x and ν_h describe the elasticities of substitution between non-housing, non-car energy consumption and car energy consumption as well as between the housing size and heating energy consumption, respectively. Further, $1/\sigma$ characterizes the intertemporal elasticity of substitution.

3.1.2 Labor Income

Household's labor income consists of the economy-wide wage w_t and an idiosyncratic productivity shock $\nu_{j,t}$, which follows an AR(1)-process. Thus, it evolves according to

$$\begin{aligned} \log y_{i,t} &= \log w_{i,t} + \nu_{i,t} \\ \nu_{i,t} &= \rho \nu_{i,t-1} + \eta_{i,t} \\ \eta_{i,t} &\sim \mathcal{N}\left(0, \sigma_\eta^2\right) \quad \text{ i.i.d.}, \end{aligned}$$

where $p \in [0,1]$ describes the persistence of the idiosyncratic component and σ_η^2 the variance of the innovations. Note, that this process is the same for households in rural and urban regions which is motivated by very similar income paths for them in the EVS dataset.[10]

3.1.3 Budget constraint

Besides their labor income y, households may receive two kinds of governmental transfers. First, the government extracts firm's profits from constructing and renovating housing which it recycles back to households within regions in lump-sum transfers T^π. Second, in case the government introduces carbon taxes, the resulting tax revenues might be paid back to households in transfers T^τ, depending on the policy. They can spend this total income on four consumption goods, the housing size h, car energy e^c, heating energy e^h and non-car energy, non-housing consumption x (the numeraire). The price for renting one unit of housing is given by the location - and housing type-specific renting rate $\rho(l, \lambda^h)$. The variable λ^h takes on values of 1 and 0 if the household lives in a house with a clean and dirty technology, respectively. The renting price is endogenous and will be determined on one the four segmented renting markets. If a household lives in a clean house, he pays the exogenous price p^{eh} per unit of heating consumption. This price is constant over time and the same for households in rural and urban regions.[11] If a household lives in a dirty house, he has to pay the carbon tax τ on each ton of carbon emissions he emits, where ξ_l^h translates heating energy consumption into carbon emissions. Note that ξ_l^h is location-specific because households in rural regions use heating technologies which emit more carbon emissions, as shown in the empirical section of this paper. The costs for car energy are modeled analogously. If a household drives a clean car, indicated by $\lambda^c = 1$, he pays the economy-wide and constant exogenous price p^{ec}. Households driving a dirty car need to pay the carbon tax τ,

[10] I further checked whether income levels of rural and urban households might be heterogeneously impacted by carbon taxes. Känzig (2021) finds that the impact of carbon taxes on household's income is strongest in sectors with a high sensitivity to changes in aggregate demand, such as retail or hospitality. Based on his classification, I grouped sectors into those with lower and higher demand sensitivity but did not find any significant differences in employment shares for rural and urban households.

[11] Even though, as shown in the empirical part, households in rural and urban regions use different heating technologies, the prices per unit of heating energy are very similar.

where ξ^c transforms car energy consumption into carbon emissions. Lastly, households can decide to buy a new car. The costs associated with this adjustment are denoted E_i and depend on whether the household buys a car with a clean ($j = cl$) or dirty technology ($j = di$). Hence, the household's budget constraint reads

$$x = y + T^\pi + T^\tau - \rho(l, \lambda^h)h - [p^{eh} + (1-\lambda^h)\xi_l^h\tau]e^h - [p^{ec} + (1-\lambda^c)\xi^c\tau]e^c - E^j$$

where $E^j = p^{c,cl}$, $E^j = p^{c,di}$, or $E^j = 0$ in case the household buys a clean, dirty, or no new car, respectively. The prices for new cars $p^{c,cl}$ and $p^{c,di}$ will be further specified in Section 3.2.3.

3.1.4 Recursive formulation of the dynamic decision problem

Besides their continuous choices for the four consumption goods, housing size h, heating energy e^h, car energy e^c and non-car energy non-housing consumption x, households need to take four discrete decisions every period. They need to decide whether they want to change their location, whether they want to move to a house with another technology, whether they want to buy a new car and if yes whether they want to buy a car with an dirty or clean technology. In total, there are 12 different combinations of these decisions. For each of the four discrete decisions, there is a extreme value type-1 shock to smooth them. The timing is as follows. First, households enter the period with their state variables from last period and observe their income shock. Second, they make contingent consumption plans for each of the 12 combinations of discrete decisions. Third, they receive the extreme value shocks. First, they receive the car-type shock, then the car-adjustment, the housing type and moving shocks. Thus, households take their car-type decision conditional on the car-adjustment, the housing type and the moving decision, the car-adjustment decision conditional on the housing type and the moving decision and finally the housing type decision conditional on their moving decision. These decisions become effective immediately. Lastly, consumption takes place and households transition to the next period.

For brevity, I will focus on the discrete decision whether households buy a dirty or clean car conditional on buying a new car and neither changing the housing technology nor the region in which they live. The recursive formulations of the other discrete choices are analogous and are shown in the Appendix B.1. The value functions of adjusting to a dirty car (DCA) and clean car (CCA) conditional on not moving (NM) and not adjusting the housing type (NHA) are given by

$$V_t^{\text{NM,NHA,DCA}}(l_t, y_t, \kappa, \lambda_t^c, \lambda_t^h) = \max_{\{h_t, e_t^c, e_t^h\}} u(x_t, h_t, e_t^h, e_t^c, l_{t+1}) + \beta\,\mathbb{E}\left[V_{t+1}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right]$$

$$\text{s.t.} \quad x_t = y_t + T_t^\pi + T_t^\tau - \rho(l_{t+1}, \lambda_{t+1}^h)h_t - [p_t^{eh} + (1-\lambda_{t+1}^h)\xi_l^h\tau]e_t^h - [p^{ec} + (1-\lambda_{t+1}^c)\xi^c\tau]e_t^c - p_t^{c,cl}$$

$$l_{t+1} = l_t, \quad \lambda_{t+1}^c = 0, \quad \lambda_{t+1}^h = \lambda_t^h$$

$$V_t^{\text{NM,NHA,CCA}}(l_t, y_t, \kappa, \lambda_t^c, \lambda_t^h) = \max_{\{h_t, e_t^c, e_t^h\}} u(x_t, h_t, e_t^h, e_t^c, l_{t+1}) + \beta\,\mathbb{E}\left[V_{t+1}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right]$$

$$
\begin{aligned}
s.t. \qquad & x_t = y_t + T_t^{\pi} + T_t^{\tau} - \rho(l_{t+1}, \lambda_{t+1}^h) h_t - [p_t^{eh} + (1-\lambda_{t+1}^h)\xi_l^h \tau] e_t^h - [p^{ec} + (1-\lambda_{t+1}^c)\xi^c \tau] e_t^c - p_t^{c,di} \\
& l_{t+1} = l_t, \quad \lambda_{t+1}^c = 1, \quad \lambda_{t+1}^h = \lambda_t^h
\end{aligned}
$$

respectively. The expected value function will also be further specified in Appendix B.1.

3.2 Firm sector and production

There are three representative and competitive firms in the economy. First, a construction firm which builds dirty and clean housing and transforms dirty into clean housing. Second, a renting firm which buys the housing stock from the construction firm and rents it out to households. Last, a production firm which produces cars, energy, and the non-housing, non-energy good.

3.2.1 Construction Sector

The construction firm can built houses of both technologies and can renovate dirty into clean housing. For the housing production function, I follow Kaplan et al. (2020) and assume that the firm uses land permits and labor services as inputs. Thus, the firm's problem for constructing housing of type j in region l in time period t, reads

$$
\max_{I_{l,t}^j} q_{l,t}^j I_{l,t}^j - w_{l,t}^j N_{l,t}^j \quad \text{s.t} \quad I_{l,t}^j = \psi_t^{h,j} \left(N_{l,t}^j\right)^{\alpha_l^h} \overline{L_l}^{1-\alpha_l^h} \quad \text{with} \quad \psi_t^{h,cl} = \Omega_t^h \psi^{h,di}
$$

where $q_{l,t}^j$ and $I_{l,t}^j$ are the housing price and the number of housing units built for housing type j in location l in time period t. Further, $w_{l,t}^j$ is the wage for one unit of labor services, $N_{l,t}^j$ is the quantity of labor services employed, and $\overline{L_l}$ is the number of new permits for buildable land in region l.[12] Following Favilukis et al. (2017), I assume that these permits are sold by the government competitively to the construction firm and that its number is exogenous and constant over time. Hence, in equilibrium all rents from housing production accrue to the government and the construction firm makes no profits by building houses. Next, α_l^h and $\psi_t^{h,j}$ describe the relative share of labor services for producing housing and the total factor productivity of constructing housing, respectively. Based on empirical estimates from the literature, I assume that clean housing production is initially more costly but that, due to exogenous technological process, its productivity converges to the one of dirty housing production which is assumed to be constant over time. The parameter Ω_t^h describing this convergence in productivities over the transition period will be further specified when calibrating the model in Section 3.4.5.

Solving the firm's problem gives the optimal level of housing construction of type j of[13]

$$
I_{l,t}^j = \left(\alpha_l^h \frac{q_{l,t}^j}{w_{l,t}^j}\right)^{\frac{\alpha_l^h}{1-\alpha_l^h}} \left(\psi_t^{h,j}\right)^{\frac{1}{1-\alpha_l^h}} \overline{L_l}.
$$

[12]In equilibrium all wages in this economy will be equal to one, as will be discussed in Section 3.2.3

[13]The full derivation is provided in Appendix B.2.1.

Hence, the number of new houses build increases one-to-one with the number of new land permits issued. Further note, that the housing supply elasticity, meaning how strongly housing construction reacts to housing price changes, is given by $\frac{\alpha_l^h}{1-\alpha_l^h}$.

Having derived the optimal levels for dirty and clean housing production, the construction firm needs to decide which housing type to build on the newly permitted land. Comparing both profit functions[14] shows that the firm builds the clean housing stock if

$$q_{l,t}^{h,cl} > \frac{q_{l,t}^{h,di}}{\Omega_t^h}.$$

Thus, the construction firm builds clean houses if the selling price for one unit of clean housing is higher than the selling price for one unit of dirty housing adjusted by the relative productivity in clean housing construction.

Besides constructing new housing, the construction firm can also transform dirty into clean housing. For this renovation process, the firm solves:

$$\max_{I_{l,t}^{ren}} q_{l,t}^{cl} I_{l,t}^{ren} - w_{l,t}^{ren} N_{l,t}^{ren} - q_{l,t}^{di} h_{l,t}^{di} \quad \text{s.t} \quad I_{l,t}^{ren} = \psi_{l,t}^{ren} \min\{\left(N_{l,t}^{ren}\right)^{\alpha^{ren}}, h_{l,t}^{di}\} \quad \text{with} \quad \psi_{l,t}^{ren} = \Omega_t^{ren} \psi_l^{ren}$$

Thus, the construction firm uses dirty houses, which it buys from the renting firm for the market price $q_{l,t}^{di}$ and labor services, for which it pays wage $w_{l,t}^{ren}$, to produce clean housing of size $I_{l,t}^{ren}$, which it can sell to the renting firm for the market price $q_{l,t}^{cl}$. I assume a Leontief production function with a TFP-parameter of $\psi_{l,t}^{ren}$, for which I assume the same speed of technological process as for the TFP-parameter for constructing clean houses $\psi_{l,t}^{h,cl}$. Solving this problem, the optimal level of renovations is given by[15]

$$I_{l,t}^{ren} = \psi_{l,t}^{ren} \left[\frac{\alpha^{ren}}{w_{l,t}^{ren}} \left(\psi_{l,t}^{ren} q_{l,t}^{cl} - q_{l,t}^{di} \right) \right]^{\frac{\alpha^{ren}}{1-\alpha^{ren}}}.$$

Hence, this specification of the production function makes sure that the elasticity of renovating dirty housing with respect to the effective renovating price $\psi_{l,t}^{ren} q_{l,t}^{cl} - q_{l,t}^{di}$ is $\frac{\alpha^{ren}}{1-\alpha^{ren}}$, which I calibrate to match the overall housing supply elasticity. The construction firm will renovate dirty housing as long as this yields positive profits, meaning $\psi_{l,t}^{ren} q_{l,t}^{cl} > q_{l,t}^{di}$, and there is still dirty housing, meaning $H_{l,t}^{di} > 0$. I assume that the government accrues all profits from the renovation process.

3.2.2 Renting Firm

The competitive, representative renting firm decides how much dirty and clean housing to buy and how much dirty housing to renovate in each location. Hence, it maximizes

[14] Again, full derivations are provided in Appendix B.2.1.
[15] Full derivations are provided in Appendix B.2.2.

$$
\begin{aligned}
V(H^{di}_{l,t}, H^{cl}_{l,t}) &= \max_{h^{cl}_{l,t}, h^{di}_{l,t}, h^{ren}_{l,t}} \quad \rho^{cl}_{l,t}(H^{cl}_{l,t} + h^{cl}_{l,t} + h^{ren}_{l,t}) + \rho^{di}_{l,t}(H^{di}_{l,t} + h^{di}_{l,t} - h^{ren}_{l,t}/\psi^{ren}_{l,t}) \\
&- \quad q^{cl}_{l,t}h^{cl}_{l,t} - q^{di}_{l,t}h^{di}_{l,t} - q^{ren}_{l,t}h^{ren}_{l,t} + \frac{1}{1+r}E[V(H^{di}_{l,t+1}, H^{cl}_{l,t+1})] \\
\text{s.t} \quad H^{cl}_{l,t+1} &= (1-\delta^h)(H^{cl}_{l,t} + h^{cl}_{l,t} + h^{ren}_{l,t}) \\
H^{di}_{l,t+1} &= (1-\delta^h)(H^{di}_{l,t} + h^{di}_{l,t} - h^{ren}_{l,t}/\psi^{ren}_{l,t}) \\
H^{cl}_{l,t+1}, H^{di}_{l,t+1} &\geq 0.
\end{aligned}
$$

where $h^{cl}_{l,t}$, $h^{di}_{l,t}$, and $h^{ren}_{l,t}$ are the number of clean, dirty, and renovated houses bought, $q^{cl}_{l,t}$, $q^{di}_{l,t}$, and $q^{ren}_{l,t}$ are the respective prices and $\rho^{cl}_{l,t}$ and $\rho^{di}_{l,t}$ are the renting rates for clean and modern housing. Finally, $H^{di}_{l,t}$ and $H^{cl}_{l,t}$ are the housing stocks of both types which must be non-negative and depreciate at rate δ^h every period. Solving this problem yields an one-to-one mapping between renting rates and housing prices, where the latter are given by the infinitely, discounted and depreciated sum of the former[16]

$$
q^{cl}_{l,t} = \rho^{cl}_{l,t} + \sum_{j=1}^{\infty} \left(\frac{1-\delta}{1+r}\right)^j E\left[\rho^{cl}_{l,t+j}\right], \quad q^{di}_{l,t} = \rho^{di}_{l,t} + \sum_{j=1}^{\infty} \left(\frac{1-\delta}{1+r}\right)^j E\left[\rho^{cl}_{l,t+j}\right].
$$

3.2.3 Car, energy, and non-housing, non-energy production

Lastly, there is a representative and competitive firm which produces cars, energy and the non-housing, non-energy good. All three goods are produced with a constant returns to scale technology, implying that the production firm does not make any profits. For the non-housing, non-energy good, the production function reads

$$
X_{l,t} = N^x_{l,t}
$$

where $N^x_{l,t}$ is the number of efficient units of labor employed. As the price for the non-housing, non-energy good is the numeraire in this economy, the competitive wage is given by $w^x_{l,t} = 1$. Since labor is assumed to be perfectly mobile across sectors within regions, all wages in this economy are equal to one.

The production function for cars of technology j is given by

$$
C^j_{l,t} = \psi^{c,j}_t N^{c,j}_{l,t}, \quad \text{with} \quad \psi^{c,cl}_t = \Omega^c_t \psi^{c,di}
$$

where Ω^c_t describes the productivity differences between producing clean and dirty cars. Similarly to the construction of housing, I assume also for the production of clean cars exogenous technological improvements while I assume no productivity changes for producing dirty cars. Thus, the price for cars of technology j is given by $p^{c,j}_t = \frac{1}{\psi^{c,j}_t}$.

The production functions of car and heating energy also follow a constant returns to scale technology and read

$$
E^c_{l,t} = \psi^{ec} N^{ec}_{l,t} \quad \text{and} \quad E^h_{l,t} = \psi^{eh} N^{eh}_{l,t},
$$

[16]This relationship can easily be rewritten into the user cost formula, i.e. $\rho^{cl}_{l,t} = q^{cl}_{l,t} - \frac{1-\delta}{1+r}E\left[q^{cl}_{l,t+1}\right]$ and $\rho^{di}_{l,t} = q^{di}_{l,t} - \frac{1-\delta}{1+r}E\left[q^{di}_{l,t+1}\right]$.

implying prices of $p^{ec} = \frac{1}{\psi^{ec}}$ and $p^{eh} = \frac{1}{\psi^{eh}}$. For the production of energy, I do not assume any technological process, such that these prices stay constant over the transition period.

3.3 Government

In the baseline scenario without carbon taxation, the government collects revenues from two sources. First, it owns the land permits in both locations and thus extracts all profits from constructing housing. Second, it further extract all profits from renovating dirty houses. These revenues are transferred back to households within regions in a lump-sum way such that the governmental budget is balanced every period. Hence, the governmental budget constraints read

$$\pi^{h,cl}_{u,t} + \pi^{h,di}_{u,t} + \pi^{ren}_{u,t} = \int_i T^{\pi}_{l,t} l_{i,t}$$

with $\quad \pi^{h,cl}_{u,t} = q^{cl}_{u,t} I^{cl}_{u,t} - w^{cl}_{u,t} N^{cl}_{u,t}, \quad \pi^{h,di}_{u,t} = q^{di}_{u,t} I^{di}_{u,t} - w^{di}_{u,t} N^{di}_{u,t}$ and $\pi^{ren}_{u,t} = q^{cl}_{u,t} I^{ren}_{u,t} - w^{ren}_{u,t} N^{ren}_{u,t} - q^{di}_{u,t} h^{di}_{u,t}$

and

$$\pi^{cl}_{h,r,t} + \pi^{h,di}_{r,t} + \pi^{ren}_{r,t} = \int_i T^{\pi}_{l,t}(1 - l_{i,t})$$

with $\quad \pi^{h,cl}_{r,t} = q^{cl}_{r,t} I^{cl}_{r,t} - w^{cl}_{r,t} N^{cl}_{r,t}, \quad \pi^{h,di}_{r,t} = q^{di}_{r,t} I^{di}_{r,t} - w^{di}_{r,t} N^{di}_{r,t}$ and $\pi^{ren}_{r,t} = q^{cl}_{r,t} I^{ren}_{r,t} - w^{ren}_{r,t} N^{ren}_{r,t} - q^{di}_{r,t} h^{di}_{r,t}$

for both regions, where $l_{i,t}$ indicates whether household i lives in the city in year t.

3.4 Calibration

I calibrate the initial stationary equilibrium of this model, in which by assumption no clean technologies exist, to the German economy in 2018. I take the year 2018 as starting point because of two reasons. First, the main dataset for the calibration, the EVS-dataset, is from 2018. Second, in 2018 the shares of electric cars and heat pumps relative to all cars and heating systems were, according to the German Federal Motor Transport Authority (*Kraftfahrtsbundesamt*) and the German Federal Association of the Energy and Water Industry (*Bundesverband der Energie- und Wasserwirtschaft*), only 0.1 and 2.2 percent, respectively, which allows me to interpret 2018 as steady state without these goods. Further, I calibrate the share of urban households to the share of tenants living in cities of at least 100,000 inhabitants, which is around 47.4 percent.

My calibration strategy follows a three-step procedure. First, I take a set of parameters from the literature and directly from the data. A second set of parameters is calibrated in closed form directly matching empirical moments. Lastly, I calibrate the remaining parameters by applying a simulated method of moments. In the following, I describe this procedure in more detail.

3.4.1 Utility Function

One period in the model corresponds to one year in the data. The discount rate and coefficient of relative risk aversion are set to standard values of $\beta = 0.98$ and $\sigma = 2.0$, respectively. Exploiting the nested CES-structure of the utility function, I can calibrate the weights in the utility function on the non-housing composite γ, the non-housing non-car energy consumption μ_c, and the housing size μ_h in closed-form to the respective empirical expenditures shares using the first-order conditions. This procedure gives me values of $\gamma = 0.77$, $\mu_c = 0.99$, and $\mu_h = 0.99$. The mean of the city amenities is set up match the share of households in the city, which results in a value of $\mu_\kappa = -0.0000004$. The elasticity of substitution between non-car energy non-housing consumption and gasoline consumption, denoted ν_c is calibrated to match the own price elasticity of car energy consumption. A price increase in car energy will result in a larger drop in car energy consumption if both goods are close substitutes than if they are close complements. As target, I take the own-price elasticity for gasoline of -0.35 estimated by Frondel and Vance (2009) for Germany, which results in a value of $\nu_c = 0.45$. For the elasticity of substitution between the housing size and heating energy, I proceed analogously. I target the own-price elasticity of heating energy of -0.2 estimated by Auffhammer and Rubin (2018) and receive a value of $\nu_h = 0.1$. Both targets are well in line with other estimates in the literature (Brons et al., 2008; Davis and Muehlegger, 2010; Goetzke and Vance, 2021). Lastly, I need to calibrate the subsistence levels of car-energy consumption in both regions. For the rural region, I take the difference in car-energy consumption in both region, which is $\underline{e^{c,r}} = 4.5$, and for the rural region, I use $\underline{e^{c,u}} = 0$ as normalization as I already match the overall car-energy consumption by calibrating the utility weights.

3.4.2 Preference Shocks

For each of the four discrete decisions, there is a location and scale parameter to calibrate. To calibrate the migration shock, I exploit data from the German Socio-Economic Panel (SOEP). The SOEP is a longitudinal survey of around 40,000 individuals in Germany from 1984 to 2021. It contains information on many socio-economic variables and on the county in which an individual lives. Based on the classification of the Federal Institute for Research on Building, Urban Affairs and Spatial Development, I group these 401 counties into rural and urban region. Lastly, I compute the share of households in the SOEP with moves between these two regions every period, which is around 0.79%. I calibrate the location parameter of the mobility shock to match this share and receive a value of $\mu_{\epsilon,l} = -0.00054$. I calibrate the scale parameter of the moving shock, denoted $\sigma_{\epsilon,l}$, to match the moving semi-elasticity with respect to income shocks, indicating by how many percentage points net migration rates increase if wages increase by one percent. The implicit assumption is that households react to expenditure shocks, resulting from carbon prices, in the same way as they react to income shocks. As target, I take -0.2 estimated by Monras (2018) which gives me a value of $\sigma_{\epsilon,l} = 0.00015$. Next, I calibrate the location and scale parameters of the housing-type shock to match the share of households changing the heating technology each year and the observed price path for clean vs. dirty houses in the data. I use the data from the German Federal Association of the Energy and Water Industry (*Bundesverband der Energie- und Wasserwirtschaft*) and get values of $\mu_{\epsilon,h} = -0.00001$ and $\sigma_{\epsilon,h} = 0.000005$, respectively. For the car-adjustment and car-type shocks, I proceed similarly. I calibrate the location parameter to match the share of overall cars bought and electric cars bought, where I exploit data from the German

Federal Motor Transport Authority (*Kraftfahrtsbundesamt*). The scale parameters are set to match the price elasticities of clean and dirty cars. As empirical targets, I take estimates from Fridstrøm and Østli (2021) for Norway.

3.4.3 Labor Income

Average yearly household income in the model provides a normalization and is set to 30,821 Euro in line with the EVS data for tenants. For the idiosyncratic shock process, I use estimates from Fehr et al. (2013) for the persistence parameter ρ and calibrate the variance of the income shock σ_η^2 to match the Gini coefficient for net household income in Germany, resulting in values of $\rho = 0.957$ and $\sigma_\eta^2 = 0.031$.

3.4.4 Housing Construction

The labor intensity parameter in the housing production function is a key parameter of the model as it determine the housing supply elasticity in region l, which is given by $\alpha_l^h/(1-\alpha_l^h)$. Exploiting this direct mapping, I calibrate α_l^h to estimates of the housing supply elasticity in both regions. As targets, I take estimates from Beze (2023) for Germany who finds values of 0.285 and 0.204 for rural and urban regions.[17] These estimates are well in line with other estimates in the literature (Baum-Snow and Han, 2024; Lerbs, 2014). This gives me values of $\alpha_r^h = 0.3986$ and $\alpha_u^h = 0.2563$. The numbers of land permits issued are calibrated to match the initial steady state renting rates in both regions, respectively, and yield $\overline{L_r} = 362$ and $\overline{L_u} = 154$. As, in the initial steady state housing construction and the total housing supply are directly linked through the depreciation rate, the number of land permits determines also the number of housing units constructed and the TFP-parameter for dirty housing construction $\psi^{h,di}$ can be normalized to 1. For the initial productivity of clean housing production relative to dirty housing production, I take estimates from the Bavarian Construction Association which estimates that constructing clean housing was 147 Euros per square meter more expensive in 2018, resulting in an initial productivity discount of 5.77 percent. For its convergence along the transition period, I take estimates from LCP Delta who forecast that the price for heat pumps, a key component of clean housing, drop by 40 percent within 10 years (Delta, 2021). I then extrapolate this convergence rate over the entire transition period.[18] The TFP-parameters for housing renovations are calibrated to match the number of renovations. According to Cischinsky and Diefenbach (2018), the renovation rate, characterizing the share of housing renovations in a given year relative to the total housing stock, was 0.99% in Germany in 2016 which yields values of $\psi_r^{ren} = 0.67$ and $\psi_u^{ren} = 0.57$. For the productivity improvements of housing renovations, captured by Ω^{ren}, I assume the same speed as for housing construction. I calibrate the depreciation rate for housing by matching the share of newly build housing units relative to the total housing stock. According to the German Statistical Office these numbers were 287,400 and 42,400,000 in 2018, respectively, which results in a depreciation rate for housing of $\delta_h = 0.0068$. Lastly, I calibrate the interest rate to match the housing-price-to-rent ratio. According to the German Bundesbank, the ratio was 28 in 2018 resulting in an interest rate of $r = 0.03$.

[17] These estimates for urban and rural housing supply elasticities refer to the estimates of the 25th and 75th percentile of the land development intensity distribution.

[18] Figures showing the technological process along the transition can be found in Appendix C.1.

3.4.5 Technical Parameters

For calibrating the parameters translating consumption units of heating energy into carbon emissions, I employ estimates from the GEMIS Version 5.0 analysis of the International Institute for Sustainable Analysis and Strategy (IINAS and Strategy, 2021). I multiply their estimates of carbon emissions for different heating systems with empirical shares of rural and urban households using these heating systems. I get estimates of $\xi_r^h = 0.247$ and $\xi_u^h = 0.232$ tons of carbon emissions per MWh of heating consumption. For translating car energy consumption into carbon emissions, I take estimates from the Helmholtz Institute for diesel and gasoline, weight them with their respective empirical shares and get a value of $\xi^c = 0.251$ tons of carbon emissions per 100 liters of gasoline. I calibrate the productivity parameters for producing heating and car energy by matching their prices in 2018, which I get from the German Federal Statistical Office. For car energy, I weight the average prices for diesel and gasoline with their respective shares in 2018, resulting in a price of $p^{ec} = 0.0045$ and a productivity of $\psi^{ec} = 222$. Analogously, I also weight the different heating sources by their respective empirical shares and get a price for heating energy of $p^{eh} = 0.0023$ and a productivity of $\psi^{eh} = 435$. The productivity of producing dirty cars is calibrated to match the expenditure share on new car purchases and results in a price of $p^{eh} = 0.0023$ and a productivity of $\psi^{eh} = 435$. For the production of clean cars, I calibrate the productivity difference relative to dirty cars and its convergence over time by matching the observed share of electric vehicles bought in Germany until 2023. I then again extrapolate this convergence rate over the entire transition period. Finally, I need to calibrate the parameters governing the efficiency differences of transforming raw car and heating energy into vehicles miles traveled and temperature between clean and dirty technologies. For cars, Lévay et al. (2017) estimate that costs for fossil fuels are 25 percent higher than for electric fuels, resulting in a value of $\phi^c = 0.33$. For heating energy, Taruttis and Weber (2022) estimate that the average energy consumption in for houses with heat pumps and all other houses are 51 kWh/m^2a and 174 kWh/m^2a, respectively. Thus, the efficiency premium for clean houses is $\phi^h = 2.412$.

All parameters and the respective moments are summarized in Table 1.

Table 1: List of parameters

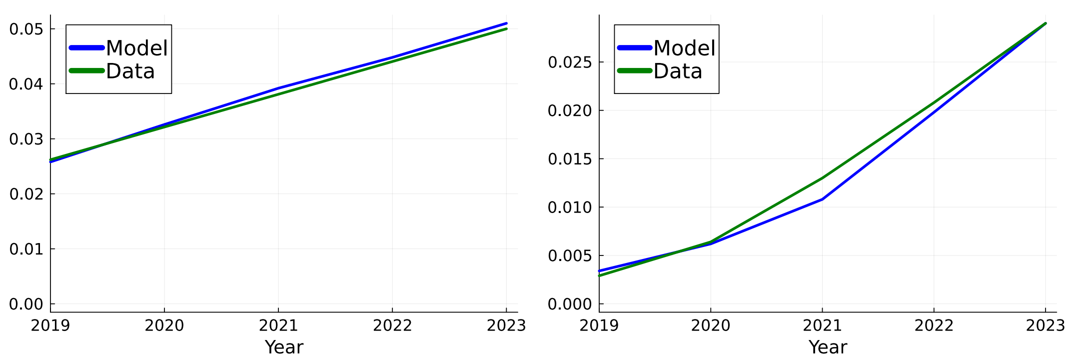
Symbol	Description	Value	Source/Target
Utility function			
β	Discount rate	0.98	Standard value
σ	CRRA-coefficient	2.0	Standard value
γ	Weight on non-housing composite good	0.77	Expenditure share of non-housing
μ_c	Weight on non-housing non-car energy consumption	0.99	Average exp. on non-housing non-car energy
μ_h	Weight on housing size	0.99	Average expenditures rent
μ_κ	Mean city amenities	-0.0000004	Share of HHs in city
ν_c	Elasticity of subst. c vs. e^c	0.45	Own price elasticity of car energy
ν_h	Elasticity of subst. h vs. e^h	0.1	Own price elasticity of heating energy
$\underline{e^{c,r}}$	Min. car energy consumption - rural	4.5	Difference car energy consump. urban vs. rural
$\underline{e^{c,u}}$	Min. car energy consumption - urban	0.0	Normalization
Preference shocks			
$\mu_{\epsilon,l}$	Location parameter of location pref. shock	-0.00054	Share of households moving across regions
$\mu_{\epsilon,c}$	Location parameter of car pref. shock	-0.000075	Share of overall car purchases
$\mu_{\epsilon,ct}$	Location parameter of car technology pref. shock	-0.00054	Share of electric car purchases
$\mu_{\epsilon,h}$	Location parameter of housing type pref. shock	-0.00001	Number of new heating systems
σ_l^2	Scale parameter of location pref. shock	0.00015	Elasticity of moving
σ_c^2	Scale parameter of car pref. shock	0.00003	Elasticity overall car purchases
σ_{ct}^2	Scale parameter of car technology pref. shock	0.0002	Elasticity electric car purchases
σ_h^2	Scale parameter of housing type pref. shock	0.000005	Price path clean/dirty houses
Labor income			
ρ	Persistence of income shock	0.957	Fehr et al. (2013)
σ_η^2	Variance of income shock	0.031	Gini-Coefficient
Housing construction			
a_r	Labor intensity housing production - rural	0.3986	Housing supply elasticity in Beze (2023)
a_u	Labor intensity housing production - urban	0.2563	Housing supply elasticity in Beze (2023)
$\overline{L_r}$	Number of issued land permits - rural	362	Initial steady state renting rate - rural
$\overline{L_u}$	Number of issued land permits - urban	154	Initial steady state renting rate - urban
$\psi^{h,di}$	Productivity dirty housing construction	1.0	Normalization
Ω^h	Technological progress clean housing construction	see A2b	See text
ψ_r^{ren}	Productivity of renovations - rural	0.67	Number of renovations
ψ_u^{ren}	Productivity of renovations - urban	0.57	Number of renovations
Ω^{ren}	Technological progress housing renovations	see A2b	See text
δ	Depreciation rate for housing	0.0068	Housing construction-to-housing stock ratio
r	Interest rate	0.03	Housing price-to-rent ratio
Technical parameters			
ξ_r^h	Heating energy to emissions translation - rural	0.247	See text
ξ_u^h	Heating energy to emissions translation - urban	0.232	See text
ξ^c	Car energy to emissions translation	0.251	See text
ψ^{ec}	Inverse of price car energy per liter	222	German Federal Statistical Office
ψ^{eh}	Inverse of price gas per MWh	435	German Federal Statistical Office
ψ^c	Productivity dirty car production	61.6	Average exp. car purchases
Ω^c	Technological progress clean car production	see A2a	See text
ϕ^c	Relative car energy efficiency clean cars	0.33	Lévay et al. (2017)
ϕ^h	Relative heating energy efficiency clean houses	2.412	Taruttis and Weber (2022)

3.4.6 Model fit

Overall, the model is able to match key data moments very well. I can, for example, match the expenditure shares for the different consumption goods exactly by exploiting the nested CES utility function. But also the own-price elasticities for energy, which are used to calibrate the elasticities of substitution between these different goods, are very close to their empirical counterparts. When introducing a carbon tax of 300 Euro per ton, which doubles the price for heating energy and increases the one for car energy by around 50 percent, energy consumption decreases by x and y percent, which is very close the reduction of x and y percent which empirical estimates would suggest. Besides these intensive consumption adjustments, the model is also able to match the extensive, discrete technology adjustments very well. Figure 4 plots the share of households with clean goods in the model, without carbon taxes, and in the data. We can see that the level and the speed of adjustments are matched very well.

Figure 4: Model fit technology adjustments

(a) Housing (b) Cars



Notes: Panel (a) compares the shares of households with clean housing technologies in the model and in the data. The data points are weighted equally by the share of households with heat pumps and those who live in apartments of energy class A+ or A. Panel (b) shows the corresponding values for cars. The empirical values show the share of households with electric vehicles, not including hybrids.

4 Policy experiments

I will now use the calibrated model to study the consequences of introducing carbon taxes on the level of redistribution across income groups and space, on the speed of transitioning to clean technologies and on their political support depending on the way the carbon tax revenues are paid back to households. I will start in an initial stationary equilibrium in which only dirty technologies exist and introduce clean technologies exogenously. As clean technologies are more efficient in transforming raw car and heating energy into vehicles miles traveled and temperature, this will start a transitional process towards clean technologies, even without any policy intervention. I compare this transition without policy with a transition in which I introduce a carbon tax of 300 Euros per ton. This level is well in line with estimates about future carbon taxes within the Emission Trading System 2 (ETS2) proposed by the European Union (Kalkuhl et al., 2023). I compare three ways of recycling back the carbon tax revenues. First, as carbon taxes have been shown to be regressive as poor households spend a higher share of their total expenditures on carbon intensive goods, like energy, the literature has identified lump-sum transfers as a way to avoid this regressive redistribution. Generally, reimbursing households with lump-sum transfers has been found to be even progressive as households' carbon footprints increase with income (Douenne, 2020). When compensating households with lump-sum transfers, the governmental budget constraint in period t reads

$$\int_i \left[e^h_{i,t}(1-\lambda^h_{i,t})(l_{i,t}\xi^h_u + (1-l_{i,t})\xi^h_r) + e^c_{i,t}(1-\lambda^c_{i,t})\xi^c\right]\tau di = NT^\tau_t,$$

where N describes the constant number of households in the economy.

Second, carbon footprints differ substantially across space between rural and urban regions, as having shown in the empirical part of this paper. Thus, lump-sum reimbursements redistribute from rural to urban regions. Hence, I will look at place-based transfers where transfers are set such that no redistribution between regions happens. The governmental budget constraints for both regions read

$$\int_i \left[e^h_{i,t}(1-\lambda^h_{i,t})(1-l_{i,t})\xi^h_r + e^c_{i,t}(1-\lambda^c_{i,t})(1-l_{i,t})\xi^c\right]\tau di = \int_i (1-l_{i,t})NT^\tau_{r,t}di$$

and

$$\int_i \left[e^h_{i,t}(1-\lambda^h_{i,t})l_{i,t}\xi^h_u + e^c_{i,t}(1-\lambda^c_{i,t})l_{i,t}\xi^c\right]\tau di = \int_i l_{i,t}NT^\tau_{u,t}di.$$

for the rural and urban region, respectively.

Last, I will use carbon tax revenues to finance subsidies on clean housing renovations, where the subsidies are paid on the labor costs. Hence, the governmental budget reads

$$\int_i \left[e^h_{i,t}(1-\lambda^h_{i,t})(l_{i,t}\xi^u + (1-l_{i,t})\xi^r) + e^c_{i,t}(1-\lambda^c_{i,t})\xi^c\right]\tau di = \left[w_{l,t}N^{ren}_{r,t} + w_{l,t}N^{ren}_{u,t}\right]\psi_t,$$

where ψ_t characterizes the percentage subsidy in period t. While the first two policies are concerned with redistribution across income groups and space, the third policies aims at increasing the speed of transitioning to clean technologies and implicitly redistributes between early and late clean housing type adjusters.

I will start by comparing the long-run consequences of these different policies. For doing so, I will compare their stationary equilibria. Thereafter, I will study the consequences of these policies along the transition. I will evaluate them in terms of redistributional and welfare consequences in order to draw conclusions for their political support. Lastly, I will evaluate the consequences of varying the level of the carbon tax.

4.1 Long-run consequences of carbon taxes

I will start by comparing the stationary equilibria of these policies in order to evaluate their long-run consequences. As the prices for clean and dirty houses have converged in the long-run, housing renovations are not profitable anymore. Thus, I will focus on reimbursing households via lump-sum and place-based transfers for this long-run analysis. Table 2 shows the percentage changes in key variables for the different policy scenarios relative to the initial stationary equilibrium without clean technologies. As clean technologies are more efficient in transforming raw energy into temperature and vehicle miles traveled, the vast majority of households uses clean technologies in the final steady states. In the scenario without carbon taxes this is true for around 93 to 95 percent of households and when introducing carbon taxes, this number even increases to 99 to 100 percent. There are hardly any differences across regions and policies. As the share of these clean, energy-efficient technologies increase, energy consumption decreases. Without policy intervention the decrease is around 10 and 60 percent for car and heating energy. With carbon taxes these numbers increase in absolute terms to 11 and 65 percent. While for heating energy consumption, there are no substantial differences across space, for car energy consumption, the reductions are around 5 percentage points larger in urban regions. This difference results from a higher subsistence level and thus for a given consumption level a higher marginal utility resulting in a lower price-elasticity of car energy consumption in rural regions, which is consistent with empirical findings (Santos and Catchesides, 2005; Wadud et al., 2010). As a result from this shift towards clean technologies, carbon footprints decrease by around 91 percent without and by 99 percent with carbon taxes. Thus, (almost) complete decarbonization can only be achieved with carbon taxes. As heating energy and housing size are complementary goods, a higher efficiency in the heating technology also increases demand for housing. Thus, the average housing size increases by around 2.2 percent without policy intervention and by 2.4 percent with carbon taxes. The larger increase with carbon taxes results from a higher share of clean houses and thus more efficient heating technologies. This increase in housing demand leads to an increase in the renting rate by around 5 to 7 percent as the housing supply exhibits decreasing returns to scale due to the constant number of land permits issued every period. Consumption of the non-housing, non-energy good increases slightly increases and there is almost no spatial redistribution from carbon taxes as carbon footprints have fallen by 99 percent.

Table 2: Changes from initial to final stationary equilibrium

	No policy		Lump-sum		Place-based	
	Rural	Urban	Rural	Urban	Rural	Urban
Clean cars (ppts.)	92.55	93.22	100	100	100	100
Clean housing (ppts.)	95.53	95.04	99.36	98.80	99.36	98.81
Car energy (%)	-8.18	-11.21	-8.55	-13.64	-8.50	-13.77
Heating energy (%)	-61.13	-59.97	-65.28	-64.83	-65.16	-64.99
Carbon emissions (%)	-91.18	-90.48	-99.57	-98.90	-99.56	-98.91
Housing size (%)	2.27	2.17	2.49	2.34	2.53	2.36
Renting rate (%)	6.88	5.53	7.13	5.42	7.12	5.42
Further consumption (%)	0.17	0.86	0.36	1.14	0.35	1.14
Net transfers (Euro)	0	0	2.58	-2.99	0.02	-0.02
Population (ppts.)	1.23	-1.23	1.34	-1.34	1.35	-1.35
Welfare (CEV)	2.97	2.79	3.13	2.93	3.11	2.92

As effective energy becomes cheaper in the final steady state due to more efficient technologies, there is net migration to the rural region where households consume more energy. Without carbon taxes the rural population increases by 1.23 percent of the total population, corresponding to around one million individuals for the German population. With carbon taxes the share of clean technologies is higher and consequently the rural population share increases by around 1.34 percent. Lastly, household's welfare, measured as the consumption equivalent variation (CEV) in terms of the non-housing, non-energy good, increases by around 2.87 and 3.03 percent without and with carbon taxes, relative to the initial stationary equilibrium without clean technologies. This welfare gain results from the usage of more efficient technologies, which allow to decrease raw energy consumption and spend more on the other consumption goods.

4.2 Results along the transition

Next, I study how the results change along the transition between the two stationary equilibria. I start by documenting how key variables, like consumption levels, carbon footprints, and the level of spatial redistribution evolve along the transition. Thereafter, I will show how the endogenous housing prices change and how the construction and renovation of housing react to the different policy scenarios. Lastly, I will zoom into the distributional consequences of the different policy scenarios across income groups and regions in order to evaluate the implications for their political support.

4.2.1 Consumption and spatial redistribution

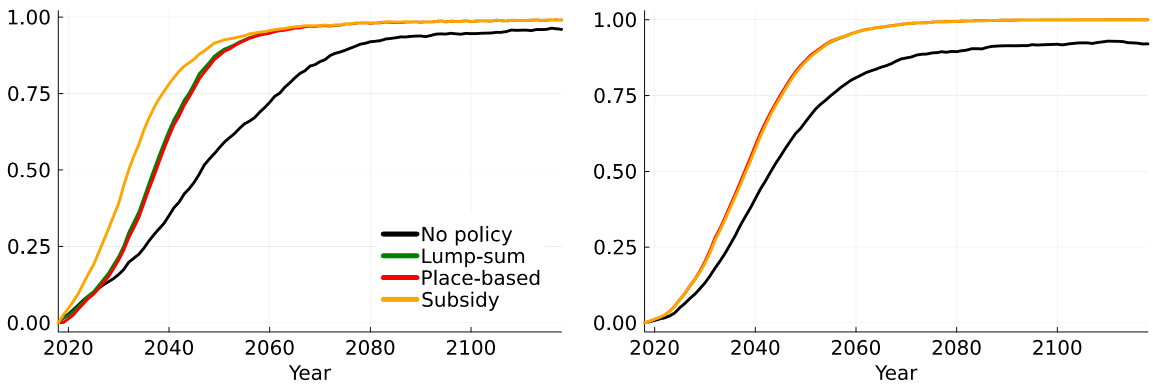
Figure 5 shows the share of households with clean houses and cars along the transition. Without any policy intervention the share of households with clean houses increases steadily along the transition to around 30 and 73 percent in 2040 and 2060 before it converges to the new equilibrium at around 96 percent in 2085. Introducing a carbon tax of 300 Euros per ton speeds up the transition substantially. When spending the tax revenue on transfers to households, the share of clean houses is 55 and 90 percent in 2040 and 2060, before it converges to the new equilibrium at 99 percent in 2080. When spending the carbon tax revenues on subsidies for renovations, the transition is further accelerated, such that in 2040 and 2060 77 and 92 percent of the housing stock is clean. Note that the subsidy on renovations is initially around 70 percent before it falls at the same speed as carbon emissions.[19] For cars the transition looks similar. Without policy intervention, the share of clean cars is around 35 and 81 percent in 2040 and 2060 before it converges to the new equilibrium at 92 percent. With carbon taxes, the share is around 54 and 95 percent in 2040 and 2060 and converges to 100 percent. The share of clean cars does not depend on how the carbon tax revenues are spent.

[19] Figure A3 in the Appendix shows the level of the subsidy along the transition period.

Figure 5: Share of clean technologies along the transition

(a) Housing

(b) Cars



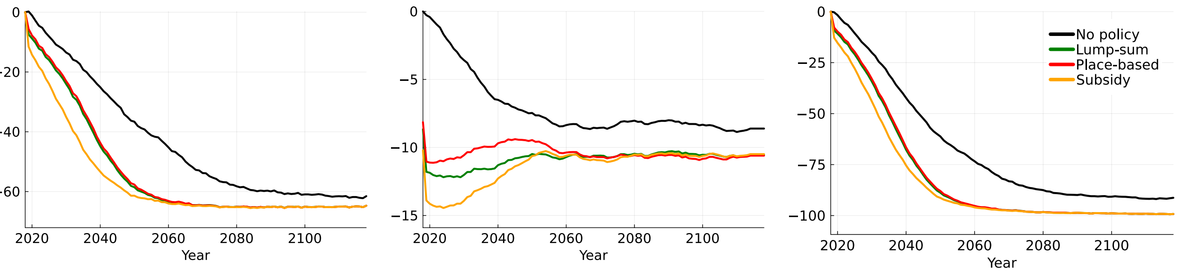
Notes: Panel (a) shows the share of households with clean houses along the transition period for the different policy scenarios considered. Panel (b) shows the corresponding share of households with clean cars along the transition.

As households adopt the clean and more efficient technologies, their raw energy consumption falls. Figure 6 shows the percentage changes of energy consumption and carbon footprints relative to the initial steady state along the transition. First, note that energy consumption falls after introducing carbon taxes by around 10 to 15 percent due to a price effect, as a carbon tax of 300 Euros per ton increases the price for heating and car energy by around 100 and 60 percent, respectively. As car energy consumption is lower in the city and lump-sum transfers redistribute from rural to urban regions, leading to an increase in the urban population share, the reduction in car energy consumption is one percentage points larger.[20] When spending the carbon tax revenues on subsidies for renovations reduction is another two percentage point larger due to the negative income effect of smaller direct transfers. As households move into clean houses the housing energy steadily falls until it converges the new equilibrium around 63 percent lower than the initial equilibrium. When introducing carbon taxes, car energy consumption stays at the level of the initial drop around 11 percent lower than in the initial equilibrium. Without carbon taxes fewer households adopt clean cars and hence car energy consumption falls only by 8 percent. The resulting carbon footprints fall faster with carbon taxes. With carbon taxes they fall by around 70 to 75 percent and by 95 percent until 2040 and 2060, without them these share are 45 and 74 percent, respectively. Note again that without carbon taxes, there is no complete decarbonization.

[20]These results will be shown next.

Figure 6: Percentage changes over the transition

(a) Heating energy (b) Cars energy (c) Carbon footprints

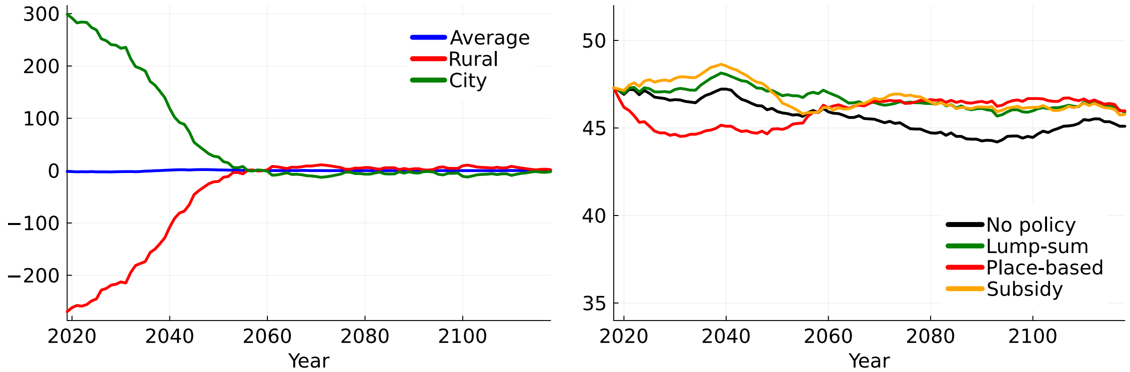


Notes: Panel (a) and Panel (b) show the percentage change in heating and car energy along the transition for the considered policy scenarios. Panel (c) depicts the resulting path of carbon footprints. Lastly, Panel (d) shows the annual level of net transfers for rural and urban households if the carbon tax revenues are used for lump-sum transfers.

Lastly, Figure 7 shows the resulting level of spatial redistribution and the urban population share along the transition. For readability, I focus on the redistribution with lump-sum transfers, all other policies are shown in Figure A4 in the Appendix. Upon introducing the carbon tax with lump-sum transfers, an average households in the city receives annual net transfers of 300 Euros while rural households have to pay around 270 Euros. As households adopt clean technologies over time the level of redistribution falls until there is no more spatial redistribution from 2055 onwards. The net present value of this difference in net transfers is around 8,000 Euros, around 25 percent of an average household's annual net income. This spatial redistribution has implications on the location choice of households. For the first years of the transition, the urban population share increases from 47.1 percent initially to around 48.5 percent in the year 2040. This net migration of 1.4 percent of the population corresponds to around 1.2 million people. Over time this share falls again and converges to the new stationary equilibrium at around 46.3 percent. When spending the carbon tax revenues on subsidies for renovations, the initial increase in the urban population share is similar as the renovation rate is similar in both regions as will be shown in the next subsection. When spending the carbon tax revenues on place-based transfers and without any carbon taxes, there is no such net migration to the city but only the net migration to the rural regions.

Figure 7: Location choices and redistribution over the transition

(a) Redistribution with lump-sum transfers (in Euros) (b) Share of households in urban region (%)



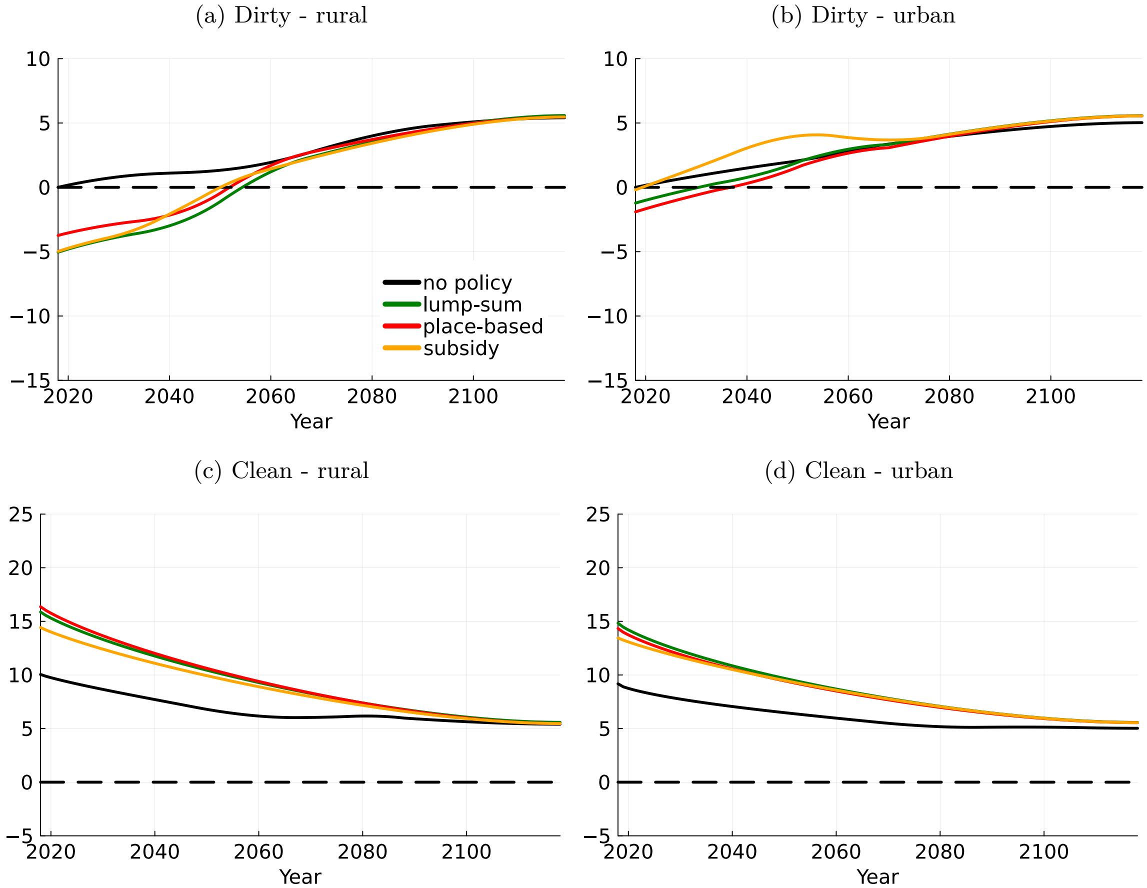
Notes: Panel (a) shows the annual level of net transfers for rural and urban households if the carbon tax revenues are used for lump-sum transfers. Figure A4 in the Appendix shows the level of spatial redistribution for all policy scenarios. Panel (b) shows the share of the population that lives in the urban region along the transition for the different policy scenarios.

4.2.2 Housing sector

Next, I will document how the housing prices and the level of housing construction and renovation change over the transition. Figure 8 shows the percentage changes for the four endogenous housing prices relative to the initial stationary equilibrium in the same region. Without any policy intervention, the endogenous price premium for clean houses jumps to 10 and 9 percent for rural and urban regions upon introducing clean technologies, where the price for dirty houses remains unchanged. This price increase is slightly higher in the rural region as carbon tax payments and thus the demand for clean houses is higher. As the construction firm builds new clean houses and renovates dirty into clean houses, we see that the prices for clean and dirty houses converge and stabilize around 6 percent higher than in the initial stationary equilibrium after around 100 years.

When introducing a carbon tax of 300 Euros per ton of carbon emissions and transferring the collected tax revenues back to households in a lump sum way, the initial rise in the price premium for clean houses is slightly higher, around 14 and 12 percent in rural and urban regions, but also the prices for dirty houses drop relative to the initial steady state by 11 and 7 percent for rural and urban regions. This drop results from a decrease in housing demand as heating energy, a complementary good to the housing size, becomes more expensive due to the carbon tax. As rural households consume more heating energy and use heating technologies which emit more carbon emissions, this effect is stronger for rural households. When spending the carbon tax revenues on place-based transfers without spatial redistribution, the price paths look very similar. When spending the carbon tax revenues on subsidies for renovating dirty into clean houses, the initial rise in the price premium for clean houses is substantially smaller, around 8 and 7 percent in rural and urban regions.

Figure 8: Housing price changes over the transition



Notes: Panel (a) shows the percentage change of the housing price for dirty houses in the rural region relative to the price in the initial stationary equilibrium for dirty housing along the transition for the different policy scenarios. Panels (b) - (d) show the corresponding paths for dirty houses in the urban region and for clean houses in the dirty and clean region, respectively.

Next, I document how housing supply, which is the construction and renovation of houses, reacts to the different policy scenarios. Figure 9 depicts the construction and renovation rates, defined as the share of newly constructed and renovated houses in a given period relative to the total housing stock in the initial steady state in the same region.[21] As the price for clean houses is larger with carbon taxes, the housing construction is around 0.02 percentage points larger. Housing construction rates increase to around 0.71 percent in 2050 before they fall to the final stationary equilibrium due to falling housing prices. Subfigure 9a shows that the rate of renovations strongly varies with the policy scenario. Without any policy scenario the renovation rate first increases from around 1 percent to 1.3 percent in 2035 because of technological progress. But over time the prices for clean and dirty houses converge, making renovations less profitable and thus decreasing the renovation rate until there are no more renovations from 2115 onwards. With carbon taxes, we saw that the price difference between clean and dirty houses is larger, leading to a renovation rate of around 2 percent until 2040, before it falls until all dirty houses are renovated in 2070. Lastly, with subsidies on renovations the rate increases

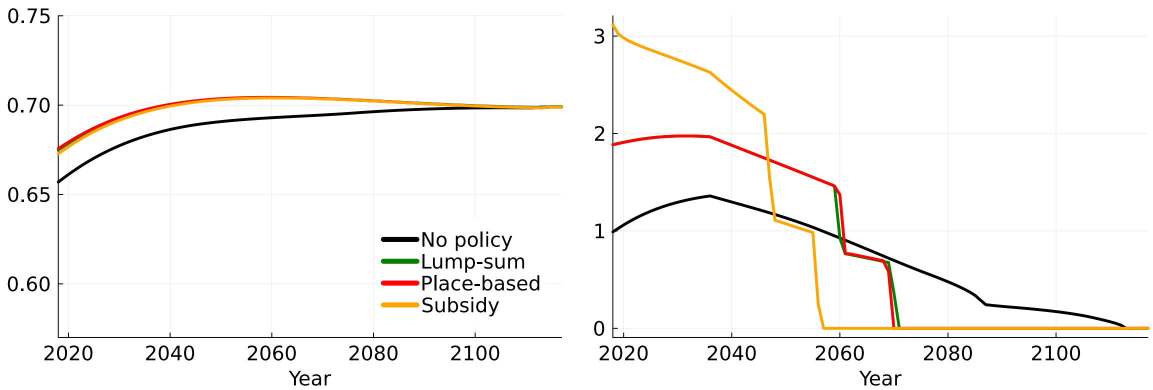
[21]Subfigure 9a shows the total level of newly constructed housing irrespective of the technology. Figure x in the Appendix shows the construction rates of both technology types along the transition.

to around 3 percent before it falls to zero in 2058. The initial subsidy on renovations is around 70 percent and its path over the transition is shown in Figure A3 in the Appendix.

Figure 9: Housing construction and renovations over the transition

(a) Housing construction rate (%)

(b) Housing renovation rate (%)

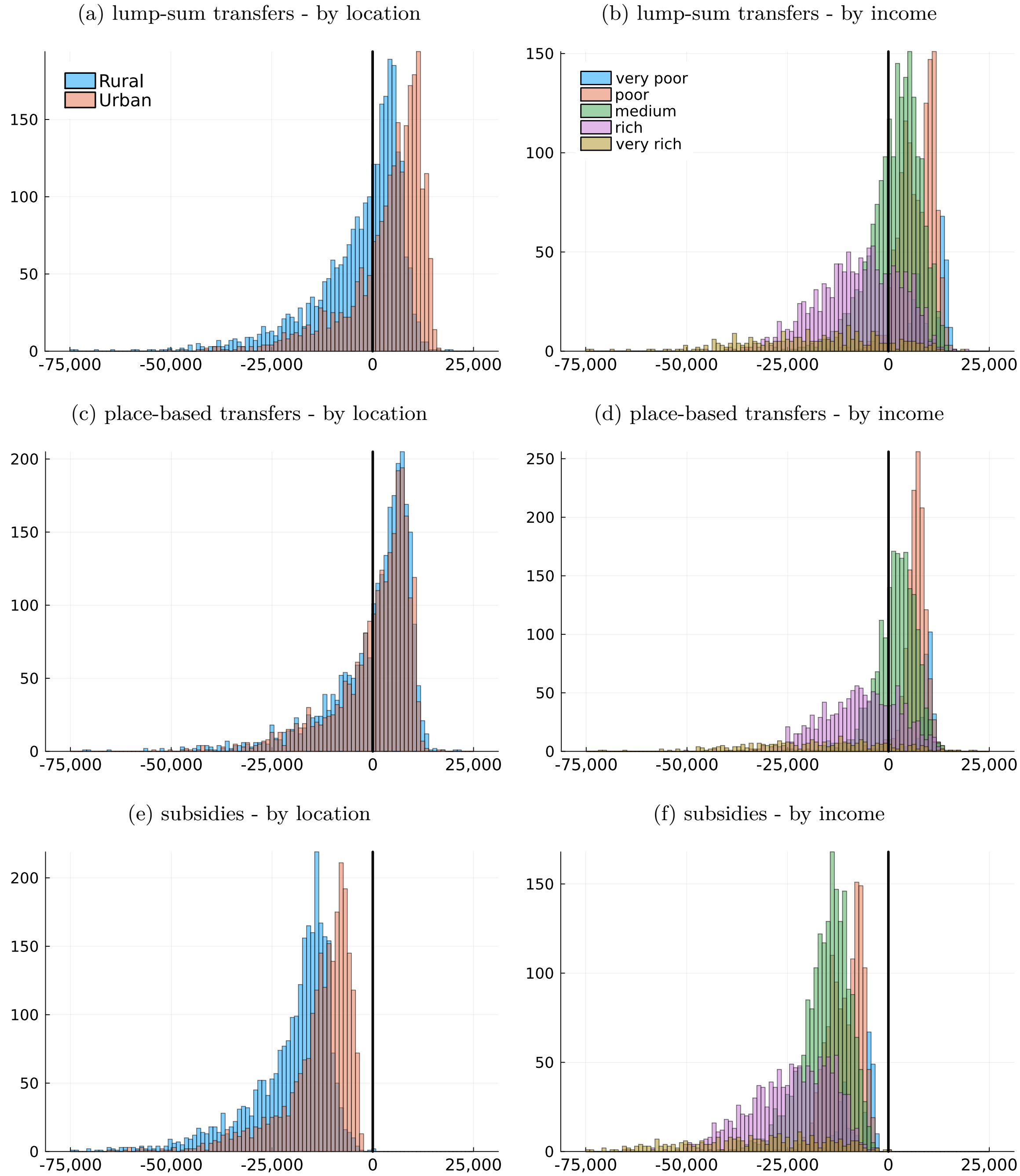


Notes: Panel (a) shows the share of newly constructed housing relative to the stock of housing in the initial stationary equilibrium for the different policy scenarios along the transition. Panel (b) shows the share of housing renovations relative to the stock of housing in the initial stationary equilibrium for the different policy scenarios along the transition.

4.2.3 Distributional consequences and political support

Having documented the average paths of key economic variables along the transition, the next step is to zoom into the distributional effects of the different policy scenarios across income groups and regions in order to evaluate their political support. I will start by evaluating these policies based on their monetary consequences for households. Thus, for each household in the economy, I compute its net transfers, defined by subtracting their carbon tax payments from the direct transfers they receive, including the transfers by the government from firm's profits, for each year. I will then compute the net present value of these net transfers. Figure 10 plots the distribution of these net present values depending on the region and income level of households in the first period of the transition for the different policy scenarios. First, observe that the distribution is left-skewed for all three policy scenarios. When spending the carbon tax revenues on transfers, there are many households with positive net present values of up to 15,000 Euros, while some households loose up to 40,000 Euros.

Figure 10: Distributional effects (net transfers) with lump-sum transfers



(a) lump-sum transfers - by location

(b) lump-sum transfers - by income

(c) place-based transfers - by location

(d) place-based transfers - by income

(e) subsidies - by location

(f) subsidies - by income

Notes: Panel (a) shows distribution of the net present value of net transfers depending on the location in which households lived in the first period of the transition. Panel (b) shows the same distribution for different income groups.

With lump-sum transfers, urban households on average have higher net present values than rural households implying a redistribution from rural to urban regions. This is not surprising as the empirical part of this paper has shown that rural households have higher carbon footprints. With place-based transfers, this spatial redistribution disappears by construction and households in both regions have very similar distributions of net present values. When spending the carbon taxes on subsidies for renovations, the net transfers are always negative as the transfers from firm's profits are lower than household's carbon tax payments.

Table 3 shows the resulting share of the population which benefits, i.e. which have a positive net present value, for the different policy scenarios. Besides the baseline carbon tax of 300 Euros per ton, I evaluate a low and a high carbon tax scenario with tax levels of 100 Euros and 500 Euros per ton, respectively.[22]

The share of households who benefit from a given policy hardly changes with the level of the carbon tax. For the two transfers scenarios it only depends on the share of households who emits less than the population average or the region specific average. Across the different recycling schemes, there are, however, substantial differences. For lump-sum transfers the share of benefiting households in rural and urban regions is around 48 and 76 percent while with place-based transfers it is around 63 percent in both regions. Thus, when evaluating these policies entirely based on their monetary consequences for households, place-based transfers have a majority in both regions whereas lump-sum transfers do not. With subsidies, by construction, no household benefits as there are no direct transfers to households.

Table 3: Political support for climate policies - monetary decision

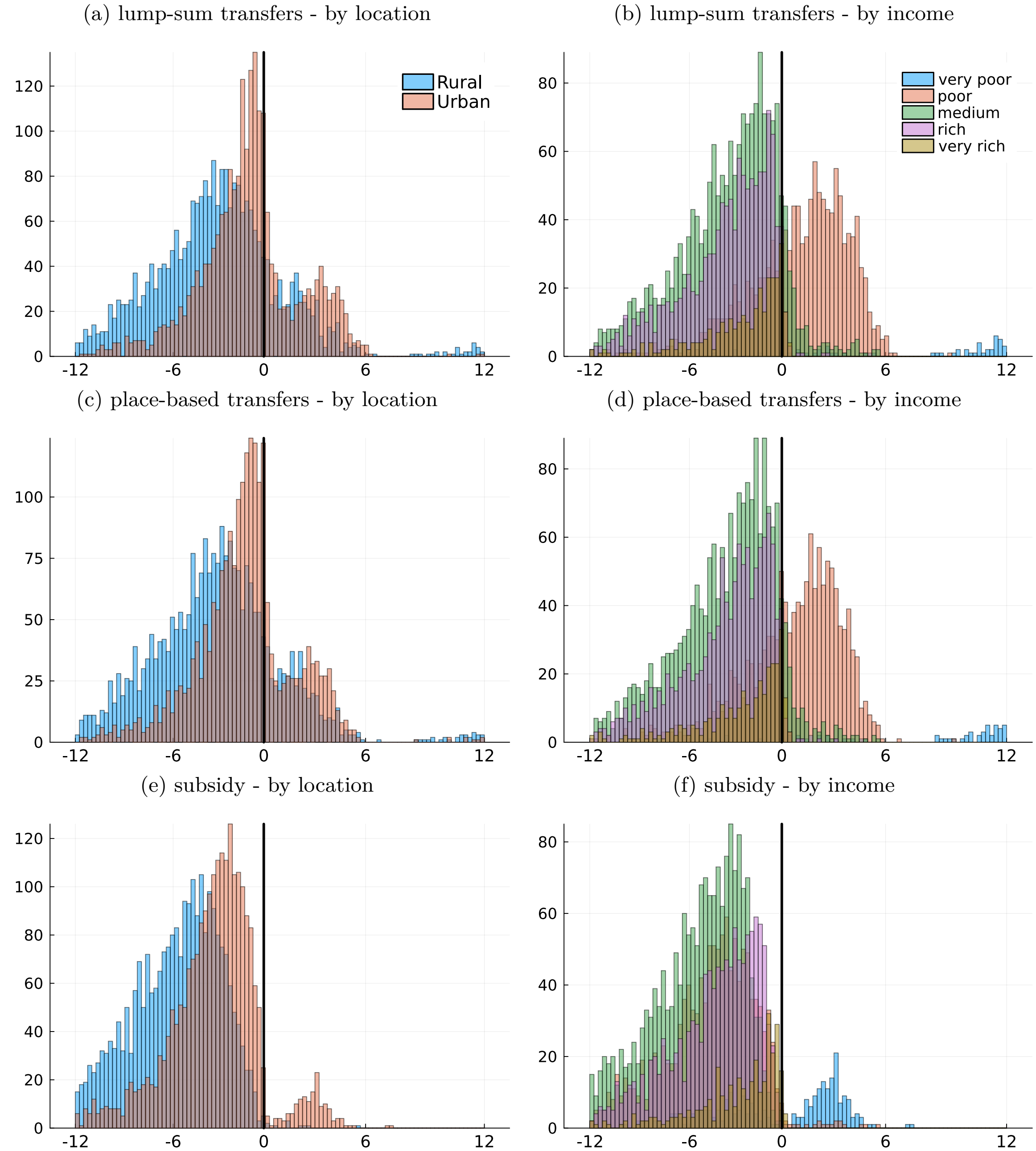
	Lump-sum			Place-based			Subsidy		
$\tau =$	100	300	500	100	300	500	100	300	500
All	60.1	59.9	60.1	62.2	62.4	62.1	0.00	0.0	0.0
Rural	48.2	48.1	48.1	63.3	63.2	63.0	0.0	0.0	0.0
Urban	73.3	73.1	73.2	61.0	61.3	61.1	0.0	0.0	0.0

Notes: This table shows the share of households who benefit from a given policy in monetary terms.

Next, I want to look at the welfare consequences of these policies. Thus, I do not only look at the monetary consequences of these policies but also include their distortionary effects. Figure 11 plots the welfare effects, measured as the consumption equivalent variation (CEV) in terms of the non-housing, non-energy good relative to the transition without policy intervention.

[22] The reason for this is that, even though a carbon tax of 300 Euros per ton is well in line with empirical estimates for the expected carbon tax within the European Emission Trading System (ETS) II, which is explicitly targeted at emissions from heating and car energy, there is still a high uncertainty on its exact level and path. This uncertainty comes from the fact that the European Union does not fix a carbon price but rather fixes the number of issued certificates which are required for emitting carbon emission. Hence, the price for these certificates, which will constitute the carbon tax, will be determined on the market depending on their supply and demand. Appendix C further provides all results for these two carbon tax scenarios.

Figure 11: Welfare effects (CEV)



(a) lump-sum transfers - by location
(b) lump-sum transfers - by income
(c) place-based transfers - by location
(d) place-based transfers - by income
(e) subsidy - by location
(f) subsidy - by income

Notes: Panel (a) shows the share of households heating with oil for different in cities sizes. These averages are computed controlling for the age of the main earner, households net disposable income, the household size, and the survey year. Similarly, Panel (b) shows the share of households using district heating for different in cities sizes. Again, I control for the age of the main earner, households net disposable income, the household size, and the survey year.

First, note that the political support for carbon taxes when being evaluated based on the welfare effects is lower than when being evaluated based on monetary outcomes. This is because the carbon tax distorts households' consumption decision and thus lowers their welfare. Further, note that the support for a carbon tax with lump sum reimbursements is still higher in the urban region. The welfare effects along the income dimension are not as monotone as it was the case for the monetary effects as monetary changes have generally smaller welfare implications the higher the income is. Overall, the differences in the political support between rural and urban regions depend less on the way of recycling carbon taxes. This is because the distortions from carbon taxes are more severe in rural regions, where energy consumption is higher, reducing their support for carbon taxes independently from the way carbon tax revenues are reimbursed.

The last step of the analysis is to not only capture the distortionary effects of a carbon tax but to also account for the benefits of reducing carbon emissions by modelling them in reduced-form based on estimates from the literature. It is important to note that there is a high uncertainty of these estimates, in particular on the damages from climate change and how they impact households' welfare. Consequently, the level of potential benefits is also highly uncertain, implying that this last subsection becomes more qualitative than quantitative. I need to make two assumptions. First, I assume the social costs of carbon to be 500 Euros per ton, which is at the higher end of what people traditionally use but within the range of recent estimates (Bilal and Känzig, 2024; Rennert et al., 2022). Second, I assume that this tax is set for the European Union and that households within this union do not care about households from other regions.[23]

Table 4: Political support for climate policies - welfare decision

	Lump-sum			Place-based			Subsidy		
$\tau =$	100	300	500	100	300	500	100	300	500
Baseline									
All	36.3	29.1	11.3	40.2	32.4	14.0	14.1	8.1	4.0
Rural	31.1	22.3	2.0	38.8	30.2	9.0	11.7	6.1	2.0
Urban	41.1	36.1	19.8	42.9	34.1	19.2	16.6	10.1	6.0
Positive ext.									
All	46.2	39.9	18.3	50.1	41.9	22.1	25.1	16.3	11.0
Rural	41.2	32.6	7.1	48.9	40.0	17.1	22.5	13.2	7.1
Urban	51.1	46.8	27.0	52.4	43.6	26.8	27.1	19.3	15.0

Table 4 presents the main results on the political support for the different policies with and without benefits from reducing carbon emissions. As already noted for the histograms, the political support is lower when evaluating carbon taxes based on welfare than when evaluating them based on monetary terms as the distortionary effects of the taxes are accounted for. Thus, the political support decreases with the level of the carbon tax. Spending carbon taxes on subsidies for housing renovations only finds little support among the electorate. The support for spending the carbon tax revenues on place-based transfers is higher than for spending them on lump-sum transfers, in particular for rural households as this policy avoids spatial redistribution from rural to urban regions. Thus, place-based transfers seem to preferable to lump-sum transfers if the government wants to find support in both regions. When

[23]The histograms depicting the distributions of welfare effects accounting for the benefits of reducing carbon emission can be found in Figure x in the Appendix.

accounting for the positive externalities from reducing carbon emissions, the overall support for carbon taxes increases by around 10 percentage points. The qualitative results do not change.

4.3 Sensitivity Analysis

The current analysis considers already a variety of different policy scenarios which are evaluated regarding their consequences on the spatial redistribution and their political support. For the main analysis, I assume a constant carbon tax of 300 Euros per ton of carbon emissions. Even though this tax is well in line with empirical estimates for the expected carbon tax within the European Emission Trading System (ETS) II, which is explicitly targeted at emissions from heating and car energy, there is still a high uncertainty on its exact level and path. This uncertainty comes from the fact that the European Union does not fix a carbon price but rather fixes the number of issued certificates which are required for emitting carbon emission. Hence, the price for these certificates, which will constitute the carbon tax, will be determined on the market depending on their supply and demand. Thus, C.2 provides an extensive sensitivity analysis with respect to the level and path of the carbon tax as well as for extreme ways of recycling carbon tax revenues where either only rural or only urban households receive transfers. First, consistent with the scenarios considered for the political support of policies, I show the results for a lower carbon tax scenario of 100 Euros per ton and a higher tax scenario of 500 Euros per ton. The sensitivity analysis shows that all qualitative results remain unchanged. For the quantitative results, I observe, not surprisingly, that lower (higher) carbon taxes decrease (increase) the size of the effects. Lower (higher) carbon taxes decrease (increase) the speed of the transition and lead to less (more) spatial redistribution when being used for lump-sum transfers. While in the baseline scenario with a carbon tax of 300 Euros per ton, the share of households with clean cars and houses were around x and y in 2040 and x and y in 2060, these numbers decrease with taxes of 100 Euros per ton (increase with taxes of 500 Euros per ton) to x and y (x and y) percent in 2040 and to x and y (x and y) percent in 2060. At the same time the difference in the net present values of net lump-sum transfers between rural and urban households decreases (increases) from 8,000 Euros to x (y) with carbon taxes of 100 Euros per ton (500 Euros per ton).

Next, the carbon tax within the ETS II, will most likely not be constant over time but will rather be slightly increasing (Kalkuhl et al., 2023). Thus, I will check the sensitivity of the constant carbon tax in the baseline with an increasing price path. I will start with a relatively low tax of 100 Euros per ton in 2019, which then increases linearly to 250 Euros in 2030 and to 520 Euros in 2045, where it remains.[24] These numbers are based on the price scenario estimated by Kalkuhl et al. (2023) for the ETS II. Lastly, I check the sensitivity of the baseline results relative to two very extreme ways of transferring the carbon revenues back to households. I transfer the revenues either exclusively to rural or exclusively to urban households. In these very extreme scenarios the average paths hardly change. But the level of redistribution increases. If transferring carbon tax revenues only to rural households, their net present value of net transfers is x while for urban households it decreases to y. When only reimbursing urban households their net present value of net transfers changes to x while for rural households it falls to y.

[24]Figure A5 plots this price path over the transition period.

5 Conclusion

This paper presents new empirical estimates on the heterogeneity of carbon footprints across income groups in rural and urban regions in the German household sector. Further, it builds a novel and rich theoretical framework to study the distributional consequences of carbon taxes along the transition to clean technologies between regions. I use this framework to evaluate different recycling schemes for carbon tax revenues with respect to their spatial redistribution and the implications for their political support. I show empirically that rural households consume more heating and car energy and use heating technologies which pollute more. As a consequence their carbon footprint is around 2.2 tons higher, around 12 percent of an average household's carbon footprint in Germany in 2018. Thus, spending carbon taxes on lump-sum transfers redistributes from rural to urban regions. Based on the quantitative model, the difference in the net present value of net transfers is around 8,000 Euros, around 25 percent of an average household's annual net income. Place-based transfers avoid this spatial redistribution without decreasing the speed of transitioning to clean technologies. Subsidies on housing renovations lead to a faster transition but only find little support among the electorate, even when accounting for the positive externalities of reducing carbon emissions. Hence, place-based transfers are useful if policy makers want to find support for carbon taxes in rural and urban regions. In this model, carbon taxes lead to sizeable general equilibrium effects on the level of housing prices of different technologies. As the present paper models households as tenants, these effects are accounted for in the renting prices. An avenue for future research is to adopt this framework to model homeowners and landlords who are affected by these price changes via wealth effects.

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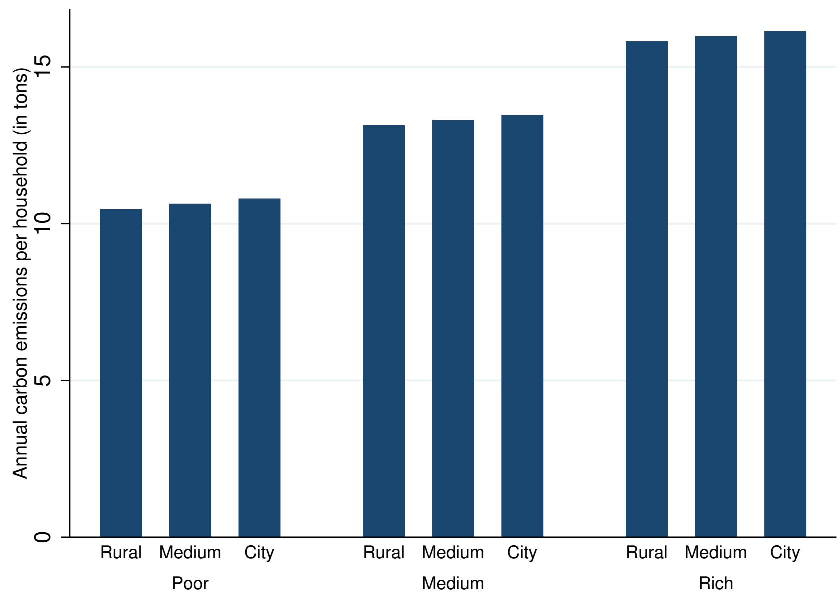
A Additional empirical results

Figure A1: Indirect carbon footprints across space and income



Notes: This figure shows the average indirect carbon emissions for households of different income and city size groups. First, I divide the total sample of households into three equally-sizes groups according to their net disposable household income. Second, I group households within these income groups based on whether they live in small villages of less than 20,000 inhabitants, in small cities with population sizes between 20,000 and 100,000, or in large cities with more than 100,000 inhabitants.

B Additional derivations for the model

B.1 Full dynamic household problem

This subsection presents the full dynamic household problem. Each period households have 12 alternatives to choose from where the value functions depending on the four discrete decisions, moving (M) vs. not moving (NM) to the other region, adjusting the housing type (HA) vs. not adjusting it (NHA), buying a new car (CA) vs. not buying a new car (NCA), and buying a dirty car (DCA) vs. buying a clean car (CCA), are given by

$$V_t^{\text{NM,NHA,NCA}}(l_t, y_t, \kappa, \lambda_t^c, \lambda_t^h) = \max_{\{h_t, e_t^c, e_t^h\}} \quad u(x_t, h_t, e_t^h, e_t^c, l_{t+1}) + \beta\, \mathbb{E}\left[V_{t+1}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right]$$

$$\begin{aligned} s.t. \qquad & x_t = y_t + T_t^{\pi} + T_t^{\tau} - \rho(l_{t+1}, \lambda_{t+1}^h) h_t - [p_t^{eh} + (1-\lambda_{t+1}^h)\xi_l^h \tau] e_t^h - [p^{ec} + (1-\lambda_{t+1}^c)\xi^c \tau] e_t^c \\ & l_{t+1} = l_t, \quad \lambda_{t+1}^c = \lambda_t^c, \quad \lambda_{t+1}^h = \lambda_t^h \end{aligned}$$

$$V_t^{\mathrm{NM,NHA,DCA}}(l_t, y_t, \kappa, \lambda_t^c, \lambda_t^h) = \max_{\{h_t, e_t^c, e_t^h\}} \quad u(x_t, h_t, e_t^h, e_t^c, l_{t+1}) + \beta\,\mathbb{E}\left[V_{t+1}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right]$$

$$\begin{aligned} s.t. \qquad & x_t = y_t + T_t^\pi + T_t^\tau - \rho(l_{t+1}, \lambda_{t+1}^h)h_t - [p_t^{eh} + (1-\lambda_{t+1}^h)\xi_l^h\tau]e_t^h - [p^{ec} + (1-\lambda_{t+1}^c)\xi^c\tau]e_t^c \\ & l_{t+1} = l_t, \quad \lambda_{t+1}^c = 0, \quad \lambda_{t+1}^h = \lambda_t^h \end{aligned}$$

$$V_t^{\mathrm{NM,NHA,CCA}}(l_t, y_t, \kappa, \lambda_t^c, \lambda_t^h) = \max_{\{h_t, e_t^c, e_t^h\}} \quad u(x_t, h_t, e_t^h, e_t^c, l_{t+1}) + \beta\,\mathbb{E}\left[V_{t+1}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right]$$

$$\begin{aligned} s.t. \qquad & x_t = y_t + T_t^\pi + T_t^\tau - \rho(l_{t+1}, \lambda_{t+1}^h)h_t - [p_t^{eh} + (1-\lambda_{t+1}^h)\xi_l^h\tau]e_t^h - [p^{ec} + (1-\lambda_{t+1}^c)\xi^c\tau]e_t^c \\ & l_{t+1} = l_t, \quad \lambda_{t+1}^c = 1, \quad \lambda_{t+1}^h = \lambda_t^h \end{aligned}$$

$$V_t^{\mathrm{NM,HA,NCA}}(l_t, y_t, \kappa, \lambda_t^c, \lambda_t^h) = \max_{\{h_t, e_t^c, e_t^h\}} \quad u(x_t, h_t, e_t^h, e_t^c, l_{t+1}) + \beta\,\mathbb{E}\left[V_{t+1}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right]$$

$$\begin{aligned} s.t. \qquad & x_t = y_t + T_t^\pi + T_t^\tau - \rho(l_{t+1}, \lambda_{t+1}^h)h_t - [p_t^{eh} + (1-\lambda_{t+1}^h)\xi_l^h\tau]e_t^h - [p^{ec} + (1-\lambda_{t+1}^c)\xi^c\tau]e_t^c \\ & l_{t+1} = l_t, \quad \lambda_{t+1}^c = \lambda_t^c, \quad \lambda_{t+1}^h \neq \lambda_t^h \end{aligned}$$

$$V_t^{\mathrm{NM,HA,DCA}}(l_t, y_t, \kappa, \lambda_t^c, \lambda_t^h) = \max_{\{h_t, e_t^c, e_t^h\}} \quad u(x_t, h_t, e_t^h, e_t^c, l_{t+1}) + \beta\,\mathbb{E}\left[V_{t+1}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right]$$

$$\begin{aligned} s.t. \qquad & x_t = y_t + T_t^\pi + T_t^\tau - \rho(l_{t+1}, \lambda_{t+1}^h)h_t - [p_t^{eh} + (1-\lambda_{t+1}^h)\xi_l^h\tau]e_t^h - [p^{ec} + (1-\lambda_{t+1}^c)\xi^c\tau]e_t^c \\ & l_{t+1} = l_t, \quad \lambda_{t+1}^c = 0, \quad \lambda_{t+1}^h \neq \lambda_t^h \end{aligned}$$

$$V_t^{\mathrm{NM,HA,CCA}}(l_t, y_t, \kappa, \lambda_t^c, \lambda_t^h) = \max_{\{h_t, e_t^c, e_t^h\}} \quad u(x_t, h_t, e_t^h, e_t^c, l_{t+1}) + \beta\,\mathbb{E}\left[V_{t+1}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right]$$

$$\begin{aligned} s.t. \qquad & x_t = y_t + T_t^\pi + T_t^\tau - \rho(l_{t+1}, \lambda_{t+1}^h)h_t - [p_t^{eh} + (1-\lambda_{t+1}^h)\xi_l^h\tau]e_t^h - [p^{ec} + (1-\lambda_{t+1}^c)\xi^c\tau]e_t^c \\ & l_{t+1} = l_t, \quad \lambda_{t+1}^c = 1, \quad \lambda_{t+1}^h \neq \lambda_t^h \end{aligned}$$

$$V_t^{\mathrm{M,NHA,NCA}}(l_t, y_t, \kappa, \lambda_t^c, \lambda_t^h) = \max_{\{h_t, e_t^c, e_t^h\}} \quad u(x_t, h_t, e_t^h, e_t^c, l_{t+1}) + \beta\,\mathbb{E}\left[V_{t+1}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right]$$

$$\begin{aligned} s.t. \qquad & x_t = y_t + T_t^\pi + T_t^\tau - \rho(l_{t+1}, \lambda_{t+1}^h)h_t - [p_t^{eh} + (1-\lambda_{t+1}^h)\xi_l^h\tau]e_t^h - [p^{ec} + (1-\lambda_{t+1}^c)\xi^c\tau]e_t^c \\ & l_{t+1} \neq l_t, \quad \lambda_{t+1}^c = \lambda_t^c, \quad \lambda_{t+1}^h = \lambda_t^h \end{aligned}$$

$$V_t^{\mathrm{M,NHA,DCA}}(l_t, y_t, \kappa, \lambda_t^c, \lambda_t^h) = \max_{\{h_t, e_t^c, e_t^h\}} \quad u(x_t, h_t, e_t^h, e_t^c, l_{t+1}) + \beta\,\mathbb{E}\left[V_{t+1}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right]$$

$$s.t. \qquad x_t = y_t + T_t^{\pi} + T_t^{\tau} - \rho(l_{t+1}, \lambda_{t+1}^h)h_t - [p_t^{eh} + (1-\lambda_{t+1}^h)\xi_l^h \tau]e_t^h - [p^{ec} + (1-\lambda_{t+1}^c)\xi^c \tau]e_t^c$$
$$l_{t+1} \neq l_t, \quad \lambda_{t+1}^c = 0, \quad \lambda_{t+1}^h = \lambda_t^h$$

$$V_t^{\text{M,NHA,CCA}}(l_t, y_t, \kappa, \lambda_t^c, \lambda_t^h) = \max_{\{h_t, e_t^c, e_t^h\}} \quad u(x_t, h_t, e_t^h, e_t^c, l_{t+1}) + \beta\,\mathbb{E}\left[V_{t+1}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right]$$

$$s.t. \qquad x_t = y_t + T_t^{\pi} + T_t^{\tau} - \rho(l_{t+1}, \lambda_{t+1}^h)h_t - [p_t^{eh} + (1-\lambda_{t+1}^h)\xi_l^h \tau]e_t^h - [p^{ec} + (1-\lambda_{t+1}^c)\xi^c \tau]e_t^c$$
$$l_{t+1} \neq l_t, \quad \lambda_{t+1}^c = 1, \quad \lambda_{t+1}^h = \lambda_t^h$$

$$V_t^{\text{M,HA,NCA}}(l_t, y_t, \kappa, \lambda_t^c, \lambda_t^h) = \max_{\{h_t, e_t^c, e_t^h\}} \quad u(x_t, h_t, e_t^h, e_t^c, l_{t+1}) + \beta\,\mathbb{E}\left[V_{t+1}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right]$$

$$s.t. \qquad x_t = y_t + T_t^{\pi} + T_t^{\tau} - \rho(l_{t+1}, \lambda_{t+1}^h)h_t - [p_t^{eh} + (1-\lambda_{t+1}^h)\xi_l^h \tau]e_t^h - [p^{ec} + (1-\lambda_{t+1}^c)\xi^c \tau]e_t^c$$
$$l_{t+1} \neq l_t, \quad \lambda_{t+1}^c = \lambda_t^c, \quad \lambda_{t+1}^h \neq \lambda_t^h$$

$$V_t^{\text{M,HA,DCA}}(l_t, y_t, \kappa, \lambda_t^c, \lambda_t^h) = \max_{\{h_t, e_t^c, e_t^h\}} \quad u(x_t, h_t, e_t^h, e_t^c, l_{t+1}) + \beta\,\mathbb{E}\left[V_{t+1}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right]$$

$$s.t. \qquad x_t = y_t + T_t^{\pi} + T_t^{\tau} - \rho(l_{t+1}, \lambda_{t+1}^h)h_t - [p_t^{eh} + (1-\lambda_{t+1}^h)\xi_l^h \tau]e_t^h - [p^{ec} + (1-\lambda_{t+1}^c)\xi^c \tau]e_t^c$$
$$l_{t+1} \neq l_t, \quad \lambda_{t+1}^c = 0, \quad \lambda_{t+1}^h \neq \lambda_t^h$$

$$V_t^{\text{M,HA,CCA}}(l_t, y_t, \kappa, \lambda_t^c, \lambda_t^h) = \max_{\{h_t, e_t^c, e_t^h\}} \quad u(x_t, h_t, e_t^h, e_t^c, l_{t+1}) + \beta\,\mathbb{E}\left[V_{t+1}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right]$$

$$s.t. \qquad x_t = y_t + T_t^{\pi} + T_t^{\tau} - \rho(l_{t+1}, \lambda_{t+1}^h)h_t - [p_t^{eh} + (1-\lambda_{t+1}^h)\xi_l^h \tau]e_t^h - [p^{ec} + (1-\lambda_{t+1}^c)\xi^c \tau]e_t^c$$
$$l_{t+1} \neq l_t, \quad \lambda_{t+1}^c = 1, \quad \lambda_{t+1}^h \neq \lambda_t^h$$

where the expected value is given by

$$\begin{aligned}
&\mathbb{E}\left[V_{t+1}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right] = \\
&\max\{\mathbb{E}\left[V_{t+1}^{\text{NM}}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right], \mathbb{E}\left[V_{t+1}^{\text{M}}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right] + \epsilon_l\}
\end{aligned}$$

where

$$\begin{aligned}
&\mathbb{E}\left[V_{t+1}^{\text{NM}}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right] = \\
&\max\{\mathbb{E}\left[V_{t+1}^{\text{NM, NHA}}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right], \mathbb{E}\left[V_{t+1}^{\text{NM, HA}}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right] + \epsilon_h\} \\
&\mathbb{E}\left[V_{t+1}^{\text{M}}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right] = \\
&\max\{\mathbb{E}\left[V_{t+1}^{\text{M, NHA}}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right], \mathbb{E}\left[V_{t+1}^{\text{M, HA}}(l_{t+1}, y_{t+1}, \kappa, \lambda_{t+1}^c, \lambda_{t+1}^h) \mid y_t\right] + \epsilon_h\}
\end{aligned}$$

and further

$$
\begin{aligned}
&\mathbb{E}\left[V^{\text{NM, NHA}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right] = \\
&\max\{\mathbb{E}\left[V^{\text{NM, NHA, NCA}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right], \mathbb{E}\left[V^{\text{NM, NHA, CA}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right] + \epsilon_c\} \\
&\mathbb{E}\left[V^{\text{NM, HA}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right] = \\
&\max\{\mathbb{E}\left[V^{\text{NM, HA, NCA}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right], \mathbb{E}\left[V^{\text{NM, HA, CA}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right] + \epsilon_c\} \\
&\mathbb{E}\left[V^{\text{M, NHA}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right] = \\
&\max\{\mathbb{E}\left[V^{\text{M, NHA, NCA}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right], \mathbb{E}\left[V^{\text{M, NHA, CA}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right] + \epsilon_c\} \\
&\mathbb{E}\left[V^{\text{M, HA}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right] = \\
&\max\{\mathbb{E}\left[V^{\text{M, HA, NCA}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right], \mathbb{E}\left[V^{\text{M, HA, CA}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right] + \epsilon_c\}
\end{aligned}
$$

and finally

$$
\begin{aligned}
&\mathbb{E}\left[V^{\text{NM, NHA, CA}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right] = \\
&\max\{\mathbb{E}\left[V^{\text{NM, NHA, CA, DC}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right], \mathbb{E}\left[V^{\text{NM, NHA, CA, CC}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right] + \epsilon_{ct}\} \\
&\mathbb{E}\left[V^{\text{NM, HA, CA}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right] = \\
&\max\{\mathbb{E}\left[V^{\text{NM, HA, CA, DC}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right], \mathbb{E}\left[V^{\text{NM, HA, CA, CC}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right] + \epsilon_{ct}\} \\
&\mathbb{E}\left[V^{\text{M, NHA, CA}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right] = \\
&\max\{\mathbb{E}\left[V^{\text{M, NHA, CA, DC}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right], \mathbb{E}\left[V^{\text{M, NHA, CA, CC}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right] + \epsilon_{ct}\} \\
&\mathbb{E}\left[V^{\text{M, HA, CA}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right] = \\
&\max\{\mathbb{E}\left[V^{\text{M, HA, CA, DC}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right], \mathbb{E}\left[V^{\text{M, HA, CA, CC}}(l_{t+1}, y_{t+1}, \kappa, \lambda^c_{t+1}, \lambda^h_{t+1}) \mid y_t\right] + \epsilon_{ct}\}.
\end{aligned}
$$

Each period consists of four stages. First, households enter the period with their state variables from last period, observe the realizations of the income shock and solve the maximization problem for all combinations of the four discrete decisions. Second, they observe the moving preference shock ϵ_l and decide whether or not to move to the other region. This problem has already been shown in the main text. Next, households observe the housing type shock and decide whether or not to change the technology type of their house, conditional on their moving decision. Thus, their value functions read

B.2 Housing Construction Firm

This subsection describes the relevant decisions for constructing and renovating houses in detail.

B.2.1 Housing Construction

The representative construction firm has to decide how much dirty and clean housing it wants to build in the rural and urban region. Hence, the problem reads

$$
\max_{I^j_{l,t}} q^j_{l,t} I^j_{l,t} - w^j_{l,t} N^j_{l,t} \quad \text{s.t} \quad I^j_{l,t} = \psi^{h,j}_{l,t} \left(N^j_{l,t}\right)^{\alpha^h_l} \overline{L_l}^{1-\alpha^h_l}
$$

rewriting the budget constraint and plugging it into the objective function yields

$$\max_{I^j_{l,t}} q^j_{l,t} I^j_{l,t} - w^j_{l,t} \left(\psi^{h,j}_{l,t}\right)^{-\frac{1}{\alpha^h_l}} I_{l,t}^{\frac{1}{\alpha^h_l}} \overline{L_l}^{\frac{\alpha^h_l - 1}{\alpha^h_l}}$$

which gives the first-order condition

$$q^j_{l,t} - \left(\psi^{h,j}_{l,t}\right)^{-\frac{1}{\alpha^h_l}} \frac{w^j_{l,t}}{\alpha^h_l} I_{l,t}^{\frac{1-\alpha^h_l}{\alpha^h_l}} \overline{L_l}^{\frac{\alpha^h_l - 1}{\alpha^h_l}} = 0.$$

Hence, the optimal level of newly build housing is given by

$$I^j_{l,t} = \left(\alpha^h_l \frac{q^j_{l,t}}{w^j_{l,t}}\right)^{\frac{\alpha^h_l}{1-\alpha^h_l}} \left(\psi^{h,j}_{l,t}\right)^{\frac{1}{1-\alpha^h_l}} \overline{L_l}.$$

Last, the firm needs to decide whether to use built clean or dirty housing. For doing so it compares the profits in both cases. Note, that the firm will always either only build clean or only dirty houses. Hence, the condition for building dirty housing is given by

$$\begin{aligned}
\tau^{cl} &> \tau^{di} \\
\leftrightarrow q^{cl}_{l,t} I^{cl}_{l,t} - w^{cl}_{l,t} N^{cl}_{l,t} &> q^{di}_{l,t} I^{di}_{l,t} - w^{di}_{l,t} N^{di}_{l,t} \\
\leftrightarrow q^{cl}_{l,t} I^{cl}_{l,t} - w^{cl}_{l,t} \left(\psi^{h,di}_{l,t}\right)^{-\frac{1}{\alpha^h_l}} I^{\frac{1}{\alpha^h_l}} \overline{L_l}^{\frac{\alpha^h_l - 1}{\alpha^h_l}} &> q^{di}_{l,t} I^{di}_{l,t} - w^{di}_{l,t} \left(\psi^{h,di}_{l,t}\right)^{-\frac{1}{\alpha^h_l}} I^{\frac{1}{\alpha^h_l}} \overline{L_l}^{\frac{\alpha^h_l - 1}{\alpha^h_l}} \\
\leftrightarrow q^{cl}_{l,t} \left[\left(\alpha^h_l \frac{q^{cl}_{l,t}}{w^{cl}_{l,t}}\right)^{\frac{\alpha^h_l}{1-\alpha^h_l}} \left(\psi^{h,cl}_{l,t}\right)^{\frac{1}{1-\alpha^h_l}} \overline{L_l}\right] &- w^{cl}_{l,t} \left(\psi^{h,di}_{l,t}\right)^{-\frac{1}{\alpha^h_l}} \left[\left(\alpha^h_l \frac{q^{cl}_{l,t}}{w^{cl}_{l,t}}\right)^{\frac{\alpha^h_l}{1-\alpha^h_l}} \left(\psi^{h,cl}_{l,t}\right)^{\frac{1}{1-\alpha^h_l}} \overline{L_l}\right]^{\frac{1}{\alpha^h_l}} \overline{L_l} \\
> q^{di}_{l,t} \left[\left(\alpha^h_l \frac{q^{di}_{l,t}}{w^{di}_{l,t}}\right)^{\frac{\alpha^h_l}{1-\alpha^h_l}} \left(\psi^{h,di}_{l,t}\right)^{\frac{1}{1-\alpha^h_l}} \overline{L_l}\right] &- w^{di}_{l,t} \left(\psi^{h,di}_{l,t}\right)^{-\frac{1}{\alpha^h_l}} \left[\left(\alpha^h_l \frac{q^{di}_{l,t}}{w^{di}_{l,t}}\right)^{\frac{\alpha^h_l}{1-\alpha^h_l}} \left(\psi^{h,di}_{l,t}\right)^{\frac{1}{1-\alpha^h_l}} \overline{L_l}\right]^{\frac{1}{\alpha^h_l}} \overline{L_l} \\
\leftrightarrow \left(q^{cl}_{l,t}\psi^{h,cl}_{l,t}\right)^{\frac{1}{1-\alpha^h_l}} \left(\frac{\alpha^h_l}{w^{di}_{l,t}}\right)^{\frac{\alpha^h_l}{1-\alpha^h_l}} \overline{L_l} - \left(\alpha^h_l \psi^{h,cl}_{l,t} \frac{q^{cl}_{l,t}}{w^{cl}_{l,t}}\right)^{\frac{1}{1-\alpha^h_l}} \overline{L_l} &> \left(q^{di}_{l,t}\psi^{h,di}_{l,t}\right)^{\frac{1}{1-\alpha^h_l}} \left(\frac{\alpha^h_l}{w^{di}_{l,t}}\right)^{\frac{\alpha^h_l}{1-\alpha^h_l}} \overline{L_l} - \left(\alpha^h_l \psi^{h,di}_{l,t} \frac{q^{di}_{l,t}}{w^{di}_{l,t}}\right)^{\frac{1}{1-\alpha^h_l}} \overline{L_l} \\
\leftrightarrow \left(q^{cl}_{l,t}\psi^{h,cl}_{l,t}\right)^{\frac{1}{1-\alpha^h_l}} \overline{L_l} \left[\left(\frac{\alpha^h_l}{w^{cl}_{l,t}}\right)^{\frac{\alpha^h_l}{1-\alpha^h_l}} - \left(\frac{\alpha^h_l}{w^{cl}_{l,t}}\right)^{\frac{1}{1-\alpha^h_l}}\right] &> \left(q^{di}_{l,t}\psi^{h,di}_{l,t}\right)^{\frac{1}{1-\alpha^h_l}} \overline{L_l} \left[\left(\frac{\alpha^h_l}{w^{di}_{l,t}}\right)^{\frac{\alpha^h_l}{1-\alpha^h_l}} - \left(\frac{\alpha^h_l}{w^{di}_{l,t}}\right)^{\frac{1}{1-\alpha^h_l}}\right] \\
\leftrightarrow q^{cl}_{l,t}\psi^{h,cl}_{l,t} &> q^{cl}_{l,t}\psi^{h,cl}_{l,t} \\
\leftrightarrow q^{cl}_{l,t} &> q^{di}_{l,t} \left(\frac{\psi^{di}_{l,t}}{\psi^{cl}_{l,t}}\right),
\end{aligned}$$

where the step from the third-to-last to the second-to-last equation comes from the fact that in equilibrium all wages in the economy are equal to one.

B.2.2 Housing Renovations

For renovating old into modern housing, the construction firm solves the following problem:

$$\max_{I_{l,t}^{ren}} q_{l,t}^{cl} I_{l,t}^{r} - w_{l,t}^{ren} N_{l,t}^{ren} - q_{l,t}^{di} h_{l,t}^{di} \quad \text{s.t} \quad I_{l,t}^{ren} = \lambda_{l,t}^{r} \min\{\left(N_{l,t}^{ren}\right)^{\alpha_l^h}, h_{l,t}^{di}\}$$

The construction firm transform one unit of dirty housing into one unit of clean housing. The labor productivity for this process is determined by a time-varying and a constant productivity parameter, λ_l. Hence, optimality requires that

$$\begin{aligned} h_{l,t}^{di} &= \frac{I_{l,t}^{ren}}{\lambda_{l,t}^{r}} \\ N_{l,t}^{ren} &= \left(\frac{I_{l,t}^{ren}}{\lambda_{l,t}^{r}}\right)^{\frac{1}{\alpha_l^h}} \end{aligned}$$

Hence, the construction firm solves

$$\max_{I_{l,t}^{ren}} q_{l,t}^{cl} I_{l,t}^{r} - w_{l,t}^{ren} \left(\frac{I_{l,t}^{ren}}{\lambda_{l,t}^{r}}\right)^{\frac{1}{\alpha_l^h}} - q_{l,t}^{di} \frac{I_{l,t}^{ren}}{\lambda_{l,t}^{r}}$$

which gives the first-order-condition

$$q_{l,t}^{cl} - \frac{w_{l,t}^{ren}}{\alpha_l^h \lambda_{l,t}^{r}} \left(I_{l,t}^{ren}\right)^{\frac{1-\alpha_l^h}{\alpha_l^h}} - \frac{q_{l,t}^{di}}{\lambda_{l,t}^{r}} = 0.$$

Thus, plugging in the equilibrium wage of $w_{l,t}^{ren} = 1$ the optimal level of housing renovating dirty into clean houses is given by

$$I_{l,t}^{ren} = \lambda_{l,t}^{r} \left[\alpha_l^h \left(\lambda_{l,t}^{r} q_{l,t}^{cl} - q_{l,t}^{di}\right)\right]^{\frac{\alpha_l^h}{1-\alpha_l^h}}.$$

Last, the profits from renovations are given by

$$\pi^r = q_{l,t}^{cl} I_{l,t}^{r} - \left(\frac{I_{l,t}^{ren}}{\lambda_{l,t}^{r}}\right)^{\frac{1}{\alpha_l^h}} - q_{l,t}^{di} \frac{I_{l,t}^{ren}}{\lambda_{l,t}^{r}} \quad \text{with} \quad I_{l,t}^{ren} = \lambda_{l,t}^{r} \left[\alpha_l^h \left(\lambda_{l,t}^{r} q_{l,t}^{cl} - q_{l,t}^{di}\right)\right]^{\frac{\alpha_l^h}{1-\alpha_l^h}}.$$

Plugging $I_{l,t}^{r}$ into the profits equations gives

$$
\begin{aligned}
\pi^r &= I^{ren}_{l,t}\left(q^{cl}_{l,t} - \left(I^{ren}_{l,t}\right)^{\frac{1-\alpha^h_l}{\alpha^h_l}}\left(\lambda^r_{l,t}\right)^{-\frac{1}{\alpha^h_l}} - q^{di}_{l,t}\left(\lambda^r_{l,t}\right)^{-1}\right) \\
&= \lambda^r_{l,t}\left[\alpha^h_l\left(\lambda^r_{l,t}q^{cl}_{l,t} - q^{di}_{l,t}\right)\right]^{\frac{\alpha^h_l}{1-\alpha^h_l}}\left(q^{cl}_{l,t} - \left(\left[\lambda^r_{l,t}\left[\alpha^h_l\left(\lambda^r_{l,t}q^{cl}_{l,t} - q^{di}_{l,t}\right)\right]^{\frac{\alpha^h_l}{1-\alpha^h_l}}\right]\right)^{\frac{1-\alpha^h_l}{\alpha^h_l}}\frac{1}{\left(\lambda^r_{l,t}\right)^{\frac{1}{\alpha^h_l}}} - \frac{q^{di}_{l,t}}{\lambda^r_{l,t}}\right) \\
&= \lambda^r_{l,t}\left[\alpha^h_l\left(\lambda^r_{l,t}q^{cl}_{l,t} - q^{di}_{l,t}\right)\right]^{\frac{\alpha^h_l}{1-\alpha^h_l}}\left(q^{cl}_{l,t} - \left(\lambda^r_{l,t}\right)^{\frac{1-\alpha^h_l-1}{\alpha^h_l}}\left(\left[\alpha^h_l\left(\lambda^r_{l,t}q^{cl}_{l,t} - q^{di}_{l,t}\right)^{\frac{\alpha^h_l}{1-\alpha^h_l}}\right]\right)^{\frac{1-\alpha^h_l}{\alpha^h_l}} - \frac{q^{di}_{l,t}}{\lambda^r_{l,t}}\right) \\
&= \left[\alpha^h_l\left(\lambda^r_{l,t}q^{cl}_{l,t} - q^{di}_{l,t}\right)\right]^{\frac{\alpha^h_l}{1-\alpha^h_l}}\left(\lambda^r_{l,t}q^{cl}_{l,t} - \alpha^h_l\left(\lambda^r_{l,t}q^{cl}_{l,t} - q^{di}_{l,t}\right) - q^{di}_{l,t}\right)
\end{aligned}
$$

B.3 Definition of Stationary Equilibrium

to be continued

There are eight markets in the economy. There are four segmented rental markets for each combination of the two different housing stocks and the two regions. The two corresponding rental market clearing conditions for each location l read

$$
I^{cl}_l + I^r_l = h^{cl}_l + h^r_l
$$

$$
I^r_l = h^r_l
$$

Additionally, there is a labor market which needs to satisfy

$$
N^c + N^h_r + N^h_u = 1. \tag{1}
$$

Lastly, market clearing in the final good sector reads

$$
\begin{aligned}
Y = &\int c d\phi^T + \int c d\phi^L + \int \mathbb{1}\{(h' \neq h) \cup (l' = l)\}\psi^h d\phi^T + \int \mathbb{1}\{l' = l\}\psi^r d\phi^T \\
&+ \int \mathbb{1}\{h'^o \neq h^{di}\}\psi^h d\phi^L + \int \mathbb{1}\{\widetilde{h} < h^{di} + h^{cl}\}\psi_m d\phi^T + NX
\end{aligned}
$$

where the first two terms of the right side are expenditures for the non-housing good of tenants and landlords, respectively. The following four terms describe the moving costs within regions, across regions, the adjustment costs for updating dirty housing and the monitoring costs. The last term describes losses/profits of the foreign financial agents who supply the safe asset. Note that I assume that heating energy and car energy are produced abroad and are not part of the final good production.

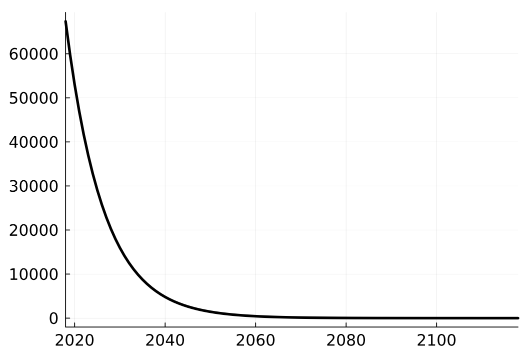
C Additional model results

This section provides more results on the quantitative analysis. First, I will show further details on the calibration and the transitional paths of key variables for rural and urban regions. Thereafter, I will check the robustness of the baseline results by providing an extensive sensitivity analysis.

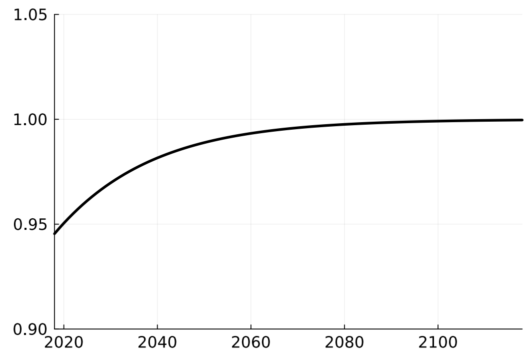
C.1 Additional results for the baseline model

Figure A2: Convergence of productivities and prices of clean technologies

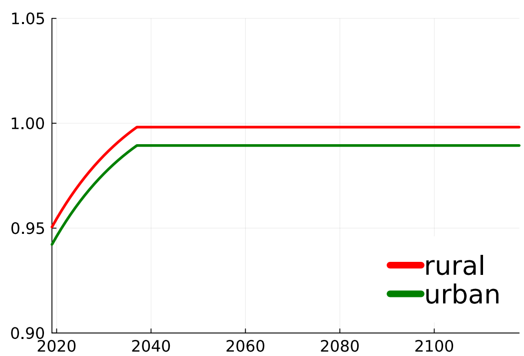
(a) Price premium for clean cars



(b) Productivity discount for clean housing

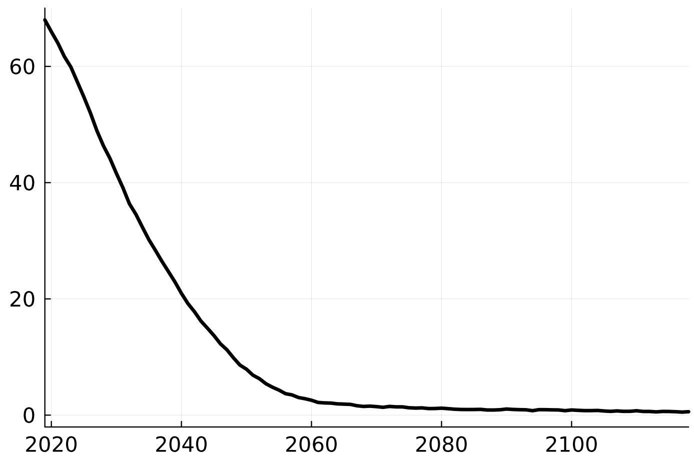


(c) Productivity for housing renovations



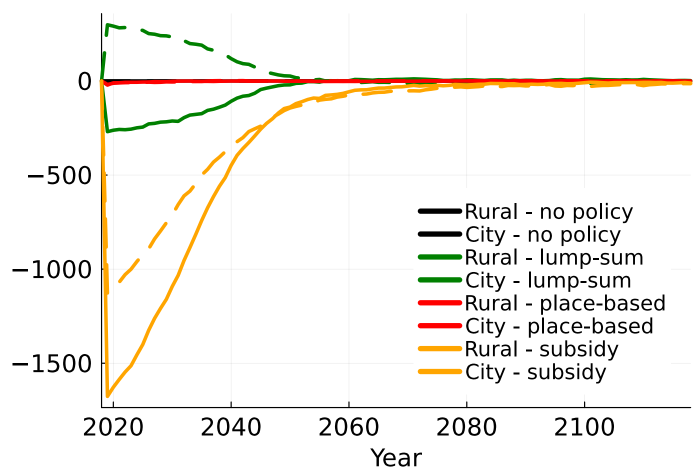
Notes: Panel (a) shows the price premium of clean cars relative to dirty cars in Euros over the transition period. Similarly, Panel (b) shows the productivity of clean housing construction relative to dirty housing construction over the transition period. Lastly, Panel (b) shows the productivity of transforming dirty into clean houses along the transition for both regions.

Figure A3: Subsidy level (%) for carbon tax of 300 Euros per ton



Notes: This figure shows the percentage subsidy on housing renovations for a carbon tax of 300 Euros per ton.

Figure A4: Spatial redistribution along the transition period



Notes: This figure shows the net transfers of rural and urban households for the different policy scenarios along the transition.

C.2 Sensitivity analysis

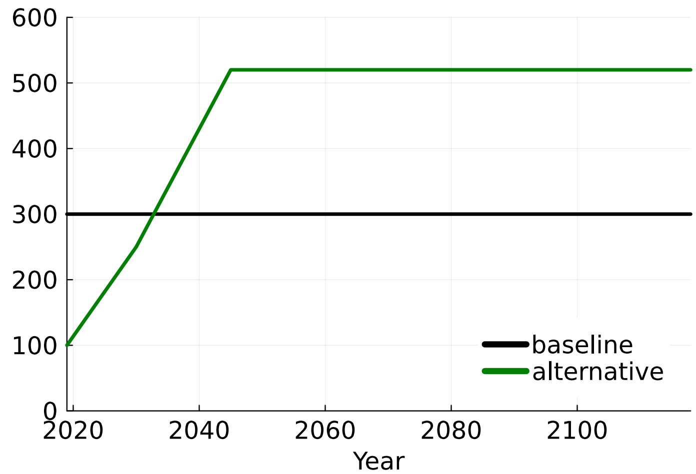
For the policy analysis in the main part, I focus on a constant carbon tax of 300 Euros per ton of carbon emissions. Within the European Emission Trading System (ETS), there is, however, no fixed price per ton of emissions but rather a given number of issued certificates which allow firms to emit carbon emissions. These certificates are traded on the market such that the resulting price per ton of carbon emissions is endogenous and depends on the demand and supply of these certificates. The carbon tax of 300 Euros per ton is well in line with empirical estimates but there is also a high uncertainty about its exact level and path (Kalkuhl et al., 2023). Hence, this subsection tests the robustness of my main results with respect to this type of uncertainty. Further, the main part looks, besides the subsidies on renovations, lump-sum and place based transfers. In this sensitivity analysis, I will look at two additional, very extreme, scenarios. I will either pay all carbon tax revenues to in a lump-sum way to rural or urban households in order to check how sensitive the main results are with respect to these extreme forms of spatial redistribution.

C.2.1 Carbon taxes of 100 Euros per ton of carbon emissions

C.2.2 Carbon taxes of 500 Euros per ton of carbon emissions

C.2.3 Increasing path of carbon emissions

Figure A5: Alternative path of carbon taxes



Notes: This figure shows the alternative, increasing carbon price path and the baseline constant carbon tax path.

C.3 Transfers only to rural households

C.4 Transfers only to urban households