

# **Wage Flexibility and Household Heterogeneity in a Small Open Economy**

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## Abstract

In this paper, I use a small open economy heterogeneous agent New Keynesian (HANK) model to investigate the role of wage flexibility when facing adverse external shocks. Structural reforms that increase labour market flexibility have conventionally been believed to facilitate employment stability. These stability benefits depend on the responsiveness of employment to labour cost adjustments. In a standard New Keynesian model, this responsiveness depends on the endogenous reaction of monetary policy. Galí and Monacelli (2016) find that in a monetary union, changes in labour costs therefore have only minimal effects on employment and that increasing wage flexibility may be welfare reducing in some cases. Recent advances in the HANK literature, however, have shown that the presence of borrowing constrained heterogeneous households can alter relevant transmission dynamics. I examine whether these altered dynamics affect the results of Galí and Monacelli (2016) and repeat their analysis using a heterogeneous agent version of their model. To determine the responsiveness of employment to a labour cost reduction, I first simulate a temporary payroll tax cut and compare responses for a Taylor rule and a monetary union. To determine the role of wage flexibility in accommodating adverse shocks, I then simulate both a foreign interest rate hike and a foreign demand drop at varying levels of wage flexibility and assess respective welfare losses.

The simulation of a payroll tax cut reveals that employment is responsive to labour cost reductions under both monetary regimes. The response is driven primarily by the effect of higher income on consumption and the mutual reinforcement of consumption and output, i.e., the typical multiplier effect in HANK models. These effects are independent of the endogenous response of monetary policy, breaking with traditional New Keynesian logic. Poorer households are affected disproportionately more, leading to a decrease in consumption inequality in the short term. Over time, however, assets of wealthier households grow more strongly and raise wealth inequality. The welfare analysis for adverse shocks yields three conclusions: (i) Wage flexibility does increase the responsiveness of employment in the face of adverse foreign shocks. (ii) In a monetary union, wage flexibility results in substantial relative welfare losses, especially for poorer households, driven by income and multiplier effects. For a Taylor rule, welfare effects are smaller and co-determined by the monetary policy response. (iii) Under both monetary regimes, a one for one increase of price and wage flexibility has only minimal welfare effects, while a decrease of flexibility results in relative gains for an interest rate shock and relative losses for a foreign demand shock. Together these findings cast new light on the transmission of labour cost adjustments in small open economies as well as on potential risks that structural labour market reforms may entail for aggregate stability and household welfare.

# 1 Introduction

In this paper, I investigate the role of wage flexibility when facing adverse external shocks in the context of a small open economy heterogeneous agent New Keynesian model. In the wake of recent interest rate hikes, countries in the European periphery are experiencing a widening of sovereign bond yield spreads, putting under strain their borrowing ability. The risk of financial instability brings back memories of the sovereign debt crisis and with it discussions about policies to strengthen the economic resilience of EU member states. Structural reforms aimed at making labour markets more flexible have long been recommended as an avenue of intervention, especially when facing asymmetric shocks (De Grauwe, 2020). In addition to providing potential gains in competitiveness, policies that increase wage flexibility, are seen as facilitators of economic stability: By making labour costs more sensitive to cyclical conditions, wages can adjust downwards more quickly. At lower wage levels, employers can afford to hire more workers, thus (partially) offsetting drops in employment and output.

The macroeconomic intuition behind the response of employment to changes in labour costs, however, hinges on modelling assumptions. While in classical business cycle models, labour demand is affected by wages directly, it is determined by the level of production and thus by aggregate demand in a New Keynesian model with nominal rigidities. Here, lower wages affect employment only via the endogenous response of monetary policy to decreasing inflation, since interest rate cuts incentivize consumption and raise aggregate demand. Galí and Monacelli (2016) follow this logic and show that countries without monetary sovereignty do not reap stability benefits from labour cost reductions. They find that employment in a New Keynesian model of a country with pegged interest rate barely reacts to the introduction of a payroll tax cut. After analysing welfare losses at varying levels of wage rigidity, they conclude that wage flexibility may not be particularly desirable in a monetary union.

Recent advances in the development of heterogeneous-agent New Keynesian or HANK (an abbreviation proposed by Kaplan et al. (2018)) models cast doubt on these findings. The HANK literature compliments New Keynesian models featuring monopolistic competition and nominal rigidities with households that face idiosyncratic income risk and borrowing constraints. Investigations of the transmission of monetary and fiscal policy in HANK models have found that output and employment responses are driven to a large part by direct effects on household income as well as multiplier effects, i.e., the mutual reinforcement of consumption and output (see, e.g., Kaplan et al., 2018; Auclert et al., 2018; Hagedorn et al., 2019). Since income and multiplier effects in a HANK model can

be triggered by changes in real wages, labour cost reductions of the kind assumed by Galí and Monacelli (2016) could elicit employment reactions, irrespective of the endogenous response of monetary policy. Furthermore, the link between output and consumption may hold additional implications for the role of wage flexibility, since a swifter reduction in wage income could reduce aggregate demand and amplify output contractions and welfare losses in response to adverse shocks.

To test these hypotheses, I repeat the analysis of Galí and Monacelli (2016) by employing a heterogeneous agent extension to their small open economy New Keynesian model, following a similar model introduced by Auclert et al. (2021b). I calibrate the model to match salient features of a typical southern EU member state.<sup>2</sup> The analysis consists of two parts: (i) First, I simulate a temporary payroll tax cut and compare the reactions of an economy under a standard Taylor rule and a monetary union. (ii) I then simulate adverse shocks to the foreign interest rate and to foreign demand at varying levels of wage rigidity and compare respective model responses and losses in household welfare.

The results of the first part of the analysis show that reductions in labour costs in the form of a payroll tax cut have significant impact on output and employment under both monetary regimes. A decomposition of the consumption response suggests that in both cases income and multiplier effects are the primary drivers of these adjustments. In addition, these effects disproportionately affect poorer households, leading to a decrease in consumption inequality in the short term. Over time, however, a stronger increase in assets among wealthier households increases wealth inequality. The results of the second part of the analysis lead to three additional conclusions. (i) Wage flexibility does increase the responsiveness of employment in the face of adverse foreign shocks. (ii) In a monetary union, wage flexibility results in substantial relative welfare losses, especially for poorer households, driven by income and multiplier effects. For a Taylor rule, welfare effects are smaller and co-determined by the monetary policy response. Here, reducing flexibility results in relative losses for the foreign interest rate shock and relative gains for the foreign demand shock. (iii) Under both monetary regimes, a one for one increase of price and wage flexibility has only minimal welfare effects, while a decrease of flexibility results in relative gains for an interest rate shock and relative losses for a foreign demand shock. Together these findings cast new light on the transmission of labour cost adjustments in small open economies as well as on potential risks that structural labour market reforms may entail for aggregate stability and household welfare.

The rest of the paper is structured as follows. Section 2 reviews related work on wage flex-

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<sup>2</sup>Following Galí and Monacelli (2016), the model matches average characteristics of Greece, Italy, Portugal, and Spain.

ibility in open economy New Keynesian models and introduces recent contribution to the open economy HANK literature. Section 3 presents the setup of the small open economy HANK model. Section 4 introduces the numerical methods used to solve the model. Section 5 presents the model calibration. Section 6 reviews transmission dynamics in HANK models and presents the analysis of the response to a temporary payroll tax cut. Section 7 presents the response of the model to a foreign interest rate and a foreign demand shock as well as the analysis of relative welfare losses. Section 8 discusses implications and potential limitations of the prior sections. Section 9 concludes the analysis.

## 2 Related Literature

This thesis is related to two strands of literature within the New Keynesian tradition. Following the European debt crisis, several studies have employed small open economy models to study potential policy responses (see, e.g., Farhi et al., 2014, on fiscal devaluations). Among these works are analyses of the role of labour market rigidities in small open economies (Galí and Monacelli, 2016; Okano, 2020; Bustos and Uras, 2021), which this thesis builds on. Additionally, an emerging literature has started to introduce heterogeneous agent extensions to the "New open economy macroeconomics" framework. So far, these studies have explored aggregate and distributional responses to foreign and domestic monetary policy shocks (Guo et al., 2020; Auclert et al., 2021b; Otten, 2021; Oskolkov, 2021), foreign credit supply shocks (De Ferra et al., 2020), fiscal devaluations (Giagheddu, 2020), and deficit-financed fiscal transfers (Aggarwal et al., 2022).

### 2.1 Wage Flexibility in Open Economy New Keynesian Models

The inclusion of wage rigidities has become a staple in many applications of New Keynesian models. Prominent examples of such applications in a closed economy context include, Blanchard and Galí (2007), who show that in the presence of such rigidities central banks face a trade-off between stabilizing the welfare-relevant output gap and stabilizing inflation. Galí (2013) study the impact of price flexibility on output and employment and find that higher degrees of flexibility may be conducive of volatility and welfare losses if monetary policy does not react strongly to inflation. Similar findings have been reported for an economy at the zero lower bound (Bhattarai et al., 2018; Billi and Galí, 2020). As for the open economy, a long standing consensus view, starting with Friedman (1953), has been that frictionless price adjustment is desirable and, in the presence of nominal rigidities, can be accommodated by flexible exchange rates. Extensions of the New Keynesian



framework to the small open economy case, however, have challenged this view.<sup>3</sup>

Galí and Monacelli (2016) argue that the conventional wisdom that less flexible labour markets hinder the adjustment of wages necessary for output and employment stability disregards the role of monetary policy in New Keynesian style economies. Echoing some of the closed economy results in Galí (2013), they stress the centrality of the "endogenous policy channel" in small open economies with nominal rigidities: Wages affect employment through their sequential impact on marginal costs and inflation and thus, via the ensuing reaction of monetary policy, the response of aggregate demand. To investigate this channel, the authors analyse the effect of a payroll tax cut under two monetary regimes, inflation targeting and a monetary union. The findings show that under inflation targeting, a payroll cut causes declines in interest rates, boosting output and employment. In a monetary union, however, the observed responses are minimal. Galí and Monacelli (2016) conclude that policies aimed at making employment more responsive to cyclical adjustments by increasing the flexibility of wages are likely ineffective in a monetary union. They proceed to evaluate relative welfare losses associated with more flexible wages in the context of domestic and foreign shocks. For a foreign interest rate shock, the relationship between flexibility and welfare losses in a monetary policy follows a hump shape, increasing relative losses at low levels of rigidity and decreasing losses at higher levels of rigidity. For a foreign export shock, however, more flexibility almost always reduces welfare losses. Under a Taylor rule, the observed relationship is overall stronger and wage flexibility always reduces relative losses.

Two articles have since revisited the results of Galí and Monacelli (2016). (i) First, Okano (2020) expands the model by introducing a government budget constraint with endogenous fiscal policy. The government now refinances debt with a pro-cyclical output tax. Like Galí and Monacelli (2016), Okano (2020) finds that the effect of labour cost adjustments on employment is much smaller under a currency peg. In addition, the presence of the fiscal rule makes employment more elastic with respect to domestic price inflation, thus decreasing relative welfare losses when wage become more flexible. Here, households only suffer relative losses associated with more flexible wages when facing a domestic demand shock. (ii) Second, Bustos and Uras (2021) extend the model of Galí and Monacelli (2016) by adding a sector in which prices are always perfectly flexible. However, they only consider a domestic demand shock, for which the hump-shaped relation between rigidity and welfare loss remains.

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<sup>3</sup>Schmitt-Grohé and Uribe (2016), who investigate output stabilization under downward wage rigidity, have a different focus but should be mentioned given the thematic overlap.

## 2.2 Open Economy HANK Models

The rapidly growing HANK literature combines Aiyagari-Bewley-Hugget-Imrohoroglu style households (Bewley, 1976; Imrohoroglu, 1989; Huggett, 1993; Aiyagari, 1994), i.e., a continuum of households facing idiosyncratic risk and borrowing constraints, with general equilibrium models featuring monopolistic firms and nominal rigidities. This combination has given rise to salient features like a higher marginal propensity to consume (MPC) and precautionary saving induced by pro-cyclical income risks (Acharya and Dogra, 2020). Pioneering work like Werning (2015), Gornemann et al. (2016), McKay et al. (2016), or Kaplan et al. (2018), as well as the authors that followed, has since employed HANK models to explore various aspects of monetary and fiscal policy. More recently, authors have expanded the HANK framework to explore foreign shocks by incorporating heterogeneous households into small open economy New Keynesian models like Galí and Monacelli (2005).

The contribution to this literature that is most relevant for my analysis was made by Auclert et al. (2021b), who investigate the transmission and aggregate effects of monetary policy and exchange rate shocks in a small open economy. Similar to Auclert et al. (2018), they leverage the sequence space representation of the model to recover households' intertemporal MPCs (iMPCs), i.e., the response of consumption to a change in real income at a given date. The authors use these iMPCs to decompose the output response into real income, expenditure switching, multiplier, and interest rate effects. The practical decomposition of shock responses is discussed in section 6.1 and replicated in Appendix C.1. As will be seen later, the distinct channels play a central role in the analysis of wage flexibility since they fundamentally alter the transmission of labour cost reductions. Auclert et al. (2018) find that while expenditure switching, the mechanism by which consumers substitute towards home or foreign goods following a change in relative prices, is present in representative-agent models, the real income effect, i.e., changes in the purchasing power of households, and the multiplier effect, i.e., the mutual reinforcement of consumption and output, are only found in the heterogeneous agent case. Calibrating their model to represent features of a typical Latin American economy, Auclert et al. (2021b) show that the relative strengths of the multiplier and the real income effect depend on the openness and the trade elasticities of an economy. They establish benchmark values for these parameters, under which the responses to a monetary policy and to an exchange rate shock in the representative agent model and in the HANK model are equivalent. In light of the central role of monetary policy in the transmission of labour cost adjustments observed by Galí and Monacelli (2016), these equivalence results are relevant for the comparison of the results in section 6 with the standard New Keynesian case.

Additional contributions to the open economy HANK literature have focused on distributional impacts of monetary policy, fiscal policy, or credit supply shocks. Guo et al. (2020) investigate the distributional aspects of foreign and domestic monetary policy shocks in an economy where households differ in their integration with international goods and financial markets. De Ferra et al. (2020) investigate distributional effects of a sustained expansion and sudden reversal of the current account when households hold debt denominated in foreign currency. Similarly, Otten (2021) investigates the role of heterogeneity in the transmission of interest rate shocks when households invest in foreign bonds. Oskolkov (2021) employs a model with international goods but exclusively domestic financial markets to explore distributional effects of foreign interest rate shocks. Giagheddu (2020) investigates aggregate and disaggregate effects of a fiscal devaluation in peripheral euro area economies. Lastly, Aggarwal et al. (2022) explore consequences of deficit financed fiscal transfers. Although thematically related, the focus of most of these investigation lies on optimal policy design rather than the role of transmission dynamics or wage flexibility. While not directly related to the focus of my analysis, relevant results of these works are taken into account in the discussion of implications and limitations in section 8.

### 3 Model

This section describes the key elements of the open economy HANK model used in the analyses of sections 6 and 7. Like the representative agent model used by Galí and Monacelli (2016) to analyse the effect of wage flexibility in a small open economy, the model features monopolistic competition, as well as staggered wage and price setting. The addition of HANK features largely follows the model introduced by Auclert et al. (2021b). I further extend the baseline model of Auclert et al. (2021b) by introducing nominal price rigidities, a foreign demand shifter, and two alternative monetary policy regimes, a standard Taylor rule and a monetary union. The government is effectively absent in that it does not spend or tax households and keeps domestic bonds at zero net supply. The domestic economy is part of a world economy consisting of a continuum of foreign countries. Variables corresponding to the world economy are denoted with a star superscript. Given the negligible size of the home economy, aggregate foreign consumption and interest rates are effectively exogenous. Capital is absent from the model and financial markets are not subject to frictions. Furthermore, imports are assumed to be denominated in the currency of the producing country and there is perfect pass-through of exchange rates into the price of domestic goods, i.e., the law of one price holds.

### 3.1 Domestic Households

The model is set in discrete time and runs from  $t = 0$  to  $\infty$ . A continuum of households inhabits the home economy. While aggregate risk is absent, each household faces idiosyncratic uncertainty in the form of ability states  $e_{it}$ . These idiosyncratic ability states follow a first order Markov process with the fixed transition matrix  $\Pi$ . The mass of households in an given state  $e_{it}$  is equal to the probability  $\pi(e_{it})$ , where the average level of ability is normalized, so that  $\sum_{it} \pi(e) e = 1$ . Idiosyncratic ability affects real household income  $z_{it}$ :

$$z_{it} \equiv \frac{W_t}{P_t} e_{it} N_t \quad (1)$$

In each period  $t$ , households decide on the optimal allocation of income between saving and consumption to maximize their expected utility. Consumption is split between "foreign" goods  $F$ , which are produced abroad and imported, and domestically produced "home" goods  $H$ , which can be exported. Saved assets  $a$  are invested into a domestic mutual fund and yield returns which do not depend on ability states. The behavior of a household with ability state  $e$  and asset level  $a$  at time  $t$  is characterized by the dynamic programming problem expressed by the following Bellman equation:

$$\begin{aligned} V_t(a, e) &= \max_{c_F, c_H, a'} u(c_F, c_H) - v(N_t) + \beta E_t [V_{t+1}(a', e')] \\ \text{s.t.} \quad &\frac{P_{Ft}}{P_t} c_F + \frac{P_{Ht}}{P_t} c_H + a' = (1 + r_t^p) a + e \frac{W_t}{P_t} N_t \\ &a' \geq 0 \end{aligned} \quad (2)$$

Here,  $P_{Ft}$  refers to the nominal price of foreign goods in units of domestic currency,  $P_{Ht}$  refers to the price of domestic goods,  $P_t$  refers to the level of the consumer price index (CPI),  $r_t^p$  denotes the ex-post returns from the mutual fund in units of the CPI,  $W_t$  is the nominal wage,  $N_t$  refer to the labour supply, which is common across households and determined by union demand specified below, and  $a' \geq 0$  is the zero borrowing constraint faced by households. Utility from the consumption of foreign and domestic products as well as the disutility from labour is characterized across households according to

$$u(c_F, c_H) = \frac{c^{1-\sigma}}{1-\sigma}, \quad v(N) = \psi \frac{N^{1+\varphi}}{1+\varphi} \quad (3)$$

where the parameter  $\sigma > 0$  is the inverse elasticity of intertemporal substitution,  $\varphi > 0$  refers to the inverse Frisch elasticity of labour supply, and  $\psi$  is a scaling parameter, relevant in the characterization of the steady state in Appendix A.6.  $c$  is the consumption

basket of foreign and domestic products given by

$$c = \left[ \alpha^{1/\eta} c_F^{(\eta-1)/\eta} + (1-\alpha)^{1/\eta} c_H^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} \quad (4)$$

Here,  $\alpha$  is the measure of openness of the economy, i.e., it specifies a preference for the consumption of foreign goods  $c_F$ .  $\eta > 0$  denotes the elasticity of substitution between home and foreign goods. Both the consumption basket and the CPI are the same across households. The latter is defined as follows:

$$P_t \equiv \left[ \alpha P_{Ft}^{1-\eta} + (1-\alpha) P_{Ht}^{1-\eta} \right]^{1/(1-\eta)} \quad (5)$$

Minimizing total household expenditure subject to the consumption basket yields standard results for the domestic demand schedules. The derivation of these results can be found in Appendix A.1. A household with ability state  $e$  and assets  $a$  splits its consumption  $c_t(a, e)$  between foreign and home goods according to

$$c_{Ft}(a, e) = \alpha \left( \frac{P_{Ft}}{P_t} \right)^{-\eta} c_t(a, e) \quad (6)$$

$$c_{Ht}(a, e) = (1-\alpha) \left( \frac{P_{Ht}}{P_t} \right)^{-\eta} c_t(a, e) \quad (7)$$

Foreign monetary policy, specified below, holds the price of foreign goods denoted in foreign currency, constant at  $P_{Ft}^* = P_t^* = 1$ . Given the assumptions that imports are denominated in foreign currency and that exchange rates pass-through into domestic prices perfectly, the price of foreign goods in domestic currency is equal to the nominal exchange rate,  $P_{Ft} = \mathcal{E}_t$ . The real exchange rate can then be expressed as

$$Q_t \equiv \frac{\mathcal{E}_t}{P_t} \quad (8)$$

### 3.2 Foreign Agents

The rest of the world economy consists of a unit mass of symmetric foreign countries, each inhabited by a representative agent. Because the domestic economy is infinitesimal, foreign aggregates do not have to be treated as endogenous. Below, I derive the foreign shocks used to shift the foreign interest rate and foreign export demand. The representative household in each country seeks to maximize its expected utility:

$$\max E_0 \sum_{t=0}^{\infty} (\beta^*)^t \mathcal{B}_t \{u(C_t^*) - v(N_t^*)\} \quad (9)$$

Here, utility from consumption and disutility from labour are the same as in equation 3 and  $\beta^*$  is the foreign discount factor. Following Auclert et al. (2021b),  $\mathcal{B}_t$  is introduced as a utility modifier capturing time-varying patience. Its initial value is  $\mathcal{B}_{-1} = 1$ . The utility modifier is nonnegative, bounded, and reverts to a value of 1 in the long run, so that  $\mathcal{B}_t \in (0, \bar{\mathcal{B}})$  for  $\bar{\mathcal{B}} > 0$  and  $\lim_{t \rightarrow \infty} \mathcal{B}_t = 1$ . The consumption basket of the aggregate foreign economy is the complement to the one in the domestic economy, with the openness parameter  $\alpha$  now denoting preference for goods produced in the domestic economy. In the absence of household heterogeneity, the export demand for home goods, derived analogously to equations (7) and (6) in Appendix A.1, is given by

$$C_{Ht}^* = \alpha \left( \frac{P_{Ht}^*}{P_t^*} \right)^{-\gamma} C_t^* \mathcal{E}_t \quad (10)$$

Here, the only new feature is the introduction of  $\mathcal{E}_t$ , representing a foreign demand shifter that is discussed below.  $\gamma > 0$  is the elasticity of substitution across countries, and  $P_{Ht}^*$  denotes the foreign-currency price for home goods. Assuming the law of one price holds for foreign products, the latter is equal to the cost of the home good in foreign currency units:

$$P_{Ht}^* = \frac{P_{Ht}}{\mathcal{E}_t} \quad (11)$$

Foreign countries each produce their own final good by aggregating inputs from monopolistic intermediate firms under constant elasticity of substitution (CES). Monopolists feature constant returns to scale and a linear technology, so that  $Y_t^* = Z^* N_t^*$ . Given flexible prices and wages, foreign prices are a constant markup over wages  $P_t^* = \mu W^*$ . The foreign household's first order condition then implies:

$$\frac{v'(C^*/Z^*)}{u'(C^*)} = \frac{W_t^*}{P_t^*} = \frac{1}{\mu} \quad (12)$$

That is, the level of foreign consumption as well as, given the market clearing condition in the world economy  $C_t^* = Y_t^*$ , the level of foreign output are constant. Foreign monetary policy targets a constant foreign price index  $P^*$ , setting the nominal foreign interest rate according to a price-level targeting rule with the foreign real rate  $r_t^*$  as intercept:

$$i_t^* = r_t^* + \phi \log \left( \frac{P_t^*}{P^*} \right) \quad (13)$$

In equilibrium, the foreign nominal and real interest rate are thus equal and, given the constant level of consumption, directly related to the foreign discount factor and the utility modifier  $\mathcal{B}_t$  via the Euler equation:

$$1 + i_t^* = 1 + r_t^* = \frac{1}{\beta^*} \frac{\mathcal{B}_t}{\mathcal{B}_{t+1}} \quad (14)$$

Shocks to the foreign utility modifier  $\mathcal{B}_t$  thus directly affect the foreign nominal and real interest rate  $i^*$ , while exports are affected only via the endogenous response of the terms of trade. I therefore refer to  $\mathcal{B}_t$  as a foreign interest rate shifter from here on. The foreign demand shifter  $\mathcal{E}_t$ , on the other hand, leaves interest rates unchanged and only shifts the level of global consumption. This distinction follows Galí and Monacelli (2016) and serves the isolation of the effect of the respective shock on the domestic economy. Constructing such isolated shocks is undoubtedly a strong assumption since the global level of consumption and the world interest rate are themselves endogenous variables and likely correlated. The two shocks are assumed to follow independent, exogenous  $AR(1)$  processes

$$\mathcal{B}_t = \rho_{\mathcal{B}} \mathcal{B}_{t-1} + \varepsilon_t^{\mathcal{B}} \quad (15)$$

$$\mathcal{E}_t = \rho_{\mathcal{E}} \mathcal{E}_{t-1} + \varepsilon_t^{\mathcal{E}} \quad (16)$$

where, for  $i \in [\mathcal{B}, \mathcal{E}]$ ,  $\rho_i$  represents the persistence of shocks and  $\varepsilon^i$  represents white noise error terms.

### 3.3 Financial Sector

Following Auclert et al. (2021b), all investment activities are conducted by a financial intermediary which issues claims to household and invests in domestic and foreign stocks and bonds. Both domestically and across international borders, capital flows are frictionless. As shown in Appendix A.2, at the beginning of each period  $t$ , the real value of outstanding household claims must be equal to the liquidation value of the fund portfolio, that is:

$$(1 + r_t^p) A_{t-1} = (D_t + p_t) s_{t-1}^H + (1 + i_{t-1}) \frac{P_{t-1}}{P_t} \frac{B_{t-1}^H}{P_{t-1}} + (D_t^* + p_t^*) Q_t s_{t-1}^F + (1 + i_{t-1}^*) Q_t \frac{B_{t-1}^F}{P^*} \quad (17)$$

Here,  $s_{t-1}^H$  and  $s_{t-1}^F$  denote the shares of domestic and foreign firms respectively, while  $D_t$  and  $D_t^*$  denote real dividends issued by them.  $i_{t-1}$  is the nominal interest rate for domestic bonds  $B_{t-1}^H$  and  $i_{t-1}^*$  is the nominal interest rate for foreign bonds  $B_{t-1}^F$ . The end of period value of all newly purchased bonds and shares must be equal to the value of newly issued liabilities:

$$A_t = p_t s_t^H + \frac{B_t^H}{P_t} + p_t^* Q_t s_t^F + Q_t \frac{B_t^F}{P^*} \quad (18)$$

The problem of the financial intermediary is to choose the size of its investment in the four asset types to maximize the expected real rate of return on its liabilities  $r_{t+r}^p$  for all  $t \geq 0$ . Optimally then implies that the returns on all asset types are equal in equilibrium and given by:

$$1 + r_{t+1}^p = 1 + r_t = \frac{D_{t+1} + p_{t+1}}{p_t} = \frac{(D_{t+1}^* + p_{t+1}^*) Q_{t+1}}{p_t^* Q_t} = \frac{(1 + i_t^*) Q_{t+1}}{Q_t} \quad (19)$$

Where the ex-ante real interest rate is defined as

$$1 + r_t \equiv (1 + i_t) \frac{P_t}{P_{t+1}} \quad (20)$$

The ex-post return on household claims  $r_0^p$  in period 0 is indeterminate, since it depends on the initial asset allocation of the mutual fund. To resolve this indeterminacy, I follow Auclert et al. (2021b) in assuming that at the beginning of period 0, the financial intermediary is entirely invested in domestic firm shares. The nominal version of the standard uncovered interest parity (UIP) condition follows from the equality of expected nominal returns (59):

$$1 + i_t = (1 + i_t^*) \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \quad (21)$$

The real version of the UIP condition, in turn, follows from the equality of expected real returns in (19):



$$1 + r_t = (1 + i_t^*) \frac{Q_{t+1}}{Q_t} \quad (22)$$

Lastly, net foreign assets are defined as the difference between the value of domestically accumulated assets,  $A_t$ , and the total value of assets that are in net supply domestically. Assuming that domestic bonds are in zero net supply, the foreign asset position can be written as

$$\text{nfa}_t \equiv A_t - p_t \quad (23)$$

### 3.4 Domestic Firms

The specification of firms follows Auclert et al. (2020) and features a final good producer as well as monopolistic intermediate good firms and Calvo pricing. Unlike the model in Auclert et al. (2020), however, intermediate good production does not feature capital and the homogeneous output good  $Y_t$  is produced by a final good firm following the CES production function

$$Y_t = \left( \int_0^1 Y_{jt}^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \quad (24)$$

where  $Y_{jt}$  is the intermediate good produced by firm  $j$  in period  $t$ .  $\varepsilon_p > 1$  denotes the elasticity of substitution between intermediate products. Intermediate goods producers operate under constant returns to labour according to

$$Y_{jt} = N_{jt} Z \quad (25)$$

where  $Z$  is the constant level of technology, common to all intermediate good producers.  $N_{jt}$  is the equal amount of labour employed by each firm, summing to aggregate labour  $N_t$ . Therefore, all firms face the same nominal marginal costs  $s_t$ :

$$s_t = \frac{W_t (1 + \tau_t)}{Z P_{Ht}} \quad (26)$$

As in Galí and Monacelli (2016),  $\tau_t$  is a proportional payroll tax. Like the previously introduced shocks, it follows an exogenous  $AR(1)$  process with persistence  $\rho_\tau$  and the white noise error term  $\varepsilon^\tau$ . The tax is used as an external device to model the temporary payroll tax cut in section 6. Importantly, tax cuts do not have to be repaid at a later point.

Prices of intermediate producers are sticky à la Calvo. Each period, firms can reset the price of intermediate goods  $P_{Hjt}$  with a probability of  $1 - \theta_p$ . Conditional on resetting their price, firms face stock prices  $p_{jt}(P_{Hjt})$  from the mutual fund and choose their reset price so as to maximize their respective firm value, given by the sum of stock prices  $p_{jt}(P_{Hjt})$  and nominal dividends. Nominal dividends are given by

$$D_{jt} = \left( \frac{P_{Hjt}}{P_{Ht}} - s_t \right) Y_{jt} \quad (27)$$

Given the common production function 25, aggregate dividends in terms of the CPI 5 can be expressed as:

$$D_t = \frac{\int D_{jt} dj}{P_t} = \frac{P_{Ht} Y_t - W_t N_t}{P_t} \quad (28)$$

Appendix A.3 shows that the optimal price setting of firms delivers the New Keynesian Phillips Curve

$$\pi_{Ht} = \kappa_H (\mu s_t - 1) + \frac{1}{1+r} E_t [\pi_{H,t+1}] \quad (29)$$

where inflation in domestic prices  $\pi_{Ht} \equiv \log \left( \frac{P_{Ht}}{P_{Ht-1}} \right)$ ,  $\kappa_H = (1 - \theta_H) \left( 1 - \frac{1}{1+r} \theta_H \right) / \theta_H$ , the markup  $\mu = \frac{\varepsilon_p - 1}{\varepsilon_p}$ , and the discount factor  $r$ , which, following (19), is given by the ex-ante real return on assets.

### 3.5 Unions

Following prior work in the New Keynesian sticky-wage literature (Erceg et al., 2000; Schmitt-Grohé and Uribe, 2005; Christiano et al., 2005; Auclert et al., 2018; Auclert et al., 2020), domestic wage setting, is subject to nominal rigidities emerging from the union problem. The introduction of such rigidities not only accommodates the investigation of the effect of wage flexibility but has been shown to be a crucial addition to nominal price rigidities, especially in the HANK context (see, e.g., Auclert et al., 2021a; Broer et al., 2020). Labour hours supplied by agents in the domestic economy are determined by the demand of unions. Each worker  $i$  supplies  $n_{ikt}$  units of labour to each of the unions in the continuum of  $k$  unions with unit mass. The total labour effort for each household is then given by  $n_{it} \equiv \int n_{ikt} dk$ . Each union aggregates the efficient units of labour into a union-specific task  $N_{kt} = \int e_{it} n_{ikt} di$ . Finally, the individual tasks are packaged into aggregate employment by a competitive labour packer according to the CES function

$$N_t = \left( \int_k N_{kt}^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dk \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}} \quad (30)$$

and sold to the final good firm at price  $W_t$ . In each period  $t$ , unions can reset the wages they demand with a Calvo probability of  $1 - \theta_w$ . Here, the parameter  $\theta_w$  represents the fraction of nominal wages that remain unchanged in each period  $t$  and can thus be seen as an index of nominal wage rigidity. Variations in  $\theta_w$  are used to analyse the effects of changing wage flexibility in section 7. Union wages  $W_{kt}$  are set to maximize the average utility of union members. Appendix A.4 shows that the union problem leads to the New Keynesian wage Phillips curve

$$\pi_{wt} = \kappa_w \left( \frac{v'(N_t)}{\frac{1}{\mu_w} \frac{W_t}{P_t} u'(C_t)} - 1 \right) + \beta \pi_{wt+1} \quad (31)$$

Here, wage inflation  $\pi_{wt} \equiv \log \left( \frac{W_t}{W_{t-1}} \right)$ ,  $\kappa_w = \frac{(1 - \beta \theta_w)(1 - \theta_w)}{\theta_w}$ , and wage markup  $\mu_w = \frac{\varepsilon_w - 1}{\varepsilon_w}$ . Contrary to the approach in Auclert et al. (2021b), the consumption level entering the wage Phillips curve takes into account inequality in labour earnings, with  $u'(C_t) = \int e_{it} u'(c_{it}) d\mathcal{D}_{it}$ .

### 3.6 Monetary Regimes

Following Galí and Monacelli (2016), two monetary regimes are considered, a domestic-inflation-based Taylor rule and the case of a monetary union. Under the Taylor rule, the monetary authority aims to stabilize inflation in domestic prices,  $\pi_{Ht}$ , by setting the nominal interest rate according to

$$i_t = r_{ss} + \phi_H \pi_{Ht} \quad (32)$$

with the Taylor parameter  $\phi_H = 1.5$ . Lastly, monetary policy in a monetary union is modelled as a credible and indefinite peg to the world currency,  $i = i^*$ , which, given the UIP condition (21) implies

$$\mathcal{E}_t = 1 \quad (33)$$

for all  $t$ .

### 3.7 Equilibrium Definition

Given the exogenous sequences for the foreign interest rate shifter  $\{\mathcal{B}\}$ , the foreign export demand shifter  $\{\mathcal{E}\}$ , and the payroll tax  $\tau$ , as well as an initial wealth distribution  $\mathcal{D}_0(a, e)$  and an initial portfolio allocation for the mutual fund, a competitive equilibrium is a path of policies  $\{c_{Ht}(a, e), c_{Ft}(a, e), c_t(a, e), a_{t+1}(a, e)\}$  for households, distributions  $\mathcal{D}_t(a, e)$ , prices  $\{\mathcal{E}_t, Q_t, P_t, P_{Ht}, P_{Ft}, W_t, p_t, i_t, r_t, r_t^p\}$ , and aggregate quantities  $\{C_t, C_{Ht}, C_{Ft}, Y_t, A_t, D_t, nfa_t\}$ , such that:

1. The real uncovered interest rate parity condition (22) holds.
2. The relative prices  $\frac{P_{Ht}}{P_t}$  and  $\frac{P_{Ht}}{P_t^*}$  are consistent with the real exchange rate  $Q_t$  and the pricing equation for foreign goods  $P_{Ft} = \mathcal{E}_t$  holds.
3. Household choices are optimal given each household's real income (1) and their aggregates are given by  $\{C_t, A_t\}$ .
4. Domestic price and wage inflation satisfy (29) and (31).
5. The nominal interest rate  $i_t$  is consistent with the domestic monetary policy regime, that is, one of (32), or (33).
6. The domestic goods market clears:

$$C_{Ht} + C_{Ht}^* = Y_t \quad (34)$$

More,  $C_t \equiv \sum_e \pi_e \int c_t(a, e) \mathcal{D}_t(a, e)$  denotes aggregate consumption by domestic households,  $C_{Ht} \equiv \sum_e \pi_e \int c_{Ht}(a, e) \mathcal{D}_t(a, e)$  denotes their aggregate consumption of home goods,  $C_{Ft} \equiv \sum_e \pi_e \int c_{Ft}(a, e) \mathcal{D}_t(a, e)$  denotes their consumption of foreign goods, and  $A_t$  are aggregate assets. Only those equilibria are considered, in which the long-run real exchange rate returns to its steady state level,  $Q_\infty = Q_{ss}$ . In such an equilibrium, the value of net exports of the economy is given by

$$NX_t \equiv C_t - \frac{P_{Ht}}{P_t} Y_t \quad (35)$$

which, as derived in Appendix A.5, is related in equilibrium to the level of net foreign assets 23 via the current account identity:

$$nfa_t - nfa_{t-1} = NX_t + r_{t-1} nfa_{t-1} \quad (36)$$

### 3.8 Steady State

The steady state considered for the analysis features a constant level of prices and wages, that is,  $\pi$ ,  $\pi_H$ , and  $\pi_w$  are equal to 0. As in Galí and Monacelli (2016) and Auclert et al. (2021b), prices  $P_{ss}$ ,  $P_{Hss}$ ,  $P_{Fss}$ ,  $P_{Hss}^*$ ,  $\mathcal{C}_{ss}$ , and  $Q_{ss}$  are normalized to 1, as are productivity  $Z_{ss}$  and domestic and foreign consumption,  $C_{ss}$  and  $C^*$ . This implies that also  $Y_{ss} = 1$ , and labour  $N_{ss} = 1$ . The steady state level of domestic consumption of home goods then corresponds to the factor of home bias  $1 - \alpha$  and consumption of foreign goods is equal to  $\alpha$ . Given the initial portfolio allocation of the mutual fund laid out in Appendix A.2, net foreign assets are 0 in steady state. The solution to the remaining stationary parameters is presented in Appendix A.6.

## 4 Numerical Method

This section introduces the numerical methods used to solve the small open economy HANK model. Advances in the successful application of heterogeneous-agent models over the past decade have been driven in large part by the overcoming of computational hurdles. Dynamic models with household heterogeneity and aggregate shocks pose particular challenges, mainly related to their high degree of dimensionality. Equilibrium involves state variables like aggregate household assets, which are in turn dependent on the high-dimensional cross-sectional distribution of households in each period. Iterating multiple times through all individual states at each point in time to find the effect of an aggregate shock on the model economy thus comes at an infeasible computational cost.

Early advances in the solution methods for heterogeneous-agent models were made by Den Haan (1997) and Krusell and Smith (1998). These authors, like many that followed in their footsteps, assume that in representing the distribution by a finite number of moments, the dimensionality of the model can be reduced while retaining a sufficient degree of accuracy. Krusell and Smith (1998), for instance, find that the mean of the capital distribution at time  $t$  is enough to predict the future value of aggregate capital. Such representations, however, fall short when the cross-sectional distribution of households takes a more central role in the model solution, which requires keeping track of a more accurate representation.

To retain a high-dimensional representation of the distribution, Reiter (2009) proposes to linearize the solution with regard to aggregate shocks. Crucially though, this is done after solving for a stationary solution that includes the idiosyncratic shocks which characterize the distribution. First, the steady state of the model is computed using projection

methods following Algan et al. (2008). Then, following perturbation methods by Kim et al. (2010) and Preston and Roca (2007), a linear approximation of the model dynamics around its steady state is derived. In this way, rich interactions between the distribution of households and macroeconomic outcomes are captured.

Authors since have taken similar approaches, only linearizing with respect to aggregates, but have provided more efficient techniques for the linearization and the computation of impulse responses. Boppart et al. (2018) first solve for the stationary equilibrium using the endogenous grid point method of Carroll (2006), which is presented below, to solve the household problem. They then solve non-linearly for impulse responses to a single small unexpected shock that hits the economy at its steady state, a so called "MIT shock". The sequence of reactions of a variable to a small shock represents a numerical derivative that incorporates distributional dynamics and can be treated as a linear impulse response function.

An efficient way to solve directly for these sequences of linear impulse responses has been proposed by Auclert et al. (2021c). Exploiting the consistency of households' steady state behavior as well as the consistency of the law of motion of their distribution, they derive an algorithm to quickly calculate the derivatives of equilibrium mappings between aggregate sequences of variables around the steady state. These objects, which they call "sequence-space Jacobians", summarize dynamic and distributional aspects of the model around the steady state and can thus be used to directly construct impulse responses. The technique revolves around three key components: (i) Expressing the model in the reduced form of the sequence space Jacobians, (ii) employing the endogenous grid point method by Carroll (2006) to find the steady state solution to the household problem, and (iii) using the so called "fake news" algorithm to effectively compute the sequence space Jacobians. These three components presented in more detail below.

## 4.1 General Equilibrium in the Sequence Space

The small open economy HANK model outlined above can be expressed in the sequence space by writing it as a system of nonlinear equations

$$\mathbf{F}(\mathbf{X}, \mathbf{Z}) = 0 \tag{37}$$

Here,  $\mathbf{Z}$  denotes the path of the vector of exogenous shocks  $\{\mathcal{B}\}$ ,  $\{\mathcal{E}\}$ , and  $\{\tau\}$  at each time  $t$ .  $\mathbf{X}$  denotes the path of the vector of all  $n_X$  endogenous variables at each time  $t$ . Assuming that the model has an equal number of equations and endogenous variables

and is locally determinate, (37) can be truncated to the time horizon  $T$  and becomes a nonlinear system of  $n_X \times T$  equations. The solution to this system delivers the general equilibrium impulse response to changes  $d\mathbf{Z}$  in the path of the vector of exogenous shocks  $\mathbf{Z}$  relative to the steady state  $\mathbf{Z}^{ss}$  (Auclert et al., 2021c). Arriving at the impulse responses requires a first order Taylor expansion of  $\mathbf{F}(\mathbf{X}, \mathbf{Z})$  around the steady state, which can then be solved for the responses of the endogenous variables  $d\mathbf{X}$  to the shocks  $d\mathbf{Z}$ :

$$\begin{aligned} 0 &= \frac{\partial \mathbf{F}(\mathbf{X}, \mathbf{Z})}{\partial \mathbf{X}^{ss}} d\mathbf{X} + \frac{\partial \mathbf{F}(\mathbf{X}, \mathbf{Z})}{\partial \mathbf{Z}^{ss}} d\mathbf{Z} \\ d\mathbf{X} &= - \left( \frac{\partial \mathbf{F}(\mathbf{X}, \mathbf{Z})}{\partial \mathbf{X}^{ss}} \right)^{-1} \frac{\partial \mathbf{F}(\mathbf{X}, \mathbf{Z})}{\partial \mathbf{Z}^{ss}} d\mathbf{Z} \\ d\mathbf{X} &\equiv \mathbf{G} d\mathbf{Z} \end{aligned} \quad (38)$$

Here,  $\frac{\partial \mathbf{F}(\mathbf{X}, \mathbf{Z})}{\partial \mathbf{X}^{ss}} = \mathbf{F}_X$  and  $\frac{\partial \mathbf{F}(\mathbf{X}, \mathbf{Z})}{\partial \mathbf{Z}^{ss}} = \mathbf{F}_Z$  are the Jacobians of  $\mathbf{F}$  evaluated at the steady state and  $\mathbf{G}$  linearly maps shocks  $d\mathbf{Z}$  to the general equilibrium response  $d\mathbf{X}$ . Depending on the complexity of the model, that is the size of  $\mathbf{X}$  and  $T$ ,  $\mathbf{F}$  can take very high dimensions. To lessen the computational burden, Auclert et al. (2021c) suggest reducing the dimensionality by explicitly solving for some variables in terms of others. Separating  $\mathbf{F}$  into  $\mathbf{F}_1$  and  $\mathbf{F}_2$ , where  $\mathbf{F}_2(\mathbf{X}, \mathbf{Z}) = 0$  can be solved as a function of a smaller vector of endogenous variables  $\mathbf{U}$  to obtain  $\mathbf{X}$ , i.e.,  $\mathbf{X} = \mathbf{M}(\mathbf{U}, \mathbf{Z})$ , one can define

$$\mathbf{H}(\mathbf{U}, \mathbf{Z}) \equiv \mathbf{F}_1(\mathbf{M}(\mathbf{U}, \mathbf{Z}), \mathbf{Z}) = 0 \quad (39)$$

As before, the implicit function theorem can be used to solve for the responses of the endogenous variables  $d\mathbf{X}$  to the shocks  $d\mathbf{Z}$ :

$$\begin{aligned} d\mathbf{U} &= -\mathbf{H}_U^{-1} \mathbf{H}_Z d\mathbf{Z} \\ d\mathbf{X} &= \mathbf{M}_U d\mathbf{U} + \mathbf{M}_Z d\mathbf{Z} = \mathbf{G} d\mathbf{Z} \end{aligned} \quad (40)$$

Here,  $\mathbf{H}_U$ ,  $\mathbf{H}_Z$ ,  $\mathbf{M}_U$ , and  $\mathbf{M}_Z$  again denote Jacobians of the respective functions  $\mathbf{H}$  and  $\mathbf{M}$  around the steady state. Applying this structure to the small open economy HANK model introduced in section 3 not only reduces the number of unknowns but also allows for the separate evaluation of equilibrium relationships. The construction of impulse responses then consists of assembling the Jacobians calculated for each block according to the chain rule (Auclert et al., 2021c).

Another important application of this structure is the decomposition of response paths. The complex relationship between household consumption and the real rate of return on

assets  $r^p$ , for example, can be expressed separately from the general model response of the rate of return in response to a shock to the payroll tax  $\tau$ . Combining the two relationships and repeating this process for all inputs required to solve the household problem, facilitates the decomposition of the general equilibrium response of consumption into the real income and the real rate effect presented in 6.2.

## 4.2 Method of Endogenous Gridpoints

Computing the Jacobians of the household problem requires, in a first step, to solve for its stationary equilibrium. The dynamic programming problem in (2) is solved on a grid, using the endogenous grid point method of Carroll (2006). The method exploits the endogenous dependencies between current consumption and future asset value implicitly expressed by the Euler equation, which applies whenever the household is not borrowing constrained:

$$u'(c) = \beta(1 + r^p)E_t[u'(c')] \quad (41)$$

Here,  $u'(c) = c^{-\sigma}$  and expectations are taken over ability states  $e$ . The Envelope theorem implies that the marginal utility of next period consumption  $u'(c')$  is equal to the discounted expected marginal value of next period assets  $\frac{1}{1+r^p}v(a')$ . Using this and the Euler equation in (41) thus allows to express current period consumption in terms of future asset value according to

$$c = (\beta v(a'))^{-\frac{1}{\sigma}} \quad (42)$$

Next, the budget constraint in (2) can be written as

$$c\Phi + a' = (1 + r_t^p)a + z_t \quad (43)$$

where  $\Phi = \alpha \left(\frac{P_{Ft}}{P_t}\right)^{1-\eta} + (1-\alpha) \left(\frac{P_{Ht}}{P_t}\right)^{1-\eta}$  is an adjustment factor reflecting the impact of relative prices for home and foreign goods on total consumption. The adjustment factor is derived by inserting the domestic demand schedules (6) and (7) into the budget constraint.  $z_t$  denotes real income as defined in (1). Now, starting with an initial marginal asset value  $v(a)$ , the level of current consumption  $c$  is given by (42), while cash on hand  $(1 + r_t^p)a + z_t$  is constructed over an initial asset grid  $a$ . With these elements in hand, future asset values  $a'$  can be derived via linear interpolation of the budget constraint, where interpolated values below zero are replaced with the minimum permitted asset



level  $a_t = 0$ . Then, using  $a'$ , (43) can be solved for  $c$ . Finally,  $c$  yields the discounted expected marginal value of current period assets,  $\frac{1}{1+r^p}v(a) = c^{-\sigma}$  which, as before, is used to calculate previous period consumption via (42). Iterating backwards in this way until the marginal value of assets becomes stationary delivers the optimal policy rules for consumption and saving that solve the household problem. Finally, combining the optimal policy rules with the Markov transition matrix  $\Pi$  for ability states  $e$  and iterating forward from an (arbitrary) initial distribution eventually yields the steady state distribution which determines the aggregates.

### 4.3 Fake News Algorithm

Auclert et al. (2021c) calculate the associated Jacobian of the household problem by means of an algorithm that exploits the consistency of household behavior in steady state as well as the consistency of the law of motion of the household distribution over time. The heterogeneous-agent problem is expressed as system of equations:

$$\begin{aligned} \mathbf{v}_t &= v(\mathbf{v}_{t+1}, \mathbf{X}_t) \\ \mathcal{D}_{t+1} &= \Lambda(\mathbf{v}_{t+1}, \mathbf{X}_t)' \mathcal{D}_t \\ \mathbf{Y}_t &= y(\mathbf{v}_{t+1}, \mathbf{X}_t)' \mathcal{D}_t \end{aligned}$$

Here,  $\mathbf{v}_t$  can be interpreted as the value function in (2) which is solved by the optimal consumption and saving policies derived via the endogenous grid method above.  $\Lambda$  is the transition matrix representing the discretized law of motion of the distribution  $\mathcal{D}_t$  and is derived from the policy function for assets and the transition matrix  $\Pi$ . Combining the distribution with the matrix of individual outcomes  $y$  yields aggregate outcomes  $\mathbf{Y}_t$ . To compute the Jacobian representing the steady state aggregate response of households at any time  $t$  to a shock at any time  $s$  Auclert et al. (2021c) use that individual outcomes  $y_t^s$  and the behavior of the transition matrix  $\Lambda_t^s$  depend only on the distance to a past shock  $t - s$ , not on the actual periods  $s$  and  $t$ . Therefore, the behavior of all individual policy functions and all relevant transition matrices in response to a shock can be computed from a single backwards iteration starting from a shock at  $T - 1$ . However, while the policy and transition matrix will behave the same regardless of when a shock took place, households will also act in anticipation of a shock, which impacts the distribution in period  $s$ . To derive the aggregate response to a shock, Auclert et al. (2021c) start by defining the response to a "fake news" shock that is announced at  $t - 1$  and retracted again at  $t$  as

$$\mathcal{F}_{t,s} \cdot dx \equiv dY_t^s - dY_{t-1}^{s-1} \quad (44)$$

The fake news response is the difference of impulse responses to infinitesimal shocks in any two consecutive periods. Exploiting the symmetrical behavior of individual policies and distribution transition matrices in response to shocks, they then show that the difference between aggregate outcomes in (44) can be computed from the steady state policies, the transition matrix, and the distribution in period 1, according to:

$$\mathcal{F}_{t,s} \cdot dx = \mathbf{y}_{ss}' (\Lambda_{ss}')^{t-1} d\mathcal{D}_1^s$$

Because the fake news response is defined as the difference between responses in consecutive periods, any Jacobian  $\mathcal{J}_{t,s}$  for responses in period  $t$  to a shock at  $s$  can be calculated as the sum of the Jacobian for a response and a shock in the previous period and the fake news response in the current period. From this recursion follows the definition of the Jacobian as the sum of all fake news responses in previous periods:

$$\mathcal{J}_{t,s} = \sum_{k=0}^{\min s,t} \mathcal{F}_{t-k,s-k} \quad (45)$$

The first-order aggregate response of the households to a shock in any period  $s$  can thus be described as a function of the aggregate response at  $t = 0$ , for which no fake news response is defined, the response of the distribution at date  $t = 1$ , and the stationary solutions for the policy function and the transition matrix. Given the steady state solution to the household problem derived via the endogenous grid method introduced earlier, the algorithm for computing the Jacobian consists of four steps: (i) Calculating individual policies from a single backwards iteration from  $T - 1$  and combining them with the initial steady state distribution at  $t = 0$  to derive the initial response of aggregates and the change in distribution for a shock at time  $s$ , (ii) calculating the expectations vector  $\mathbf{E}_t \equiv (\Lambda_{ss})^t \mathbf{y}_{ss}$  which, combined with the initial response of the distribution, forms the fake news responses, (iii) constructing the matrix of fake news responses for each  $s$  and  $t$ , and (iv) constructing the Jacobian according to (45). The Jacobian can then be combined with the Jacobians of the remaining equilibrium equations to construct the impulse responses as described in section 4.1.

## 5 Calibration

This section presents the calibration of the model parameters based on the relevant literature. The model is calibrated at quarterly frequency. Similar to Galí and Monacelli (2016), the aim of the calibration is to capture average characteristics of Greece, Italy, Portugal, and Spain (GIPS) during the 21st century. In light of the importance of income and consumption dynamics for model responses in a HANK context, the calibration of distributional parameters and MPCs takes a central role. The calibration is summarized in Table 1.

Table 1: Calibration of model parameters

Parameter	Value	Target
Intertemporal elasticity of substitution	$\sigma = 1$	Standard value
Inverse Frisch elasticity of labour supply	$\varphi = 2$	Standard value
Elasticity of substitution between home and foreign goods	$\eta = 1$	Feenstra et al. (2018)
Elasticity of substitution between goods of foreign countries	$\gamma = 1$	Auclert et al. (2021b)
Domestic discount factor	$\beta = 0.973$	Marginal propensity to consume
Distribution factor spread	$\delta = 0.012$	Net foreign assets
Foreign discount factor	$\beta^* = 0.99$	Real interest rate
Openness	$\alpha = 0.3$	Galí and Monacelli (2016)
Price markup	$\mu = 1.03$	Asset distribution
Wage markup	$\mu_w = mu$	Smets and Wouters (2007)
Calvo price adjustment parameter	$\theta_H = 0.8$	Galí and Monacelli (2016)
Calvo wage adjustment parameter	$\theta_w = 0.8$	Galí and Monacelli (2016)
Average annual MPC	$MPC = 0.46$	Drescher et al. (2020)
Steady state net foreign assets	$nfa = 0$	Auclert et al. (2021b)
Steady state annual real interest rate	$r_{ss} = 0.4$	Galí and Monacelli (2016)
Taylor rule coefficient	$\phi_H = 1.5$	Galí and Monacelli (2016)
Standard deviation of income process	$\sigma_e = 0.6$	Auclert et al. (2021b)
Persistence of income process	$\rho_e = 0.94$	Guntin et al. (2020)
Persistence of exogenous shock processes	$\rho_i = 0.9$	Galí and Monacelli (2016)

### 5.1 Aggregate Household Parameters

The parameters of the household utility function (3) take the standard values used in the HANK literature. The intertemporal elasticity of substitution  $\sigma$  is equal to 1. Given L'Hôpital's rule, the limit of the CRRA utility of consumption is thus equal to log utility,  $\lim_{\sigma \rightarrow 1} u(c_t) = \ln(c_t)$ . The inverse Frisch elasticity of labour supply  $\varphi$  is set to 2.

Gross income is assumed to follow an  $AR(1)$  process. Estimates for the persistence of the wage process vary, with typical estimates for the US between 0.91 (Floden and Lindé, 2001) and 0.96 (Storesletten et al., 2004), and EU estimates ranging from 0.81 in Sweden (Floden and Lindé, 2001) to 0.97 in Hungary. In the absence of estimates for other GIPS economies, I rely on values for Italy by Guntin et al. (2020), who estimate a persistence of  $\rho_e = 0.94$ . Calibration choices for the standard deviation of the income process exhibit an even wider range. Estimates from the studies cited above are around 0.2. However, because of the low data frequency behind most estimates, Kaplan et al. (2018) infer high

frequency dynamics from higher order moments of changes in annual earnings, arriving at a value of 0.92. I adopt the estimate of Auclert et al. (2021b), who scale down the value by Kaplan et al. (2018) to correct for tax progressivity. This yields  $\sigma_e = 0.6$ .

Together with the additional parameters, these values deliver a good fit for the average distribution of wealth observed in the GIPS economies, as well as for the size and heterogeneity of consumption responses to income shocks, both of which are discussed below. The income process is discretized on a 7-point Markov chain using the method proposed by Rouwenhorst (2021). In addition to idiosyncratic shocks, agents are assumed to face a borrowing constraint with  $a = 0$ , following McKay et al. (2016).

## 5.2 Asset Distribution and MPCs

A central goal of the calibration is to accurately match the average distribution of wealth and the marginal propensity to consume in GIPS economies. The ability of HANK models to accommodate realistic MPCs and distributions is especially important in light of the key role that consumption dynamics take in the transmission of labour cost adjustments analysed in the next section. The left panel of Figure 1 plots the average share of wealth held by the bottom 50, 80, 90, 95, and 99 percent of the population, based on calculations by Arrondel et al. (2014) for Greece and Portugal, Acciari et al. (2020) for Italy, and Martínez-Toledano (2020) for Spain in 2010. Although considerable wealth inequality exists, the slope of the Lorenz curve is flatter than for other advanced economies typically analysed. The share of total net wealth held by the top 1% for instance, was about 20% in GIPS economies versus upwards of 25% in France and Germany (Arrondel et al., 2014) and upwards of 30% in the USA (Saez and Zucman, 2020).

Estimating MPCs requires the identification of unexpected transitory income shocks and the measurement of a resulting response in consumption. Widely accepted approaches include exploiting natural experiments like lottery winnings or tax rebates (Johnson et al., 2006; Parker et al., 2013; Misra and Surico, 2014; Broda and Parker, 2014; Fagereng et al., 2021), relying on survey evidence on spending out of hypothetical income surges (Jappelli and Pistaferri, 2014; Drescher et al., 2020), or exploring the joint dynamics of income and consumption data by imposing a theory-guided covariance structure (Blundell et al., 2008; Kaplan and Violante, 2014; Hong, 2020). While findings vary slightly, typical MPC estimates lie around 0.25 quarterly (Kaplan and Violante, 2014) and 0.5 annually (Fagereng et al., 2021).

In addition to aggregate values, several authors report on the heterogeneity of MPCs across the wealth distribution. For the US, Broda and Parker (2014) find that less liq-

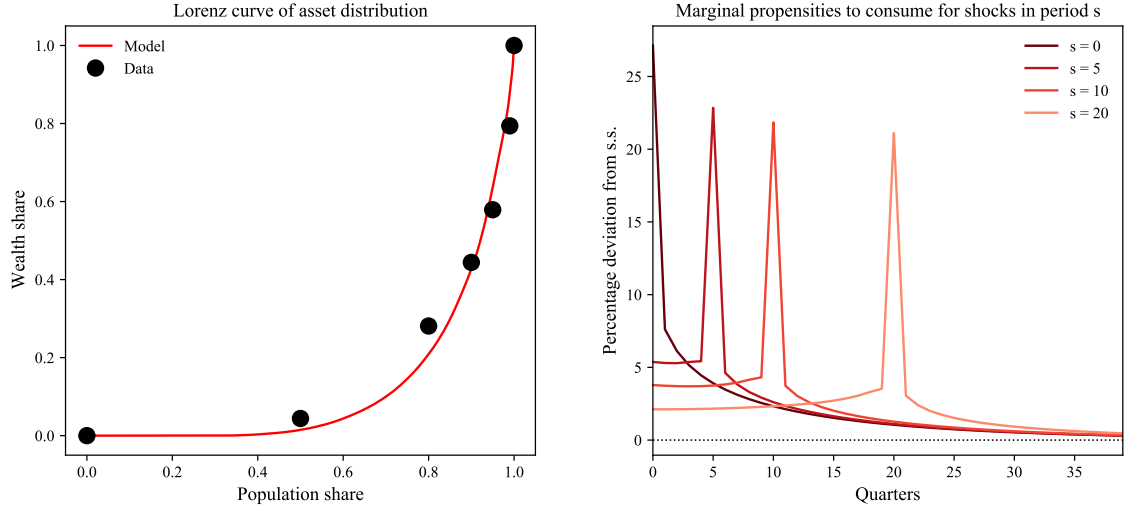
uid households responded much stronger to the 2008 stimulus payments, while Misra and Surico (2014) also document that some wealthy households exhibit high MPCs, a finding shared by Kaplan et al. (2014). Using Norwegian administrative data to examine the propensity to consume out of lottery winnings, Fagereng et al. (2021) find higher MPCs for households with low liquid asset positions, even if these have a higher stock of illiquid assets. Jappelli and Pistaferri (2014) and Hong (2020) report MPCs along the entire income distribution and find that the consumption response declines at higher income levels. For Italy, Jappelli and Pistaferri (2014) report average MPCs of 0.48. Here, the average values for the bottom and top third of the distribution lie around 0.55 and 0.35 respectively.

To capture both the uneven distribution of wealth in the GIPS economies as well as the characteristics of MPCs, I assume that households are subject to discount factor heterogeneity. Following Auclert et al. (2021b), there are three equally sized groups of households. The discount factors across household groups follows a three point distribution at  $\beta - \frac{\delta}{2}, \beta, \beta + \frac{\delta}{2}$ , with  $\delta$  representing the discount factor spread between the household group with the lowest and the group with the highest average asset holdings. Given the discretized income process, steady state real wages, and the annualized steady state real interest rate of  $r_{ss} = 0.4\%$  assumed by Galí and Monacelli (2016), I adjust the domestic discount factor and its spread to hit the targets of zero net foreign assets and an average annual MPC of 0.46. The latter value is the average of the EU survey evidence for Greece, Italy, and Portugal presented by Drescher et al. (2020). This delivers  $\beta = 0.973$  and  $\delta = 0.012$ .

The structure of the household Jacobian discussed in section 4.1 facilitates the computation of iMPCs, i.e., the response of consumption to a period  $s$  real income shock Auclert et al. (2018). The derivation of these iMPCs is discussed in section 6.1. The right panel of Figure 1 reports iMPCs for shocks in various periods  $s$ . The average quarterly MPC to a contemporaneous shock is 0.27, close to the value estimated by Kaplan and Violante (2014) and slightly below the value implied by Fagereng et al. (2021). Anticipated shocks elicit smaller responses, since agents increase their spending in the lead up to the shock. Plots of the iMPCs for the three household groups can be found in Appendix B. With an average annual MPC of 0.57 for the low liquidity households group, 0.46 for the middle group, and 0.34 for high liquidity households, the distribution of MPCs matches the findings by Jappelli and Pistaferri (2014).

As laid out in Appendix A.6, it follows from (23) that the zero net foreign asset target requires assets to be equal to the share value of firms  $p$ , which in steady state is given by  $\frac{1}{r} \left(1 - \frac{1}{\mu}\right)$ . Because of this equality, firm markup  $\mu$  not only affects returns on assets but

Figure 1: Calibration of asset distribution and MPC



*Notes:* The left panel plots the asset distribution of the small open economy HANK model against the average wealth shares in Greece, Italy, Portugal, and Spain. The right panel plots the average consumption response of households to income shocks in different quarters.

also the overall level of steady liquidity in the model. While setting  $\mu = 1.03$  delivers the best fit for the wealth distribution, the implied asset to GDP ratio is only around 75%, much lower than the average of 437% reported by the OECD Wealth Distribution Database. However, given the absence of capital in the model and the negative correlation between illiquid wealth and MPCs reported by Fagereng et al. (2021), I follow Auclert et al. (2021b) in interpreting assets as illiquid wealth.<sup>4</sup> I choose wage and firm markup to be identical at  $\mu_w = \mu$ , a common approach in the literature (see, e.g., Smets and Wouters, 2007).

### 5.3 Openness and Trade Elasticity

Following Galí and Monacelli (2016), I set the openness parameter in correspondence with average import-to-GDP and export-to-GDP ratios, which yields  $\alpha = 0.3$ . Aggarwal et al. (2022) follow the same approach using data from the World Development Indicators (WDI) between 2015 and 2019 and arrive at a similar value. Together with the elasticity of substitution between home and foreign goods,  $\eta$ , and the elasticity of substitution between varieties of foreign goods,  $\gamma$ , openness is a critical factor for the response of the economy to foreign shocks. Auclert et al. (2021b) define the overall trade elasticity as

<sup>4</sup>Notably, the ratio of financial assets to GDP reported by the OECD for GIPS economies between 2000 and 2020 is around 190%. I accept this caveat given the fit of the distributional features.

$$\chi \equiv \eta(1 - \alpha) + \gamma \quad (46)$$

and show that variations in the parameters can dampen or amplify exchange rate and monetary policy shocks, a result which proofs relevant for the analyses in section 6 and 7.<sup>5</sup> Studies have attempted to estimate the relevant elasticities for several decades (for comprehensive reviews see, e.g., Marquez, 2002; Gallaway et al., 2003; Feenstra et al., 2018). While macro estimates for the overall elasticity of substitution by Heathcote and Perri (2002) and Bergin (2006) lie around unity, Imbs and Mejean (2015) show that the sector aggregation in such estimates may lead to a downward bias. Feenstra et al. (2018) instead analyse disaggregate trade data. Although they find elasticities between varieties of foreign goods that are larger than one in several sectors, they fail to reject the hypothesis of unit elasticity at the aggregate level.

As a result of the range of estimates, the related small open economy New Keynesian literature takes different approaches to the calibration of trade elasticities. Some authors assume that the elasticities of substitution between home and foreign and between varieties of foreign goods are equal and report results for elasticities ranging from 0.5 to 2 (e.g., Gali and Monacelli, 2005; Auclert et al., 2021b; Galí and Monacelli, 2016). Other studies follow the macro evidence and choose an elasticity of substitution between home and foreign goods around 1, while choosing higher values for the elasticity between varieties of foreign goods (e.g., Oskolkov, 2021). Finally, some authors choose high values for both elasticities (e.g., Guo et al., 2020).

Given the disparity between estimates, the main part of the analysis relies on elasticities of unity,  $\eta = 1$  and  $\gamma = 1$ , which, given the choice of  $\alpha$ , implies  $\chi = 2 - \alpha$ . Auclert et al. (2021b) show that this level establishes equivalence between the effect of monetary transmission in a stylized open economy HANK model and the representative agent case. Additional values considered for robustness are:  $\eta = 0.6$  and  $\gamma = 0.6$ , which yields  $\chi \approx 1$ ,  $\eta = 1$  and  $\gamma = 2$ ,  $\eta = 2$  and  $\gamma = 2$ , and  $\eta = 1$  and  $\gamma = 3$ .<sup>6</sup>

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<sup>5</sup> $\chi$  is defined as the sum of the elasticities of domestic consumption of foreign goods and foreign consumption of home goods to relative prices:  $\frac{\partial \log C_{Ft}}{\partial \log P_{Ft}/P_{Ht}} = -\eta \frac{\partial \log P_{Ft}/P_t}{\partial \log P_{Ft}/P_{Ht}} = -\eta(1 - \alpha)$  and  $\frac{\partial \log C_{Ht}^*}{\partial \log P_{Ht}^*/P_{Ft}^*} = -\gamma \frac{\partial \log P_{Ht}^*/P_t^*}{\partial \log P_{Ht}^*/P_{Ft}^*} = -\gamma$ .

<sup>6</sup>To avoid division by zero in the computation, I accept small deviation from unit elasticity and set  $\eta = 0.99$  and  $\gamma = 1.01$ .



## 5.4 Phillips Curve and Additional Parameters

Estimates of the level of wage and price flexibility in Europe typically rely on theory based estimations of the New Keynesian Phillips curve (e.g., Galí et al., 2001) or on survey and price data for the size and frequency of price and wage adjustments (e.g., Alvarez et al., 2006; Druant et al., 2012; Knell, 2013; Izquierdo et al., 2017). Typical estimates for both wage and price rigidity lie between 0.7 and 0.8, implying adjustments every 10 to 15 months. However, Knell (2013) notes that wage adjustments exhibit significant variation across countries, sectors, and time. Their findings underline the need for analyses of consequences of varying adjustment speeds. Following Galí and Monacelli (2016), I choose  $\theta_H = 0.8$  and  $\theta_w = 0.8$  as baseline values. The welfare analysis in section 7 then considers various levels of rigidity.

Finally, the Taylor rule coefficient is set to  $\phi_H = 1.5$ , following Galí and Monacelli (2016). The foreign discount factor is set to target the steady state interest rate of 4%, which gives  $\beta^* = 0.99$ . The persistence of external shock processes is 0.9, following Galí and Monacelli (2016).

## 6 Revisiting the Effectiveness of Labour Cost Reductions

As a means to stabilize output and employment in the face of adverse shocks, EU member states have in the past been recommended to pursue structural reforms aimed at increasing the sensitivity of labour costs to cyclical conditions. Following conventional wisdom, a swift drop in wages can facilitate a quick adjustment in employment and, assuming no movement in the nominal exchange rate, boost exports by raising competitiveness (De Grauwe, 2020). The extent to which such reforms may play a stabilizing role thus crucially depends on the responsiveness of employment to changes in wages or other labour cost components. This section reviews the transmission mechanism between labour costs and employment in a HANK context, before analysing the effect of a labour cost reduction in the form of a temporary payroll tax cut under a Taylor rule and in a monetary union.

### 6.1 Labour Costs and the New Keynesian Transmission Mechanism

The way that labour cost reductions affect output and employment differs fundamentally across classical business cycle models, standard New Keynesian models, and HANK models. Galí (2013) addresses differences in the transmission of wage effects in closed



economy real business cycle models and a New Keynesian model with wage rigidity. In classical models, firms' demand for labour is determined by the equality between the wage and the marginal product of labour. A reduction of labour costs in the form of wages, therefore, has a direct positive effect on employment. This direct effect is independent of the monetary policy rule in place. In a New Keynesian model, however, labour demand is determined, in the short run and for a given production technology, by the level of production and thus by aggregate demand. The effect of a wage reduction thus depends on the consumption response of households. In a representative agent setup, changes in consumption are primarily driven by intertemporal substitution and therefore determined by interest rates. A reduction in labour costs via a temporary payroll tax cut would reduce firms' marginal costs. Lower marginal costs, via the New Keynesian Phillips curve (29), exert downwards pressure on inflation. With active inflation targeting, monetary policy would react by lowering the nominal interest rate. In the face of lower returns on savings, the perfectly insured representative agent of the standard New Keynesian model chooses to substitute towards contemporaneous consumption, thus raising aggregate demand. Given goods market clearing and the production technology, labour demand and employment increase.

In a New Keynesian framework, the response of monetary policy thus becomes the key determinant of the effect on employment. Galí and Monacelli (2016) call this mechanism the "endogenous policy channel". They find that with a Taylor type rule, a reduction in the payroll tax,  $\tau$ , results in an increase of employment of just above 3%. This response is driven by a drop in the nominal and the real rate by 3%. Under a currency peg, however, the endogenous policy channel is muted. With interest rates largely unchanged, agents do not change their behavior of intertemporal substitution and employment levels remain relatively unaffected. The small change in employment and output that is observable under an interest rate peg follows from what Galí and Monacelli (2016) call the "competitiveness channel": A drop in marginal costs lowers domestic prices. Given the constant nominal exchange rate, this leads to an appreciation of the real exchange rate and thus an increase in export demand and production.

In the presence of household heterogeneity and borrowing constraints, the transmission of shocks via monetary policy and the consumption of households functions fundamentally different. Several authors have used stylized versions of HANK models to analytically clarify how the novel model features affect the propagation of different shocks in closed economies (see, e.g., Werning, 2015; Acharya and Dogra, 2020; Bilbiie, 2020; Debor-toli and Galí, 2017; Auclert, 2019; Bernstein, 2019; Bilbiie et al., 2022). The central difference between between HANK models and their representative agent counterparts lies in the diminished importance of intertemporal substitution and the centrality of real

household income. The assumption of borrowing constraints directly impedes the ability of agents to borrow to increase consumption following a rate cut. However, consumption is now affected directly by movements in real income and output. Therefore, general equilibrium effects are also transmitted via changes in income as well as the ensuing mutual reinforcement between consumption and output. Because output also impacts firm dividends, it can further affect consumption via a change in real returns.

Auclert et al. (2021b) use stylized version of the model in section 3 to isolate these new effects and demonstrate how they come into play. Assume that nominal prices are perfectly flexible and monetary policy follows a CPI-based Taylor rule that holds the real interest rate constant by setting the nominal interest rate according to  $i_t = r_{ss} + \pi_{t+1}$ , where  $r_{ss}$  is the target for the steady state real interest rate. This rule is employed to partial out the effects of monetary policy.<sup>7</sup> Under flexible prices, equation (26) rewrites to  $P_{Ht} = \mu \frac{W_t}{Z}$ . Combining this with the production function (25), real wage income is given by

$$\frac{W_t}{P_t} N_t = \frac{1}{\mu} \frac{P_{Ht}}{P_t} Y_t \quad (47)$$

Real wages affect consumption via households' budget constraint in (2). In addition, combining (47) with equation (28), real dividends can be written as

$$D_t = \left(1 - \frac{1}{\mu}\right) \frac{P_{Ht}}{P_t} Y_t \quad (48)$$

Dividends affect the share value of home firms and thus, given the constant real interest rate  $r$ , the equality of returns (19), and the initial portfolio allocation of the mutual fund, determine the real rate of return on household savings  $r_0^p$ . In this setup, real wages and real dividends pin down real income and returns on savings and are thus sufficient to solve the household problem and derive optimal consumption at each time  $t$ . By aggregating up (47) and (48), Auclert et al. (2021b) define the consumption function  $C_t = \mathcal{C}_t \left( \left\{ r_s, \frac{P_{Hs}}{P_s} Y_s \right\}_{s=0}^{\infty} \right)$ . The Jacobian of  $\mathcal{C}$  with regard to  $\frac{P_{Hs}}{P_s} Y_s$  is defined by  $\mathbf{M}$ . Its elements characterize the date  $t$  consumption responses to changes in real income at date  $s$ . These elements are the iMPCs introduced earlier.

To conceptualize how the endogeneity of output in consumption affects the general transmission of shocks, consider first the case of a closed economy. Auclert et al. (2018) derive what they call the "generalized intertemporal Keynesian cross", a general solution for the response of output and, given the production function in (25), employment. With a con-

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<sup>7</sup>Similar setups have been used to analyse effects of exchange rate and aggregate demand shocks (see, e.g., Auclert et al., 2018; Auclert et al., 2021b; Aggarwal et al., 2022; McKay et al., 2016; Woodford, 2011).

stant real interest rate and in the absence of government and investment, i.e.,  $Y_t = C_t$ , the response of output is determined on the one hand by the direct effect of the shock on consumption,  $\partial \mathbf{C}$ , and on the other hand by the multiplier effect,  $\mathbf{M}d\mathbf{Y}$ , which, since output enters the consumption function, describes the feedback effect between rising consumption and output:

$$d\mathbf{Y} = \partial \mathbf{C} + \mathbf{M}d\mathbf{Y} \quad (49)$$

Here,  $\mathbf{C} \equiv (\mathcal{C}_{t,\vartheta}d\vartheta)$  is the response of consumption to any given shock  $\vartheta$ .<sup>8</sup> Applying this framework to the example of a payroll tax cut clarifies the central difference between the representative agent and the HANK model. In a representative agent model, the second term of (49) is absent, while the first term is affected primarily by the endogenous policy channel. The payroll tax cut leads to decreasing inflation which induces a rate cut and raises consumption via intertemporal substitution. In a monetary union, this channel is absent and the response disappears. In a HANK model, both terms of (49) are active. Here, consumption is affected both by changes in the rate of return and direct changes in income. The effect of a payroll tax cut thus depends on three factors: (i) Its impact on real returns, (ii) its effect on real income, and (iii) the multiplier. Importantly, the response of the real rate of return is affected both by monetary policy and by firm dividends. Since the response of dividends, incomes, and the multiplier are all active in a monetary union, a payroll tax cut can be expected to affect employment irrespective of the endogenous policy response. The overall size of the effect under the two monetary regimes then depends on the relative strength of the three factors. Werning (2015) shows that the weaker intertemporal substitution effect in HANK models can be offset by the income and multiplier effects.

Whether the smaller effect of interest rates can also be offset in the small open economy case, depends on the interplay of three additional factors (Auclert et al., 2021b). (i) First, the competitiveness channel pointed out by Galí and Monacelli (2016) is also present in the HANK context. To the extent that external shocks have an effect on relative prices, domestic and foreign consumers will substitute towards the relatively cheaper good. The size of this effect depends both on the country specific preference for foreign goods  $\alpha$  as well as on the relevant consumption elasticities  $\eta$  and  $\gamma$ . (ii) Second, because the consumer price index enters into (47) and (48), changes of relative prices directly affect the components of real household income and, indirectly, the returns on their savings. Given the definition of the price index (5), this effect likewise depends on the preference for foreign goods  $\alpha$  and the elasticity of substitution between home and foreign goods  $\eta$ .

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<sup>8</sup>For the proof behind equation (49), see Appendix B in Auclert et al. (2018).

(iii) Third, with a share of consumption falling on foreign goods, the size of the multiplier in open economies reduces to  $(1 - \alpha)\mathbf{M}d\mathbf{Y}$ .<sup>9</sup> However, Auclert et al. (2021b) show at the example of a monetary policy shock that if the overall trade elasticity  $\chi$  defined in (5.3) equals  $2 - \alpha$ , the weaker interest rate channel and the effects of relative prices on real income are exactly offset by the output multiplier, thus establishing equivalence between the representative agent and the HANK result. This corresponds to the commonly-studied parametrization by Cole and Obstfeld (1991), in which  $\eta = \gamma = 1$ , the baseline case of my analysis. At higher values of elasticities, however, the output response of the HANK model is greater than that of the standard New Keynesian model. Because higher levels of elasticity mean that consumers react more strongly to changes in relative prices, the substitution towards domestic goods and therefore the significance of the competitiveness channel increases. The added demand for home goods in turn amplifies the multiplier effect. Conversely, at lower levels of elasticity, these effects and thus the response of output are reduced.

To sum up, the employment response to a labour cost reduction in a representative agent New Keynesian model depends on the endogenous response of monetary policy and, to a lesser extent, the relative competitiveness of domestic firms. In a HANK model, the response of real returns is co-determined by dividends and effects on employment are driven by additional income and multiplier effects. Two hypotheses can be derived from this theoretical review: (i) In a monetary union, the effect of a payroll tax cut on employment is likely stronger than in a standard new Keynesian model. (ii) Under a Taylor rule, the relative strength of the effect in a HANK model may depend on the level of trade elasticities and openness.

## 6.2 The Effect of a Payroll Tax Cut

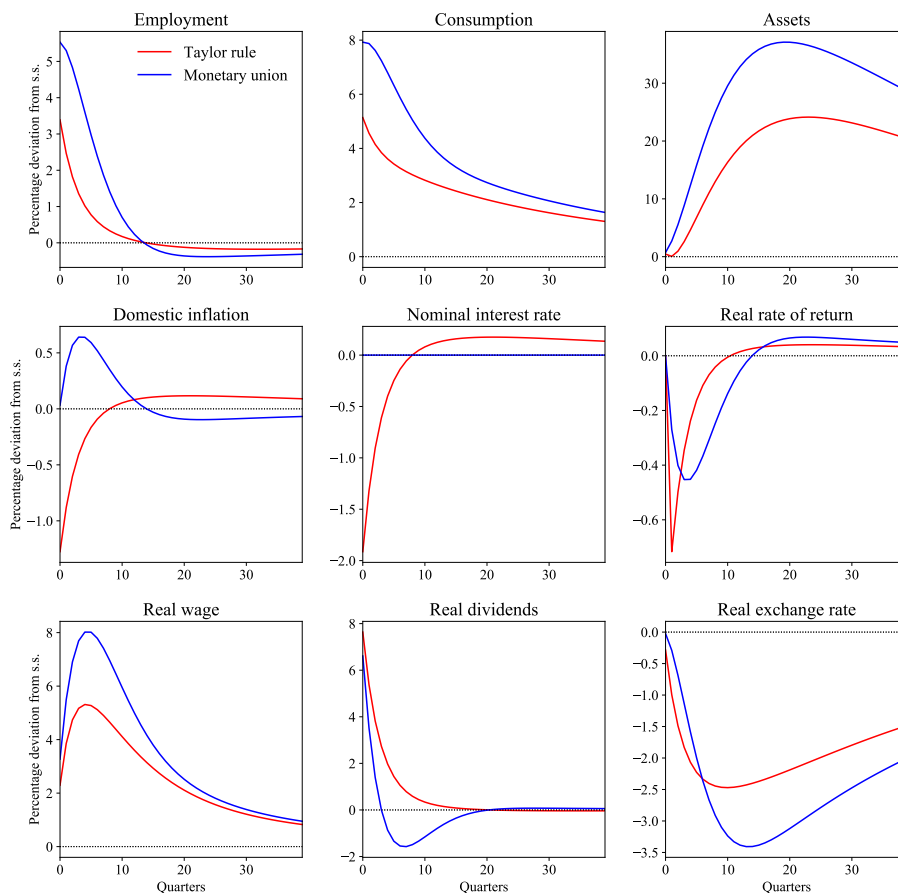
The simulation of an exogenous temporary reduction in the payroll tax under the two monetary policy regimes confirms the hypotheses derived in the previous section. Galí and Monacelli (2016) argue that this experiment serves as a fitting example for policies advocated in southern EU member states which, due to the monetary union and EU fiscal rules, are constrained in their monetary and fiscal response. Figure 2 shows impulse responses of the model described in section 3 to a 10 percent reduction in the payroll tax with a persistence of 0.9 for a 10-year horizon. The case of the Taylor rule (32) is depicted in red. The case of the monetary union (33) is shown in blue. The respective responses

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<sup>9</sup>Auclert et al. (2021b) demonstrate that although these effects can be found in a representative agent model with incomplete markets, their size increases substantially in the heterogeneous agent case. Appendix C.1 replicates and explores this result.

are discussed below.

Figure 2: Dynamic responses to a temporary payroll tax cut



*Note:* The above plots show impulse responses for an exogenous 10% cut in the payroll tax with a persistence of 0.9 for the case of a Taylor rule and for a monetary union.

## 6.2.1 The Case of Inflation Targeting

Consider first the case of the Taylor rule. In response to the payroll tax cut, employment increases by 3.4% on impact relative to its steady state value. It returns to and slightly undershoots its steady state level after 15 quarters. Consumption jumps by 5.1% on impact and slowly falls over the ten year horizon. Asset levels begin to elevate three quarters after the shock, peaking at 2.4% above steady state level after 25 quarters. Domestic inflation drops on impact by 1.3% and rises to a relatively constant level of 0.1% above steady state over the first 10 quarters. The path of the nominal interest rate follows this response, dropping by 191 basis points on impact, as does the real rate of return, which drops by 72 basis points in the second quarter. Real wages jump by 2.3% on impact but don't peak until quarter 5. Real dividends jump by 7.6% on impact and revert back to steady state

around quarter 15. The real exchange rate drops on impact by 0.2% and continue to fall to around negative 2.5%, 10 quarters in. Impulse responses of additional model variables can be found in Appendix C.2.

The responses of employment and the nominal interest rate resemble the results by Galí and Monacelli (2016). The endogenous policy channel is active: In response to the tax cut, firms face reduced marginal costs (26), which, via the New Keynesian Phillips curve (29), causes deflationary pressure. Monetary policy reduces the nominal interest rate, pushing down the real interest rate via the Fisher equation (20). The arbitrage condition implicit in (19) dictates that with a lower real interest rate and high future dividends, the share value of firms rises. Given the initial allocation of the mutual fund, this pushes down the ex-post rate of return on assets and causes households to substitute intertemporally towards consumption. The resulting rise in aggregate demand increases production and thus employment. Here, the path of the employment response is almost identical to the one seen in the representative agent case.<sup>10</sup> As expected, the competitiveness channel does not play a significant role if the nominal exchange rate is flexible. Although a decrease in domestic prices would, all else equal, raise export demand, it is offset by a drop in import prices: Falling nominal rates push down the nominal exchange rate via the UIP condition (21) which, given the law of one price, causes import prices to drop. General equilibrium effects likely contribute to the relatively smaller decrease in domestic prices, with higher wages partially offsetting the reduction in marginal costs caused by the payroll tax cut. The improvement in the terms of trade caused by these changes in relative prices is analogous to the results of Galí and Monacelli (2016).

In addition to the endogenous policy and the competitiveness channel, real income and multiplier effects contribute to the large consumption response. With rising production and labour demand, the marginal disutility of average work increases. As this disutility starts to exceed the product of marginal consumption utility and marked-down real wage in the wage Phillips curve (31), unions set higher wages. The resulting increase in real income drives the increase in aggregate demand as well as households' ability to save. In addition, the sluggishness of the wage response caused by nominal rigidities likely contributes to the continuously elevated consumption levels. The multiplier stems from the fact that real income drives, and is itself driven by, output.

To determine the relative significance of the real income and multiplier effect, I follow Auclert et al. (2018), who suggest a decomposition of the consumption response into a real wage and a real rate of return component.<sup>11</sup> As described in section 4.1, the respective

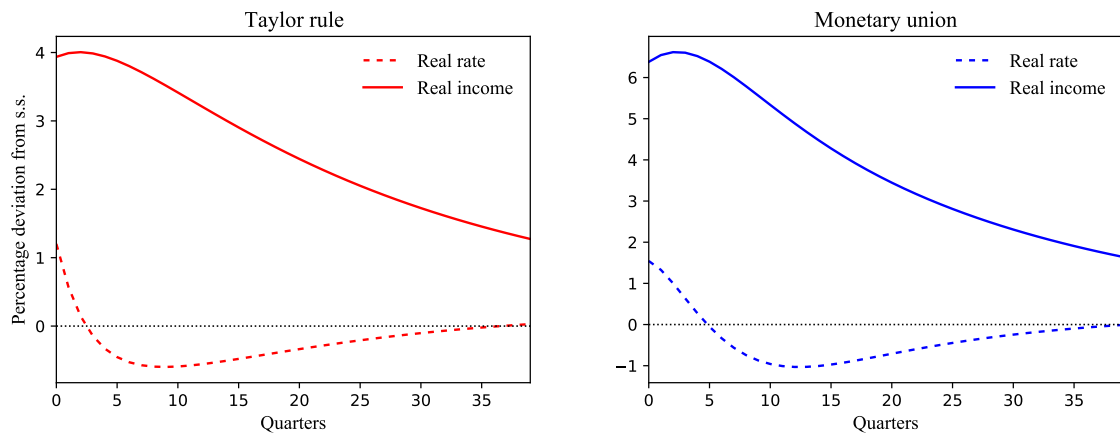
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<sup>10</sup>For the impulse responses of the representative agent New Keynesian model, see Figure 1a. in Galí and Monacelli (2016).

<sup>11</sup>For a formal decomposition of the consumption response in a closed economy HANK model, see

Jacobian of household consumption with regard to the real wage as well as the ex-post real rate of return can be isolated. Multiplying the Jacobian with the equilibrium response of the entire model yields a response path revealing the importance of the respective input for the general equilibrium response of consumption. The left panel of Figure 3 presents the decomposition into these response paths for the Taylor rule case. Here, the sum of the two channels is equal to the equilibrium response of consumption in Figure 2. The drop in the real rate of return raises consumption by only 1.2% on impact. As the real rate rises over consecutive quarters, its contribution to consumption falls and turns negative. The real income channel raises consumption by 3.9% on impact, which accounts for 77% of the initial response. From the second quarter onward, real income accounts for 100% of the consumption increase. The decomposition confirms both the diminished role of intertemporal substitution in the transmission of labour cost reductions in a HANK model, as well as the significant effect of real income and multiplier effects.

Figure 3: Decomposition of the consumption responses



*Note:* The figure plots respective absolute shares of the consumption response to an exogenous 10% cut in the payroll tax under a Taylor rule (left panel) and a monetary union (right panel) accounted for by changes in real wages and the real rate of return on household assets.

## 6.2.2 The Case of a Monetary Union

For the case of a monetary union, the ten percent temporary payroll tax cut raises employment and consumption on impact by 5.5% and 7.9% respectively, more than in the case of inflation targeting (see Figure 2). The results of the HANK model thus stand in stark contrast to the results of the representative agent model analysed by Galí and Monacelli (2016), who found only a minimal response of employment. Other responses are likewise

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Kaplan et al. (2018).



stronger in the HANK case. While the nominal interest rate remains constant given the interest rate peg, real ex-post returns on household savings drop by about 45 basis points over the first year. Assets peak at 37.1%. The response of domestic inflation is now positive and hump-shaped. Real wages show a stronger response than they did under a Taylor rule. Real dividends increase by 6.6% on impact, before dropping to negative 1% in quarter 12 and then converging back to steady state. Finally, over time the real exchange rate appreciates more than it did under the Taylor rule.

The model responses demonstrate the significance of real income effects and dividends in the absence of an endogenous response of monetary policy. With the domestic interest rate pegged, the endogenous policy channel is effectively muted and does not actively drive down the real return on assets. The competitiveness channel, although no longer compromised by an appreciation in the nominal exchange rate, appears to be offset by forces driving up domestic inflation, again resulting in the terms of trade improving over time. A possible effect path that sets in motion the changes in output and consumption is related to the initial drop in the value of firm shares, on display in Figure C.2 in Appendix C.2. This suggests that the drop in the real rate of return and thus an initial uptick in consumption caused by intertemporal substitution is now driven by rising dividends. As before, lower marginal costs can induce deflationary pressure which, in the absence of a monetary policy reaction, would raise the real interest rate. Given this response, the upwards jump in real (future) dividends causes the observed reduction in firm share value. The composition of the mutual fund at  $t = 0$  then implies that a drop of the real rate of return on household assets, on display in Figure 2. From here, the effects are analogous to the case of inflation targeting. Households substitute towards consumption, which raises aggregate demand, output, and employment. Unions set higher wages, increasing consumption via the real income channel and ensuing multiplier effects. High dividends continue to contribute to the fall in share value and real returns until the effect is offset by rising inflation. The latter increases over time since rising wages put upwards pressure on marginal costs.

The right panel of Figure 3 shows the contribution of changes in the real rate of return and real wages to the response of consumption under monetary policy. On impact, the increase in the real rate increases consumption by 1.5%. Real wages increase consumption by 6.4%, thus accounting for more than 80% of the initial response. The increased strength of the real income effect relative to the case of inflation targeting is explained by the overall stronger response of real wages and the weaker response of real returns in the absence of an endogenous policy channel. By the fifth quarter, the rise in consumption is entirely driven by the real income effect. The similarities between the composition of consumption responses under a Taylor rule and a monetary union is an important result



for the understanding of the transmission mechanism behind labour cost reductions in HANK models. Two findings are central: (i) The adjustment of employment in response to labour cost reductions does not require an active endogenous policy channel.<sup>12</sup> (ii) Real income and multiplier effects play a dominant role in the transmission under both monetary regimes.

### 6.3 Heterogeneous Effects on Households

Heterogeneity in households' ability states, discount factors, MPCs, and initial asset holdings leads to unequal effects of the payroll tax cut across the distribution. The Euler equation implies that changes in consumption driven by intertemporal substitution are increasing in the discount factor and thus stronger among wealthier households. While these households also benefit relatively more from wage increases given their higher productivity, they rely relatively more on financial income and thus experience a larger compression of financial wealth when real returns fall. On the other hand, the higher marginal propensity to consume among poorer households as well as the greater reliance on wage income at low asset levels strengthens the effect transmitted via the real income channel.<sup>13</sup>

Figure C.2 in Appendix C.2 includes individual impulse responses for relatively poor, middle, and wealthy households. Under a Taylor rule, poor households see a jump in consumption by 1.9% on impact, households in the middle by 1.7%, and wealthy households by 1.5%. The greater increase in consumption among households with lower initial wealth suggests that the equalizing force of the income effect outweighs the uneven effects of intertemporal substitution. The decomposition of the consumption response into the real rate and the real income effect for each of the household groups in Figure 13 of Appendix C.2 further illustrates the distributional importance of the income effect. While the effect of intertemporal substitution is relatively stable across household groups, the effect of the income channel reduces substantially with higher wealth levels, raising consumption by 1.5% for poor, 1.3% for middle, and 1.1% for wealthy households. Given their higher real income, households partially substitute back towards saving as real returns converge back to steady state. This effect is increasing in the wealth level. Assets peak at 16% above their steady state level for poor, at 23% for middle, and at 39% for wealthy households.

The case of the monetary union in Figure C.2 of Appendix C.2 presents a similar picture.

<sup>12</sup>Appendix C.3 explores the response of a model with muted dividend channel and finds that even without a direct effect of the payroll tax on dividends, labour cost reductions are effectively transmitted.

<sup>13</sup>Auclert et al. (2021b) and Otten (2021) show that abstracting from the assumption of homothetic preferences induces additional distributive effects via changes in the terms of trade, which I discuss in section 8.

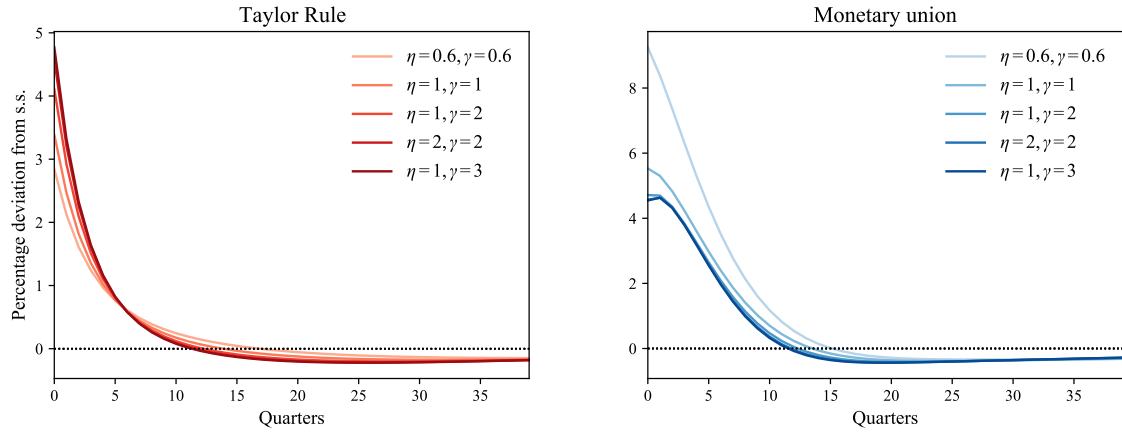
Discrepancies between the household groups for increases in consumption and assets, however, are stronger than under the Taylor rule. Total consumption initially increases by 2.9% for poor, 2.7% for middle, and 2.3% for wealthy households. Here, the real income channel accounts for a consumption increase of 2.5% for the poorer, 2.1% for middle and 1.8% for the wealthy group. Assets peak at 26.6% above steady state for poor, 36.2% for middle, and 50.4% for wealthy households. Overall, the temporary reduction in the payroll tax reduces consumption inequality in the short run. Over the long run, however, relatively higher saving among wealthy households exacerbates wealth inequality. While this effect is seen under both monetary policy regimes, it is especially pronounced in a monetary union, where income and multiplier effects are stronger.

## 6.4 The Role of Trade Elasticity

As laid out in section 6.1, the overall response to shocks in an open economy HANK model may depend on the level of trade elasticities and openness. So far, the analysis assumed unit elasticity for the substitution between home and foreign goods,  $\eta$ , as well as between varieties of foreign goods,  $\gamma$ . This calibration co-insides with the level of overall trade elasticity  $\chi = 2 - \alpha$ , which Auclert et al. (2021b) identify as the level establishing equivalence between the representative and the heterogeneous agent case. Figure 15 shows the response of employment under the two monetary policy regimes for varying levels of trade elasticities. Apart from the baseline case, both lower and higher levels of elasticities are considered. Responses of additional variables are reported in Figure 15 of Appendix C.3.

Although unit elasticity does not establish exact equivalence between the HANK model considered here and the representative agent model of Galí and Monacelli (2016) for the case of a Taylor rule, the responses are similar. In both models, employment rises by close to three percent on impact. As expected, lower levels of trade elasticities result in a smaller response of employment. With  $\eta = 0.6$  and  $\gamma = 0.6$ , which yields  $\chi \approx 1$ , the increase on impact reduces to 2.8%. At higher elasticities, the initial jump in employment increases to 4.1% with  $\eta = 1$  and  $\gamma = 2$ , 4.6% with both  $\eta$  and  $\gamma$  equal to 2, and 4.8% with  $\eta = 1$  and  $\gamma = 3$ . The intuition behind these responses is straightforward: Lower levels of elasticity mean that consumers at home and abroad react less strongly to the relative decrease in the price for home goods. This causes weaker substitution towards home goods and thus a weakening of the competitiveness channel. As a greater share of consumption falls on foreign goods, the multiplier effect is weakened and with it the response of output and employment. At higher levels of elasticity, this effect is reversed.

Figure 4: The response of consumption under different trade elasticities



*Note:* The above plots show impulse responses of consumption to an exogenous 10% cut in the payroll tax under a Taylor rule (left panel) and a monetary union (right panel) for various levels of the elasticity of substitution between home and foreign goods,  $\eta$ , as well as between varieties of foreign goods,  $\gamma$ .

In the case of a monetary union, the response of employment is strongest at low levels of elasticity. With  $\eta = 0.6$  and  $\gamma = 0.6$ , the increase on impact is to 9.3%, compared to the baseline of 5.5% and values around 4.5% for higher elasticities. Although the effect points in the opposite direction, the intuition behind the results runs analogous to the case of the Taylor rule. As seen in section 6.2.2, a labour cost reduction under a monetary union leads to an increase in domestic prices. With a constant nominal exchange rate, however, the price for foreign goods faced by domestic households remains constant. At higher levels of elasticities, this incentivizes consumers to substitute towards foreign goods, thus weakening the output multiplier and the response of employment at home. At low levels of elasticity, however, rising domestic prices do not lead consumers to substitute towards foreign goods so strongly. As a result, the multiplier remains strong and the response of employment rises. While the strong response of employment at elasticity below unity helps to shine light of the role of relative prices, such low values are not necessarily supported by the empirical literature (see e.g. Feenstra et al., 2018). When considering elasticities at or above unity, relative differences between monetary regimes and elasticity levels are much smaller.

Overall, the differences between the responses to a one-off exogenous payroll tax cut of the two monetary regimes stand in stark contrast to the results by Galí and Monacelli (2016). Galí and Monacelli (2016) come to the conclusion that a country in a monetary union may not reap any benefits by making labour cost components more sensitive to cyclical conditions, pointing to the lack of an endogenous policy channel. The results in

this section, however, have shown that the output response is primarily driven by direct income and multiplier effects which do not rely on a monetary policy response. Therefore, an economy that is part of a monetary union (or has adopted a hard peg) may succeed in achieving faster employment adjustment by increasing wage flexibility. An analysis of the potential welfare implications of such policies is the subject of the next section.

## 7 Wage Flexibility and Welfare

The previous section has focused on an exogenous reduction in the payroll tax to assess the transmission mechanism between changes in labour costs and the level of employment and output. While payroll tax cuts are relatively rare events, adverse shocks to foreign demand or foreign interest rates are viewed as common sources of business cycle fluctuations in small open economies (Galí and Monacelli, 2016). The relevance of the transmission mechanism, therefore, lies primarily with the endogenous adjustment of labour costs during such fluctuations. As argued above, the sensitivity of wages to changes in economic conditions, which in New Keynesian models is expressed in terms of the degree of wage rigidity,  $\theta_w$ , has been commonly viewed as a central determinant of employment stability.

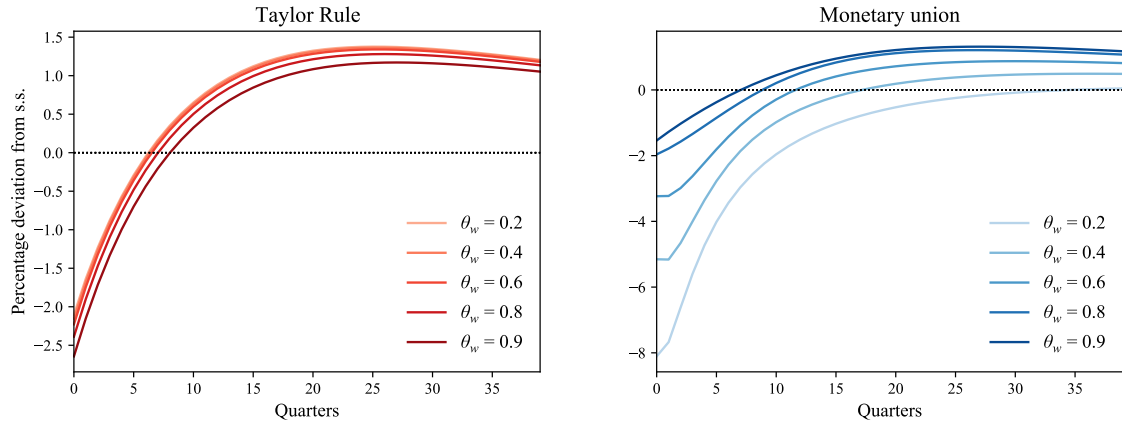
The intuition of standard New Keynesian models suggests that the pass-through of a reduction in labour costs to the overall level of employment relies primarily on the endogenous response of monetary policy. If this were the case, economies in a monetary union would be more susceptible to adverse shocks, irrespective of the level of wage rigidity. While such fluctuations are costly for households, increases in wage flexibility may be costly in their own right, as they exacerbate wage and price fluctuations. Consequently, wage flexibility in monetary unions can result in relative welfare losses, a central finding of Galí and Monacelli (2016). However, the findings of the previous section have shown that due to income and multiplier effects, employment in a small open economy HANK model does response to changes in labour costs, even in a monetary union.

To find out how the novel transmission mechanism affects the results of Galí and Monacelli (2016), I repeat their analysis using a small open economy HANK model faced with adverse shocks to the foreign interest rate and foreign demand. Below, I first assess overall responses at varying levels of wage rigidity. I then present an analysis of relative welfare losses.

## 7.1 Wage Flexibility and the Response to Adverse Shocks

To investigate the role played by wage flexibility in the transmission of foreign shocks, this section compares impulse responses to adverse foreign shocks under different levels of nominal wage rigidity. Consider first, the response of a small open economy to a shock to the foreign interest rate, induced by movement in the foreign interest rate shifter  $\mathcal{B}_t$ . The response of aggregate household utility (3) is shown in Figure 5, while responses of additional relevant variables are plotted in Figure 16 of Appendix D.1. Both under a Taylor rule and under a monetary union, a temporary increase of the foreign interest rate by 100 basis points and a persistence of 0.9 results in an initial reduction of utility. Under a Taylor rule, the severity of the reduction is slightly dampened by higher wage flexibility. In a monetary union, flexibility strongly exacerbates the initial drop. The intuition behind these diverging relationships is explored below.

Figure 5: Utility response to a foreign interest rate shock under varying wage rigidity



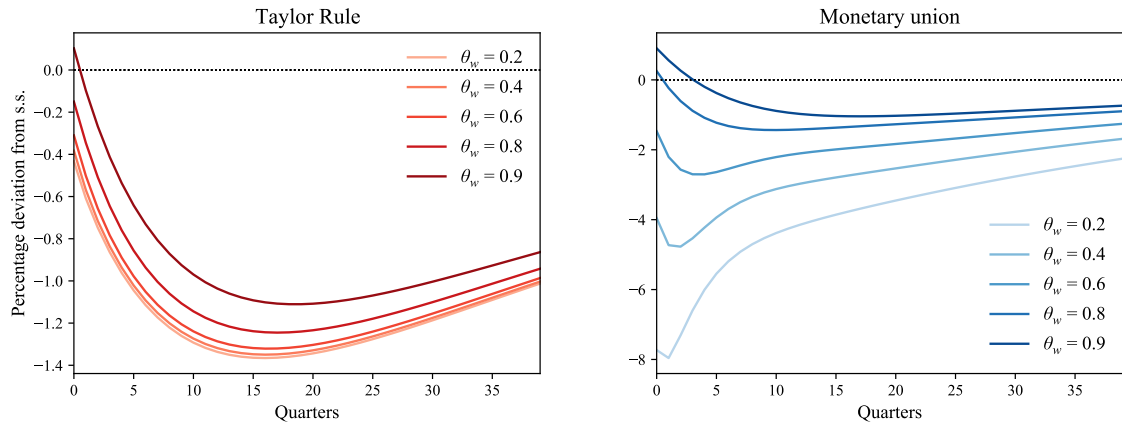
*Note:* The above figure shows impulse responses of aggregate household utility under a Taylor rule (left panel) and a monetary union (right panel), to an increase in the foreign interest rate by 100 basis points at a persistence of 0.9 for various levels of nominal wage rigidity  $\theta_w$ .

Under a Taylor rule, the increase in foreign interest rates leads to an upwards jump of the nominal exchange rate and thus deteriorating terms of trade. This change in relative prices has opposing effects on consumption and employment. It raises the demand for exports and thus production and employment, while decreasing the real income at the disposal of households and with it consumption. Since rising production causes domestic inflation and interest rate hikes, real income is further reduced and the substitution towards saving incentivized. This further compresses consumption. The response of utility depends on the relative size of employment and consumption responses as well as on the additional parameters of the utility function. How do these reactions depend on the

level of wage rigidity? If wages are flexible, unions can raise them more as employment increases, which increases marginal costs for firms. This exacerbates inflation and interest rate hikes, compressing consumption relatively more via the real income and the real rate channel. The negative effect this has on output and employment is amplified by the multiplier effect discussed in section 6.1. The relatively weaker increase in employment under flexible wages reduces households' disutility from labour and dampens the negative utility response, resulting in a positive correlation between flexibility and utility.

In a monetary union, the domestic nominal interest rate moves one for one with the foreign interest rate. This elicits a rise in real returns and incentivizes households to substitution away from consumption. The reduction in aggregate demand pulls down output and employment, setting in motion additional income and multiplier effects. If wages are flexible, these effects are stronger. Given the drop in employment levels, flexible wages reduce more sharply, further reducing real income and consumption. In addition, lower wages reduce marginal costs and domestic prices. This stimulates export demand, partly offsetting the initial drop in output and employment and increasing it over time. As a result, employment is relatively less affected by wage flexibility while consumption levels contract relatively more, giving rise to the negative correlation of utility and wage flexibility.

Figure 6: Utility response to a foreign demand shock under varying wage rigidity



*Note:* The above figure shows impulse responses of aggregate household utility under a Taylor rule (left panel) and a monetary union (right panel), to a decrease in foreign demand by 10% at a persistence of 0.9 for various levels of nominal wage rigidity  $\theta_w$ .

Figure 6 shows the response of aggregate household utility to a temporary drop in foreign demand by 10% at a persistence of 0.9, induced by movements in the foreign demand shifter  $\mathcal{E}_t$ . Responses of additional relevant variables are shown in Figure 17 of Appendix D.1. Under both monetary regimes, the drop in foreign demand results in a reduction of

utility over time. The level of utility, both initially and over time, is negatively correlated with wage flexibility. In a monetary union, however, this correlation is significantly stronger and the overall response is almost an order of magnitude larger.

Under a Taylor rule, the drop in foreign exports leads to a contraction of output and a drop in employment. The contraction sets in motion negative income and multiplier effects that lead to a prolonged reduction in consumption. Importantly though, deflationary pressure elicits an accommodative monetary policy response, partly offsetting the fall in consumption and employment. Higher wage flexibility affects this response in two ways. First, negative income effects are exacerbated by a stronger drop in wages. However, lower wages reduce marginal costs, which adds deflationary pressure and results in a stronger monetary policy response. These two opposing forces result in relatively higher employment and marginally higher consumption at low levels of flexibility. The relatively weak negative correlation between utility and flexibility is thus driven by rising disutility from labour.

The intuition for the case of a monetary union is similar to the Taylor rule case. The drop in exports leads to an output contraction and thus reductions in employment and consumption driven by real income and multiplier effects. In the absence of a monetary policy, deflationary pressure no longer results in an interest rate cut, leaving the contraction more severe. Wage flexibility exacerbates the drop in consumption and employment since real wages decrease significantly more on impact, adding to the negative income and multiplier effects. However, lower wages also increase deflationary pressure via falling marginal costs, thus increasing the competitiveness of firms. This results in a relatively swifter surge in exports and a quicker recovery of output and employment. The relatively more severe drop in consumption and the quicker convergence of employment back to steady state explain the strong negative correlation between utility and wage flexibility.

The above examples illustrate differences in the role of wage flexibility for the transmission of shocks under the two monetary regimes. Under a Taylor rule, wage flexibility amplifies income and multiplier effects as well as the response of monetary policy. If a shock causes inflationary pressure, these forces jointly dampen increases in employment. If a shock causes deflationary pressure, the two forces oppose each other, leaving consumption less affected by the level of flexibility. In either case, variations in the utility responses are relatively small and primarily driven by changes in employment. Under a monetary union, strong negative income and multiplier effects exacerbate drops in consumption when wages are flexible, while drops in employment are partly offset by rising firm competitiveness. Utility responses, therefore, exhibit a strong negative correlation with wage flexibility. Despite raising the responsiveness of employment and competitive-

ness in a monetary union, wage flexibility thus appears to exacerbate the adverse effect of foreign shocks due to negative income and multiplier effects.

## 7.2 Welfare Analysis

To offer a more complete picture of the welfare effects associated with the level of wage flexibility when facing foreign shocks, I examine the sum of utility deviations over a ten year horizon, relative to the baseline wage rigidity level of  $\theta_w = 0.8$ . The calculation of these welfare losses differs from the approaches in the representative agent literature because it does not rely on a first order approximation of the utility function. Galí and Monacelli (2016) use a first order approximation that allows for the expression of welfare as a linear combination of the variances of the employment gap, price inflation, and wage inflation. While this approach facilitates the decomposition of welfare losses into distinct channels, it is not easily applicable to the heterogeneous agent case, where individual consumption levels are contingent on ability states and asset levels. A simple aggregation based on price and wage inflation would disregard these differences and therefore present an inaccurate picture of aggregate welfare effects. Therefore, I instead calculate welfare for each individual household as the sum of utility deviation from its respective steady state level over a ten year horizon.<sup>14</sup> This is done for the same shock at various levels of wage rigidity  $\theta_w$ . Relative welfare loss is then calculated by normalizing each welfare value with the welfare under the baseline level of wage rigidity,  $B$ , where  $\theta_w = 0.8$  according to:

$$L_{\theta_w} = -\log \left( \frac{\sum_{t=0}^{40} \left[ \left( u_t \left( C_t^{\theta_w} \right) - v_t \left( N_t^{\theta_w} \right) \right) - (u(C_{ss}) - v(N_{ss})) \right] + k}{\sum_{t=0}^{40} \left[ \left( u_t \left( C_t^B \right) - v_t \left( N_t^B \right) \right) - (u(C_{ss}) - v(N_{ss})) \right] + k} \right) \quad (50)$$

Here,  $L_{\theta_w}$  is the relative welfare loss at the respective level of wage rigidity  $\theta_w$  and  $k = 10$  is a constant that is sufficiently large to ensure both the nominator and the denominator are positive. Without the addition of the constant, ratios between two values with negative sign would be identical to ratios between the same values with positive sign. Expressing relative losses on the log scale, the relative welfare loss at baseline rigidity,  $L_B$ , is normalized to 0.

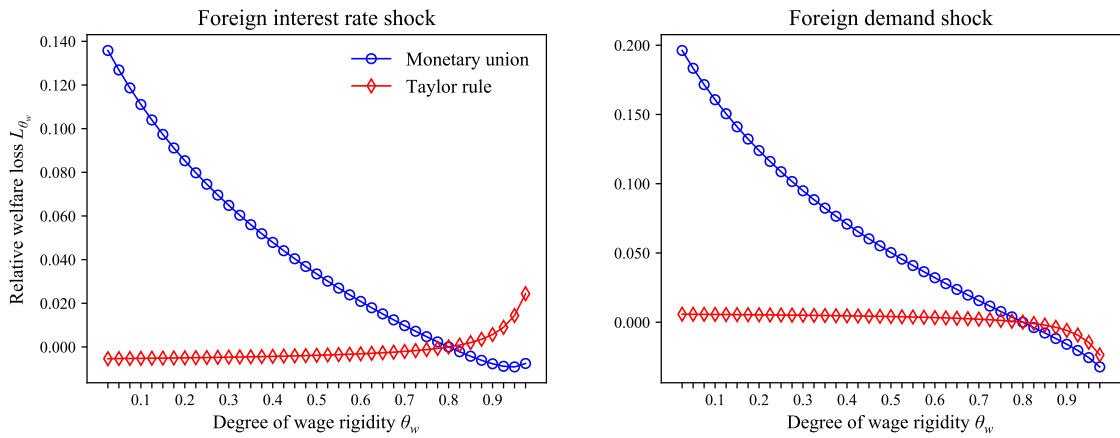
The left panel of Figure 7 shows losses associated with the foreign interest rate shock introduced earlier. In the case of a Taylor rule, relative welfare losses are monotonically

<sup>14</sup>Cho and Ma (2022) follow a similar approach to compare welfare effects of business cycles between individual households. For approximations of the welfare function in a stylized two agent HANK models see, e.g., Bilbiie (2018) or Bilbiie and Ragot (2021).



increasing in rigidity, varying relatively little at lower levels of rigidity but increasing as rigidity levels approach unity. As seen above, the reduction in losses is driven by relatively weaker increases in employment, which reduce disutility from labour. If wages are flexible, increases in employment caused by surging exports are partly offset by stronger monetary policy as well as by income and multiplier effects. In a monetary union, the observed relation between wage rigidity and relative welfare loss is strong and negative. Here, more flexible wages decrease utility by amplifying negative income and multiplier effects on consumption in a near linear fashion, while rising firm competitiveness elevates disutility from labour over time.

Figure 7: Welfare losses from foreign shocks under varying nominal wage rigidity



*Note:* The above figure plots relative welfare losses at various levels of nominal wage rigidity, defined as the sum of utility deviations over a ten year horizon, relative to the baseline wage rigidity level,  $\theta_w = 0.8$ , in response to a temporary increase in the foreign interest rate by 100 basis points (left panel) as well as a temporary decrease in foreign demand by 10% (right panel).

Welfare losses associated with an adverse shock to foreign demand are plotted in the right panel of Figure 7. The Taylor rule case again exhibits a rather flat slope, with losses decreasing slightly at high levels of wage rigidity. As seen above, the relatively weak correlation between flexibility and welfare is caused by the opposing forces of income and monetary policy effects. Here, flexible wages exacerbated inflationary pressure and cause both a reduction in real income and a stronger rate cut. Reductions in welfare loss are primarily driven by lower disutility from labour. The case of the monetary union again exhibits a strong negative correlation between relative welfare loss and wage rigidity. As before, income and multiplier effects drive down consumption and rising competitiveness raises employment over time. Both effects are strengthened by wage flexibility, increasing relative welfare losses.

The results on the heterogeneous effects of a payroll tax cut in section 6.3 demonstrate

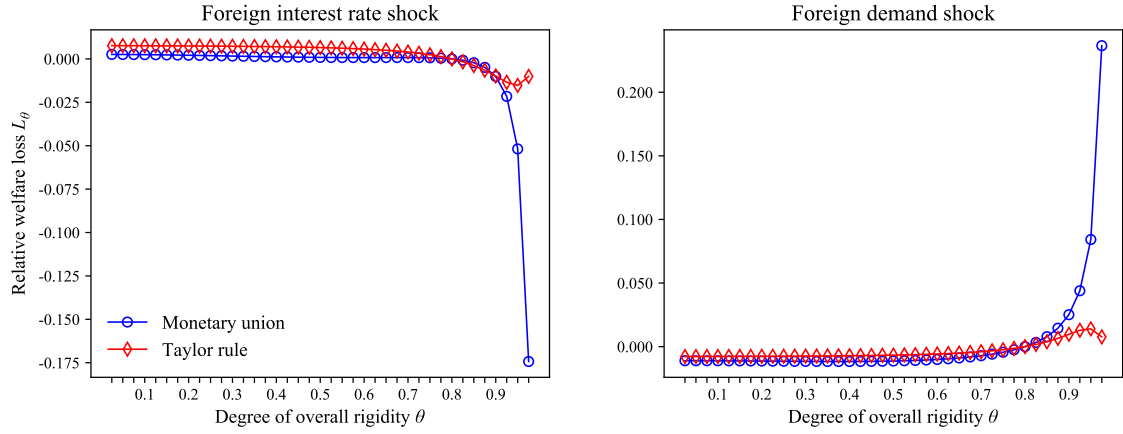
that real rate and real income effects differ across household groups. The consumption of poorer households are more affected by income and multiplier effects, while wealthy households exhibit stronger saving responses over time. These effects apply also to the response to foreign shocks. Figure 22 in Appendix D.2 plots relative welfare losses for the three household groups along the wealth distribution. Under a Taylor rule, relative losses are nearly identical between groups, while under a monetary union, the welfare loss of poorer households is more sensitive to wage flexibility. This disparity is explained by the greater exposure of households in a monetary union to income and multiplier effects. Thus, not only does the welfare of households in a monetary union react stronger to changes in wage rigidity, it reacts non-uniformly across wealth shares of the population and harms poorer households disproportionately more.

Given the results on the effect of trade elasticity in section 6.4 and the nature of the utility function in (3), the elasticity parameters  $\eta$  and  $\gamma$ , as well as the inverse Frisch elasticity,  $\varphi$ , are potentially important factors in determining welfare effects. To ensure robustness, the analysis is therefore repeated at varying levels of the three parameters. The results of these robustness checks are reported in Appendix D.2. Results for the Taylor rule case are robust to variations of the parameters. Here, only welfare responses to a foreign demand shock show a slight decrease in sensitivity at higher levels of trade elasticity. The general shape of the loss curve, however, remains unchanged. Results for the monetary union likewise remain true to the shape of the baseline curve. However, both higher values of the inverse Frisch elasticity and higher values of trade elasticities reduce the sensitivity of relative welfare losses to increasing wage flexibility for all shocks. The intuition behind these results is simple. First, a high inverse Frisch elasticity raises the importance of disutility from labour, which, given the relatively stronger negative response of employment under flexible wages, reduces overall welfare losses. Second, high trade elasticities raise the importance of relative prices to households, resulting in a stronger substitution towards home goods as domestic prices fall. This substitution weakens the drop in consumption and the negative multiplier, offsetting part of the welfare loss under flexible wages.

In addition to analysing the impact of changes in the degree of wage rigidity while keeping the rigidity of prices unchanged at  $\theta_H=0.8$ , Galí and Monacelli (2016) consider the welfare effects of changing wage and price rigidity in unison. Figure 8 shows the replication of this analysis for the HANK case. Welfare losses are plotted for identically varied values of  $\theta_w$  and  $\theta_H$ , denoted as overall rigidity  $\theta$ . The result that stands out is that the shape of the welfare curve now exhibits a similar shape for both monetary regimes. Variations in rigidity below the baseline level lead to very little variation in relative welfare losses. At rigidities closer to unity, however, both regimes experience relative welfare gains in response to a foreign interest rate shock and relative losses in response to a for-

eign demand shock. To help understand these results, figures 18 and 19 in Appendix D.2 plot response functions for varying levels of overall elasticity. For the interest rate shock, the relative differences of consumption and employment responses is small if wages are flexible. Very rigid wages, however, increase consumption gains over time and thus result in relative welfare gains. Here, price rigidity dampens inflation, resulting in less restrictive monetary policy under a Taylor rule and a smaller adverse effect on real wages under both monetary regimes. In responses to the foreign demand shock, high rigidity means lower consumption levels. Here, rigidity reduces deflationary pressure. Under a Taylor rule, this limits positive monetary policy and income effects. In a monetary union, positive effects of lower domestic prices likewise exacerbate drops in real income and reduce gains in competitiveness.

Figure 8: Relative welfare loss at varying overall rigidity



*Note:* The above figure plots relative welfare losses for various identical levels of nominal wage rigidity and nominal price rigidity, defined as the sum of utility deviations over a ten year horizon, relative to the baseline rigidities,  $\theta_w = \theta_H = 0.8$ , in response to a temporary increase in the foreign interest rate by 100 basis points (left panel) as well as a temporary decrease in foreign demand by 10% (right panel).

The results of the analyses above stand in contrast to the findings by Galí and Monacelli (2016), who report a monotonically increasing relationship between nominal wage rigidity and welfare loss in the case of a Taylor rule for both foreign shocks and a non-monotonic relationship for the case of a monetary union. When jointly varying both wage and price rigidity, Galí and Monacelli (2016) find that flexibilization is desirable up to a threshold of  $\theta$  between 0.7 and 0.8. The differences with the results presented here confirm the significance of the novel transmission dynamics in a small open economy HANK model. While variations in welfare are relatively small in the case of a Taylor rule, they underline the importance of interactions between monetary policy with income and multi-

plier effects, all of which are amplified by wage flexibility. The case of a monetary union puts the significance of transmission dynamics on clear display. Here, wage flexibility facilitates strong consumption responses, driven by unhinged income and multiplier effects. These dynamics amplify negative responses to adverse external shocks. While wage flexibility does raise firm competitiveness and thus the responsiveness of employment, the stronger contractions in consumption and output lead to substantial welfare losses relative to higher degrees of rigidity. Poorer households, who are subject to the greatest fluctuations in consumption under flexible wages, suffer the greatest losses. When jointly changing wage and price rigidity, both monetary regimes show only minuscule variations in utility at low rigidities. High levels of rigidities, however, result in relative welfare gains for a foreign interest rate shock and relative losses for a foreign demand shock.

A central conclusion by Galí and Monacelli (2016) is that wage flexibility may not be particularly desirable in a monetary union. The here presented results support this statement, albeit with an important qualification. Three conclusions can be drawn for a small open economy faced by adverse foreign shocks: (i) Wage flexibility does increase the responsiveness of employment in the face of adverse foreign shocks. (ii) In a monetary union, wage flexibility results in substantial relative welfare losses, especially for poorer households, driven by income and multiplier effects. For a Taylor rule, welfare effects are smaller and co-determined by the monetary policy response. Here, reducing flexibility results in relative losses for the foreign interest rate shock and relative gains for the foreign demand shock. (iii) Under both monetary regimes, a joint decrease of price and wage flexibility results in relative gains for an interest rate shock and relative losses for a foreign demand shock, while increasing flexibility has only minimal effects.

## 8 Discussion

This section discusses some implications as well as potential limitations of the analyses presented in sections 6 and 7.

### 8.1 Implications

Section 6 found that reductions in labour costs in the form of a temporary payroll tax cut significantly impact output and employment. This finding holds for the case of an economy under a Taylor rule and, contrary to the standard New Keynesian small open economy model of Galí and Monacelli (2016), for the case of a monetary union. In both cases, the primary driving forces behind the adjustments are income and multiplier

effects, stemming from the direct impact of output on wages and consumption. Since these effects have a stronger impact on poorer households, labour cost reductions also have distributional consequences. Two implications of these results are worth underlining.

First, the importance of the novel transmission mechanism in HANK models has the potential to cast new light on policy interventions in small open economies. While the explicit role played by income and multiplier effects has been the subject of extensive study in the closed economy HANK literature, open economy applications have thus far focused primarily on general effects of monetary policy and its distributional implications. Auclert et al. (2021b), whose decomposition of transmission effects informs part of the analysis in section 6, is an important exceptions. They establish benchmarks for the equivalence of shock responses in representative and heterogeneous agent models for a Taylor rule. Given the strength of income and multiplier effects explored in section 6, similar equivalence is unlikely in a monetary union. As a result, policies which, like the here considered payroll tax cut, are reliant on the endogenous response of monetary policy in a standard New Keynesian setup, could prove effective when viewed through the lens of a HANK model.

Second, although aggregate results under a Taylor rule are similar to the representative agent case, the prominence of new driving forces has distributional implications under for all monetary regimes. Income and multiplier effects have a stronger impact on poorer households, who have a higher marginal propensity to consume and cannot rely on asset returns as a source of income to the same degree as wealthier households. The payroll tax cut, therefore, raises the consumption of poorer households, especially in a monetary union. This finding is in line with distributional effects reported in the open economy HANK literature. Guo et al. (2020), as well as Otten (2021), for instance, find a trade-off between aggregate stabilization provided by a Taylor rule, and the reduction of consumption inequality under a currency peg. A contributing factor to this dynamic that has not been considered in the analysis is that poorer households spend a relatively larger share of their income on imports (see, e.g., Carroll and Hur, 2020), a fact that Auclert et al. (2021b) and Otten (2021) incorporate into their models by assuming non-homothetic preferences. Given the observed improvement in the terms of trade, the consumption of poorer household could therefore rise even more in response to labour cost reductions. In addition to the effects on consumption, the results show a relatively stronger asset increase for wealthy households over time. This implies that economies might in fact face a trade-off between consumption equality on one side and stability as well as wealth equality on the other.

The findings of section 6 point to the question whether policies that increase wage flexibil-

ity, aimed at making labour costs more sensitive to cyclical conditions, could prove an effective stabilization tool in a HANK model. Following Galí and Monacelli (2016), section 7, therefore, assessed impulse responses and welfare effects for adverse foreign shocks at various levels of wage rigidity. The findings show that employment does become more responsive if wages are flexible. Negative income and multiplier effects, however, are likewise amplified and have adverse implications for output, welfare, and equality in a monetary union. These results are an important addition to the findings of the representative agent literature like Galí and Monacelli (2016), Bustos and Uras (2021), or Okano (2020). As the example of wage flexibility shows, HANK models reveal the risk of structural reforms amplifying negative consumption and output dynamics. Indeed, structural reforms like the EU/IMF adjustment programmes in Greece and Portugal initially under-shot anticipated effects on employment and output while observing considerable drops in domestic consumption (EC et al., 2011; EC et al., 2014). While these trends have a multitude of driving factors, HANK models can contribute to their understanding by providing plausible theoretical intuition from a general equilibrium point of view.

## 8.2 Limitations

The model setup gives rise to several limitations. Three of the modelling choices, which have been shown to have potential relevance for the transmission of shocks and the distribution of wealth, are worth highlighting. (i) First, several authors choose to explicitly model capital and public investment. This choice allows the introductions of capital adjustment costs which, although irrelevant for aggregate results, have been shown to reduce the importance of labour income in the transmission of interest rate shocks (Alves et al., 2020). In addition, the presence of public investment could lead to higher interest rates and crowd out private investment, altering the consumption and saving behavior of households (see, e.g., Hagedorn et al., 2019, on the effect of crowding out on the fiscal multiplier). (ii) A second modeling choice that affects investment concerns the limits, composition, and denomination of household portfolios. De Ferra et al. (2020) adapts a looser borrowing constraint and finds that households who hold much of their debt in foreign currency experience a disproportional reduction in wealth if the domestic currency depreciates. Because the exchange rate is affected by the shocks considered in section 7, the ability to borrow in foreign currencies could affect aggregate and distributional results. Oskolkov (2021) entirely precludes the ability for households to invest in foreign assets. Given the substantial rise in net foreign assets on display in section 6.2, a limit on investment could alter household behavior. However, the responses to a foreign interest rate shock reported by Oskolkov (2021) are in line with my results for both monetary

regimes. In addition, Oskolkov (2021), too, finds stronger income effects for poor households, lending support to the distributional implications of my analysis. (iii) A third issue concerns assumptions related to prices and exchange rates. Following Galí and Monacelli (2016), I assume producer currency pricing throughout the analysis. However, several authors stress that import prices are increasingly invoiced in dollar or euro, especially among importers in the euro area (see, e.g., Boz et al., 2020). Auclert et al. (2021b) show that under dollar currency pricing, output reacts less to changes in the exchange rate since the expenditure switching channel becomes weaker. The direction and the order of magnitude of the effect, however, remains unchanged. An additional assumption has been the perfect pass-through of exchange rates to imports and exports. Auclert et al. (2021b) find that abandoning this assumption decreases the strength of real income and competitiveness effects, further reducing the overall impact of an exchange rate depreciation. Abandoning these assumption, i.e., adopting Euro invoicing of imports and imperfect exchange rate pass-through, would likely reduce the size of the output effect in the analysis. A general reversal of the effects and more far reaching implications for welfare, however, are unlikely.

Additional limitations concern the experimental setup of the analysis, i.e., the assumption of an exogenous payroll tax cut, as well as the interpretation of the results. While the payroll tax cut serves as a clear benchmark to assess the effects of a reduction in labour costs, implications beyond the identification of transmission dynamics have to be judged with care. In practice, additional fiscal policy measures might be introduced to accompany labour market reforms. Okano (2020), for instance, introduces government investment as well as a cyclically adjusted output tax and shows that this makes employment more elastic with respect to domestic inflation. Another possible addition to the policy intervention could be the introduction of a tax on imports to complement the payroll tax cut. Farhi et al. (2014) show that such policies can affect the terms of trade while protecting domestic real prices (see Giagheddu, 2020, for a heterogeneous agent application). However, given the approach in Galí and Monacelli (2016), the assumption of an isolated payroll tax cut is necessary to identify differences in the transmission dynamics between representative and heterogeneous agent models. A related limitation concerns the results of the welfare analysis in 7. Here, the different nature of the model itself necessitates abstracting from the welfare loss function employed by Galí and Monacelli (2016). While this does not change the qualitative and relative comparison of welfare implications, it prevents a comparison of the absolute level of welfare loss in the two models.<sup>15</sup> The interpretation of results and the identification of driving forces are subject to another limitation that plagues

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<sup>15</sup>A comparison of consumption and employment effects is likewise impossible since Galí and Monacelli (2016) do not report impulse responses for foreign shocks.



most applications of HANK models: The degree of complexity makes them theoretically intractable. While transmission mechanisms can be identified in decompositions of more stylized model responses as in Appendix C.1, the interpretation of transmission dynamics in the here presented analysis is subject to a some degree of uncertainty given their general equilibrium nature.

## 9 Conclusion

In this paper, I use a small open economy HANK model to investigate the role of wage flexibility when facing adverse external shocks. Since potential stability benefits of flexibility rely on the responsiveness of employment to adjustments of labour costs, I first simulate a temporary payroll tax cut and compare reactions of an economy under a standard Taylor rule and a monetary union. Both monetary regimes exhibit significant responsiveness of output and employment. These adjustments are driven primarily by the effects of higher income on consumption as well as by a multiplier effect, i.e., the mutual reinforcement of consumption and output typically seen in HANK models. These effects function independent of the endogenous response of monetary policy, breaking with the intuition of standard New Keynesian models. An additional finding is that the consumption of poorer households is affected disproportionately more, leading to a decrease in consumption inequality in the short term. Wealth inequality, however, increases over time due to a stronger increase in assets among wealthier households.

In light of these novel transmission dynamics, I examine the role of wage flexibility in accommodating a foreign interest rate hike as well as a drop in foreign demand, assessing respective welfare losses. The analysis yields three central conclusions: (i) First, the responsiveness of employment increases with wage flexibility. (ii) Second, income and multiplier effects cause substantial welfare losses in a monetary union if wage flexibility increases. The effect is especially detrimental to poorer households. For a Taylor rule, welfare effects are smaller and co-determined by the monetary policy response. Here, reducing flexibility results in relative losses for the foreign interest rate shock and relative gains for the foreign demand shock. (iii) Third, under both monetary regimes, a one for one increase of price and wage flexibility has only minimal welfare effects, while a decrease of flexibility results in relative gains for an interest rate shock and relative losses for a foreign demand shock.

Together these findings cast new light on the transmission of labour cost adjustments in small open economies as well as on potential risks that structural labour market reforms may entail for aggregate stability and household welfare. Since such reforms could result



in a stronger contractions of aggregate demand, they have the potential to exacerbate recessions. The findings thus also point towards the potential importance of policies like social safety nets in mitigating drops in aggregate demand and providing output stability. Investigating the joint effects of such policies and structural labour market reforms in a HANK context is an avenue for future research. Two additional avenues for future research can be highlighted: First, as discussed in section 8, the model structure could be enriched by incorporating dollar currency pricing, imperfect exchange rate pass-through, or non-homothetic preferences. Second, analysing the responses to additional shocks like movements in domestic demand or productivity could yield further insights into welfare and transition dynamics.

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## A Appendix to section 3

### A.1 Household Demand Schedules

The derivation of household demand schedules in the domestic economy is analogous to Auclert et al. (2021b). However, rather than consuming a continuum of goods in the home economy, agents in this model only consume the aggregate product of the final good firm. The corresponding problem for the foreign economy is given below countries. Consumption  $c(a, e)_t$  of a household indexed by its state  $(a, e)$  aggregates the final home good  $H$  and a composite foreign good  $F$  with elasticity of substitution  $\eta > 0$ ,

$$c_t(a, e) = \left[ (1 - \alpha)^{\frac{1}{\eta}} (c_{Ht}(a, e))^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (c_{Ft}(a, e))^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (51)$$

$c_{HT}(a, e)$  is the consumption of the domestically produced final good, while foreign consumption aggregates goods produced in a continuum of countries  $k$ :

$$c_{Ft}(a, e) = \left( \int_0^1 c_{kt}(a, e)^{\frac{\gamma-1}{\gamma}} dk \right)^{\frac{\gamma}{\gamma-1}} \quad (52)$$

with elasticity of substitution between foreign countries  $\gamma > 0$ . Prices in foreign countries are identical due to symmetry. Given (52), the budget constraint of the household can thus be expressed as

$$\frac{P_{Ht}}{P_t} c_{Ht}(a, e) + \frac{P_{Ft}}{P_t} c_{Ft}(a, e) + a_{it+1} \leq (1 + r_t^p) a_{it} + e_{it} \frac{W_t}{P_t} N_t$$

where  $P_{Ft}$  is the price of the foreign aggregate good in terms of local currency and  $P_t$  is the CPI. Each household minimizes its total expenditure subject to the consumption basket (51). The corresponding Lagrangian and first order conditions are then given by

$$\begin{aligned} \mathcal{L} = & P_{Ht} c_{Ht}(a, e) + P_{Ft} c_{Ft}(a, e) + \lambda_t [c_t(a, e) \\ & - \left( \left[ (1 - \alpha)^{\frac{1}{\eta}} (c_{Ht}(a, e))^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (c_{Ft}(a, e))^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \right) \end{aligned} \quad (53)$$



$$c_{Ft}(a, e) = \alpha \left( \frac{P_{Ft}}{\lambda_t} \right)^{-\eta} c_t(a, e) \quad (54)$$

$$c_{Ht}(a, e) = (1 - \alpha) \left( \frac{P_{Ht}}{\lambda_t} \right)^{-\eta} c_t(a, e) \quad (55)$$

Inserting the first order conditions back into the expression for the consumption basket (51) and solving for  $\lambda$  yields the expression for the consumer price index (5):

$$\lambda_t = \left[ \alpha P_{Ft}^{1-\eta} + (1 - \alpha) P_{Ht}^{1-\eta} \right]^{1/(1-\eta)} = P_t$$

Reinserting into the first order conditions then yields the domestic demand schedules for home and foreign goods (6) and (7). The problem the representative household in each foreign country is analogous. Using the elasticity of substitution between foreign countries  $\gamma$  and noting the preference for domestic goods  $\alpha$ , the Lagrangian for the household in each of the foreign countries reads

$$\mathcal{L} = P_{Ht}^* c_{Ht}^* + P_{Ft}^* c_{Ft}^* + \lambda_t \left[ c_t^* - \left( \left[ \alpha^{\frac{1}{\gamma}} (c_{Ht}^*)^{\frac{\gamma-1}{\gamma}} + (1 - \alpha)^{\frac{1}{\gamma}} (c_{Ft}^*)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}} \right) \right]$$

Inserting the first order conditions into the foreign consumption basket to solve for the foreign consumer price index and reinserting yields the foreign demand for exports

$$c_{Ht}^* = \alpha \left( \frac{P_{Ht}^*}{P_t^*} \right)^{-\gamma} c_t^*$$

Which, after aggregating across symmetric foreign countries and adding the export demand shifter  $\mathcal{E}_t$ , corresponds to equation (10).

## A.2 Mutual Fund

This section derives the real value and real return for of household assets, as well as the UIP condition and the expression of net foreign assets from the problem of the domestic mutual fund. The same results can be found in the appendix of Auclert et al. (2021b). Letting  $\mathcal{A}_t$  be the fund's nominal liabilities and  $i_t^p$  the nominal return on these liabilities, then the nominal liquidation value of the financial intermediary at the beginning of period  $t$  is given by

$$(1 + i_t^p) \mathcal{A}_{t-1} = (D_t + p_t) P_t s_{t-1}^H + (1 + i_{t-1}) B_{t-1}^H + (D_t^* + p_t^*) P_t^* \mathcal{E}_t s_{t-1}^F + (1 + i_{t-1}^*) \mathcal{E}_t B_{t-1}^F \quad (56)$$

and the end of period  $t$  the value must be

$$\mathcal{A}_t = p_t P_t s_t^H + B_t^H + p_t^* P_t^* \mathcal{E}_t s_t^F + \mathcal{E}_t B_t^F. \quad (57)$$

The ex-post real return to the mutual fund is given by:

$$1 + r_t^p = (1 + i_t^p) \frac{P_{t-1}}{P_t} \quad (58)$$

The fund maximizes the expected real return on its liabilities,  $r_{t+1}^p$ , for all  $t \geq 0$ , which is equivalent to maximizing the expected nominal return  $i_{t+1}^p$ , and yields the equality of nominal returns on assets

$$\frac{(D_{t+1} + p_{t+1}) P_{t+1}}{p_t P_t} = 1 + i_t = \frac{(D_{t+1}^* + p_{t+1}^*) P_{t+1}^* \mathcal{E}_{t+1}}{p_t^* P_t^* \mathcal{E}_t} = \frac{(1 + i_t^*) \mathcal{E}_{t+1}}{\mathcal{E}_t} \quad (59)$$

Combining the nominal value of claims at the beginning of period  $t$  (56) and the equality of returns in (59), it follows that for all  $t \geq 0$ ,

$$1 + i_{t+1}^p = 1 + i_t$$

The ex-ante real risk free rate  $1 + r_t$  is defined in 20. Taking the definition of the real exchange rate in (8), real exchange rate depreciation between time  $t - 1$  and  $t$  can be expressed as

$$\frac{Q_t}{Q_{t-1}} = \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \frac{P_{t-1}}{P_t} \quad (60)$$

Combining expressions (58), (59), (20) and (60), and noting that  $P_t^* = 1$ , one can derive the expression for equality of real returns

$$1 + r_{t+1}^p = 1 + r_t = \frac{D_{t+1} + p_{t+1}}{p_t} = \frac{(D_{t+1}^* + p_{t+1}^*) Q_{t+1}}{p_t^* Q_t} = \frac{(1 + i_t^*) Q_{t+1}}{Q_t}$$

which is equation (19). Lastly, defining the real asset position of the mutual fund as

$$A_t \equiv \frac{\mathcal{A}_t}{P_t}$$

Equations (56) and (57) rewrite in real terms to

$$(1 + r_t^p) A_{t-1} = (D_t + p_t) s_{t-1}^H + (1 + i_{t-1}) \frac{P_{t-1}}{P_t} \frac{B_{t-1}^H}{P_{t-1}} + (D_t^* + p_t^*) Q_t s_{t-1}^F + (1 + i_{t-1}^*) Q_t \frac{B_{t-1}^F}{P^*}$$

and

$$A_t = p_t s_t^H + \frac{B_t^H}{P_t} + p_t^* Q_t s_t^F + Q_t \frac{B_t^F}{P^*}$$

which are equations (17) and (18). From (18) it follows that the value of the initial mutual fund portfolio  $s_{-1}^H, s_{-1}^F, \frac{B_{-1}^H}{P_{-1}}$  and  $\frac{B_{-1}^F}{P^*}$ , in steady state must add up to  $A_{ss}$ . Assuming an initial steady state where the fund holds only domestic stocks, it therefore follows that  $A_{ss} = p_{ss}$ , and the net foreign asset position  $nfa_{-1} = 0$ .

### A.3 Intermediate Goods Firm Price-Setting

The derivation of the Keynesian Phillips Curve is a special case of the price setting problem in Auclert et al. (2020). As apposed to Auclert et al. (2020), prices awaiting reset are not indexed to inflation and in assuming a Kimball superelasticity of 0, the Kimball aggregator reduces to the standard CES case. Intermediate good firms are local monopolists. Using the formulation for final good production (24), the final good firm solves the following problem:

$$\max_{Y_{jt}} P_{Ht} \left( \int_0^1 Y_{jt}^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}} - \int_0^1 P_{Hjt} Y_{jt} dj$$

The first order condition reduces to:

$$\frac{P_{Hjt}}{P_{Ht}} = \left( \int_0^1 Y_{jt}^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di \right)^{\frac{1}{\varepsilon_p - 1}} Y_{jt}^{-\frac{1}{\varepsilon_p}}$$

Raising both sides to the power of  $-\varepsilon_p$  and using (24) to aggregate product varieties yields the static demand curve faced by intermediate goods producers  $j$ :

$$Y_{jt} = \left( \frac{P_{Hjt}}{P_{Ht}} \right)^{-\varepsilon_p} Y_t$$

Next, inserting the demand into the equation for dividends of intermediate firms (27) to get

$$D(P_{Hjt}, P_{Ht}, s_t, Y_{jt}) = \left( \frac{P_{Hjt}}{P_{Ht}} - s_t \right) \left( \frac{P_{Hjt}}{P_{Ht}} \right)^{-\varepsilon_p} Y_t$$

and taking the derivative with respect to the current price of intermediate good  $P_{Hjt}$  yields the cost-minimising first order condition:

$$\begin{aligned} \frac{\partial D}{\partial P_{Hjt}} &= \left( \frac{1}{P_{Ht}} \right) \left( \frac{P_{Hjt}}{P_{Ht}} \right)^{\varepsilon_p} Y_t + \left( \frac{P_{Hjt}}{P_{Ht}} - s_t \right) (-\varepsilon_p) \left( \frac{P_{Hjt}}{P_{Ht}} \right)^{-\varepsilon_p - 1} Y_t \\ &= \left( \frac{P_{Hjt}}{P_{Ht}} \right)^{-\varepsilon_p} Y_t \left( \left( \frac{1}{P_{Ht}} \right) + \left( \frac{P_{Hjt}}{P_{Ht}} - s_t \right) (-\varepsilon_p) \frac{1}{P_{Hjt}} \right) \\ &= \left( \frac{P_{Hjt}}{P_{Ht}} \right)^{-\varepsilon_p} Y_t \left( \left( \frac{1}{P_{Ht}} \right) + \left( \frac{s_t}{P_{Hjt}} - \frac{1}{P_{Ht}} \right) \varepsilon_p \right) \end{aligned}$$

Conditional on setting the optimal price  $P^*$  in period  $t$ , intermediate good producers seek to maximize their firm value  $D(P_{Ht}^*; P_{Ht}, s_t, Y_t) + P_{Ht}(P_{Ht}^*)$ , where share price  $p_t$  can be expressed in terms of the ex-ante stock return, future dividends, and future share price as

$$p_t(P_t^*) = \frac{1}{1 + r_t} (D(P_{t+1}^*; P_{Ht+1}, s_{t+1}, Y_{t+1}) + p_{t+1}(P_t^*)),$$

which, under the assumption of Calvo price staggering with a probability of resetting prices  $1 - \theta_H$ , becomes:

$$\begin{aligned} p_t(P_t^*) &= \frac{1}{1 + r_t} [\theta_H (D(P_{Ht}^*; P_{Ht+1}, s_{t+1}, Y_{t+1}) + p_{t+1}(P_t^*)) + \\ &\quad (1 - \theta_H) \max_{\hat{p}} (D(\hat{p}; P_{Ht+1}, s_{t+1}, Y_{t+1}) + p_{t+1}(\hat{p}))] \end{aligned}$$

Neglecting the second part of the equation since it references firms who can make next period price adjustments, and removing recursiveness yields the share value for firms that can readjust their prices in period  $t$ :

$$p_t(P_t^*) = \max_{P^*} E_t \left[ \sum_{k=0}^{\infty} \theta_H^k \left( \prod_{q=t}^{t+k+1} \frac{1}{1 + r_q} \right) D(P_t^*; P_{Ht+k}, Y_{t+k}, s_{t+k}) \right].$$

The share price maximizing first order condition is:

$$\frac{\partial p_t(P_t^*)}{\partial P_t^*} = \sum_{k=0}^{\infty} \theta_H^k \left( \prod_{q=t}^{t+k+1} \frac{1}{1+r_q} \right) \left( \frac{1}{P_{Ht+k}} \left( \frac{P_t^*}{P_{Ht+k}} \right)^{-\varepsilon_p} Y_t + \left( \frac{P_t^*}{P_{Ht+k}} - s_t \right) \frac{P_t^{*- \varepsilon_p - 1}}{P_{Ht+k}^{\varepsilon_p}} (-\varepsilon_p) Y_{t+k} \right) = 0 \quad (61)$$

$$= \sum_{k=0}^{\infty} \theta_H^k \left( \prod_{q=t}^{t+k+1} \frac{1}{1+r_q} \right) \left( \frac{P_t^*}{P_{Ht+k}} \right)^{-\varepsilon_p} Y_{t+k} \left( \frac{1}{P_{Ht+k}} + \left( \frac{P_t^*}{P_{Ht+k}} - s_t \right) \frac{1}{P_t^*} (-\varepsilon_p) \right) = 0 \quad (62)$$

$$= \sum_{k=0}^{\infty} \theta_H^k \left( \prod_{q=t}^{t+k+1} \frac{1}{1+r_q} \right) \left( \frac{P_t^*}{P_{Ht+k}} \right)^{-\varepsilon_p} Y_{t+k} \left( \frac{1}{P_{Ht+k}} + \left( s_t - \frac{P_t^*}{P_{Ht+k}} \right) \frac{1}{P_t^*} (\varepsilon_p) \right) = 0 \quad (63)$$

Multiplying by  $P_t^*$  gives the first order condition in terms of the relative reset price  $P_t^*/P_{Ht+k}$  and corresponds to equation (51) in Auclert et al. (2020):

$$\begin{aligned} \frac{\partial p_t(P_t^*)}{\partial P_t^*} = E_t \left( \sum_{k=0}^{\infty} \theta_H^k \left( \prod_{q=t}^{t+k+1} \frac{1}{1+r_q} \right) \left( \frac{P_t^*}{P_{Ht+k}} \right)^{-\varepsilon_p} \right. \\ \left. Y_{t+k} \left( \frac{P_t^*}{P_{Ht+k}} + \left( s_t - \frac{P_t^*}{P_{Ht+k}} \right) (\varepsilon_p) \right) \right) = 0 \end{aligned} \quad (64)$$

Here, the latter part of the derivation can be defined as

$$f\left(\frac{P_t^*}{P_{Ht+k}}, s_t\right) = \left( \frac{P_t^*}{P_{Ht+k}} + \left( s_t - \frac{P_t^*}{P_{Ht+k}} \right) (\varepsilon_p) \right) \quad (65)$$

to be evaluated separately before total differentiating the first order condition. At the zero inflation steady state where future domestic price level and optimal firm price are constant at  $P_{Ht+k} = P_t^* = P_H$  and marginal cost is equal to the inverse markup implied by equation (26),  $s_t = s_{ss} = \frac{\varepsilon_p - 1}{\varepsilon_p}$ , function (65) and its partial derivatives are given by:

$$f(1, s_{ss}) = 0 \quad (66)$$

$$\frac{\partial f(\cdot)}{\partial \frac{P_t^*}{P_{Ht+k}}} \left( 1, \frac{\varepsilon_p - 1}{\varepsilon_p} \right) = -\varepsilon_p \left( 1 + \left( \frac{\varepsilon_p - 1}{\varepsilon_p} - 1 \right) \varepsilon_p \right) + (1 - \varepsilon_p) = (1 - \varepsilon_p) \quad (67)$$

$$\frac{\partial f(\cdot)}{\partial s_t} \left( 1, \frac{\varepsilon_p - 1}{\varepsilon_p} \right) = \varepsilon_p \quad (68)$$

The total derivative of the first order condition of the price setting problem (64), with

aggregate output and the discount factor in steady state, is

$$E_t \left[ \sum_{k=0}^{\infty} \theta_H^k \frac{\frac{\partial p_t(P_t^*)}{\partial P_t^*}}{\partial \left( \left( \prod_{q=t}^{t+k+1} \frac{1}{1+r_q} \right) \left( \frac{P_t^*}{P_{Ht+k}} \right)^{-\varepsilon_p} Y_{t+k} \right)} d \left( \left( \prod_{q=t}^{t+k+1} \frac{1}{1+r_q} \right) \left( \frac{P_t^*}{P_{Ht+k}} \right)^{-\varepsilon_p} Y_{t+k} \right) f(1, s_{ss}) \right. \\ \left. + \sum_{k=0}^{\infty} \theta_H^k \left( \frac{1}{1+r} \right)^k \left( \frac{1}{1} \right)^{-\varepsilon_p} Y^{ss} \frac{\partial f(\cdot)}{\partial \frac{P_t^*}{P_{Ht+k}}} \left( 1, \frac{\varepsilon_p - 1}{\varepsilon_p} \right) d \left( \frac{P_t^*}{P_{Ht+k}} \right) \right. \\ \left. + \sum_{k=0}^{\infty} \theta_H^k \left( \frac{1}{1+r} \right)^k \left( \frac{1}{1} \right)^{-\varepsilon_p} Y^{ss} \frac{\partial f(\cdot)}{\partial s_t} \left( 1, \frac{\varepsilon_p - 1}{\varepsilon_p} \right) d(s_{t+k}) \right] = 0.$$

Using (66), (67), and (68) from above, the total derivative reduces to:

$$E_t \left[ \sum_{k=0}^{\infty} \left( \frac{\theta_H}{1+r} \right)^k Y^{ss} (1 - \varepsilon_p) d \left( \frac{P_t^*}{P_{Ht+k}} \right) + \sum_{k=0}^{\infty} \left( \frac{\theta_H}{1+r} \right)^k Y^{ss} (\varepsilon_p) d(s_{t+k}) \right] = 0 \\ (\varepsilon_p - 1) E_t \left[ \sum_{k=0}^{\infty} \left( \frac{\theta_H}{1+r} \right)^k Y^{ss} d \left( \frac{P_t^*}{P_{Ht+k}} \right) \right] = \varepsilon_p \left[ \sum_{k=0}^{\infty} \left( \frac{\theta_H}{1+r} \right)^k Y^{ss} d(s_{t+k}) \right] \\ (\varepsilon_p - 1) E_t \left[ \sum_{k=0}^{\infty} \left( \frac{\theta_H}{1+r} \right)^k d \left( \frac{P_{Ht}}{P_{t+k}} \right) \right] = \varepsilon_p E_t \left[ \sum_{k=0}^{\infty} \left( \frac{\theta_H}{1+r} \right)^k d(s_{t+k}) \right]$$

Linearizing around the zero inflation steady-state yields  $d(P_t^*/P_{Ht+k}) = p_t^* - p_{Ht+k}$ , where  $p_t^* = \log(P_t^*)$  and  $p_{Ht+k} = \log(P_{Ht+k})$ . Inserting the linearized values and solving for  $p_{Ht+k}$  gives the following price setting equation

$$(\varepsilon_p - 1) E_t \left[ \sum_{k=0}^{\infty} \left( \frac{\theta_H}{1+r} \right)^k p_t^* \right] = \varepsilon_p E_t \left[ \sum_{k=0}^{\infty} \left( \frac{\theta_H}{1+r} \right)^k (d(s_{t+k}) + p_{Ht+k}) \right] \\ (\varepsilon_p - 1) \frac{1}{1 - \frac{\theta_H}{1+r}} p_t^* = \varepsilon_p E_t \left[ \sum_{k=0}^{\infty} \left( \frac{\theta_H}{1+r} \right)^k (d(s_{t+k}) + (\varepsilon_p - 1) p_{Ht+k}) \right] \\ p_t^* = \left( 1 - \frac{\theta_H}{1+r} \right) E_t \left[ \sum_{k=0}^{\infty} \left( \frac{\theta_H}{1+r} \right)^k \left( \frac{\varepsilon_p}{\varepsilon_p - 1} d(s_{t+k}) + p_{Ht+k} \right) \right]$$

Which, in recursive form, reads

$$p_t^* = \left( 1 - \frac{\theta_H}{1+r} \right) \left( \frac{\varepsilon_p}{\varepsilon_p - 1} d(s_{t+k}) + p_{Ht} \right) + \frac{\theta_H}{1+r} E_t [p_{t+1}^*]. \quad (69)$$

Lastly, linearizing the Calvo price index for domestic products

$$\begin{aligned}
P_{Ht}^{1-\varepsilon_p} &= \theta_H P_{Ht-1}^{1-\varepsilon_p} + (1-\theta_H) P_{Ht}^{*1-\varepsilon_p} \\
1 &= \theta_H \left( \frac{P_{Ht-1}}{P_{Ht}} \right)^{1-\varepsilon_p} + (1-\theta_H) \left( \frac{P_{Ht}^*}{P_{Ht}} \right)^{1-\varepsilon_p} \\
0 &= \theta_H (1-\varepsilon_p) (P_{Ht-1} - P_{Ht}) + (1-\theta_H) (1-\varepsilon_p) (P_t^* - P_{Ht}) \\
P_t^* &= \frac{\theta_H}{1-\theta_H} P_{Ht-1} + P_{Ht} \\
P_t^* &= \frac{1}{1-\theta_H} \pi_{Ht} + P_{Ht-1}.
\end{aligned}$$

and inserting the linearized Calvo index into the recursive price setting equation (69), yields the New Keynesian Phillips curve as a function of inflation and marginal cost:

$$\begin{aligned}
\pi_{Ht} \frac{1}{1-\theta_H} + P_{Ht-1} &= \left( 1 - \frac{\theta_H}{1+r} \right) \left( P_{Ht} + \frac{\varepsilon_p}{\varepsilon_p-1} d(s_t) \right) + \frac{\theta_H}{1+r} E_t \left[ \pi_{Ht+1} \frac{1}{1-\theta_H} + P_{Ht} \right] \\
\pi_{Ht} \frac{1}{1-\theta_H} + P_{Ht-1} - P_{Ht} &= \left( 1 - \frac{\theta_H}{1+r} \right) \left( \frac{\varepsilon_p}{\varepsilon_p-1} d(s_t) \right) + \frac{\theta_H}{1+r} E_t \left[ \pi_{Ht+1} \frac{1}{1-\theta_H} \right] \\
\pi_{Ht} + (1-\theta_H)(-\pi_{Ht}) &= (1-\theta_H) \left( 1 - \frac{\theta_H}{1+r} \right) \left( \frac{\varepsilon_p}{\varepsilon_p-1} d(s_t) \right) + \frac{\theta_H}{1+r} E_t [\pi_{Ht+1}] \\
\pi_{Ht} &= \frac{(1-\theta_H) \left( 1 - \frac{\theta_H}{1+r} \right)}{\theta_H} \frac{\varepsilon_p}{\varepsilon_p-1} \left( s_t - \frac{\varepsilon_p-1}{\varepsilon_p} \right) + \frac{1}{1+r} E_t [\pi_{Ht+1}] \\
\pi_{Ht} &= \frac{(1-\theta_H) \left( 1 - \frac{\theta_H}{1+r} \right)}{\theta_H} \left( \frac{\varepsilon_p}{\varepsilon_p-1} s_t - 1 \right) + \frac{1}{1+r} E_t [\pi_{Ht+1}]
\end{aligned}$$

Which can be stated as equation (29). In the special case with full price flexibility, i.e.,  $\theta_H = 0$ , all intermediate firms set the same price at a constant markup  $\varepsilon_p - 1/\varepsilon_p$  over nominal marginal costs, corresponding to equation (11) in Auclert et al. (2021b).

## A.4 Union Wage-Setting

This section derives the wage Phillips curve from the union problem. Similar to A.3, the derivation is analogous to the one in Auclert et al. (2020), albeit with simple CES labour aggregation and without wage inflation indexing. At any time  $t$ , union  $k$  seeks to maximize the utility of the employees it represents by setting the optimal wage  $W_{kt}$ . Because of Calvo wage staggering, the union can reset the wage with a probability of  $1 - \theta_w$  and therefore must account for those future states in which wages cannot be adjusted. It thus

sets the wage to maximize

$$\sum_{\ell=0}^{\infty} \beta^{\ell} \theta_w^{\ell} \int \{u(c_{it}) - v(n_{it})\} d\mathcal{D}_{it} \quad (70)$$

while taking as given the initial distribution of households over idiosyncratic states  $\mathcal{D}_{it}$  and the labour packer's demand curve for tasks. The latter can be derived by solving the labour packer's problem, who seeks to minimize  $\int N_{kt} dk$  subject to the labour aggregation function (30). Combining the first order condition and the aggregation function yields the demand curve

$$N_{kt} = \left( \frac{W_{kt}}{W_t} \right)^{-\varepsilon_w} N_t \quad (71)$$

where  $W_t = \left( \int_0^1 W_{kt}^{\varepsilon_w} dk \right)^{\frac{1}{1-\varepsilon_w}}$  is the price index for aggregate employment services. To solve the union problem, first find the derivatives of its components with regard to  $W_{kt}$ . Because each union is infinitesimal, it takes into account only its marginal effect on household consumption and labour supply. Following the envelope theorem, indirect utility can then be evaluated as if all income from the union wage change is consumed, so  $\frac{\partial c_{it}}{\partial W_{kt}} = \frac{\partial z_{it}}{\partial W_{kt}}$ . Here, using the demand curve (71) and the fact that all households are required to supply the same amount of labour  $n_{ikt} = N_{kt}$  to each union, real household earnings are given by

$$\begin{aligned} z_{it} &= e_{it} \frac{\int_0^1 W_{kt} N_{kt} dk}{P_t} \\ &= e_{it} \frac{1}{P_t} \int_0^1 W_{kt} \left( \frac{W_{kt}}{W_t} \right)^{-\varepsilon_w} N_t dk \\ &= e_{it} \frac{1}{P_t} \int_0^1 \frac{W_{kt}^{1-\varepsilon_w}}{W_t^{-\varepsilon_w}} dk N_t. \end{aligned}$$

And the marginal effect of the wage paid by union  $j$  on the real income of the household is:

$$\frac{\partial z_{it}}{\partial W_{kt}} = \frac{e_{it} N_t}{P_t} (1 - \varepsilon_w) \left( \frac{W_{kt}}{W_t} \right)^{-\varepsilon_w} \quad (72)$$

Total labour provided by household  $i$  is given by

$$n_{it} = \int_0^1 \left( \frac{W_{kt}}{W_t} \right)^{-\varepsilon_w} N_t dk$$

and the marginal effect of a wage change is thereby



$$\frac{\partial n_{it}}{\partial W_{kt}} = -\varepsilon_w \left( \frac{W_{kt}}{W_t} \right)^{-\varepsilon_w} \frac{N_t}{W_{kt}} \quad (73)$$

Next, using the partial derivatives in (72) and (73) from above, the marginal effect of a union wage change on household utility  $\int \{u(c_{it}) - v(n_{it})\} d\mathcal{D}_{it}$  is

$$\begin{aligned} \frac{\partial U_t}{\partial W_{kt}} &= \int \left\{ \left( \frac{e_{it} N_t}{P_t} (1 - \varepsilon_w) \left( \frac{W_{kt}}{W_t} \right)^{-\varepsilon_w} u'(c_{it}) + \varepsilon_w \left( \frac{W_{kt}}{W_t} \right)^{-\varepsilon_w} \frac{N_t}{W_{kt}} v'(n_{it}) \right) \right\} d\mathcal{D}_{it} \\ &= N_t \left( \frac{W_{kt}}{W_t} \right)^{-\varepsilon_w} \left( \int \frac{e_{it}}{P_t} (1 - \varepsilon_w) u'(c_{it}) d\mathcal{D}_{it} + \int \varepsilon_w \frac{1}{W_{kt}} v'(n_{it}) dP_{it} \right) \\ &= N_t \left( \frac{W_{kt}}{W_t} \right)^{-\varepsilon_w} \left( (1 - \varepsilon_w) \frac{1}{P_t} \int e_{it} u'(c_{it}) d\mathcal{D}_{it} + \varepsilon_w \frac{1}{W_{kt}} \int v'(n_{it}) d\mathcal{D}_{it} \right) \\ &= N_t \left( \frac{W_{kt}}{W_t} \right)^{-\varepsilon_w} \left( (1 - \varepsilon_w) \frac{1}{P_t} MU + \varepsilon_w \frac{1}{W_{kt}} MV \right) \\ &= N_t \left( \frac{W_{kt}}{W_t} \right)^{-\varepsilon_w} \left( \frac{MU}{P_t} + \varepsilon_w \left( \frac{MV}{W_{kt}} - \frac{MU}{P_t} \right) \right) \end{aligned}$$

where  $MU = \int e_{it} u'(c_{it}) d\mathcal{D}_{it}$  and  $MV = \int v'(n_{it}) d\mathcal{D}_{it}$ . Finally, using this result, the first order condition for the initial union problem (70) is given by

$$\begin{aligned} 0 &= E \left[ \sum_{\ell=0}^{\infty} \beta^\ell \theta_w^\ell N_{t+\ell} \left( \frac{W_{kt}^*}{W_{t+\ell}} \right)^{-\varepsilon_w} \left( \frac{MU_{t+\ell}}{P_{t+\ell}} + \varepsilon_w \left( \frac{MV_{t+\ell}}{W_{kt}^*} - \frac{MU_{t+\ell}}{P_{t+\ell}} \right) \right) \right] \\ &= E \left[ \sum_{\ell=0}^{\infty} \beta^\ell \theta_w^\ell N_{t+\ell} \left( \frac{W_{kt}^*}{W_{t+\ell}} \right)^{-\varepsilon_w} \frac{1}{W_{jt}^*} \left( \frac{MU_{t+\ell} W_{kt}^* W_{t+\ell}}{P_{t+\ell} W_{t+\ell}} + \varepsilon_w \left( MV_{t+\ell} - \frac{MU_{t+\ell} W_{kt}^* W_{t+\ell}}{P_{t+\ell} W_{t+\ell}} \right) \right) \right] \\ &= E \left[ \sum_{\ell=0}^{\infty} \beta^\ell \theta_w^\ell N_{t+\ell} \frac{MU_{t+\ell} W_{t+\ell}}{P_{t+\ell}} \left( \frac{W_{kt}^*}{W_{t+\ell}} \right)^{-\varepsilon_w} \left( \frac{W_{kt}^*}{W_{t+\ell}} + \varepsilon_w \left( \frac{MV_{t+\ell}}{\frac{MU_{t+\ell} W_{t+\ell}}{P_{t+\ell}}} - \frac{W_{kt}^*}{W_{t+\ell}} \right) \right) \right] \quad (74) \end{aligned}$$

Following Auclert et al. (2020), let  $s_{wt}$  be the inverse wage markup

$$s_{wt+\ell} = \frac{MV_{t+\ell}}{\frac{MU_{t+\ell} W_{t+\ell}}{P_{t+\ell}}} = \frac{\int v'(n_{it}) d\mathcal{D}_{it}}{\frac{W_{t+\ell}}{P_{t+\ell}} \int e_{it+\ell} u'(c_{it+\ell}) d\mathcal{D}_{it}} = \frac{v'(N_t)}{\frac{W_{t+\ell}}{P_{t+\ell}} u'(C_{t+\ell}^*)} \quad (75)$$

For the linearization of the first order condition around the steady state, define the desired steady state wage markup as  $\frac{\varepsilon_n}{\varepsilon_w - 1}$  and the inverse steady state markup as  $s_w = \frac{\varepsilon_w - 1}{\varepsilon_w}$ . Next, use (75) and the second part of the first order condition to define the function  $f\left(\frac{W_{kt}^*}{W_t}, s_{wt}\right)$

and its partial derivatives:

$$f\left(\frac{W_{kt}^*}{W_t}, s_{wt}\right) = \left(\frac{W_{kt}^*}{W_t}\right)^{-\varepsilon_w} \left(\frac{W_{kt}^*}{W_t} + \varepsilon_w \left(s_{wt} - \frac{W_{kt}^*}{W_t}\right)\right)$$

$$\frac{\partial f(\cdot)}{\partial \frac{W_{jt}^*}{W_t}} = \left(\frac{W_{kt}^*}{W_t}\right)^{-\varepsilon_w} \left((1 - \varepsilon_w) + \varepsilon_w \left(-\varepsilon_w s_{wt} \left(\frac{W_{kt}^*}{W_t}\right)^{-1} - (1 - \varepsilon_w)\right)\right)$$

$$\frac{\partial f(\cdot)}{\partial s_{wt}} = \left(\frac{W_{kt}^*}{W_t}\right)^{-\varepsilon_w} \varepsilon_w$$

Using that in steady state  $W_{kt}^* = W_t = W^{ss}$ , the partial derivatives become

$$\frac{\partial f(\cdot)}{\partial W^{ss}} = (1 - \varepsilon_w), \quad \frac{\partial f(\cdot)}{\partial s_{wt}} \left(\frac{W^{ss}}{W^{ss}}, s^{ss}\right) = \varepsilon_w$$

Total differentiating the first order condition (74) using the new steady state values delivers

$$0 = E_t \left[ 0 + \sum_{\ell=0}^{\infty} \theta_w^\ell \beta^\ell N_{ss} \frac{MU^{ss} W^{ss}}{P_{ss}} (1 - \varepsilon_w) d\left(\frac{W_{kt}^*}{W_t}\right) + \sum_{\ell=0}^{\infty} \theta_w^\ell \beta^\ell N_{ss} \frac{MU^{ss} W^{ss}}{P_{ss}} (\varepsilon_w) d(s_{wt}) \right]$$

$$(\varepsilon_w - 1) E_t \left[ \sum_{\ell=0}^{\infty} \theta_w^\ell \beta^\ell d\left(\frac{W_{kt}^*}{W_t}\right) \right] = (\varepsilon_w) \left[ \sum_{\ell=0}^{\infty} \theta_w^\ell \beta^\ell d(s_{wt}) \right]$$

$$(\varepsilon_w - 1) E_t \left[ \sum_{\ell=0}^{\infty} \theta_w^\ell \beta^\ell (w_{kt}^* - w_t) \right] = (\varepsilon_w) \left[ \sum_{k=0}^{\infty} \theta_w^\ell \beta^\ell d(s_{wt}) \right]$$

where  $d\left(\frac{W_{kt}^*}{W_t}\right) = d \log\left(\frac{w_{kt}}{w_t}\right) = (w_{kt}^* - w_t)$ . In recursive form the linearized first order condition reads

$$w_{kt}^* = (1 - \beta \theta_w) \left( w_t + \frac{\varepsilon_w}{\varepsilon_w - 1} d(s_{wt}) \right) + \beta \theta_w E_t [w_{kt+1}^*]$$

For the last step, rewriting from the Calvo wage staggering condition and differentiating

around the steady state yields

$$\begin{aligned}
W_t^{1-\varepsilon_w} &= \theta_w W_{t-1}^{1-\varepsilon_w} + (1-\theta_w) W_t^{1-\varepsilon_w} \\
1 &= \theta_w \left( \frac{W_{t-1}}{W_t} \right)^1 \left( \frac{W_{t-1}}{W_t} \right)^{-\varepsilon_w} + (1-\theta_w) \left( \frac{W_t^*}{W_t} \right)^1 \left( \frac{W_t^*}{W_t} \right)^{-\varepsilon_w} \\
0 &= \theta_w (1-\varepsilon_w) d \left( \frac{W_{t-1}}{W_t} \right) + (1-\theta_w) (1-\varepsilon_w) d \left( \frac{W_t^*}{W_t} \right) \\
w_t^* &= \frac{1}{1-\theta_w} w_t - \frac{\theta_w}{1-\theta_w} w_{t-1}
\end{aligned}$$

Combining the linearized Calvo condition and the linearized first order condition then leads to the wage Phillips curve

$$\begin{aligned}
\frac{1}{1-\theta_w} w_t - \frac{\theta_w}{1-\theta_w} w_{t-1} &= (1-\beta\theta_w) \left( w_t + \frac{\varepsilon_w}{\varepsilon_w-1} d(s_{wt}) \right) + \beta\theta_w E_t \left[ \frac{1}{1-\theta_w} w_{t+1} - \frac{\theta_w}{1-\theta_w} w_t \right] \\
w_t - \theta_w w_{t-1} &= (1-\theta_w) (1-\beta\theta_w) \left( w_t + \frac{\varepsilon_w}{\varepsilon_w-1} d(s_{wt}) \right) + \beta\theta_w E_t [w_{t+1} - \theta_w w_t] \\
w_t + \beta\theta_w^2 w_t - (1-\theta_w) (1-\beta\theta_w) w_t - \theta_w w_{t-1} - \beta\theta_w E_t [w_{t+1}] &= (1-\theta_w) (1-\beta\theta_w) \left( \frac{\varepsilon_w}{\varepsilon_w-1} d(s_{wt}) \right) \\
(\theta_w + \beta\theta_w) w_t - \theta_w w_{t-1} - \beta\theta_w E_t [w_{t+1}] &= (1-\theta_w) (1-\beta\theta_w) \left( \frac{\varepsilon_w}{\varepsilon_w-1} d(s_{wt}) \right) \\
\pi_t^w &= \frac{(1-\theta_w) (1-\beta\theta_w)}{\theta_w} \frac{\varepsilon_w}{\varepsilon_w-1} (d(s_{wt})) + \beta E_t [\pi_{t+1}^w] \\
\pi_t^w &= \frac{(1-\theta_w) (1-\beta\theta_w)}{\theta_w} \frac{\varepsilon_w}{\varepsilon_w-1} \left( s_{wt} - \frac{\varepsilon_w-1}{\varepsilon_w} \right) + \beta E_t [\pi_{t+1}^w] \\
\pi_t^w &= \frac{(1-\theta_w) (1-\beta\theta_w)}{\theta_w} \left( \frac{\varepsilon_w}{\varepsilon_w-1} s_{wt} - 1 \right) + \beta E_t [\pi_{t+1}^w]
\end{aligned}$$

and corresponds to (31).

## A.5 Current Account Identity

Following, Auclert et al. (2021b), start by aggregating up household budgets in equation (2), using  $Ee_{it} = 1$ ,

$$\frac{P_{Ft}}{P_t} C_{Ft} + \frac{P_{Ht}}{P_t} C_{Ht} + A_t = (1+r_t^p) A_{t-1} + w_t N_t$$

Taking the definition of the net foreign assets (23) and writing the aggregate expost return

as

$$\begin{aligned}
(1 + r_t^p) A_{t-1} &= (1 + r_{t-1}) A_{t-1} + (r_t^p - r_{t-1}) A_{t-1} \\
&= (1 + r_{t-1}) (p_{t-1} + nfa_{t-1}) + (r_t^p - r_{t-1}) A_{t-1} \\
&= D_t + p_t + (1 + r_{t-1}) nfa_{t-1} + (r_t^p - r_{t-1}) A_{t-1}
\end{aligned}$$

to obtain

$$\frac{P_{Ft}}{P_t} C_{Ft} + \frac{P_{Ht}}{P_t} C_{Ht} + p_t + nfa_t = D_t + p_t + (1 + r_{t-1}) nfa_{t-1} + (r_t^p - r_{t-1}) A_{t-1} + w_t N_t$$

Substituting in the value of dividends  $D_t$  from (28) yields

$$\underbrace{\frac{P_{Ft}}{P_t} C_{Ft} + \frac{P_{Ht}}{P_t} C_{Ht}}_{=C_t} + nfa = \frac{P_{Ht}}{P_t} \underbrace{(C_{Ht} + C_{Ht}^*)}_{=Y_t} + (1 + r_{t-1}) nfa_{t-1} + (r_t^p - r_{t-1}) A_{t-1}$$

Noting that following (19)  $r_t^p = r_{t-1}$ , this delivers

$$nfa_t - nfa_{t-1} = \underbrace{\frac{P_{Ht}}{P_t} Y_t - C_t}_{NX_t} + r_{t-1} nfa_{t-1}$$

which is equation (36).

## A.6 Steady State Solution

The steady state solution is similar to Auclert et al. (2021b), who show that the level of the real exchange rate  $Q_t$  and aggregate domestic and foreign spending  $C_t$  and  $C_t^*$  uniquely determine the level of domestic output and net exports. Normalizing their steady state values to 1 implies that the prices  $P, P_H, \mathcal{E}$  are also 1. Given the model parameters presented in Table 1 and assuming constant levels of model aggregates  $C, C_H, C_F, C^*, Y, A, N, p, D, nfa$ , prices  $P, P_H, Q, W, r, i^*$ , and shocks  $\mathcal{B}, \mathcal{E}$ , the solution then consists of (i) deriving the implied stationary values for the foreign discount factor  $\beta^*$ , real wages  $\frac{W}{P}$ , share prices  $p$ , real dividends  $D$ , and exports  $C_H^*$ , (ii) solving the household problem for the level and the distribution of discount factors so as to deliver the zero net foreign asset as well as the MPC target, and (iii) solving the New Keynesian wage Phillips curve for the normalization parameter  $\psi$ . Combining the real UIP condition (22) with the foreign Euler equation

(14) implies that the foreign discount factor must satisfy

$$r = i^* = \frac{1}{\beta^*} - 1$$

Taking the equation for firm's marginal costs (26), with  $s = \frac{1}{\mu}$ , and  $P_H$ ,  $P$  and  $Z$ , which are all equal to 1, yields the constant levels of real and nominal wage as

$$\frac{W}{P} = W = \frac{1}{\mu}$$

Which in turn gives real dividends following equation (28) as

$$D = 1 - \frac{1}{\mu}$$

The equality of real returns of the mutual fund (19) implies that the share value of firms is given by

$$p = \frac{1}{r} \left( 1 - \frac{1}{\mu} \right)$$

With  $P_H$  and  $\mathcal{E}$  normalized to 1, (10) implies

$$C_H^* = \alpha C^*$$

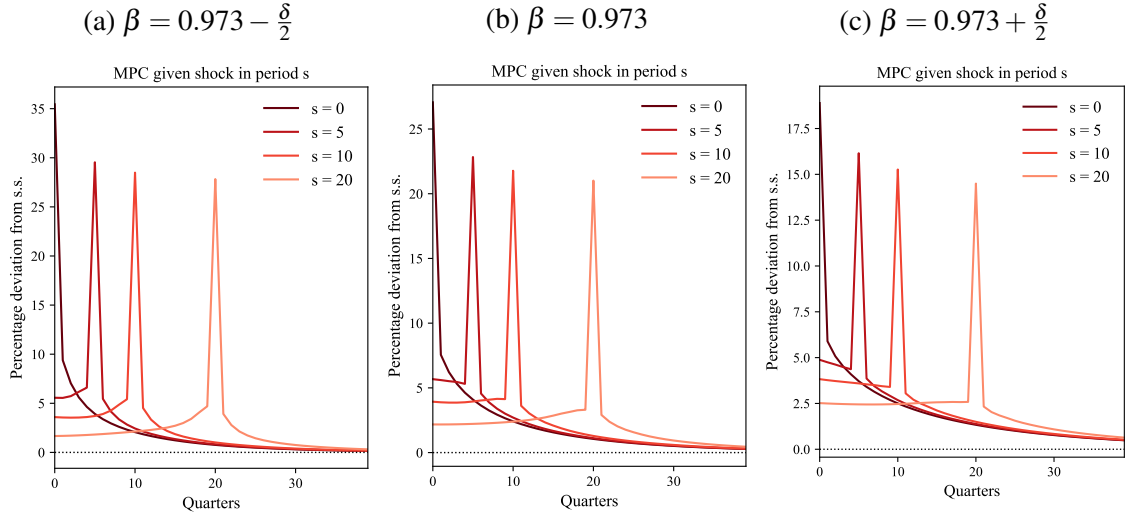
Then, given that the real return on assets is  $r^p = r$ , as implied by (19) and the real wage, the stationary solution to the household problem delivers the aggregate levels of consumption  $C$ ,  $C_H$ ,  $C_F$  and assets  $A$ . Using the discount factor  $\beta$  to target an asset level equal to the share price  $p$  ensures that net foreign assets are zero, while the discount factor spread  $\delta$  is used to target the average MPC across households. Given the market clearing condition (34) and normalizing  $Z = 1$ , the production function (25) yields the level of employment  $N = 1$ . Lastly, the stationary of wages implies  $\pi_w = 0$ . Using this,  $N$ , and the solution to the household problem, the wage Phillips curve (31) can be solved for the adjustment factor  $\psi$  which ensures that the following relationship holds:

$$\psi = \frac{1}{\mu_w \mu} \frac{u'(C_t)}{1^\varphi}$$

## B Appendix to section 5

This section contains plots of iMPCs for individual household groups. The reported iMPCs in Figure 9 are in line with the empirical findings for Italy by Jappelli and Pistaferri (2014).

Figure 9: MPCs for household groups with distinct discount factors



## C Appendix to section 6

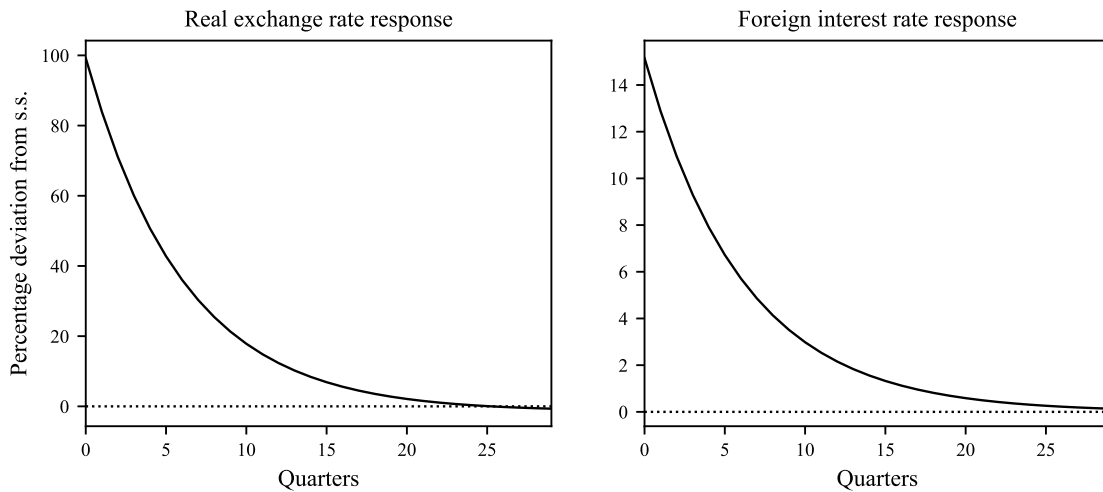
### C.1 The Effect of an Exchange Rate Shock

This section replicates the decomposition of the effect of a real exchange rate depreciation in Auclert et al. (2021b) to demonstrate the role of competitiveness and real income effects in a small open economy HANK models. In the stylized version of the model without nominal price rigidity and the constant real rate rule described in 6.1, consider a shock to the foreign interest rate  $i_t^*$  in (14), caused by a shock to preferences shifter of foreign households  $\mathcal{B}_t = \prod_{s \geq t} \left( \frac{1+i_s^*}{1+r_{ss}} \right)$ . Combining the real UIP condition 22, the fact that  $Q_\infty = 1$ , and the constant real rate, the real exchange can be shown to be equal to the preference shifter:

$$Q_t = \prod_{s \geq t} \left( \frac{1+i_s^*}{1+r_{ss}} \right) = \mathcal{B}_t$$

Intuitively, when foreign households become more impatient, that is, an increase in  $\mathcal{B}_t$ , they push up foreign interest rates  $i_t^*$ , leading to capital outflows that depreciate the exchange rate, i.e., an increase in  $Q_t$ . The real exchange rate is thus effectively exogenous. Figure 10 AR(1) shock to  $i_t^*$  with persistence 0.85 and the corresponding impulse response of the real exchange rate  $Q_t$ . The shock is normalized so that the real exchange rate depreciates by 1% on impact.

Figure 10: The exchange rate shock



Given the absence of a monetary policy intervention, the impulse response of consump-

tion to a shock to the real exchange rate  $dQ$  is given by

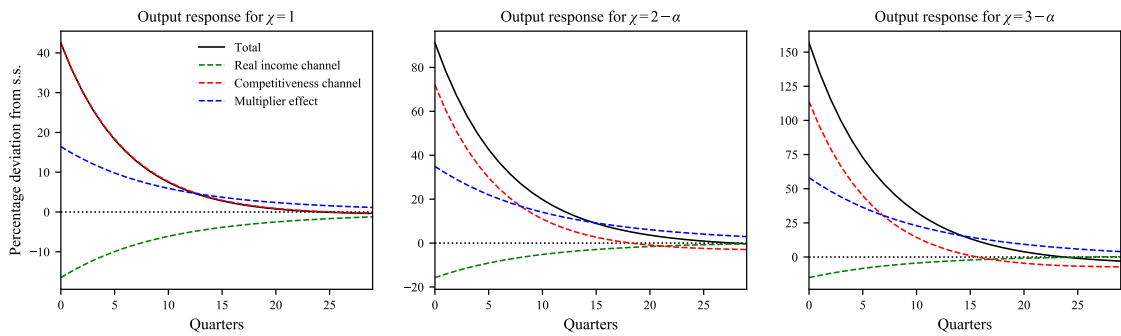
$$dC = - \underbrace{\frac{\alpha}{1-\alpha} M dQ}_{\text{Real income channel}} + \underbrace{M dY}_{\text{Multiplier}}$$

and the impulse response of output  $dY$  is determined by an "international Keynesian cross"<sup>16</sup>:

$$dY = \underbrace{\frac{\alpha}{1-\alpha} \chi dQ}_{\text{Competitiveness channel}} - \underbrace{\alpha M dQ}_{\text{Real income channel}} + \underbrace{(1-\alpha) M dY}_{\text{Multiplier}} \quad (76)$$

Here, the left term is the competitiveness channel alluded to by Galí and Monacelli (2016), which Auclert et al. (2021b) refer to as the "expenditure switching channel". The term in the middle is the effect that an exchange rate shock exerts on real household income via changes in relative prices. The left term is the familiar output multiplier, now reduced by a factor of  $(1 - \alpha)$  given the share of consumption that falls on foreign goods. Figure 11 shows this decomposition for a 100% shock to the real exchange rate like the one depicted in Figure 10 and different values of overall elasticity  $\chi$ . Apart from the competitiveness channel, which is also active in a representative agent model, the decomposition reveals the substantial role played by the multiplier effect and the real income channel in determining the overall effect on output. As becomes clear from equation (C.1), the size of these shock components depends on openness  $\alpha$  and the sum of trade elasticities  $\chi$ .

Figure 11: The effect of a real depreciation



In the special case in the left panel of Figure 11,  $\chi$  takes the value 1 which leads the multiplier and the real income effect to offset each other perfectly, thus establishing equivalence between the HANK and the representative agent case (Auclert et al., 2021b). Given

<sup>16</sup>For a proof of the international Keynesian cross, see Appendix B.1 and B.2 in Auclert et al. (2021b).



(5.3), setting the elasticity of substitution between home and foreign goods  $\eta$  and the elasticity of substitution between varieties of foreign goods  $\gamma$  close to unity, results in  $\chi = 2 - \alpha = 1.7$ , which is the equivalence case for a monetary policy shock. Finally, the left panel shows the response for  $\eta$  close to unity and  $\gamma$  equal to 2, following empirical estimates suggesting higher elasticities between varieties of foreign goods (see, e.g., Feenstra et al., 2018). Because higher levels of elasticity mean that consumers react more strongly to changes in relative prices, the substitution towards domestic goods and therefore the significance of the competitiveness channel increases. The added demand for home goods further amplifies the multiplier effect to where it outweighs the potentially negative effect that relatively more expensive foreign goods have on real household income.

## C.2 Additional Impulse Response Functions

This section features impulse responses to a 10% payroll tax cut of additional model variables (Figure 12), as well as decompositions of the consumption responses to the same shock for the three household groups (Figure 13).

## C.3 Payroll Tax Cut Robustness Checks

Figure 14 shows additional robustness checks for the effect of a payroll tax cut in a model in which the payroll tax no longer has a direct impact on firm dividends. Here, the left panel plots impulse responses, while the right panel plots the respective decomposition of the consumption responses. The significant positive impact of a reduction of labour costs on employment is robust to the muting of the dividend channel under both monetary regimes. Given the fall in firm dividends, the drop in the real rate of return is now driven by a much stronger reduction in the value of firm shares. Accordingly, the decomposition of the consumption response closely resembles the baseline case. The effect of labour costs on aggregate demand and employment therefore is not reliant on an endogenous policy channel, even if the direct effect on firm dividends is muted. In addition, Figure 15 reports impulse responses at varying levels of trade elasticities  $\eta$  and  $\gamma$ .

Figure 12: Additional responses to a temporary payroll tax cut

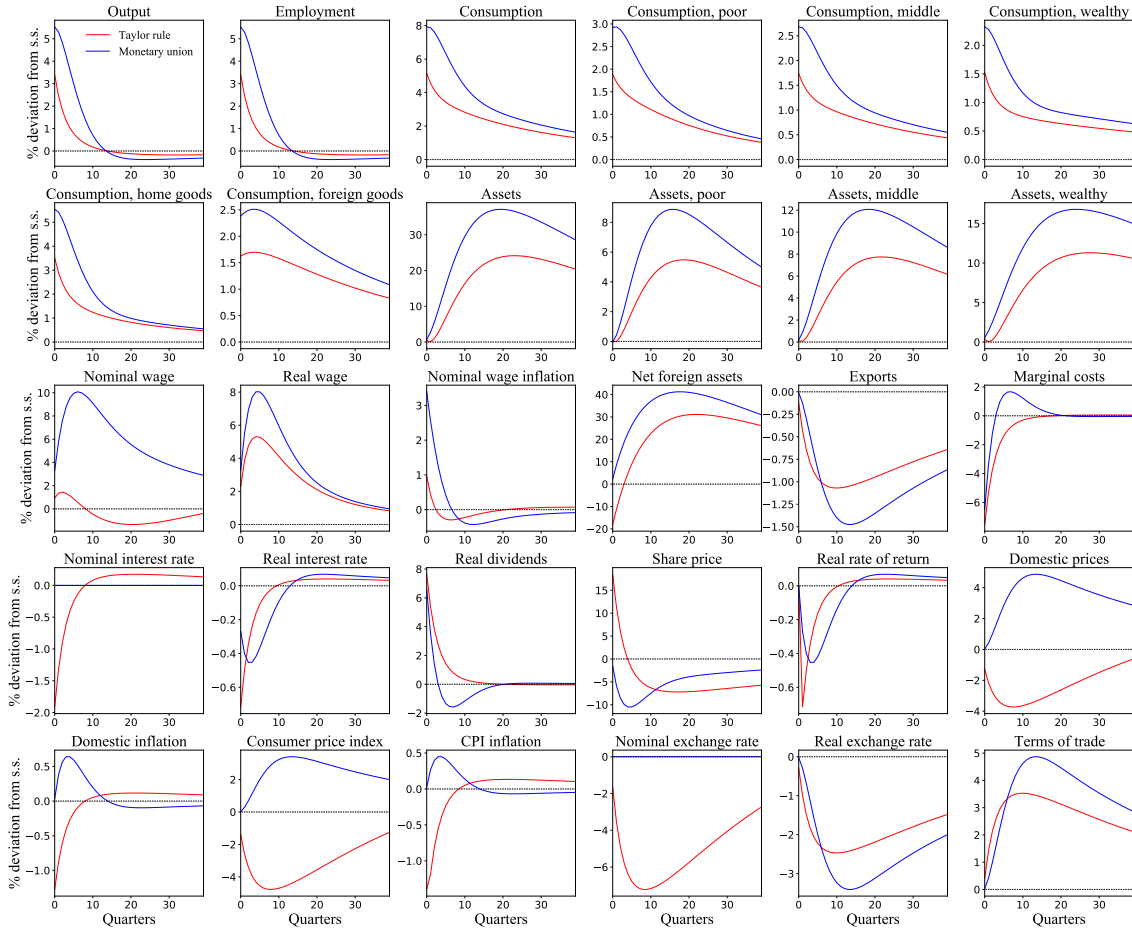


Figure 13: Decomposition of the consumption response across household groups

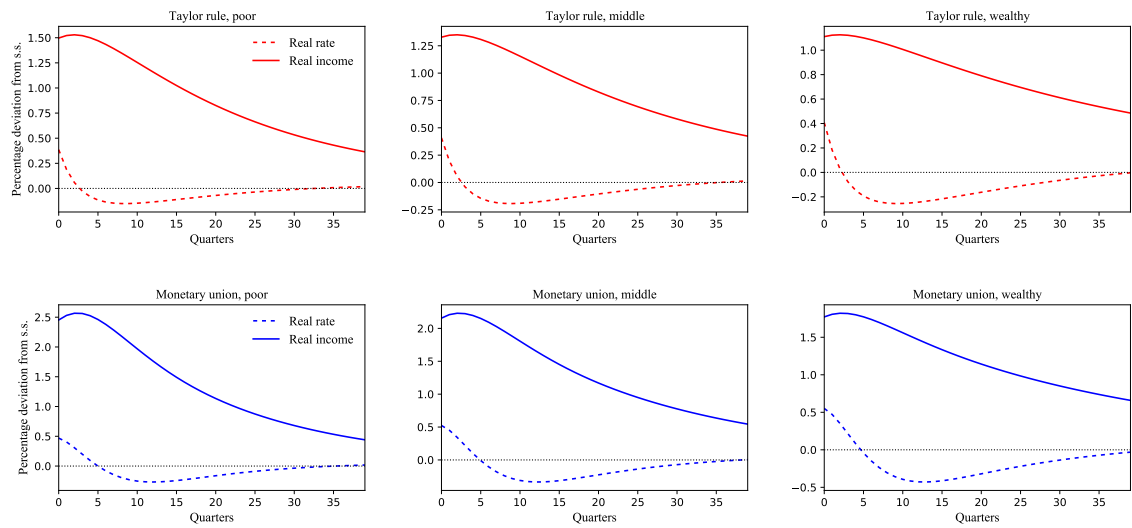


Figure 14: Responses to a payroll tax cut with muted dividend channel

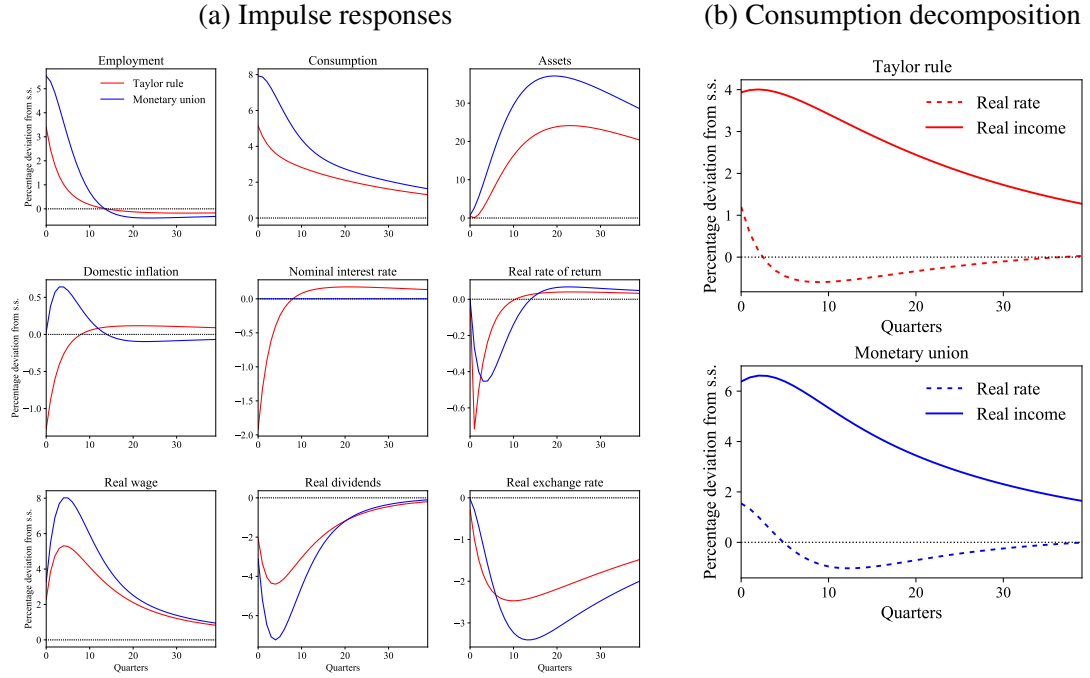
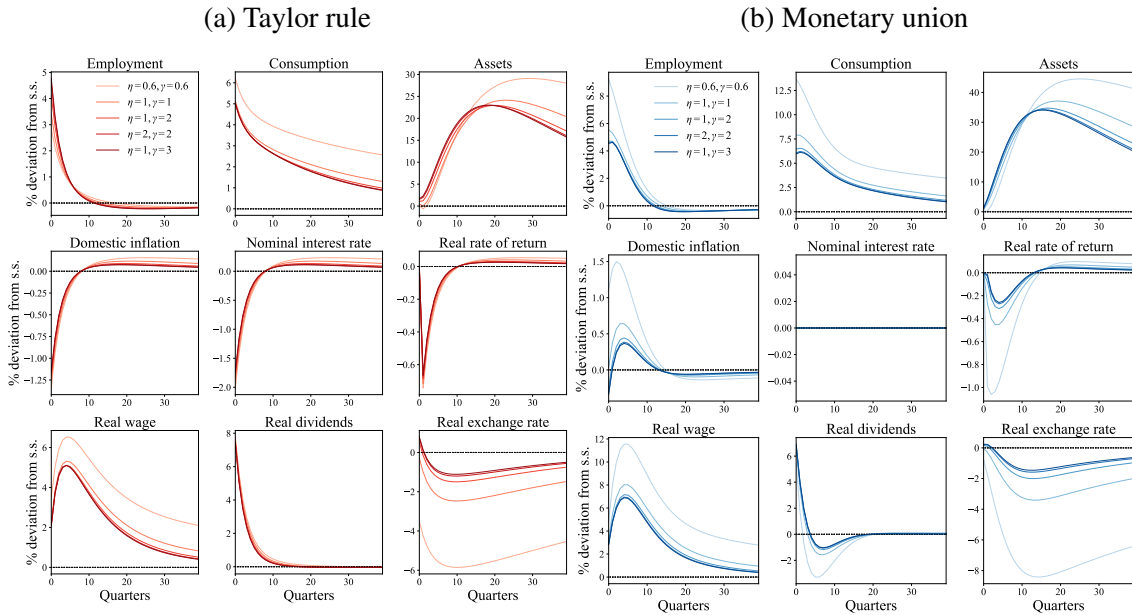


Figure 15: Responses to a payroll tax cut under different trade elasticities

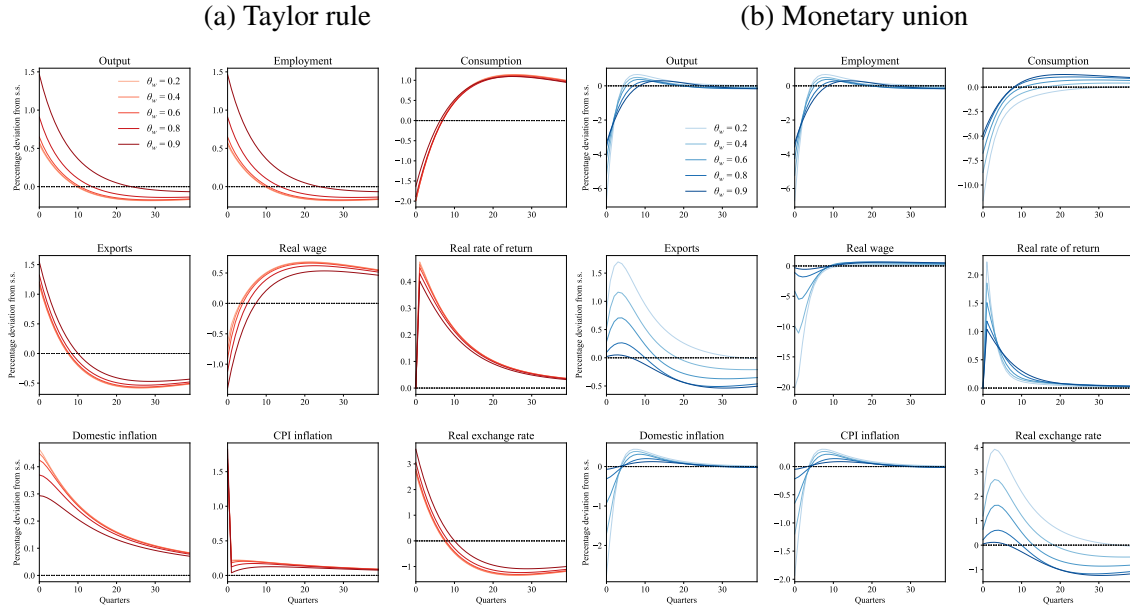


## D Appendix to section 7

### D.1 Additional Impulse Responses to Foreign Shocks

This section contains additional impulse response for a small open economy hit by a positive 100 basis points foreign interest rate shock and by a negative 10% global demand shock. Calibrations featuring varying levels of nominal wage rigidity,  $\theta_w$ , are considered for both the case of a Taylor rule and the case of a monetary union (Figures 16 and 17). In addition, Figures 16 and 19 show impulse responses at varying levels of overall rigidity, i.e. one for one changes in  $\theta_w$  and  $\theta_H$ .

Figure 16: Responses to foreign interest rate hike under varying levels of wage rigidity



### D.2 Welfare Analysis Robustness Checks

This section contains additional robustness checks for the analysis of welfare effects of wage flexibility. Given the central role of disutility from work for overall welfare, Figure 20 shows the welfare loss at varying inverse Frisch elasticities  $\phi$ . In light of the significance of trade elasticities for the transmission of shocks in small open economies, Figure 21 plots relative welfare losses at varying degrees of trade elasticities,  $\eta$  and  $\gamma$ . Finally, Figure 22 plots the relative loss of welfare for individual household groups at different points of the wealth distribution.

Figure 17: Responses to a global demand drop under varying levels of wage rigidity

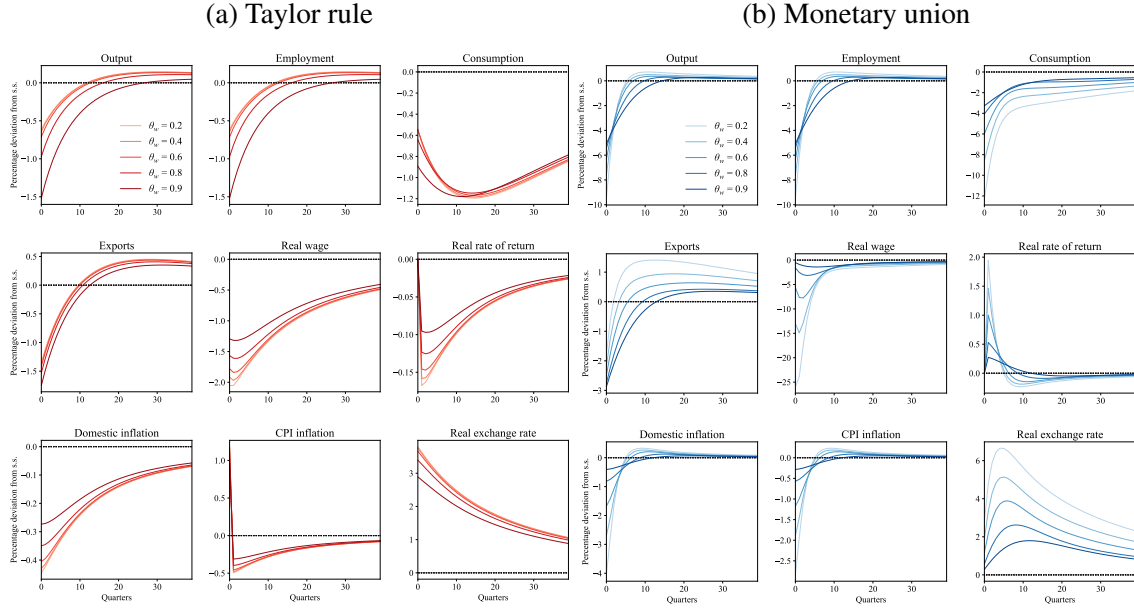


Figure 18: Responses to foreign interest rate hike under varying levels of overall rigidity

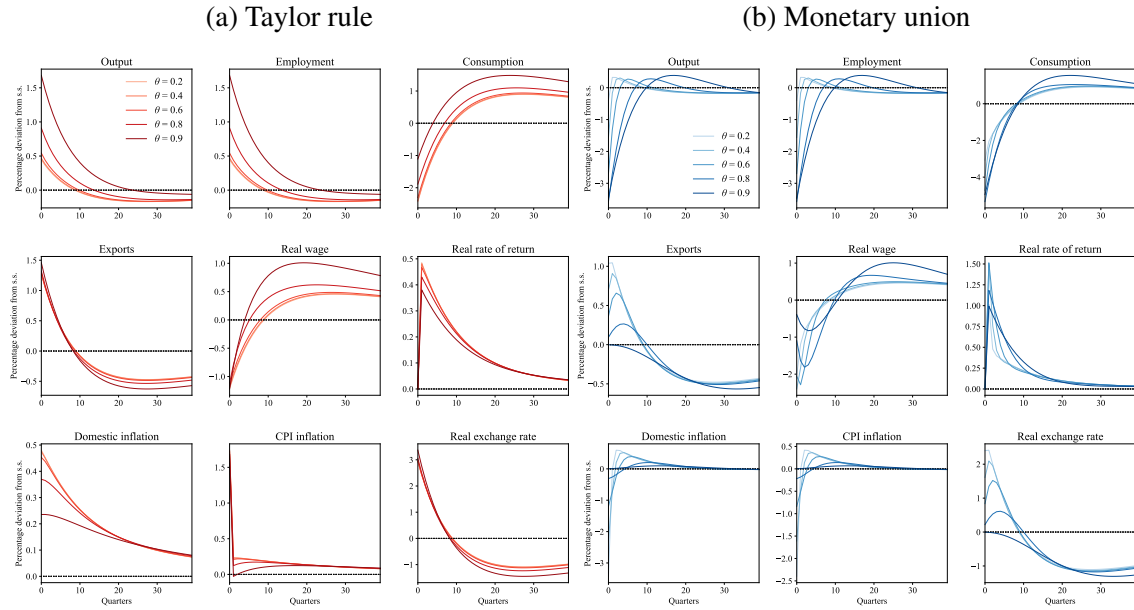


Figure 19: Responses to a global demand drop under varying levels of overall rigidity

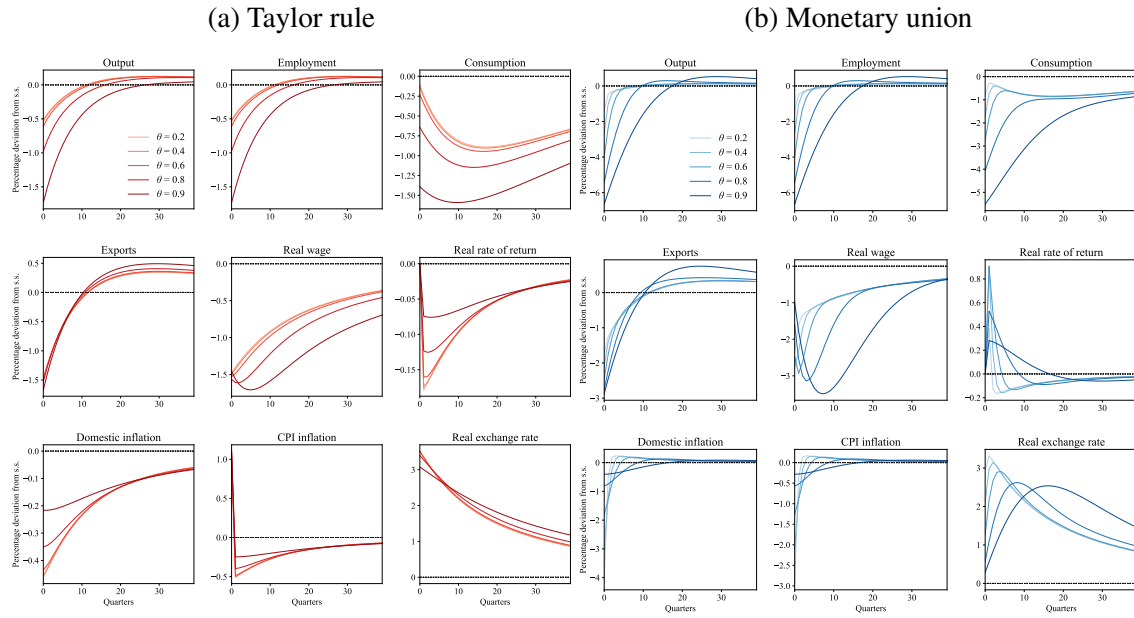


Figure 20: Relative welfare loss at varying levels of inverse Frisch elasticity

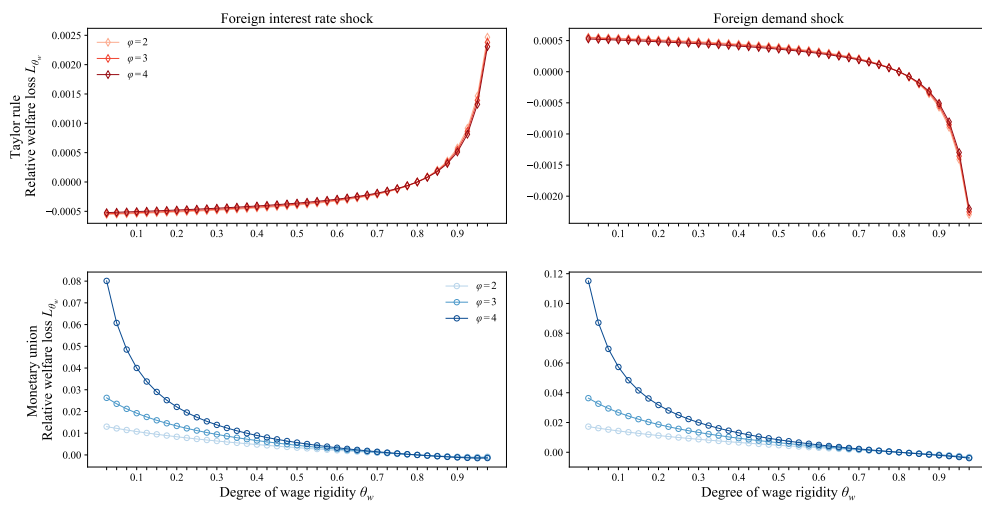


Figure 21: Relative welfare loss at varying levels of trade elasticities

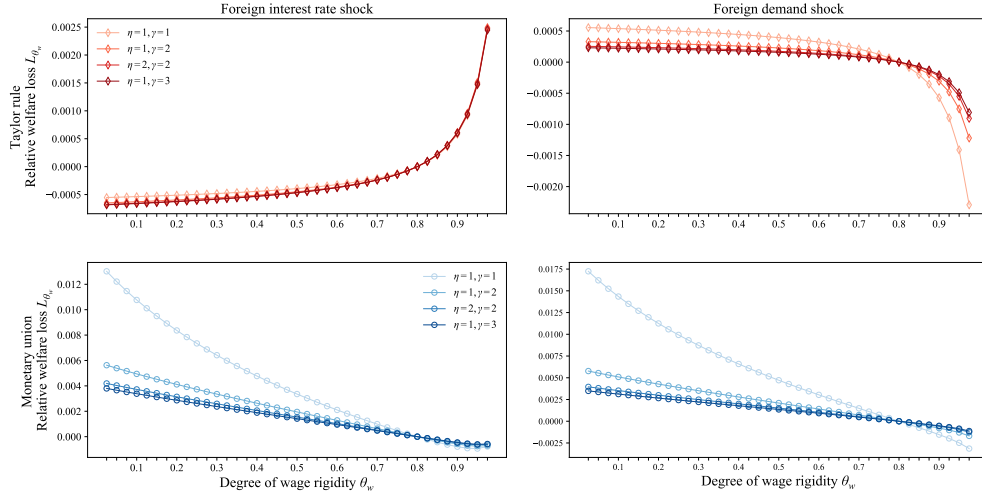


Figure 22: Relative welfare loss for individual household groups

