

Master's Thesis  
MSC-XXX



# **An approach to agile Maneuvering with Hydrobatic Micro Underwater Robots**

**Ein Ansatz für  
agiles Manövrieren mit  
kleinen hydrobatischen Unterwasserrobotern**

by  
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# Chapter 1

## Introduction

- Definition micro auv [Watson12]
- Definition of hydrobatic [BhatStenius18] (bridge between fully actuated hover style AUVs and the traditional ones)
- Definition of agile [Dücker22]
- 

### 1.1 Motivation

### 1.2 Problem Statement

Consider the scenario of the highly agile and underactuated HippoCampus *micro autonomous underwater vehicle* ( $\mu$ AUV) maneuvering in a confined environment – such as a water tank – to reach a time dependent goal state. Such a time dependent goal state can be represented by a moving ring, that is to be caught by the  $\mu$ AUV. Additionally, there might be obstacles present, which make a method for collision avoidance mandatory. Due to the physical limitations of wireless underwater communication [Bettale08, GeistEtAl16], all algorithms have to be executed on-board and in real time to allow for autonomous maneuvering.

From this scenario we can derive the following requirements for a trajectory generation and tracking system:

- real time capability of the trajectory generation algorithm, when executed by the on-board hardware

- verification of the trajectories' feasibility based on the dynamics of the  $\mu$ AUV
- a control scheme to track the planned trajectory
- dynamic collision avoidance due to obstacles
- make clear what I mean by real time capable (i.e. fast enough to keep up with the update rate, but no real time guarantees in worst case execution time)

Goal of this thesis is to develop such a system meeting the above-mentioned requirements and assess its performance by carrying out simulations and lab experiments.

## 1.3 Contribution and Outline

Based on a literature review in Section 2.2 on existing underwater path planning approaches in general and agile path planning – including publications in the field of aerial vehicles – the suitability of different approaches is discussed.

By leveraging the synergies between aerobatic *unmanned aerial vehicles* (UAVs) and hydrobatic  $\mu$ AUVs, the approach of computational efficient trajectory generation for quadcopters in [MuellerHehnD'Andrea15] is transferred to the underwater domain in Chapter 3. This includes taking the differences in modelling the vehicle's dynamics into account and addressing them with respect to computational efficiency, robustness of the trajectory tracking performance, and the sampling strategy.

The dynamic model of the HippoCampus  $\mu$ AUV is derived in Section 3.2 and applied to the sampling-based trajectory generation framework presented in Section 3.3 and Section 3.4. It is complemented by an attitude controller as proposed in Section 3.6. The framework's implementation details are presented in this thesis are given in Section 3.7

Schon mal sagen, welche Erkenntnisse konkret aus den Experimenten gewonnen werden. Muss abgestimmt werden, mit dem, was ich am Ende tatsaechlich dort stehen habe

# Chapter 2

## Fundamentals

### 2.1 Platform Overview

### 2.2 Review on Path Planning

#### 2.2.1 Underwater Path Planning

A\*

Potential Fields

Rapidly-exploring Random Tree

Model Predictive Control

Discussion

- Vehicles are large/slow
- Have more computational power or algorithm in general not real timecapable.

#### 2.2.2 Review on Agile Path Planning Methods

- Natural to look at quadcopters to transfer solutions to the underwater domain because of the similarity of the design.

### 2.2.3 Discussion



# Chapter 3

## An Approach to Agile Maneuvering

### 3.1 Overview

### 3.2 System Dynamics

#### 3.2.1 Choice of Reference Frames

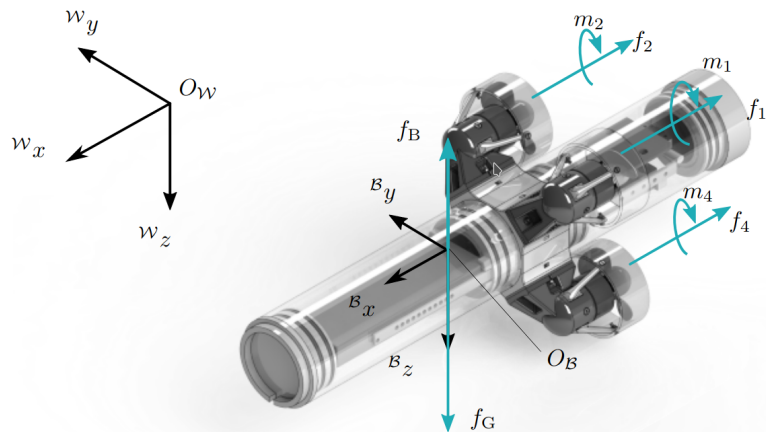


Figure 3.1: Definition of the world-fixed inertial frame of reference  $\mathcal{W}$  and the body-fixed frame  $\mathcal{B}$ .

### World-fixed Frame

### Body-fixed Frame

## 3.2.2 Equations of Motion

The vector of absolute linear and angular velocities expressed in the body-fixed frame  $\mathcal{B}$

$$\boldsymbol{\nu} = \begin{bmatrix} {}^{\mathcal{B}}\mathbf{v} \\ {}^{\mathcal{B}}\boldsymbol{\omega} \end{bmatrix} \quad (3.1)$$

with

$${}^{\mathcal{B}}\mathbf{v} = [u \quad v \quad w]^\top \quad \text{and} \quad {}^{\mathcal{B}}\boldsymbol{\omega} = [p \quad q \quad r]^\top \quad (3.2)$$

Forces and moments with respect to the body-fixed frame.

$$\boldsymbol{\tau} = \begin{bmatrix} {}^{\mathcal{B}}\mathbf{f} \\ {}^{\mathcal{B}}\mathbf{m} \end{bmatrix} \quad (3.3)$$

with

$${}^{\mathcal{B}}\mathbf{f} = [X \quad Y \quad Z]^\top \quad \text{and} \quad {}^{\mathcal{B}}\mathbf{m} = [K \quad M \quad N]^\top \quad (3.4)$$

The vehicle's absolute pose  $\boldsymbol{\eta}$ , i.e. position and orientation, expressed in  $\mathcal{W}$ , is

$$\boldsymbol{\eta} = \begin{bmatrix} {}^{\mathcal{W}}\mathbf{p} \\ {}^{\mathcal{W}}\boldsymbol{\Theta}_{\mathcal{WB}} \end{bmatrix} \quad (3.5)$$

with

$${}^{\mathcal{W}}\mathbf{p} = [x \quad y \quad z]^\top \quad \text{and} \quad \boldsymbol{\Theta}_{\mathcal{WB}} = [\phi \quad \theta \quad \psi]^\top, \quad (3.6)$$

where  ${}^{\mathcal{W}}\mathbf{p}$  is the vehicle's position, i.e. the position of body frame origin  $O_{\mathcal{B}}$ , with respect to the world frame origin  $O_{\mathcal{W}}$ , and  $\boldsymbol{\Theta}_{\mathcal{WB}}$  denotes the euler angle vector describing the transformation between  $\mathcal{W}$  and  $\mathcal{B}$  following the extrinsic  $x$ - $y$ - $z$  convention.

We write the relation between the linear and angular velocities  ${}^{\mathcal{B}}\mathbf{v}$  and  ${}^{\mathcal{B}}\boldsymbol{\omega}$  expressed in the body-fixed frame and the corresponding velocities in the world-fixed frame as

$$\begin{bmatrix} {}^{\mathcal{W}}\dot{\mathbf{p}} \\ \dot{\boldsymbol{\Theta}}_{\mathcal{WB}} \end{bmatrix} = \underbrace{\begin{bmatrix} {}^{\mathcal{W}}\mathbf{R}_{\mathcal{B}} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{T}_{\boldsymbol{\Theta}} \end{bmatrix}}_{:=\mathbf{J}_{\boldsymbol{\Theta}}} \begin{bmatrix} {}^{\mathcal{B}}\mathbf{v} \\ {}^{\mathcal{B}}\boldsymbol{\omega} \end{bmatrix}, \quad (3.7)$$

with  $\mathbf{J}_{\boldsymbol{\Theta}}$  is the rotation matrix

$${}^{\mathcal{W}}\mathbf{R}_{\mathcal{B}} = \begin{bmatrix} c\psi c\theta & c\psi s\theta s\phi - s\psi c\phi & s\psi s\phi + c\psi c\phi s\theta \\ s\psi c\theta & c\psi c\phi + s\phi s\theta s\psi & s\theta s\psi c\phi - c\psi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \quad (3.8)$$

and the transformation matrix

$$\mathbf{T}_\Theta = \begin{bmatrix} 1 & s\phi t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & \frac{s\phi}{t\theta} & \frac{c\phi}{c\theta} \end{bmatrix}, \quad (3.9)$$

where  $s\cdot$ ,  $c\cdot$  and  $t\cdot$  represent the functions  $\sin(\cdot)$ ,  $\cos(\cdot)$  and  $\tan(\cdot)$ , respectively.

$$\mathbf{M}_{\text{RB}}\dot{\boldsymbol{\nu}} + \mathbf{C}_{\text{RB}}(\boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{\tau} \quad (3.10)$$

$$\mathbf{M}_{\text{RB}} = \begin{bmatrix} m\mathbf{I}_{3\times 3} & \mathbf{0} \\ \mathbf{0} & \mathbf{J} \end{bmatrix} \quad (3.11)$$

Because *principal axes of inertia* coincide with body frame axes,  $\mathbf{J}$  will be diagonal, i.e.  $\mathbf{J} = \text{diag}(J_x, J_y, J_z)$ .

$$\mathbf{M}_{\text{RB}}\dot{\boldsymbol{\nu}} + \mathbf{C}_{\text{RB}}(\boldsymbol{\nu})\boldsymbol{\nu} + \underbrace{\mathbf{M}_{\text{A}}\dot{\boldsymbol{\nu}} + \mathbf{C}_{\text{A}}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{D}_{\text{A}}(\boldsymbol{\nu})\boldsymbol{\nu}}_{\text{hydrodynamic loads}} + \underbrace{\mathbf{g}(\boldsymbol{\eta})}_{\text{hydrostatic load}} = \boldsymbol{\tau} \quad (3.12)$$

$$\mathbf{M} = \mathbf{M}_{\text{RB}} + \mathbf{M}_{\text{A}} \quad (3.13)$$

make sure to  
reference  
[Fossen11]

Replace  $\boldsymbol{\tau}$  with  ${}^{\mathcal{B}}\mathbf{u}$ ? Assumption of no other external loads?

$${}^{\mathcal{B}}\mathbf{u} = \begin{bmatrix} u_1 & 0 & 0 & u_2 & u_3 & u_4 \end{bmatrix}^\top \quad (3.14)$$

$$\widetilde{\mathbf{M}} = (m\mathbf{I} + \mathbf{M}_{v,\text{A}}), \text{ with } \mathbf{M}_{\text{A}} = \text{diag}(\mathbf{M}_{v,\text{A}}, \mathbf{J}_{\text{A}}) \quad (3.15)$$

$$\widetilde{\mathbf{J}} = (\mathbf{J} + \mathbf{J}_{\text{A}}), \text{ with } \mathbf{D}_{\text{A}} = \text{diag}(\mathbf{D}_{v,\text{A}}, \mathbf{D}_{\omega,\text{A}}) \quad (3.16)$$

$$\widetilde{\mathbf{M}}{}^{\mathcal{B}}\dot{\mathbf{v}} = {}^{\mathcal{B}}\mathbf{v} \times \widetilde{\mathbf{M}}{}^{\mathcal{B}}\boldsymbol{\omega} - \mathbf{D}_{v,\text{A}}{}^{\mathcal{B}}\mathbf{v} - \mathbf{g}(\boldsymbol{\eta}) + u_1\mathbf{e}_1 \quad (3.17)$$

$$\widetilde{\mathbf{J}}{}^{\mathcal{B}}\dot{\boldsymbol{\omega}} = {}^{\mathcal{B}}\mathbf{v} \times \widetilde{\mathbf{M}}{}^{\mathcal{B}}\boldsymbol{\omega} - {}^{\mathcal{B}}\boldsymbol{\omega} \times \widetilde{\mathbf{J}}{}^{\mathcal{B}}\boldsymbol{\omega} - \mathbf{D}_{\omega,\text{A}}{}^{\mathcal{B}}\boldsymbol{\omega} - \mathbf{g}(\boldsymbol{\eta}) + \begin{bmatrix} u_2 & u_3 & u_4 \end{bmatrix}^\top \quad (3.18)$$

Muss es nicht  
sein? die obere  
Haelfte von  $\mathbf{g}$   
sein?  
Muesste  $\widetilde{\mathbf{M}}$   
nicht vor  ${}^{\mathcal{B}}\mathbf{v}$   
stehen?

We make the following assumptions:

- Symmetry with respect to  $xz$ ,  $yz$  and  $xy$  planes.
- The center of gravity lies in the origin  $O_{\mathcal{B}}$  of the body-fixed frame  $\mathcal{B}$ .

- The difference in the magnitudes of buoyancy force and gravitational force is zero, i.e. the vehicle is neutrally buoyant. Therefore, the restoring force is assumed to be zero.
- The center of buoyancy and the center of gravity coincide. The resulting restoring moment will be zero.
- The vehicle's velocity is relatively small.
- The motion of the vehicle is uncoupled.
- The principal axes of inertia coincide with the body-fixed frame axes.

Using body symmetry for  $xz$ ,  $yz$  and  $xy$  planes

$$\mathbf{M}_A = \text{diag}(X_{\dot{u}}, Y_{\dot{v}}, Z_{\dot{w}}, K_{\dot{p}}, M_{\dot{q}}, N_{\dot{r}}) \quad (3.19)$$

In general,

$$\mathbf{D}(\boldsymbol{\nu}) = -\text{diag}(X_u, Y_v, Z_w, K_p, M_q, N_r) \quad (3.20)$$

Fossen 7.5.5

Nicht-linearen Teil lass ich direkt weg, weil der auch nicht mal simuliert wird.

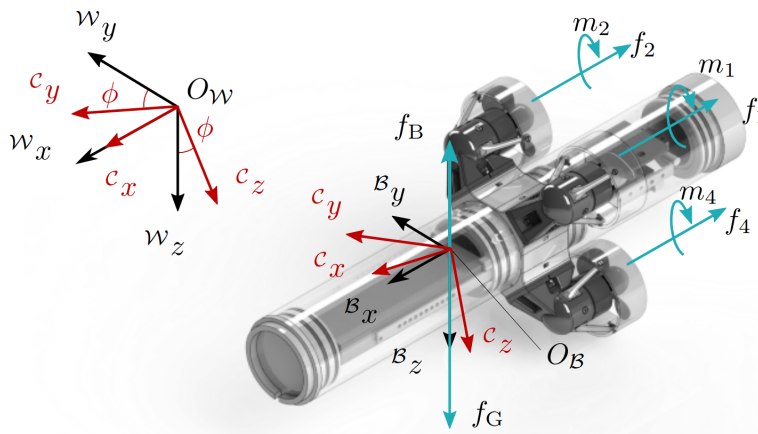


Figure 3.2: Free body diagram of the HippoCampus  $\mu$ AUV with buoyancy force  $\mathbf{f}_B$ , gravitational force  $\mathbf{f}_G$ , thruster forces  $\mathbf{f}_{1:4}$ , and thruster torques  $\mathbf{m}_{1:4}$ .

- Rigid body dynamics -> added hydrodynamic terms -> assumptions and simplifications -> yields simplified dynamic model
- Thruster model
  - assume motor speed is reached instantaneously (fast in comparison to the body dynamics)

- assume quadratic thrust curve (Hastedt)
- neglect dead band
- neglect forward/reverse asymmetry.

### 3.2.3 Differential Flatness (Optional)

show diff. flatness and motivate it by highlighting that it is useful for the trajectory-generation section

## 3.3 Trajectory Generation

- make clear, that we do not care for any constraints (affine translational or input) for now. refer to the next section.
- Define the state variable consisting of translational variables and derivatives

$$\mathbf{s} = \begin{bmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{a} \end{bmatrix} \quad (3.21)$$

$$\mathbf{s}_i = \begin{bmatrix} p_i \\ v_i \\ a_i \end{bmatrix} \quad (3.22)$$

$$\dot{\mathbf{s}}_i = \begin{bmatrix} v_i \\ a_i \\ \dot{j}_i \end{bmatrix} \quad (3.23)$$

### 3.3.1 Kinematic Derivation of the jerk-optimal Trajectory

- Define the cost function
- derive the minimum jerk solution per axis
-

$$J_{\Sigma} = \frac{1}{T} \int_0^T \|\mathbf{j}(t)\|^2 dt \quad (3.24)$$

$$J_{\Sigma} = \sum_{i=1}^3 J_i, \text{ where } J_i = \frac{1}{T} \int_0^T j_i(t)^2 dt \quad (3.25)$$

$$\mathbf{s}^* = \begin{bmatrix} \frac{\alpha}{120}t^5 + \frac{\beta}{24}t^4 + \frac{\gamma}{6}t^3 + \frac{a_0}{2}t^2 + v_0t + p_0 \\ \frac{\alpha}{24}t^4 + \frac{\beta}{6}t^3 + \frac{\gamma}{2}t^2 + a_0t + v_0 \\ \frac{\alpha}{6}t^3 + \frac{\beta}{2}t^2 + \gamma t + a_0 \end{bmatrix} \quad (3.26)$$

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 720 & -360T & 60T^2 \\ -360T & 168T^2 & -24T^3 \\ 60T^2 & -24T^3 & 3T^4 \end{bmatrix} \begin{bmatrix} \Delta p \\ \Delta v \\ \Delta a \end{bmatrix} \quad (3.27)$$

$$\begin{bmatrix} \Delta p \\ \Delta v \\ \Delta a \end{bmatrix} = \begin{bmatrix} p_f - p_0 - v_0T - \frac{1}{2}a_0T^2 \\ v_f - v_0 - a_0T \\ a_f - a_0 \end{bmatrix} \quad (3.28)$$

### 3.3.2 Sampling Strategy

- how to choose the final state to reach the high level goal, i.e. catching the ring as proposed scenario.
  - vehicle tip position inside the ring
  - different final attitudes possible (sampling on a section of a sphere around final tip position)
  - calculate actual vehicle's position based on that
- higher variety of final states increases chance to generate a feasible solution  
-> refer to following section
- introduce additional rotated inertial system to specify axis components or let them unspecified
- stress out the difference to MuellerHehn15, due to velocity dependency.

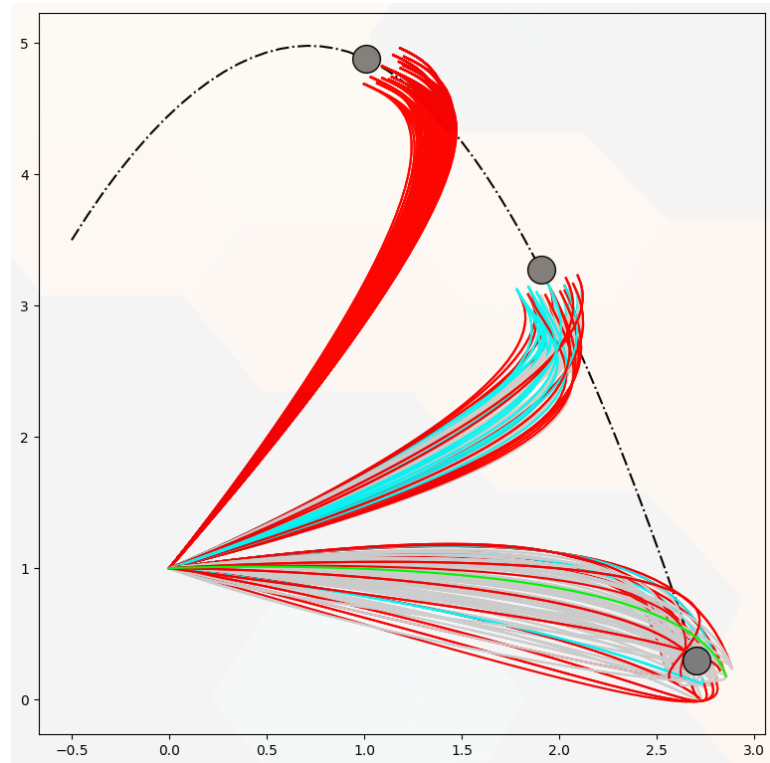


Figure 3.3: Generated trajectories for different final states and time horizons.

## 3.4 Feasibility of Trajectories

### 3.4.1 Input Feasibility

$$0 \text{ N} \leq f_{\min} \leq f \leq f_{\max} \quad (3.29)$$

$$\|\omega\| \leq \omega_{\max} \quad (3.30)$$

#### Thrust Input Feasibility

- cubic instead of quadratic function is to be solved
- stress out, that feasibility criterium is quite coarse -> feasible solution might be classified as indeterminable

$$\max_{t \in \mathcal{T}} f(t)^2 \leq f_{\max}^2 \quad (3.31)$$

$$\min_{t \in \mathcal{T}} f(t)^2 \geq f_{\min}^2 \quad (3.32)$$

### Body Rates Input Feasibility

- Insert dynamics into definition of jerk

### 3.4.2 Position Feasibility

## 3.5 Obstacle/Collision Avoidance

- Make clear, that obstacle avoidance is not built into the trajectory generation itself.
- can be seen as subsequent feasibility check.

## 3.6 Control

- one could use the min jerk trajectory approach simply for generation -> feed-back control to keep the vehicle on track
- computational efficiency and real-time capability of approach enables trajectory generation to be used for implicit feedback. Regenerate trajectories in each control time step.
- body rates will change rather slowly compared to quadrocopters -> instead of using body rates as in MuellerHehn15, use target attitude and attitude control (actually the body rates are derived from a target attitude anyway)

## 3.7 Implementation



# Chapter 4

## Analysis

- Test the dynamic behavior of the body rates to assess whether the assumptions that they are reached slowly and by far not instantaneously holds.
- real time capability on limited hardware is required for deployment -> analysis of computation times required for assessment
- single trajectory tracking without implicit feedback, to analyze performance of (half) open loop trajectory tracking. while keeping disturbances at a minimum, this should show how susceptible the tracking performance is for errors in the assumed model parameters.
- Multi Trajectory/Implicit Feedback should show the full capability of the approach. Collision avoidance can be added optionally

### 4.1 Body Rates Analysis

- Useful for myself to get reasonable values for the body rate limit
- Either use the body rate controller or just feed through
- Use step function and compute the time constant?
- Only test pitch/yaw. Roll is irrelevant

## 4.2 Computational Performance

- Measure the following
  - generation time (expected to be constant, small variance due to running on a best effort machine)
  - input feasibility (a recursive algorithm. maybe have a special look on worst case)
  - position feasibility (only extrema need to be considered. small variance expected)
  - collision avoidance ()
- Tabular, or BoxPlot? Or Barplot?

## 4.3 Single Trajectory Tracking

## 4.4 Implicit Feedback

# Chapter 5

## Conclusion

### 5.1 Summary

### 5.2 Future Work

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## **Erklärung**

Ich, Thies Lennart Alff (Student des Theoretischen Maschinenbaus an der Technischen Universität Hamburg, Matrikelnummer 21380139), versichere, dass ich die vorliegende Masterarbeit selbstständig verfasst und keine anderen als die angegebenen Hilfsmittel verwendet habe. Die Arbeit wurde in dieser oder ähnlicher Form noch keiner Prüfungskommission vorgelegt.

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Unterschrift

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Datum