sheet08

January 9, 2024

Sheet08

Ex. 15

(a)

Die Response-Matrix beschreibt allgemein den Messprozess und ist ein linearer Operator $A : Af = g$. Die Matrix bildet die wahre Verteilung auf die gemessene Verteilung ab. Das Matrixelement A_{ij} ist die Wahrscheinlichkeit, das Event x aus Bin j nach der Messung als y in Bin i zu finden. Im konkreten Beispiel beschreibt die Matrix eine Verschmierung in benachbarte Bins (da auch Nebendiagonalelemente).

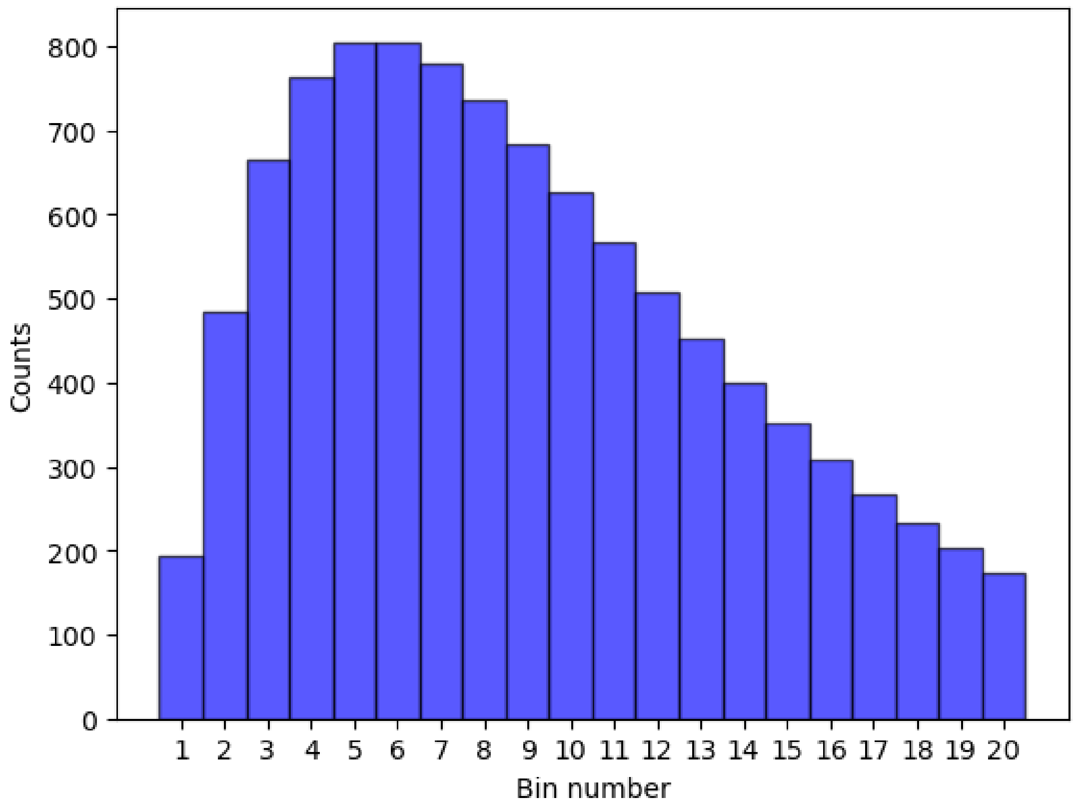
[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
```

[ ]:
```
def response_matrix_A(n, epsilon = 0.23):
    A = np.zeros((n, n))
    for i in range(n):
        for j in range(n):
            if i == j:
                A[i, j] = 1-2*epsilon
            elif i == j + 1:
                A[i, j] = epsilon
            elif i == j - 1:
                A[i, j] = epsilon
    return A
```

(b)

[ ]:
```
bin_values = np.array([193, 485, 664, 763, 804, 805, 779, 736, 684, 626, 566,␣
 ↪508, 452, 400, 351, 308, 268, 233, 202, 173])
bins = np.arange(1, 21, 1)
plt.bar(bins, bin_values, width=1, color='b', edgecolor='k', alpha=0.65)
plt.xticks(bins)
plt.xlabel('Bin number')
plt.ylabel('Counts')
```

[ ]:
```
Text(0, 0.5, 'Counts')
```



[ ]:
```python
def measurement(f):
    n = len(f)
    A = response_matrix_A(n)
    g = A @ f
    g_measured = np.random.poisson(g)
    return g_measured, A
```

[ ]:
```python
import numpy as np
import matplotlib.pyplot as plt


def diagonalize(A):
    ew, ev = np.linalg.eig(A)
    idx = ew.argsort()[::-1]
    ew = ew[idx]
    ev = ev[:,idx]
    U = ev
    U_inv = np.linalg.inv(U)
    D = U_inv @ A @ U
    np.round(D, 3, out=D)
    np.round(U, 3, out=U)
```

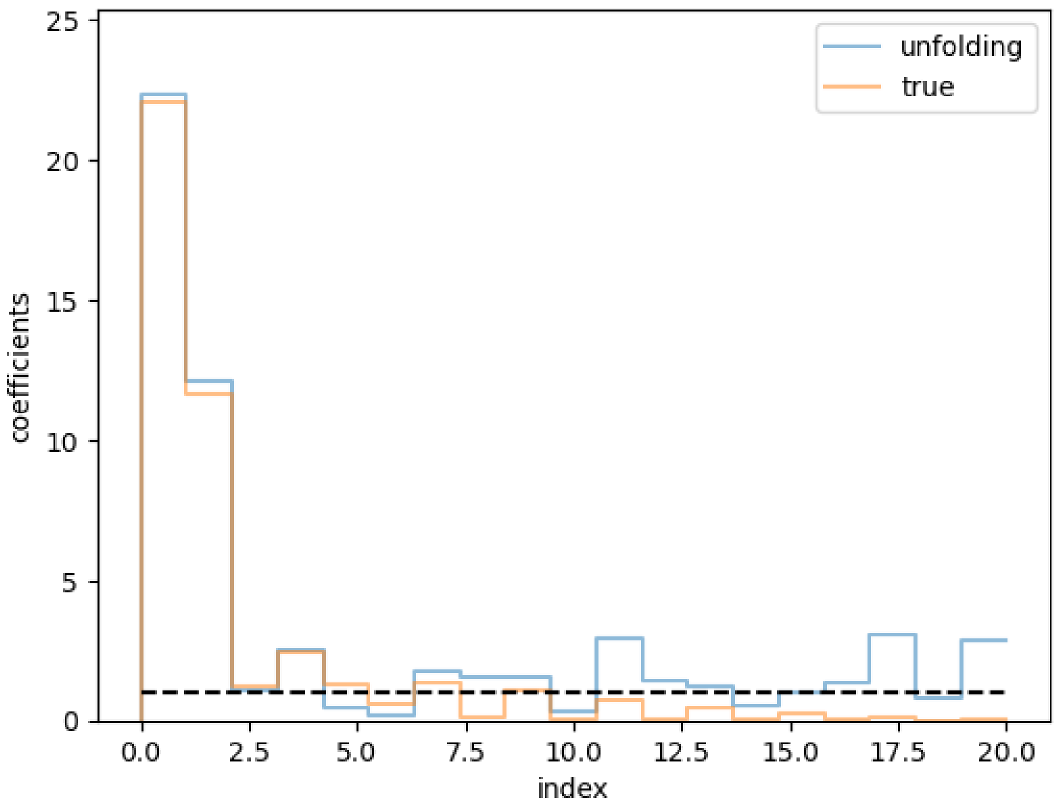
```
    np.round(U_inv, 3, out=U_inv)
    return D, U, U_inv
```

(d)

```
[ ]: f = bin_values
g_measured, A = measurement(f)
D, U, U_inv = diagonalize(A)
#coefficients c and b
c = U_inv @ g_measured
b = U_inv @ f
b_measured = np.linalg.inv(D) @ np.linalg.inv(U) @ g_measured
#bvb formula for covariance
cov_b_measured = np.linalg.inv(D) @ np.linalg.inv(U) @ np.diag(g_measured) @␣
 ↪(np.linalg.inv(U) @ np.linalg.inv(D)).T
cov_b = np.linalg.inv(D) @ np.linalg.inv(U) @ np.diag(f) @ (np.linalg.inv(U) @␣
 ↪np.linalg.inv(D)).T
#scale to standard deviation
norm_b_measured = b_measured / np.sqrt(cov_b_measured.diagonal())
norm_b = b / np.sqrt(cov_b.diagonal())
#basis change
f = U @ b
g_measured = U @ c
norm_b_measured = np.array(norm_b_measured)
norm_b = np.array(norm_b)
norm_b_measured = norm_b_measured.reshape(20,1)
norm_b = norm_b.reshape(20,1)
norm_b_measured = np.abs(norm_b_measured)
norm_b_measured[0] = -norm_b_measured[0]
norm_b = np.abs(norm_b)
norm_b[0] = -norm_b[0]
```

```
[ ]: x = np.linspace(0, 20, 20)
plt.step(x, norm_b_measured, label='unfolding',alpha=0.5)
plt.step(x, norm_b, label='true',alpha=0.5)
plt.legend()
plt.xlabel('index')
plt.ylabel('coefficients')
plt.ylim(0, max(norm_b_measured) + 3)
plt.hlines(1, 0, 20, color = 'black', linestyle = 'dashed')
```

```
[ ]: <matplotlib.collections.LineCollection at 0x7fce900bc400>
```

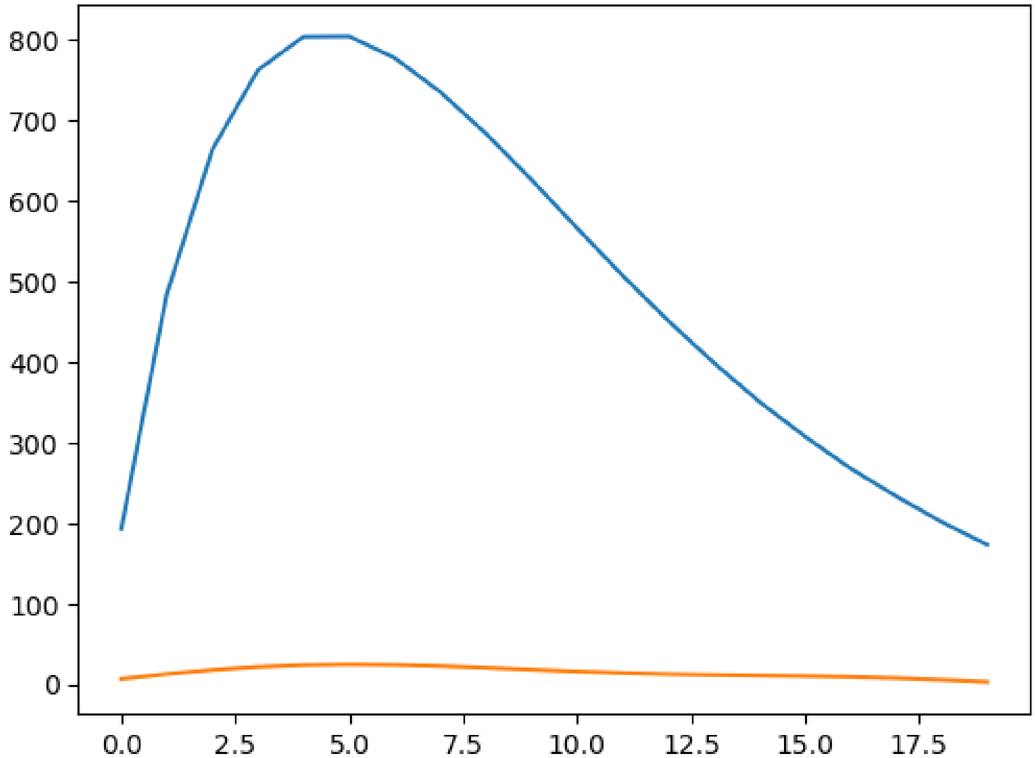


(e)

```
[ ]: for i in range(20):
         if norm_b_measured[i+1] < 1:
             norm_b_measured[i:] = 0
             index = i
             break
     U_reg = U[:, :index]
     norm_b_measured_reg = norm_b_measured[:index]
     f_reg = U_reg @ norm_b_measured_reg
     #keine ahnung iwas funktioniert nicht
```

```
[ ]: plt.plot(f)
     plt.plot(f_reg)
```

```
[ ]: [<matplotlib.lines.Line2D at 0x7fce90146b30>]
```



Die regularisierte Lösung sollte die Oszillationen entfernen.