V64

Modern Interferometry

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Contents

1 Objective **3**

2 Theory **3**
2.1 Light as an electromagnetic wave 3
2.1.1 Polarisation 3
2.1.2 Coherence 4
2.1.3 Interference 4
2.2 Contrast of the interferometer 6
2.3 Calculation of the refractive indices 6

3 Structure of the experiment **8**

4 Conduction of the experiment **9**

5 Evaluation **10**
5.1 Determination of contrast 10
5.2 Refractive index of glass 12
5.3 Refractive index of air 13

6 Discussion **15**
6.1 Contrast 15
6.2 Refractive index of glass 15
6.3 Refractive index of air 15

References **16**

1 Objective

Objective of this experiment is to get to know the Sagnac Interferometer, as well as the measurement of its contrast and the refraction index of glass and air.

2 Theory

2.1 Light as an electromagnetic wave

Light is a electromagnetic wave, which can be derived from the maxwell equations. Its differential equation describes a simple harmonic motion of the field vector in time and space and results in what is called a planar wave function

$$\vec{E}\left(\vec{x},t\right) = \vec{E}_0 \exp\left(i\left(\vec{x}\cdot\vec{k} - \omega t\right)\right) \tag{1}$$

where $\vec{k}$ is the direction of propagation and ω is the frequency of the wave. For a fixed place $\vec{x} = \vec{x}_0$, the electromagnetic field reduces to a harmonic oscillation in time given by the function

$$\vec{E}\left(t\right) = \vec{E}_0 \cos\left(\omega t + \delta\right) \tag{2}$$

where the place component has bin absorbed into a arbitrary phase shift δ.

2.1.1 Polarisation

Light is a transversal wave with its electric and magnetic field vector oszillating perpendicular to its direction of propagation $\vec{k}$. This gives the field vector two degrees of freedom x and y if $\vec{k}$ is parallel to z in an arbitrary frame of reference. One can then differentiate 3 cases:

1 **linear polarisation**: The field components in x, y direction oscillate with arbitrary amplitudes and a constant phase shift of 0° or 180°, the field vector oszillates in a constant plane.

2 **circular polarisation**: The field components in x, y direction oscillate with equal amplitudes and a constant phase shift of 90°, the field vector describes a circular motion around its direction of propagation.

3 **no polarisation**: The field components in x, y have arbitrary amplitudes and no constant phase shift, the field vector does not oscillate in a constant plane or describes a circular motion.

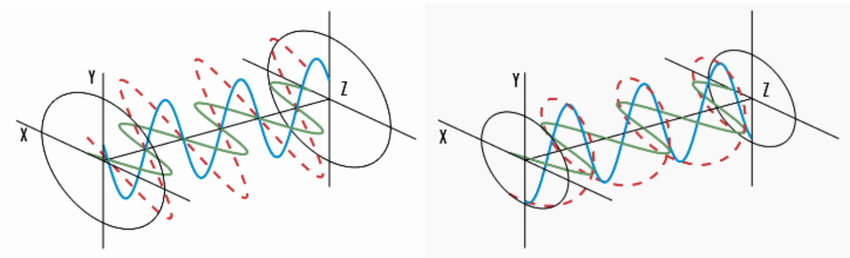

Figure 1: Polarisation modes of light (linear with $\Delta\delta = \pi$ and circular, left to right). [7]

Figure 1 shows linear polarisation on the left and circular polarisation on the right. The red dottet line shows the movement of the electric field vector. For two electromagnetic waves to interfer they have to be polarised in the same direction.

2.1.2 Coherence

Light is coherent when its direction $\vec{k}$, frequency ω and phase δ is the same for every wavepackege that makes up the total planar wave. In this experiment coherent light is produced by a HeNe Laser, which works by induced photon emission. If a photon emission is induced by another photon, the induced photon is coherent with the other photon. By amplifying the amount of induced photons in an active medium by using a so called resonator, the HeNe achieves a coherent beam of light. The distance, in which the phase of two wavefronts of a single light beam diverge by such an amount, that no visible interference is achievable with this lightbeam is called coherence length. It is defined by

$$l_c = \tau_c \cdot \frac{c}{n} \tag{3}$$

where $\frac{c}{n}$ is the speed of light in a specific medium with refraction index n, and τ_c is the coherence time. τ_c describes the time interval in which a stationary interference pattern is achievable with two lightbeams from the same source of delay Δt. Short: the maximum time delay in which two beams of light from the same source are coherent. As the coherence length of a good HeNe Lasers lies in the orders of $10^3 m$ and the used Sagnac Interferometer in this experiments has combined arm lengthes in the meter region, coherence of the Laser beam will be given in this experiment.

2.1.3 Interference

If two coherent beams of light, described by planar wave functions combine in a fixed point $\vec{x} = \vec{x_0}$, interference happens. The electric fields of two beams of the same frequency, each described by a function of the form

$$\vec{E}\left(t\right) = \vec{E}_{0,1/2} \cos\left(\omega t + \delta_{1/2}\right) \tag{4}$$

in a fixed point, simlpy add up like

$$\vec{E}_{1,2}\left(t\right) = \vec{E}_1\left(t\right) + \vec{E}_2\left(t\right) \ . \tag{5}$$

The intensity however is given by

$$I = \langle \vec{E}^2 \rangle \,, \tag{6}$$

$\langle\rangle$ being the mean over one period $T = \frac{2\pi}{\omega}$. Assuming, that both beams have the same amplitude and hit a given point under the same angle, the intensity is given by an Integral of the form

$$I_{1,2} = \frac{1}{T}\int_{t'}^{t'+T} \left(E_0 \cos\left(\omega t' + \delta_1\right)\right)^2 + \left(E_0 \cos\left(\omega t' + \delta_2\right)\right)^2 + 2E_0^2 \cos\left(\omega t' + \delta_1\right) \cdot \cos\left(\omega t' + \delta_2\right) \mathrm{d}t' \,. \tag{7}$$

where the two interfering beams of light are polarised in the same direction. The resulting intensity $I_{1,2}$ then reduces to

$$I_{1,2} = I_0 \left(1 + \cos\left(\delta_1 - \delta_2\right)\right) \tag{8}$$

Which results in destructive interference of the two wavefront for $\mathit{\Delta}\delta = (2n-1) \cdot \pi$ and a constructive interference for $\mathit{\Delta}\delta = 2n\pi$, $n \in \mathbb{N}^+$. For the case that the two beams are perpendicular to each other before entering a polariser, with a polarisation angle $\mathit{\Phi}$ relative to the polarisation axis of the first beam, $I_{1,2}$ has to be modified. The first beam then has an amplitude proportional to $E_{0,1}\left(\mathit{\Phi}\right) \propto \cos\left(\mathit{\Phi}\right)$ after travelling through the polariser while the other beams amplitude is proportional to $E_{0,2}\left(\mathit{\Phi}\right) \propto \cos\left(\mathit{\Phi} + \pi\right) = \sin\left(\mathit{\Phi}\right)$ resulting in a modification in the second term of Equation 8 to

$$\begin{aligned} I_{1,2} &= I_0 \left(1 + 2\cos\left(\delta_1 - \delta_2\right)\cos\left(\mathit{\Phi}\right)\sin\left(\mathit{\Phi}\right)\right) \\ \Longrightarrow I_{\mathrm{max,min}} &= I_0 \left(1 \pm 2\cos\left(\mathit{\Phi}\right)\sin\left(\mathit{\Phi}\right)\right) \end{aligned} \tag{9}$$

for the destructive and constructive phase shifts of the two wavefronts.

2.2 Contrast of the interferometer

The contrast of the interferometer is a measure of visibility of the interference pattern. It is given by

$$C = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \tag{10}$$

and can be seen exemplary in Figure 2.

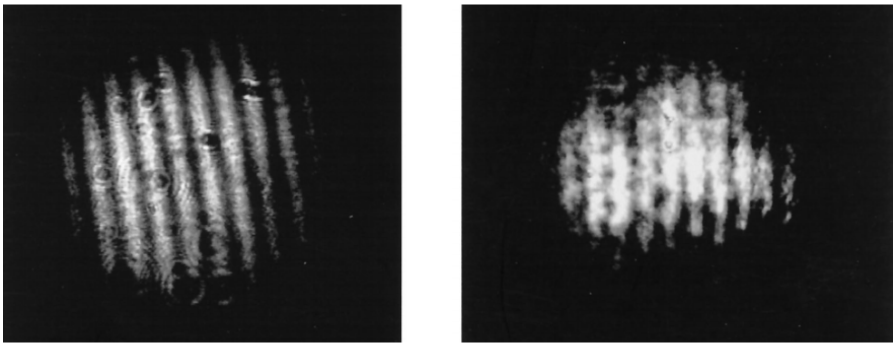

Figure 2: High and low contrast of an interference pattern. [12]

When plugging Equation 9 into Equation 10, the contrast as a function of the polarisation angle Φ is given by

$$C\left(\Phi\right) = \left|\sin\left(2\Phi\right)\right|. \tag{11}$$

2.3 Calculation of the refractive indices

The refractive index n of a medium is an intrinsic property directly dependend of the electromagnetic properties of the medium. The speed of light in a medium changes accordingly to

$$c_m = \frac{c}{n_m} \tag{12}$$

which induces a phase shift relative to a beam which is in a vacuum given by

$$\Delta\delta = \frac{2\pi L}{\lambda_{\text{vac}}} \Delta n\,. \tag{13}$$

When using an interferometer, the number of maxima, which are measured during a change of optical length of one of the beams, is dependend on the total phase shift of the Laser. It can be calculated using

$$M = \frac{\Delta\delta}{2\pi}\,, \tag{14}$$

where M is the number of intensity maxima. When measuring M, $\Delta\delta$ has to be known as a function of the refractive index in order to obtain information about it. For two pieces of glass with thickness T under a rotation angle θ, relative to a perpendicular axis to the beam, the the relative phase shift of both beams can be approximated by the expression

$$\Delta\delta\,(n) = \frac{2\pi T}{\lambda_{\mathrm{vac}}} \frac{n-1}{2n} \Delta\theta^2\,. \tag{15}$$

In the case of this experiment, there are two glass plates in both beams with an initial angle of $\theta_0 = 0.174533\,\mathrm{rad}$ in oposite direction. Equation 15 then has to be modified to

$$\begin{aligned} \Delta\delta\,(n) &= \frac{2\pi T}{\lambda_{\mathrm{vac}}} \frac{n-1}{2n} \left((\theta+\theta_0)^2 - (\theta-\theta_0)^2\right) \\ &= \frac{4\pi T(n-1)}{\lambda_{\mathrm{vac}}} \theta_0\theta \end{aligned} \tag{16}$$

which can be put into Equation 14 and solved for n to get

$$n = \left(1 - \frac{\lambda_{\mathrm{vac}} M}{2\theta\theta_0 T}\right)^{-1}\,. \tag{17}$$

For the refractive index of air, the refractive index can be calculated by phase shift analogous to Equation 13 or as a function of preassure p. for the latter the Lorentz-Lorenz-Law

$$A = \frac{RT}{p} \frac{n^2-1}{n^2+2} \tag{18}$$

can be used, which expresses the molrefraction A as a function of the refractive index n and preassure p. Under the assumption that A can be approximated in linear order around $n = 1$ the equation simplifies to

$$A = \frac{2RT}{3p}\,(n-1) + \mathcal{O}\left(n^2\right) \tag{19}$$

under the use of a taylor series in n around $n = 1$. Solving for n results in the equation

$$n = \frac{3Ap}{2RT} + 1 \tag{20}$$

which is analogous to Equation 13 put into Equation 14 and solved for n_{air} to

$$n_{\mathrm{air}} = \frac{\lambda_{\mathrm{vac}} M}{L} + 1\,. \tag{21}$$

3 Structure of the experiment

For the experiment a Sagnac Interferometer is used, which can be seen in Figure 3.

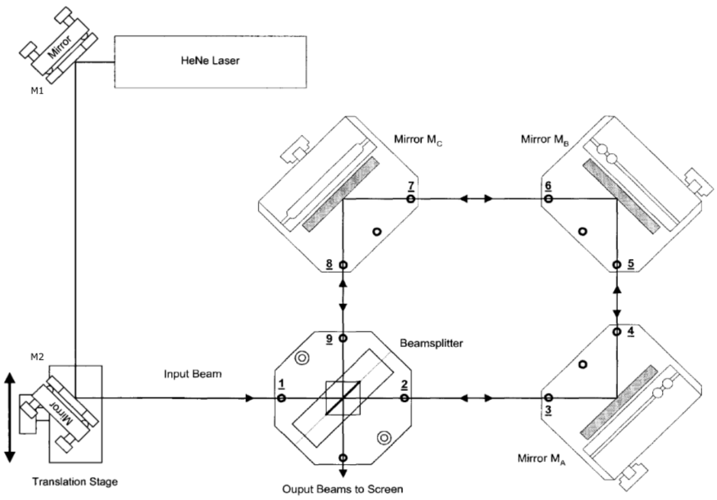

Figure 3: illustration of the Sagnak interferometer [10]

The interferometer consists of 6 mirrors, of which M1 is used to calibrate the Laser beam directly without moving the Laser itself, M2 is used to split the beam into two parallel beams inside the interferometer, and MA to MC are mirrors of the interferometer itself and are used to align the beam correctly. The interferometer is powered by a HeNe Laser, which has a wavelength of $\lambda = 632.8\,\mathrm{nm}$ and a linear polarisation axis of fourtyfive degrees relative to the interferometer plane. Between M2 and MA the Polarising Beam Splitter Cube, short PBSC, can be seen, which splits the incoming beam into two beams, where the beam that is let through is polarised parallel to the interferometer plane and the beam that is reflected is polarised perpendicular to the interferometer plane. The two beams are running in opposite directions in the interferometer and are then recombined by the PBSC. For the measurement of the beams intensity another PBSC which is tilted by fourtyfive degrees is used. The tilted PBSC splits the beam to be measured by two seperate photodiodes, of which one experiences anihilation while the other experiences amplification. The two diodes are connected to a modern interferometry controller, which measures the difference of voltage to correct for stray light. For callibration there are callibration plates which can be placed infront of the mirrors A-C and the first PBSC. For the polarisation measurements two polarisers are used and a vacuum pump configuration is installed for the measurement of the refractive index of air as well as two glass plates for the measurement of the refractive index of glass.

4 Conduction of the experiment

The first step of the experiment is the calibration of the interferometer. For this the HeNe is turned on and the mirror M1 is calibrated to point directly into the center of M2. After that a rigerous calibration process starts which is explained in detail in source [10]. Once the interferometer is calibrated, a polariser is installed infront of the PBSC and the glass plates between MC and the PBSC, in order to measure intensity maximum and minimum. The intensity maximum and minimum is then manually measured by a volt meter for 10 degree increments of the polariser from 0° to 180° 3 times. The polarisation angle is then set to the angle of highest contrast and intensity for the rest of the experiment. For the measurement of the refractive indices of glass, the glass plates are installed in the interferometer and M2 is moved so that there are two seperate beams in the interferometer. The interferometer is readjusted and the modern interferometry controller is connected to the photodiodes and an oscilloscope, which displays the differential voltage of the two photodiodes. The glass plates are then turned by a total of 10 degrees and the number of maxima are measured by the modern interferometry controller. This procedure is done several times, as well for the measurement of the refractive index of air which works almost analogous. For the measurement of the refractive index of air, the difference is that the glass plates are replaced by a vacuum pump configuration, where a vacuum chamber lies in one of the split beams. The vacuum pump is then turned on and the vacuum chamber is vacuumated to almost 0 mbar. Air is slowly let into the chamber and the number of maxima is measured for 50 mBar increments.

5 Evaluation

The graphics and calculations shown in section 5 were created using the Python libraries Matplotlib [4], Scipy [11] and Numpy [3].

5.1 Determination of contrast

The measured values for the voltages U_{max} and U_{min} for the different polarizer angles ϕ are shown in Table 1. The average value is formed from the 3 measurements and the contrast is thus calculated according to Equation 10.

ϕ	U_{max1}	U_{min1}	U_{max2}	U_{min2}	U_{max3}	U_{min3}
0	2.00	1.76	1.91	1.68	1.83	1.68
15	1.74	0.85	1.69	0.85	1.67	0.85
30	1.43	0.45	1.56	0.33	1.44	0.34
45	1.51	0.22	1.59	0.16	1.53	0.15
60	1.81	0.24	1.87	0.21	1.90	0.35
75	2.38	0.73	2.39	0.57	2.46	0.74
90	2.23	1.86	2.22	1.61	2.48	1.86
105	3.63	1.53	3.57	1.59	3.53	1.70
120	5.27	0.62	5.31	0.54	5.24	0.86
135	6.49	0.25	6.00	0.31	5.87	0.44
150	5.59	0.64	5.37	0.70	5.37	0.61
165	4.02	1.41	3.61	1.37	3.83	1.33
180	2.04	1.76	2.14	1.80	1.72	2.16

Table 1: Measured voltages for different polarizer angles.

In Figure 4 the contrast is plotted against the polarizer angle ϕ. Also the fit of the data points to a sinusoidal function of the form

$$f(x) = a \cdot \sin(b \cdot x + c) \tag{22}$$

is shown. The fit parameters are

$$\begin{aligned} a &= (0.867 \pm 0.016), \\ b &= (2.022 \pm 0.015), \\ c &= (-0.127 \pm 0.018). \end{aligned}$$

The maximum contrast occurs at 135°, where the contrast is $C = (0.897 \pm 0.024)$.

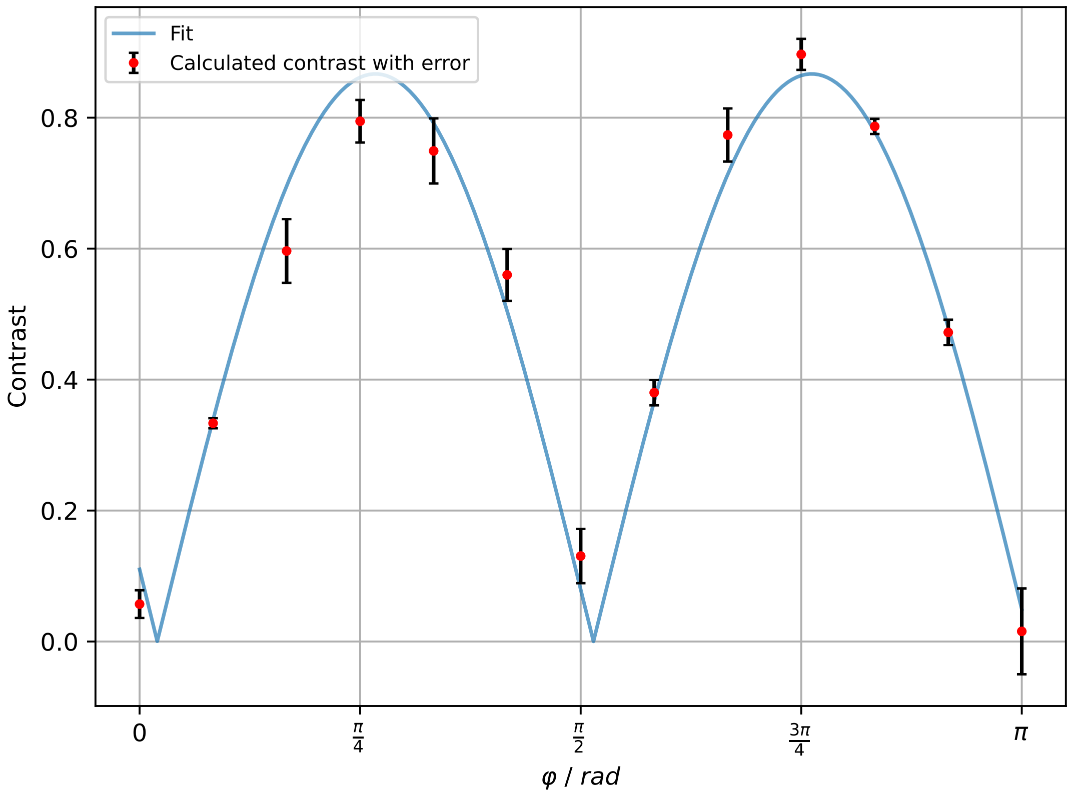

Figure 4: Contrast plotted against the polarizer angle ϕ.

5.2 Refractive index of glass

To determine the refractive index of the glass plate, **??** is used. Needed for this calculation are the maxima counts listed in Table 2 and the wavelength of the light source $\lambda = 632.8\,\text{nm}$. Furthermore, the thickness of the glass plates is $d = 1\,\text{mm}$ and the angles are $\theta = \theta_0 = 10°$.

counts	counts
34	34
30	34
30	33
34	31
34	32
34	32

Table 2: Maxima counts for the refractive index measurement.

The values are averaged to $M_{mean} = (32.4 \pm 1.6)$ and the refractive index is calculated to be $n = (1.51 \pm 0.04)$.

5.3 Refractive index of air

In Table 3 the measured values for p and the corresponding maxima counts are shown.

p / mBar	count1	count2	count3	count4	count5
50	3	2	2	3	2
100	6	4	4	4	4
150	8	6	6	6	6
200	10	8	8	8	8
250	12	10	10	10	10
300	14	12	12	12	12
350	16	14	14	14	14
400	18	17	16	16	16
450	20	19	18	18	18
500	22	21	20	20	21
550	25	23	23	23	23
600	27	25	25	25	25
650	29	27	27	27	27
700	32	29	29	29	29
750	34	31	31	31	31
800	36	33	33	33	33
850	38	36	36	36	35
900	44	38	38	38	37
950	46	40	40	40	40
991	48	41	41	41	41

Table 3: Measured maxima counts for different air pressures.

The average of the counts is calculated for each row. To determine the refractive index of air Equation 21 is used. The values for the refractive index are plotted against the corresponding air pressure in Figure 5. Also plotted are the fit of the data points to a linear function of the form

$$f(x) = a \cdot x + b \tag{23}$$

and the theory curve according to the Lorentz-Lorenz Equation 20.

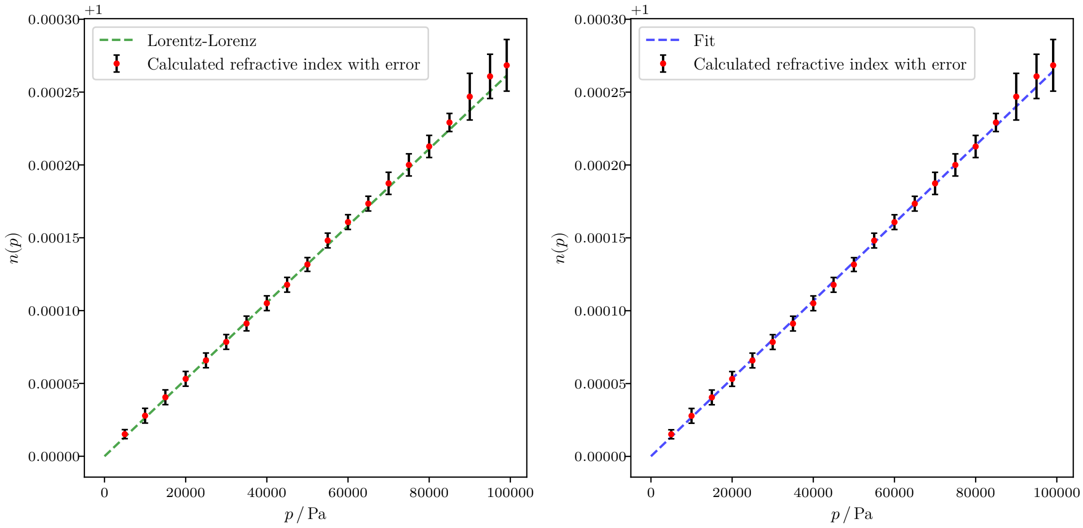

Figure 5: Refractive index of air plotted against the air pressure, theory curve and fit.

The fit parameters are

$$a = (2.66 \pm 0.02) \times 10^{-9} \, \frac{1}{\text{mbar}},$$
$$b = 1.00.$$

For the Lorentz-Lorenz model following values are used:

$$A \approx 0.21 \cdot A_{oxygen} + 0.79 \cdot A_{nitrogen} \approx (4.2915 \pm 0.0024) \times 10^{-6} \, \text{m}^3/\text{mol},$$
$$T = (293.550 \pm 0.001) \, \text{K},$$
$$R = 8.314 \, \text{J/mol/K}.$$

The values for A_{oxygen}[1] and $A_{nitrogen}$[6] are estimated to be constant. Now values for the refractive index at normal pressure can be calculated using regression and the Lorentz-Lorenz law. With $p = 101\,300\,\text{Pa}$ and $T = 288.15\,\text{K}$ the refractive index with the Lorentz-Lorenz law is $n_{LL} = 1.000\,27$ and with the regression $n_{reg} = 1.000\,27$.

6 Discussion

In the following, the percentage deviations are calculated with

$$\mathit{\Delta} = |\frac{exp - theo}{theo}| \cdot 100\%. \tag{24}$$

6.1 Contrast

The maximum contrast is measured to be $C = (0.897 \pm 0.024)$ at an angle of $\phi = 135°$. Expected is a maximum value at either 135° or 45° since the contrast should follow $C\ |\sin(2x)|$. Therefore the experimental value is in agreement with the theoretical prediction.

6.2 Refractive index of glass

The refractive index of the glass plate is measured to be $n = (1.51 \pm 0.04)$. The theoretical value for the refractive index of glass is $n = 1.5168$ [9]. The percentage deviation is calculated to be $\mathit{\Delta} = 0.44\,\%$. This deviation is negligible and the theoretical value even lies in the uncertainty of the experimental value.

6.3 Refractive index of air

For the refractive index of air, the theoretical value is $n = 1.00027$ [8]. This is again in perfect agreement with on the one hand the experimental value and on the other hand the value from Lorentz-Lorenz law:

$$n_{LL} = n_{reg} = 1.000\,27.$$

As one can see, there isn't much to discuss about the results. Overall, it can be said that the measurement appears to be very accurate. This indicates good adjustment of the optical structure. In addition, the difference measurement and averaging over several measurement series appear to eliminate or limit statistical fluctuations.

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