Monday 2-5
Go Forth and Measure

Comparison of Vibrational Sweet Spots of Tennis Rackets

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Abstract

Tennis rackets have evolved greatly and as a result of this evolution there now exists a large set of rackets with variability in length, weight, material composition and price. The goal of this experiment was to find a measure for comparing rackets based solely on the area of the vibrational sweet spot. Using an accelerometer and an impact to measure the vibrations of the string bed for a Babolat, a Head and a Prince racket, this experiment found that the Head racket had an area of 0.2251 units^2 that is 0.1049 and 0.1467 units^2 larger than the areas of the Babolat and the Prince rackets respectively. This new measure for comparing rackets can help identify rackets that are both less likely to cause injury and also more likely to offer the easier use.

1. Introduction

The first tennis racket was made in London in 1874 out of solid wood. [1] Since then, rackets have evolved in both size and material composition. Solid wood gave way to steel and in 1975, the first oversized aluminum racket was created. In 1987, Wilson introduced the first "widebody" racket that had a larger beam width. In 1990, Wilson also introduced a racket with a heavier head. In more recent years, rackets have been getting increasingly lighter and have been made using composite materials. Rackets are both compared quantitatively using head size, weight, length, and string tension and also qualitatively using swing speed, stroke style and power level.

But throughout this time, there haven't been good measures for objectively comparing different rackets. Because of their different compositions and geometry, rackets are hard to compare and a lot of it relies on testing them out yourself and finding the one that feels best. Another pain point when comparing rackets is the price. Due to the materials used and the engineering that goes into manufacturing them, rackets can range from 20 to several hundred dollars. As a result, anyone considering purchasing a racket for regular to extensive use will take a long time to pick a racket to make sure they invest their money wisely. The goal of this experiment is to derive a new measure for comparing tennis rackets based on the area of the sweet of a tennis racket as a ratio to the overall dimension of the racket string bed.

2. Background

The sweet spot of a tennis racket is defined as an area on the racket where the force transmitted to the player's hand is relatively small. On any given racket there are usually three different sweet spots along the axis of the handle. One corresponds to minimal initial shock, another to minimal vibration, and the third corresponds to the maximum coefficient of restitution and results in the most power [2]. For this experiment, the focus was solely on sweet spot corresponding to minimal vibration.

The minimum vibration sweet spot is defined as the nodal point of the first elastic mode of vibration. For a system, a mode is a sinusoidal motion. Each mode has a fixed frequency associated with it, also known as its natural or resonant frequency. In the case of the tennis racket, the first elastic mode, which occurs at a lowest frequency, typically produces vibrations of highest amplitude [2]. This makes it a top priority because strong vibrations can lead to player injury.

One common tennis injury is called tennis elbow. Tennis elbow (also known as lateral epicondylitits) is caused by injury, abrupt or subtle, of the area where the muscles and tendons of the forearm attach on the outside of the elbow. Although the main cause in tennis player is stress from overusing the forearm, low-frequency vibrations (such as those of the first elastic mode) aid the injury formation process. Li et al. [3] have in the past used finite element simulations to observe the effects of vibration on a tennis racket during a racket collision with a tennis ball. Specifically they focused on investigating how string tension and impact location affected the racket deformation and resultant force on a player's hand as a way to look at ergonomics and injury prevention.

Another past study by Buechler [4] was very similar to the experiment I performed. Buechler analyzed vibration characteristics of a tennis racket through the use of a physical setup aided by modal software analysis to identify mode shapes of the racket with and without commercial dampeners [4]. The study also used finite element simulation to match the experimental data to mode shapes. Although this study gives a more precise location of the sweet spot associated with minimal vibration, it focuses more on quantifying the effects of string tension on sweet spot and mode shapes as well as the effect of vibrational dampeners on the sweet spot. While this focus does help to better quantify the sweet spot as a racket performance measure and give information as to which rackets may be better, it does not look at the area of the sweet in comparison to the overall area of the racket head. Thus, this inquiry into the ratio of sweet spot area to racket head area of various rackets aims to measure the ratio area of the sweet spot which maybe offer insight into a more objective measure of racket performance since racket and string materials vary.

These studies by Mohanty [2], Li et al. [3] and Buechler [4] have done a solid job at creating models for measuring the vibrational sweet spot and observing how different parameters like string tension and accessories like vibrational dampeners affect the sweet spot, providing a great base for what the sweet spot is and how it changes. Unfortunately they fail to look at the difference in racket manufacturing (in terms of racket and string material properties) and size of the racket head and how that affects not only the size and location of the spot. This experiment aims to use dimensions of the racket head to compute a normalized value for the area of the sweet spot that can be used to compare different racket types (or brands) and even among the same racket at different tension levels.

3. Experimental Design

In order to quantify the vibrational sweet spot of different tennis rackets, an experiment was conducted using an accelerometer, an impact hammer, and three different rackets.

3.1 Testing Set-up

[Note to readers: better labeled pictures of the setup will be added to the final draft.]

For all three rackets the setup was the same. The rackets were clamped down at the handle using a large industrial C-Clamp close to the lower end of the racket and a large plastic clamp nearer to the throat of the racket. On the lower end of the tennis racket head, above the throat, a Vernier 3-axis accelerometer was attached with cable ties in between the two middle longitudinal strings of the racket. The accelerometer was used to measure the vibration of the racket head. Only the Z-axis channel was used for this sensor that can recognize a maximum acceleration of ±5 g. Input force was measured using an Impact Hammer PCB model 086D05. The impact hammer has a max force of 22.2 kN with a 0.23 mV/N sensitivy. Voltage measured with the impact hammer first went through a Vernier Instrumentation Amplifier with a gain setting of 0-1 V. All data was collected by the Vernier LabQuest Mini and the Logger Pro 3.10.1 software, at 5000 samples/second for 1 second per sample.

Figure 1: Photo of setup during original data collection for Babola AeroPro Drive racket.

3.3 Data Collection

For this experiment, data was taken from three different commercially sold rackets: Babolat AeroPro Drive, Head Graphene Prestige Midplus, and Prince Tour 98. After the accelerometer was attached to the racket throat and the impact hammer set-up, data samples were collected from each of the eleven locations by impacting the racket string bed with the hammer. Every round of data collection was triggered by the voltage transmitted from the force of the impact hammer increasing across 10 mV. All data analysis was done in MATLAB R2015b and Microsoft Excel 2016.

4. Results and Discussion

To compare the rackets, the distances from center had to first be normalized against the dimensions of each racket string bed. To simplify analysis, the racket string beds were assumed to be perfect circles. Lengths of central axes were measured to find the radii for the ellipse. Distances from the center were measured for the two points to the left of the center and three points up from the center. Because we're assuming circular areas, the distances from center to the left are the same as the distances from center to the right and there is a similar relationship between distances from center up and down. Table one shows the distances from the center of each racket to each testing location. Since this analysis is of the racket as a circle, the locations of interest are the center (distance equals 0), Down1, Down2, Down3, Up1, Up2, and Up3.

Location	Babolat Racket	Head Racket	Prince Racket
Down3	-0.721153846	-0.744897959	-0.705882353
Down2	-0.451923077	-0.43877551	-0.431372549
Down1	-0.221153846	-0.204081633	-0.215686275
Left1	-0.261437908	-0.268456376	-0.251968
Left2	-0.549019608	-0.55033557	-0.503936
Right1	0.261437908	0.268456376	0.251968
Right2	0.549019608	0.55033557	0.503936
Up1	0.221153846	0.204081633	0.215686275
Up2	0.451923077	0.43877551	0.431372549
Up3	0.721153846	0.744897959	0.705882353

Table 1: Distances from center to tested locations for each racket bed. Distances are unitless because they've been normalized against dimensions of each racket bed, allowing for inter racket comparison.

At each of these locations samples of acceleration and input force using the accelerometer and the impact hammer were collected. The input (force in the form of voltage) and output (acceleration response to striking the racket bed) were used to estimate a transfer function for the sample. The max gain was extracted from each sample and then averaged by location. Figure 2 shows the average maximum gain at each of the longitudinal (up-down) locations and Figure 3 shows the average maximum gain at each of the latitudinal (left-right) locations.

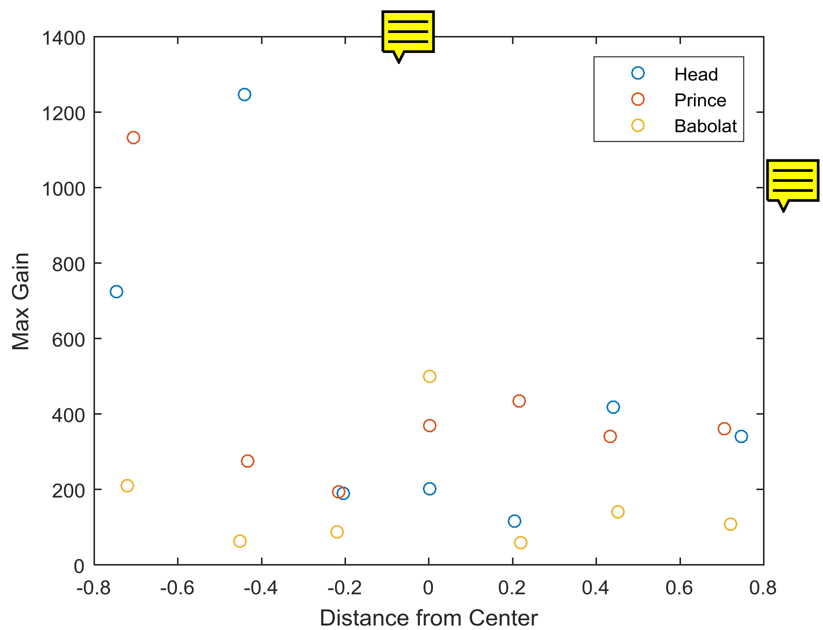

Figure 2: Plot of average max gain for each racket at the 7 locations in the longitudinal direction (the center is counted in the seven locations).

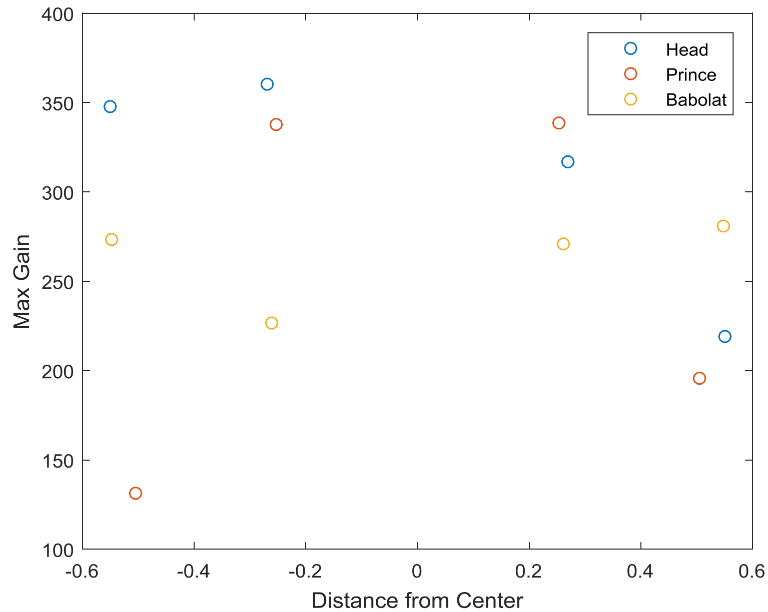

Figure 3: Plot of average max gain for each racket at the 4 locations in the latitudinal direction.

[Note to readers: In the final draft, I will present alternative fits and an elliptical analysis but I have some issues with some of the data being abnormally off and makes the elliptical analysis with a parabola or 4th degree polynomial impossible. Some data had to be discarded because of the abnormalities and will be retaken for the final draft.]

Since the tennis racket string bed resembles an ellipse in shape, one might think that the deformation of the racket bed when struck follows some paraboloid-like shape. For this reason, each set of points was fitted first to a parabola in the longitudinal direction. Figures 4, 5, and 6 show these fits for the Babolat, Head, and Prince rackets respectively.

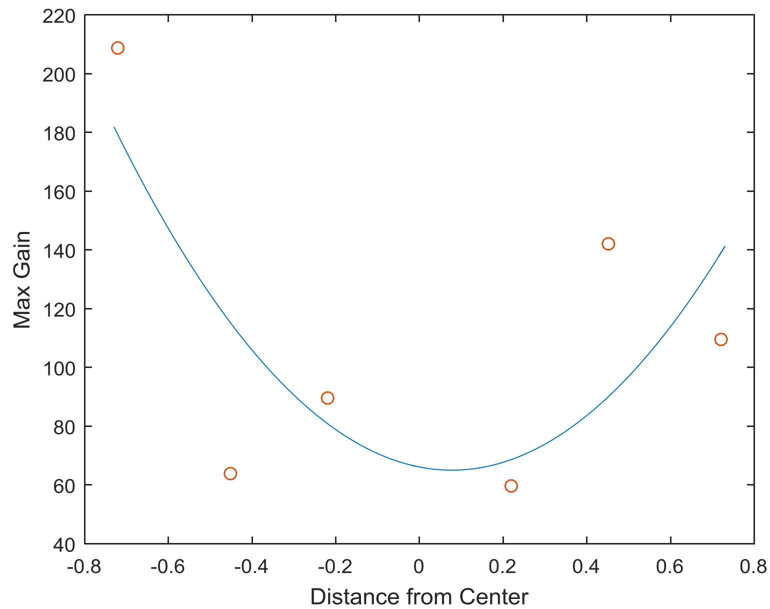

Figure 4: Plot of the parabolic fit of Max Gain vs. Distance from center in the longitudinal direction for the Babolat racket. The blue lines is the fit equation. For the Babolat racket, the parabola has equation: $Gain = 179.1 * x^2 - 27.84 * x + 66.12$. In this, x is the Distance from Center.

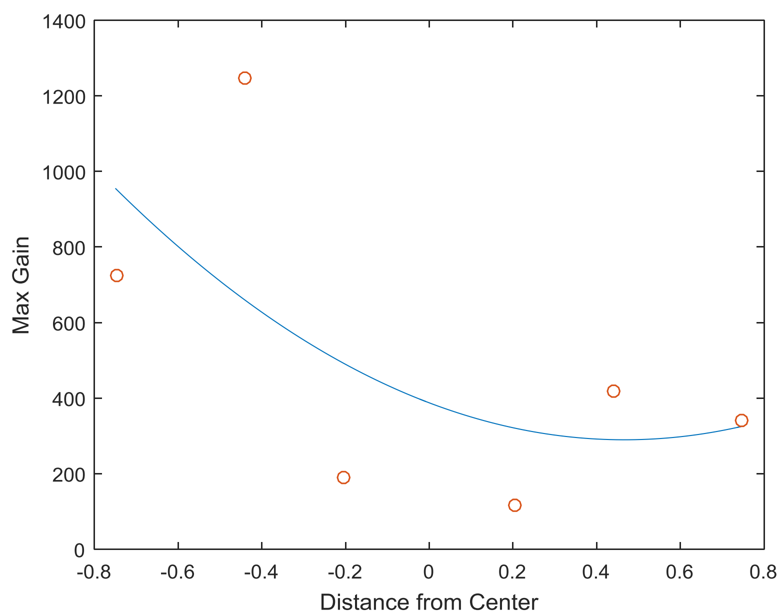

Figure 5: Plot of the parabolic fit of Max Gain vs. Distance from center in the longitudinal direction for the Head racket. The blue lines is the fit equation. For the Head racket, the parabola has equation: $Gain = 449.9 * x^2 - 419.6 * x + 387.5$. In this, x is the Distance from Center.

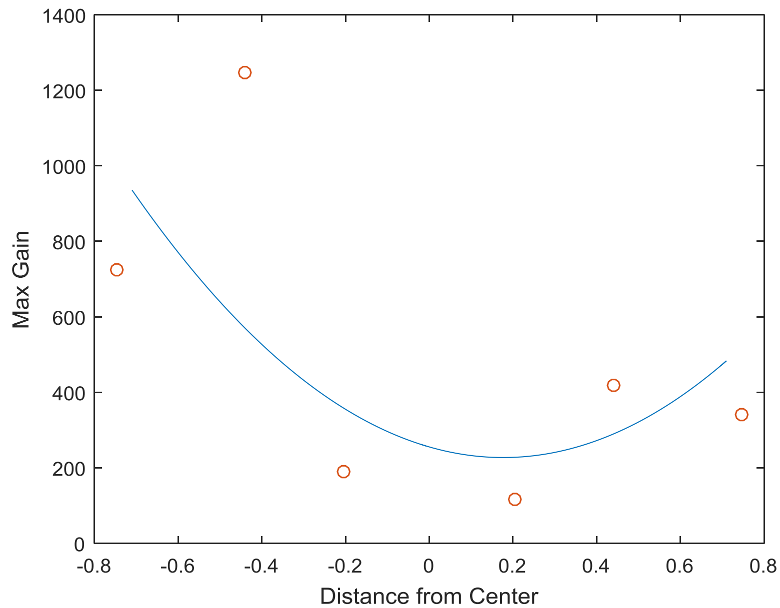

Figure 6: Plot of the parabolic fit of Max Gain vs. Distance from center in the longitudinal direction for the Prince racket. The blue lines is the fit equation. For the Prince racket, the parabola has equation: $Gain = 900.7 * x^2 - 318.5 * x + 255.3$. In this, x is the Distance from Center.

With these fit equations, the minimum gain or vibrations across the racket's longitudinal axis can be found. The location of the vibrational sweet spot is the same as the location for the minimum gain. To find the sweet spot area, a cutoff of 10% gain increase from the minimum gain was set. Table 2 shows the minimum gains, along with the 10% cutoff gain for the sweet spot and the location of both gains.

Racket	**Minimum Gain**	**Distance from center to Minimum Gain**	**10% Cutoff Gain**	**Distance from center to 10% Cutoff Gain**
Babolat	65.04	0.07273	71.54	0.2683
Head	289.7	0.4525	318.67	0.7202
Prince	227.2	0.1778	249.92	0.3358

Table 2: This table shows the minimum gain calculated from the fit equation, as well as the distance from center to the minimum gain, the 10% cutoff gain for the sweet spot area and distance from center to the cutoff gain. The difference in distances between minimum and cutoff gain are used to calculate the radius of the sweet spot area as a circle calculation.

With the distances from center to the location of minimum gain and the location of the 10% cutoff gain, we can find the radius of the vibrational sweet spot area. And with that radius, we can find the area of the vibrational sweet spot. Table 3 lists the radii and areas of the vibrational sweet spots for the three rackets.

Racket	Radius of Vibrational Sweet Spot Area	Area of Vibrational Sweet Spot
Babolat	0.19557	0.1202
Head	0.2677	0.2251
Prince	0.1580	0.0784

Table 3: Table of radii and areas of the vibrational sweet spot.

From the analysis, we can conclude that the Head Graphene Prestige Midplus racket has the largest vibrational sweet spot area when compared to the Babolat AeroPro Drive and the Prince Tour 98 rackets. The Head Graphene Prestige Midplus racket has a vibrational sweet spot area that of 0.2251 units^2 that is 0.1049 units^2 and 0.1467 units^2 larger than the areas of the Babolat AeroPro Drive and the Prince Tour 98 rackets respectively.

The main limitations to this approach are the inability to accurately model the racket head and the inability to eliminate string tension as a factor. Modern tennis rackets have heads shaped like ellipses, and an estimation of the vibrational sweet spot area as an ellipse and not a circle would be more accurate. Also, all three rackets had different amounts of tension in the string bed, and my experiment currently doesn't account for this. During initial data collection, there were samples of data that resulted in very large maximum gain values and this was most likely a result of the string tension being different in each racket. String tension is how tense or stiff the strings. If the tension is high, the strings are less likely to move significantly when struck.

5. Conclusions

Through looking at the transfer function at various locations on a tennis racket string bed and creating a parabolic fit to the maximum gains along the longitudinal axis of a racket, this experiment was able to create a measure for comparing rackets based on their vibrational sweet spot area. When comparing the Prince Tour 98, Babolat AeroPro Drive and the Head Graphine Midplus rackets, the latter racket had the largest vibrational sweet spot area of 0.2251 units^2. The Babolat AeroPro Drive had a vibrational sweet spot area of 0.1202 units^2, and the Prince Tour 98 had a vibrational sweet spot area of 0.0784 units^2.

These results have interesting implications for tennis players who are looking to buy a tennis racket. When purchasing a racket, the rule of thumb is that a larger racket head means a larger sweet spot. This experiment gives us a new way to compare rackets and find the one with the biggest vibrational sweet spot area. The be[illegible]t of this is that we can compare rackets across the same price range, and thus pick a racket that is less likely to lead to injury if used extensively.

Acknowledgments

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References

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[4] Miles A Buechler, L. A. E., "Vibration modeling and supression in tennis racquets."