The dead spot of a tennis racket

Rod Cross

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It is shown that a tennis racket has a dead spot, but it does not have a well-defined sweet spot, when measured in terms of the rebound of a tennis ball. A ball dropped onto the center of the strings bounces to about 30% its original height. The bounce is much weaker near the tip of the racket, being almost zero at the dead spot. These effects are explained in terms of the effective mass and rotational inertia of the racket, and by reference to the behavior of other cantilevered beams. It is concluded, somewhat paradoxically, that the best place to hit a serve or smash is at the dead spot.

I. INTRODUCTION

In discussing the physics of baseball bats and tennis rackets, various authors[1–5] have identified a special impact point known as the sweet spot. This spot describes, in general terms, the point where a ball should be hit to prevent jarring of the hands. There are three different ways to define such a spot, all leading to a slightly different location for each spot.[2] One definition is that it is a vibration node. Another is that it is the center of percussion. The definition of most interest in this paper is that it is the spot where the coefficient of restitution, e, is a maximum. A standard technique to measure e is to clamp the handle, drop a ball onto the bat or racket and measure the ratio of the rebound velocity v_2 to the incident velocity v_1. Alternatively, one can measure the rebound height h_r and drop height h_d, in which case $e = v_2/v_1 = \sqrt{h_r/h_d}$. A more formal definition of e, that takes into account the recoil velocity of the bat or the racket, is that e is the ratio of the relative velocity of the ball and bat (or racket) before the collision to their relative velocity after the collision. However, for the purposes of this paper, the less formal definition will be used since, as described below, some collisions can involve multiple impacts.

In attempting such a measurement, I discovered to my amazement that there is a dead spot where a tennis ball dropped onto the strings of a racket tends to stick to the strings without bouncing, or bounces to only a very small height. I also discovered that this effect is not peculiar to a tennis racket or a tennis ball, since the same effect can easily be demonstrated by dropping any ball onto any relatively rigid object clamped at one end and free at the other. For example, if a 30-cm wood ruler is clamped by hand so that it hangs over the edge of a desk, the dead spot for a tennis ball is about 8 cm from the edge of the desk. If a small superball is dropped onto the ruler, the whole central region acts a dead region and there is a sweet spot a few cm from the free end. When a tennis ball is dropped onto a 9-mm-thick, 40-mm-wide aluminium bar, of length 30 cm or more, the dead spot is in the most unexpected place, at the free end of the bar, and the bounce is so weak that the ball abruptly comes to rest on the end of the bar. With a tennis racket, the dead spot is near the tip of the racket or displaced slightly toward the center of the strings.

If a tennis ball is dropped onto a rigid surface, such as a concrete slab, it compresses and then expands elastically, remaining in contact with the surface for a dwell time of about 5 ms. It is easy to estimate the dwell time in terms of the mass of the ball (about 60 g) and the spring constant of the ball (about 1.2×10^4 N/m). If the ball is dropped onto a more flexible surface, the dwell time can be shorter or longer depending on the mass and stiffness of the surface. If the ball is dropped onto the strings of a racket clamped at the handle end only, then the impact will set the racket frame itself into an oscillatory mode. The fundamental mode of oscillation of the frame has a node at the clamped end, an antinode at the free end, and a period of about 30 ms. For this mode, the ball can leave the strings at a time when the frame is nearing its maximum deflection downwards, in which case all the energy that was expended in bending the frame is lost. The only energy that can be recovered is the energy expended in compressing the ball plus the energy expended in stretching the strings. However, as explained by Brody,[1] only part of the ball energy is recovered since there is a net loss of energy in compressing and then expanding a tennis ball. This line of argument suggests that tennis rackets are not optimally designed and that stiffer frames should be used, with a fundamental period of oscillation about 10 ms. However, as explained below, there are several reasons why stiffer frames are not required for this purpose, one being that the fundamental mode of a hand-held racket *does* have a period of about 10 ms, another being that the mode is not excited if the ball strikes a node.

When a ball strikes a sweet spot on a cantilevered beam, most of the energy imparted to the beam is rapidly returned to the ball. A sweet spot can therefore be regarded as a point that maximizes the coefficient of restitution and minimizes the induced vibrations. By contrast, all or most of the kinetic energy of a ball, when it strikes a dead spot, is imparted to the beam, and none or very little of the energy is returned to the ball. A dead spot therefore minimizes e and maximizes the induced vibrations. There are several possible explanations that spring to mind for a dead spot. One is that the beam pulls away from the ball at the instant the ball reaches its maximum compression, in which case the ball has nothing to expand against and loses all its compressed state energy in internal modes of oscillation. Another possible explanation is that the ball remains in contact with the beam during the impact, but after an initial compression, the ball then expands at a rate that matches the beam's movement away from the ball. A related effect can be demonstrated by a tennis player who catches a ball on the strings of the racket or who catches a ball in his or her hands, but in these situations, the ball is unlikely to be compressed to any significant extent by the impact. Another related effect, familiar to snooker and pool players, occurs when two identical balls collide on a horizontal surface. If one of the balls is initially stationary, and the collision is head-on, then the incident ball transfers all its kinetic energy to the stationary ball. In fact, it will be shown below that this effect actually gives rise to a sweet spot on a cantilevered beam, since the beam returns all its energy to the ball on its way back to the equilibrium position. A dead spot results when the mass of the ball is about half the effective mass of the beam, since the ball bounces just enough to stay clear of the subsequent vibrations of the beam.

The dynamics of an impact can also be considered in terms of the propagation of a pulse away from the point of impact, and reflections of the pulse back toward the impact point from the boundaries. Consider for example the impact of a tennis ball on a wood or metal beam clamped at one end.

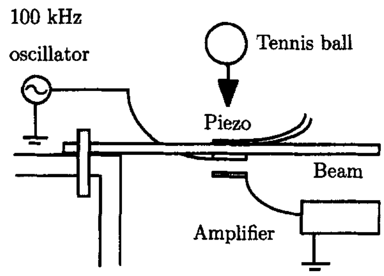

Fig. 1. Beam deflection apparatus.

The impact will generate a transverse pulse in the beam that suffers a 180° phase reversal from the clamped end, but no reversal from the free end.The pulse will also be dispersive, the high frequency components traveling faster than the low frequency components (due to the increased stiffness of a beam at small wavelengths). The speed of a transverse (or flexural) wave on a beam is typically of order 100 m/s, and therefore the wave travels about 50 cm during the 5-ms dwell time of a tennis ball. The impact can therefore include one or possibly two reflections of the pulse, from the ends of a 50-cm beam, back to the point of impact. As the ball bounces off the beam, the net effect of the interfering pulses at the point of impact will depend on the phasing, and this effect can either assist or retard the motion of the ball.

II. EXPERIMENTAL ARRANGEMENTS

In order to examine the factors contributing to a dead spot, a simple experiment was set up to study the dynamics of a ball bouncing off a uniform, rectangular cross-section beam. A rectangular beam is much simpler in construction than a racket, and the results are therefore easier to interpret, at least in terms of the theoretically predictable modes of oscillation. No special attempt was made to match the mass or dimensions of the beam with that of a racket since it was felt that a wide variation of parameters might help in extracting an explanation for the dead spot.

Several different beams were examined, including a standard 1-m wood rule, some aluminium bars (several different cross sections) and a carbon composite beam made up from three 7-mm-diameter circular tubes joined side by side (with epoxy). The carbon composite beam was studied because it is often used in rackets and it has a significantly higher ratio of Young's modulus (E) to density (ρ) than wood, aluminum or steel, the last three materials having almost identical E/ρ ratios. The carbon composite tubes were obtained as kite spars from a hobby shop, since rectangular beams are not readily available. The beam was clamped to the edge of a bench with a length l, typically about 50 cm, protruding over the edge, as shown in Fig. 1. Each beam was clamped flat on the bench for maximum flexibility (i.e., with the larger area face flat on the bench). The beam was then instrumented to measure, as a function of time, (a) the deflection at various points along the beam and (b) the force exerted by the ball on the beam.

To measure the deflection, a 2 cm$\times$2 cm piece of 1-mm-thick single-sided circuit board was taped to the underside of the beam, and connected with a light lead to the output of a 100 kHz sine wave generator. The voltage on the circuit board was sensed by means of a metal plate (in the form of a 2 cm$\times$10 cm circuit board with copper on part of one side) held in a clamp about 5–10 mm below the other plate in such a way that the two plates formed a capacitor (with $C\sim5$ pF). The fixed plate was wired directly either to the input of a storage oscilloscope or to a high input impedance amplifier using a length of coaxial cable with a BNC connector on the instrument end and with the inner conductor soldered to the circuit board. This way, the lead was shielded along most of its length.

With the beam at rest, the signal detected at the oscilloscope was about 0.5 V in amplitude. When the beam moved toward the plate, the signal amplitude increased (to about 2 V), and when the beam moved away from the plate, the signal decreased (to about 0.3 V). Despite the slightly nonlinear (1/distance) response of this arrangement, it provided a simple and effective method of measuring the local deflection of the beam when subject to the impact of a ball. Because of the flexibility of the beam, a large response was obtained when the ball was dropped from a height of only a few cm. This made it easy to drop the ball accurately on a predetermined spot and to catch the ball on rebound without having to chase it around the room. To generate the results shown below, the 100-kHz signal was amplified and then rectified to monitor the displacement more directly, but the rf envelope also provides a simple indication of the deflection. The polarity of the rectified signal was chosen so that a negative signal corresponds to a deflection downwards.

In order to measure the force of the ball on the beam, a thin (0.3-mm-thick), 20-mm-diameter piezoelectric crystal was taped to the upper side of the beam. The crystal and connecting leads were extracted from a cheap piezo buzzer, available from most electronics stores. The crystal was housed in a plastic case, and was extracted by carefully breaking apart the seal around the case with a pair of pliers. The crystal was attached to a thin brass plate, but this plate formed one electrode and was not removed. The voltage induced in the crystal when a ball was dropped onto it was measured with a standard $\times10$ voltage probe, held in a clamp above the beam, and connected via light leads to the crystal. The output from the crystal was surprisingly large, typically about 50 V, when a tennis ball was dropped from a height of a few cm. The crystal was also used as a simple accelerometer to measure vibrations in the frame of a tennis racket, in which case the output was typically about 50 mV. The crystal behaves electrically as a charged capacitor with a charge proportional to the applied force. When connected to a $\times10$ voltage probe, it had a time constant of 200 ms, which was sufficiently long to measure short impacts faithfully. As an accelerometer, it even provided a useful output at frequencies as low as 1 Hz.

III. RESULTS FOR A WOOD RULE

Results obtained by dropping a 60-g tennis ball from a height of a few cm onto a wood rule, of cross section 29 mm$\times$5 mm, and with a cantilever length 50 cm and mass 58 g, are shown in Figs. 2(a)–(d). For each of the drop positions, the piezo crystal and the displacement monitor were both moved to a point directly under the path of the ball. The long-term behavior of the rule is dominated by the fundamental mode at 15.4 Hz (period 65 ms) and the next highest mode at 91 Hz (period 11 ms). For a uniform beam of length l, cross section $a\times b$, and clamped at one end, the vibration frequencies are given by[6]

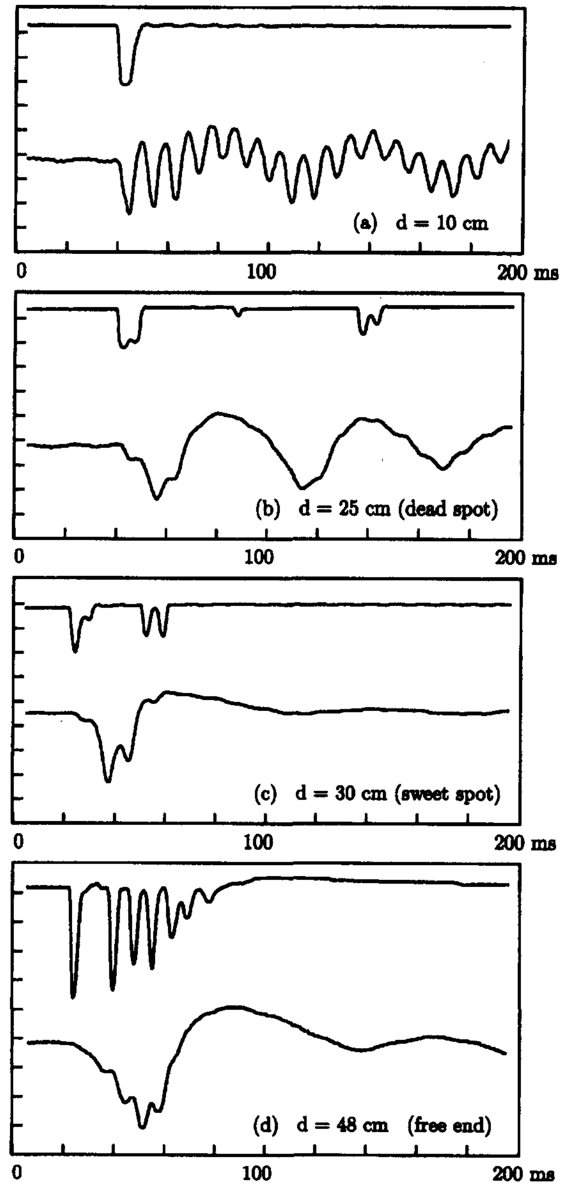

Fig. 2. Results of dropping a tennis ball on a 50-cm-length wood rule. Upper trace is the piezo signal, lower trace is the rule deflection measured at a point under the ball.

$$\omega=\frac{G^2a}{l^2}\sqrt{\left(\frac{E}{12\rho}\right)}, \tag{1}$$

where $G=1.875$, 4.694, 7.853, 10.996 for the first four modes, E is Young's modulus, ρ is the density and a is the dimension in the direction of the vibration. The first three modes are shown in Fig. 3(a). The node of the second mode is located at a position $x/l=0.78$ from the clamped end, and the nodes of the third mode are located at $x/l=0.50$ and 0.87. In the case of a beam that is freely suspended with both ends free, the first three modes are given by Eq. (1), but with $G=4.730$, 7.853 and 10.996. The first two modes are shown in Fig. 3(b). The nodes of the fundamental mode are located at $x/l=0.22$ and 0.78. This mode is therefore very similar in frequency and node location to the second mode of a beam clamped at one end.

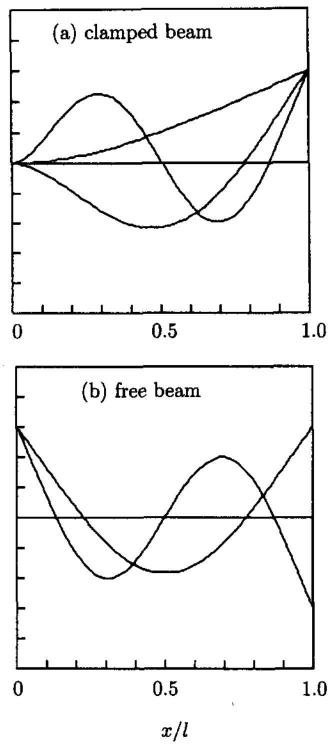

Fig. 3. (a) The first three modes of oscillation of a cantilever beam clamped at $x=0$, (b) the first two modes for a beam that is free at both ends.

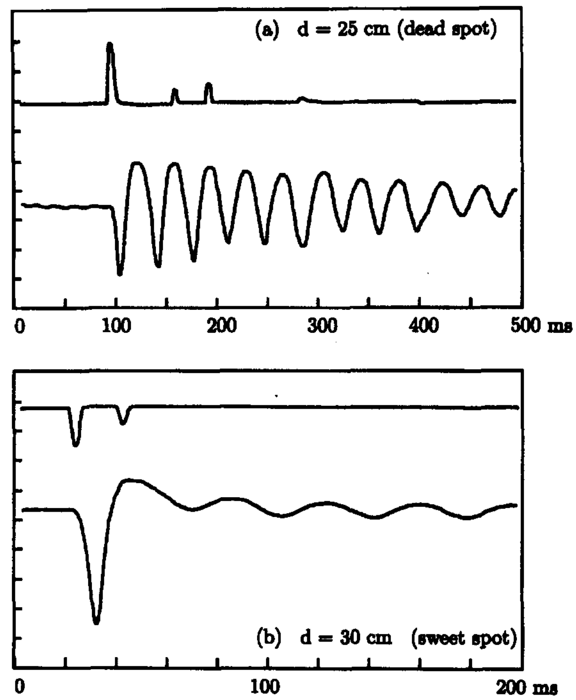

Fig. 4. Results of dropping a tennis ball on a 40-cm-length aluminum beam at (a) 25 cm and (b) 30 cm from the clamped end. The upper trace is the piezo signal and the lower trace is the beam displacement at a point directly under the ball. Different piezo crystals, of opposite polarity, were used here.

For wood, $E \sim 2\times 10^{10}$ N/m^2 and $\rho \sim 800$ kg/m^3, and the first two modes are expected at frequencies of 16.1 and 101 Hz, reasonably close to the observed frequencies. The third mode was not observed when dropping a tennis ball onto the rule, but it was observed clearly when a small, 28-mm-diameter superball was dropped near the node of the second mode ($d=39$ cm) or the antinode of the third mode ($d=35$ cm).

Figure 2(a) shows the result of dropping the ball at a distance $d=10$ cm from the clamped end. The rule is quite stiff near the clamped end, so there is a clean bounce with $e=0.6$, exciting both the first and second modes of oscillation with roughly equal amplitude.

Figure 2(b) shows the results at $d=25$ cm, corresponding to a dead spot. At this position, $e=0.2$, and most of the ball energy is transferred to the rule, as evidenced by the fact that the free end of the rule vibrates with large amplitude for about 5 s after the impact. The collision of the ball with the rule involves two double impacts and a very weak single impact between the two double impacts. The impacts separated by only 11 ms involve an interaction with the 91-Hz mode, and the impacts separated by 90 ms correspond to the ball dropping back onto the rule after the first dead bounce. The second bounce is also very weak, as is the single impact where the ball brushes the rule very lightly.

Figure 2(c), for $d=30$ cm, represents a sweet spot bounce (with $e=0.8$) where two double impacts occur in rapid succession, but the rule does not vibrate significantly afterwards. In other words, most of the energy transferred to the rule after the first two impacts is given back to the ball during the next two impacts.

Figure 2(d) shows the result of dropping the ball at $d=48$ cm, close to the free end of the rule. The ball bounces cleanly as judged by eye, about 20° away from the vertical, but the sensors detect seven distinct impacts before the ball actually bounces clear of the rule. This effect is explained below (see Fig. 8).

IV. RESULTS FOR AN ALUMINUM BEAM

Results for an aluminum beam of dimensions 40 cm×25 mm×6 mm and mass 162 g are shown in Fig. 4 for a tennis ball dropped onto the dead spot ($d=25$ cm) and onto the sweet spot ($d=30$ cm). For this beam (and all other aluminum beams investigated), modes higher than the first are excited only if the ball is dropped near the clamped end, an effect possibly associated with the relative softness of aluminum. The dead and sweet spots happen to be at exactly the same locations as for the longer wood rule, but this is somewhat coincidental. The dead- and sweet-spot locations for a tennis ball dropped onto other beams are shown in Fig. 5. Since the second mode is not excited, the results in Fig. 4 are easier to interpret. As for the wood rule, the dead spot corresponds to a case where the first mode is excited strongly by the first and subsequent impacts, and the sweet spot corresponds to a case where the second impact acts to transfer most of the beam energy back to the ball. For some of the beams tested, it was possible to find a sweet spot where all of the energy in the beam was returned to the ball since there was no discernible oscillation of the beam after the second impact. The results shown in Figs. 2 and 4 are analyzed in terms of a theoretical model in the following section.

V. BALL ON SPRING MODEL

Some insights into the behavior of a ball bouncing off a cantilever beam can be obtained by treating both the ball and

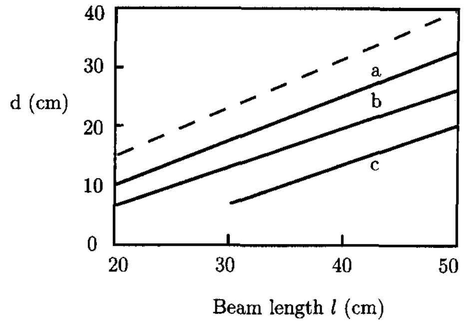

Fig. 5. Sweet- and dead-spot locations for various beams. d is the distance from the clamped end. The dashed line is the sweet-spot location for an aluminum beam of cross section 25×6 mm, and line (a) is the dead-spot location for this beam. Line (b) is the dead-spot location for the 29×5-mm wood rule. Line (c) is the dead-spot location for the 21×7-mm carbon composite beam which has a sweet-spot region extending over a 15-cm length at the free end of the beam.

the beam as simple springs, each with a different spring constant. As shown in Fig. 6, the ball can be modeled as a mass m_1 attached to a massless spring, with a spring constant k_1. The beam is modeled as a mass m_2 attached to the upper end of a massless spring, with a spring constant k_2, the spring being attached at the lower end to a rigid support. The basis for treating the beam as a spring is that (a) the deflection of the beam is directly proportional to the applied force, (b) the static deflection curve of a beam loaded at the free end[7] varies by less than 1% from the dynamic deflection curve for the fundamental mode of oscillation, and (c) the correct oscillation frequency of the fundamental mode can be calculated to within 1%, as described below, in terms of the equivalent spring constant. While the two systems remain in contact, they can be described by the relations

$$m_1 \frac{d^2 y_1}{dt^2} = m_1 g - k_1 x_1, \tag{2}$$

$$m_2 \frac{d^2 y_2}{dt^2} = k_1 x_1 - k_2 x_2, \tag{3}$$

where y_1 is the displacement of m_1, y_2 is the displacement of m_2, x_1 is the compression of the spring attached to m_1, and x_2 is the compression of the spring attached to m_2 (above that due to the equilibrium compression $x_2 = m_2 g/k_2$). In this situation, $y_2 = x_2$ and $y_1 = x_1 + x_2$. These two equations can be solved numerically, with initial conditions $dy_1/dt = v_0$, $dy_2/dt = 0$, $y_1 = 0$ and $y_2 = 0$. The equations must be modified, by setting $x_1 = 0$, if at any integration step it is found that $x_1 = y_1 - y_2$ is zero or negative. In numerical calculations, it is unlikely that x_1 will ever be identically zero, so x_1 was equated to zero at the first integration step with $x_1 < 0$. In other words, the ball exerts no force on m_2 if x_1 is zero or negative, since the ball is not physically attached to m_2. While x_1 remains zero, the ball is subject only to the gravitational force, and m_2 undergoes simple harmonic motion. If the ball makes a subsequent collision with m_2, then $x_1 = y_1 - y_2$ becomes positive, and Eqs. (2) and (3) are re-used to describe the subsequent motion. As a check on numerical accuracy, it was verified that the total (kinetic plus potential) energy was conserved during the collision.

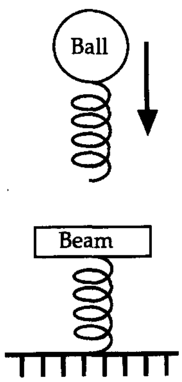

Fig. 6. Model of a ball bouncing onto a beam.

The spring constant for a rectangular cross-section beam clamped at one end and free at the other can be estimated in terms of the static deflection, y, when a force, F, is applied at the free end. The deflection at the free end of a uniform beam is given by[7] $y = 4Fl^3/(Eba^3)$ where l is the length of the beam, E is Young's modulus, b is the width of the beam and a is the thickness (in the bending direction). The spring constant, k_0, is therefore given by $k_0 = F/y = Eba^3/4l^3$, at least if the beam is loaded at its free end. If the same force is applied at any other position along the beam, the effective spring constant is larger, since the deflection is smaller. The static deflection, y, of a beam loaded by a force F applied at a distance d from the clamped end is given by $y = 4Fd^3/(Eba^3)$, where y is measured at the point of application of F. For example, if a beam is loaded at $d = l/2$, the deflection at this point is 1/8 of the value obtained when the beam is loaded at $d = l$, and the effective spring constant is 8 times larger.

The elastic potential energy of a beam, loaded at the free end by a force F, is given by $U = k_0 y^2/2$ where y is the deflection at the free end, and k_0 is the effective spring constant at the free end, as described above. If the force is suddenly released, the beam will oscillate at a frequency, ω. The frequency can be estimated by equating U to the kinetic energy of the beam when it passes through the equilibrium position. The deflection, y, at any point distant x from the clamped end of a beam, loaded at the free end, is given by $y = 2Fx^2(3l - x)/Eba^3$. If it is assumed that each point in the beam oscillates with an amplitude given by this static deflection relation, and with a velocity amplitude ωy, then the total kinetic energy of the beam can be obtained by integration along the beam. The result of this calculation is that $\omega = 1.03(a/l^2)\sqrt{E/\rho}$, about 1% higher than that given by Eq. (1).

If a mass, m, is located at the free end of a beam, and the mass of the beam is negligible, the system will oscillate at a frequency $\omega = \sqrt{k_0/m}$. However, if there is no mass at the end of the beam, and m is the mass of the beam itself, the lowest frequency mode of oscillation is given, from Eq. (1), by $\omega = 2.03\sqrt{k_0/m}$. If this mode is excited by dropping a ball on the beam, then the motion of the beam can be modeled by treating the beam as a mass–spring system with a k_2/m_2 ratio that generates the correct oscillation frequency, but where the required k_2 and m_2 values are different at different

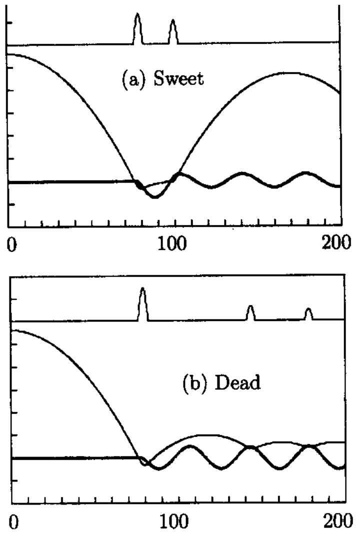

Fig. 7. Results of beam model calculations for an aluminum bar of dimensions 400 mm×25 mm×6 mm with (a) $m_2=95$ g, $k_2=2700\ \mathrm{N\,m^{-1}}$ and (b) $m_2=165$ g, $k_2=4700\ \mathrm{N\,m^{-1}}$. The lower curves show the trajectory of the ball (y_1 vs t) and the displacement of the beam (thicker curve, y_2 vs t). The upper curve represents the force of the ball on the beam, and is proportional to the compression of the ball.

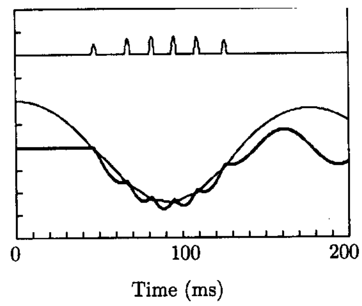

Fig. 8. Calculated impact for a tennis ball dropped, from a height of 1 cm, onto the end of a light beam, with $m_1=60$ g, $k_1=12\,000\ \mathrm{N\,m^{-1}}$, $m_2=15$ g, $k_2=140\ \mathrm{N\,m^{-1}}$. The upper curve shows the force on the ball. The thicker curve shows the displacement of the beam. The center curve shows the trajectory of the ball. There are only four impacts if the drop height is increased to 100 cm.

impact points along the beam. At the free end of the beam, $k_2=Eba^3/4l^3$ and $m_2=$mass of beam/2.03^2. Since the effective mass is about four times less than the actual mass of the beam itself, a light ball colliding with a heavy beam can couple a surprisingly large amount of energy to the beam. At other positions along the beam, k_2 and m_2 are both larger than the free-end values, by the same numerical factor. At the clamped end of the beam, k_2 and m_2 are effectively infinite. Higher frequency modes of oscillation of the beam could probably be modeled in terms of additional mass–spring systems, but this was not attempted. A mass–spring racket model that includes string motion, as well as damping, has previously been described by Leigh and Lu.[8]

The model behavior of a tennis ball dropped onto a 0.4-m-length aluminum beam of mass 162 g, cross section 25 mm×6 mm, is shown in Fig. 7, assuming ball parameters $m_1=60$ g, $k_1=1.2\times10^4\ \mathrm{N\,m^{-1}}$, with beam parameters (a) $m_2=95$ g and $k_2=2700\ \mathrm{N\,m^{-1}}$ and (b) $m_2=165$ g and $k_2=4700\ \mathrm{N\,m^{-1}}$. For each of these beam parameters, the fundamental period of oscillation corresponds to the experimentally observed period of 38 ms. The drop height of the ball was taken as $h=28$ mm, and the ball first lands on the beam at $t=\sqrt{2h/g}=76$ ms. The results in Fig. 7(a) describe conditions where the ball is dropped on a sweet spot, and Fig. 7(b) describes a dead spot. Agreement with the experimental data in Fig. 4 is seen to be quite good, and the assumed values of m_2 and k_2 are consistent with the expected $(l/d)^3$ weighting factor. The results even provide a good description of the relative magnitudes of each impact.

The interaction in each case can be understood qualitatively in terms of billiard-ball-type collisions since the ball continues in its initial direction if $m_1>m_2$, it reverses its direction of motion if $m_1<m_2$ and it stops dead if $m_1=m_2$. The subsequent motion of the ball depends on whether it makes another collision with the beam. In general, it is found that:

(a) If m_2 is much smaller than m_1, then the ball makes several rapid impacts with the beam before bouncing clear, within one period of oscillation of the beam. The theoretical e value is within the range 0.85–1.00 depending on k_2. The number of these impacts decreases as k_2 increases since the beam recoils more quickly following the initial impact when k_2 is large. An example of this type of interaction is shown in Fig. 8 for comparison with the experimental results in Fig. 2(d).

(b) If $m_2\sim m_1$ then there are usually two distinct impacts, the first slowing the ball to near zero speed, and transferring most of the initial ball energy to the beam. The second impact occurs when the beam collides with the ball on its return to the equilibrium position or soon after it passes through the equilibrium position. The second impact acts to transfer the beam energy back to the ball, with an e value typically in the range 0.75–1.00. The e value depends slightly on the initial ball velocity before impact, since gravity acts to effect the timing of the second impact. At large values of k_2, there is only one impact and $e\rightarrow1.00$ as $k_2\rightarrow\infty$.

(c) When $m_2\sim2m_1$, the ball bounces with sufficient velocity to just clear the beam after only one impact, but e is low, typically about 0.3–0.4, if $k_2\leqslant k_1$. At large values of k_2, $e\rightarrow1.00$ as $k_2\rightarrow\infty$.

(d) If $m_2>3m_1$, then $e\rightarrow1.00$ as $m_2\rightarrow\infty$, regardless of the value of k_2.

(e) There is no special interaction between the ball and the beam when the dwell time of the ball coincides with a half period of oscillation of the beam.

Any condition leading to e near unity can be equated with a sweet spot, while only condition (c) leads to a dead spot. In the absence of any loss processes, and if one waits long enough for additional bounces, even condition (c) will yield $e=$unity. However, damping of the oscillations in the beam will ensure that e remains quite low following the initial impact. Furthermore, energy losses in the ball itself have been entirely neglected in the above simple model. As shown in Fig. 9, about 30% of the initial ball energy is lost during a

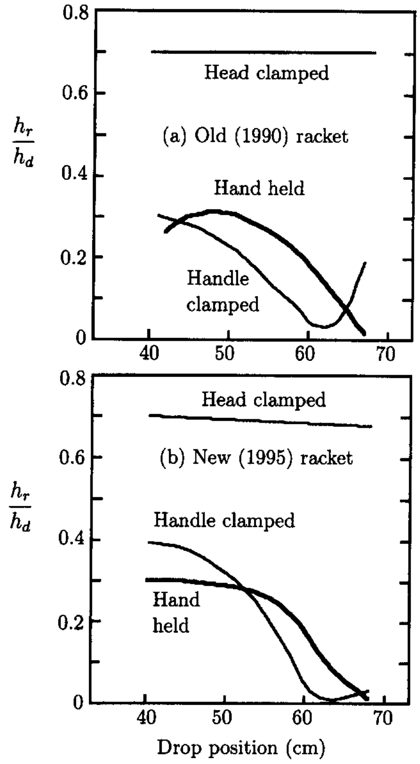

Fig. 9. Ratio of rebound height, h_r, to drop height h_d vs drop position, d, for a tennis ball dropped onto the strings of (a) the 1990 racket, and (b) the 1995 racket. h_r and h_d were measured from the strings to the bottom of the ball, and d was measured from the end of the handle.

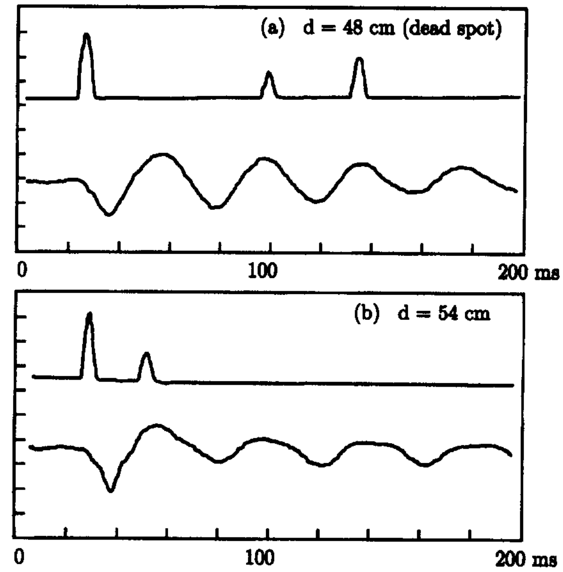

Fig. 10. Results of dropping a tennis ball onto the strings of a tennis racket with the handle clamped. The upper trace is the piezo signal and the lower trace is the racket deflection measured at a point directly under the path of the ball. The center of the strings is at $d=43$ cm and the free end of the racket is at $d=59$ cm.

single bounce off the strings of a head-clamped racket. Losses in the ball are even larger for a bounce off a more rigid surface or at higher velocities.

VI. RESULTS FOR A RACKET

A tennis racket was instrumented in the same manner as in Fig. 1, by taping a piezo crystal either onto the strings or onto the handle and locating a 2 cm×2 cm deflection sensor underneath the racket, one of the plates being tied to the strings with light wire. The data highlighted the fact, previously observed by Brody,[9] that a racket clamped by its handle onto a bench does not behave in the same manner as a hand-held racket. In fact, the behavior is so different that one should conclude that bench tests on a clamped racket are really only of academic interest. Furthermore, when clamping an expensive racket to the bench, considerable care needs to be taken not to crack the handle, since most modern rackets have hollow handles. Nevertheless, the sweet spot, or the power region,[2] of a racket has previously been identified as a point where the coefficient of restitution is a maximum, using tests on a clamped handle.

Measurements of bounce height are shown in Fig. 9 for (a) a 1990 vintage graphite composite racket of mass 370 g and length 685 mm, and (b) a lighter 1995 graphite composition racket of mass 325 g and length 690 mm. These results were obtained by dropping a tennis ball from a height of 50 cm onto the strings with (a) the handle clamped to the edge of a bench, or (b) the head and handle both clamped flat on the floor so that only the strings could vibrate, or (c) the racket held firmly in a horizontal position by hand. The maximum string length was 305 mm, and the ball was dropped at positions $d=40$ cm (near the throat of the racket) to $d=67$ cm (near the free end) along the central axis through the handle of the racket. The rebound height, h_r, was measured by allowing the ball to cut a horizontal laser pointer beam near the top of its path. The beam was detected with a photodiode and the trace recorded on a storage oscilloscope to monitor the times at which the ball crossed the beam. Given the difficulty of aligning the beam exactly at the maximum rebound height, h_r was calculated in terms of the height of the beam, y, and the time interval, τ, during which the beam was blocked or unblocked depending on whether the beam was aligned near the top or bottom of the ball. From these data, $h_r=y+\tau^2 g/8$. No significant differences were observed with old or new balls, despite the fact that old balls are softer than new balls. This finding is consistent with solutions of Eqs. (2) and (3), which indicate that the bounce height decreases by only about 1% if k_1 is halved.

Figure 9 shows the rebound height, h_r, as a fraction of the original drop height, h_d. The maximum e (0.84) is obtained when the head of the racket is clamped, and the response is quite uniform over the whole area of the strings. The response for a hand-held or handle-clamped racket is more variable and can vary significantly from one racket to the next. However, the general trend is that the ball bounces best near the throat of the racket and worst near the tip, as reported previously.[1,2] There is no well-defined spot where the ball bounces particularly well on all rackets, but there is a well-defined region on all rackets, near the tip, where the bounce is particularly weak.

Piezo crystal and deflection results, with the handle clamped, are shown in Fig. 10 (all results in Figs. 10–14 were obtained with the 1990 racket). The drop height of the ball was 10 cm rather than 50 cm to prevent the strings from hitting the deflection sensor, but no qualitative differences

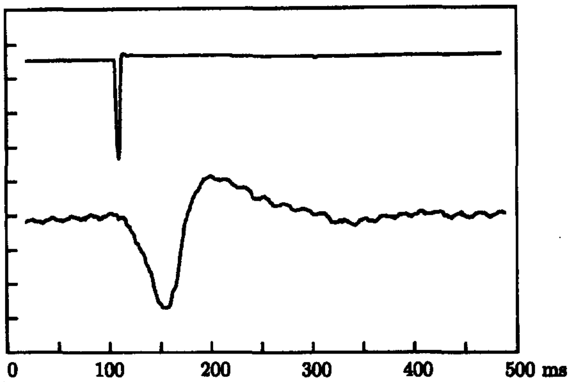

Fig. 11. Results of dropping a tennis ball onto the center of the strings of a hand-held racket. The upper trace is the piezo signal and the lower trace is the racket deflection measured at a point directly under the path of the ball.

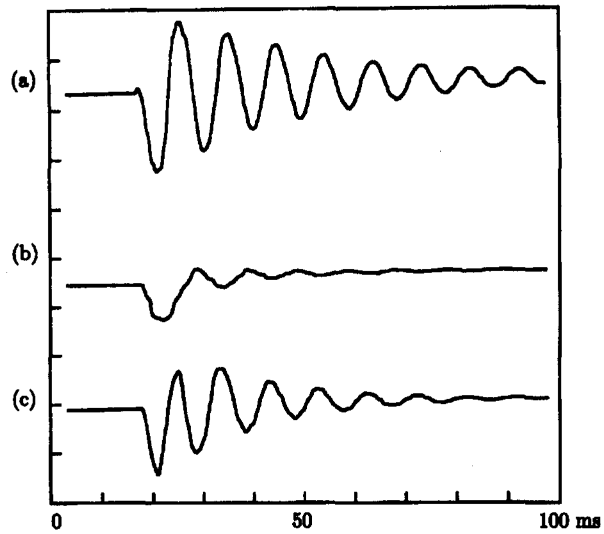

Fig. 12. Handle vibrations induced by dropping a tennis ball onto the strings of a hand-held racket, as measured by a piezo crystal taped to the handle. The ball was dropped (a) near the free end of the racket, (b) in the center of the strings and (c) near the handle end of the strings.

were observed when the drop height was varied. The ball was dropped directly onto the piezo crystal, taped to the strings. These results are consistent with the previous results obtained for simple beams. When a ball is dropped on or near the dead spot, the racket absorbs most of the energy of the ball at the first impact, and the ball falls back onto the strings after one or two oscillations of the racket frame. The fundamental mode of oscillation of the racket frame occurs at a frequency of 23 Hz. When the ball is dropped onto the strings near the tip of the racket, the initial bounce is so weak that it is hit by the racket on its first return past the equilibrium position, thereby transferring some of the racket energy back to the ball. The same result was reported by Brody[3] using strobe photography.

The data in Fig. 10 show that the racket continues to vibrate with substantial amplitude after the ball has bounced, so e is relatively low even near the tip of the racket. Near the throat of the racket, the bounce is higher due to the increased stiffness and effective mass of the frame, and there is only one impact.

The result of dropping a ball onto the center of the strings of a hand-held racket is shown in Fig. 11. In this case, the ball makes a single impact with a dwell time of about 6 ms. The racket is displaced downwards by the impact, and then returns to the equilibrium position, after a slight overshoot, over an interval of about 200 ms. This result is quite reproducible and appears to be independent of any conscious effort of the person holding the racket to respond to the impact. Indeed, 200 ms is about the normal reaction delay before someone can respond to a surprise event. There is also no evidence in Fig. 11 of any instinctive reaction to hit the ball, since the initial deflection is downwards, not upwards.

It will be noticed in Fig. 11 that there is no indication of any vibration of the racket (the small amplitude oscillations at 50 Hz prior to and after the impact are due to main power noise). There is in fact a small amplitude vibration, at 100 Hz, that can be detected by taping a piezo crystal to the racket handle. The piezo acts as a simple accelerometer, responding to the second derivative of the displacement, and is therefore more sensitive to high frequency vibrations than the capacitive displacement sensor. These results are shown in Fig. 12 for a hand-held racket, and in Fig. 13 for a handle-clamped racket. The piezo was located on a flat section of the racket handle, halfway between the end of the handle and the strings of the racket. By comparing these two sets of results, we see the following:

(1) The fundamental mode of oscillation of a handle-clamped racket, at 23 Hz ($T=43$ ms), is entirely absent for a hand-held racket. Exactly the same effect can be demonstrated with a ruler. The ruler can be made to vibrate by holding or clamping one end on a table and twanging the other end. It is impossible to get a hand-held ruler to vibrate this way. The reader is encouraged to try it!

(2) The second mode of a handle-clamped racket, at 125 Hz, is shifted down in frequency to 100 Hz when the racket is hand held. The downshift in frequency is the result of increasing the length of the racket to its free length. The

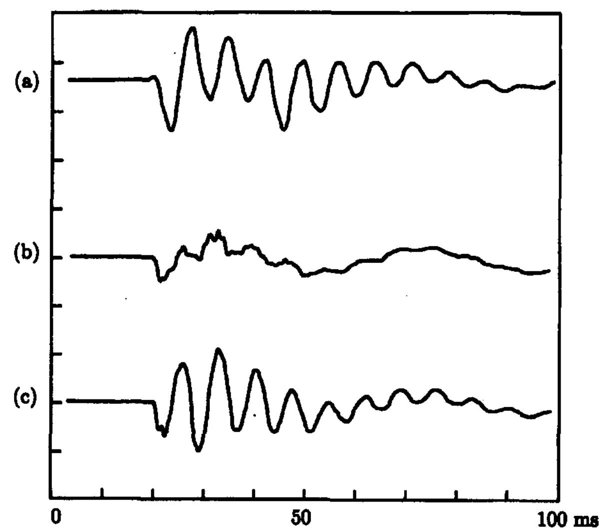

Fig. 13. The corresponding set of traces to those in Fig. 12, but with the handle clamped on a bench. The main difference is the absence of the fundamental mode for a hand-held racket.

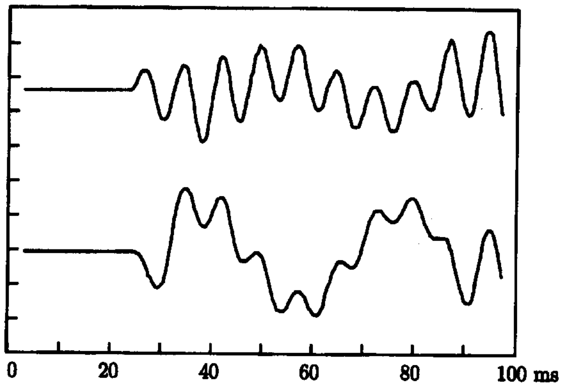

Fig. 14. Vibrations induced in the frame of a racket by dropping a tennis ball onto the strings, with the racket handle clamped. The upper trace is the signal from a piezo crystal halfway along the handle. The lower trace is the signal from a piezo located on the frame at the tip of the racket. Both traces were recorded simultaneously to check the phase, using piezos of opposite polarity. The fundamental mode is in phase at the two locations, and the higher frequency mode is out of phase.

racket is shortened by 95 mm when the handle is held in a rigid clamp. There is also a slight (1%) upshift effect, due to the fact that the mode shown in Fig. 12 actually corresponds to the fundamental mode of a beam that is free at both ends. To establish this result, two piezo crystals were mounted at the extreme ends of the racket, one on the end of the handle, the other taped to the frame at the tip of the racket. When the handle was gripped firmly by hand, both crystals responded in phase for the 100-Hz mode, regardless of where the ball was dropped.

(3) The 100- and 125-Hz modes are not excited significantly when the ball is dropped onto the center of the strings, so these modes have a node near the center of the strings. The other node, for a hand-held racket, is located under the hand, near the thumb.

(4) There is no additional damping when the racket is hand held, at least when compared with a handle-clamped racket. However, the damping is much larger than that for a freely suspended racket. By suspending the racket horizontally with light wire at the two nodes, the fundamental mode of the freely suspended racket was found to be 110 Hz and the quality factor Q was 130, the amplitude of the oscillations decreasing to $e^{-\pi}$ or 4%, after Q cycles, at $t=1.2$ s. Clamping or holding the handle therefore increases the damping of the high frequency modes, by a factor of about 10, and holding the handle decreases the fundamental frequency of the free racket from 110 to 100 Hz.

(5) There is no obvious phase difference between waveforms (a) and (c), despite the fact that these waveforms were generated by dropping a ball on opposite sides of the node, leaving the piezo crystal in the same location on the handle. Furthermore, Brody[9] shows data for a freely suspended racket where there is a clear 180° phase difference on opposite sides of the node. To resolve this discrepancy, two piezo crystals were used to obtain simultaneous measurements of vibrations in the handle and at the tip of the frame. A typical result, for a case where the ball is dropped onto the strings near the throat of the racket, is shown in Fig. 14. It was found that the initial deflection of the racket is always downwards at all locations (at least when the ball is dropped on the strings) and that there is a propagation delay of several ms as the deflection propagates through the racket, shorter than the dwell time of the ball on the strings. After a few vibration cycles, the phase differences between waveforms (a) and (c) in Figs. 12 and 13 take on their expected values, being in phase for the fundamental clamped mode at 23 Hz and 180° out of phase for the 100- and 125-Hz modes.

VII. HAND-HELD RACKET

The behavior of a hand-held racket is clearly of more practical interest than the behavior of a racket whose handle is rigidly clamped. The hand plays an obvious role in ensuring that the racket does not fly out of the court when it is struck by a ball, but the above results show conclusively that a hand-held racket behaves, with respect to vibrational modes, as if it were essentially free at both ends. In this section, we examine the role of the hand during the initial impact to determine the effect, if any, on the coefficient of restitution.

The results in Figs. 9 and 11 indicate that when a ball strikes a hand-held racket, there is a significant transfer of energy from the ball into rotational energy of the racket. The axis of rotation may be through the center of mass of the racket, or it may be a pivot point in the wrist, or the elbow, or the shoulder, or perhaps a combination of all these axes. If we assume for the moment that the racket is free to rotate about a point at the end of the handle, and also assume that the racket is perfectly rigid or that vibrational modes absorb a negligible amount of energy, at least when the ball strikes a node, then we can obtain an estimate for the coefficient of restitution using conservation of angular momentum and energy. There is a significant loss of energy in the ball itself, as evidenced by the head-clamped data in Fig. 9, but this can also be ignored for the purposes of the following discussion. If the racket has a moment of inertia, I, about the pivot point, is initially at rest, and is hit at a distance d from the pivot point by a ball of mass m_1 moving at speed v_1, then it is easy to show that the rebound velocity, v_2, of the ball is given by

$$e=\frac{v_2}{v_1}=\frac{I-m_1d^2}{I+m_1d^2}. \qquad (4)$$

The impact can be likened to the head-on impact with a stationary ball of mass m_2, in which case $e=(m_2-m_1)/(m_2+m_1)$. A racket that is pivoted at one end therefore has an effective mass I/d^2 that varies with d, being less than the actual mass of the racket near the free end and greater than the actual mass near the pivoted end. It can be seen that e is small at the free end of the racket and increases toward unity as the impact parameter d tends to zero. The value of e drops to zero when $I/d^2=m_1$ and is negative when $I/d^2<m_1$. When $e=0$, all of the initial angular momentum of the ball is transferred to the racket. For most rackets, $e=0$ when $d\sim 1$ m, placing the dead spot well beyond the tip of the racket. However, if a similar calculation is performed assuming realistic energy losses, the dead spot is located close to the tip of the racket. If a fraction f of the initial energy is lost in the collision, then the dead-spot location is given by $d^2=I(1-f)/m_1$. Typically, f varies from about 0.3 in the center of the strings to about 0.5 near the tip of the racket. For all practical purposes, the bounce is dead when $e<0.2$ since the ball then rebounds to a height less than 4% of the original drop height.

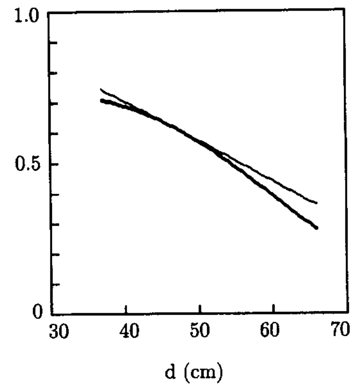

Fig. 15. Coefficient of restitution, e vs distance, d, from the end of the handle, as given by Eq. (4) (thin curve, handle pivoted) or Eq. (5) (thick curve, freely suspended racket). The impact parameter d_0 in Eq. (5) is taken as $d-33$ cm.

A measurement of I for the 0.37-kg racket was obtained by timing the period of oscillation about the end of the handle, giving $I=0.056$ kg m^2. This result agrees with measured values reported by Brody,[10] although the values quoted by Brody are smaller since they refer to an axis that is 76 mm away from the end of the handle. A 60-g ball hitting in the center of the strings at $d=52$ cm would then have an e value of 0.55, meaning that the ball would rebound to a height of 30% of its initial height. This is consistent with the observed height at the center of the strings, indicating that the inertia of the hand or the arm is not significant. The data in Fig. 9 were taken by attempting to keep the wrist firm and holding the racket horizontal, with the elbow bent at 90°. However, it was observed visually that the wrist, rather than the elbow, acted as the main pivot point.

Other authors[5,8,9,11,12] have claimed that the hand plays no role at all in determining either the vibrational modes of the racket or the coefficient of restitution of the ball. A theoretical basis for this claim can be seen by calculating e for a racket that is lightly suspended, or balanced vertically on its handle without any other support, so that it is free to rotate about its center of mass and also free to translate. From conservation of linear momentum, angular momentum and total energy, it is easy to show in this case that

$$e=\frac{m_2I_0-m_1(I_0+m_2d_0^2)}{m_2I_0+m_1(I_0+m_2d_0^2)}, \tag{5}$$

where m_1 is the mass of the ball, m_2 is the mass of the racket, I_0 is the moment of inertia of the racket about an axis through the center of mass, and d_0 is the impact parameter (i.e., the distance from the point of impact to the center of mass). The relevant moment of inertia in this case[10] is about 0.017 kg m^2. Equations (4) and (5) are plotted in Fig. 15 using the parameters listed above, and using the measured distance, 33 cm, of the center of mass from the end of the handle. The e values are surprisingly similar and are therefore not easily distinguished experimentally. The similarity is due to the fact that if a freely suspended racket is hit in the center of the strings, the combined effect of rotation about the center of mass, plus translation in the direction of the incident ball, is almost indistinguishable from a simple rotation of the racket about a point near the end of the handle. This is because the center of the strings is close to the center of percussion, defined as another sweet spot by Brody[2] since the impact transmitted to the hand is minimized. The e values in Eqs. (4) and (5) are equal when $d_0=I_0/m_2h$, where $h=d-d_0$, meaning that for this value of d_0 the impact point coincides exactly with the center of percussion.

One way to determine if a hand-held racket rotates about the center of mass, rather than (or as well as) the end of the handle, is to strike the racket at a point between the hand and the center of mass, observing the motion at the tip of the racket. It was found, by dropping a ball from heights up to 2 m, that the tip of the racket deflects downwards, indicating that the racket rotates about the end of the handle. If this is attempted with a more flexible 1-m wood rule, the tip of the rule drops down if a ball is dropped from a small height, but it deflects upwards if the ball is dropped from a large height. This effect is easily demonstrated if a stop is located just above the tip of the rule. By holding one end of the rule in one hand and slapping the rule with the palm of the other, the tip either smacks the stop or it doesn't, depending on how hard the rule is slapped. There may be a slight initial upwards movement with a light slap, but it is much smaller than the subsequent motion downwards. The explanation is that for a static deflection or an infinitely stiff rule, the rule can only rotate about a pivot in the hand, but for the dynamic deflection of a flexible rule, it also rotates about the center of mass. It seems likely that the same effect will be observed for a racket if the ball velocity is high enough, but high velocity impacts were not examined in this study.

VIII. IMPLICATIONS FOR A SERVE OR SMASH

As described above, a tennis racket whose handle is clamped has a dead spot that lies between the center of the strings and the tip of the racket. At this spot, the effective mass of the racket is about twice the mass of the ball and the ball bounces just clear of the racket vibrations. If the racket is hand held, the dead spot is closer to the tip of the racket where the effective mass of the racket is about equal to the mass of the ball. In the latter case, the ball transfers all or most of its angular momentum to the racket and the racket rotates clear of the ball.

The velocity ratio v_2/v_1 can be quite different if the racket is not initially at rest, but has rotational kinetic energy about a pivot point in the wrist. If the ball is moving much faster than the racket just prior to the collision, then Eq. (4) is not significantly modified, and the best place to hit the ball, for maximum rebound speed, is near the throat of the racket. However, if the racket has a similar speed to the ball, in the laboratory frame, then e is only a weak function of d and the best place to hit the ball is in the center of the strings (in order to minimize the induced vibrations).

In the case of a serve or smash, where $v_1=0$, then $v_2=0$ at $d=0$ and v_2 is a maximum when $I=m_1d^2$, at least in the absence of energy losses. If the fractional energy loss during the collision is f, the impact parameter that maximizes v_2 is given by $d^2=I(1-f)/m_1$. In other words, the best place to hit a serve or smash is at the dead spot on the racket, even if the collision involves significant energy losses in the ball and the racket. This somewhat paradoxical result is easily explained in terms of the effective mass of the racket. If the racket is rotating and strikes a stationary ball, it transfers all its angular momentum to the ball when the effective mass of the racket is equal to the mass of the ball. Instead of the ball stopping dead, the racket stops dead. In practice, the racket will not stop dead during a serve or smash since it is gripped firmly by the hand and since it is carried forward by the momentum of the arm and the body. Nevertheless, it can be shown that the best place to hit a serve or smash in this case is also at the dead spot. Under these conditions, the angular velocity of the racket about the wrist drops to zero, and the tip of the racket translates, immediately after the impact, with the same linear velocity as the handle.

Given that the tip of the racket is the worst place to hit a ball in terms of frame vibrations, it would be interesting to consider the performance of a modified racket with a dead-spot bounce coinciding with a sweet-spot node. Such a racket could be constructed if the moment of inertia were sufficiently small, and it would have a mass of about 280 g if the length was about 0.7 m. Alternatively, a conventional racket would have such a dead spot if the mass of the ball were increased from 60 to about 95 g.

IX. CONCLUSIONS

There is no sweet spot on a tennis racket where the ball bounces particularly well. The best bounce is obtained when the head is clamped, since negligible energy is lost by vibration or rotation of the frame. If the racket handle is clamped or is hand held, the best spot is between the throat and the center of the strings, but when compared with a light meter rule or an aluminum or carbon composite beam, the really sweet spots are missing. This is because the mass of a racket is significantly larger than that of a tennis ball so the ball usually bounces clear of the racket while the racket absorbs a significant fraction of the incident energy of the ball. However, the mass of a racket is still small enough that a dead spot can be found, where essentially all of the incident energy is transferred to vibrational modes of the racket, provided the handle is clamped. The dead spot is located between the center of the strings and the tip of the racket. At the tip of the racket, there is a double bounce where some of the vibrational energy in the racket is returned to the ball, but there is no really sweet spot where all of the vibrational energy is returned to the ball. If the handle is hand held, there is a dead spot, near the tip of the racket, where most of the energy of the ball is transferred to the racket, in the form of vibrational plus rotational energy.

A racket has a sweet spot near the center of the strings, in the form of a node for vibrations near 100 Hz, regardless of whether the handle is clamped or hand held. Despite this sweet spot, there is no significant change in the bounce properties at or near the node. In the case of a clamped handle, this can be explained by the fact that fundamental mode does not have a node, and by the fact that most of the energy lost by the ball is coupled to the fundamental mode at 23 Hz rather than to the next (125-Hz) mode (see Fig. 10). In the case of a hand-held racket, the bounce of the ball is determined mainly by energy transfer to rotation of the racket about one or more axes. The rotational energy of the racket can be calculated either by assuming that the end of the handle acts as a pivot point, or by considering rotation about the center of mass. Both calculations lead to essentially the same result, that the ball bounces, near the center of the racket, to about 30% of its original height.

No attempt was made in this paper to quantify experimentally the energy transferred from the ball to the racket, in terms of the vibrational and rotational components, but this would be a useful extension of the present study. For example, if a comparison is made between the theoretical e values in Fig. 15 (for zero energy loss) and the experimental rebound heights in Fig. 9, it is clear that energy losses play a significant role particularly near the tip and throat of the racket. A measurement of these losses would help explain the surprising result that, between the center of the strings and the tip of the racket, the e values are lower for a handle-clamped racket than for a hand-held racket. One might expect intuitively that a handle-clamped racket, being more rigid than a hand-held racket, should yield higher e values at all points on the strings. However, the effective mass at the tip of a handle-clamped racket, treated as a uniform beam, is equal to the actual mass of the racket divided by 4.12. By comparison, the effective mass at the free end of a uniform beam, pivoted at one end, is the actual beam mass divided by 3 (since $I/L^2 = M/3$ for a beam of mass M and length L). On this basis, a hand-held racket should have a higher e value than a clamped racket near the free end, at least if vibration losses in a hand-held racket are negligible. A clamped racket can be expected to have higher e values than a hand-held racket near the throat since its effective mass varies as $1/d^3$, whereas the effective mass of a hand-held racket varies as $1/d^2$.

The dead spot of a racket defines the point where an incident ball stops dead on a stationary racket. For the same reason, it also defines the point where a rotating racket stops dead on a stationary ball. In theory, the best place to serve or smash a ball is at the dead spot since this will impart maximum speed to the ball. Since this will also lead to some discomfort due to the induced vibrations and a tendency to wrench the racket out of the hand, most players should probably aim to strike the ball closer to the center of the strings.

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