Measuring Sweet Spots of Tennis Rackets

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Abstract

Measuring and analysis procedures are presented in this paper to study the first two sweet spots of a tennis racket. After a brief description of sweet spots in racket, we describe the experimental procedures applied in this work. Reproducing realistically the boundary conditions at the grip of the racket is a major difficulty that is tackled here in a simple manner. Instrumentation and signal processing issues are discussed. Using digital filtering to extract time domain signals and comparing time signals from different locations on a racket, mode shapes have been identified. Test results are presented and sweet spots are identified for rackets with large and small heads, and for high or low string tension.

Nomenclature

L	Distance between pivot point and sweet spot 1
g	Acceleration due to Gravity
ω	Pendular Angular Frequency
PSD	Power Spectral Density
CG	Center of Gravity
COP	Center of Percussion
SD	Standard Deviation

Introduction

Sweet spots provide a way of defining the playing quality of a tennis racket. Due to the ever-increasing competitiveness of the game, it has become essential to identify sweet spots of a racket accurately in order to provide players with high performance and comfortable rackets. Hence for manufacturers, designing rackets that combine in an optimum way low vibration and shock level, and maximum power is a challenge that requires advanced simulation and experimental analysis.

When a tennis racket hits a ball, the player first feels an initial shock, then the vibration of the whole structure. Due to the flexibility of the racquet and of its strings, the rebound might be powerful or weak, depending on the location of the hit. Therefore, it is common practice in the tennis world to define three sweet spots on the rackets corresponding to 1. minimal initial shock (sweet spot 1, also called Center of Percussion, COP), 2. minimal vibration (sweet spot 2) and 3. maximum coefficient of restitution (COR, sweet spot 3). Typical locations for these points are depicted in Fig. 1.

In this paper, one experimental method will be discussed for identification of the first two sweet spots when using accelerometers and data analysis. The third sweet spot, which represents the location of maximum restitution of ball speed, is not discussed in this paper.

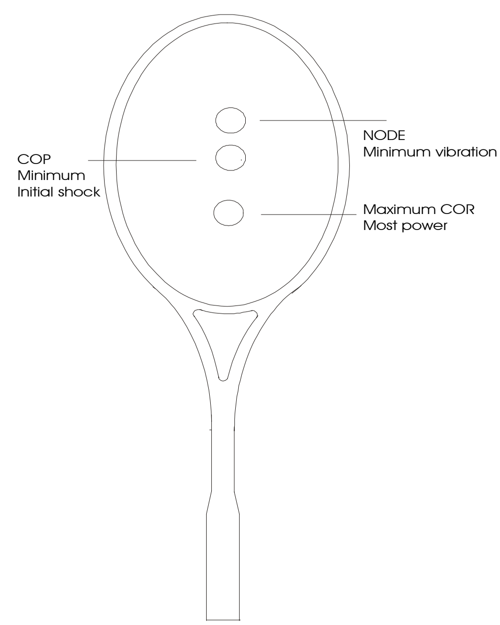

Figure 1 Sweet spots on a tennis racquet

Theory

Sweet Spot 1: the first sweet spot is defined as a point on the net of a racket such that when the ball impacts on it, no gross translation force is transmitted to the hand.

Let us assume that the racquet is rigid. As shown in Figure 2, when the ball impacts at the center of mass (CM) of the racket, the latter undergoes a translation motion. But, when the impact point is away from the CM, both translation and rotation are initiated in the racket. In particular, when the ball hits above the CM, the rotation and translation motions compensate. Hence, for a specific impact point, namely the first sweet spot [1], the net displacement at the grip is zero. In reality, the racquet is not free but it is held in the hand of the player. When the ball impacts on the first sweet spot, the player must not produce any force to counteract the impact, but only a moment to restrain the rotation of the racquet. This impact point on the net is also called COP (center of percussion) since it does not induce any initial shock in the hand of the player.

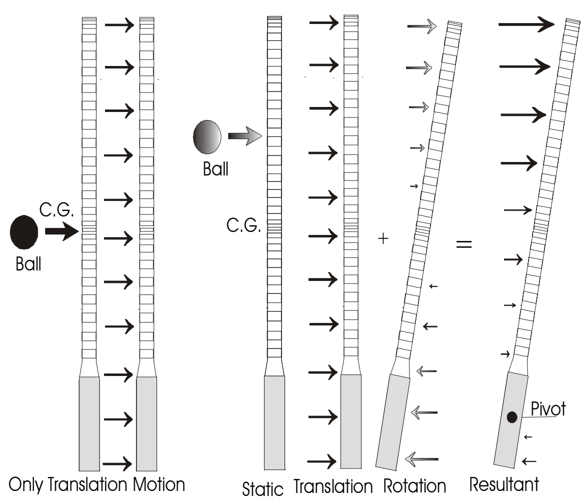

Figure 2: Definition of sweet spot 1

Sweet Spot 2: In reality, a racquet is not fully rigid. Hence, when a ball hits the racket, oscillations appear on top of the rigid body behavior. Note that because the total response is a combination of elastic oscillations and rigid body motion, the impact on the net will be felt on the grip only after a finite time, equivalent for the strain waves to travel to the grip. The amplitude and frequency of the vibrations depend on the impact location and on the structural characteristics of the racquet. Indeed, if a ball hits an anti-node point of a vibration mode of the racquet, that mode will be strongly excited. But when the ball impacts on a nodal point of a mode, that particular mode will not be triggered (see Figure 3). It has been observed that the first few modes of vibrations are very annoying for the player, hindering his ability to control the racquet and even causing injuries. Since the first mode (i.e. the one of lowest frequency) is easily excited, it typically produces the highest amplitudes of vibration. Therefore the second sweet is commonly defined as the nodal point of the first elastic mode of vibration [1]. It is thus the point on the net where an impact will not excite the first elastic mode of the racquet. Note however that higher frequency modes can also be detrimental and minimizing their excitability by the ball impact is also a practical concern.

As already mentioned before, the racquet is in reality not free but more or less loosely held at the grip. The boundary conditions at the grip will influence the vibration modes that are exhibited by the racquet.

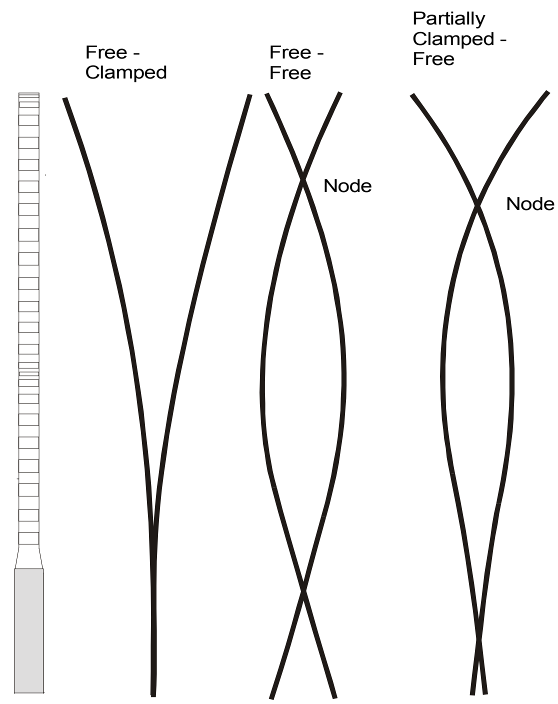

Figure 3: Definition of Sweet Spot 2

Considering the racquet as a beam-like structure, Figure 3 depicts the mode shapes of the first vibration mode depending on the boundary conditions at the grip. The case where one end is partially clamped is representative of the boundary condition in a player's hand since the grip is surrounded by soft tape and because of the skin and muscles surrounding the bones in the hand.

In this study, we assumed that the holding conditions of the racquet corresponds to some loose clamping such that elastic vibration modes are only slightly affected while the rigid body modes are transformed in low frequency modes of the rigid racquet on its soft support.

Boundary Conditions

One of the most difficult issues in carrying out experiments on a racquet is to properly define the boundary conditions on the grip. In reality, a person should hold the racket, which is quite unpractical since it is totally impossible to apply the same grip all the time. For that reason, we simulated similar boundary conditions by holding the racquet in a clamp while putting soft foam between the jaws of the clamp and the grip of the racquet.

We verified that this experimental setup gives results similar to the case when the racquet is actually held by a player. We also made sure that when removing and clamping again a racquet, the clamping conditions could be reproduced in so far as the resulting dynamic measurements were identical.

Using this procedure, a mode where the racquet is vibrating nearly rigidly around its soft clamping zone was identified at about 10 Hz. It is important to reproduce realistic boundary condition, especially for the second sweet spot measurement. Indeed, although the clamping is rather loose, we found that the frequency of the first elastic mode of the racquet is about 15% higher for our clamping conditions compared to the fully free racquet. In turn, the boundary condition will also influence the position of the vibration nodes and thus of the second sweet spot.

The Experiments

The aim of our experiments was to identify in a simple manner both the first and second sweet spots. The experiments were performed on four different racquets. Two racquets had a wide head (one with a high tension on the string and one with lower tension). Two other racquets had a smaller head (again one with high and one with low string tension). We first defined three locations on the net of the racquet: one in the middle of the net (*M*), one higher up (*T*) and one lower down (*O*). (Figure 4).

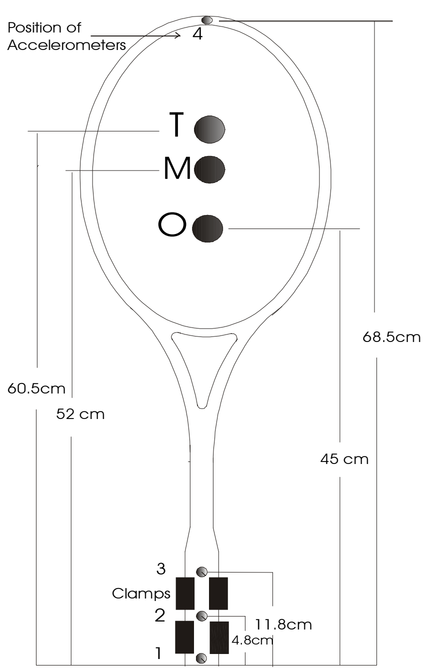

Figure 4: Impacting locations and positions of accelerometers

To create the actual playing conditions, a blower was used to throw balls to the predefined positions on the net. Three accelerometers were placed on the grip (Figure 4). Out of the three accelerometers, one was located above the jaws of the clamp, one below and one in the middle of the clamped part (which roughly corresponds to the middle point of the holding zone). The fourth accelerometer was attached at the tip of the racquet in order to identify the modal response of the structure.

Data acquisition was done with the SigLab 20-42 Dynamic Signal and System Analyzer. The bandwidth was set to 2KHz after observing that our maximum frequency of interest is much lower than 2KHz. The corresponding sampling rate was 5120 Hz. PCB 338M12 ICP accelerometers were used as sensors. A blower was used to throw the ball and a speedometer was installed to check the repeatability of the impact speed of the ball. The same ball was used during the entire tests.

Since the repeatability of the experiment was not perfect due to variations in ball speed, every experiment was repeated ten-times so that averaged results could be computed.

Procedure to identify the sweet spots

In our analysis, we considered only the signal measured 0.01 seconds after the impact. This corresponds to the free response since we observed that forced transient response related to the impact of the ball last for less than 10 ms.

First Sweet Spot:

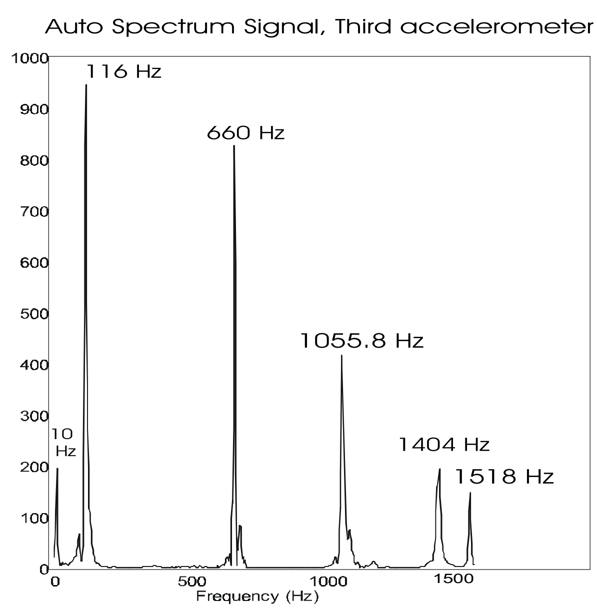

Figure 5: Auto-spectrum of signal measured by accelerometer number 3

In Figure 5, we show a typical auto-spectrum obtained during our test on the third accelerometer, clearly showing the presence of a low frequency quasi-rigid mode and vibration modes of the racquet.

Resonance frequencies were computed from the auto-spectrum diagram for each experiment. Band-pass digital filter was used to filter signal associated to the quasi-rigid mode frequency, i.e. at ≈ 10 Hz. For the first elastic mode, i.e. at ≈ 116 Hz band-pass was also used to filter out the signal. The same filtering was also applied for the third mode around 660 Hz. Comparing then, for every resonance frequency, the filtered time-domain signals at the four positions, we can directly identify the mode shapes of the racket.

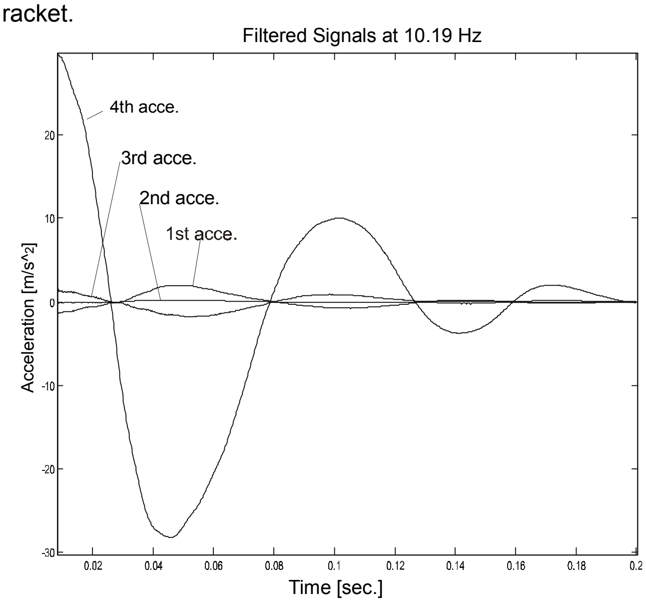

Figure 6: Filtered accelerations around 10.19 Hz

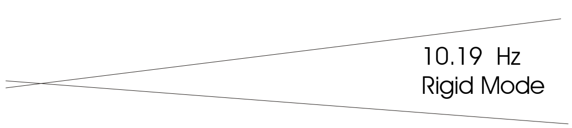

Figure 7: Mode identified at 10.19 Hz

Applying the filtering around 10 Hz, one finds for the first three accelerometers the time signal depicted in Figure 6. As shown in Figure 7, the mode associated to 10 Hz corresponds to the rotation of the nearly rigid racquet around its clamping point. From Figure 6, it can also be seen that accelerations measured by the first and third sensor have opposite. Accelerations measured by the second sensors are comparatively small. This indicates that the pivot point is located close to the second accelerometer, between the first and third.

To compute the pivot point, for each discrete time we fit a quadratic curve through the acceleration measured at the three locations on the grip, assuming that the racket may not be vibrating in a fully rigid manner (see Figure 8).

Zero displacement (acceleration) pivot points were computed in that manner for a time interval of one complete period, approximately 0.1 sec. In Figure 9, we show a histogram describing the computed distance between the rotation point and the base of the handle. Finally taking moments of all the pivots, we are able to define the best approximate to the pivot location. Once we have computed the pivot locations for one experiment, the same experiment is repeated 10 times and an averaged value is computed.

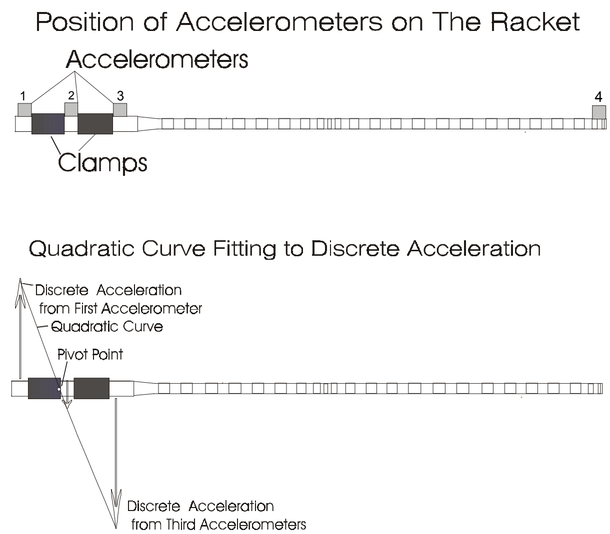

Figure 8: Computing the pivot point for the first sweet spot

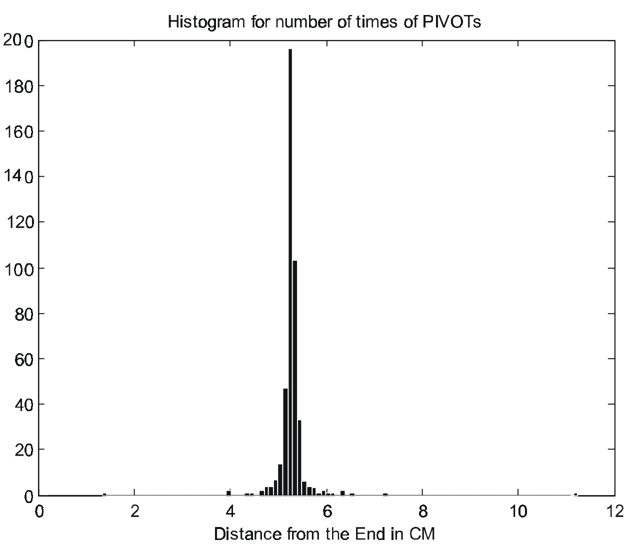

Figure 9: Histogram of PIVOT location for an experiment.

The first sweet spot was also calculated by a pendular procedure [1], where the racket is hanged freely at the assumed pivot. Measuring the oscillation time, the corresponding sweet spot can be computed as

$$L = \frac{g}{\omega^2}$$

From the calculation it was found that the first sweet spot of 100-Hard racket is around 50 cm from the base of the racket, where as by our method, we found to be at 52 cm from the base.

Second Sweet Spot:

From the auto spectrum diagram (Figure 5), it was found that the second peak is close to 116Hz. These frequencies were computed for each experiment. Digital filtering was applied to extract the signals at that frequency for all four accelerometers. Comparing time-domain signals, the associated mode-shapes could be found (Figure 10).

Figure 10: Identified mode shape at ≈ 116 Hz

This mode clearly corresponds to the first elastic mode of the racquet. When the ball impacts on the vibration node of that mode, vibrations should be minimal. Hence, the PSD (power spectral density) at the particular resonance frequency should be close to zero when the ball is hitting at that point (second sweet spot). The PSD was calculated for all the experiments for the resonance frequency of 116 Hz considering a time interval of 0.1 seconds. It was than averaged for all 10 experiments to get the averaged PSD. The PSD obtained when the ball impacts at location *T, M* and *O* are then compared to identify the second sweet spot. A summary of the results is given in the next section.

Results

In the following tables we report the results obtained when impacting the four rackets at three different locations.

Racket names:

100 hard: wide head, high tension
100 soft : wide head, low tension
90 hard : small head, high tension
90 soft : small head, low tension

Impact locations on the net:

Top = T
Middle = M
Bottom = O

We indicate, for each case, the location of the pivot point (zero displacement for the quasi-rigid mode) with respect to the base of the racket. We also show the PSD of the accelerations measured by accelerometer 3 :

PSD(1) = Power Spectral Density for first elastic mode at accelerometer 3 [m^2/s^4].
PSD(2) = Power Spectral Density for second elastic mode at accelerometer 3 [m^2/s^4].

As indicated before, the values listed are averages obtained over 10 measurements per case.

Table 1: Pivot location relative to the base of the racket and PSD for accelerometer 3 (100-Hard Racket)

Points	pivot [*cm*]	PSD(1)	PSD(2)
T	6.2	9.5 e2	7.1
M	4.9	2.6 e2	1.8
O	5.1	2.5 e3	2.6 e1

Table 2: pivot location relative to the base of the racket and PSD for accelerometer 3 (100-Soft Racket)

Points	pivot [cm]	PSD(1)	PSD(2)
T	5.1	1.1 e3	1.0 e1
M	4.9	3.8 e2	2.91
O	5.2	4.0 e3	7.9 e1

Table 3: pivot location relative to the base of the racket and PSD for accelerometer 3 (90-Hard Racket)

Points	pivot [cm]	PSD(1)	PSD(2)
T	6.3	1.3 e3	5.9 e2
M	5.1	6.7 e2	6.4 e2
O	5.0	5.3 e3	8.5 e2

Table 4: pivot location relative to the base of the racket and PSD for accelerometer 3 (90-Soft Racket)

Points	pivot [cm]	PSD(1)	PSD(2)
T	5.4	1.3 e3	7.4 e1
M	5.0	3.8 e2	2.3 e1
O	4.9	2.5 e3	9.7 e1

First analysis of the results

***First sweet spot*:**
As discussed previously, the pivot obtained from our tests correspond to the zero displacement (acceleration) location on the grip when the ball hits a given point on the net. Tables 1-4 show that the pivot point for all rackets and all impact locations is situated on the grip. For the 100-Soft and the 90-Soft racquets, the pivot point is always close to 5cm from the base, whatever the impact location of the ball on the racket. For the 100-Hard and the 90-Hard racquets, the pivot point moves up higher to about 6 cm from the racket base if the ball impacts at the highest point on the net (T).

Let us note that in theory, the pivot point should move up as the impact location moves up. This is however not observed in Tables 1 and 2. Also, the pivot point for a given impact location depends in theory only on the rotation inertia and total mass of the racket. But we observe a difference of pivot points for identical racquets with different string tension. This indicates that our procedure is not perfectly precise for determining the first sweet spot, probably due to measurement problems and because the impact conditions can not be fully controlled with our blower.

Second sweet spot:
From Table 1-4, we observe that the PSD associated to the first and second elastic modes are minimal when the ball impacts in the middle of the net (M). Only for the racquet 90-hard, the PSD for the second mode is slightly lower for the top position (T) compared to impacts on (M). As mentioned earlier, only the first few elastic modes of vibration are important for analyzing the second sweet spot. In our case, the third elastic mode has a high (above f1000 Hz) and is not considered in this analysis. From the above results, we can conclude that for all four rackets, the second sweet spot is at the middle location.

From these results, it appears that when a player holds the racket at the base of the grip, the pivot point is located close to the middle of the holding zone whatever the impact point. Therefore nearly any point on the racket is a sweet spot 1. However only when the ball impacts in the middle of the net (M), the vibration is minimal and thus (M) is the sweet spot 2. Hence, the rackets are properly designed so as to have the sweet spots at the middle locations provided the player holds the racket around 5 cm above its base.

Further analysis of the results

From results of Table 1-4 we have concluded that the overall vibration level corresponding to the first and second elastic modes is minimal when the ball impacts at point (M), meaning that location (M) is close to a vibration node of these modes. However it is also interesting to analyze the vibration level actually transmitted to the hand of the player. Indeed, even if the vibration level is not small, the player might feel nearly no vibration if he holds the racket at another vibration node.

Since the speed of the ball was slightly different for each experiment, every measurement is first normalized such that its PSD at accelerometer 4 for a given frequency is unity. Then averages are computed for all experiments carried out for a given racket and a given impact point. The mean values and standard deviation (S.D) compared to the mean was calculated for the relative PSD of all four accelerometers. The results are listed in Table 5 for the 90-Hard racket.
From Table 5, it is observed that for all impact locations i.e. top (T), middle (M) and bottom (O), relative PSD values at the second accelerometer are very small compared to the other measurement locations. Hence, we can conclude that the second accelerometer is located close to a vibration node of the first and second elastic modes. Therefore if a player is holding the racket near the pivot point, he will feel very little vibration even if the ball impacts away from point (M) that was identified as the second sweet spot. Thus, also the sweet spot 2 in the classical sense was found at location (M), any impact spot on the net will be rather “sweet” as long as there exist also a vibration node at the location where the player is holding the racket.

Table 5: PSD normalized with respect to accelerometer 4 (90-Hard Racket)

	First Elastic Mode				Second Elastic Mode			
	1st acce.	**2nd acce.**	**3rd acce.**	**4th acce.**	**1st acce.**	**2nd acce.**	**3rd acce.**	**4th acce.**
	Top (T)				**Top (T)**			
mean	0,04	1,58E-03	0,05	1,00	0,23	2,93E-03	0,27	1,00
SD	2%	25%	2%	0%	2%	19%	3%	0%
	Middle (M)				**Middle (M)**			
mean	0,03	9,18E-04	0,05	1,00	0,27	1,05E-02	0,27	1,00
SD	4%	15%	4%	0%	9%	26%	4%	0%
	Bottom (O)				**Bottom (O)**			
mean	0,05	1,01E-03	0,06	1,00	0,39	1,08E-02	0,52	1,00
SD	6%	5%	6%	0%	11%	18%	6%	0%

Conclusion

In this paper, a simple procedure to identify sweet spots 1 and 2 of tennis rackets is described. Reproducing properly the boundary conditions at the grip was found to be an important issue. Using accelerometers and simple digital filtering, vibration modes could be identified in a way similar to operational modal analysis. From the identified modes and the corresponding PSD, the quality of the racket could be rapidly assessed.

The procedure used was found to be suitable to quickly measure simultaneously the first and second sweet spots. In this procedure, realistic boundary conditions and speeds for the ball were used so that the measured vibrations are similar to those actually experienced by a player. In particular the vibration modes and thus the characteristic of the racket according to its second sweet spot can be influenced by the boundary conditions. Although free boundary conditions are commonly assumed, using proper clamping of the grip is believed to yield results closer to reality.

Due for instance to the bad repeatability of the machine throwing the balls, the present procedure is however not as precise as the classical pendulum method to measure the first sweet spot.

For the rackets tested, we found that the first sweet spot (minimum initial shock) was spread over the net whereas the second sweet spot (minimum vibration) is located close to the middle of the net. However we also observed that a vibration node for the first and second elastic mode was present about 5 cm above the base of the racket on the grip. Hence, if the player holds the racket at that location he will feel low levels of vibration even if the ball impacts away form the second sweet spot.

Acknowledgement

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Reference

[1] **Brody, H.** Tennis Science for Tennis Players, University of Pennsylvania Press, 1987