Gramatici LL(k) tari. Derivare descendent recursiva

Ce e gramatica LL(k)? - reaminitire

O gramatica independenta de context $G = (T, N, P, Z)$ este $LL(k)$ pentru un $k \geq 0$ daca pentru derivari arbitrare

$$Z \Rightarrow^L \mu X \chi \Rightarrow \mu \nu \chi \Rightarrow^* \mu \gamma$$

$$Z \Rightarrow^L \mu X \chi \Rightarrow \mu \omega \chi \Rightarrow^* \mu \gamma'$$

$$unde\ \mu, \gamma, \gamma' \in T^*, \nu, \chi, \omega \in V^*, X \in N$$

avem urmatoarea proprietate: $k : \gamma = k : \gamma'$ implica $\nu = \omega$
Observatie: Dependenta de μ obliga pastrarea in situatiile $[X \to \alpha.\beta; \omega]$ a contextului dreapta. Daca se elimina aceasta dependenta: gramatici **LL(k) tari**

Gramatici LL(k) tari

O gramatica independenta de context $G = (T, N, P, Z)$ este o gramatica $LL(k)$ tare pentru un $k > 0$ daca pentru derivari arbitrare

$$Z \Rightarrow^L \mu X \chi \Rightarrow \mu \nu \chi \Rightarrow^* \mu \gamma$$

$$Z \Rightarrow^L \mu' X \chi' \Rightarrow \mu' \omega \chi' \Rightarrow^* \mu' \gamma'$$

$$\textit{unde } \mu, \mu', \gamma, \gamma' \in T^*, \nu, \chi, \omega \in V^*, X \in N$$

avem urmatoarea proprietate: $k : \gamma = k : \gamma'$ implica $\nu = \omega$

$$Fie\ G\ cu\ P = \{ \quad \begin{aligned} &Z \to X \\ &X \to aAab | bAbb \\ &A \to a | \varepsilon \} \end{aligned}$$

$Z \Rightarrow X \Rightarrow aAab \stackrel{A \to \varepsilon}{\Rightarrow} aab$

$Z \Rightarrow X \Rightarrow aAab \Rightarrow aaab$

$Z \Rightarrow X \Rightarrow bAbb \Rightarrow bbb$

$Z \Rightarrow X \Rightarrow bAbb \stackrel{A \to a}{\Rightarrow} babb$

Este LL(1)? Este LL(2)? Este strong LL(2)?

$$Fie\ G\ cu\ P = \{ \quad \begin{array}{l} Z \to X \\ X \to aAab|bAbb \\ A \to a|\varepsilon\} \end{array}$$

$Z \Rightarrow X \Rightarrow aAab \overset{A\to\varepsilon}{\Rightarrow} aab$

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Este LL(1)? Este LL(2)? Este strong LL(2)?

$Z \Rightarrow X \Rightarrow aAab \Rightarrow aab$

$Z \Rightarrow X \Rightarrow bAbb \Rightarrow babb$

pt LL(k) tare: $k : \gamma = k : \gamma \Rightarrow$ aceeasi productie pt A; dar aici contextul stanga conteaza

Conditia strong LL(k)

O gramatica independenta de context G este strong LL(k) daca pentru orice pereche de productii $X \to \chi$, $X \to \chi', \chi \neq \chi'$ urmatoarea conditie este adevarata:

$$FIRST_k(\chi FOLLOW_k(X)) \cap FIRST_k(\chi' FOLLOW_k(X)) = \emptyset$$

exemplu

$$Fie\ G\ cu\ P = \{ \quad \begin{array}{l} Z \to X \\ X \to aAab|bAbb \\ A \to a|\varepsilon\} \end{array}$$

pt A : $FIRST_2(a\{ab, bb\}) \cap FIRST_2(\varepsilon\{ab, bb\}) = \{ab\}$

Strong LL(k)

NU e necesar niciun context pt a decide productia pentru nonterminalul X. Nu trebuie tinuti minte pasii anteriori din derivarea stanga, cei care au condus la nonterminalul X.

Algoritmul LL(k) - reamintire

Fie $G = (T, N, P, Z)$. Pt automatul stiva se determina Q si tranzitiile R:

1. $Q = \{q_0\}$ si $R = \emptyset$ cu $q_0 = [Z \to .S, \{\#\}]$
   Obs: $FOLLOW_k(Z) = \{\#\}$. q_0 starea initiala si a stivei. Automatul se opreste daca aceasta stare se intalneste din nou, stiva este vida, simbolul de intrare urmator este $\#$.
2. fie $q = [X \to \mu.\nu; \Omega]$ un element al lui Q care inca nu a fost tratat
3. Daca $\nu = \varepsilon$ atunci se include $q\varepsilon \to \varepsilon$ in R.
4. Daca $\nu = t\gamma$, $t \in T$ si $\gamma \in V^*$, fie $q' = [X \to \mu t.\gamma; \Omega]$. Adauga q' in Q si $qt \to q'$ in R.
5. Daca $\nu = Y\gamma$, $Y \in N$ si $\gamma \in V^*$,
   - fie $q' = [X \to \mu Y.\gamma; \Omega]$
   - si $H = \{[Y \to .\beta_i; FIRST_k(\gamma\Omega)] | Y \to \beta_i \in P\}$.
   - actualizeaza $Q = Q \cup \{q'\} \cup H$
   - si $R = R \cup \{q\tau_i \to q' h_i \tau_i | h_i \in H, \tau_i \in FIRST_k(\beta_i\gamma\Omega)\}$
6. daca toate starile din q au fost analizate, stop. Altfel continua cu 2.

Algoritm LL(k) tare

Daca $\nu = Y\gamma$, $Y \in N$ si $\gamma \in V^*$ in loc de pasul 5 din LL(k)

- fie $q' = [X \to \mu Y.\gamma; \Omega]$
- si $H = \{[Y \to .\beta_i; FIRST_k(\gamma\Omega)] | Y \to \beta_i \in P\}$.
- actualizeaza $Q = Q \cup \{q'\} \cup H$ si
- $R = R \cup \{q\tau_i \to q' h_i \tau_i | h_i \in H, \tau_i \in FIRST_k(\beta_i\gamma\Omega)\}$

se poate folosi pentru strong LL(k)

- fie $q' = [X \to \mu Y.\gamma; \Omega]$
- si $H = \{[Y \to .\beta_i; FOLLOW_k(Y)] | Y \to \beta_i \in P\}$.
- actualizeaza $Q = Q \cup \{q'\} \cup H$ si
- $R = R \cup \{q\tau_i \to q' h_i \tau_i | h_i \in H, \tau_i \in FIRST_k(\beta_i FOLLOW_k(Y))\}$

Toate situatiile distincte anterior doar prin context dreapta apartin intotdeauna aceleiasi stari.

LL(1) tare

Fie $Z \to E,\ E \to E + F|F,\ F \to i|(E)$
Prin eliminarea recursivitatii stanga:
$Z \to E,\ E \to FE_1,\ E_1 \to \varepsilon| + FE_1,\ F \to i|(E)$

simbol	$FIRST_1(X)$	$FOLLOW_1(X)$
E	$\{(, i\}$	$\{), \#\}$
E_1	$\{+, \varepsilon\}$	$\{), \#\}$
F	$\{(, i\}$	$\{+, \#,)\}$

Conditie LL(1) tare:
pt E_1:

$$FIRST_1(\varepsilon FOLLOW(E_1)) \cap FIRST_1(+FE_1 FOLLOW(E_1)) = \emptyset$$

pt F:

$$FIRST_1(iFOLLOW(F)) \cap FIRST_1((E)FOLLOW(F)) = \emptyset$$

$Z \to E,\ E \to FE_1,\ E_1 \to \varepsilon | + FE_1,\ F \to i | (E)$

stari noi	tranzitii noi
$q_0 = [Z \to .E; \#]$	

$Z \to E,\ E \to FE_1,\ E_1 \to \varepsilon | + FE_1,\ F \to i | (E)$

	stari noi	tranzitii noi
	$q_0 = [Z \to .E; \#]$	
q_0	$q' = [Z \to E.; \#] = q_1$ $H = \{[E \to .FE_1; \#] = q_2\}$	$\tau \in FIRST_1(FE_1 FOLLOW_1(E)) = \{i, (\}$ $q_0 i \to q_1 q_2 i$ $q_0 (\to q_1 q_2 ($

$Z \to E,\ E \to FE_1,\ E_1 \to \varepsilon | + FE_1,\ F \to i|(E)$

	stari noi	tranzitii noi
	$q_0 = [Z \to .E; \#]$	
q_0	$q' = [Z \to E.; \#] = q_1$ $H = \{[E \to .FE_1; \#] = q_2\}$	$\tau \in FIRST_1(FE_1 FOLLOW_1(E)) = \{i, (\}$ $q_0 i \to q_1 q_2 i$ $q_0 (\to q_1 q_2 ($
q_1		$q_1 \varepsilon \to \varepsilon$

$Z \to E,\ E \to FE_1,\ E_1 \to \varepsilon | + FE_1,\ F \to i | (E)$

	stari noi	tranzitii noi
	$q_0 = [Z \to .E; \#]$	
q_0	$q' = [Z \to E.; \#] = q_1$ $H = \{[E \to .FE_1; \#] = q_2\}$	$\tau \in FIRST_1(FE_1 FOLLOW_1(E)) = \{i, (\}$ $q_0 i \to q_1 q_2 i$ $q_0 (\to q_1 q_2 ($
q_1		$q_1 \varepsilon \to \varepsilon$
q_2	$[E \to F.E_1] = q_3$ $H = \{[F \to .i, FOLLOW_1(F)] = q_4$ $[F \to .(E); FOLLOW_1(F)] = q_5\}$	$\tau \in FIRST_1(iFOLLOW_1(F))$ $q_2 i \to q_3 q_4 i$ $q_2 (\to q_3 q_5 ($

fiind LL(1) strong, capetele din situatii nu le mai pastram (se pot deduce din situatie)

$Z \to E,\ E \to FE_1,\ E_1 \to \varepsilon | + FE_1,\ F \to i | (E)$

	stari noi	tranzitii noi
	$q_0 = [Z \to .E; \#]$	
q_0	$q' = [Z \to E.; \#] = q_1$ $H = \{[E \to .FE_1; \#] = q_2\}$	$\tau \in FIRST_1(FE_1 FOLLOW_1(E)) = \{i, (\}$ $q_0 i \to q_1 q_2 i$ $q_0(\to q_1 q_2($
q_1		$q_1 \varepsilon \to \varepsilon$
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q_3	$[E \to FE_1.] = q_6$ $H = \{[E_1 \to .\varepsilon] = q_7$ $[E_1 \to . + FE_1] = q_8\}$	$\tau \in FIRST_1(\varepsilon FOLLOW(E_1))$ $q_3) \to q_6 q_7)$ $q_3\# \to q_6 q_7 \#$ $q_3+ \to q_6 q_8 +$

$Z \to E,\ E \to FE_1,\ E_1 \to \varepsilon | + FE_1,\ F \to i | (E)$

	stari noi	tranzitii noi
	$q_0 = [Z \to .E; \#]$	
q_0	$q' = [Z \to E.; \#] = q_1$ $H = \{[E \to .FE_1; \#] = q_2\}$	$\tau \in FIRST_1(FE_1 FOLLOW_1(E)) = \{i, (\}$ $q_0 i \to q_1 q_2 i$ $q_0 (\to q_1 q_2 ($
q_1		$q_1 \varepsilon \to \varepsilon$
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q_4	$[F \to i.] = q_9$	$q_4 i \to q_9$

$Z \to E,\ E \to FE_1,\ E_1 \to \varepsilon | + FE_1,\ F \to i | (E)$

	stari noi	tranzitii noi
	$q_0 = [Z \to .E; \#]$	
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q_4	$[F \to i.] = q_9$	$q_4 i \to q_9$
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q_4	$[F \to i.] = q_9$	$q_4 i \to q_9$
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q_6		$q_6 \varepsilon \to \varepsilon$

$Z \to E,\ E \to FE_1,\ E_1 \to \varepsilon | + FE_1,\ F \to i|(E)$

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q_2	$[E \to F.E_1] = q_3$ $H = \{[F \to .i, FOLLOW_1(F)] = q_4$ $[F \to .(E); FOLLOW_1(F)] = q_5\}$	$\tau \in FIRST_1(iFOLLOW_1(F))$ $q_2 i \to q_3 q_4 i$ $q_2(\to q_3 q_5($
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q_5	$[F \to (.E)] = q_{10}$	$q_5(\to q_{10}$
q_6		$q_6 \varepsilon \to \varepsilon$
q_7		$q_7 \varepsilon \to \varepsilon$

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q_4	$[F \to i.] = q_9$	$q_4 i \to q_9$
q_5	$[F \to (.E)] = q_{10}$	$q_5 (\to q_{10}$
q_6		$q_6 \varepsilon \to \varepsilon$
q_7		$q_7 \varepsilon \to \varepsilon$
q_8	$[E_1 \to +.FE_1] = q_{11}$	$q_8 + \to q_{11}$

$Z \to E,\ E \to FE_1,\ E_1 \to \varepsilon \mid +FE_1,\ F \to i \mid (E)$

	stari noi	tranzitii noi
	$q_0 = [Z \to .E; \#]$	
q_0	$q' = [Z \to E.; \#] = q_1$ $H = \{[E \to .FE_1; \#] = q_2\}$	$\tau \in FIRST_1(FE_1 FOLLOW_1(E)) = \{i, (\}$ $q_0 i \to q_1 q_2 i$ $q_0 (\to q_1 q_2 ($
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q_4	$[F \to i.] = q_9$	$q_4 i \to q_9$
q_5	$[F \to (.E)] = q_{10}$	$q_5 (\to q_{10}$
q_6		$q_6 \varepsilon \to \varepsilon$
q_7		$q_7 \varepsilon \to \varepsilon$
q_8	$[E_1 \to +.FE_1] = q_{11}$	$q_8 + \to q_{11}$
q_9		$q_9 \varepsilon \to \varepsilon$

$Z \to E,\ E \to FE_1,\ E_1 \to \varepsilon | + FE_1,\ F \to i | (E)$

	stari noi	tranzitii noi
	$q_0 = [Z \to .E; \#]$	
q_0	$q' = [Z \to E.; \#] = q_1$ $H = \{[E \to .FE_1; \#] = q_2\}$	$\tau \in FIRST_1(FE_1 FOLLOW_1(E)) = \{i, (\}$ $q_0 i \to q_1 q_2 i$ $q_0 (\to q_1 q_2 ($
q_1		$q_1 \varepsilon \to \varepsilon$
q_2	$[E \to F.E_1] = q_3$ $H = \{[F \to .i, FOLLOW_1(F)] = q_4$ $[F \to .(E); FOLLOW_1(F)] = q_5\}$	$\tau \in FIRST_1(iFOLLOW_1(F))$ $q_2 i \to q_3 q_4 i$ $q_2 (\to q_3 q_5 ($
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q_3	$[E \to FE_1.] = q_6$ $H = \{[E_1 \to .\varepsilon] = q_7$ $[E_1 \to .+FE_1] = q_8\}$	$\tau \in FIRST_1(\varepsilon FOLLOW(E_1))$ $q_3) \to q_6 q_7)$ $q_3 \# \to q_6 q_7 \#$ $q_3 + \to q_6 q_8 +$
q_4	$[F \to i.] = q_9$	$q_4 i \to q_9$
q_5	$[F \to (.E)] = q_{10}$	$q_5 (\to q_{10}$
q_6		$q_6 \varepsilon \to \varepsilon$
q_7		$q_7 \varepsilon \to \varepsilon$
q_8	$[E_1 \to +.FE_1] = q_{11}$	$q_8 + \to q_{11}$
q_9		$q_9 \varepsilon \to \varepsilon$
q_{10}	$[F \to (E.)] = q_{12}$ $H = \{[E \to .FE_1] = q_2\}$	$\tau \in FIRST_1(FE_1 FOLLOW(E))$ $q_{10} (\to q_{12} q_2 ($ $q_{10} i \to q_{12} q_2 i$

$Z \to E,\ E \to FE_1,\ E_1 \to \varepsilon | + FE_1,\ F \to i | (E)$

	stari noi	tranzitii noi
	$q_0 = [Z \to .E; \#]$	
q_0	$q' = [Z \to E.; \#] = q_1$ $H = \{[E \to .FE_1; \#] = q_2\}$	$\tau \in FIRST_1(FE_1 FOLLOW_1(E)) = \{i, (\}$ $q_0 i \to q_1 q_2 i$ $q_0 (\to q_1 q_2 ($
q_1		$q_1 \varepsilon \to \varepsilon$
q_2	$[E \to F.E_1] = q_3$ $H = \{[F \to .i, FOLLOW_1(F)] = q_4$ $[F \to .(E); FOLLOW_1(F)] = q_5\}$	$\tau \in FIRST_1(iFOLLOW_1(F))$ $q_2 i \to q_3 q_4 i$ $q_2 (\to q_3 q_5 ($
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q_3	$[E \to FE_1.] = q_6$ $H = \{[E_1 \to .\varepsilon] = q_7$ $[E_1 \to . + FE_1] = q_8\}$	$\tau \in FIRST_1(\varepsilon FOLLOW(E_1))$ $q_3) \to q_6 q_7)$ $q_3 \# \to q_6 q_7 \#$ $q_3 + \to q_6 q_8 +$
q_4	$[F \to i.] = q_9$	$q_4 i \to q_9$
q_5	$[F \to (.E)] = q_{10}$	$q_5 (\to q_{10}$
q_6		$q_6 \varepsilon \to \varepsilon$
q_7		$q_7 \varepsilon \to \varepsilon$
q_8	$[E_1 \to +.FE_1] = q_{11}$	$q_8 + \to q_{11}$
q_9		$q_9 \varepsilon \to \varepsilon$
q_{10}	$[F \to (E.)] = q_{12}$ $H = \{[E \to .FE_1] = q_2\}$	$\tau \in FIRST_1(FE_1 FOLLOW(E))$ $q_{10} (\to q_{12} q_2 ($ $q_{10} i \to q_{12} q_2 i$
q_{11}	$[E_1 \to +F.E_1] = q_{13}$ $H = \{[F \to .i, FOLLOW_1(F)] = q_4$ $[F \to .(E); FOLLOW_1(F)] = q_5\}$	$\tau \in FIRST_1(iFOLLOW_1(F))$ $q_{11} i \to q_{13} q_4 i$ $q_{11} (\to q_{13} q_5 ($

$Z \to E,\ E \to FE_1,\ E_1 \to \varepsilon | + FE_1,\ F \to i|(E)$

	stari noi	tranzitii noi
	$q_0 = [Z \to .E; \#]$	
q_0	$q' = [Z \to E.; \#] = q_1$ $H = \{[E \to .FE_1; \#] = q_2\}$	$\tau \in FIRST_1(FE_1 FOLLOW_1(E)) = \{i, (\}$ $q_0 i \to q_1 q_2 i$ $q_0 (\to q_1 q_2 ($
q_1		$q_1 \varepsilon \to \varepsilon$
q_2	$[E \to F.E_1] = q_3$ $H = \{[F \to .i, FOLLOW_1(F)] = q_4$ $[F \to .(E); FOLLOW_1(F)] = q_5\}$	$\tau \in FIRST_1(iFOLLOW_1(F))$ $q_2 i \to q_3 q_4 i$ $q_2 (\to q_3 q_5 ($
	fiind LL(1) strong, capetele din situatii nu le mai pastram (se pot deduce din situatie)	
q_3	$[E \to FE_1.] = q_6$ $H = \{[E_1 \to .\varepsilon] = q_7$ $[E_1 \to .+FE_1] = q_8\}$	$\tau \in FIRST_1(\varepsilon FOLLOW(E_1))$ $q_3) \to q_6 q_7)$ $q_3 \# \to q_6 q_7 \#$ $q_3 + \to q_6 q_8 +$
q_4	$[F \to i.] = q_9$	$q_4 i \to q_9$
q_5	$[F \to (.E)] = q_{10}$	$q_5 (\to q_{10}$
q_6		$q_6 \varepsilon \to \varepsilon$
q_7		$q_7 \varepsilon \to \varepsilon$
q_8	$[E_1 \to +.FE_1] = q_{11}$	$q_8 + \to q_{11}$
q_9		$q_9 \varepsilon \to \varepsilon$
q_{10}	$[F \to (E.)] = q_{12}$ $H = \{[E \to .FE_1] = q_2\}$	$\tau \in FIRST_1(FE_1 FOLLOW(E))$ $q_{10} (\to q_{12} q_2 ($ $q_{10} i \to q_{12} q_2 i$
q_{11}	$[E_1 \to +F.E_1] = q_{13}$ $H = \{[F \to .i, FOLLOW_1(F)] = q_4$ $[F \to .(E); FOLLOW_1(F)] = q_5\}$	$\tau \in FIRST_1(iFOLLOW_1(F))$ $q_{11} i \to q_{13} q_4 i$ $q_{11} (\to q_{13} q_5 ($
q_{12}	$[F \to (E).] = q_{14}$	$q_{12}) \to q_{14}$

$Z \to E,\ E \to FE_1,\ E_1 \to \varepsilon | +FE_1,\ F \to i | (E)$

	stari noi	tranzitii noi
	$q_0 = [Z \to .E; \#]$	
q_0	$q' = [Z \to E.; \#] = q_1$ $H = \{[E \to .FE_1; \#] = q_2\}$	$\tau \in FIRST_1(FE_1 FOLLOW_1(E)) = \{i, (\}$ $q_0 i \to q_1 q_2 i$ $q_0 (\to q_1 q_2 ($
q_1		$q_1 \varepsilon \to \varepsilon$
q_2	$[E \to F.E_1] = q_3$ $H = \{[F \to .i, FOLLOW_1(F)] = q_4$ $[F \to .(E); FOLLOW_1(F)] = q_5\}$	$\tau \in FIRST_1(iFOLLOW_1(F))$ $q_2 i \to q_3 q_4 i$ $q_2 (\to q_3 q_5 ($
	fiind LL(1) strong, capetele din situatii nu le mai pastram (se pot deduce din situatie)	
q_3	$[E \to FE_1.] = q_6$ $H = \{[E_1 \to .\varepsilon] = q_7$ $[E_1 \to .+FE_1] = q_8\}$	$\tau \in FIRST_1(\varepsilon FOLLOW(E_1))$ $q_3) \to q_6 q_7)$ $q_3 \# \to q_6 q_7 \#$ $q_3 + \to q_6 q_8 +$
q_4	$[F \to i.] = q_9$	$q_4 i \to q_9$
q_5	$[F \to (.E)] = q_{10}$	$q_5 (\to q_{10}$
q_6		$q_6 \varepsilon \to \varepsilon$
q_7		$q_7 \varepsilon \to \varepsilon$
q_8	$[E_1 \to +.FE_1] = q_{11}$	$q_8 + \to q_{11}$
q_9		$q_9 \varepsilon \to \varepsilon$
q_{10}	$[F \to (E.)] = q_{12}$ $H = \{[E \to .FE_1] = q_2\}$	$\tau \in FIRST_1(FE_1 FOLLOW(E))$ $q_{10} (\to q_{12} q_2 ($ $q_{10} i \to q_{12} q_2 i$
q_{11}	$[E_1 \to +F.E_1] = q_{13}$ $H = \{[F \to .i, FOLLOW_1(F)] = q_4$ $[F \to .(E); FOLLOW_1(F)] = q_5\}$	$\tau \in FIRST_1(iFOLLOW_1(F))$ $q_{11} i \to q_{13} q_4 i$ $q_{11} (\to q_{13} q_5 ($
q_{12}	$[F \to (E).] = q_{14}$	$q_{12}) \to q_{14}$
q_{13}	$[E_1 \to +FE_1.] = q_{15}$ $H = \{[E_1 \to .\varepsilon] = q_7$ $[E_1 \to .+FE_1] = q_8\}$	$\tau \in FIRST_1(\varepsilon FOLLOW(E_1))$ $q_{13}) \to q_{15} q_7)$ $q_3 \# \to q_6 q_7 \#$ $q_{13} + \to q_{15} q_8 +$

$$Z \to E,\ \ E \to FE_1,\ \ E_1 \to \varepsilon | + FE_1,\ \ F \to i | (E)$$

	stari noi	tranzitii noi
	$q_0 = [Z \to .E; \#]$	
q_0	$q' = [Z \to E.; \#] = q_1$ $H = \{[E \to .FE_1; \#] = q_2\}$	$\tau \in FIRST_1(FE_1 FOLLOW_1(E)) = \{i, (\}$ $q_0 i \to q_1 q_2 i$ $q_0 (\to q_1 q_2 ($
q_1		$q_1 \varepsilon \to \varepsilon$
q_2	$[E \to F.E_1] = q_3$ $H = \{[F \to .i, FOLLOW_1(F)] = q_4$ $[F \to .(E); FOLLOW_1(F)] = q_5\}$	$\tau \in FIRST_1(iFOLLOW_1(F))$ $q_2 i \to q_3 q_4 i$ $q_2 (\to q_3 q_5 ($
	fiind LL(1) strong, capetele din situatii nu le mai pastram (se pot deduce din situatie)	
q_3	$[E \to FE_1.] = q_6$ $H = \{[E_1 \to .\varepsilon] = q_7$ $[E_1 \to . + FE_1] = q_8\}$	$\tau \in FIRST_1(\varepsilon FOLLOW(E_1))$ $q_3) \to q_6 q_7)$ $q_3 \# \to q_6 q_7 \#$ $q_3 + \to q_6 q_8 +$
q_4	$[F \to i.] = q_9$	$q_4 i \to q_9$
q_5	$[F \to (.E)] = q_{10}$	$q_5 (\to q_{10}$
q_6		$q_6 \varepsilon \to \varepsilon$
q_7		$q_7 \varepsilon \to \varepsilon$
q_8	$[E_1 \to +.FE_1] = q_{11}$	$q_8 + \to q_{11}$
q_9		$q_9 \varepsilon \to \varepsilon$
q_{10}	$[F \to (E.)] = q_{12}$ $H = \{[E \to .FE_1] = q_2\}$	$\tau \in FIRST_1(FE_1 FOLLOW(E))$ $q_{10} (\to q_{12} q_2 ($ $q_{10} i \to q_{12} q_2 i$
q_{11}	$[E_1 \to +F.E_1] = q_{13}$ $H = \{[F \to .i, FOLLOW_1(F)] = q_4$ $[F \to .(E); FOLLOW_1(F)] = q_5\}$	$\tau \in FIRST_1(iFOLLOW_1(F))$ $q_{11} i \to q_{13} q_4 i$ $q_{11} (\to q_{13} q_5 ($
q_{12}	$[F \to (E).] = q_{14}$	$q_{12}) \to q_{14}$
q_{13}	$[E_1 \to +FE_1.] = q_{15}$ $H = \{[E_1 \to .\varepsilon] = q_7$ $[E_1 \to . + FE_1] = q_8\}$	$\tau \in FIRST_1(\varepsilon FOLLOW(E_1))$ $q_{13}) \to q_{15} q_7)$ $q_3 \# \to q_6 q_7 \#$ $q_{13} + \to q_{15} q_8 +$
q_{14}		$q_{14} \varepsilon \to \varepsilon$
q_{15}		$q_{15} \varepsilon \to \varepsilon$

$q_0:$	$[Z \to \bullet E]$	$q_8:$	$[E_1 \to \bullet + F E_1]$
$q_1:$	$[Z \to E \bullet]$	$q_9:$	$[F \to i \bullet]$
$q_2:$	$[E \to \bullet F E_1]$	$q_{10}:$	$[F \to (\bullet E)]$
$q_3:$	$[E \to F \bullet E_1]$	$q_{11}:$	$[E_1 \to + \bullet F E_1]$
$q_4:$	$[F \to \bullet i]$	$q_{12}:$	$[F \to (E \bullet)]$
$q_5:$	$[F \to \bullet (E)]$	$q_{13}:$	$[E_1 \to + F \bullet E_1]$
$q_6:$	$[E \to F E_1 \bullet]$	$q_{14}:$	$[F \to (E) \bullet]$
$q_7:$	$[E_1 \to \bullet \epsilon]$	$q_{15}:$	$[E_1 \to + F E_1 \bullet]$

$$
\begin{array}{lll}
q_0 i \to q_1 q_2 i, & q_0 (\to q_1 q_2 (, & \\
q_1 \to \epsilon, & & \\
q_2 i \to q_3 q_4 i, & q_2 (\to q_3 q_5 (, & \\
q_3 \# \to q_6 q_7 \#, & q_3) \to q_6 q_7), & q_3 + \to q_6 q_8 +, \\
q_4 i \to q_9, & & \\
q_5 (\to q_{10}, & & \\
q_6 \to \epsilon, & & \\
q_7 \to \epsilon, & & \\
q_8 + \to q_{11}, & & \\
q_9 \to \epsilon, & & \\
q_{10} i \to q_{12} q_2 i, & q_{10} (\to q_{12} q_2 (, & \\
q_{11} i \to q_{13} q_4 i, & q_{11} (\to q_{13} q_5 (, & \\
q_{12}) \to q_{14}, & & \\
q_{13} \# \to q_{15} q_7 \#, & q_{13}) \to q_{15} q_7), & q_{13} + \to q_{15} q_8 +, \\
q_{14} \to \epsilon, & & \\
q_{15} \to \epsilon & &
\end{array}
$$

$q_0 q_0$	$(i+i)\#$	$q_0(\to q_1 q_2($
$q_0 q_1 q_2$	$(i+i)\#$	$q_2(\to q_3 q_5($
$q_0 q_1 q_3 q_5$	$(i+i)\#$	$q_5(\to q_{10}$
$q_0 q_1 q_3 q_{10}$	$i+i)\#$	$q_{10} i \to q_{12} q_2 i$
$q_0 q_1 q_3 q_{12} q_2$	$i+i)\#$	$q_2 i \to q_3 q_4 i$
$q_0 q_1 q_3 q_{12} q_3 q_4$	$i+i)\#$	$q_4 i \to q_9$
$q_0 q_1 q_3 q_{12} q_3 q_9$	$+i)\#$	$q_9 \to \varepsilon$
$q_0 q_1 q_3 q_{12} q_3$	$+i)\#$	$q_3 + \to q_6 q_8 +$
$q_0 q_1 q_3 q_{12} q_6 q_8$	$+i)\#$	$q_8 + \to q_{11}$
$q_0 q_1 q_3 q_{12} q_6 q_{11}$	$i)\#$	$q_{11} i + \to q_{13} q_4 i$
$q_0 q_1 q_3 q_{12} q_6 q_{13} q_4$	$i)\#$	$q_4 i \to q_9$
$q_0 q_1 q_3 q_{12} q_6 q_{13} q_9$	$)\#$	$q_9 \to \varepsilon$
$q_0 q_1 q_3 q_{12} q_6 q_{13}$	$)\#$	$q_{13}) \to q_{15} q_7)$
$q_0 q_1 q_3 q_{12} q_6 q_{15} q_7$	$)\#$	$q_7 \to \varepsilon$
$q_0 q_1 q_3 q_{12} q_6 q_{15}$	$)\#$	$q_{15} \to \varepsilon$
$q_0 q_1 q_3 q_{12} q_6$	$)\#$	$q_6 \to \varepsilon$
$q_0 q_1 q_3 q_{12}$	$)\#$	$q_{12}) \to q_{14}$
$q_0 q_1 q_3 q_{14}$	$\#$	$q_{14} \to \varepsilon$
$q_0 q_1 q_3$	$\#$	$q_3 \# \to q_6 q_7 \#$
$q_0 q_1 q_6 q_7$	$\#$	$q_7 \to \varepsilon$
$q_0 q_1 q_6$	$\#$	$q_6 \to \varepsilon$
$q_0 q_1$	$\#$	$q_1 \to \varepsilon$
q_0	$\#$	$q_1 \to \varepsilon$

$$q_0 = [Z \to .E]$$
$$q_1 = [Z \to E.],\ \ q_2 = [E \to .FE_1]$$
$$q_3 = [E \to F.E_1],\ \ q_5 = [F \to .(E)]$$
$$q_{10} = [F \to (.E)]$$
$$q_{12} = [F \to (E.)], q_2 = [E \to .FE_1]$$
$$q_3 = [E \to F.E_1], \ldots$$

Algoritm derivator LL(1)

Convertirea automatului LL(1) in proceduri recursive: Descendenta recursiva (Recursive descent)

- derivator descendent recursiv: starea automatului este o pozitie din derivator
- stiva - locatii de unde derivatorul poate relua executia
- daca starea e $[X \to \mu.B\nu;\omega]$, $B \in N$: se pune pe stiva informatia despre $[X \to \mu B.\nu;\omega]$ inainte de a lua in considerare $B \to \beta$.
- daca folosim limbaje de programare cu suport pt recursivitate: procedura pt fiecare nonterminal B + mecanismul standard de recursivitate pentru a implementa stiva automatului

Schema de program

$q \to \varepsilon$	q: end
$qt \to q'$	q: if symbol = t then next_symbol else error; q'
	q : X; q' :...
	....
	proc X:
$qt_1 \to q' q_1 t_1$	*begin*
....	*case symbol of*
$qt_m \to q' q_m t_m$	t_1 : *begin* q_1 : ... *end*;
	....
	t_m: *begin* q_m : ... *end*;
unde	otherwise error
$q = [Y \to \mu.X\nu;]$	*end*
	end

Reguli de transformare

1. nonterminal X - procedura X; simbolul de start - programul principal
2. corpul functiei X:
   - ramificare case pt productiile cu X in partea stanga
   - fiecare nonterminal din partea dreapta a productiei - apel al procedurii corespunzatoare
   - fiecare terminal din partea dreapta a productiei - verificare a presentei terminalului, urmat de apel al $next\_symbol$
3. daca niciunul dintre terminalele asteptate nu e prezent - apel functia de tratare a erorilor

- Pt tranzitii $qt_1 \to q'q_1t1\ldots$
- schema program indica:
  $q : F(); q'$
  procedura $F()$ - case pt toate t_i
- $q_2 i \to q_3 q_4 i$, $q_2(\to q_3 q_5($
  $q_4 i \to q_9$, $q_9 \to \varepsilon$, $q_5(\to q_{10}$,
  $q_{10} i \to q_{12} q_2 i$, $q_{10}(\to q_{12} q_2($,
  $q_{12}) \to q_{14}$,
- $q_2 = [E \to .FE_1]$, $q_3 = [E \to F.E_1]$, $q_{10} = [F \to (.E)]$

```
q2: F(); q3

procedure F()
{ case symbol of
   'i' : { q4:  if (symbol == 'i') then next_symbol else
        error();
            q9:  ;}
   '(' : { q5:  if (symbol == '(') then next_symbol else
        error();
            q10: E();
            q12: if (symbol == ')') then next_symbol else
                error();
            q14: ;}
   otherwise error();   }
```

```
derivator()                          procedure E()
{ q0: E()                            { q2: F();
  q1: if (symbol != '#')               q3: E1();
        error();                       q6: ;
}                                     }
procedure E1()
{ case symbol of
    '#' , ')' : q7: ;
    '+' : {
          q8:  if (symbol == '+')  next_symbol(); else error
                 ();
          q11: F();
          q13: E1;
          q15: ;
    otherwise    : error();
}
procedure F()
{ case symbol of
   'i' : { q4:  if (symbol == 'i') then next_symbol else
         error();
             q9:   ;}
   '(' : { q5:  if (symbol == '(') then next_symbol else
         error();
             q10: E();
             q12: if (symbol == ')') then next_symbol else
                    error();
             q14: ;}
    otherwise error();    }
```

Parsing table - tabel de derivare

- Ullman 4.4 . Nonrecursive predictive parsing
- Table-driven predictive parsing: input, stiva, parsing table.
- Tabel de derivare: M[A,a] - A nonterminal, a - terminal sau #

Exemplu de tabel de derivare

	lookahead					
	i	+	*	(	)	#
E	$E \to TE'$			$E \to TE'$		
E'		$E' \to +TE'$			$E' \to \varepsilon$	$E' \to \varepsilon$
T	$T \to FT'$			$T \to FT'$		
T'		$T' \to \varepsilon$	$T' \to *FT'$		$T' \to \varepsilon$	$T' \to \varepsilon$
F	$F \to i$			$F \to (E)$		

$$
\begin{aligned}
P = \{E &\to TE' \\
E' &\to +TE' | \varepsilon \\
T &\to FT' \\
T' &\to *FT' | \varepsilon \\
F &\to (E) | id\}
\end{aligned}
$$

Algoritm de derivare predictiva cu tabel de derivare

```
  #S (simbol de start) pe stiva, string# la intrare
set ip  to point to the first symbol of input string
repeat
  let X be the top stack symbol and a the symbol pointed to
      by ip
  if X is a terminal or # then
     if X = a then
        pop X from the stack and advance ip
     else error()
  else
     if M[X,a] = X-> Y1 Y2 ...Yk then begin
        pop X fro the stack
        push Yk, Yk-1, ...Y1 onto the stack, with Y1 on top
        output the production X-> Y1 Y2 ...Yk

     else error()
until X=#
```

Algoritm de derivare predictiva cu tabel de derivare

```
  #S (simbol de start) pe stiva, string# la intrare
set ip  to point to the first symbol of input string
repeat
  let X be the top stack symbol and a the symbol pointed to
      by ip
  if X is a terminal or # then
     if X = a then
        pop X from the stack and advance ip
     else error()
  else
     if M[X,a] = X-> Y1 Y2 ...Yk then begin
        pop X fro the stack
        push Yk, Yk-1, ...Y1 onto the stack, with Y1 on top
        output the production X-> Y1 Y2 ...Yk

     else error()
until X=#
```

$\{tqt \to q | t \in T\} \cup$
$\{Xq \to x_n....x_1 q | X \to x_1 x_2 ... x_n \in P, n \geq 0, X \in N, X_i \in V\}$

Exemplu de tabel de derivare

	lookahead					
	id	$+$	$*$	(	)	#
E	$E \to TE'$			$E \to TE'$		
E'		$E' \to +TE'$			$E' \to \varepsilon$	$E' \to \varepsilon$
T	$T \to FT'$			$T \to FT'$		
T'		$T' \to \varepsilon$	$T' \to *FT'$		$T' \to \varepsilon$	$T' \to \varepsilon$
F	$F \to id$			$F \to (E)$		

$$
\begin{aligned}
P = \{ E &\to TE' \\
E' &\to +TE' | \varepsilon \\
T &\to FT' \\
T' &\to *FT' | \varepsilon \\
F &\to (E) | id \}
\end{aligned}
$$

simbol	$FIRST_1(X)$	$FOLLOW_1(X)$
E	$\{(, id\}$	$\{), \#\}$
E'	$\{+, \varepsilon\}$	$\{), \#\}$
T	$\{(, id\}$	$\{+, \#,)\}$
T'	$\{*, \varepsilon\}$	$\{+, \#,)\}$
F	$\{(, id\}$	$\{*, +, \#,)\}$

1. for each production $A \to \alpha$ do steps 2 and 3
2. for each terminal a in $FIRST(\alpha)$, add $A \to \alpha$ to $M[A, a]$
3. if $\varepsilon \in FIRST(\alpha)$, add$A \to \alpha$ to $M[A, b$ for each terminal $b \in FOLLOW(A)$. if $\varepsilon \in FIRST(\alpha)$ and $\# \in FOLLOW(A)$, add $A \to \alpha$ to $M[A, \#]$
4. Make each undefined entry of M be error

test it online

First, follow sets, predict set

Algoritm $First_1$ - gramatici fara recursivitate stanga

Nu intra la examen
Se aplica urmatoarele reguli pana cand nu se mai poate reduce nimic

- $FIRST(a) = \{a\}$
- $X \to \varepsilon$: $FIRST(X) = \varepsilon$
- $X \to Y_1 Y_2 Y_3$:
  - daca $\varepsilon \notin FIRST(Y_1)$ then $FIRST(X) = FIRST(Y_1)$
  - daca $\varepsilon \in FIRST(Y_1)$ then $FIRST(X) = (FIRST(Y_1) - \{\varepsilon\}) \cup FIRST(Y_2 Y_3)$

Algoritm $Follow_1$

Nu intra la examen

- penrtu simbolul de start: se adauga $\{\#\}$ in follow(Z)
- $X \rightarrow \alpha Y$: $FOLLOW(Y) = FOLLOW(X)$
- $X \rightarrow \alpha Y \beta$:
  - daca $\varepsilon \notin FIRST(\beta)$ then $FOLLOW(Y) = FIRST(\beta)$
  - daca $\varepsilon \in FIRST(\beta)$ then $FOLLOW(Y) = (FIRST(\beta) - \{\varepsilon\}) \cup FOLLOW(X)$