

Parsare ascendenta (Bottom-up) 1.

# Parsare ascendenta

- ▶ necesita luarea unei decizii **dupa** analiza sirului derivat dintr-o productie
- ▶ mai multa informatie pt decizie →
  - ▶ clasa larga de gramatici
  - ▶ pret: cresterea complexitatii procedurii de analiza si a automatului rezultat

# Automat stiva - analiza ascendenta

Fie  $G = (T, N, P, Z)$  o CFG si automatul stiva

$A = (T, \{q\}, R, q, \{q\}, V, \epsilon)$  cu  $V = T \cup N$  si  $R$ :

(alfabet, stari, productii, stare initiala, stari finale, alfabet stiva, continut initial stiva)

$$\{x_1x_2...x_nq \rightarrow Xq \mid X \rightarrow x_1x_2...x_n \in P, n \geq 0, X \in N, X_i \in V\} \cup$$

$$\{qt \rightarrow tq \mid t \in T\} \cup$$

$$\{Zq \rightarrow q\}$$

Automatul accepta un sir din  $L(G)$  lucrând inapoi printr-o derivare cea mai din dreapta a sirului.

# Comparatie automat stiva in analiza descendenta vs ascendenta pt $G = (T, N, P, Z)$ o CFG

- ▶ **descendenta** - cursuri anterioare

$A = (T, \{q\}, R, q, \{q\}, V, Z)$  cu  $V = T \cup N$  si  $R$ :

$$\{tqt \rightarrow q \mid t \in T\} \cup$$

$$\{Xq \rightarrow x_n \dots x_1 q \mid X \rightarrow x_1 x_2 \dots x_n \in P, n \geq 0, X \in N, X_i \in V\}$$

- ▶ **ascendenta**

$A = (T, \{q\}, R, q, \{q\}, V, \epsilon)$  cu  $V = T \cup N$  si  $R$ :

$$\{x_1 x_2 \dots x_n q \rightarrow Xq \mid X \rightarrow x_1 x_2 \dots x_n \in P, n \geq 0, X \in N, X_i \in V\} \cup$$

$$\{qt \rightarrow tq \mid t \in T\} \cup$$

$$\{Zq \rightarrow q\}$$

# Exemplu

Fie  $G_1 = (T, N, E, P)$

- ▶  $T = \{+, *, (, ), i\}$ ,  $N = \{E, T, F\}$
- ▶ cu productiile P
  - ▶  $(1, 2) E \rightarrow T | E + T$
  - ▶  $(3, 4) T \rightarrow F | T * F$
  - ▶  $(5, 6) F \rightarrow i | (E)$

Automatul stiva:

- ▶  $T = \{+, *, (, ), i\}$ ,  $Q = \{q\}$ ,  
 $q_0 = q$ ,  $F = \{q\}$ ,  $S = \{+, -, *, (, ), i, E, T, F\}$ ,  $s_0 = \epsilon$
- ▶ cu productiile R
  1.  $Tq \rightarrow Eq$ ,  $E + Tq \rightarrow Eq$ ,
  2.  $Fq \rightarrow Tq$ ,  $T * Fq \rightarrow Tq$ ,
  3.  $iq \rightarrow Fq$ ,  $(E)q \rightarrow Fq$ ,
  4.  $q+ \rightarrow +q$ ,  $q* \rightarrow *q$ ,  $q(\rightarrow (q, q) \rightarrow q)$ ,  $qi \rightarrow iq\}$
  5.  $Eq \rightarrow q\}$

## Derivarea gasita: $i+i*i$

stiva	stare	intrare	derivarea cea mai din dreapta
	q	$i + i * i$	$i+i*i$
i	q	$+i * i$	
F	q	$+i * i$	$F+i*i$
T	q	$+i * i$	$T+i*i$
E	q	$+i * i$	$E+i*i$
E+	q	$i * i$	
E+i	q	$*i$	
E+F	q	$*i$	$E+F*i$
E+T	q	$*i$	$E+T*i$
E+T*	q	$i$	
E+T*i	q	$i$	
E+T*F	q		
E+T	q		$E+T*F$
E	q		$E+T$
	q		$E$

# Observatii

- ▶ coloana din dreapta este inversul derivarii celei mai din dreapta
- ▶ ascendenta - traseaza derivarea de jos la simbolul de start
- ▶ stiva contine la fiecare pas un **sir** din care **se poate deriva** portiunea de **sir deja citita**
- ▶ informatia semnificativa: perechea  $(\rho, \sigma)$ , unde
  - ▶  $\rho \in V^*$  - continutul stivei,
  - ▶  $\sigma \in T^*$  - restul sirului de la intrare

LL	LR
Does a leftmost derivation.	Does a rightmost derivation in reverse.
Starts with the root nonterminal on the stack.	The last nonterminal on the stack is the root nonterminal.
Ends when the stack is empty.	Starts with an empty stack.
Uses the stack for designating what is still to be expected.	Uses the stack for designating what is already seen.
Builds the parse tree top-down.	Builds the parse tree bottom-up.
Continuously pops a nonterminal off the stack, and pushes the corresponding right hand side.	Tries to recognize a right hand side on the stack, pops it, and pushes the corresponding nonterminal.
Expands the non-terminals.	Reduces the non-terminals.
Reads the terminals when it pops one off the stack.	Reads the terminals while it pushes them on the stack.
Pre-order traversal of the parse tree.	Post-order traversal of the parse tree.



# Clase de echivalență pentru perechile $(\rho, \sigma)$

Pentru  $p \in 1..n$ , fie  $X_p \rightarrow \chi_p$  producția a  $p$ -a a gramaticii independente de context  $G = (T, N, P, Z)$ . Clasele de reducere  $R_j, j \in 0, ..n$  sunt definite de

$$R_0 = \{(\rho, \sigma) | \rho = \mu\gamma, \sigma = \nu\omega \text{ a.i. } Z \Rightarrow^R \mu Y \omega, Y \Rightarrow^{R'} \gamma \nu, \nu \neq \varepsilon\}$$

$$R_p = \{(\rho, \sigma) | \rho = \mu\chi_p, Z \Rightarrow^R \mu X_p \sigma, X_p \Rightarrow \chi_p\}$$

unde  $Y \Rightarrow^{R'} \alpha$  este  $Y \Rightarrow^R \alpha$  si ultimul pas din derivare nu ia forma  $Y_1 \alpha \Rightarrow \alpha$

## Clase de reducere - continuare

- ▶ clasele de reducere - perechile de siruri care ar putea sa apara in timpul analizei ascendente a unei propozitii din  $L(G)$  de catre automatul stiva
- ▶ clasa de reducere careia ii apartine o pereche caracterizeaza tranzitia efectuata de catre automat cand acea pereche apare ca o configuratie
  1.  $(\rho, \sigma) \in R_0$  - fraza simpla  $\chi$  nu e complet in stiva; se aplica  $qt \rightarrow tq$  cu  $t = 1 : \sigma$  **tranzitie de deplasare**
  2.  $(\rho, \sigma) \in R_p, p \in 1..n$  - fraza simpla  $\chi_p$  e complet in stiva; se aplica  $\chi_p q \rightarrow X_p q$  **tranzitie de reducere**  
Obs: pt  $p = 1$  tranzitia  $Zq \rightarrow q$  si automatul se opreste
  3.  $(\rho, \sigma) \notin R_j, j \in 0..n$ . nu mai sunt posibile alte tranzitii; sirul de intrare nu apartine  $L(G)$

# Clase stiva k

- Pentru un  $k \geq 0$ , multimile  $R_{j,k}$ ,  $k \in 0..n$  se numesc clase stiva k al gramaticii G daca

$$R_{j,k} = \{(\rho, \tau) | \exists(\rho, \sigma) \in R_j, \tau = k : \sigma\}$$

- Daca clasele stiva k sunt mutual disjuncte atunci automatul stiva este determinist chiar si cand examinarea inainte este limitata la  $k$  simboluri

# Gramatica LR(k)

O gramatica independenta de context  $G = (T, N, P, Z)$  este  $LR(k)$  pentru un  $k \geq 0$  dat daca pentru derivari arbitrare

$$Z \Rightarrow^R \mu X \omega \Rightarrow \mu \chi \omega \quad \mu \in V^*, \omega \in T^*, X \rightarrow \chi \in P$$

$$Z \Rightarrow^R \mu' Y \omega' \Rightarrow \mu' \gamma \omega' \quad \mu' \in V^*, \omega' \in T^*, Y \rightarrow \gamma \in P$$

$(|\mu \chi| + k) : \mu \chi \omega = (|\mu' \gamma| + k) : \mu' \gamma \omega'$  implica  
 $\mu = \mu', X = Y, \chi = \gamma$

# LR(k)

## Automatul

- ▶ baleiaza sirul de intrare de la stanga la dreapta (Left to right)
- ▶ traversand inversa celei mai din dreapta derivari (Right)
- ▶ fara sa examineze mai mult de  $k$  simboluri de intrare intr-un pas

# Teorema

O gramatica independenta de context este LR( $k$ ) daca si numai  
daca clasele sale stiva  $k$  sunt mutual disjuncte.

# Verificarea proprietatii LR(k) prin intersectarea claselor stiva

NU intra la examen

- ▶ clasele stiva  $k$  contin o infinitate de perechi  $(\rho, \tau)$ ,  $\# \tau$  fiind finit datorita limitei de lungime, insa lungimea stivei nefiind limitata,  $\# \rho$  este infinitate
- ▶ pentru fiecare clasa stiva  $k$   $R_{j,k}$  se poate preciza o gramatica regulata  $G_j$  a.i.

$$L(G_j) = \{(\rho \& \tau) \mid (\rho, \tau) \in R_{j,k}\}$$

- ▶ exista algoritmi pt a determina daca doua limbaje regulate sunt distincte

# Situatii si inchidere nonterminal

Gramaticile regulate care genereaza clase stiva k:  
Simbolurile nonterminale:

$$W = \{[X \rightarrow \mu.\nu; \omega] | X \rightarrow \mu\nu \in P, \omega \in FOLLOW_k(X)\}$$



Gramatici care genereaza clasele stiva k, fara a fi regulate

$$G'_j = (V \cup \{\&, \#\}, W, P' \cup P'' \cup P_j, [Z \rightarrow .S; \#])$$

$$P' = \{[X \rightarrow \mu.\nu\gamma; \omega] \rightarrow \nu[X \rightarrow \mu\nu.\gamma; \omega] \quad | \nu \in V\}$$

$$P'' = \{[X \rightarrow \mu.Y\gamma; \omega] \rightarrow [Y \rightarrow .\beta; \tau] \quad | Y \rightarrow \beta \in P, \tau \in EFF_k(\gamma\omega)\}$$

$$P_0 = \{[X \rightarrow \mu.\nu; \omega] \rightarrow \&\tau \quad | \nu \neq \varepsilon, \tau \in EFF_k(\nu\omega)\}$$

$$P_p = \{[X_p \rightarrow \chi_{p.}; \omega] \rightarrow \&\omega \quad p \in 1..n\}$$

Care productii sunt permise in gramatica regulata?

Lungimile  $\&\tau$ ,  $\&\omega$  sunt finite datorita lui k;

Gramatici care genereaza clasele stiva k, fara a fi regulate

$$G'_j = (V \cup \{\&, \#\}, W, P' \cup P'' \cup P_j, [Z \rightarrow .S; \#])$$

$$P' = \{[X \rightarrow \mu.\nu\gamma; \omega] \rightarrow \nu[X \rightarrow \mu\nu.\gamma; \omega] \quad | \nu \in V\}$$

$$P'' = \{[X \rightarrow \mu.Y\gamma; \omega] \rightarrow [Y \rightarrow .\beta; \tau] \quad | Y \rightarrow \beta \in P, \tau \in EFF_k(\gamma\omega)\}$$

$$P_0 = \{[X \rightarrow \mu.\nu; \omega] \rightarrow \&\tau \quad | \nu \neq \varepsilon, \tau \in EFF_k(\nu\omega)\}$$

$$P_p = \{[X_p \rightarrow \chi_{p.}; \omega] \rightarrow \&\omega \quad p \in 1..n\}$$

Care productii sunt permise in gramatica regulata?

$P'$  si  $P_j$  unde  $j \in 0..n$

Lungimile  $\&\tau$ ,  $\&\omega$  sunt finite datorita lui k; sunt considerate simboluri terminale

# Inchiderea nonterminalului

o gramatica se poate rescrie a.i. sa nu contina productii precum cele din  $P''$

- Inchiderea unui nonterminal

$$H(X) = \{X\} \cup \{Y \mid Y_l \rightarrow Y \in P, Y_l \in H(X)\}$$

# Algoritm de rescriere a gramaticii

1. se selecteaza un  $X \in N$  pentru care  $H(X) \neq \{X\}$ .
2.  $P = P - \{X \rightarrow Y \mid Y \in N\}$
3.  $P = P \cup \{X \rightarrow \beta \mid Y \rightarrow \beta \in P, Y \in H(X), \beta \notin N\}$

Alg se termina cand nu se mai poate face nicio selectie la pasul 1

din  $G'_j$  rezulta  $G_j$ . Sirurile  $\beta$  sunt toate de forma  $\nu[..]$ ,  $\&\tau$  sau  $\&\omega$ :  
deci **gramatica regulata**

# Teorema - materie de examen

Pentru orice gramatica  $G$  de tipul  $LR(k)$  exista un automat stiva determinist  $A$  a.i.  $L(A) = L(G)$ .

Constructia automatului se bazeaza pe gramaticile  $G_j$ :

- ▶ automatul genereaza clasele stiva  $k$
- ▶ si le verifica fata de inversa celei mai din dreapta derivari a sirului
- ▶ in functie de clasa stiva  $k$  particulara, automatul
  - ▶ **stivuieste** simbolul de intrare, sau
  - ▶ **reduce** un numar de simboluri stivuite la un nonterminal

## ..continuare construire automat LR

- ▶ alg de construire genereaza treptat situatiile necesare si utilizeaza operatia de inchidere pentru evitarea productiilor din  $P''$ .
- ▶ o stare - o multime de situatii:
  - ▶ fiecare situatie dintr-o stare poate fi utilizata pentru derivarea clasei stiva k curente
- ▶ o alta formulare a inchiderii direct in functie de o multime de situatii M:

$$H(M) = M \cup \{[Y \rightarrow \cdot\beta; \tau] \mid$$

$$\exists [X \rightarrow \mu.Y\gamma; \omega] \in H(M),$$

$$Y \rightarrow \beta \in P,$$

$$\tau \in FIRST_k(\gamma\omega)\}$$

## Algoritm LR(k)-determinare $Q$ si $R$ :

1.  $Q = \{q_0\}$  si  $R = \emptyset$  cu  $q_0 = H([Z \rightarrow .S; \#])$
2. pt orice  $q \in Q$  se efectueaza pasii 3-5 pt fiecare  $\nu \in V$
3. fie  $basis(q, \nu) = \{[X \rightarrow \mu\nu.\gamma; \omega] \mid [X \rightarrow \mu.\nu\gamma; \omega] \in q\}$
4. daca  $basis(q, \nu) \neq \emptyset$  atunci  $next(q, \nu) = H(basis(q, \nu))$ . Se include  $q' = next(q, \nu)$  in  $Q$
5. daca  $basis(q, \nu) \neq \emptyset$  si  $\nu \in T$  se actualizeaza  
 $R = R \cup$   
$$\begin{cases} \{q\nu \rightarrow qq'\}, & k \leq 1 \\ \{q\nu\tau \rightarrow qq'\tau \mid [X \rightarrow \mu.\nu\gamma; \omega] \in q, \tau \in FIRST_{k-1}(\gamma\omega)\}, & k > 1 \end{cases}$$
6. daca toate elementele lui  $Q$  au fost tratate se executa pasul 7 pt fiecare  $q \in Q$  si alg se termina; altfel se continua pasul 2
7. pentru fiecare  $[X \rightarrow \chi.; \omega] \in q$ , unde  $\chi = x_1..x_n$  se face

$$\begin{aligned} R = R \cup \{ & q_1..q_nq\omega \rightarrow q_1q'\omega \mid [X \rightarrow .\chi; \omega] \in q_1, \\ & q_{i+1} = next(q_i, x_i) (i \in 1..n-1), \\ & q = next(q_n, x_n), \\ & q' = next(q_1, X) \} \end{aligned}$$

## Exemplu LR(k) cu $k=2$

$$T = \{a, b, c\}, N = \{Z.X, Y\}$$

$$P = \{(1)Z \rightarrow X,$$
$$(2, 3)X \rightarrow Y|bYa,$$
$$(4, 5)Y \rightarrow c|ca\}$$



Stare	X	Y	a	b	c
$q_0$ $H([Z \rightarrow .X; \#])$ $[Z \rightarrow .X; \#],$ $[X \rightarrow .Y; \#],$ $[X \rightarrow .bYa; \#],$ $[Y \rightarrow .c; \#],$ $[Y \rightarrow .ca; \#]$	$[Z \rightarrow X.; \#]$	$[X \rightarrow Y.; \#]$	-	$[X \rightarrow b.Ya; \#]$	$[Y \rightarrow c.; \#]$  $[Y \rightarrow c.a; \#]$
$next(q_0, \dots)$	$q_1$	$q_2$		$q_3$	$q_4$
$R$				$q_0bc \rightarrow q_0q_3c$ $FIRST_1(Ya\#)$	$q_0c\# \rightarrow q_0q_4\#$ $FIRST_1(\#)$ $q_0ca \rightarrow q_0q_4a$ $FIRST_1(a\#)$

Stare	X	Y	a	b	c
$q_0$ $H([Z \rightarrow .X; \#])$ $[Z \rightarrow .X; \#],$ $[X \rightarrow .Y; \#],$ $[X \rightarrow .bYa; \#],$ $[Y \rightarrow .c; \#],$ $[Y \rightarrow .ca; \#]$	$[Z \rightarrow X.; \#]$	$[X \rightarrow Y.; \#]$	-	$[X \rightarrow b.Ya; \#]$	$[Y \rightarrow c.; \#]$  $[Y \rightarrow c.a; \#]$
$next(q_0, \dots)$	$q_1$	$q_2$		$q_3$	$q_4$
$R$				$q_0bc \rightarrow q_0q_3c$ $FIRST_1(Ya\#)$	$q_0c\# \rightarrow q_0q_4\#$ $FIRST_1(\#)$ $q_0ca \rightarrow q_0q_4a$ $FIRST_1(a\#)$

Stare	X	Y	a	b	c
$q_0$ $H([Z \rightarrow .X; \#])$ $[Z \rightarrow .X; \#],$ $[X \rightarrow .Y; \#],$ $[X \rightarrow .bYa; \#],$ $[Y \rightarrow .c; \#],$ $[Y \rightarrow .ca; \#]$	$[Z \rightarrow X.; \#]$	$[X \rightarrow Y.; \#]$	-	$[X \rightarrow b.Ya; \#]$	$[Y \rightarrow c.; \#]$  $[Y \rightarrow c.a; \#]$
$next(q_0, \dots)$	$q_1$	$q_2$		$q_3$	$q_4$
$R$				$q_0bc \rightarrow q_0q_3c$ $FIRST_1(Ya\#)$	$q_0c\# \rightarrow q_0q_4\#$ $FIRST_1(\#)$ $q_0ca \rightarrow q_0q_4a$ $FIRST_1(a\#)$

Stare	X	Y	a	b	c
$q_0$ $H([Z \rightarrow .X; \#])$ $[Z \rightarrow .X; \#],$ $[X \rightarrow .Y; \#],$ $[X \rightarrow .bYa; \#],$ $[Y \rightarrow .c; \#],$ $[Y \rightarrow .ca; \#]$	$[Z \rightarrow X.; \#]$	$[X \rightarrow Y.; \#]$	-	$[X \rightarrow b.Ya; \#]$	$[Y \rightarrow c.; \#]$  $[Y \rightarrow c.a; \#]$
$next(q_0, \dots)$	$q_1$	$q_2$		$q_3$	$q_4$
$R$				$q_0bc \rightarrow q_0q_3c$ $FIRST_1(Ya\#)$	$q_0c\# \rightarrow q_0q_4\#$ $FIRST_1(\#)$ $q_0ca \rightarrow q_0q_4a$ $FIRST_1(a\#)$

Stare	X	Y	a	b	c
$q_0$ $H([Z \rightarrow .X; \#])$ $[Z \rightarrow .X; \#],$ $[X \rightarrow .Y; \#],$ $[X \rightarrow .bYa; \#],$ $[Y \rightarrow .c; \#],$ $[Y \rightarrow .ca; \#]$	$[Z \rightarrow X.; \#]$	$[X \rightarrow Y.; \#]$	-	$[X \rightarrow b.Ya; \#]$	$[Y \rightarrow c.; \#]$  $[Y \rightarrow c.a; \#]$
$next(q_0, \dots)$	$q_1$	$q_2$		$q_3$	$q_4$
$R$				$q_0bc \rightarrow q_0q_3c$ $FIRST_1(Ya\#)$	$q_0c\# \rightarrow q_0q_4\#$ $FIRST_1(\#)$ $q_0ca \rightarrow q_0q_4a$ $FIRST_1(a\#)$

Stare	X	Y	a	b	c
$q_0$ $H([Z \rightarrow .X; \#])$ $[Z \rightarrow .X; \#],$ $[X \rightarrow .Y; \#],$ $[X \rightarrow .bYa; \#],$ $[Y \rightarrow .c; \#],$ $[Y \rightarrow .ca; \#]$	$[Z \rightarrow X.; \#]$	$[X \rightarrow Y.; \#]$	-	$[X \rightarrow b.Ya; \#]$	$[Y \rightarrow c.; \#]$  $[Y \rightarrow c.a; \#]$
$next(q_0, \dots)$ $R$	$q_1$	$q_2$		$q_3$	$q_4$
				$q_0bc \rightarrow q_0q_3c$ $FIRST_1(Ya\#)$	$q_0c\# \rightarrow q_0q_4\#$ $FIRST_1(\#)$ $q_0ca \rightarrow q_0q_4a$ $FIRST_1(a\#)$
$q_1$ $H([Z \rightarrow X.; \#])$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

Stare	X	Y	a	b	c
$q_0$ $H([Z \rightarrow .X; \#])$ $[Z \rightarrow .X; \#],$ $[X \rightarrow .Y; \#],$ $[X \rightarrow .bYa; \#],$ $[Y \rightarrow .c; \#],$ $[Y \rightarrow .ca; \#]$	$[Z \rightarrow X.; \#]$	$[X \rightarrow Y.; \#]$	-	$[X \rightarrow b.Ya; \#]$	$[Y \rightarrow c.; \#]$  $[Y \rightarrow c.a; \#]$
$next(q_0, \dots)$ $R$	$q_1$	$q_2$		$q_3$	$q_4$
				$q_0bc \rightarrow q_0q_3c$ $FIRST_1(Ya\#)$	$q_0c\# \rightarrow q_0q_4\#$ $FIRST_1(\#)$ $q_0ca \rightarrow q_0q_4a$ $FIRST_1(a\#)$
$q_1$ $H([Z \rightarrow X.; \#])$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$q_2$ $H([X \rightarrow Y.; \#])$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

Stare	X	Y	a	b	c
$q_0$ $H([Z \rightarrow .X; \#])$ $[Z \rightarrow .X; \#],$ $[X \rightarrow .Y; \#],$ $[X \rightarrow .bYa; \#],$ $[Y \rightarrow .c; \#],$ $[Y \rightarrow .ca; \#]$	$[Z \rightarrow X.; \#]$	$[X \rightarrow Y.; \#]$	-	$[X \rightarrow b.Ya; \#]$	$[Y \rightarrow c.; \#]$  $[Y \rightarrow c.a; \#]$
$next(q_0, \dots)$	$q_1$	$q_2$		$q_3$	$q_4$
R				$q_0bc \rightarrow q_0q_3c$ $FIRST_1(Ya\#)$	$q_0c\# \rightarrow q_0q_4\#$ $FIRST_1(\#)$ $q_0ca \rightarrow q_0q_4a$ $FIRST_1(a\#)$
$q_1$ $H([Z \rightarrow X.; \#])$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$q_2$ $H([X \rightarrow Y.; \#])$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$q_3 : H([X \rightarrow$ $\quad b.Ya; \#])$ $[X \rightarrow b.Ya; \#]$ $[Y \rightarrow .c; a\#]$ $[Y \rightarrow .ca; a\#]$	$\emptyset$	$[X \rightarrow bY.a.; \#]$	$\emptyset$	$\emptyset$	$[Y \rightarrow c.; a\#]$ $[Y \rightarrow c.a; a\#]$
$next(q_3, \dots)$		$q_5$			$q_6$
R					$q_3ca \rightarrow q_3q_6a$



Stare	X	Y	a	b	c
$q_4$ $H([Y \rightarrow c.; \#], )$ $[Y \rightarrow c.a; \#]$	$\emptyset$	$\emptyset$	$[Y \rightarrow ca.; \#]$	$\emptyset$	$\emptyset$
$next(q_5...)$			$q_7$		
$R$			$q_4 a \# \rightarrow q_4 q_7 \#$		
$q_5$ $H([X \rightarrow bY.a; \#], )$ $next(q_5, ...)$	$\emptyset$	$\emptyset$	$[X \rightarrow bYa.; \#]$	$\emptyset$	$\emptyset$
			$q_8$		
$R$			$q_5 a \# \rightarrow q_5 q_8 \#$		
$q_6$ $H([Y \rightarrow c.; a\#], )$ $[Y \rightarrow c.a; a\#]$	$\emptyset$	$\emptyset$	$[Y \rightarrow ca.; a\#]$	$\emptyset$	$\emptyset$
$next(q_6, ...)$			$q_9$		
$R$			$q_6 a a \rightarrow q_6 q_9 a$		
$q_7$ $H([Y \rightarrow ca.; \#])$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$q_8$ $H([X \rightarrow bYa.; \#])$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$q_9$ $H([Y \rightarrow ca.; a\#])$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

pas 7. Pentru fiecare  $[X \rightarrow \chi.; \omega] \in q$ , unde  $\chi = x_1..x_n$  se face

$$\begin{aligned}
 R &= R \cup \{q_1..q_n q \omega \rightarrow q_1 q' \omega \mid [X \rightarrow .\chi; \omega] \in q_1, \\
 &\quad q_{i+1} = \text{next}(q_i, x_i) (i \in 1..n-1), \\
 &\quad q = \text{next}(q_n, x_n), \\
 &\quad q' = \text{next}(q_1, X)\}
 \end{aligned}$$

Pt  $q_2$ :  $H([X \rightarrow Y.; \#])$

rol de  $q_1$   $[X \rightarrow .Y; \#] \in ?$

rol de  $q$   $? = \text{next}(?, Y)$

rol de  $q'$   $? = \text{next}(?, X)$

Stare	X	Y	a	b	c
$q_0$ $H([Z \rightarrow .X; \#])$ $[Z \rightarrow .X; \#],$ $[X \rightarrow .Y; \#],$ $[X \rightarrow .bYa; \#],$ $[Y \rightarrow .c; \#],$ $[Y \rightarrow .ca; \#]$	$[Z \rightarrow X.; \#]$	$[X \rightarrow Y.; \#]$	-	$[X \rightarrow b.Ya; \#]$	$[Y \rightarrow c.; \#]$  $[Y \rightarrow c.a; \#]$
$\text{next}(q_0, \dots)$	$q_1$	$q_2$		$q_3$	$q_4$

pas 7. Pentru fiecare  $[X \rightarrow \chi.; \omega] \in q$ , unde  $\chi = x_1..x_n$  se face

$$\begin{aligned}
 R &= R \cup \{q_1..q_n q \omega \rightarrow q_1 q' \omega \mid [X \rightarrow .\chi; \omega] \in q_1, \\
 &\quad q_{i+1} = \text{next}(q_i, x_i) (i \in 1..n-1), \\
 &\quad q = \text{next}(q_n, x_n), \\
 &\quad q' = \text{next}(q_1, X)\}
 \end{aligned}$$

Pt  $q_2$ :  $H([X \rightarrow Y.; \#])$

rol de  $q_1$   $[X \rightarrow .Y; \#] \in q_0$

rol de  $q$   $q_2 = \text{next}(q_0, Y)$

rol de  $q'$   $q_1 = \text{next}(q_0, X)$

Stare	X	Y	a	b	c
$q_0$ $H([Z \rightarrow .X; \#])$ $[Z \rightarrow .X; \#],$ $[X \rightarrow .Y; \#],$ $[X \rightarrow .bYa; \#],$ $[Y \rightarrow .c; \#],$ $[Y \rightarrow .ca; \#]$	$[Z \rightarrow X.; \#]$	$[X \rightarrow Y.; \#]$	-	$[X \rightarrow b.Ya; \#]$	$[Y \rightarrow c.; \#]$  $[Y \rightarrow c.a; \#]$
$\text{next}(q_0, \dots)$	$q_1$	$q_2$		$q_3$	$q_4$

pas 7. Pentru fiecare  $[X \rightarrow \chi.; \omega] \in q$ , unde  $\chi = x_1..x_n$  se face

$$\begin{aligned}
 R &= R \cup \{q_1..q_n q \omega \rightarrow q_1 q' \omega \mid [X \rightarrow .\chi; \omega] \in q_1, \\
 &\quad q_{i+1} = \text{next}(q_i, x_i) (i \in 1..n-1), \\
 &\quad q = \text{next}(q_n, x_n), \\
 &\quad q' = \text{next}(q_1, X)\}
 \end{aligned}$$

Pt  $q_2$ :  $H([X \rightarrow Y.; \#])$

rol de  $q_1$   $[X \rightarrow .Y; \#] \in q_0$

rol de  $q$   $q_2 = \text{next}(q_0, Y)$

rol de  $q'$   $q_1 = \text{next}(q_0, X)$

$q_0 q_2 \# \rightarrow q_0 q_1 \#$

Stare	X	Y	a	b	c
$q_0$ $H([Z \rightarrow .X; \#])$ $[Z \rightarrow .X; \#],$ $[X \rightarrow .Y; \#],$ $[X \rightarrow .bYa; \#],$ $[Y \rightarrow .c; \#],$ $[Y \rightarrow .ca; \#]$	$[Z \rightarrow X.; \#]$	$[X \rightarrow Y.; \#]$	-	$[X \rightarrow b.Ya; \#]$	$[Y \rightarrow c.; \#]$  $[Y \rightarrow c.a; \#]$
$\text{next}(q_0, \dots)$	$q_1$	$q_2$		$q_3$	$q_4$

pas 7. Pentru fiecare  $[X \rightarrow \chi.; \omega] \in q$ , unde  $\chi = x_1..x_n$  se face

$$\begin{aligned}
 R &= R \cup \{q_1..q_n q \omega \rightarrow q_1 q' \omega \mid [X \rightarrow .\chi; \omega] \in q_1, \\
 &\quad q_{i+1} = \text{next}(q_i, x_i) (i \in 1..n-1), \\
 &\quad q = \text{next}(q_n, x_n), \\
 &\quad q' = \text{next}(q_1, X)\}
 \end{aligned}$$

Pt  $q_4$ :  $H([Y \rightarrow c.; \#], [Y \rightarrow c.a; \#])$

$q_1$   $[Y \rightarrow .c; \#] \in q_0$

$q$   $q_4 = \text{next}(q_0, c)$

$q'$   $q_2 = \text{next}(q_0, Y)$

Stare	X	Y	a	b	c
$q_0$ $H([Z \rightarrow .X; \#])$ $[Z \rightarrow .X; \#],$ $[X \rightarrow .Y; \#],$ $[X \rightarrow .bYa; \#],$ $[Y \rightarrow .c; \#],$ $[Y \rightarrow .ca; \#]$	$[Z \rightarrow X.; \#]$	$[X \rightarrow Y.; \#]$	-	$[X \rightarrow b.Ya; \#]$	$[Y \rightarrow c.; \#]$  $[Y \rightarrow c.a; \#]$
$\text{next}(q_0, \dots)$	$q_1$	$q_2$		$q_3$	$q_4$

pas 7. Pentru fiecare  $[X \rightarrow \chi.; \omega] \in q$ , unde  $\chi = x_1..x_n$  se face

$$R = R \cup \{q_1..q_n q \omega \rightarrow q_1 q' \omega \mid [X \rightarrow .\chi; \omega] \in q_1,$$

$$q_{i+1} = \text{next}(q_i, x_i) (i \in 1..n-1),$$

$$q = \text{next}(q_n, x_n),$$

$$q' = \text{next}(q_1, X)\}$$

Pt  $q_4$ :  $H([Y \rightarrow c.; \#], [Y \rightarrow c.a; \#])$

$$q_1 [Y \rightarrow .c; \#] \in q_0$$

$$q q_4 = \text{next}(q_0, c)$$

$$q' q_2 = \text{next}(q_0, Y)$$

$$q_0 q_4 \# \rightarrow q_0 q_2 \#$$

Stare	X	Y	a	b	c
$q_0$ $H([Z \rightarrow .X; \#])$ $[Z \rightarrow .X; \#],$ $[X \rightarrow .Y; \#],$ $[X \rightarrow .bYa; \#],$ $[Y \rightarrow .c; \#],$ $[Y \rightarrow .ca; \#]$	$[Z \rightarrow X.; \#]$	$[X \rightarrow Y.; \#]$	-	$[X \rightarrow b.Ya; \#]$	$[Y \rightarrow c.; \#]$  $[Y \rightarrow c.a; \#]$
$\text{next}(q_0, \dots)$	$q_1$	$q_2$		$q_3$	$q_4$

Pt  $q_6$ :  $H([Y \rightarrow c.; a\#], [Y \rightarrow c.a; a\#])$

$q_1$   $[Y \rightarrow .c; a\#] \in q_3$

$q$   $q_6 = next(q_3, c)$

$q'$   $q_5 = next(q_3, Y)$

$q_3 q_6 a\# \rightarrow q_3 q_5 a\#$

Pt  $q_7$ :  $H([Y \rightarrow ca.; \#])$

$q_1$   $[Y \rightarrow .ca; \#] \in q_0$

$q_2$   $q_4 = next(q_0, c)$

$q$   $q_7 = next(q_4, a)$

$q'$   $q_2 = next(q_0, Y)$

$q_0 q_4 q_7 \# \rightarrow q_0 q_2 \#$

Pt  $q_8$ :  $H([X \rightarrow bYa.; \#]$

$q_1$   $[X \rightarrow .bYa; \#] \in q_0$

$q_2$   $q_3 = \text{next}(q_0, b)$

$q_3$   $q_5 = \text{next}(q_3, Y)$

$q$   $q_8 = \text{next}(q_5, a)$

$q'$   $q_1 = \text{next}(q_0, X)$

$q_0 q_3 q_5 q_8 \# \rightarrow q_0 q_1 \#$

Pt  $q_9$ :  $H([Y \rightarrow \text{ca.}; a\#])$

$q_1$   $[Y \rightarrow .ca; a\#] \in q_3$

$q_2$   $q_6 = \text{next}(q_3, c)$

$q$   $q_9 = \text{next}(q_6, a)$

$q'$   $q_5 = \text{next}(q_3, Y)$

$q_3 q_6 q_9 a\# \rightarrow q_3 q_5 a\#$



$q_0:$   $[Z \rightarrow \bullet X; \#]$   
 $[X \rightarrow \bullet Y; \#]$   
 $[X \rightarrow \bullet bY a; \#]$   
 $[Y \rightarrow \bullet c; \#]$   
 $[Y \rightarrow \bullet ca; \#]$   
 $q_1:$   $[Z \rightarrow X \bullet; \#]$   
 $q_2:$   $[X \rightarrow Y \bullet; \#]$   
 $q_3:$   $[X \rightarrow b \bullet Y a; \#]$   
 $[Y \rightarrow \bullet c; a \#]$   
 $[Y \rightarrow \bullet ca; a \#]$

$q_4:$   $[Y \rightarrow c \bullet; \#]$   
 $[Y \rightarrow c \bullet a; \#]$   
 $q_5:$   $[X \rightarrow bY \bullet a; \#]$   
 $q_6:$   $[Y \rightarrow c \bullet; a \#]$   
 $[Y \rightarrow c \bullet a; a \#]$   
 $q_7:$   $[Y \rightarrow ca \bullet; \#]$   
 $q_8:$   $[X \rightarrow bY a \bullet; \#]$   
 $q_9:$   $[Y \rightarrow ca \bullet; a \#]$

$$\begin{aligned}
R = \{ & q_0bc \rightarrow q_0q_3c, \\
& q_0c\# \rightarrow q_0q_4\#, \\
& q_0ca \rightarrow q_0q_4a, \\
& q_3ca \rightarrow q_3q_6a, \\
& q_4a\# \rightarrow q_4q_7\#, \\
& q_5a\# \rightarrow q_5q_8\#, \\
& q_6aa \rightarrow q_6q_9a, \\
& q_0q_2\# \rightarrow q_0q_1\#, \\
& q_0q_4\# \rightarrow q_0q_2\#, \\
& q_3q_6a\# \rightarrow q_3q_5a\#, \\
& q_0q_4q_7\# \rightarrow q_0q_2\#, \\
& q_0q_3q_5q_8\# \rightarrow q_0q_1\#, \\
& q_3q_6q_9a\# \rightarrow q_3q_5a\# \}
\end{aligned}$$

$q_0$ :  $[Z \rightarrow \bullet X; \#]$   
 $[X \rightarrow \bullet Y; \#]$   
 $[X \rightarrow \bullet bY a; \#]$   
 $[Y \rightarrow \bullet c; \#]$   
 $[Y \rightarrow \bullet ca; \#]$   
 $q_1$ :  $[Z \rightarrow X \bullet; \#]$   
 $q_2$ :  $[X \rightarrow Y \bullet; \#]$   
 $q_3$ :  $[X \rightarrow b \bullet Y a; \#]$   
 $[Y \rightarrow \bullet c; a \#]$   
 $[Y \rightarrow \bullet ca; a \#]$

$q_4$ :  $[Y \rightarrow c \bullet; \#]$   
 $[Y \rightarrow c \bullet a; \#]$   
 $q_5$ :  $[X \rightarrow bY \bullet a; \#]$   
 $q_6$ :  $[Y \rightarrow c \bullet; a \#]$   
 $[Y \rightarrow c \bullet a; a \#]$   
 $q_7$ :  $[Y \rightarrow ca \bullet; \#]$   
 $q_8$ :  $[X \rightarrow bY a \bullet; \#]$   
 $q_9$ :  $[Y \rightarrow ca \bullet; a \#]$

$R = \{q_0bc \rightarrow q_0q_3c,$   
 $q_0c\# \rightarrow q_0q_4\#,$   
 $q_0ca \rightarrow q_0q_4a,$   
 $q_3ca \rightarrow q_3q_6a,$   
 $q_4a\# \rightarrow q_4q_7\#,$   
 $q_5a\# \rightarrow q_5q_8\#,$   
 $q_6aa \rightarrow q_6q_9a,$   
 $q_0q_2\# \rightarrow q_0q_1\#,$   
 $q_0q_4\# \rightarrow q_0q_2\#,$   
 $q_3q_6a\# \rightarrow q_3q_5a\#,$   
 $q_0q_4q_7\# \rightarrow q_0q_2\#,$   
 $q_0q_3q_5q_8\# \rightarrow q_0q_1\#,$   
 $q_3q_6q_9a\# \rightarrow q_3q_5a\#\}$

Cu  $k = 1$ : aceleasi stari;  $k = 0$  ar fuziona  $q_4, q_6$ , respectiv  $q_7, q_9$ .  
 Dar: un singur simbol inainte nu face distinctie intre tranzitiile de  
 deplasare (shift) si reducere (reduce) din starea 6.

$$Z \Rightarrow X \rightarrow bYa \Rightarrow bcaa$$

Stiva	Stare	intrare	$\Rightarrow^R$	tranzitie
#	$q_0$	bcaa#	bcaa	1
#	$q_0 q_3$	caa#		3
#	$q_0 q_3 q_6$	aa#		6
#	$q_0 q_3 q_6 q_9$	a#	bYa	12
#	$q_0 q_3 q_5$	a#		5
#	$q_0 q_3 q_5 q_8$	#	X	11
#	$q_0 q_1$	#	Z	

# Derivate LR

Algoritmul LR(k) - poate fi folosit atat pt a verifica daca o gramatica este LR(1) cat si pt construirea derivatorului sau

Algoritmul LR(k): numarul de stari este foarte mare

- ▶ similar cazului strong LL(k), multe tranzitii din LR(1) sunt independente de simbolul de lookahead
- ▶  $\Rightarrow$  putem construi un parser cu mai putine stari care implementeaza analiza LR(1) dar cu tranzitii mai putine, folosind lookahead doar cand este necesar
- ▶ LR(k)
- ▶ *simple* LR(k) : SLR(k)
- ▶ *lookahead* LR(k): LALR(k)

- ▶ se porneste cu  $LR(0)$ : nu examineaza deloc simbolurile dinainte
- ▶ si se foloseste lookahead doar la nevoie - simple  $LR(1)$  ( $SLR(1)$ )
- ▶ Obs: nu toate  $LR(1)$  sunt  $SLR(1)$
- ▶  $LALR(1)$  - lookahead aplicat la  $SLR(1)$

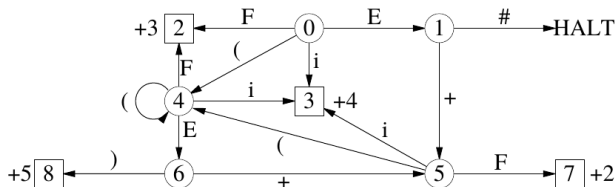
# Tabel de tranzitii

(1)  $Z \rightarrow E$  (2)  $E \rightarrow E + F$  (3)  $E \rightarrow F$  (4)  $F \rightarrow i$  (5)  $F \rightarrow (E)$

	$i$	$($	$)$	$+$	$\#$	$E$	$F$
0	3	4	.	.	.	1	2
1	.	.	.	5	*		
2	+3	+3	+3	+3	+3		
3	+4	+4	+4	+4	+4		
4	3	4	.	.	.	6	2
5	3	4	.	.	.		7
6	.	.	8	5	.		
7	+2	+2	+2	+2	+2		
8	+5	+5	+5	+5	+5		

# Diagrama de tranzitii

- ▶ suprapunerea gramaticilor regulate care corespund claselor stiva k
- ▶ starile prin care trece sunt inarcate pe stiva pana cand se ajunge intr-o stare finala
- ▶ in stare finala se face reducerea pe baza productiei  $X \rightarrow \chi$ , se elimina  $|\chi|$  stari de pe stiva si se continua **ca si cand s-ar fi citit simbolul X**



(1)  $Z \rightarrow E$  (2)  $E \rightarrow E + F$  (3)  $E \rightarrow F$  (4)  $F \rightarrow i$  (5)  $F \rightarrow (E)$

Stiva	Derivare dreapta	simbol urmator	reducere cu productia	stare urmatoare
0	.i+(i+i)#	i		3
0 3	i.+(i+i)#		4	2
0 2	F.+(i+i)#		3	1
0 1	E.+(i+i)#	+		5
0 1 5	E+. (i+i)#	(		4
0 1 5 4	E+(.i+i)#	i		3
0 1 5 4 3	E+(i.+i)#		4	2
0 1 5 4 2	E+(F.+i)#		3	6
0 1 5 4 6	E+(E.+i)#	+		5
0 1 5 4 6 5	E+(E+.i)#	i		3
0 1 5 4 6 5 3	E+(E+i.)#		4	7
0 1 5 4 6 5 7	E+(E+F.)#		2	6
0 1 5 4 6	E+(E.)#	)		8
0 1 5 4 6 8	E+(E).#		5	7
0 1 5 7	E+F.#		2	1
0 1	E.#			



- ▶ Instrumente pentru reprezentare
  - ▶ Siruri de rescriere
  - ▶ Gramatici - ierarhia lui Chomsky
  - ▶ Derivari si arbori de derivare
- ▶ Gramatici regulate si automate finite
  - ▶ Automate finite
  - ▶ Diagrame de stare si expresii regulate
- ▶ Gramatici independente de context si automate stiva:  
Automate stiva
- ▶ Analiza sintatica descendenta:
  - ▶  $LL(k)$
  - ▶ eliminare recursivitate stanga
  - ▶ Factorizare stanga
  - ▶ gramatici  $LL(k)$  tari
  - ▶ Derivator  $LL(1)$  - segmente de program
- ▶ Analiza sintatica ascendenta
  - ▶  $LR(k)$
  - ▶ Derivator  $LR(0)$  - functia de tranzitie
  - ▶  $SLR(1)$