EE 105: Voltage Supply Design and Analysis

Abstract

This lab relates to the design and simulation of a AC to DC voltage converter that is called a half-wave rectifier. The purpose of the circuit is to deliver a relatively constant voltage to a load that is modeled as a resistor.

1 Half-wave Rectifier

Figure 1 shows a circuit for implementation of a DC voltage supply. The input $u(t)$ is the household AC wall outlet that is modeled as a voltage source

$$u(t) = 120\,\sin(2\,\pi\,60\,t).$$

The circuit includes a transformer with a turns ratio of $N = 10$ and a diode.

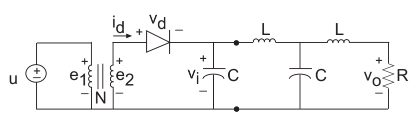

Figure 1: Voltage supply circuit.

The diode current and voltage are related by an algebraic nonlinear function:

$$i_d(t) = h(v_d(t)).$$

Because this effort-flow relationship is algebraic (i.e., no integration or differentiation), the diode is an R-element, with a nonlinear function. The nonlinear function that will be used for the simulation is defined in Table 2.

The half-wave rectifier is the diode which only allows positive current to pass. The purpose of the transformer is to decrease the peak voltage experienced on its right-hand-side. The purpose of the inductors and capacitors is to implemenpent a low pass filter (LPF) to smooth the current and voltage delivered to the load that is modeled as a resistor. Figure 2 shows the linear, LPF circuit between the diode current and the load voltage. The input is $i_d(t)$ and the output is $v_o(t)$.

2 Prelab

1. Construct and simplify the bond graph for the half-wave rectifier circuit in Fig. 1.

2. From the bond graph, define a (nonlinear) state space model: $\dot{x}(t) = f(x(t), u(t))$. Define the state vector as

$$x = [v_i\,, i_L,\, v_f,\, i_o]^\top, \tag{1}$$

where v_i is defined in Fig. 1, i_L is the current through the left inductor (positive to the right), v_f is the voltage across the right capacitor (positive on top), and i_o is the current through the right inductor (positive to the right) and the load resistor.

The output vector is

$$\mathbf{y} = [v_o, v_i, v_d, i_d]^\top. \tag{2}$$

3. Using the state definition in eqn. (1), show that the LPF circuit in Fig. 2 has a linear state space model with

$$A = \begin{bmatrix} 0 & \frac{-1}{C} & 0 & 0 \\ \frac{1}{L} & 0 & \frac{1}{L} & 0 \\ 0 & \frac{1}{C} & 0 & \frac{-1}{C} \\ 0 & 0 & \frac{1}{L} & \frac{R}{L} \end{bmatrix} \qquad B = \begin{bmatrix} \frac{1}{C} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & R \end{bmatrix} \qquad D = \ 0. \tag{4}$$

3 Linear Theory

While the half-wave rectifier is nonlinear, we can use linear systems analysis for the design of the LPF. Denote the transfer function from $i_d(t)$ to $v_o(t)$ by

$$H(s) = \frac{V_o(s)}{I_d(s)}. \tag{5}$$

Because the circuit has four independent energy storage devices, its model will be fourth order. Therefore, it would be significant work to find an analytic expesssion for the transfer function $H(s)$; instead, we will use Matlab.

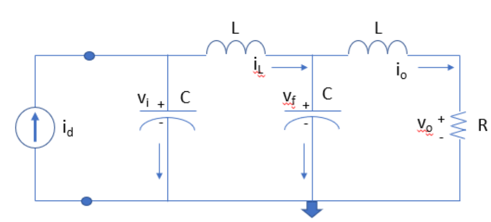

Figure 2: Voltage supply low pass filter (LPF) from $i_d(t)$ to $v_o(t)$.

The resistor has value $R = 1000$ Ohms. The capacitor has value $C = 100\mu$ Farads. The goal of this section is to analyze how the value of L affects the performance.

The diode has the effect of forcing $i_d(t)$ to be a periodic train of positive current pulses. This periodic waveform can be represented as a Fourier series:

$$i_d(t) = a_0 + \sum_{n=1}^{\infty} a_n \sin(n\,\omega\,t) \qquad (6)$$

where $\omega = 2\pi 60 \approx 377$ is the fundamental frequency and $\omega_n = n\,\omega$ for $n \geq 2$ are its harmonics. Note that the non-linear action of the diode has created an infinite numer of harmonics.

By the *frequency response* theory, we know that the LPF output will be

$$v_o(t) = a_0\,|H(0)| + \sum_{n=1}^{\infty} a_n\,|H(j\,n\,\omega)|\sin(n\,\omega\,t + \Phi(j\,n\,\omega)).$$

Because our goal is to convert AC current to DC voltage, our goal is to design $H(s)$ so that it reduces the effect of the fundamental and its harmonics. We do this by making $|H(j\,n\,\omega)| \ll |H(0)|$ for $n = 1, 2, 3, \ldots$.

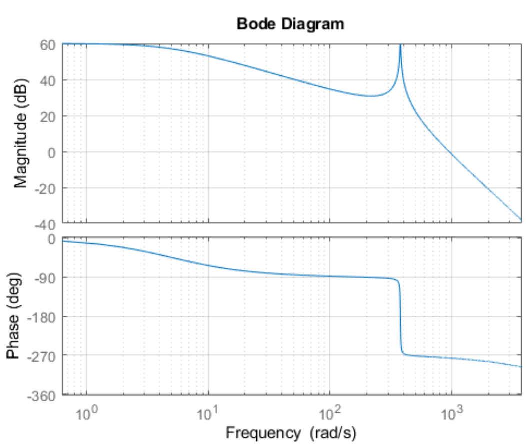

Figure 3: Voltage supply low pass filter (LPF) from $i_d(t)$ to $v_o(t)$.

Figure 3 shows the Bode plode for $L = 0.14$ H. This is a particularly bad choice because $|H(j\,\omega)| = |H(0)|$, meaning that the fundamental component of $i_d(t)$ passes perfectly through the LPF without any attenuation.

4 Lab

1. Review your Prelab with the TA.[1]

   ________Show results to TA to record approval. ________

[1]If you do not know how to read a log-scale axis, this is a good time to ask the TA. You should be able to explain that the peak in Fig. 3 occurs at $\omega = 377$ rad/sec/.

```
function [A,B,C,D,sys] = L2ABCD_sys(L)
% Input: L is inductance in Henries
R = 1000;   % Ohm
C = 100e-6;  % Farad
N = 10;     % turns

A = [0 -1/C   0  0
1/L  0  -1/L 0
0   1/C   0  -1/C
0    0   1/L -R/L]
B = [1/C; 0; 0; 0]
C = [0    0  0  R] % output is the load voltage
D = 0;

sys = ss(A,B,C,D)
```

Table 1: Matlab code to compute the Matlab state-space data structure.

2. Compute the Bode plot as shown in Fig. 3.

   (a) Table 1 shows how to compute the Matlab state-space data structure for any value of L. You need this data structure to use the Matlab Bode function.

   (b) Write a script using 'L2ABCD_sys' and Bode to produce the equivalent of Fig. 3.

   (c) To ensure that Matlab computes the Bode plot at the fundamental frequency and its harmonics use the commands:

   ```
   w =(0:0.1:600)*2*pi;
   bode(sys,w);
   grid on
   ```

   (d) Use the Matlab help facility to ensure that you understand each built-im function that you use.

   ________Show results to TA to record approval. ________

3. The goal of this step is to write a script to evaluate different values for L. The script should do the following:

   (a) Let $w = 2 * pi * (0 : 60 : 600)$. The first element is the desired DC (zero frequency component). The second element is the fundamental. The remaining elements are the first several harmonics.

   (b) Define a vector of L values between 0.01 and 4.00 Henries.

   (c) Implement a 'for' loop that for each value of L does the following:

      i. Calls '[A,B,C,D,sys]=L2ABCD_sys(L)' to compute sys for this value of L.

      ii. Uses '[mag,phase]=bode(sys,w)' to compute the magnitude and phase of $H(jw)$ at each element of w for this value L.

iii. Computes the attenuation of the fundamental relative to the DC component:
attenuation_L = mag(1,1,2)/mag(1,1,1).

Graph the attenuation as a function of L. Select a value L^* for which the attenuation is less than 0.01.

________Review your choice of L^* with the TA. ________

4. In this step, you will use simulink to simulate the half-wave rectifier. The nonlinear function describing the diode is defined in Table 2.

   (a) Implement a simulation for the state space system in Simulink. Because the model is nonlinear, you *cannot* use the Matlab State-Space block. Many other approaches exist. Two are described below.

   i. Fig. 4 shows an implementation using Matlab's *Interpreted Matlab Function* block. The specifics are as follows:
      - The output of the integrator is the signal $x(t)$.
      - A demultiplexer is used to break x into its four components so that each can be plotted with a scope.
      - The input to the integrator is $\dot{x}(t)$.
      - The Matlab function block calls an m-file that this lab will call 'f_of_v.m' that implements the function $f(x, u)$ that you found in Step 2 of the Prelab. This takes a bit of finesse, because the Matlab block only expects one input. Therefore, a multiplexer is used to define $v = [x^\top, u]^\top$. The m-file 'f_of_v.m' then looks something like the code snippet in Table 3.

   ii. Fig. 5 shows an implementation where in integrator block is used for each state of the model. For the i-th state, the output of the integrator is $x_i(t)$. The designer of the simulation must construct a function at the input of that integrator so that the derivative $\dot{x}_i(t)$ is correctly computed. For example, in Fig. 5, for the second state, the block diagram computes

$$\dot{x}_2(t) = \frac{1}{L}\left(x_1(t) - x_3(t)\right)$$

where a 'gain' block is used to compute $1/L$ and the value of L is defined in the workspace.

   Choose and implement one of these approaches.

   _Show your simulation to TA to record approval. _

   (b) Simulate the system for $L = L^*$. Plot each element of the output vector defined in eqn. (2) versus time. Discuss each graph. Your discussion should compare the various elements of the output vector. Use appropriate graph scaling to make the discussion clear. Does the ripple in the output voltage match that predicted by the linear analysis?

   (c) Simulate the system for $L = 0.14$ H. Compare the performance for this poor choice of L to the performance when $L = L^*$.

________Show results to TA to record approval. ________

function $[i_d]$=diode(v_d)
$A = 0.0007;$
if $v_d > 0.7$
$Rd = 0.01;$
$i_d = A + (v_d - 0.7)/Rd;$
else
$Rd = 1000;$
$i_d = A + (v_d - 0.7)/Rd;$
end.

Table 2: Piecewise affine diode model.

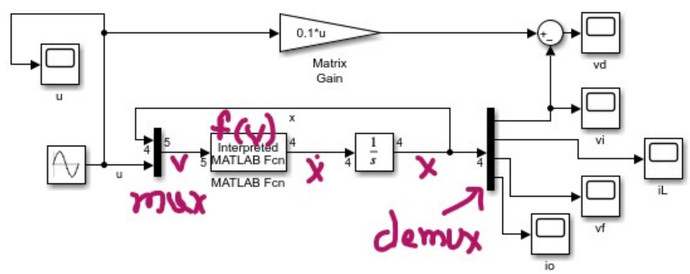

Figure 4: Simulation using the Interpreted Matlab Function block from the User-Defined Functions directory of the Simulink Library.

```
function [dxdt] = f_of_v(v)
x = v(1:4,1); % extract curent state vector
u = v(5); % extract current input
dxdt = f(x,u); % calculate state derivative
```

Table 3: Sample code structure for the user defined function in Fig. 4.

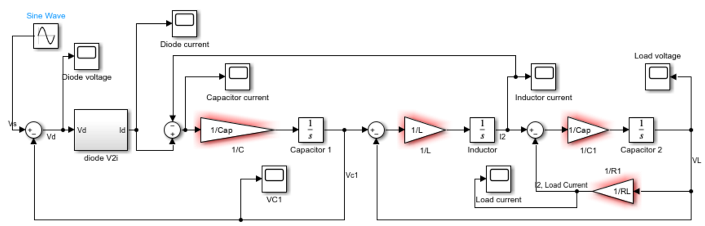

Figure 5: Simulation using the integrator blocks for each individual state. *Not that this implementation does not quite correspond to the half-wave rectifier in this lab.* The block diagram in this image only has three states, whereas the circuit for this lab has four states.