EE_105_023_23W Lab 4

Merrick Slane

TOTAL POINTS

100 / 100

QUESTION 1

1 2.2 (a,b,c,d,e) **15 / 15**

✓ **- 0 pts** *Correct*

QUESTION 2

2 2.3 (b,c) **15 / 15**

✓ **- 0 pts** *Correct*

QUESTION 3

3 2.4 (b,c,d) **15 / 15**

✓ **- 0 pts** *Correct*

QUESTION 4

4 2.4 (e) **20 / 20**

✓ **- 0 pts** *Correct*

QUESTION 5

5 3 **10 / 10**

✓ **- 0 pts** *Correct*

QUESTION 6

6 Prelab **25 / 25**

✓ **+ 25 pts** *Full prelab presented in lab*

EE105 Lab 4

Merrick Slane

February 7, 2023

1 State-Space Definition and System Parameters

1.1 Matrix Definitions

Given the A,B,C, and D matrices we found in the prelab and the values of a,b,c, and d, we use the following code to compute the values of these matrices:

```
%pt1. compute matrices
a = 10e-4;
b = 1e-6;
c = 100;
d = 40e-3;

A = [0 1/b 0 0; -1/a 0 0 -1/a; 0 0 0 1/d; 0 1/b -1/b -1/(b*c)];
B = [0;1/a;0;0];

C = [0 0 1 0];
D = 0;

%display matrix values
disp(A);
disp(B);
disp(C);
disp(D);
```

1.2 Poles of the System

In order to find the poles of the system represented by this state-space model, we use the following code:

```
%pt2. find poles
poles = eig(A);
disp(poles);
```

From this, we determine that the poles are:

$$-\frac{1}{4} \mp 4.451j \quad \text{and} \quad -\frac{1}{4} \mp 0.2516j$$

1.3 Time Constant and Settling Time

To determine the time constant and settling time of the system, we simply take the reciprocal of the real part of the closest pole to the origin using the following code:

```
%pt3. find time constant and settling time
tau = 1/abs(real(min(poles))); %closest poles determine time constant
disp(tau);

Tss = 4*tau;
disp(Tss);
```

From this, we find that $\tau = 4 \cdot 10^{-4}$ and $T_{ss} = 0.0016s$.

1.4 Transfer Function

To determine the transfer function of this system from the state space model, we use the following code:

```
%pt4 - state space to transfer function
[num,den]=ss2tf(A,B,C,D);
sys = tf(num,den)
```

From this, we find that the transfer function of this system is given by:

$$H(s) = \frac{2.5 \cdot 10^{10} s - 14.1}{s^4 - 10^4 s^3 + 2.025 \cdot 10^9 s^2 + 10^{13} s + 2.5 \cdot 10^{16}}$$

1.5 Roots Function

To verify equivalence of this transfer function to the state space model, we find its roots by calling `roots(den)` in matlab. This returns a matrix with the exact same pole values as those that we found using `eig(A)`, so we can reasonably say that this transfer function is correct.

1 2.2 (a,b,c,d,e) **15 / 15**

✓ **- 0 pts** *Correct*

2 Bode Plot

To display the bode plot of this system, we use the following code:

```
bode(num,den);
grid on;
```

This yields the bode plot of the system shown in Figure 1.

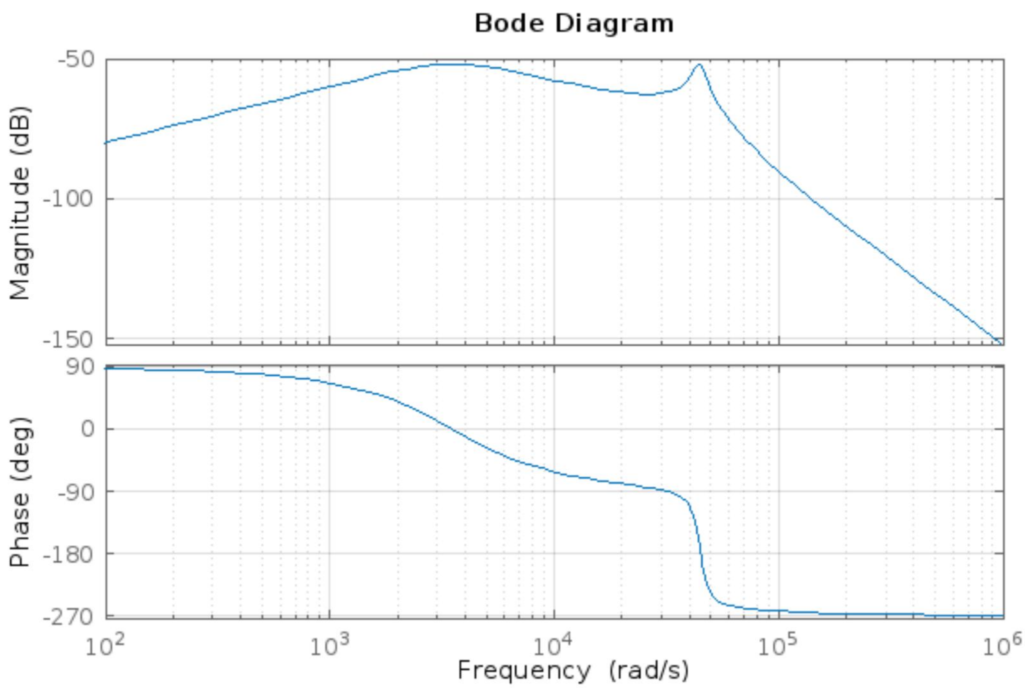

Figure 1: Bode plot of the transfer function from part 1

There are two local maxima in the amplitude response at approximately $\omega = 3.5 \cdot 10^3$ and $\omega = 4.44 \cdot 10^4$. We select these values along with three others to be evaluated in more detail and find the magnitude and phase response at these points:

Table 1: Table of selected points from the bode plot of the system

Point	ω (rad/s)	Magnitude (dB)	Phase (degrees)
f_1	$1 \cdot 10^3$	-60	67
f_{p1}	$3.5 \cdot 10^3$	-52	0
f_2	$1 \cdot 10^4$	-58	-62
f_{p2}	$4.44 \cdot 10^4$	-52	-169
f_3	$1 \cdot 10^5$	-90	-264

2 2.3 (b,c) **15 / 15**

✓ **- 0 pts** *Correct*

3 Simulink Simulation

3.1 Block Diagram

To simulate our derived state-space model in Simulink, we assemble the block diagram shown in Figure 2 using the state-space model block and set the values of the A,B,C,D matrices to those found above.

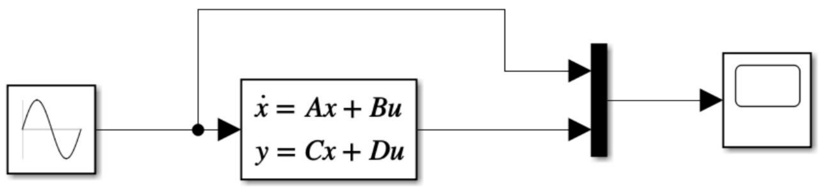

Figure 2: Simulink block diagram of the state-space model

3.2 Simulation

To simulate this system, we set all its inital conditions to 0 and set the simulation stop time to $3T_{SS} = 0.0048s$. Then, we adjust the frequency of the sine wave generator to match that of each of the five points we've selected to simulate. From the output plot, we can vusally determine the settling time, amplitude ratio, and phase shift using the methods shown in Figure 3, which is the output for $\omega = 3.5 \cdot 10^3$.

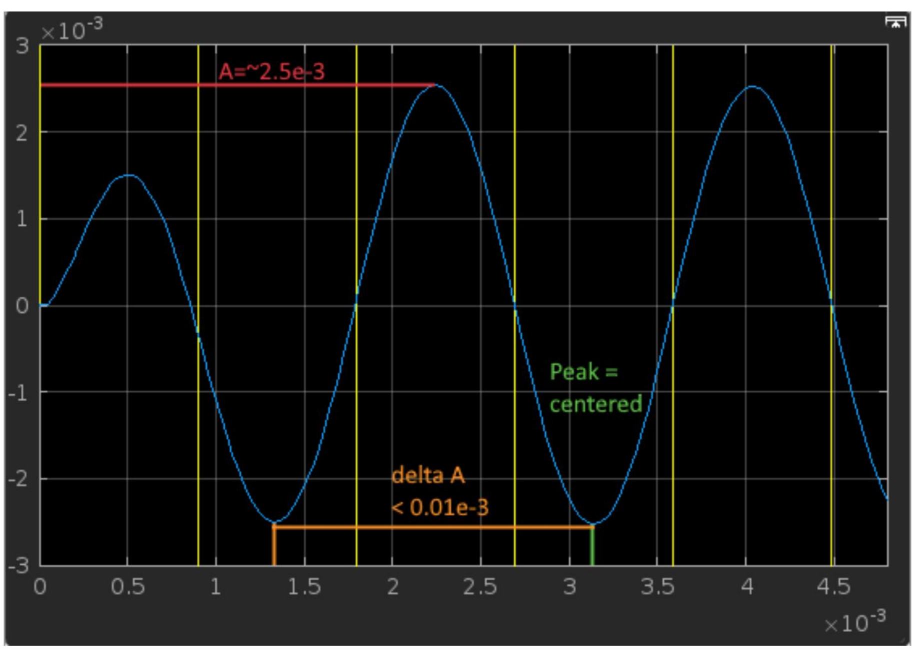

Figure 3: System response for $\omega = 3.5 \cdot 10^3$

From Figure 3, we can gather that the amplitude of the wave is approximately $2.5 \cdot 10^{-3}$. To convert this to decibels, we use the following formula:

3 2.4 (b,c,d) **15 / 15**

✓ **- 0 pts** *Correct*

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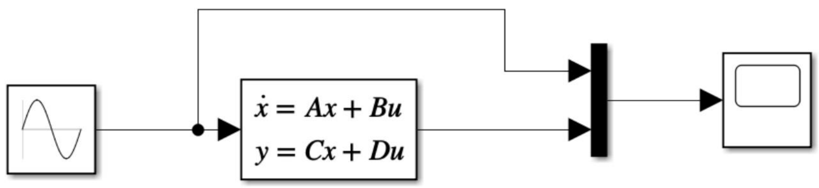

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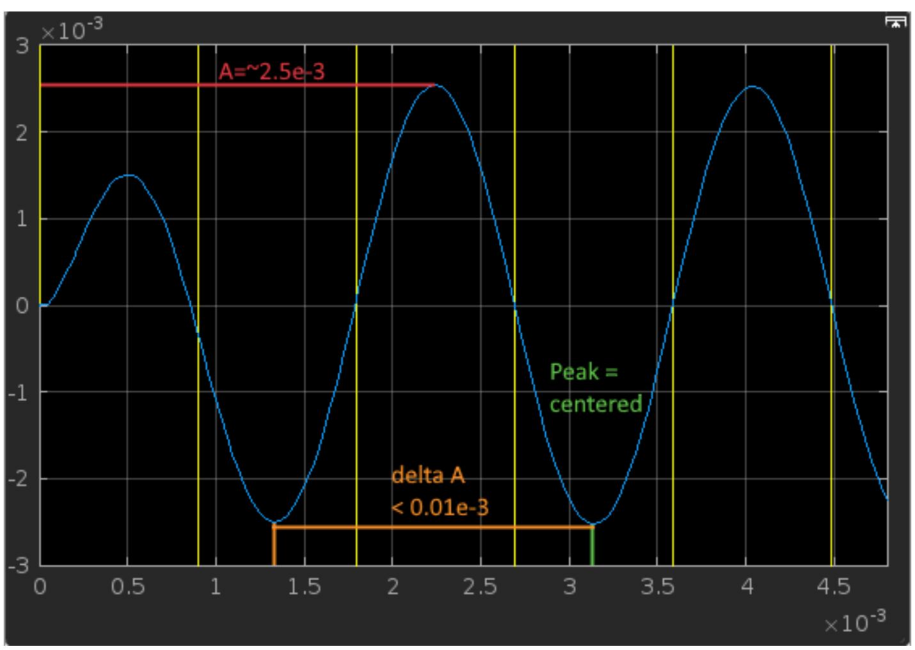

Figure 3: System response for $\omega = 3.5 \cdot 10^3$

From Figure 3, we can gather that the amplitude of the wave is approximately $2.5 \cdot 10^{-3}$. To convert this to decibels, we use the following formula:

$$A_{dB} = 20\log_{10}\left(\frac{2.5\cdot 10^{-3}}{1}\right)\text{dB} = -52.04\text{dB}$$

This nearly exactly matches our expected value of -52dB from Table 1.

To estimate the phase shift, we can evaluate the relationship between the peak of the output wave and the rising and falling edges of the input. Since the input amplitude is many orders of magnitude larger than the output magnitude, we can consider these edges to be perfectly vertical within the y-axis region of interest. In this instance, we see that the peaks of the output wave are nearly perfectly centered within the edges of the input, hence the phase shift is 0 degrees. This is exactly what we expect from Table 1.

To estimate the time constant, we can find the first point where the value of the output is equal to the value of the output 2π radians later. On this plot, we can do this by drawing a horizontal line between two peaks or troughs. We see that the difference in amplitude between the trough at $t \approx 1.4\cdot 10^{-3}$s and the trough at $t \approx 3.1\cdot 10^{-3}$s is less than $0.01\cdot 10^{-3}$, therefore we find the settling time to be $T_{ss} \approx 1.4\cdot 10^{-3}$s. This is reasonably close to the expected value of $T_{ss} = 1.6\cdot 10^{-3}$s from Part 1.

3.3 Simulated Response Data

Next, we repeat the process from section 3.2 for the other 4 angular frequency values in Table 1. The results of this are shown below in Table 2.

Table 2: Table of simulation results

Point	ω (rad/s)	Magnitude (dB)	Phase (degrees)	T_{ss} (ms)
f_1	$1\cdot 10^3$	-60	70	2
f_{p1}	$3.5\cdot 10^3$	-52.04	0	1.4
f_2	$1\cdot 10^4$	-57.7	-60	1.55
f_{p2}	$4.44\cdot 10^4$	-52.4	-170	1.5
f_3	$1\cdot 10^5$	-90.5	-270	1.7

From this, we see that the data in Table 1 and Table 2 agree reasonably well given the nature of the techniques used for deriving these parameters. Certain data points like settling time and phase shift can only be roughly determined from the output plot of the system but their results still compare favorably to the expected values.

4 2.4 (e) **20 / 20**

✓ **- 0 pts** *Correct*

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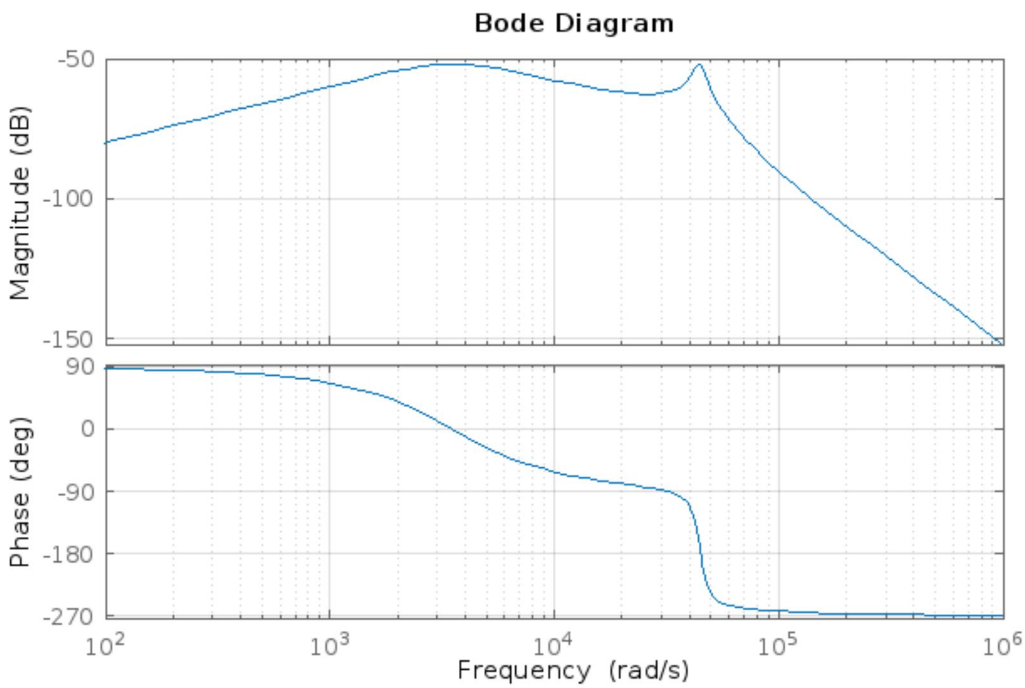

Figure 1: Bode plot of the transfer function from part 1

There are two local maxima in the amplitude response at approximately $\omega = 3.5 \cdot 10^3$ and $\omega = 4.44 \cdot 10^4$. We select these values along with three others to be evaluated in more detail and find the magnitude and phase response at these points:

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5 3 **10 / 10**

✓ **- 0 pts** *Correct*

4 Prelab

$V = IR$

$e = FR \qquad F = \frac{e}{R}$

Finish in morning

$\xrightarrow[F]{e}$

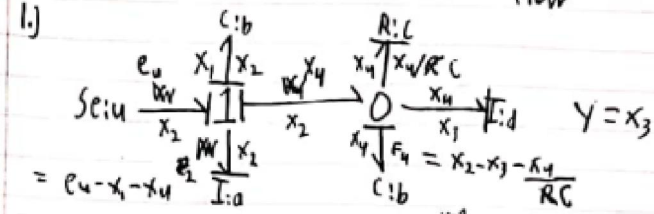
determines flow — direction of flow

0 = eq. efforts

1 = eq. flows

EE 105 Prelab #4

1.)



$Y = x_3$

$$e_u - x_1 - x_4 - e_2 = 0 \qquad x_2 - F_4 - x_3 - \frac{x_4}{R} = 0$$

$$e_2 = e_u - x_1 - x_4 \qquad x_2 - x_3 - \frac{x_4}{R} = F_4$$

$$X = \begin{bmatrix} e_b \\ F_b \\ F_d \\ e_d \end{bmatrix} = \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix}$$

$$F(t) = C\dot{e}(t) \qquad \dot{e}_C = \frac{F_C}{C}$$

$$e_I = I\dot{F}_I \qquad \dot{F}_I = \frac{e_I}{I}$$

2.)

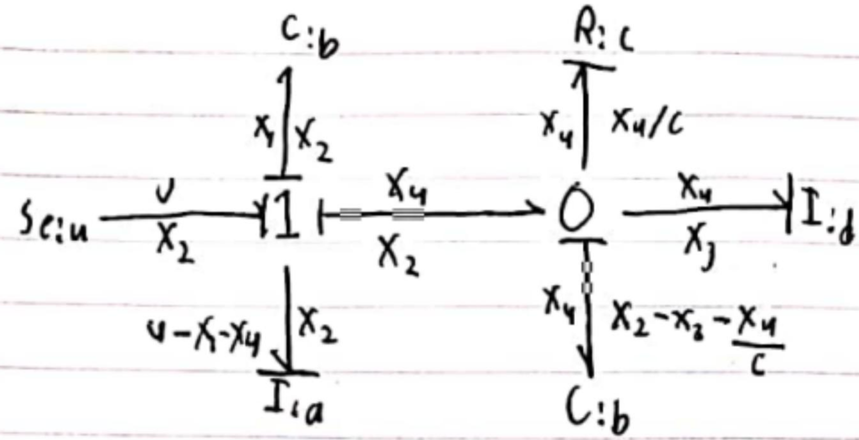
$$\dot{X} = \begin{bmatrix} \dot{e}_b \\ \dot{F}_b \\ \dot{F}_d \\ \dot{e}_d \end{bmatrix} \qquad \dot{e}_b = \frac{F_b}{b} \qquad \dot{F}_b = \dot{F}_a = \frac{e_a}{a} = \frac{e_u - x_1 - x_4}{a} \qquad \dot{x}_1 = \frac{x_2}{}$$

$$\dot{F}_d = \frac{e_d}{d} \qquad \dot{e}_d = \frac{F_b}{b} \qquad \dot{x}_1 = \frac{x_2}{b} \qquad \dot{x}_2 = \frac{x_4}{a} \qquad \dot{x}_3 = \frac{x_4}{d}$$

$$\dot{x} = \begin{bmatrix} 0 & \frac{1}{b} & 0 & 0 \\ -\frac{1}{a} & 0 & \cancel{-\frac{1}{a}} & -\frac{1}{a} \\ 0 & 0 & 0 & \frac{1}{d} \\ 0 & \frac{1}{b} & -\frac{1}{b} & -\frac{1}{bc} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{a} \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u$$

$a = 10e^{-4}, \quad b = 1e^{-6} \quad c = 100 \quad d = 40e^{-3}$



$$X = \begin{bmatrix} 0 & \frac{1}{b} & 0 & 0 \\ \frac{-1}{a} & 0 & 0 & \frac{-1}{a} \\ 0 & 0 & 0 & \frac{1}{a} \\ 0 & \frac{1}{b} & \frac{-1}{b} & \frac{-1}{bc} \end{bmatrix} X + \begin{bmatrix} 0 \\ \frac{1}{a} \\ 0 \\ 0 \end{bmatrix} u$$

$a = 10 \cdot 10^{-4}$

$b = 1 \cdot 10^{-6}$

$c = 100$

$d = 40 \cdot 10^{-3}$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \end{bmatrix} u$$

$J = 1/|re(pole)|$

6 Prelab 25 / 25

✓ **+ 25 pts** *Full prelab presented in lab*