EE105: First Order Systems in Simulink

Lab3- 1/26/16

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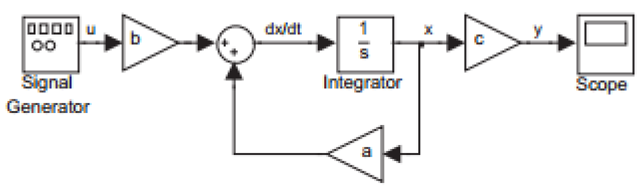
Abstract:

The purpose of this lab is to achieve familiarity with Simulink. Students will do this through applying their knowledge of system behavior and modeling it using both MATLAB and Simulink.

Section 1 Background:

Simulink is an engineering tool users can use to model and run simulation of various types of systems. In this lab you will be modeling a RC-circuit as a first-order system.

A first order system can be modeled using linear state-space models shown below in eqn. 1 and eqn. 2:



$$\frac{dx}{dt} = a\,x + b\,u \qquad (1)$$

$$y = c\,x \qquad (2)$$

Figure 1: Simulink implementation of eqns. (1-2).

A Simulink implementation of this first order system can be seen in *Figure 1* above. A first order system can also be modeled using an impulse response (eqn. 6), or its the transfer function (eqn. 5).

$$y(t) = ce^{at}x(0) \quad (4) \qquad H(s) = \frac{Y(s)}{U(s)} = \frac{cb}{s-a} \quad (5) \qquad h(t) = ce^{at}b \quad (6)$$

Figure 2: Zero-Input Response, Transfer function and Impulse Response formulations

By modeling the system in any of the aforementioned forms, we can make general conclusions about the system. For example, we may be able to find the stability of the system, the DC gain, the settling time, and even its sinusoidal response.

Section 2 More Background:

For higher order systems, the transfer function's magnitude and phase are useful for analysis. The behavior of these two characteristics of the system in response to varying frequencies can be observed through Bode Plots. This lab will focus on first-order systems.

Section 3 Prelab:

Section 3.2 Zero-Input Response:

In this section we were asked to consider the behavior of the graphs below and make an observation about their settling time.

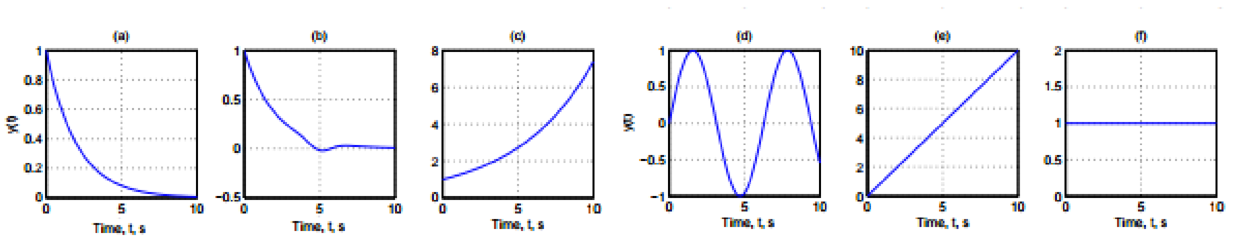


Figure 3: Prelab Graphs

Some nice properties of a Zero-Input response of a first-order linear system is the solution will be an exponential function, in the form shown above in *Figure 2*, eqn. 4.

As long as the exponential constant, *a*, is negative, the system is stable and the output eventually converges to zero. The constant *a* can be used to find the time constant τ, which is correlated to the settling time. The formula for finding τ is below:

$$\tau = \frac{1}{|a|} \quad (7)$$

If $a < 0$, τ determines the settling time. If $a > 0$, τ determines the rate of growth (i.e. doubling time).

Procedure:

For each system in *Figure 3*, we were asked to determine if the system was linear. Since we know this is a Zero-Input response, we know the system should resemble an exponential function if it is linear (refer to eqn. 4). If the system was a first order linear system, we were asked to estimate the time constant.

Results:

(a) Only graphs (a), (c), and (f) from *Figure 3* could correspond to a first order linear system, as none of the other graphs were exponential.

+5 (b) Graphs (b), (d), and (e) did not match the form of eqn. 4

(c) (f) parameter *a* has the value zero, and thus never decays.
(c) has a positive value for its parameter *a*, thus the system is unstable.
(a) has a negative value for *a*, thus it is stable. We can then use the initial value of that graph, y(0) = 1, and eqn. 4 to find the value of a, and then eqn. 7 to find τ as shown in Chapter 4 of the lecture notes.

a = <u>-.460517</u> therefore, τ = <u>2.2 seconds</u>

Section 3.3 Circuit Analysis:

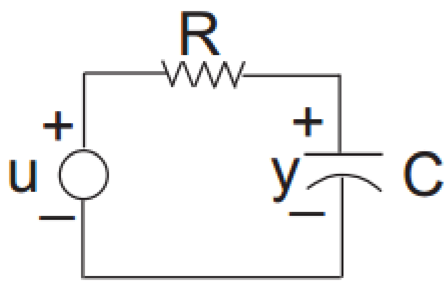


Figure 4: RC Circuit

Procedure:

In this session we were asked to model the circuit shown in *Figure 4* using both a transfer function and the state space model. We were then asked to calculate its magnitude and phase for a variety of angular frequencies and then compare our results to a 'Bode plot' of the system.

Code:

The code shown below was used for part (d) of this section. The code plots a Bode plot of the system shown in *Figure 4*.

```
%Lab3 Prelab
w = [0:.1:100];
R = 20e3;
C = 10e-6;
RC = R*C;
Num = [0 1];
Den = [RC 1];

H = tf(Num,Den);

bode(H,w);
```

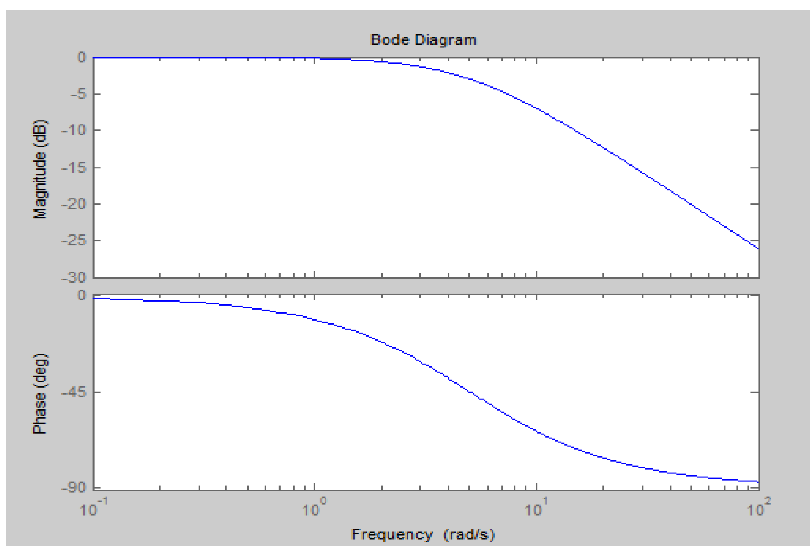


Figure 5a: MATLAB Code Bode Plot

Figure 5b: Bode Plot for circuit in Figure 4

Results:

Part (a): Transfer Function

H(s) = Y(s)/U(s) = $\frac{1}{sRC+1}$ (8) H(s) = Y(s)/U(s) = $\frac{\frac{1}{RC}}{s+\frac{1}{RC}}$ (9)

+5 By using the voltage drop across the capacitor in *Figure 4* as the output of the system, the system can be modeled as shown in eqn. 8. You can further manipulate the function to resemble eqn. 5, as shown in eqn. 9. This allows us to easily identify variable *a* and the product of variables *b* and *c*.

Part (b): State Space Model

By using the voltage drop of the capacitor as a state variable, we can model the system is a set of linear equations, known as the linear State Space Model. We can then extract the values of variables *a b* and *c* from the model. The State space model is shown below.

+5

$$\frac{dx}{dt} = \left[\frac{-1}{RC} \right] x + \left[\frac{1}{RC} \right] u$$
$$y = \left[1 \right] x$$

(10)

Part (c): Frequency Response

Using the values of R and C given in the lab, the frequency response of the transfer function can be found to be written as a function of angular frequency, as seen as in eqn.11.

H(jω) = Y(jω)/U(jω) = $\frac{25-j\omega*5}{25+\omega^2}$ (11) |H(jω)| = $\frac{5}{\sqrt{25+\omega^2}}$ (12) ∠H(jω) = arctan(-ω/5) (13)

Using eqn. 11, the magnitude and phase of the system can also be found. Eqns. 12-13 show that the magnitude and phase of the system is also a function of angular frequency. Plugging in different values of ω, leads to the results shown in *Table 1*.

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ω, rad/s	***|H(jω)|***	***20 log|H(jω)|***	**∠H(jω) rad**	**∠H(jω) deg**
0.00	*1*	*0*	*0.00*	*0°*
0.10	*.9998*	*-0.0017 dB*	*-0.200*	*-1.15°*
1.00	*.981*	*-0.167 dB*	*-0.197*	*-11.31°*
10.00	*.447*	*-6.99 dB*	*-1.11*	*-63.43°*

Table 1: Magnitude and Phase at varying ω

Using *Table 1*, we can observe the behavior of the magnitude and phase as the value of the frequency increases. *Table 1* also shows the magnitude in decibel units (dB), and the phase in both radians and degrees.

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Part (d): Bode Plot

Using the Code shown earlier in *Figure 5a* results in the Bode plot shown in *Figure 5b* above. Using MATLABS built in trace function, as shown below in *Figure 5c*, we can examine the magnitude and phase of the transfer function for varying frequencies.

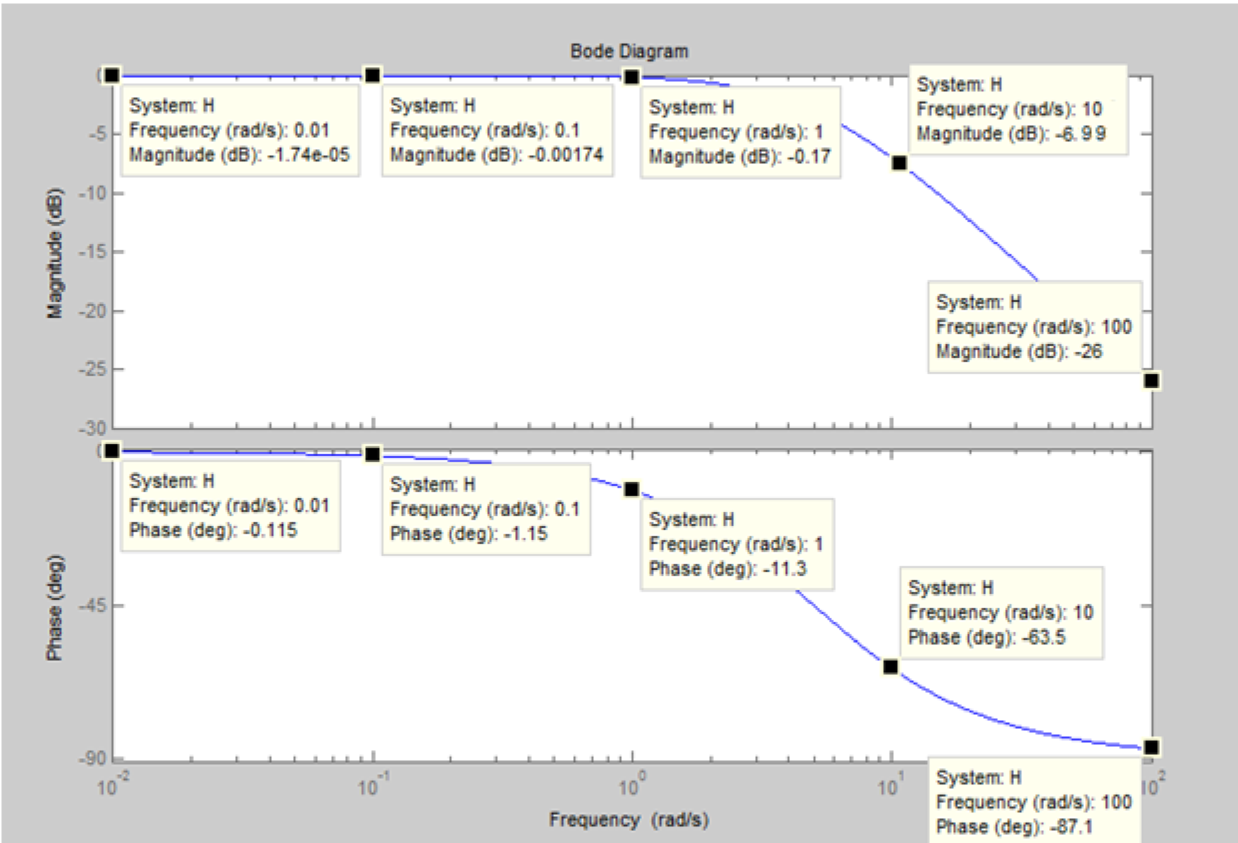

Figure 5c: Bode Plot showing Magnitude and Phase Values for varying Frequencies.

Comparing the non-zero values from part (c) to the ones in part (d), we see the values are similar. The only missing comparison would be when the frequency is 0 rad/s, as the Bode plot logarithmic x-axis scale is undefined at that point. Instead we can use the frequency at .01 rad/s to estimate the behavior. We see that the gain and phase are small and could be approximated to zero.

ω, rad/s	***Part(c) Mag***	***Part(d) Mag***	***Part(c) Mag***	***Part(d) Phase***
0.10	*-0.0017 dB*	*-0.00174 dB*	*-1.15°*	*-1.15°*
1.00	*-0.167 dB*	*-0.17 dB*	*-11.31°*	*-11.3°*
10.00	*-6.99 dB*	*-6.99 dB*	*-63.43°*	*-63.5°*

Table 2: Comparing Magnitude and Phase of Part(c) and Part (d)

Section 4 Lab:

Section 4.1 Time Constant Estimation:

In this section we will estimate the time constant for a given function using two different methods.

Procedure:

First we had to load the data points for the function using the load command built into MATLAB. Then we had to implement our methods (shown in example 4.3) for finding the time constant, τ. Method one approximated τ by using the time value at which the function reaches 37% of its initial value. Method two approximates τ by finding the zero of the line that is tangent to the function at $t = 0$.

Below is the code that was used to implement these two methods graphically.

Code:

```
%Lab3 Section 4.1 Part 1
clear;
load('EE105_Lab3data.mat')

yt1 = .37*y(1)*ones(length(t),1);% Estimation Method 1

%Estimation Method 2
m = (y(2)-y(1))/(t(2)-t(1));%Slope at y0
tau = (0-y(1))/m; %Estimation Method 2 tau value
yt2 = [y(1),0];
t2 = [0 , tau];


subplot(2,1,1)

    plot(t,y,'red',t,yt1,'blue');
    xlabel('t (sec) ');
    ylabel('y(t)');
    title('Part1-Time Constant Estimation 1');
    grid on;


subplot(2,1,2)

    plot(t,y,'red',t2,yt2,'blue',tau,0,'blue*');
    xlabel('t (sec)');
    ylabel('y(t)');
    title('Part1-Time Constant Estimation 2');
    grid on;
```

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Figure 6: Matlab Code to Find Time Constants

Results:

Running the code from *Figure 6*, we obtain the plots show below in *Figure 7*.

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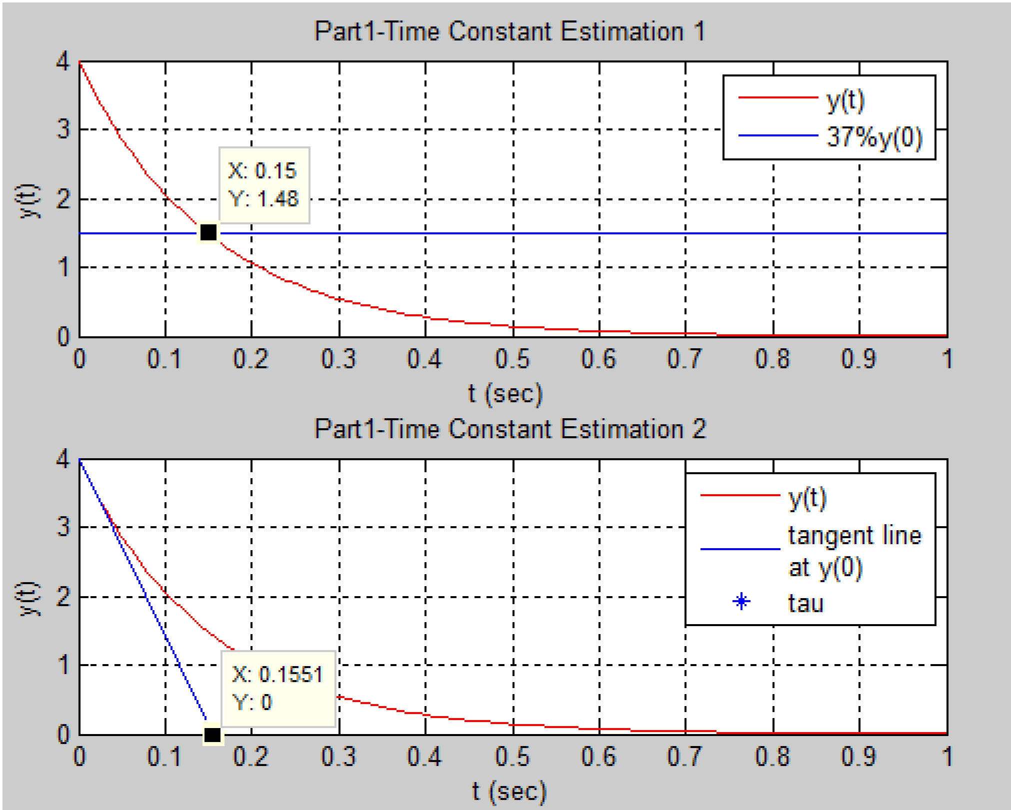

Figure 7: Plots depicting both estimation methods

+5 As one can see from Figure 7, the values of the times constant are pretty similar, with method one having a value of τ_1 = 0.150 and method two having a value of τ_2 = 0.155.

Section 4.2.1 Zero Input:

Procedure:

For this section we were asked to build a Simulink Model of the Circuit from Figure 4. After that we ran the simulation and observed the time constant for the model's output.

Code/Block Diagrams:

```
%Lab3 Section 4.2 Part 2
clear;
R = 20e3;
C = 10e-6;
RC = R*C;

a = -1/RC;
b = 1/RC;
c = 1;
```

Figure 8: Parameters for Simulink Simulation

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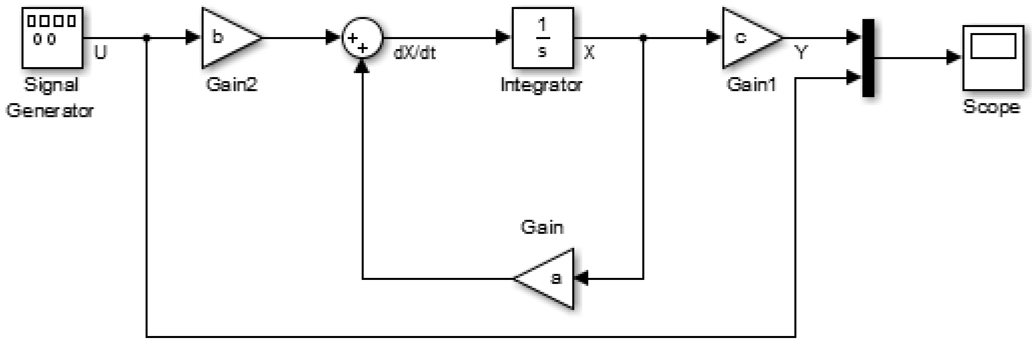

Figure 9: Simulink Model for the circuit in Figure 4

<u>Results:</u>
By implementing the block diagram showed in Figure 9, the scope outputs the graph shown below in Figure 10a.

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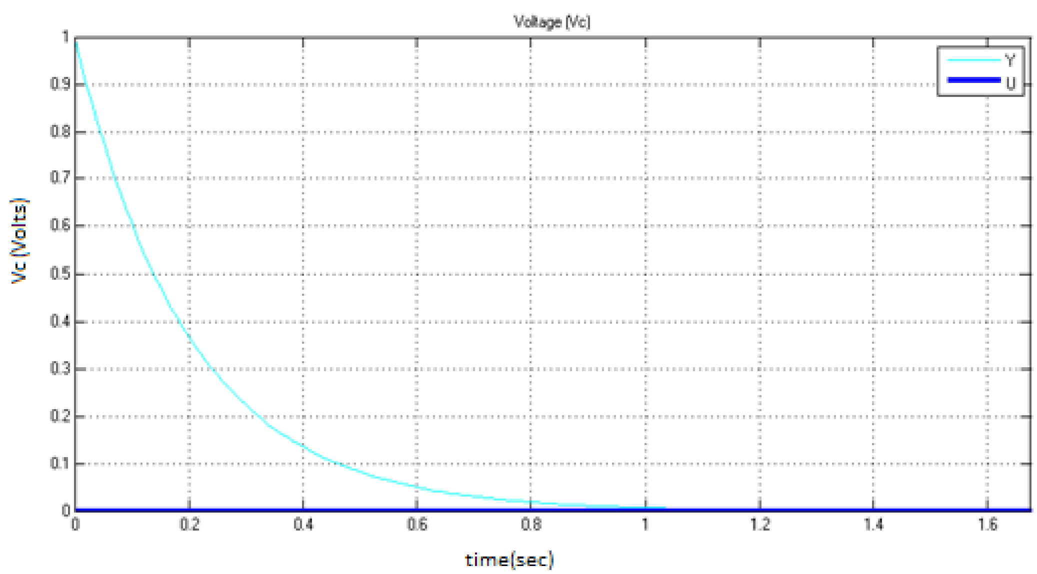


Figure 10a: Zero-Input response of Figure 4 Circuit

We can see from the graph above that the system starts with an initial with a voltage of 1 V and eventually converges to 0 V. By zooming in, we can use the 37% approximation used earlier to estimate the time constant of the system.

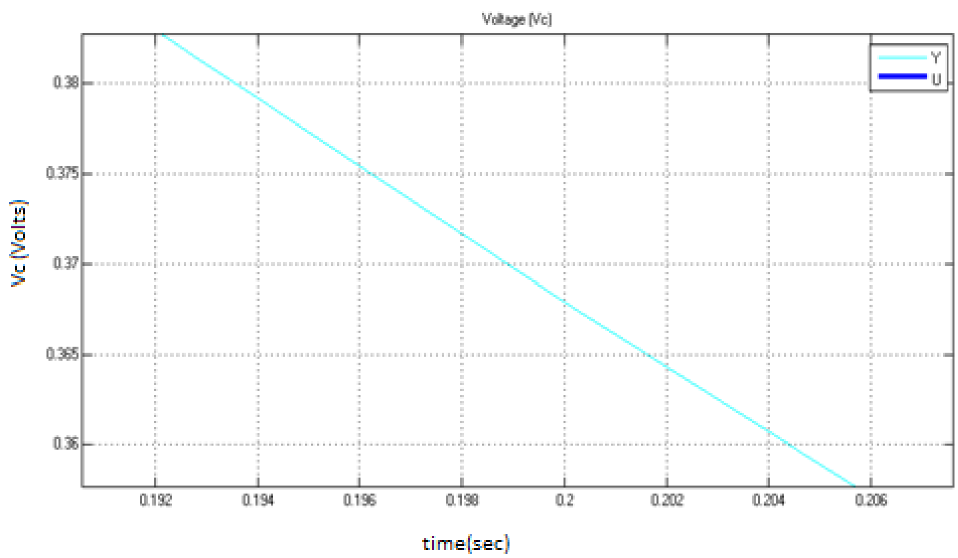


Figure 10b: Zoomed in Graph of Figure 10a

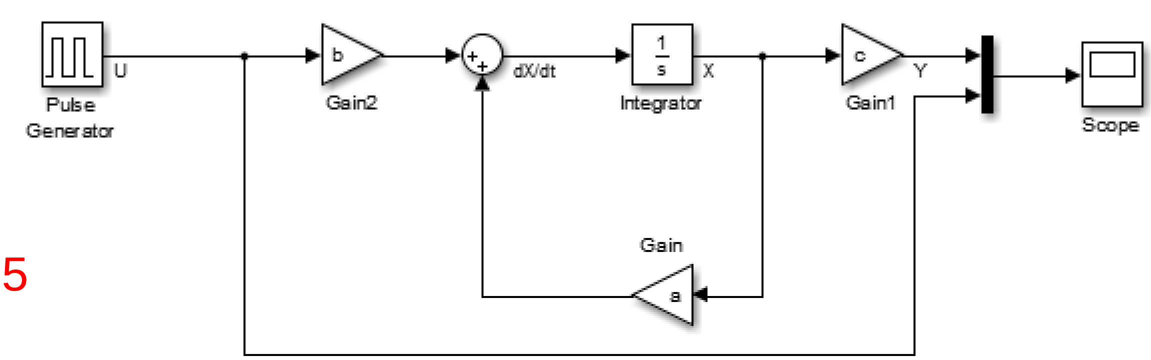
+5 From Figure 10b, we can see that the time value for which the system reaches 37% of its initial value is about τ = 0.198. Theoretically, our time constant should be τ = .199 sec, which can be found be using the value of the variable *a* from *eqn. 10* and plugging it in to *eqn. 7*. Notice how similar the two are.

<u>Section 4.2.2 Step Input:</u>

<u>Procedure:</u>
For this section we were asked to use a Step Input for the system. After that we were asked to find the settling time of the system and compare it to the theoretical value. The theoretical value should be approximately four times the time constant ($t_s = 4 * \tau$). In this case t_s = .8 seconds.

<u>Block Diagrams:</u>

Figure 11 depicts the system with a step input.



+5

Figure 11: Simulink Model with Step Input

Results:

By implementing the block diagram showed in Figure 11, the scope outputs the graph shown below in Figure 12a. Notice the output is labeled Y and the input is U.

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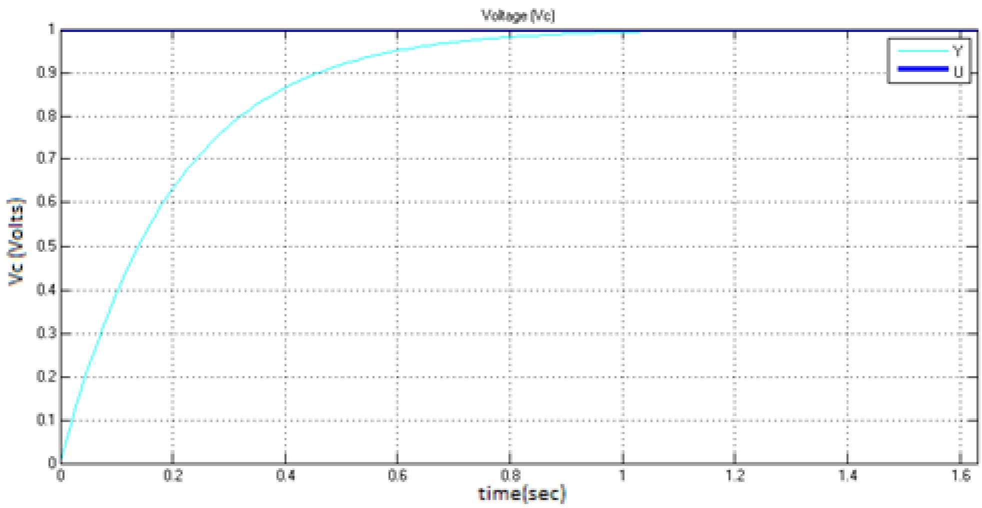

Figure 12a: Step Response Output

We can see from the graph above that the system eventually converges to a voltage of 1 V. By zooming in, we can use the time it takes to be within 2% of its final value to find the settling time.

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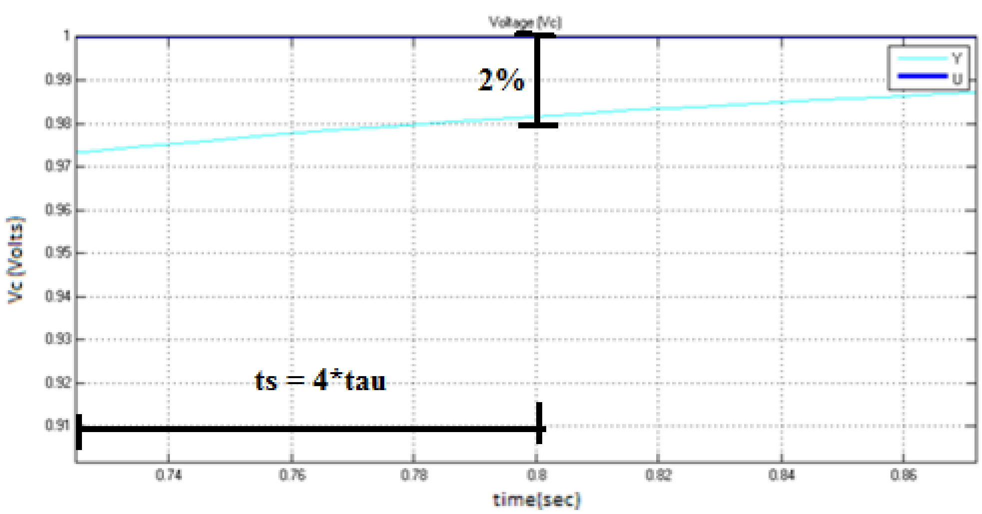

Figure 12b: Zoomed in Graph of Figure 12a

From Figure 10b, we can see that the time value for which the system reaches 2% of its final value is about t_s = 0.78 seconds. This value is comparable to the theoretical settling time found earlier, which was t_s = .8 seconds.

Section 4.2.3 Sinusoidal Input:

Procedure:

For this section we were asked to use a Sinusoidal Input for the system. After that we were asked to find the magnitude and the phase of the system for varying frequencies. To find the magnitude of the system graphically we simply take the peak to peak ratio of output to input (eqn. 14).

$|H(j\omega)| = |Y(j\omega)|/|U(j\omega)|$ (14)

$\angle H(j\omega) = \frac{d}{1}(sec) * \frac{\omega(rad)}{1(sec)} * \frac{360^o\,(\mathrm{deg})}{2\pi(rad)} = \frac{d*360^o}{2\pi/_{\omega}}$ (15)

To find the phase of the transfer function, you first need to find the time delay between the output and the input, denoted as *d* in eqn. 15. Then you can use the unit conversion in eqn. 15 to obtain the phase shift associated with that time delay. Note that if the output is shifted right, the value of *d* is delayes and therefore negative.

Block Diagrams:

Figure 13 depicts the system with a sinusoidal input.

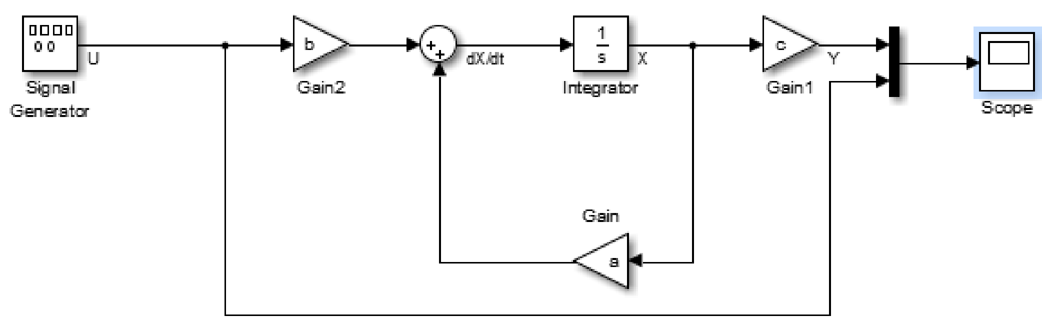

Figure 13: Simulink Model with Step Input

Results:

Figure 14a is the output of the scope after implementing the block diagram with a frequency of ω = 0.1 rad/s. Notice how similar the output and input are after the settling time.

+1

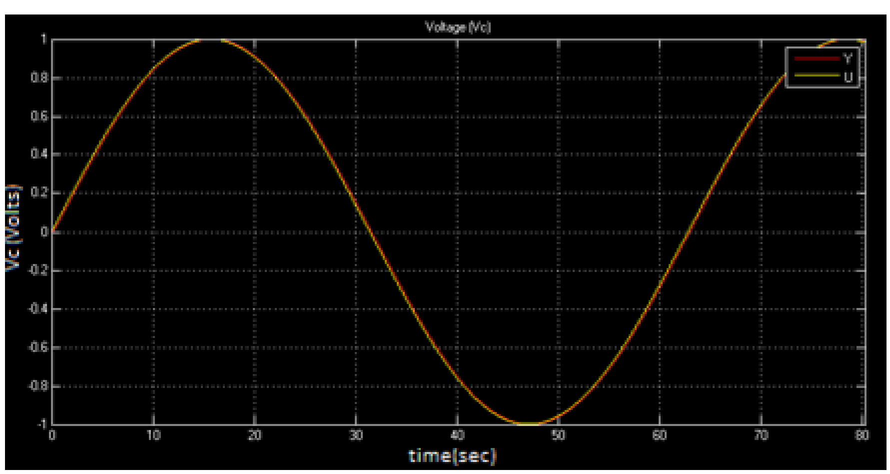

Figure 14a: Sinusoidal Response at ω = 0.1 rad/s

By zooming in, we can use the graphs to find the magnitude and phase. By observing *Figure 14b*, we can see that the magnitude of the output is |U(jω)| = .9998 and |Y(jω)| = 1. We can also see the time delay is d = -0.194 seconds.

+2

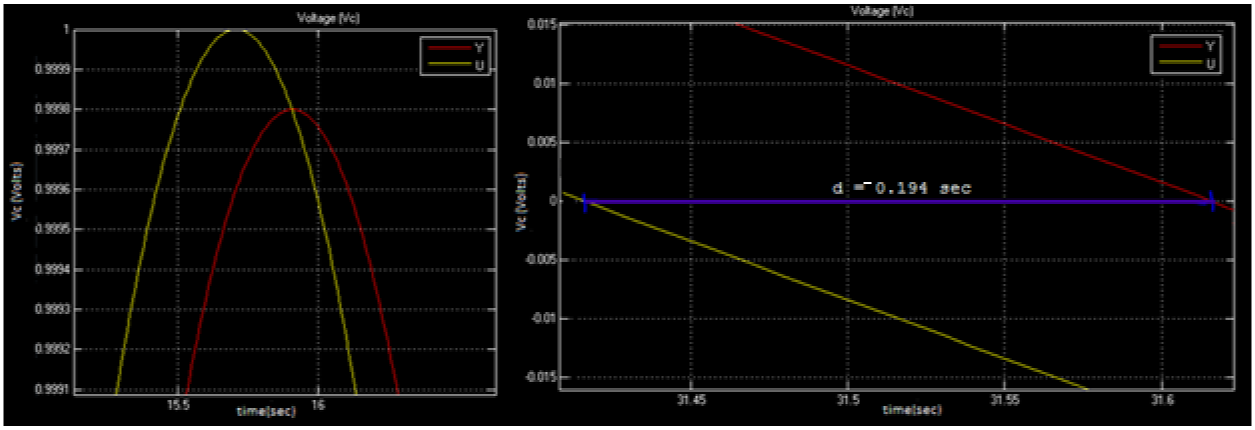

Figure 14b: Zoomed in graph used to calculate magnitude and phase of the system

By taking this values and substituting them into eqns. 14-15, we can find the magnitude and phase of the system.

+2

|H(jω)| = |Y(jω)|/|U(jω)| = |0.9998|/|1.000| = <u>0.9998</u>

$\angle H(j\omega) = \frac{d*360^o}{2\pi/\omega} = \frac{-0.194*360^o}{2\pi/0.1} = -\underline{0.111}°$

We can now repeat this for the other two non-zero frequencies. *Figure 15a* is the output of the scope after implementing the block diagram with a frequency of ω = 1.0 rad/s. Notice how the graph's input and output are still fairly similar after the settling time.

+1

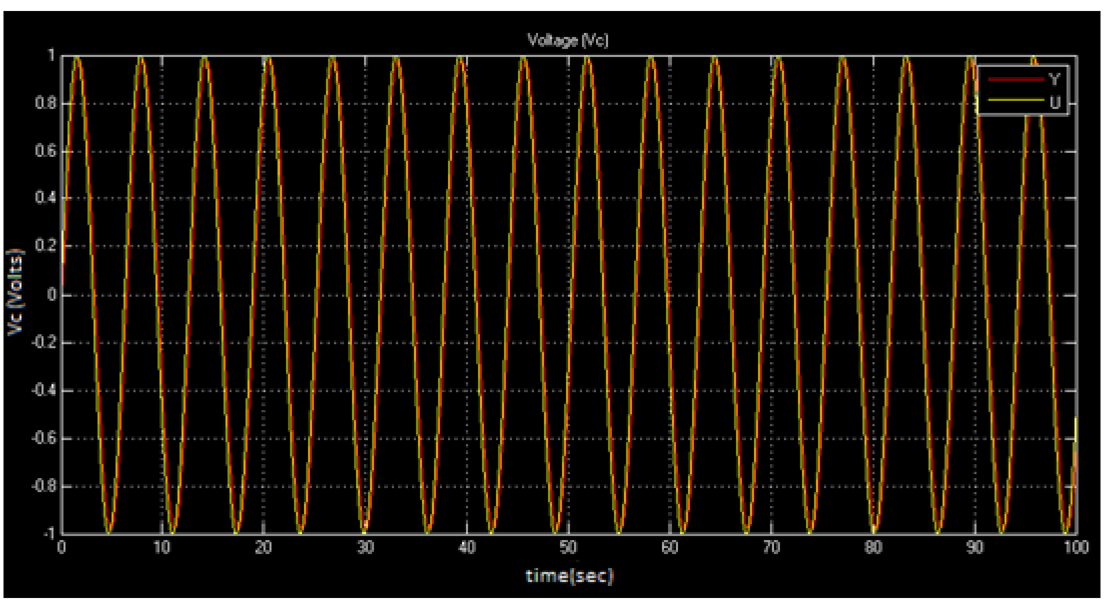

Figure 15a: Sinusoidal Response at ω = 1.0 rad/s

By observing *Figure 15b*, we can see that the magnitude of the output is $|U(j\omega)| = 1$ and $|Y(j\omega)| = .98$. We can also see the time delay is $d = -0.20$ seconds.

+2

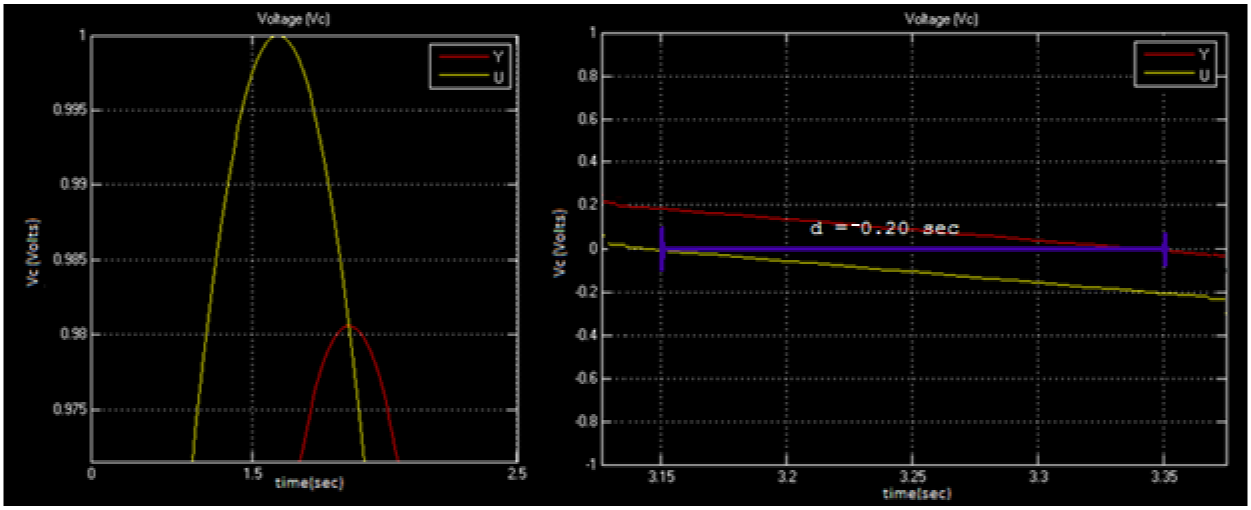

Figure 15b: Zoomed in graph used to calculate magnitude and phase of the system

By taking this values and substituting them into eqns. 14-15, we can find the magnitude and phase of the system.

$|H(j\omega)| = |Y(j\omega)|/|U(j\omega)| = |0.98|/|1.000| = \underline{0.98}$

+2

$\angle H(j\omega) = \frac{d*360^o}{2\pi/\omega} = \frac{-0.20*360^o}{2\pi/1.0} = -\underline{11.45^\circ}$

Figure 16a is the output of the scope after implementing the block diagram with a frequency of ω = 10 rad/s. Notice how now the output is noticeably different then the input.

+1

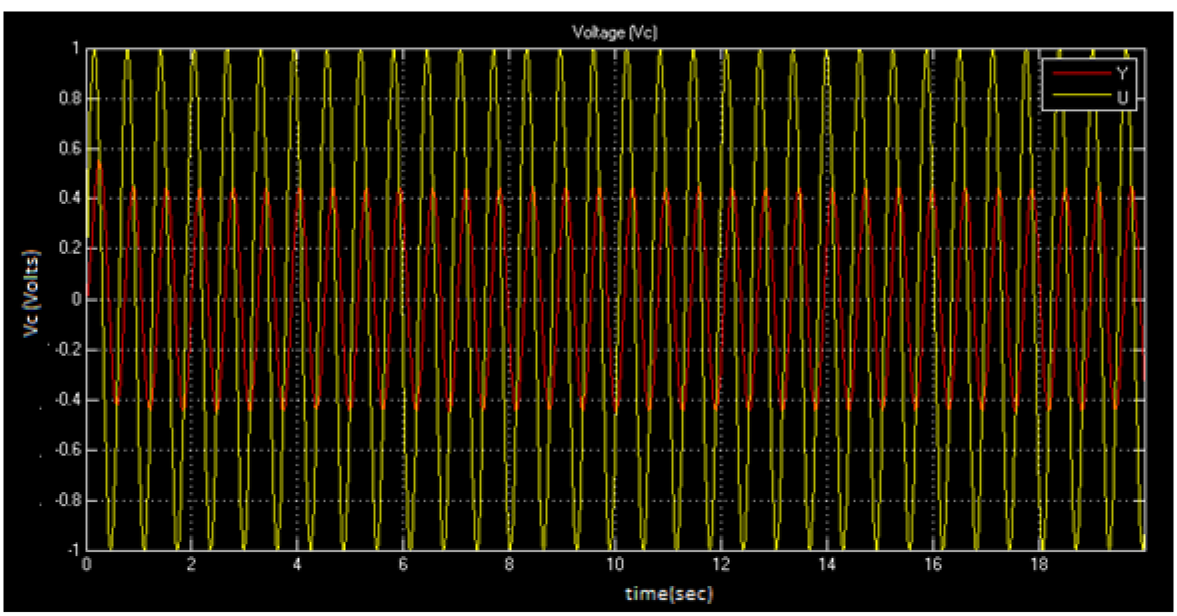

Figure 16a: Sinusoidal Response at ω = 10 rad/s

By observing *Figure 16b*, we can see that the magnitude of the output is |U(jω)| = 1 and |Y(jω)| = .45. We can also see the time delay is d = -0.11 seconds.

+2

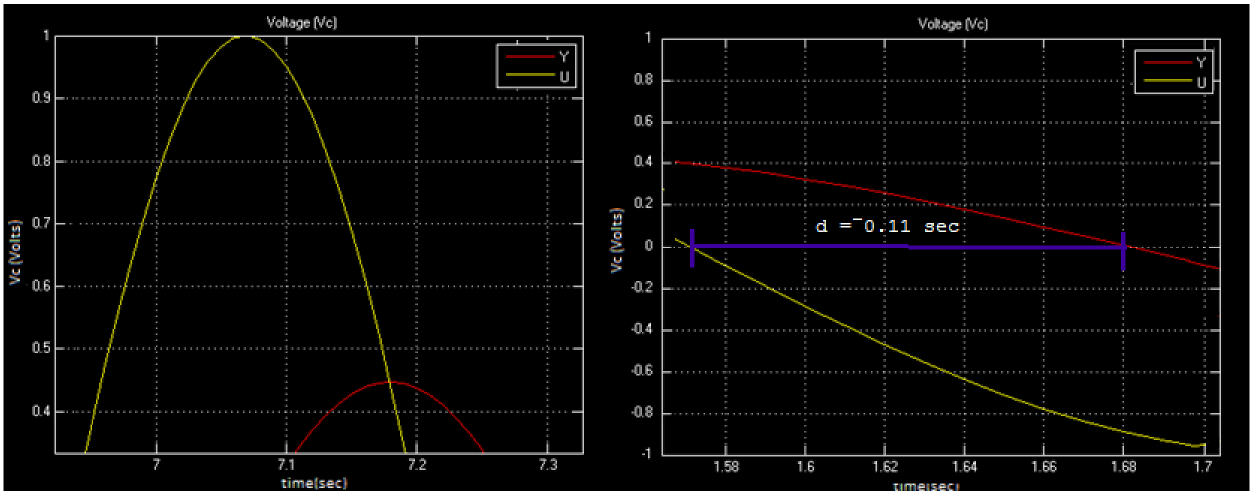

Figure 16b: Zoomed in graph used to calculate magnitude and phase of the system

By taking this values and plugging them into eqns. 14-15, we can find the magnitude and phase of the system.

|H(jω)| = |Y(jω)|/|U(jω)| = |0.45|/|1.000| = <u>0.45</u>

+2 $\angle H(j\omega) = \frac{d*360^o}{2\pi/_\omega} = \frac{-0.11*360^o}{2\pi/_{10}} = \underline{-63.03^\circ}$

Comparing this to the results obtained earlier, we see that the results are similar. Variations are most likely do to inaccuracies when obtaining and calculating the values need to find the magnitude and phase.

ω, rad/s	***Prelab |H(jω)|***	***Part 2 |H(jω)|***	**Prelab ∠H(jω)**	**Part 2 ∠H(jω)**
0.10	*.9998*	*0.9998*	*-1.15°*	*-1.111°*
1.00	*.981*	*0.98*	*-11.31°*	*-11.45°*
10.00	*.447*	*0.45*	*-63.43°*	*-63.03°*

Table 3: Comparing Magnitude and Phase from the Prelab to the values from Part 2

Conclusion:

The purpose of this lab was to get the user familiar with using Simulink to simulate state space models. Throughout the lab we compared our experimental characteristics of a system to those it should +10 theoretically have. As long as the model is correct, theory, simulation, and experiments should match.