UNIVERSITY OF CALIFORNIA, RIVERSIDE

BOURNS COLLEGE OF ENGINEERING

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

EE 105 Lab 3 Solution

First-order systems in Simulink

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1 Introduction

This laboratory introduces Simulink through the analysis and design of linear ordinary differential equations (ODEs). You will explore key systems concepts like transfer functions, time constants, pole locations, DC gain, and frequency response. The design process involves choosing system parameters to meet specific requirements, using transfer functions for analysis and state-space representation for simulation testing. The lab starts with fundamental concepts in first-order systems and progressively generalizes them to higher-order systems.

2 Pre-Lab

2.1 Part 1

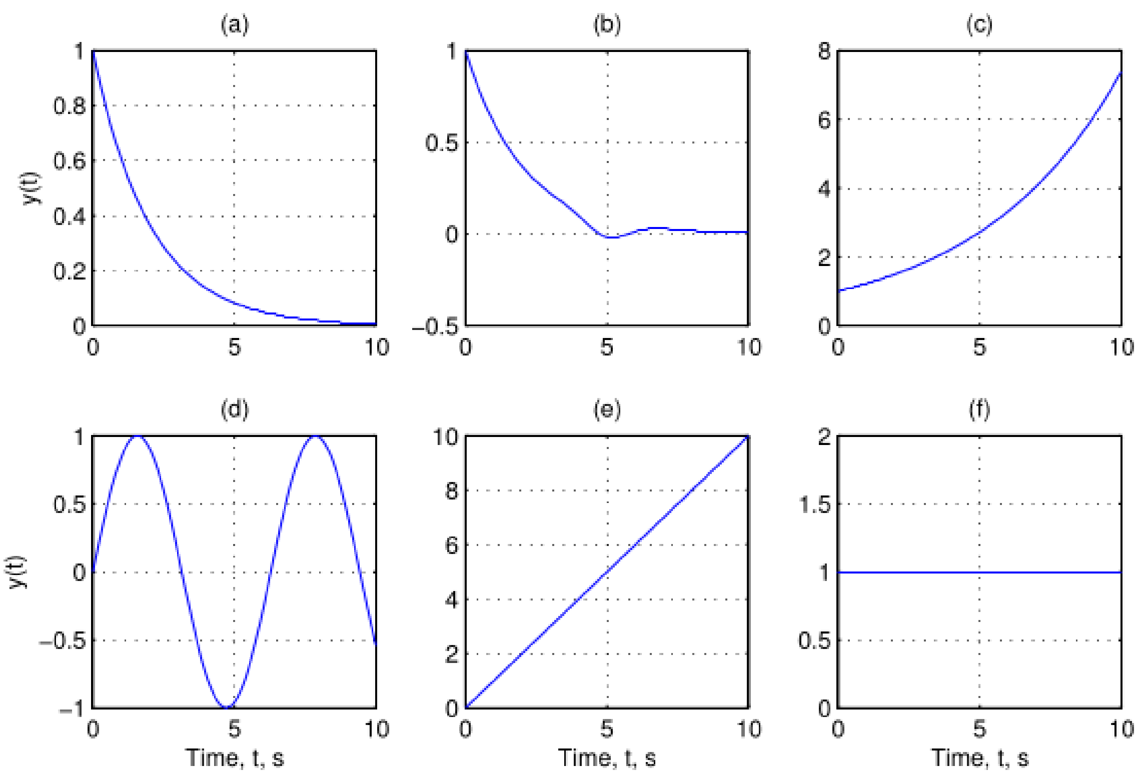

Figure 1: Figures for the Prelab Part 1

- Only graphs (a), (c), and (f) from Figure 1 could correspond to a first order linear system, as none of the other graphs were exponential.
  - (f) Parameter a has the value zero, and thus never decays.
  - (c) has a positive value for its parameter a, thus the system is unstable
  - (a) has a negative value for a, thus it is stable. We can then use the initial value of that graph, y(0) = 1, a = -.460517 therefore, τ = 2.2 seconds
- (b) Graphs (b), (d), and (e) did not match the form of a decaying function.

2.2 Part 2

The transfer function for the circuit is seen in Equation 1.

$$\begin{aligned} \mathrm{H(s) = Y(s)/U(s)} &= \frac{1}{sRC+1} \\ \mathrm{H(s) = Y(s)/U(s)} &= \frac{\frac{1}{RC}}{s+\frac{1}{RC}} \end{aligned} \tag{1}$$

$$\begin{aligned} \dot{x} &= [a]x + [b]u \\ y &= [c]x + [d]u \end{aligned} \tag{2}$$

$$\begin{aligned} \dot{x} &= \left[\frac{-1}{\mathrm{RC}}\right]x + \left[\frac{1}{\mathrm{RC}}\right]u \\ y &= [1]x + [0]u \end{aligned} \tag{3}$$

The state space for the RC circuit is shown in Equation 3. Knowing the layout of a state pace model as shown in Equation 2, a = -10, b = 10, and c = 1.

2.3 Part 3

$$M(w) = |H(j\omega)| = \frac{cb}{\sqrt{a^2+\omega^2}} \quad \text{and} \quad \Phi(\omega) = \angle H(j\omega) = \tan^{-1}\left(\frac{\omega}{a}\right) \tag{4}$$

Using equation 4 from the lab manual, the Magnitude and Phase for this RC Circuit is calculated in Equation :

$$\begin{aligned} |H(j\omega)| &= \frac{10}{\sqrt{100+\omega^2}} \\ \angle H(j\omega) &= \arctan(-\omega/10) \end{aligned} \tag{5}$$

Plugging in ω = 10 and a = -10, $|H(j\omega)| = \frac{100}{\sqrt{200}}$ = 0.707, and $\tan^{-1}\left(\frac{10}{-10}\right) = -45°$. The other ωs are calculated in Table 1.

Table 1: Magnitude and Phase for RC Circuit

ω, rad/s	$\lvert H(j\omega)\rvert$	$20\log\lvert H(j\omega)\rvert$	$\angle H(j\omega)$rad	$\angle H(j\omega)$ deg
0.00	1	0 dB	0.00	$0°$
0.01	1	0 dB	-0.001	$-0.057°$
0.10	1	0 dB	-0.01	$-0.573°$
1.00	0.995	-0.043 dB	-0.1	$-5.711°$
10.00	0.707	-3.01 dB	-0.785	$-45°$
100.00	0.1	-20.43 dB	-1.471	$-84.289°$

2.4 Part 4

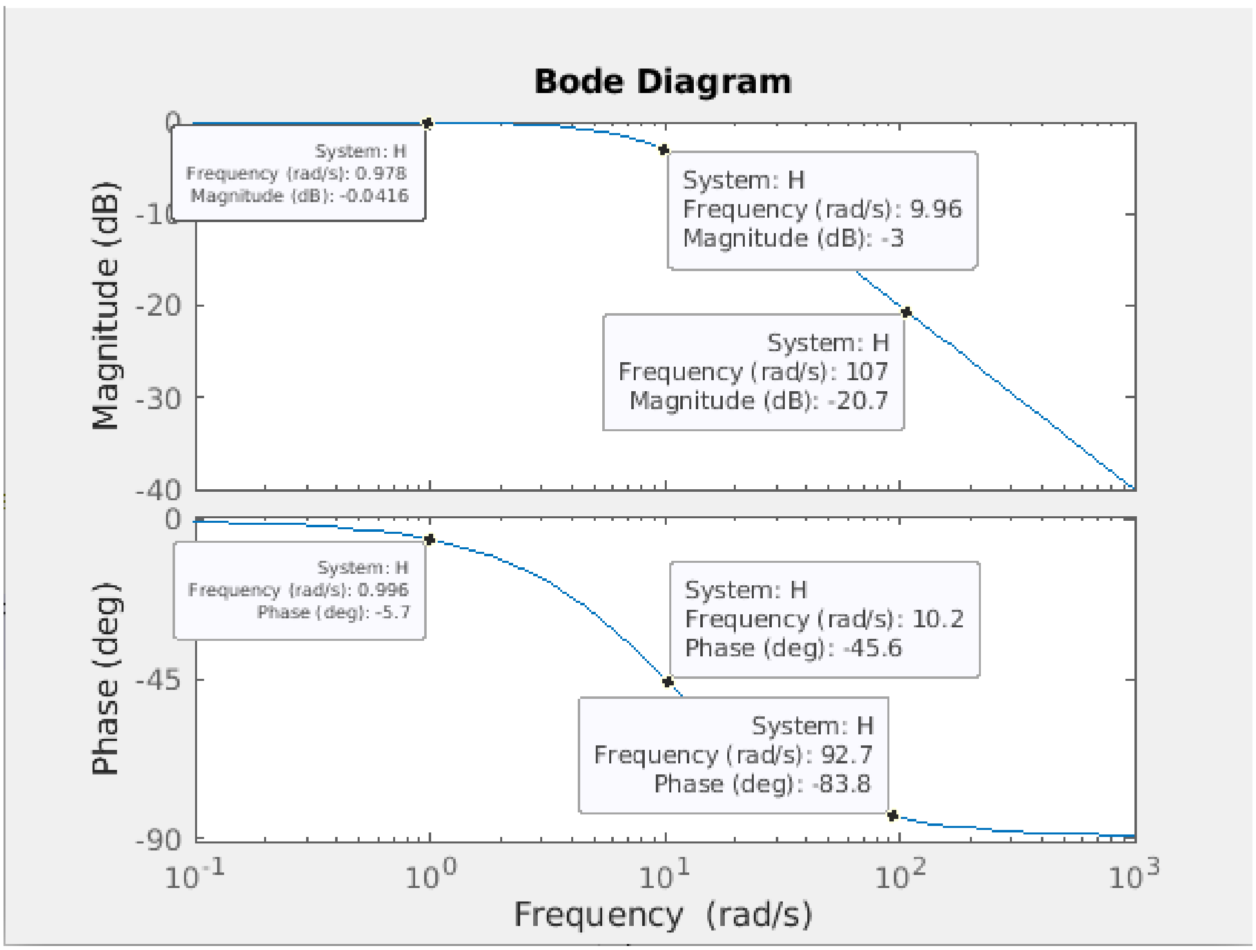

Figure 2: Bode Plot for RC Circuit

Figure 2 matches the values in Table 1.

3 Time Constant Estimation

Listing 1: Matlab Code for Time Constant Estimation

```
% Plot the .mat data
load('EE105_Lab3data.mat');

%First method
y_tau = 0.37*y(1);
index = find(abs(y_tau-y)==min(abs(y_tau-y)),1);
tau_1 = t(index);
tau_1_x_horizontal = [0, tau_1];
tau_1_y_horizontal = [y_tau, y_tau];
tau_1_x_vertical = [tau_1, tau_1];
tau_1_y_vertical = [0, y_tau];
%Second method
%Compute dy/dt
dydt = (y(2:101)-y(1:100))./(t(2:101)-t(1:100));
a = dydt./y(1:100);
%You may check a, all the same numbers
tau_2 = 1/(-a(1));
y2 = [y(1), 0];
t2 = [0, tau_2];

figure;
plot(t,y, 'blue', tau_1_x_vertical, tau_1_y_vertical, 'red', tau_1_x_horizontal, tau_1_y_horizontal, 'red');
title('First Method for Time Constant Estimation Plot', 'Interpreter', 'latex');
xlabel('$t$', 'Interpreter', 'latex');
ylabel('$y$', 'Interpreter', 'latex');
grid on;
fig = gcf; % Obtains current graphic in matlab
exportgraphics(fig, 'Fig/first_method_time_constant_plot_line.pdf', 'ContentType','vector');
plot(t,y,'blue', t2, y2, 'red');
title('Second Method for Time Constant Estimation Plot', 'Interpreter', 'latex');
xlabel('$t$', 'Interpreter', 'latex');
ylabel('$y$', 'Interpreter', 'latex');
```

```
grid on;
fig = gcf; % Obtains current graphic in matlab
exportgraphics(fig, 'Fig/second_method_time_constant_plot_line.pdf', '
    ContentType','vector');
```

τ from both methods in the code is ≈ 0.15.

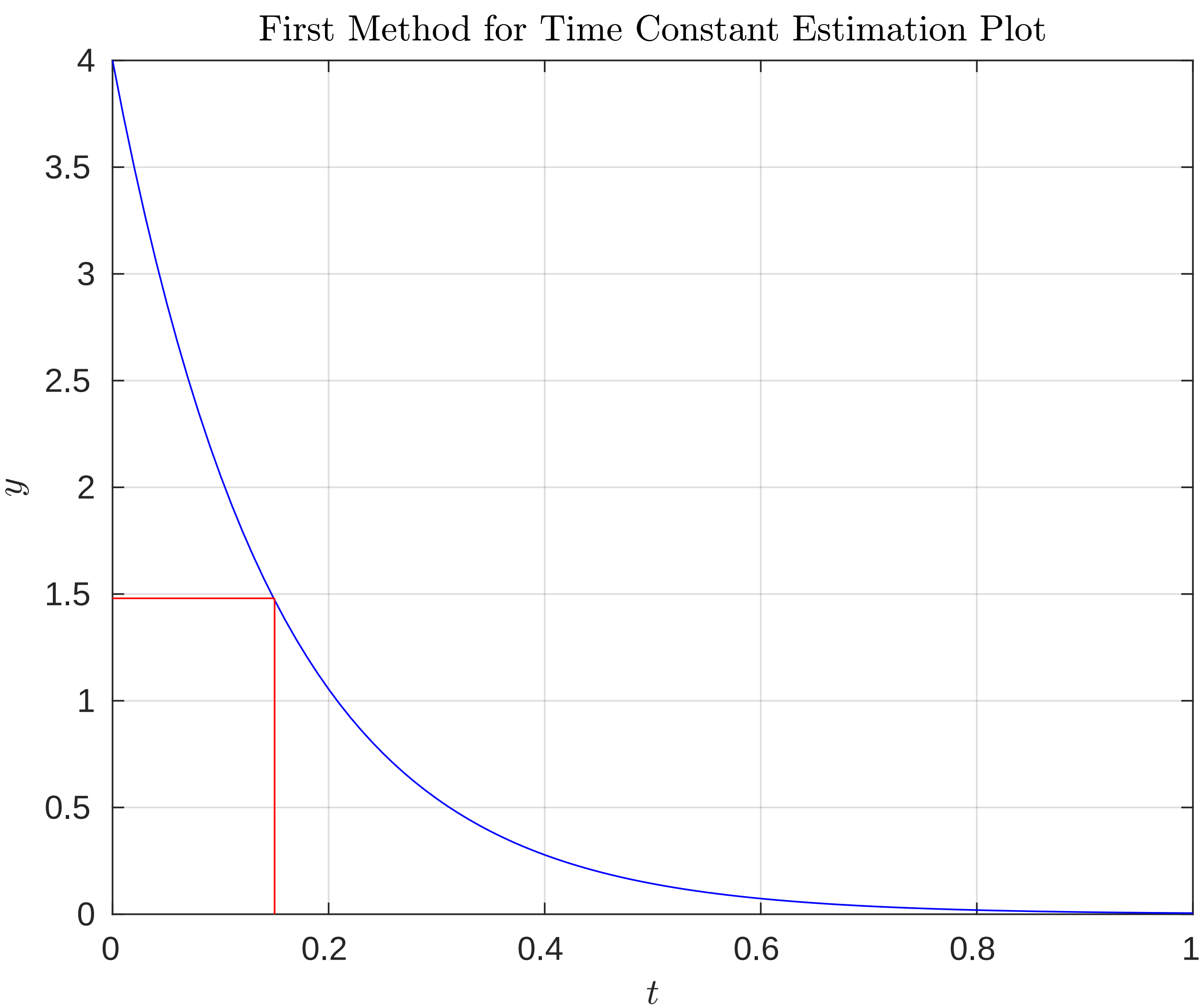

Figure 3: First Method Time Constant Estimation Plot

Figure 3 finds τ by looking for 37% of the original value. Using 0.37×4 is ≈ 1.48 to find the y value, x occurs 0.15 s.

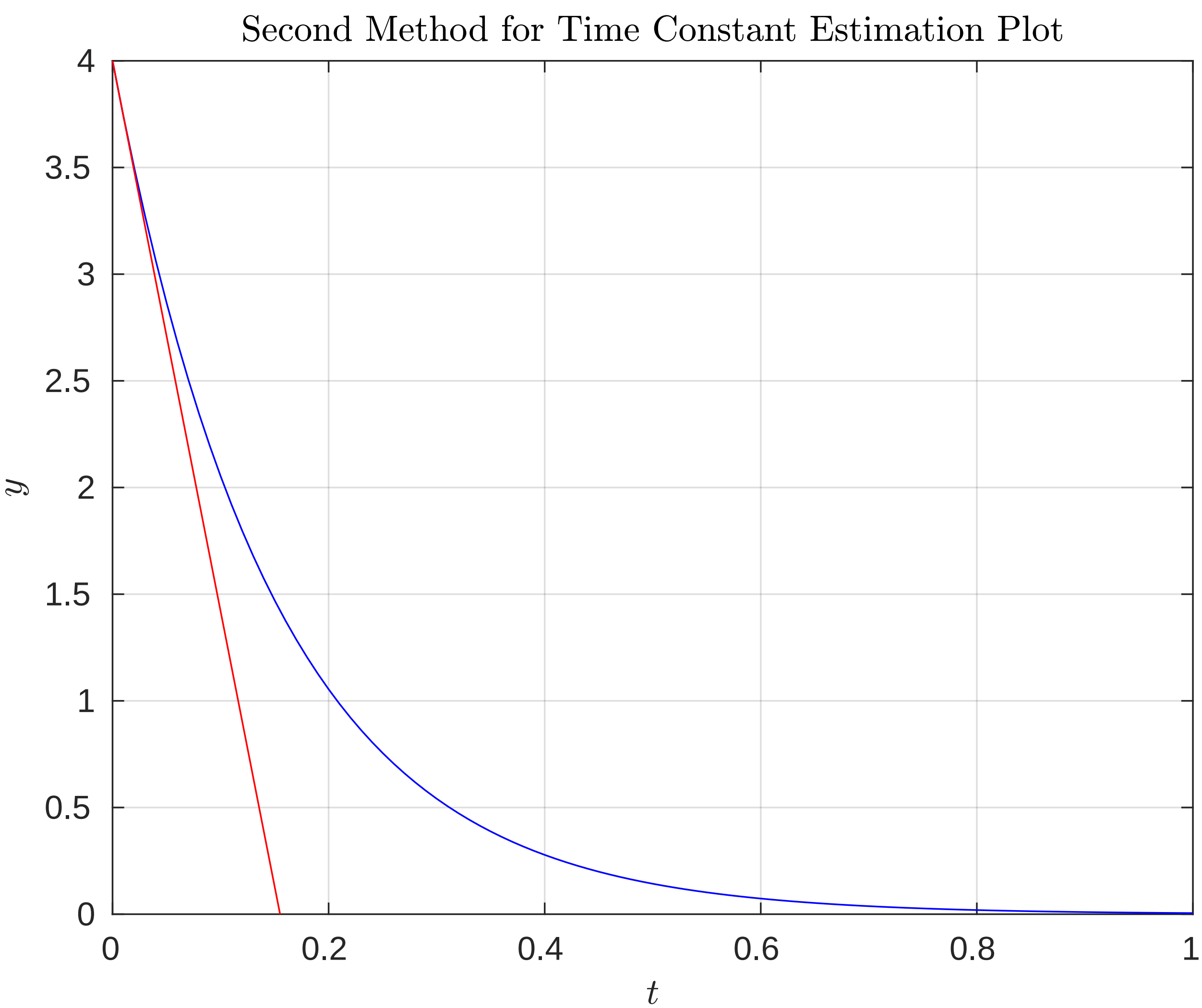

Figure 4: Second Method Time Constant Estimation Plot

Figure 4 calculates τ by finding the slope of the curve at $t = 0$. The slops crosses the x-axis at ≈ 0.15, so both methods concur that τ should be ≈ 0.15.

4 Simulation with Simulink

4.1 Zero Input Response

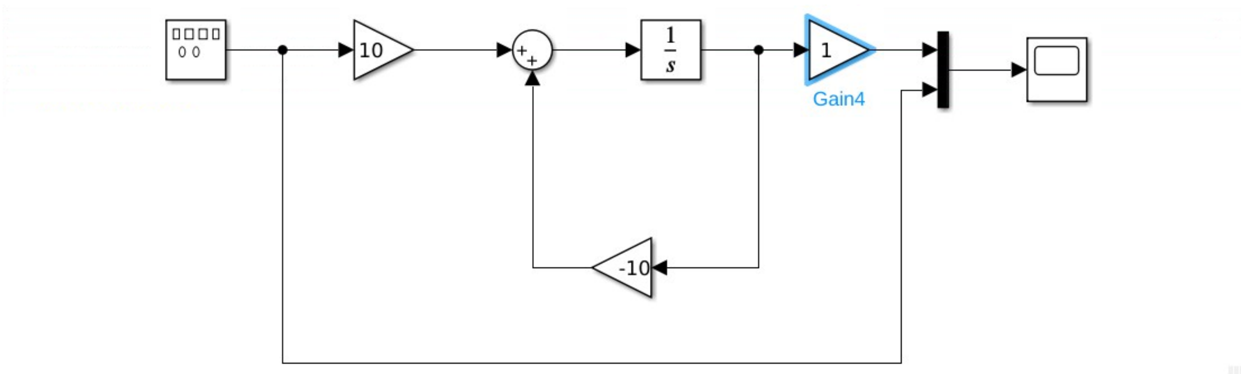

Figure 5: Simulink Diagram for Zero Input Response

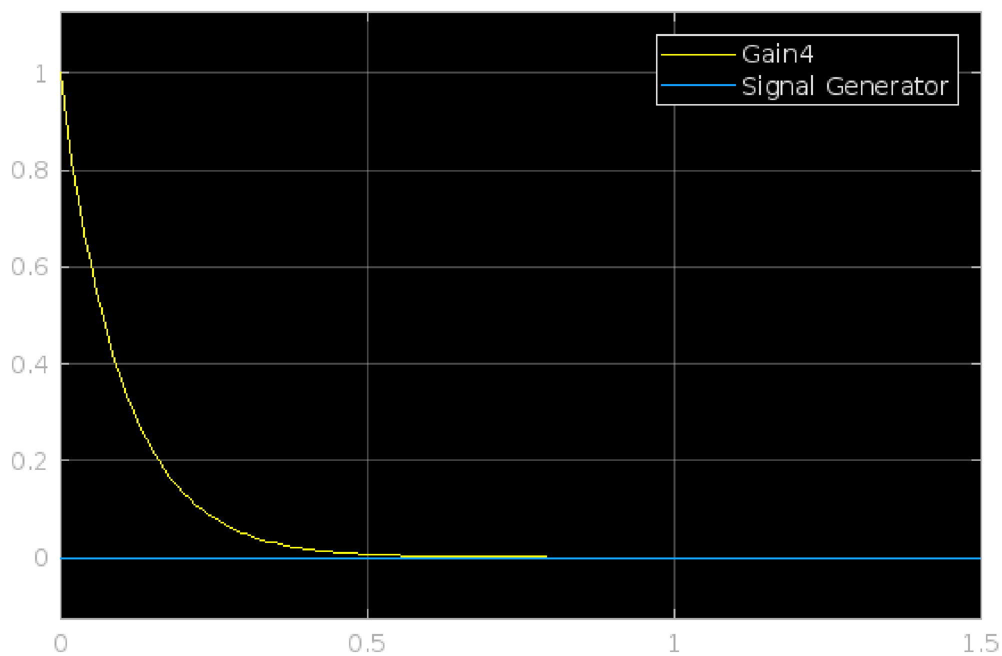

Figure 6: Oscilloscope Output of Zero Input Response

The simulation in Figure 6 confirms the settling time calculated in Figures 3 and 4 for τ is ≈ 0.15.

4.2 Forced Response: Step input

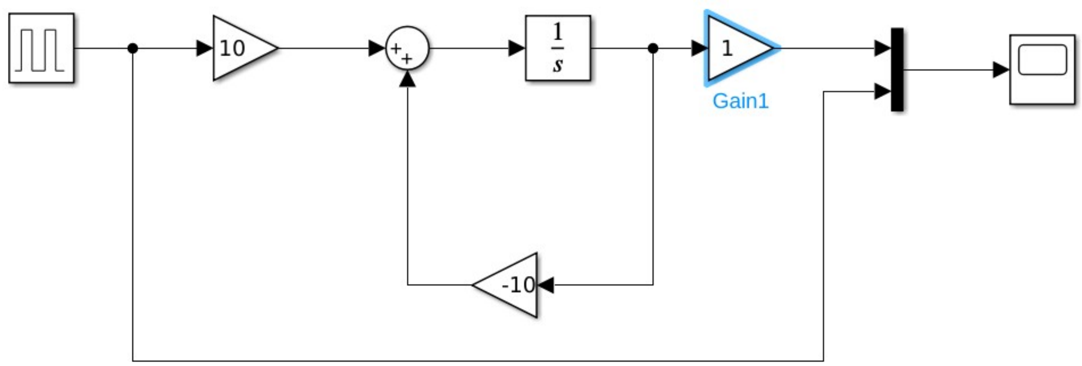

Figure 7: Simulink Diagram for Step Input Response

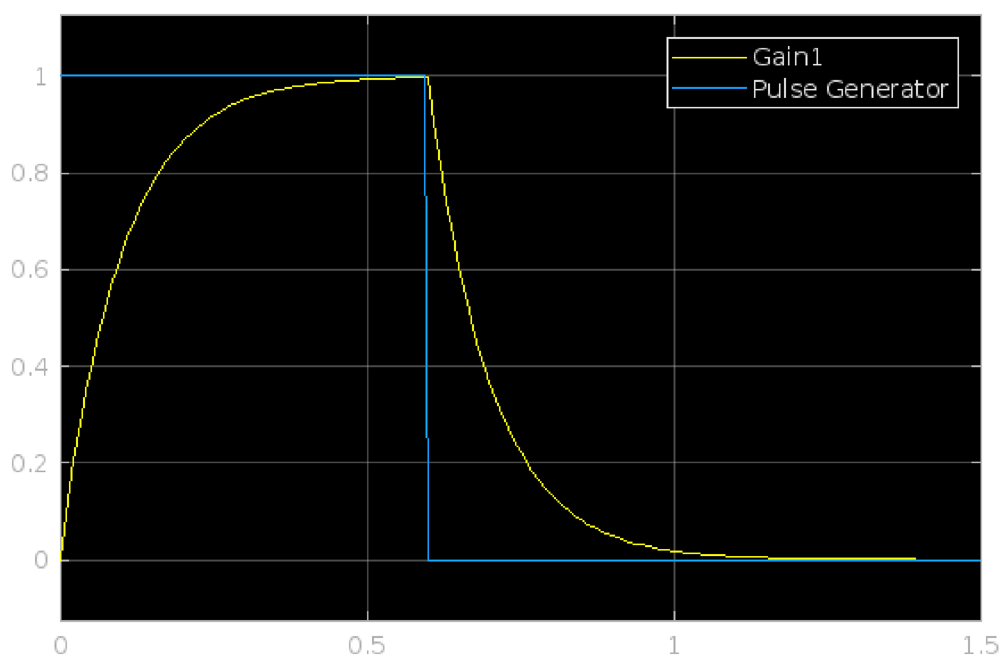

Figure 8: Oscilloscope Output of Step Input Response

After $t \approx 0.6$, the step input behaves as a zero input response. The curve settles at ≈ 1.2. Steady state is achieved in ≈ 0.6 seconds compared to τ being 0.15 seconds. Given that $T_s \approx 4\tau$, T_s aligns with what was calculated. The gain of the response with an input of 1 has a magnitude of ≈ 1. This aligns with the value calculated in the pre-lab

4.3 Forced Response: Sinusoidal Input

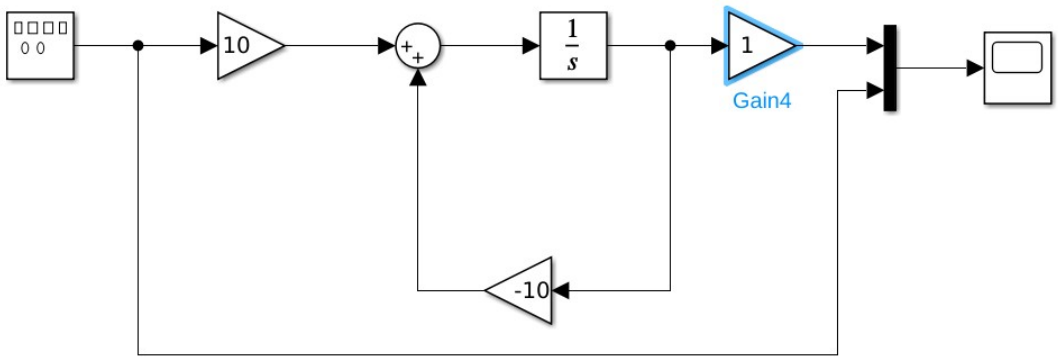

Figure 9: Simulink Diagram for Sinusoidal Input Response

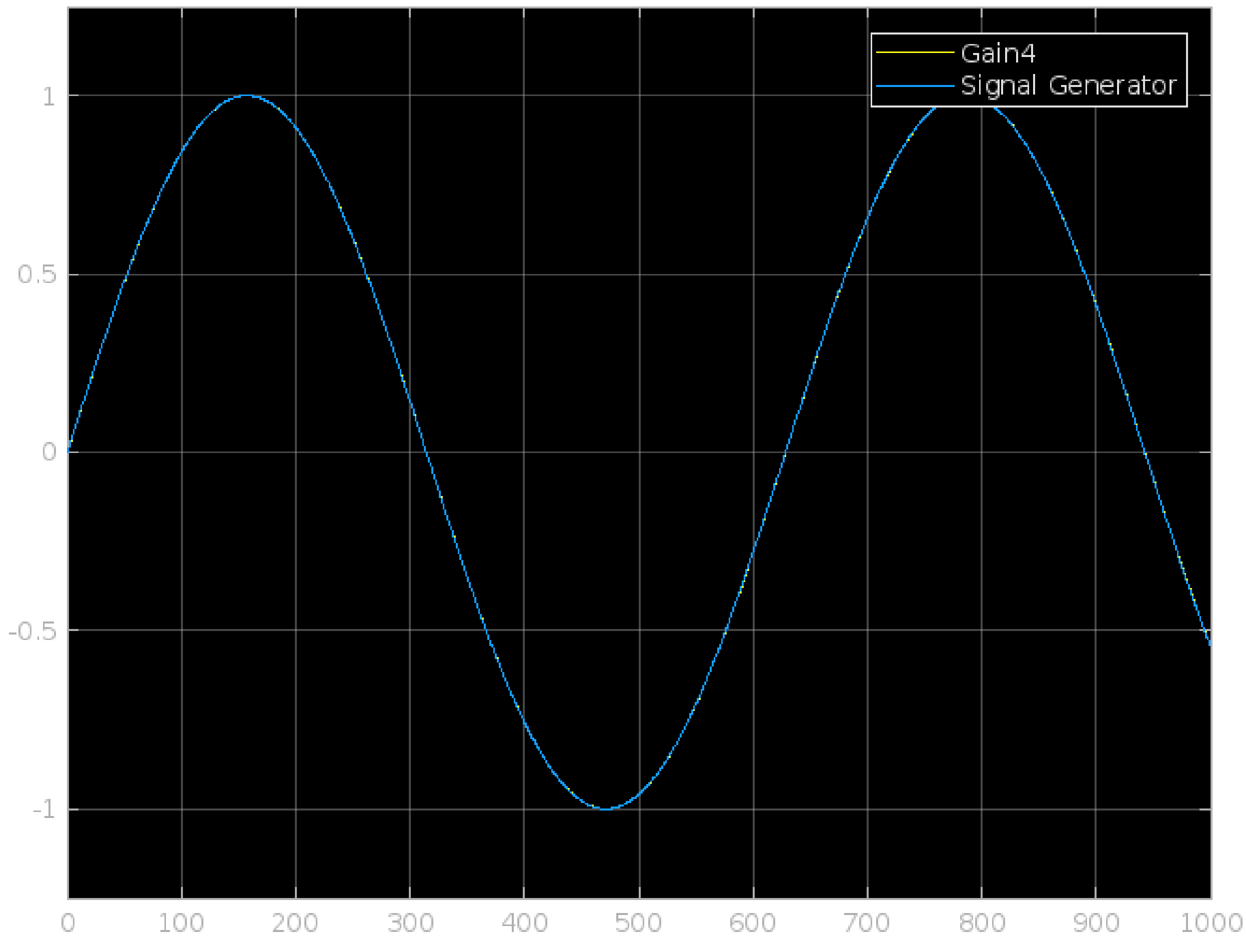

Figure 10: Oscilloscope Output of Sinusoidal Input Response $\omega = 0.01$

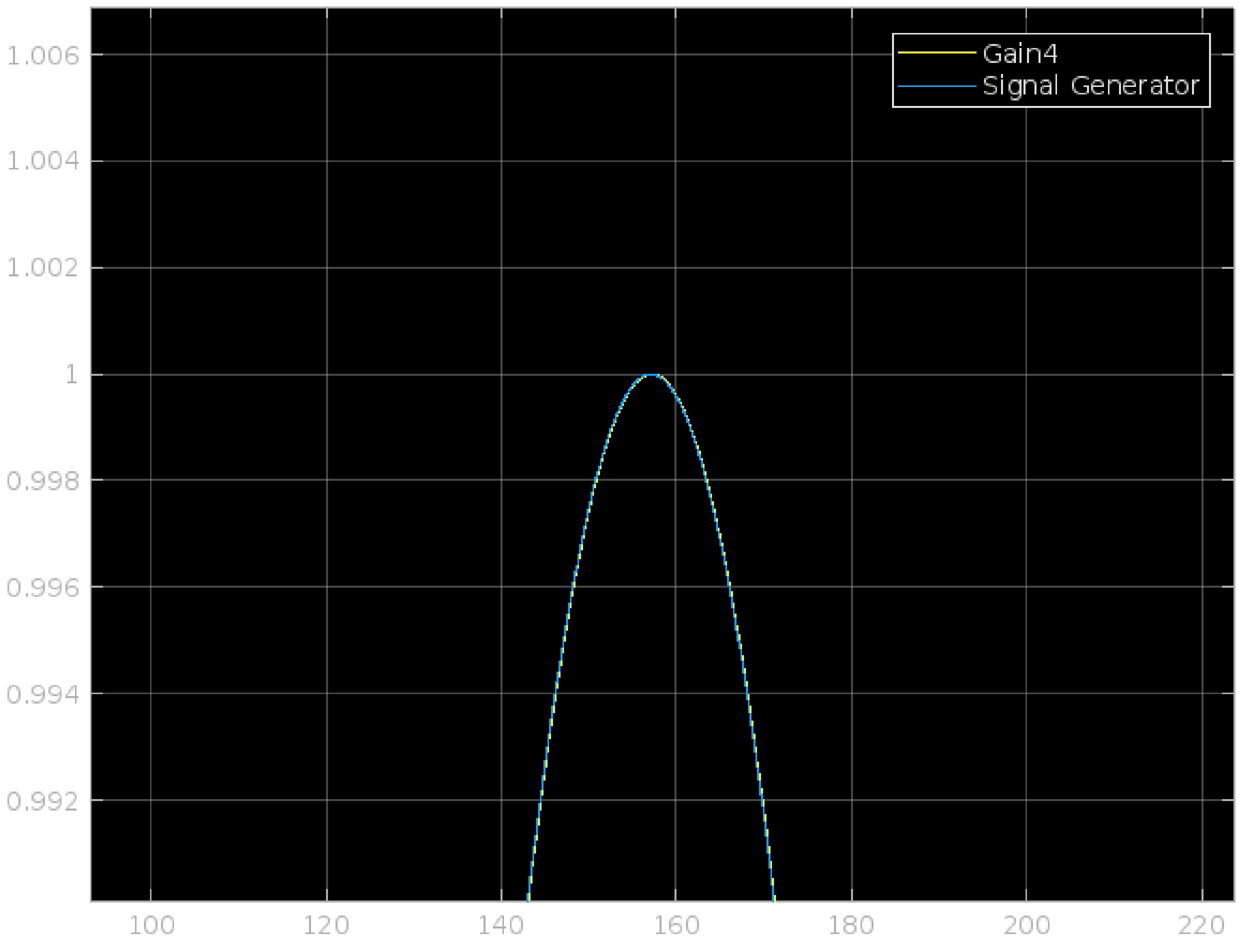

Figure 11: Oscilloscope Output of Sinusoidal Input Response $\omega = 0.01$ Overlap

The sinusoidal input and output signals depicted in Figures 10 and 11 exhibits identical magnitude and phase characteristics. This outcome aligns with the previously conducted frequency response analysis using a Bode plot. As evident in the figure, the blue and yellow lines, representing the input and output respectively, completely overlap, signifying perfect matching in both amplitude and phase characteristics.

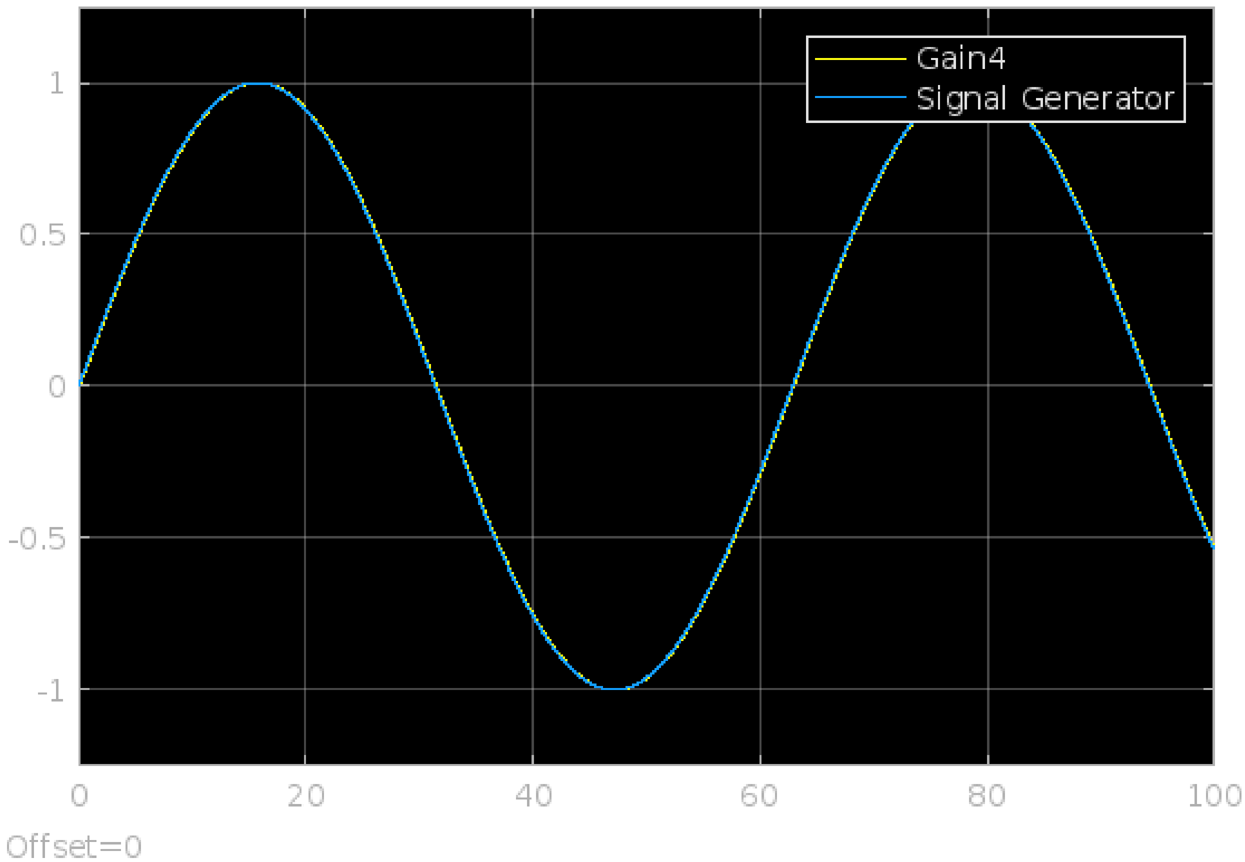

Figure 12: Oscilloscope Output of Sinusoidal Input Response $\omega = 0.10$

The sinusoidal input and output signals depicted in Figure 12 and exhibits identical magnitude and phase characteristics. This outcome aligns with the previously conducted frequency response analysis using a Bode plot. As evident in the figure, the blue and yellow lines, representing the input and output respectively, completely overlap, signifying perfect matching in both amplitude and phase characteristics.

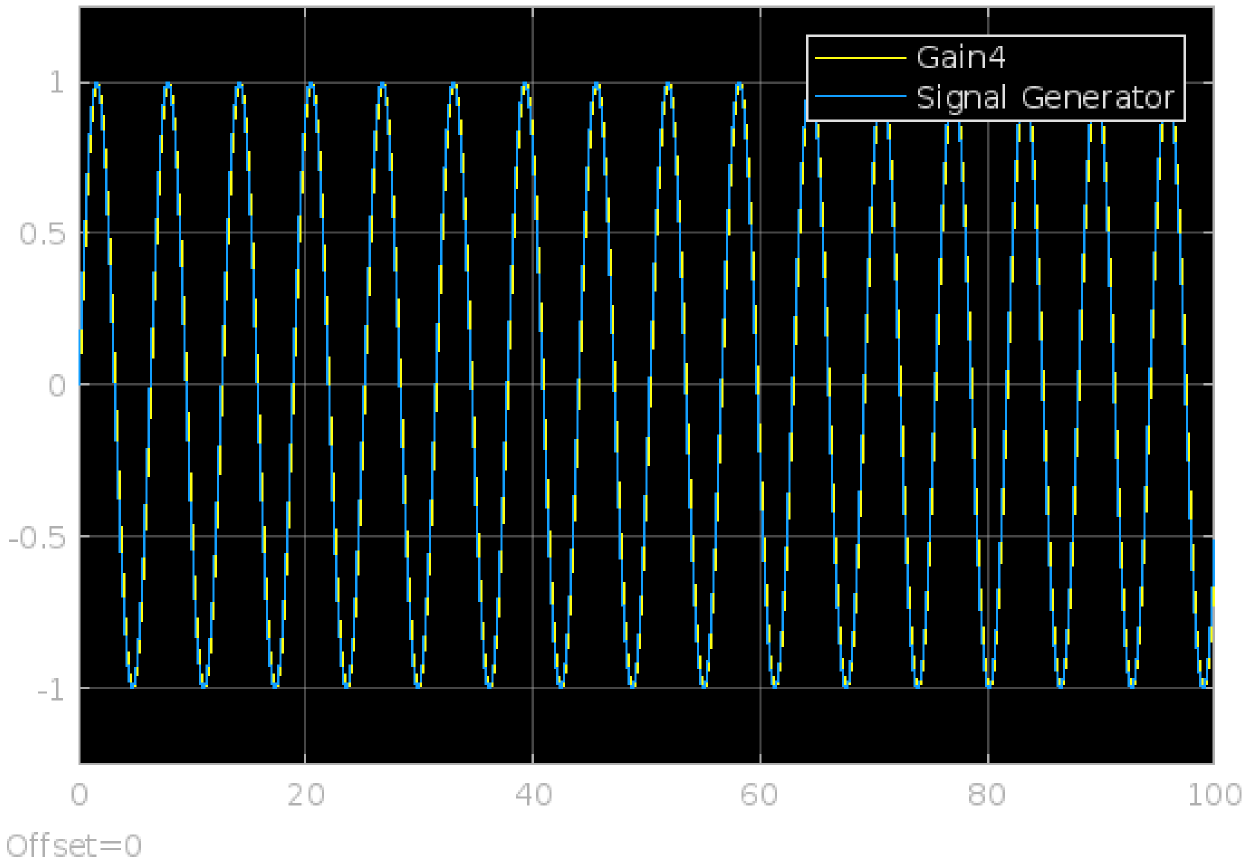

Figure 13: Oscilloscope Output of Sinusoidal Input Response $\omega = 1.00$

The sinusoidal input and output signals depicted in Figure **??** and exhibits identical magnitude and phase characteristics. This outcome aligns with the previously conducted frequency response analysis using a Bode plot. As evident in the figure, the blue and yellow lines, representing the input and output respectively, completely overlap, signifying perfect matching in both amplitude and phase characteristics.

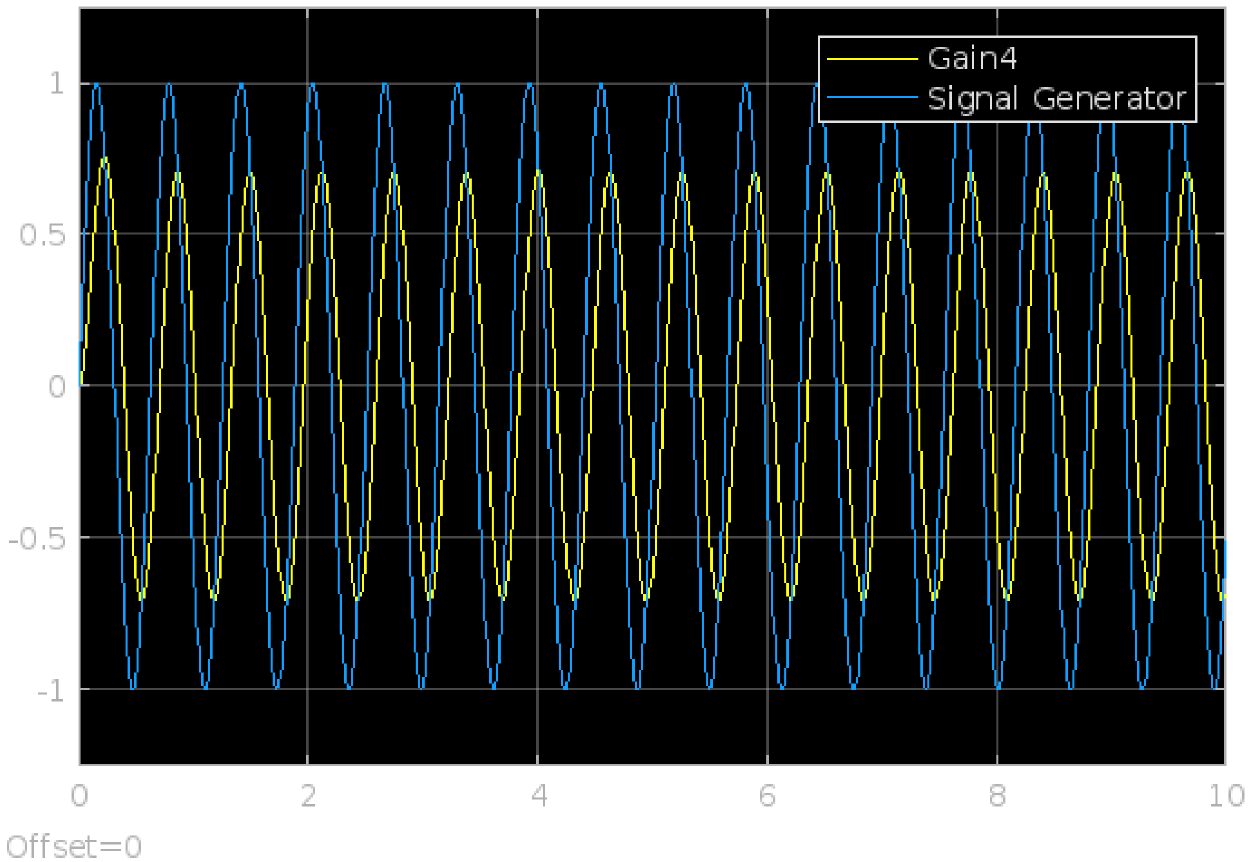

Figure 14: Oscilloscope Output of Sinusoidal Input Response $\omega = 10.00$

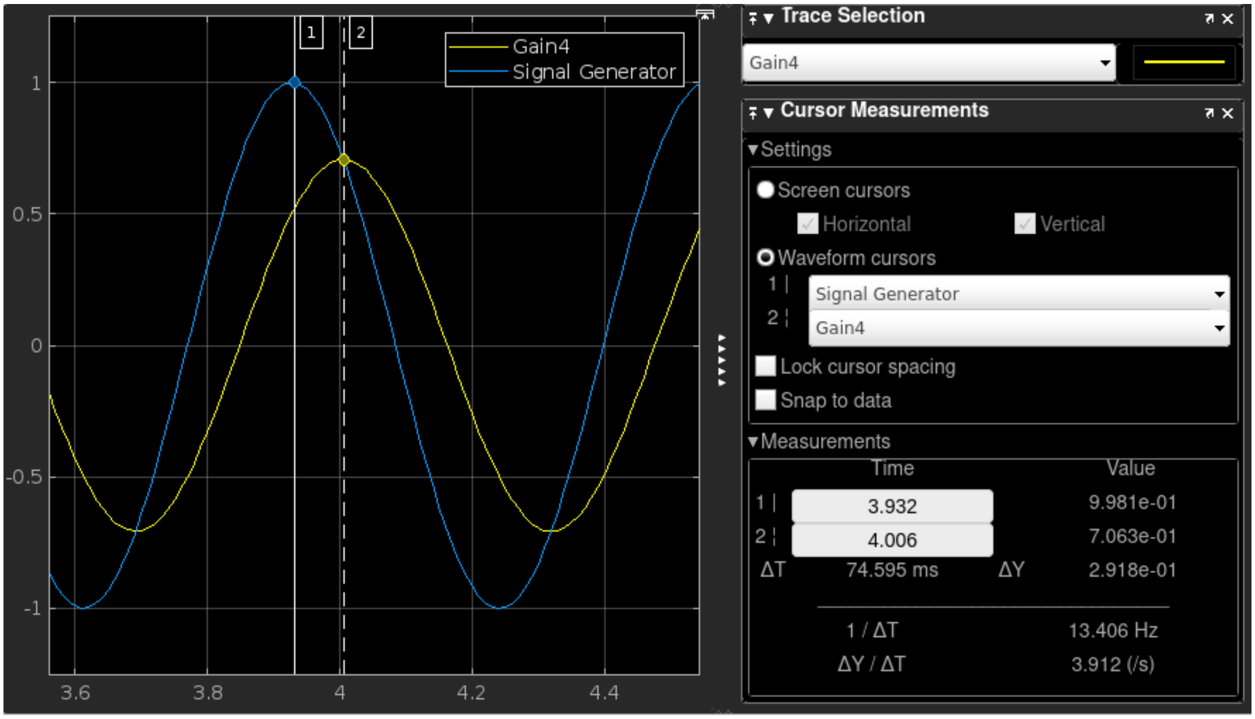

Figure 15: Oscilloscope Gain of Sinusoidal Input Response $\omega = 10.00$

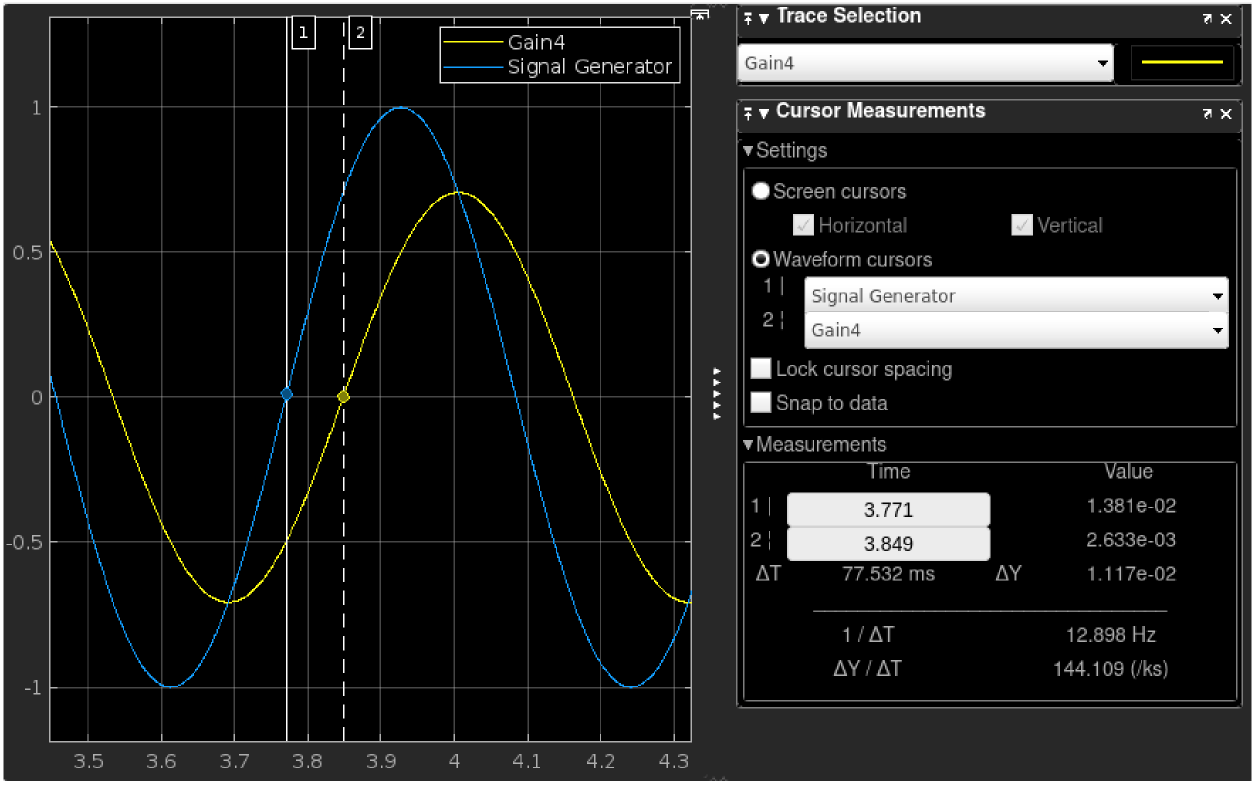

Figure 16: Oscilloscope Phase Shift of Sinusoidal Input Response $\omega = 10.00$

The magnitude difference $\frac{0.708}{0.998}$ is ≈ .0708 and the phase shift is = $\omega \times \Delta T = 10 \times$ -0.0775 s ≈ -0.775 rad.

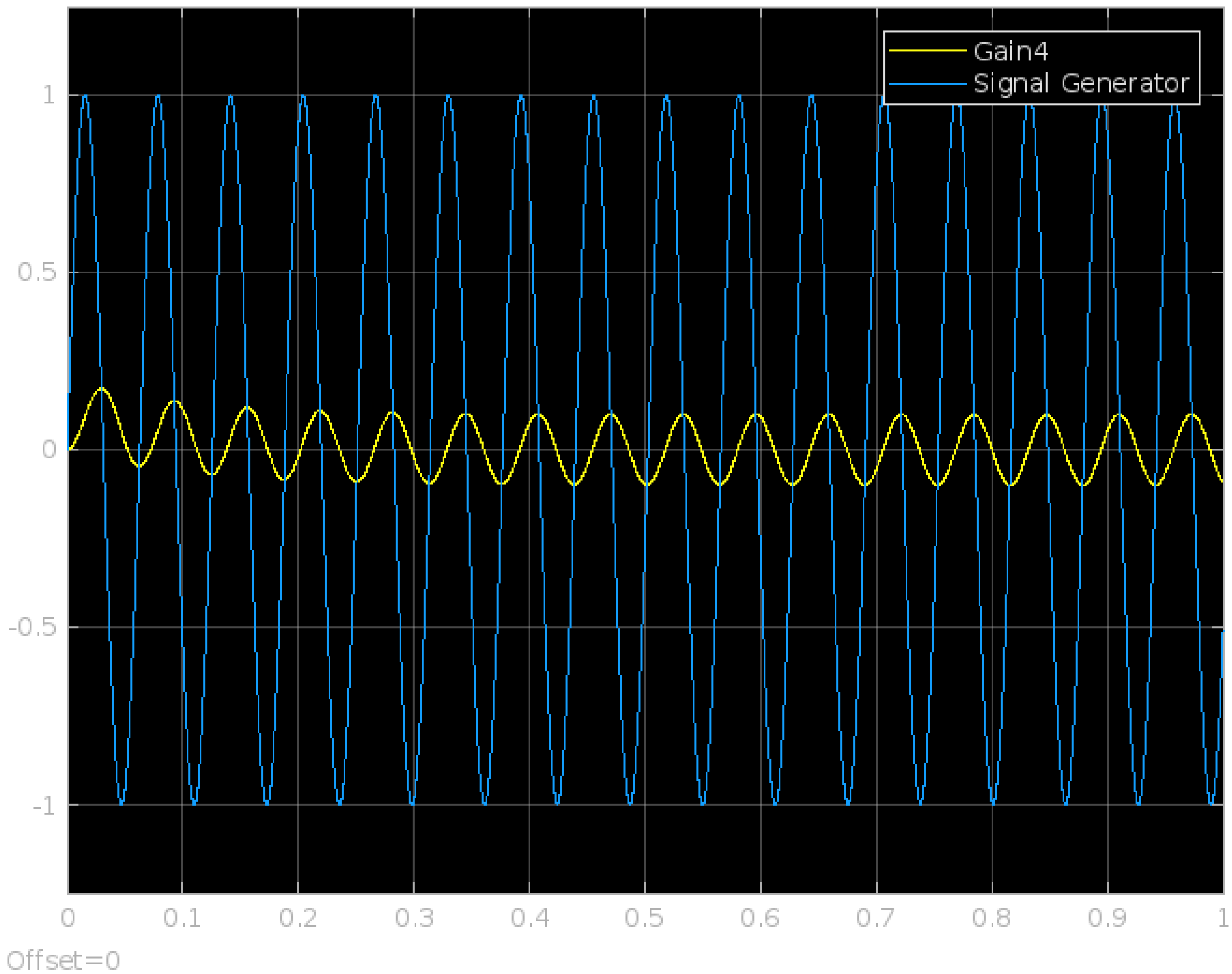

Figure 17: Oscilloscope Output of Sinusoidal Input Response $\omega = 100.00$

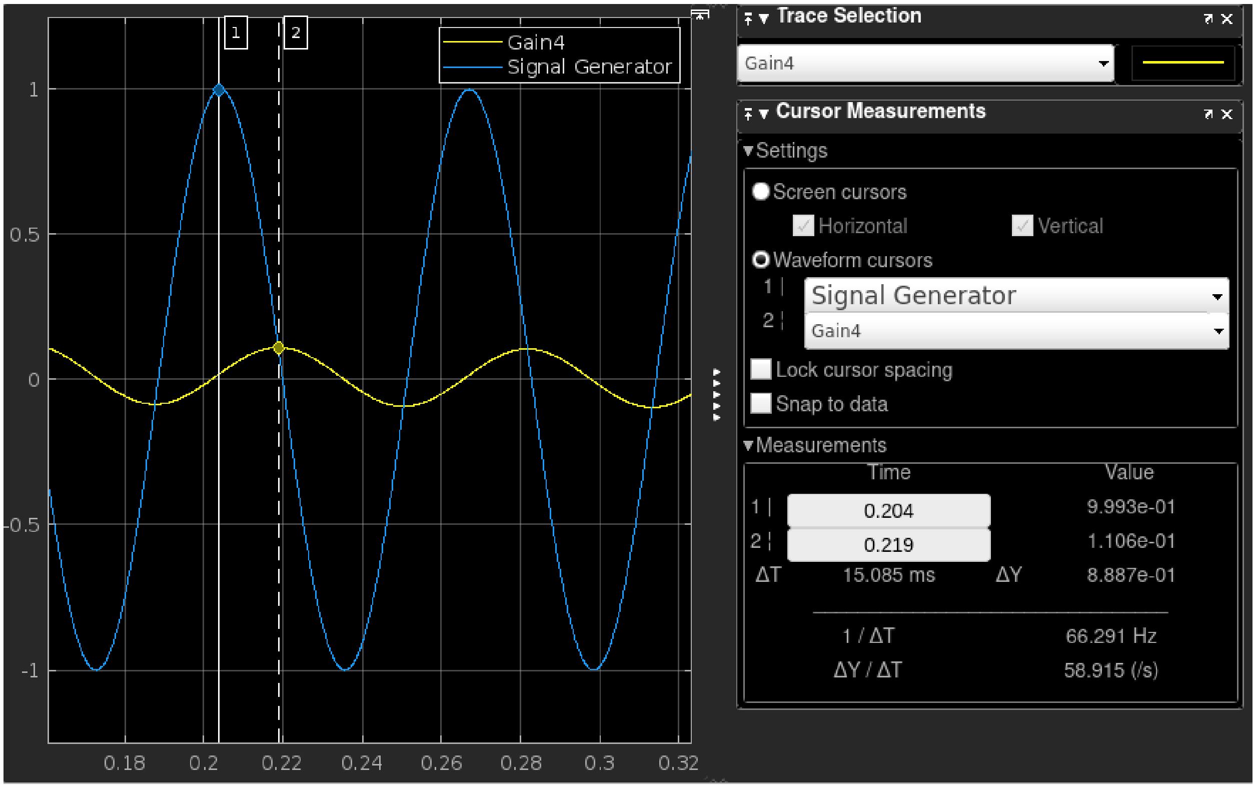

Figure 18: Oscilloscope Gain of Sinusoidal Input Response ω = 100.00

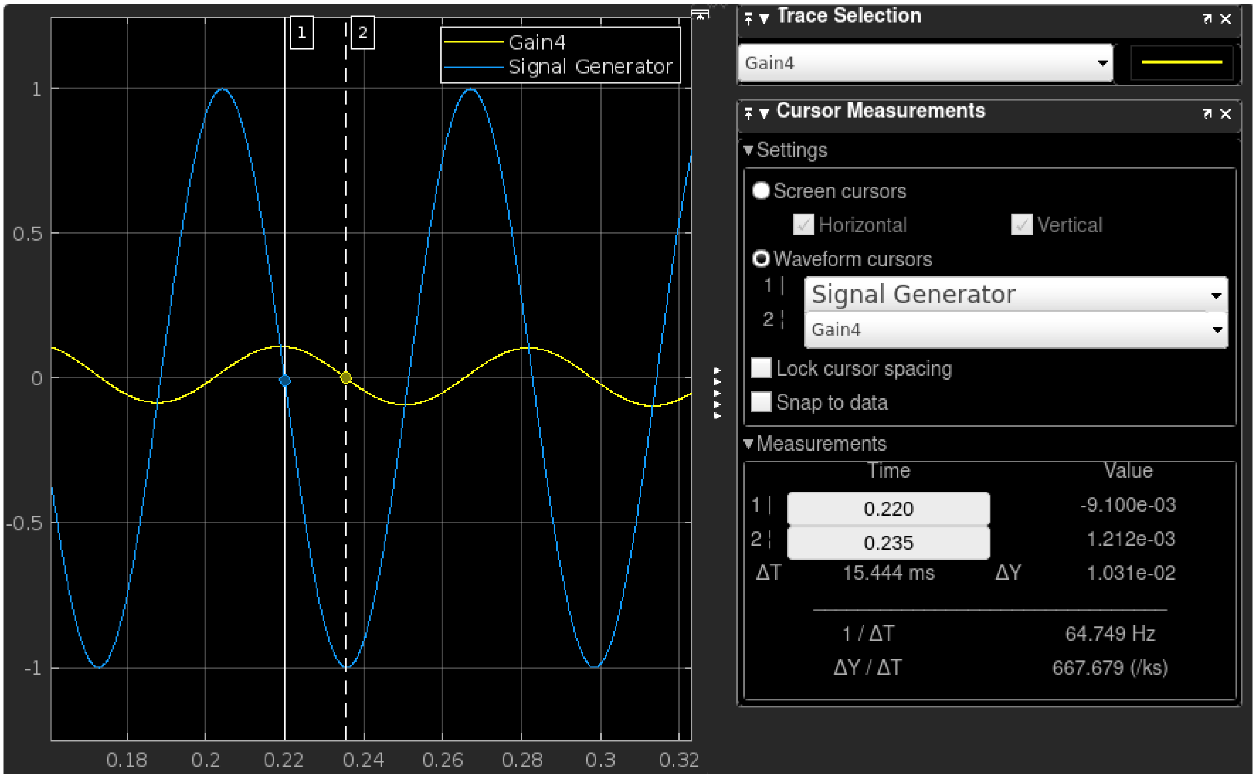

Figure 19: Oscilloscope Phase Shift of Sinusoidal Input Response $\omega = 100.00$

The magnitude difference $\frac{0.111}{0.999}$ is ≈ 0.11 and the phase shift is = $\omega \times \Delta T = 100 \times -0.0154$ s ≈ -1.54 rad.
The frequency and phase are similar to those predicted in Table 1 and Figure 2.

5 Conclusion

The comprehensive exploration and modeling of a first-order RC circuit in this laboratory exercise, employing MATLAB and Simulink for simulation, yielded insightful results. The simulation outcomes effectively corroborate the anticipated frequency response and Bode plots analysis. At nominal frequencies, the circuit demonstrated conventional decay characteristics. However, as frequencies escalated, the capacitor's influence became more pronounced, altering both magnitude and phase of input signals. Notably, the circuit exhibited low-pass behavior with negligible distortion for frequencies below $\omega \leq 1$, effectively attenuating signals beyond $\omega \geq 100$. This validation underscores the circuit's intended functionality and its applicability within specified frequency ranges.