EE 105—Pre-lab 1: Solutions
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Consider a transfer function given by

$$H(s) = \frac{16}{s^2 + 4s + 16} = \frac{G\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

1. From the above expressions, we can infer that
   - $\omega_n^2 = 16 \Longrightarrow \omega_n = 4$
   - $G = 1$
   - $\zeta = \frac{1}{2}$
   - $\omega_d = \omega_n\sqrt{1-\zeta} = 4\sqrt{\frac{1}{2}} = 2\sqrt{2}$
   - $\sigma = \zeta\omega_n = \frac{1}{2} \cdot 4 = 2$

2. $\omega = 0.1\ rad/sec$

$$H(s)\big|_{s=j\omega} = H(j\omega) = \frac{16}{(j0.1)^2 + j0.4 + 16} = 1 - \frac{j}{40}$$

$$\angle H(j\omega) = -0.025\ radians = -1.43^\circ$$

$$|H(jw)| = 1.003$$

$$y_{ss}(t) = 1.003\sin(0.1t - 0.025)$$

This means the steady-state response of the system will have virtually the same amplitude as the input signal, and it will lead just so slightly.

3.

$$\begin{aligned}
\frac{Y(s)}{U(s)} = \frac{16}{s^2+4s+16} &\Longrightarrow Y(s)(s^2+4s+16)Y(s) = 16U(s) \\
&\Longrightarrow \mathcal{L}^{-1}\left\{(s^2+4s+16)Y(s)\right\} = \mathcal{L}^{-1}\left\{16U(s)\right\} \\
&\Longrightarrow \ddot{y}(t) + 4\dot{y}(t) + 16y(t) = 16u(t) \\
&\Longrightarrow \ddot{y}(t) = 16u(t) - 4\dot{y}(t) - 16y(t)
\end{aligned}$$

By defining a state vector as follows, we obtain:

$$\mathrm{x} = \begin{bmatrix} y \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \end{bmatrix} \qquad \dot{\mathrm{x}} = \begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} \mathrm{x}_2 \\ 16u - 4\mathrm{x}_2 - 16\mathrm{x}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -16 & -4 \end{bmatrix}\mathrm{x} + \begin{bmatrix} 0 \\ 16 \end{bmatrix} u$$

Correspondingly, $y = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathrm{x} + 0 \cdot u$.
Therefore

$$A = \begin{bmatrix} 0 & 1 \\ -16 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 16 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad D = 0$$