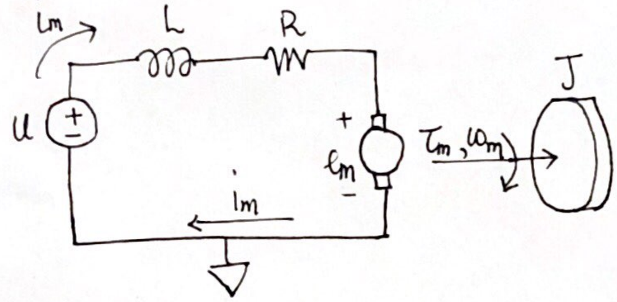
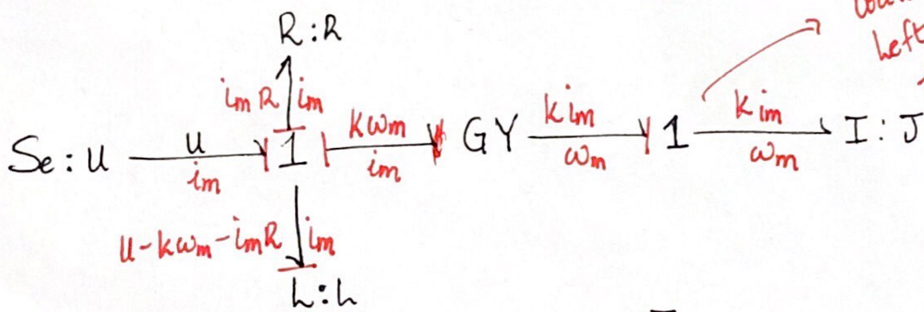
(1) $P_{in} = e_m \cdot i_m = k\omega_m i_m \quad , \quad P_{out} = \tau_m \cdot \omega_m = k i_m \omega_m$

(3)




Could be simplified. Left it like this for the possibility of the load on J.




$$x = \begin{bmatrix} i_m \\ \omega_m \end{bmatrix} \Rightarrow \dot{x} = \begin{bmatrix} \frac{1}{L}(u - k\omega_m - i_m R) \\ \frac{1}{J}(k i_m) \end{bmatrix}$$

$$\Rightarrow \dot{x} = \begin{bmatrix} -\frac{R}{L} & -\frac{k}{L} \\ \frac{k}{J} & 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u$$

$$y = \omega_m \Rightarrow y = \begin{bmatrix} 0 & 1 \end{bmatrix} x + 0 \cdot u$$

(4) $$\frac{\omega(s)}{U(s)} = \frac{k/JL}{s^2 + \frac{R}{L}s + \frac{k^2}{JL}} = \frac{A\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\Rightarrow \omega_n^2 = \frac{k^2}{JL} \Rightarrow$ *undamped natural frequency* $\Rightarrow \omega_n = \frac{k}{\sqrt{JL}}$

$A = 1/k \Rightarrow$ *DC gain*

$\zeta = \frac{R\sqrt{JL}}{2kL} \Rightarrow$ *Damping factor.*

$\sigma = \zeta\omega_n = \frac{R \cdot \sqrt{JL}}{2\cancel{k}L} \cdot \frac{\cancel{k}}{\cancel{\sqrt{JL}}} = \frac{1}{2}\frac{R}{L}. \Rightarrow$ *decay rate.*

$\omega_d = \omega_n\sqrt{1-\zeta^2} = \frac{k}{\sqrt{JL}}\sqrt{1 - \frac{R^2 \cdot JL}{4k^2L^2}} = \frac{k}{\sqrt{JL}}\sqrt{1 - \left(\frac{R}{2k}\right)^2 \frac{J}{L}} \Rightarrow$ *Damped natural frequency.*

(5) $$\omega_n = \frac{k}{\sqrt{JL}} = 1.0$$

$$\Rightarrow \sqrt{JL} = k \quad \Rightarrow JL = k^2$$

$$\Rightarrow J = \frac{k^2}{L} = \frac{(0.01)^2}{0.1} = 1\times10^{-3}\ kg\cdot m^2/s^2$$

$$\Rightarrow \text{Decay rate, } \sigma = \frac{1}{2}\cdot\frac{R}{L} = 0.5/\text{sec}$$

$$\Rightarrow \text{Time constant, } \tau = \frac{1}{\sigma} = \frac{1}{0.5} = 2 \text{ seconds}$$

(6) DC gain : $\cancel{G(s)|_{s=0}}$ $$G(s)\big|_{s=0} = \frac{k/\cancel{JL}}{k^2/\cancel{JL}} = \frac{1}{k} = 100$$

<u>Note</u> : The above value is for pulse width of 100%.
The value decreases with pulse width.

(7) $$\left|G(j\,2\pi\times100)\right| = 2.53\times10^{-4}$$

(8) Yes, if modeled correctly.