410-57, Assignment #7 Anamitra Bhattacharyya

INTRODUCTION

This assignment specifically deals with factor analysis to identify and segment sectors in the stock market data. The raw data set being used in this assignment comprises the daily returns from 20 individual stocks from a variety of market sectors. The overall goal of the assignment is to use statistical methods in SAS such as principal factor analysis, maximum likelihood expectation, with and without rotation to perform a segmentation analysis to derive the number of common factors to build a valid and appropriate model.

RESULTS

1. Data Prep

In this step of we are keeping 4 sectors and these specific stocks associated with them and dropping others:

- a) Banking e.g. BAC, JPM, WFC
- b) Oil Field Services e.g. BHI, HAL, SLB
- c) Oil Refining e.g. CVX, HES, XOM
- d) Industrial e.g. DD, DOW, HUN

The data is sorted in ascending order for the 12 stocks in the 4 sectors; the returns of the 12 stocks that are kept is shown in the output table below:

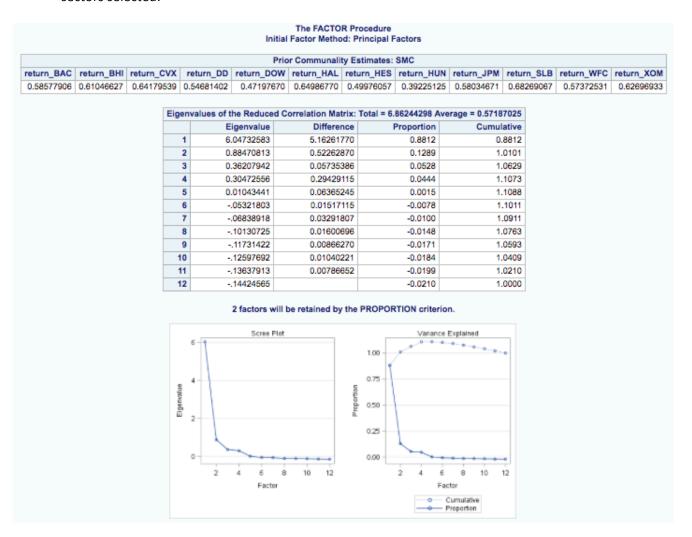


Conclusion: Created a sorted list of 12 stocks from the 4 sectors noted above (in date ascending order). The daily stock prices and calculated a log of the ratio of today's price against yesterday's price.

2. Principal Factor Analysis without rotation

This section of the assignment uses the data set that was created in (1) above; a principal factor analysis (PFA) is performed without any factor rotation. The output is shown below. From the reduced correlation matrix we can see that of the 12 variables chosen there should be 12 factors that fall out from this, but SAS retains 2 primary factors. The criterion that SAS uses to determine the number of factors to retain is the PROPORTION method, such that a sufficient number of factors is retained so that the cumulative proportion of the variance explained is >1. To this end, factor 1 explains >80% of the variance and with the factor 2, this pushes the

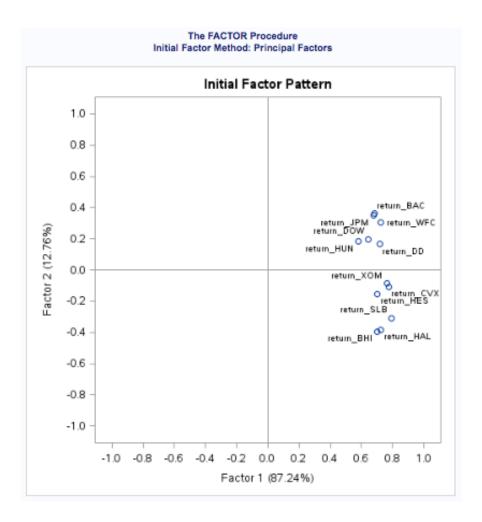
proportion of variance to over 1.0. Thus, SAS selects factors 1 and 2. The scree plot shows graphically that there is an elbow at factor=2 and the cumulative variance reveals that the first two factors push the variance >1 satisfying the default criteria for SAS for factor selection. Interestingly, We do not necessarily get the 4 factors selected coincident with the 4 original sectors selected.



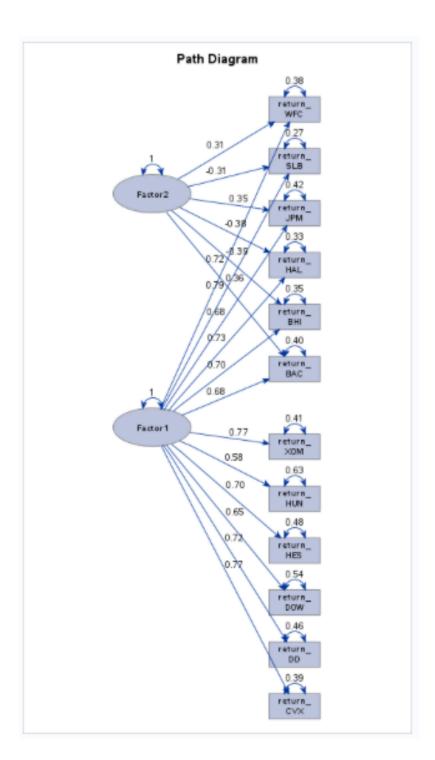
						r Pattern						
						Factor1	Facto	2				
				re	eturn_BAC	0.68475	0.3602	21				
				re	eturn_BHI (0.69984	-0.3949	8				
				re	eturn_CVX	0.77402	-0.1083	33				
				re	eturn_DD (0.71605	0.1670	13				
				re	eturn_DOW	0.64548	0.1980	11				
				re	eturn_HAL (0.72630	-0.3822	21				
				r	eturn HES	0.70361	-0.1570	9				
					_	0.58030	0.1818	86				
				n	eturn_JPM (0.67874	0.3481	3				
					_	0.79382	-0.3081					
				_	_	0.72445	0.3051					
					_	0.76500	-0.0836					
				V-	riance Explair	and but E	aab Eas	4				
				Va	Factor1		Fact					
					6.0473258	,	0.8847	081				
					mmunality Es							
_		return_CVX			return_HAL	_	_		_		return_WFC	_
0.59863104	0.64577915	0.61083713	0.54062043	0.45584934	0.67359085	0.5197	4549	0.36982204	0.58188382	0.72509913	0.61795857	0.592216

The factor pattern table shows that for the factor loading values for factor 1 are all large and significant compared to factor 2, where only some are positive. Thus, factor 1 has the most significant contribution to the variance *versus* factor 2.

To understand the factor loadings further the loadings from factor 1 are plotted against factor 2, the results from this output is shown below. Looking at the output plot, the stocks are approximately in a line with the banking and industrial stocks in the positive half of the plot and oil refining and oil services in the negative half of the plot. This is consistent with the factor 1 have high loadings and factor 2 having a combination of positive and negative values, see in the loading tables above.



The segregation of the sectors and their respective stocks for factors 1 and 2 is further illustrated in the path diagram shown below.



Conclusion: In this step SAS retained 2 out of 12 factors on the basis of proportion of variance explained by each being cumulatively >1. Plotting factors 1 versus 2 segregated the stock sectors such that banking and industrials were in the positive half of the plot, while oil related stocks were in the negative half of the plot.

3. Application of Varimax rotation to PFA

A comparison of the results with and without Varimax rotation is shown below.

Rotation of factors (Varimax)

No rotation of factors (Step 2)

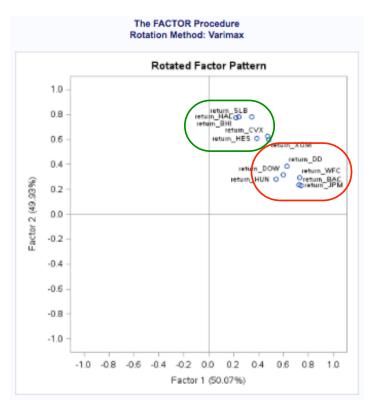
Orthogonal Transformation Matrix					
	1	2			
1	0.70781	0.70640			
2	0.70640	-0.70781			

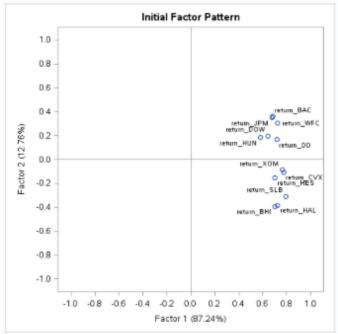
Rotated Factor Pattern					
	Factor1	Factor2			
return_BAC	0.73912	0.22875			
return_BHI	0.21634	0.77394			
return_CVX	0.47133	0.62344			
return_DD	0.62482	0.38759			
return_DOW	0.59675	0.31582			
return_HAL	0.24408	0.78359			
return_HES	0.38705	0.60822			
return_HUN	0.53921	0.28120			
return_JPM	0.72634	0.23305			
return_SLB	0.34419	0.77886			
return_WFC	0.72835	0.29575			
return_XOM	0.48241	0.59958			

Fact	or Pattern	1
	Factor1	Factor2
return_BAC	0.68475	0.36021
return_BHI	0.69984	-0.39498
return_CVX	0.77402	-0.10833
return_DD	0.71605	0.16703
return_DOW	0.64548	0.19801
return_HAL	0.72630	-0.38221
return_HES	0.70361	-0.15709
return_HUN	0.58030	0.18186
return_JPM	0.67874	0.34813
return_SLB	0.79382	-0.30815
return_WFC	0.72445	0.30517
return_XOM	0.76500	-0.08361

Rotation of factors (Varimax)

No rotation of factors (Step 2)





Did SAS retain the same number of factors?

Although SAS did not change the number of factors retained, which remains the same at 2, the loading values did change with the Varimax rotation, see results table above.

What components of the PROC FACTOR output did the rotation change? Did we obtain a 'simple structure' from our factor rotation? Did we increase the interpretability using the factor rotation?

With the absence of rotation the values for factor 1 were much higher than factor 2, and the factor 2 loadings were sometimes negative. In the presence of rotation, see table above left, the loadings were all positive for both factors. In addition, as we analyze the stocks between factors 1 and 2, sometimes the loadings for some stocks are higher for factor 1 while other stocks are higher with factor 2. For example, for BAC the loading value for factor 1 > factor 2; however, for BHI the loading for factor 2 > factor 1. Either way the loadings per stock are being maximized between the two factors. Plotting and comparing the loadings between factor 1 and 2, see plots above, one can see that with the rotation a simple structure is obtained which increases the interpretability of factors 1 and 2. Specifically, looking at the rotated factor 1 versus factor 2 plot (colored rings) the banking and industrial stocks are clearly and discretely segmented (clustered, red ring) from the oil industry sector stocks (green).

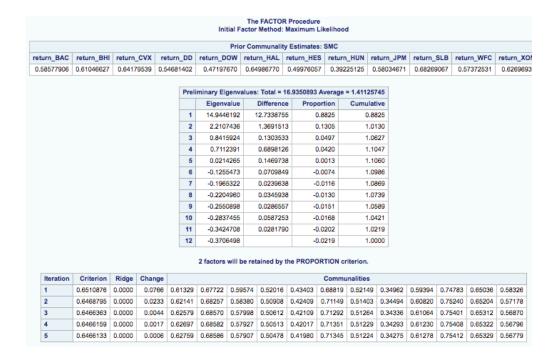
Conclusion: The Varimax rotation resulted in a simpler structure being generated, which facilitated a better interpretation of the stock data when comparing the results without factor rotation. With the rotation, discrete clusters were apparent on the plots of factor 1 versus factor 2, resulting in a segmentation of stocks falling in to these sectors (compare green and red rings above).

4. Using Maximum Likelihood Estimation to estimate common factors

In this section of the assignment a principal factor analysis is used with squared multiple correlations for the prior communality estimates (*i.e.* PRIORS=SMC), premised on a common factor model.

How many common factors does ML Factor Analysis suggest? How does ML Factor Analysis arrive at this number of factors, and in general how do we interpret the output from a ML Factor Analysis?

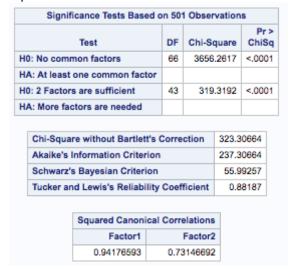
Initially SAS uses and presents the results from the PROC PROPORTIONS method, producing the following correlation matrix (see below).



Since a different method is being used (MLE) the eigenvalues are different compared to previously. Again using this initial output the first 2 factors explain the largest proportion of the variance, which leads to the initial recommendation from PROPORTION that 2 factors will be retained.

From a modeling perspective what does ML Factor Analysis provide that Principal Factor Analysis does not?

The ML factor analysis provides hypothesis testing (e.g. Chi-squared) that was not available in PFA, and one also makes assumptions about the normality of the distribution. Specifically, the hypothesis testing results shown below indicate that the p-value for null hypothesis (no common factors) is low so we reject it, suggesting common factors are required. In addition, the p-value for another hypothesis test – null hypothesis states 2 factors are required – is also low, implying > 2 factors are required.



The MLE method does perform some additional weighting on the variances for factors 1 and 2, the loadings for factors 1 and 2 are shown below (as well as the weighted/unweighted variances). Before performing any rotations, factor 1 appears to possess the more significant values for its composite loadings, whereas the factor 2 loading fluctuate from positive to negative values.

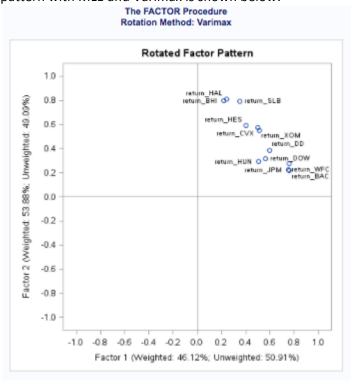
MLE without Varimax rotation

	Fact	or Patte	ern		
		Factor	1	Factor2	
return_	BAC	0.67169		0.42017	
return_	return_BHI		9	-0.37223	
return_	CVX	0.76083		-0.01315	
return_	return_DD		6	0.18472	
return_	return_DOW		4	0.20710	
return_	return_HAL			-0.35943	
return_	return_HES		9	-0.09484	
return_	return_HUN		11	0.18007	
return	JPM	0.66686		0.41017	
return_	SLB	0.82615		-0.26755	
return_	WFC	0.71228		0.38208	
return_	хом	0.75342		0.01106	
Variance	Expla	ined by	E	ach Factor	
Factor	We	ighted l		Unweighted	
Factor1	16.17	20778	6	6.02460756	
Factor2	2.72	39360	0	.94882513	

MLE with rotation

Ro	tated I	Factor	Pat	tern	
		Facto	r1	Factor2	
return_	BAC	0.7612	22	0.21969	
return_	вні	0.21664		0.79932	
return_	CVX	0.4980)6	0.57530	
return_	DD	0.5954	12	0.38748	
return_	DOW	0.56395		0.31884	
return_HAL		0.24256		0.80907	
return_HES		0.40289		0.59153	
return_	return_HUN return_JPM		88	0.29457 0.22277 0.79376	
return_			54		
return_	SLB	0.35223 0.75994 0.51113			
return_	WFC			0.27534 0.55362	
return_	хом				
Variance	Expla	ined by	Εε	ch Facto	
Factor	We	ighted	U	Unweighted	
Factor1	8.71	56851		3.55022275	
Factor2	10.18	03287	3	.42320994	

The rotated factor pattern with MLE and Varimax is shown below:

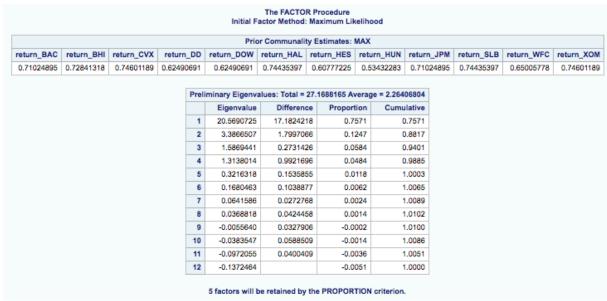


Interestingly, the factor pattern + rotation has segmented and extracted the stocks into the financial, industrial, and 2 oil industry sectors.

Conclusion: The MLE methodology has added a more nuanced view of the stock segmentation. Although suggesting a 2-factor analysis initially, the hypothesis testing indicates that > 2 factors are required, an aspect of the analysis not presented in PFA. Together with the rotation, the MLE is able to elicit separation of the 4 sectors in the starting stock portfolio.

5. Using ML factor analysis PRIORS=MAX option

In this section of the assignment as above an MLE is used with a MAX option for the prior communality estimates (*i.e.* PRIORS=MAX). This change in option (from SMC to MAX) sets the prior communality estimate for each variable to its maximum absolute correlation with any other variable. The initial output from this in SAS is shown below, and suggests 5 factors need chosen based on the PROPORTIONS criterion, at which stage the cumulative proportion will be >1.



The hypothesis-testing table for the MLE is displayed below and indicates common factors are required (low p-value for NH), and a significant p-value for the null hypothesis test that 5-factors are sufficient. However, this does negate the premise that 4-factors, for example may also be sufficient.

Test	DF	Chi-S	quare	Pr : ChiS	
H0: No common factors	66	3656	.2617	<.0001	
HA: At least one common factor					
H0: 5 Factors are sufficient	16 10.9169		0.814		
HA: More factors are needed					
Chi-Square without Bartlett's Correction 11.098156					
Akaike's Information Criterion -20.90			1844		
Schwarz's Bayesian Criterion			-88.36	7542	
Tucker and Lewis's Reliability	Coeff	icient	1.00	5840	

The AIC is lower than previous (step 4), which is good. After the Varimax rotation is performed the following loadings are obtained for the MLE with PRIORS=MAX option.

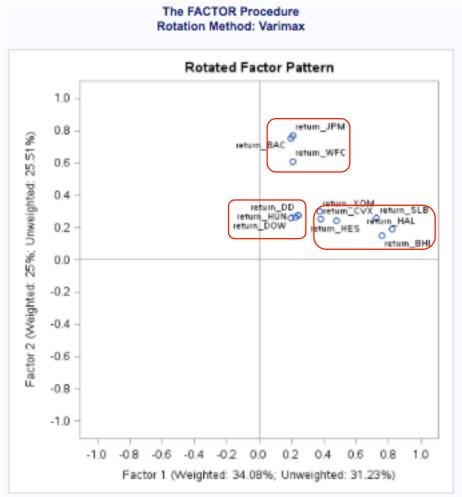
	The FACTOR Procedure Rotation Method: Varimax								
	Orthogonal Transformation Matrix								
	1	2	3	4	5				
1	0.59957	0.48473	0.43530	0.46400	0.02768				
2	-0.69192	0.66028	0.28501	-0.06233	-0.01247				
3	-0.39835	-0.43867	0.21591	0.77390	-0.05782				
4	0.02987	-0.36599	0.82461	-0.42644	-0.05762				
5	-0.04671	-0.05183	0.05170	0.00658	0.99620				

Rotated Factor Pattern								
	Factor1	Factor2	Factor3	Factor4	Factor5			
return_BAC	0.19300	0.75425	0.26803	0.17215	0.09285			
return_BHI	0.75597	0.14970	0.18684	0.24628	-0.01722			
return_CVX	0.37688	0.25354	0.26440	0.70383	0.02658			
return_DD	0.24372	0.27524	0.66859	0.31138	-0.13337			
return_DOW	0.19396	0.25931	0.64481	0.23505	-0.00701			
return_HAL	0.82071	0.18978	0.20801	0.16916	-0.00609			
return_HES	0.47834	0.23976	0.25785	0.40900	0.24903			
return_HUN	0.22592	0.26677	0.60996	0.06709	0.16770			
return_JPM	0.20547	0.77151	0.22874	0.17842	-0.03102			
return_SLB	0.72537	0.25575	0.24707	0.30301	0.05701			
return_WFC	0.20847	0.61032	0.35934	0.29285	-0.00631			
return_XOM	0.37166	0.29603	0.24083	0.66560	-0.02404			

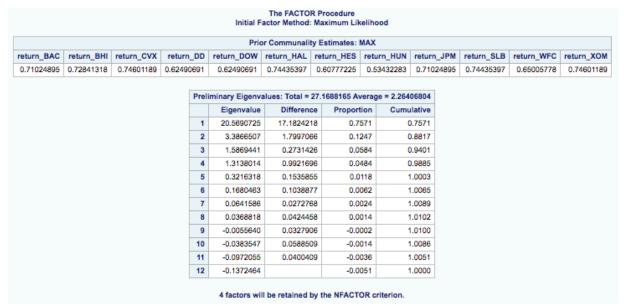
Looking at the factor loadings, there are two important points:

- 1. The other factors 1-4, has a maximum loading for at least one stock. For example,
 - a) Factor 1 has a maximum loading for BHI, HAL and SLB
 - b) Factor 2 has a maximum loading for BAC, JPM and WFC
 - c) Factor 3 has a maximum loading for DOW and HUN
 - d) Factor 4 has a maximum loading for CVX and XOM
- 2. Factor 5 has low values for all its loadings.

Since factor 5 has low loadings for all its stock loadings, it is debatable if this factor is significant to be included in the model. Using a rotation with 5 factors produces 10 rotation pattern diagrams, one is shown for factor 1 *versus* factor 2, using the MLE with PRIORS=MAX option and a Varimax rotation (see below). In this figure the sectors become segmented/clustered to different extents using the different combination of factors.



As noted above, while using this approach it was clear from the loadings from factor 5 that this factor was not required for the model. Another model was created using NFACTOR=4, see below. Note how the loading for the



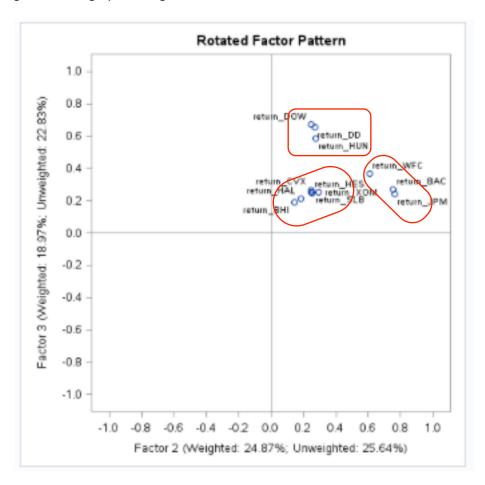
The hypothesis test results (see below) suggest that (a) common factors are required and (b) using a 4-factor model should be significant (null hypothesis p-value= 0.6211). The AIC value has become much smaller (e.g. -26 with the 4-factor model versus -20 with the 5-factor model).

Te	Test			Square	Pr >			
H0: No common	factors	66	3656.2617		<.0001			
HA: At least one	common fact	or						
H0: 4 Factors are	sufficient	24		21.2978	0.6211			
HA: More factors	are needed							
Chi-Square wi Akaike's Inford Schwarz's Bay	mation Criteri	on	ction		7791 7645			
Tucker and Le					0207			
0-	Squared Canonical Correlations							
				_				
Sq Factor1	Factor2 0.77149229		tor3	Fac	tor4			

The factor loadings with the Varimax rotation shown in the table below reveals that all the factors have positive values and each factor has at least one stock with a high loading value.

	Rotated Factor Pattern									
	Factor1	Factor2	Factor3	Factor4						
return_BAC	0.19757	0.75423	0.27205	0.17115						
return_BHI	0.75560	0.14546	0.19113	0.24328						
return_CVX	0.37940	0.24873	0.26653	0.71355						
return_DD	0.24525	0.27169	0.65220	0.30797						
return_DOW	0.19069	0.24648	0.67301	0.22569						
return_HAL	0.81890	0.18590	0.21348	0.16413						
return_HES	0.48670	0.24951	0.25485	0.39568						
return_HUN	0.23250	0.27340	0.58577	0.07364						
return_JPM	0.20509	0.76170	0.23935	0.17940						
return_SLB	0.72981	0.25515	0.24982	0.29668						
return_WFC	0.20922	0.60742	0.36668	0.29113						
return_XOM	0.37625	0.29148	0.25235	0.65233						

Once rotation is added to the model (example below shows factor 2 plotted *versus* factor 3), one can again see the graphical segmentation of the stocks in the market sectors.



Conclusion: The ML Factor Analysis shows a significant and valid model with 4 common factors, the hypothesis-testing component testing aspect of the output shown earlier reveals this. When the PRIORS option was set to MAX, the results indicate that the prior estimates of the communalities are very sensitive to the estimation of the common factors.

CONCLUSION

The use of the maximum likelihood expectation (MLE) model with its various options, including its hypothesis testing component, and significance testing, was able to produce a valid segmentation model of the 4-sector stock portfolio. While the Principal component analysis (PCA) started off with 2 factors, the MLE approach with SAS generated a 4-common factor model that was statistically valid and created through hypothesis testing of the factor number. The use of orthogonal factor rotation (Varimax) allowed for a 'simple model' representation to be produced, so that the rotated factors remain uncorrelated and the communalities are preserved. The final model produced a graphical factor pattern that displayed a teasing apart of the various stocks into their 4 different market sector families.