Axioms : Peano Axioms

Infinitely many axioms can be built by replacing φ, ψ, θ with any formulas.

Logical Axiom Schemes

1. $\varphi \to (\psi \to \varphi)$
2. $[\varphi \to (\psi \to \theta)] \to [(\varphi \to \psi) \to (\varphi \to \theta)]$
3. $(\neg\psi \to \neg\varphi) \to (\varphi \to \neg\psi)$
4. $(\forall x)(\varphi \to \psi) \to (\varphi \to (\forall x)\psi)$
5. $(\forall x)\varphi(x) \to \varphi(t)$

t is a term that can be subsituted for x in $\varphi(x)$ without any of its variables becoming unquantified.

Equality Axioms

1. $x = x$
2. $x = y \to y = x$
3. $x = y \to (y = z \to x = z)$
4. $x = y \to (x' = y')$

Non-Logical Axioms

1. $\neg(0 = x')$
2. $x' = y' \to x = y$
3. $x + 0 = x$
4. $x + y' = (x + y)'$
5. $x \cdot 0 = 0$
6. $x \cdot y' = x \cdot y + x$
7. $\varphi(0) \to [(\forall x)(\varphi(x) \to \varphi(x')) \to (\forall x)\varphi(x)]$

Induction Scheme

1. from φ and $\varphi \to \psi$, infer ψ
2. from φ, infer $(\forall x)\varphi$
3. from φ, infer $\tilde{\varphi}$

$\tilde{\varphi}$ can be any formula obtained by renaming variables in φ.