Master of Science Thesis

# Hybrid Vortex Method for 2D Vertical-Axis Wind Turbine

## A fast and accurate Eulerian-Lagrangian numerical method in python

**L. Manickathan B.Sc.**

Date TBD

Faculty of Aerospace Engineering · Delft University of Technology

Challenge the future

# Hybrid Vortex Method for 2D Vertical-Axis Wind Turbine

## A fast and accurate Eulerian-Lagrangian numerical method in python

Master of Science Thesis

For obtaining the degree of Master of Science in Aerospace Engineering at Delft University of Technology

L. Manickathan B.Sc.

Date TBD

Faculty of Aerospace Engineering · Delft University of Technology

Delft University Of Technology
Department Of
Aerodynamics and Wind Energy

The undersigned hereby certify that they have read and recommend to the Faculty of Aerospace Engineering for acceptance a thesis entitled **"Hybrid Vortex Method for 2D Vertical-Axis Wind Turbine"** by **L. Manickathan B.Sc.** in partial fulfillment of the requirements for the degree of **Master of Science**.

Dated: Date TBD

Head of department: ______________________
prof.dr.ir. G.J.W. van Bussel

Academic Supervisor: ______________________
dr.ir. C.J. Simao Ferreira

Academic Supervisor: ______________________
dr.ir. A. Palha da Silva Clerigo

Industrial Supervisor: ______________________
prof.dr.ir. I. Bennett

# Summary

This is the summary of the thesis.

# Acknowledgements

I wish to thank the following persons. . .

Delft, The Netherlands L. Manickathan B.Sc.
Date TBD

# Contents

# List of Figures

# List of Tables

# Nomenclature

## Latin Symbols

| | | |
|---|---|---|
| $p$ | Pressure | [Pa] |
| $\mathbf{u}$ | Velocity vector | $[m/s]$ |
| $\mathbf{u}_\infty$ | Free-stream velocity | $[m/s]$ |
| $\mathbf{x}_p$ | Position vector | [-] |

## Greek Symbols

| | | |
|---|---|---|
| $\Gamma$ | Circulation | $[m^2/s]$ |
| $\zeta$ | Smooth cutoff function | [-] |
| $\nu$ | Kinematic viscosity | $[m^2/s]$ |
| $\rho$ | Density | $[kg/m^3]$ |
| $\sigma$ | Core size | $[m]$ |
| $\omega$ | Vorticity | $[1/s]$ |

## Abbreviations

| | |
|---|---|
| **VAWT** | Vertical-Axis Wind Turbine |
| **VPM** | Vortex Particle Method |

# Chapter 1

# Introduction

We, the humankind, are now facing several challenges in finding a cheap and reliable energy source. Conventional energy resources such as fossil fuels and nuclear energy are not only limited supply but also pose adhere effects on the environment. Therefore, we are striving to find a cheap and renewable source of energy. Wind energy is such source of energy and is therefore getting more popular and also become more affordable and novel renewable technologies such as Vertical-Axis Wind Turbine (VAWT) is now an interested research field.

Vertical-Axis Wind turbines are unlike the normal wind turbine. Typical wind turbines are mounted on a mast away from the ground and generates energy by spinning normal to the ground. However, a VAWT spins parallel to the ground with its hub located at the ground [12]. The advantages of the vertical axis wind turbine are what makes them ideal for a source of renewable energy. As the turbine is located at the ground (unlike the Horizontal-Axis Wind Turbine), it is easily accessible and can be easily maintained. The second main advantage of the VAWT is the way it dissipates its wake [4] [11]. As the fluid past the turbine is more turbulent, the flow is able to smooth out much earlier. This means that it possible to places VAWTs much closer to each other is so in future this means that a VAWT farm can potentially give more power per area. Furthermore, operate independent of the flow direction and can operate at low wind speeds (low tip-speed ratios).

## 1.1 Motivation and Goal

However, with these advantages also comes drawbacks. As the blades passes through its own dirty air (the wake), complex wake-body interactions take places. These have adhere effect on the blade structure and therefore is more susceptible to fatigue. This happens because the blades are constantly pitching in front the free-stream flow and complex flow behaviour such as dynamic stall and constant vortex shedding occurs [10]. This complex fluid behaviours makes it hard to predict the performance of a VAWT and this is one of the reasons why VAWTs are not mainstream. In addition, as the VAWT operates at large Reynolds number, accurate numerical methods are computationally very expensive. Therefore, it is vital to have a good understanding of the flow structure evolution and the wake generation of the VAWT using not only an efficient method, but also an accurate one.

To summarize, we are now able to formulate a research goal. The key interest of this project is to develop an efficient, reliable, and an accurate numerical method for modelling the flow around a 2D VAWT. For now, only 2D problems are considered because 3D method is build upon the methodology of the 2D. Thus, once the 2D methodology is made, a 3D numerical method should be a straightforward extension.

Furthermore, the numerical method efficient at capturing both the near-wake phenomenons such as the vortex shedding, dynamic stall, & the wake-body interaction, and should be able to capture the large scale flow structure such as the evolution of the VAWT wake. From this criterias, we are able to formulate the research question.

**Research Question:** *Is it possible to develop a numerical method that is both efficient at capturing the small-scale phenomenons and the large scale phenomenons? Is is possible to apply this to a 2D VAWT?*

Similarly, the research aim of this thesis has also been summarized.

## 1.2 Research Aim and Plan

**Research aim and plan:**

- Develop a numerical method for capturing small-scale phenomenons and large scale phenomenons.
- Ensure this tool is efficient, reliable, and accurate.
- Verify, Validate the tools with model problems.
- Apply the model to the 2D flow of VAWT.

With the above formulate research question, aim and plan we are able to thoroughly perform the literature study to determine whether the research goal stated here is feasible. Finally, this report will answer why a Hybrid Eulerian-Lagrangian Vortex Particle Method will be used to the achieved the goals.

## 1.3 Thesis Outline

Chapter 2

# Lagrangian Domain: Vortex Particle Method

To model the flow around a VAWT, several approaches can be taken, Vermeer at al. (2003) [11] have also summarized in their paper. The two main approaches of investigating the flow is either employing a numerical method to simulate the flow or through experimental simulations.

Leishman (2006) [7] has shown that there are several simplified, efficient numerical tools that can be used to model the performance of a VAWT. Methods such as actuator disk theory and blade element momentum theory and deals with simplified Navier-Stokes equations and is very useful to evaluate the trend of certain design parameter. However, as they are highly simplified, complex flow phenomenons that has severe impact of the performance characteristics of the VAWT such as flow separation during dynamic stall, vortex shedding during the rotation and blade-wake interaction cannot be simulated. In order to understand them, either experimental investigation such as in wind tunnel or full Navier-Stokes simulations have to undertaken. So to understand the flow behaviour of a VAWT, several numerical research have been performed [1] [3] [6] [9] and experimental researches by Ferreira [10] [4] and others [5] [8].

All the numerical method that was grid-based struggled with dealing with large number of mesh cells for high Reynolds numbers and the numerical method that employed simplified Navier-Stokes methods had to sacrifices some accuracies.The experimental investigation also come with drawbacks as they are require more financial resources and usually only feasible to model the scaled VAWTs.

This is the main relevance of the hybrid vortex particle method for the VAWT investigations. By utilizing the two methods together, the vortex particle method away from body, and Navier-Stokes solver with turbulence model in the near-body region, one will be able to tackle the challenges in an efficient manner.

Therefore, this chapter is dedicated to given an overview on the theory of the Vortex Particle Method which we will employ with coupled Navier-Stokes solver.

!! Add chapter outline here !!

## 2.1 Introduction to Vortex Method

### 2.1.1 Vorticity

### 2.1.2 Velocity-vorticity formulation of navier-stokes equations

Vortex methods deals with the evolution of the vorticity field in the fluid domain [2]. So to derive the governing equations of 2-D Vortex Particle Method (VPM) , we must examine the 2-D incompressible Navier-Stokes equations of a viscous fluid flow. The momentum equation is given as

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u}, \tag{2.1}$$

with $\zeta$ characterizing the distribution of the vorticity, where $\mathbf{u}(\mathbf{x}, t)$ describes the velocity field of the fluid domain, $p(\mathbf{x}, t)$ describes the pressure field, and $\nu$ and $\rho$ are the kinematic viscosity and the density of the fluid respectively. We also have to satisfy the incompressiblity constraint given as

$$\nabla \cdot \mathbf{u} = 0. \tag{2.2}$$

The governing equation of the VPM is vorticity-velocity formulation of the fluid domain. Vorticity $\omega$ is defined as

$$\omega = \Delta \times \mathbf{u}, \tag{2.3}$$

and so by taking the curl of the momentum equation, we derive the vorticity transport equation,

$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = \nu \nabla^2 \omega. \tag{2.4}$$

### 2.1.3 Viscous splitting algorithm

The VPM deals with the evolution of the vorticity feild using the viscous splitting (or Fractional step) method [2]. The time stepping of the vorticity is done by dealing the viscous and the inviscid part of the transport equation separately,

- convection:
$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = 0; \tag{2.5}$$

- diffusion:
$$\frac{\partial \omega}{\partial t} = \nu \nabla^2 \omega. \tag{2.6}$$

The first substep of the evolution deals which the convection of the vorticity. The diffusion of the vorticity field is evaluated by modifying the vorticity field after the convection.

## 2.2 Spatial Discretization: Generation of Vortex Blobs

### 2.2.1 Discrete form of vorticity field

The spatial discretization of the fluid domain is done by representing the vorticity field in $N$ Lagrangian vortex particles. This is done by dividing the fluid domains into cells where the circulation of the region is assigned to the particle. This gives

$$\omega\left(\mathbf{x},t\right) \simeq \omega^h\left(\mathbf{x},t\right) = \sum_p \Gamma_p\left(t\right)\zeta_\sigma\left[\mathbf{x}-\mathbf{x}_p\left(t\right)\right], \tag{2.7}$$

### 2.2.2 Mollified vortex kernels

where $\Gamma_p$ is the estimate of the circulation around the particle $\mathbf{x}_p$ with core size $\sigma$. We must not that $\omega^h$ is an approximation to $\omega$ of the fluid.

Due to the non-zero size of the vortex elemnts, it is refered to as vortex blobs. The advantage of the vortex blobs is that the with a smooth distribution of the vorticity, the singularity disappears and so numerical instability does not happen when blobs get too close to each other.

An ideal choice for a cutoff function is a Gaussian distribution. Gaussian kernels satisfy the requirement for smooth distributon ad decays quickly. The Gaussian kernel is defined as

$$\zeta_\sigma = \frac{1}{k\pi\sigma^2}\exp\left(\frac{-\left|\mathbf{x}\right|}{k\sigma^2}\right), \tag{2.8}$$

where $k$ is 1, 2 or 4 and determines the width of the kernel.

### 2.2.3 Beale's correction: Iterative circulation processing scheme

### 2.2.4 Error in blob initialization

## 2.3 Convection of vortex blobs

### 2.3.1 Biot-savart law

The vortex transport equation evaluated using the viscous splitting algorithm. For vortex methods, it is ideal to express the equation 2.5 in Lagrangian form,

$$\frac{\mathrm{d}\mathbf{x}_p}{\mathrm{d}t} = \mathbf{u}\left(\mathbf{x}_p\right), \tag{2.9}$$

with

$$\frac{\mathrm{d}\omega_p}{\mathrm{d}t} = 0. \tag{2.10}$$

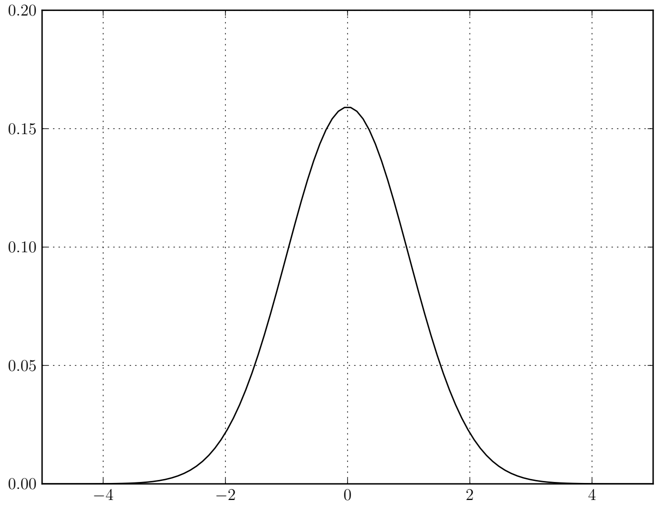


**Figure 2.1:** Vortex blob with Gaussian distribution: $[k = 2, \sigma = 1.0]$

The velocity field can be decomposed using the Helmoltz decomposition,

$$\mathbf{u} = \mathbf{u}_\infty + \mathbf{u}_\omega \tag{2.11}$$

with $\mathbf{u}_\infty$ as the free-stream velocity, $\mathbf{u}_\omega$ as the velocity of the vortical part of the flow. The velocity can be related to the vorticity using the Biot-Savart law

$$\mathbf{u} = \mathbf{u}_\infty + \mathbf{K} \star \omega, \tag{2.12}$$

$$\mathbf{u} = \mathbf{u}_\infty + \mathbf{K} \star \omega, \tag{2.13}$$

where the $\star$ represents convolution of the kernel $\mathbf{K}_p$ given by

$$\mathbf{K}_p = \frac{1}{2\pi \left|\mathbf{x}\right|^2} \left(-x_2, x_1\right). \tag{2.14}$$

The advantage of discretizing and evaluting the vorticity field in this form is that vortex elements are only needed where the vorticity is nonzero. This means that the vortex elements inherently adapts to domain of interest and does not require simulation of region where nothing happens. From equation 2.14, we see that it has a singularity when the particles approach each other and so to overcome this we can mollify the kernel using a smooth cutoff function $\zeta$. So the mollified kernel $\mathbf{K}_\epsilon$ is given as

$$\mathbf{K}_\epsilon = \mathbf{K} \star \zeta_\epsilon. \tag{2.15}$$

### 2.3.2 Remeshing scheme: Treat lagrangian grid distortion

## 2.4 Diffusion of Vortex Methods

### 2.4.1 Vorticity diffusion techniques

### 2.4.2 Modified remeshing for treating diffusion

## 2.5 Boundary conditions at solid boundary

### 2.5.1 Boundary integral equations

#### Linked boundary conditions

### 2.5.2 Panel method for treating no-slip boundary condition

### 2.5.3 Convergence study of panel method

## 2.6 Simulation acceleration techniques

### 2.6.1 Fast multi-pole Method

### 2.6.2 Parallel computation in GPU

## 2.7 Validation of lagrangian domain

## 2.8 Summary

Chapter 3

# Eulerian Domain: Finite Element Method

## 3.1 Purpose of eulerian domain

### 3.1.1 Generation of vorticity

## 3.2 Introduction to Finite Element Method

### 3.2.1 Finite element discretization

### 3.2.2 Finite element function and function spaces

## 3.3 Solving the Finite Element problem

### 3.3.1 Overview of FEniCS Project

### 3.3.2 Mesh generation using GMSH

## 3.4 Finite Element method for solving Incompressible Navier-Stokes equations

### 3.4.1 Incremental pressure correction method

### 3.4.2 Calculation of the vorticity field

### 3.4.3 Calculation of the body forces

## 3.5 Validation of eulerian domain

## 3.6 Summary

Chapter 4

# Hybrid Eulerian-Lagrangian Vortex Particle Method

## 4.1 Theory of Domain Decomposition Method

### 4.1.1 Advantage of domain decomposition

### 4.1.2 Assumptions and Limitations

### 4.1.3 Modified coupling strategy

## 4.2 Eulerian-Lagrangian coupling algorithm

### 4.2.1 Eulerian dirichlet boundary condition

### 4.2.2 Vorticity interpolation algorithm

## 4.3 Overview of pHyFlow: Hybrid solver

### 4.3.1 Program structure

## 4.4 Summary

Chapter 5

# Validation of Hybrid Method

## 5.1 Boundaryless flow

## 5.2 No-slip boundary problem

## 5.3 Impulsively started cylinder problem

## 5.4 Proof of concepts

### 5.4.1 Multiple cylinder case

### 5.4.2 Stalled airfoil

## 5.5 Summary

# Chapter 6

# Conclusion and Recommendation

## 6.1 Conclusion

### 6.1.1 Lagrangian domain

### 6.1.2 Eulerian domain

### 6.1.3 Hybrid method

### 6.1.4 Feasibility of hybrid vortex method for compressor cascade

## 6.2 Recommendations

### 6.2.1 Lagrangian domain

### 6.2.2 Eulerian domain

### 6.2.3 Hybrid method

# References


[1] K.M. Almohammadi, D.B. Ingham, L. Ma, and M. Pourkashan. Computational fluid dynamics (CFD) mesh independency techniques for a straight blade vertical axis wind turbine. *Energy*, July 2013.

[2] G H Cottet and P D Koumoutsakos. *Vortex Methods: Theory and Practice*, volume 12. Cambridge University Press, 2000.

[3] C J Simão Ferreira, H Bijl, G Van Bussel, and G Van Kuik. Simulating Dynamic Stall in a 2D VAWT: Modeling strategy, verification and validation with Particle Image Velocimetry data. *Journal of Physics: Conference Series*, 75:012023, July 2007.

[4] CJ Simão Ferreira. The near wake of the VAWT: 2D and 3D views of the VAWT aerodynamics. 2009.

[5] Robert Howell, Ning Qin, Jonathan Edwards, and Naveed Durrani. Wind tunnel and numerical study of a small vertical axis wind turbine. *Renewable Energy*, 35(2):412–422, February 2010.

[6] M Islam, D Ting, and a Fartaj. Aerodynamic models for Darrieus-type straight-bladed vertical axis wind turbines. *Renewable and Sustainable Energy Reviews*, 12(4):1087–1109, May 2008.

[7] J G Leishman. *Principles of Helicopter Aerodynamics*. Cambridge Aerospace. Cambridge University Press, 2006.

[8] Sander Mertens, Gijs van Kuik, and Gerard van Bussel. Performance of an H-Darrieus in the Skewed Flow on a Roof. *Journal of Solar Energy Engineering*, 125(4):433, 2003.

[9] Karl O. Merz. A Method for Analysis of VAWT Aerodynamic Loads under Turbulent Wind and Platform Motion. *Energy Procedia*, 24(January):44–51, January 2012.

[10] Carlos Simão Ferreira, Gijs Kuik, Gerard Bussel, and Fulvio Scarano. Visualization by PIV of dynamic stall on a vertical axis wind turbine. *Experiments in Fluids*, 46(1):97–108, August 2008.

[11] L.J. Vermeer, J.N. Sø rensen, and a. Crespo. Wind turbine wake aerodynamics. *Progress in Aerospace Sciences*, 39(6-7):467–510, August 2003.

[12] Wikipedia. Vertical-Axis Wind Turbine, July 2013.