

MASTER OF SCIENCE THESIS

Hybrid Eulerian-Lagrangian Vortex Particle Method

A fast and accurate numerical method for 2D Vertical-Axis
Wind Turbine

L. Manickathan B.Sc.

Date TBD

Faculty of Aerospace Engineering · Delft University of Technology

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For obtaining the degree of Master of Science in Aerospace
Engineering at Delft University of Technology

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DELFT UNIVERSITY OF TECHNOLOGY
DEPARTMENT OF
AERODYNAMICS AND WIND ENERGY

The undersigned hereby certify that they have read and recommend to the Faculty of Aerospace Engineering for acceptance a thesis entitled **“Hybrid Eulerian-Lagrangian Vortex Particle Method”** by **L. Manickathan B.Sc.** in partial fulfillment of the requirements for the degree of **Master of Science**.

Dated: Date TBD

Head of department:

prof.dr.ir. G.J.W. van Bussel

Academic Supervisor:

dr.ir. C.J. Simao Ferreira

Academic Supervisor:

dr.ir. A. Palha da Silva Clerigo

Industrial Supervisor:

prof.dr.ir. I. Bennett

Summary

This is the summary of the thesis.

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L. Manickathan B.Sc.

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Chapter 1

Introduction

Conventional energy resources such as fossil fuels and nuclear energy are not only limited supply but also pose adverse effects on the environment. Therefore, we are striving to find a cheap and renewable source of energy. Wind energy is such source of energy and is therefore getting more popular and also become more affordable and novel renewable technologies such as Vertical-Axis Wind Turbine (VAWT) is now an interested research field.

Vertical-Axis Wind turbines are unlike the normal wind turbine. Typical wind turbines are mounted on a mast away from the ground and generates energy by spinning normal to the ground. However, a VAWT spins parallel to the ground with its hub located at the ground [6]. The advantages of the vertical axis wind turbine are what makes them ideal for a source of renewable energy. As the turbine is located at the ground (unlike the Horizontal-Axis Wind Turbine), it is easily accessible and can be easily maintained. The second main advantage of the VAWT is the way it dissipates its wake [3] [5]. As the fluid past the turbine is more turbulent, the flow is able to smooth out much earlier. This means that it possible to places VAWTs much closer to each other is so in future this means that a VAWT farm can potentially give more power per area. Furthermore, operate independent of the flow direction and can operate at low wind speeds (low tip-speed ratios).

1.1 Motivation and Goal

However, with these advantages also comes drawbacks. As the blades passes through its own dirty air (the wake), complex wake-body interactions take places. These have adverse effect on the blade structure and therefore is more susceptible to fatigue. This happens because the blades are constantly pitching in front the free-stream flow and complex flow behaviour such as dynamic stall and constant vortex shedding occurs [4]. This complex fluid behaviours makes it hard to predict the performance of a VAWT and this is one of the reasons why VAWTs are not mainstream. In addition, as the VAWT operates at

large Reynolds number, accurate numerical methods are computationally very expensive. Therefore, it is vital to have a good understanding of the flow structure evolution and the wake generation of the VAWT using not only an efficient method, but also an accurate one.

To summarize, we are now able to formulate a research goal. The key interest of this project is to develop an efficient, reliable, and an accurate numerical method for modelling the flow around a 2D VAWT. For now, only 2D problems are considered because 3D method is build upon the methodology of the 2D. Thus, once the 2D methodology is made, a 3D numerical method should be a straightforward extension.

Furthermore, the numerical method efficient at capturing both the near-wake phenomenons such as the vortex shedding, dynamic stall, & the wake-body interaction, and should be able to capture the large scale flow structure such as the evolution of the VAWT wake. From this criterias, we are able to formulate the research question.

1.2 Research Aim and Plan

Research Question: *Is it possible to develop a numerical method that is both efficient at capturing the small-scale phenomenons and the large scale phenomenons? Is it possible to apply this to a 2D VAWT?*

Research aim and plan:

- Develop a numerical method for capturing small-scale phenomenons and large scale phenomenons.
- Ensure this tool is efficient, reliable, and accurate.
- Verify, Validate the tools with model problems.
- Apply the model to the 2D flow of VAWT.

With the above formulate research question, aim and plan we are able to thoroughly perform the literature study to determine whether the research goal stated here is feasible. Finally, this report will answer why a Hybrid Eulerian-Lagrangian Vortex Particle Method will be used to the achieved the goals.

1.3 Introduction to Hybrid Eulerian-Lagrangian Vortex Particle Method

1.3.1 Advantage of domain decomposition

!!! add picture here !!!

1.3.2 Methodology

1.4 Thesis Outline

Lagrangian Domain: Vortex Particle Method

2.1 Introduction to Vortex Method

Vortex Method (VM) is a branch of computational fluid dynamics that deals with the evolution of the vorticity of the fluid in a lagrangian description. Typically, the fluid is viewed at a fixed window where it is described as a function of space \mathbf{x} and time t . However, the lagrangian point of view regards the fluid as a collection of the particles carrying the propety of the fluid.

!!! Lagrangian vs. Eulerian fluid diagram !!!

Unlike the typical eulerian method that require discretization of all the fluid domain, vortex methods only needs fluid elements where there is vorticity. This means that the vortex method are inherently auto-adaptive method that only simulated the flow of interest. Furthermore, with the computational acceleration methods such as Fast-Multipole Method (FMM) and parallel computation on Graphics Processing Units (GPU), vortex method can be more efficient than typical eulerian methods.

2.1.1 Vorticity

Vorticity ω , the governing element of vortex method, is defined as

$$\omega = \nabla \times \mathbf{u}, \quad (2.1)$$

where \mathbf{u} is the velocity field. The circulation Γ is defined as

$$\Gamma = \int_L \mathbf{u} \cdot d\mathbf{r} = \int_S \omega \cdot \mathbf{n} dS, \quad (2.2)$$

by the stokes theorem, as represents the integral vorticity of the domain, figure 2.1

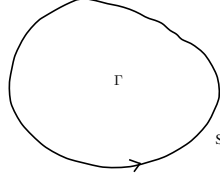


Figure 2.1: Circulation of the fluid

2.1.2 Velocity-vorticity formulation of navier-stokes equations

The governing equation of the vortex method is velocity-vorticity $\mathbf{u}-\omega$ formulation of the navier-stokes equations [2]. The 2-D incompressible navier-stokes momentum equation is given as

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u}, \quad (2.3)$$

relating the velocity field $\mathbf{u}(\mathbf{x}, t)$ to the pressure field $\mathbf{p}(\mathbf{x}, t)$, the kinematic viscosity ν and density ρ . Furthermore, we also have to satisfy the incompressibility constraint given as

$$\nabla \cdot \mathbf{u} = 0. \quad (2.4)$$

To attain the velocity-vorticity formulation, we should take the curl of the velocity-pressure $\mathbf{u}-p$ formulation of the navier-stokes equation. Taking the curl of the momentum equation 2.3, we get the vorticity transport equation

$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = \nu \nabla^2 \omega, \quad (2.5)$$

which only relates the vorticity to the velocity enabling us to neglect the pressure field.

2.1.3 Viscous splitting algorithm

Vortex method is initially used to model the evolution of incompressible, inviscid flows. However, in order to simulate a real flow, we must also deal with the viscous component of the fluid. Chorin [1] has shown that using the viscous splitting algorithm, it is possible to model a viscous flow using vortex method.

The viscous splitting algorithm is a fractional step method, where the viscous and the inviscid part of the transport equation is dealt in two subsequent steps,

- convection:

$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = 0; \quad (2.6)$$

- diffusion:

$$\frac{\partial \omega}{\partial t} = \nu \nabla^2 \omega. \quad (2.7)$$

Therefore, the evolution of the vorticity is split into two sub-steps. The first substep of the evolution deals with the convection of the vorticity and the second deals with the diffusion of the vorticity field. There are several ways of dealing with the diffusion of the vorticity field and Chorin initially employed a random walk method. This method suffers some limitations in accuracy and since then there are several methods that can be used to simulate the diffusion such as the Particle Strength Exchange (PSE) method, redistribution method and core spreading method.

2.2 Spatial Discretization: Generation of Vortex Blobs

Vortex blobs were first introduced by Chorin.

2.2.1 Discrete form of vorticity field

The spatial discretization of the fluid domain is done by representing the vorticity field in N Lagrangian vortex particles. This is done by dividing the fluid domains into cells where the circulation of the region is assigned to the particle. This gives

$$\omega(\mathbf{x}, t) \simeq \omega^h(\mathbf{x}, t) = \sum_p \Gamma_p(t) \zeta_\sigma[\mathbf{x} - \mathbf{x}_p(t)], \quad (2.8)$$

2.2.2 Mollified vortex kernels

where Γ_p is the estimate of the circulation around the particle \mathbf{x}_p with core size σ . We must not that ω^h is an approximation to ω of the fluid.

Due to the non-zero size of the vortex elements, it is referred to as vortex blobs. The advantage of the vortex blobs is that with a smooth distribution of the vorticity, the singularity disappears and so numerical instability does not happen when blobs get too close to each other.

An ideal choice for a cutoff function is a Gaussian distribution. Gaussian kernels satisfy the requirement for smooth distribution and decays quickly. The Gaussian kernel is defined as

$$\zeta_\sigma = \frac{1}{k\pi\sigma^2} \exp\left(\frac{-|\mathbf{x}|}{k\sigma^2}\right), \quad (2.9)$$

where k is 1, 2 or 4 and determines the width of the kernel.

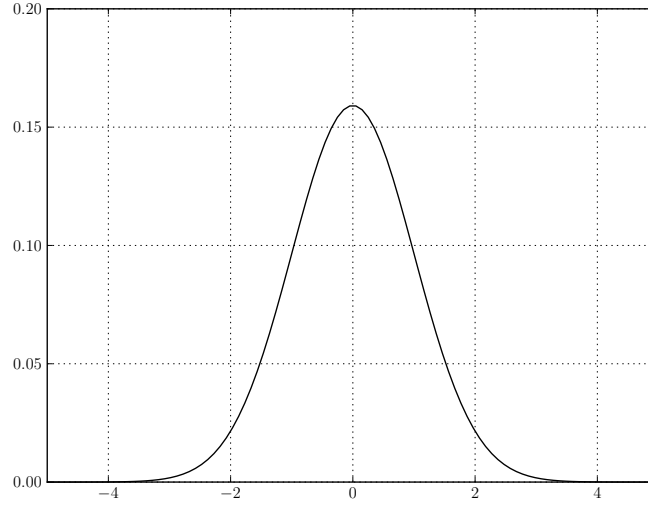


Figure 2.2: Vortex blob with Gaussian distribution: $[k = 2, \sigma = 1.0]$

2.2.3 Beale's correction: Iterative circulation processing scheme

2.2.4 Error in blob initialization

2.3 Convection of vortex blobs

2.3.1 Biot-savart law

The vortex transport equation evaluated using the viscous splitting algorithm. For vortex methods, it is ideal to express the equation 2.6 in Lagrangian form,

$$\frac{d\mathbf{x}_p}{dt} = \mathbf{u}(\mathbf{x}_p), \quad (2.10)$$

with

$$\frac{d\omega_p}{dt} = 0. \quad (2.11)$$

The velocity field can be decomposed using the Helmholtz decomposition,

$$\mathbf{u} = \mathbf{u}_\infty + \mathbf{u}_\omega \quad (2.12)$$

with \mathbf{u}_∞ as the free-stream velocity, \mathbf{u}_ω as the velocity of the vortical part of the flow.

The velocity can be related to the vorticity using the Biot-Savart law

$$\mathbf{u} = \mathbf{u}_\infty + \mathbf{K} \star \omega, \quad (2.13)$$

$$\mathbf{u} = \mathbf{u}_\infty + \mathbf{K} \star \omega, \quad (2.14)$$

where the \star represents convolution of the kernel \mathbf{K}_p given by

$$\mathbf{K}_p = \frac{1}{2\pi |\mathbf{x}|^2} (-x_2, x_1). \quad (2.15)$$

The advantage of discretizing and evaluating the vorticity field in this form is that vortex elements are only needed where the vorticity is nonzero. This means that the vortex elements inherently adapt to domain of interest and does not require simulation of region where nothing happens. From equation 2.15, we see that it has a singularity when the particles approach each other and so to overcome this we can mollify the kernel using a smooth cutoff function ζ . So the mollified kernel \mathbf{K}_ϵ is given as

$$\mathbf{K}_\epsilon = \mathbf{K} \star \zeta_\epsilon. \quad (2.16)$$

2.3.2 Remeshing scheme: Treating lagrangian grid distortion

2.4 Diffusion of Vortex Methods

2.4.1 Vorticity diffusion techniques

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VAWT, [1](#)