MASTER OF SCIENCE THESIS

Hybrid Eulerian-Lagrangian Vortex Particle Method

A fast and accurate numerical method for 2D Vertical-Axis Wind Turbine

L. Manickathan B.Sc.

Date TBD

Faculty of Aerospace Engineering · Delft University of Technology

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Challenge the future

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Master of Science Thesis

For obtaining the degree of Master of Science in Aerospace Engineering at Delft University of Technology

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Delft University Of Technology
Department Of
Aerodynamics and Wind Energy

The undersigned hereby certify that they have read and recommend to the Faculty of Aerospace Engineering for acceptance a thesis entitled **"Hybrid Eulerian-Lagrangian Vortex Particle Method"** by **L. Manickathan B.Sc.** in partial fulfillment of the requirements for the degree of **Master of Science**.

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Summary

This is the summary of the thesis.

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Nomenclature

Latin Symbols

$\mathbf{K}$	Biot-Savart kernel	$[-]$
$\mathbf{K}_\sigma$	Vortex blob kernel	$[-]$
N_p	Number of particles	$[-]$
p	Pressure	[Pa]
t	Time	$[s]$
$\mathbf{u}$	Velocity	$[m \cdot s^{-1}]$
$\mathbf{u}_\infty$	Free-stream velocity	$[m \cdot s^{-1}]$
$\mathbf{u}\left(\mathbf{x}, t\right)$	Velocity field	$[m \cdot s^{-1}]$
$\mathbf{u}_\omega$	Vortical velocity	$[m \cdot s^{-1}]$
$\mathbf{u}_\phi$	Solenoidal velocity	$[m \cdot s^{-1}]$
$\mathbf{x}$	Position vector	$[m]$
$\mathbf{x}_p$	Position vector of the particle	$[m]$

Greek Symbols

ζ_σ	Smooth cut-off function of the blob	$[-]$
Γ	Circulation	$[m^2 \cdot s^{-1}]$
Γ_p	Circulation of the particle	$[m^2 \cdot s^{-1}]$
ν	Kinematic viscosity	$[m^2 \cdot s^{-1}]$
ρ	Density	$[kg \cdot m^{-3}]$
σ	Core size	$[m]$

ω	Vorticity field	$[s^{-1}]$

Abbreviations

FMM	Fast-Multipole Method
GPU	Graphics Processing Units
PSE	Particle Strength Exchange
VAWT	Vertical-Axis Wind Turbine
VPM	Vortex Particle Method
VRM	Vortex Redistribution Method

Chapter 1

Introduction

Conventional energy resources such as fossil fuels and nuclear energy are not only limited supply but also pose adhere effects on the environment. Therefore, we are striving to find a cheap and renewable source of energy. Wind energy is such source of energy and is therefore getting more popular and also become more affordable and novel renewable technologies such as Vertical-Axis Wind Turbine (VAWT) is now an interested research field.

Vertical-Axis Wind turbines are unlike the normal wind turbine. Typical wind turbines are mounted on a mast away from the ground and generates energy by spinning normal to the ground. However, a VAWT spins parallel to the ground with its hub located at the ground [10]. The advantages of the vertical axis wind turbine are what makes them ideal for a source of renewable energy. As the turbine is located at the ground (unlike the Horizontal-Axis Wind Turbine), it is easily accessible and can be easily maintained. The second main advantage of the VAWT is the way it dissipates its wake [5] [8]. As the fluid past the turbine is more turbulent, the flow is able to smooth out much earlier. This means that it possible to places VAWTs much closer to each other is so in future this means that a VAWT farm can potentially give more power per area. Furthermore, operate independent of the flow direction and can operate at low wind speeds (low tip-speed ratios).

1.1 Motivation and Goal

However, with these advantages also comes drawbacks. As the blades passes through its own dirty air (the wake), complex wake-body interactions take places. These have adhere effect on the blade structure and therefore is more susceptible to fatigue. This happens because the blades are constantly pitching in front the free-stream flow and complex flow behaviour such as dynamic stall and constant vortex shedding occurs [7]. This complex fluid behaviours makes it hard to predict the performance of a VAWT and this is one of the reasons why VAWTs are not mainstream. In addition, as the VAWT operates at large Reynolds number, accurate numerical methods are computationally very expensive. Therefore, it is vital to have a good understanding of the flow structure evolution and the wake generation of the VAWT using not only an efficient method, but also an accurate one.

To summarize, we are now able to formulate a research goal. The key interest of this project is to develop an efficient, reliable, and an accurate numerical method for modelling the flow around a 2D VAWT. For now, only 2D problems are considered because 3D method is build upon the methodology of the 2D. Thus, once the 2D methodology is made, a 3D numerical method should be a straightforward extension.

Furthermore, the numerical method efficient at capturing both the near-wake phenomenons such as the vortex shedding, dynamic stall, & the wake-body interaction, and should be able to capture the large scale flow structure such as the evolution of the VAWT wake. From this criterias, we are able to formulate the research question.

1.2 Research Aim and Plan

Research Question: *Is it possible to develop a numerical method that is both efficient at capturing the small-scale phenomenons and the large scale phenomenons? Is is possible to apply this to a 2D VAWT?*

Research aim and plan:

- Develop a numerical method for capturing small-scale phenomenons and large scale phenomenons.
- Ensure this tool is efficient, reliable, and accurate.
- Verify, Validate the tools with model problems.
- Apply the model to the 2D flow of VAWT.

With the above formulate research question, aim and plan we are able to thoroughly perform the literature study to determine whether the research goal stated here is feasible. Finally, this report will answer why a Hybrid Eulerian-Lagrangian Vortex Particle Method will be used to the achieved the goals.

1.3 Introduction to Hybrid Eulerian-Lagrangian Vortex Particle Method

1.3.1 Advantage of domain decomposition

1.3.2 Methodology

1.4 Thesis Outline

Chapter 2

Lagrangian Domain: Vortex Particle Method

2.1 Introduction to Vortex Particle Method

Vortex Particle Method (VPM) is a branch of computational fluid dynamics that deals with the evolution of the vorticity of the fluid in a lagrangian description. Typically, the fluid is viewed at a fixed window where it is described as a function of space $\mathbf{x}$ and time t. However, the lagrangian point of view regards the fluid as a collection of the particles carrying the proprety of the fluid.

Unlike the typical eulerian method that require discretization of all the fluid domain, VPM only needs fluid elements where there is vorticity. This means that the VPM are inherently auto-adaptive method that only simulated the flow of interest. Furthermore, with the computational acceleration methods such as Fast-Multipole Method (FMM) and parallel computation on Graphics Processing Units (GPU) , VPM can be more efficient that typical eulerian methods.

2.1.1 Vorticity

Vorticity ω, the governing element of vortex particle method, is defined as

$$\omega = \Delta \times \mathbf{u}, \tag{2.1}$$

where $\mathbf{u}$ is the velocity. The circulation Γ is defined as

$$\Gamma = \int_L \mathbf{u} \cdot d\mathbf{r} = \int_S \omega \cdot \mathbf{n}\ dS, \tag{2.2}$$

by the stokes theorem, as represents the integral vorticity of the domain, figure 2.1

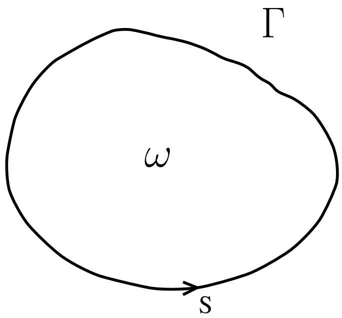

Figure 2.1: Circulation of the fluid

2.1.2 Velocity-vorticity formulation of the Navier-Stokes equations

The governing equation of the vortex particle method is velocity-vorticity $\mathbf{u} - \omega$ formulation of the Navier-Stokes equations [3]. The 2-D incompressible Navier-Stokes momentum equation is given as

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u}, \tag{2.3}$$

relating the velocity field $\mathbf{u}(\mathbf{x}, t)$ to the pressure field $\mathbf{p}(\mathbf{x}, \mathbf{t})$, the kinematic viscosity ν and density ρ. Furthermore, we also have to satisfy the incompressibility constraint given as

$$\nabla \cdot \mathbf{u} = 0. \tag{2.4}$$

To attain the velocity-vorticity formulation, we should take the curl of the velocity-pressure $\mathbf{u} - p$ formulation of the Navier-Stokes equation. Taking the curl of the momentum equation 2.3, we get the vorticity transport equation

$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = \nu \nabla^2 \omega, \tag{2.5}$$

which only relates the vorticity to the velocity enabling us to neglect the pressure field.

2.1.3 Viscous splitting algorithm

Vortex particle method was initiality used to model the evolution of incompressible, inviscid flows. However, in order to simulate a real flow, we must also deal with the viscous behaviour of the fluid. Chorin [2] has shown that using the viscous splitting algorithm, it is possible to simulate a viscous flow.

The viscous splitting algorithm is basically a fractional step method, where the viscous and the inviscid part of the transport equation is dealth in two subsquent steps,

- Sub-step 1: convection

$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = 0; \tag{2.6}$$

- Sub-step 2: diffusion

$$\frac{\partial \omega}{\partial t} = \nu \nabla^2 \omega. \tag{2.7}$$

The first sub-step of the evolution deals with the convection of the vorticity. The second sub-step deals with the diffusion of the vorticity field.

There are several advantage to this type of evolution. As the convection and diffusion are handled separately, there is minimum dispertion during the convection and furthermore, the is no restriction of the advection CFL number [9].

There are many ways of dealing with the diffusion of the vorticity field. During this project, we use a modified interpolation kernel [9] that can simultaniously treat diffusion and remesh the vortex particles, see section 2.4.

2.2 Spatial Discretization: Generation of Vortex Blobs

In order to deal with the vorticity field, we must first discretize the vorticity to vortex particles. Vortex blobs have been first introduced by Chorin and is a mollified particle carrying the local circulation. Vortex blobs describes a smooth vorticity field and are ideal because of it does not cause singularity issues when particles approach each other.

2.2.1 Discrete form of vorticity field

The spatial discretization of the fluid domain is done by representing the vorticity field in N Lagrangian vortex particles. This is done by dividing the fluid domains into cells where the circulation of the region is assigned to the particle. So,

$$\omega(\mathbf{x}, t) \simeq \omega^h(\mathbf{x}, t) = \sum_p \Gamma_p(t)\, \delta\left[\mathbf{x} - \mathbf{x}_p(t)\right], \tag{2.8}$$

where Γ_p is the estimate of the circulation around the particle $\mathbf{x}_p$ with core size σ. We must not that ω^h is an approximately equal to ω of the fluid due to the discretization. The lagrangian form of the convection equation 2.6 of the viscous-splitting algorithm is given as,

$$\frac{\mathrm{d}\mathbf{x}_p}{\mathrm{d}t} = \mathbf{u}(\mathbf{x}_p), \tag{2.9}$$

with

$$\frac{\mathrm{d}\omega_p}{\mathrm{d}t} = 0. \tag{2.10}$$

The velocity field can be decomposed using the Helmoltz decomposition, given as

$$\mathbf{u} = \mathbf{u}_\omega + \mathbf{u}_\phi, \tag{2.11}$$

where $\mathbf{u}_\omega$ is the rotational component of the veloity and $\mathbf{u}_\phi$ is the solenoidal component. In an unbounded flow we have $\mathbf{u}_\phi$ equal to the free-stream velocity $\mathbf{u}_\infty$. For bounded flow, see section 2.5.

The velocity can be related to the vorticity using the Biot-Savart law

$$\mathbf{u} = \mathbf{u}_\infty + \mathbf{K} \star \omega, \tag{2.12}$$

where the $\star$ represents convolution of the kernel $\mathbf{K}$ given by

$$\mathbf{K} = \frac{1}{2\pi \left|\mathbf{x}\right|^2} \left(-x_2, x_1\right). \tag{2.13}$$

From equation 2.13, we see that it has a singularity when the particles approach each other and so to overcome this we can mollify the kernel, removing the singularity.

2.2.2 Mollified vortex kernels

A vortex particle with a non-zero core-size is refered to as vortex blobs. The advantage of the vortex blobs is that the with a smooth distribution of the vorticity, the singularity disappears and so numerical instability does not happen when blobs get too close to each other. An ideal choice for a cutoff function is a Gaussian distribution, figure 2.2.

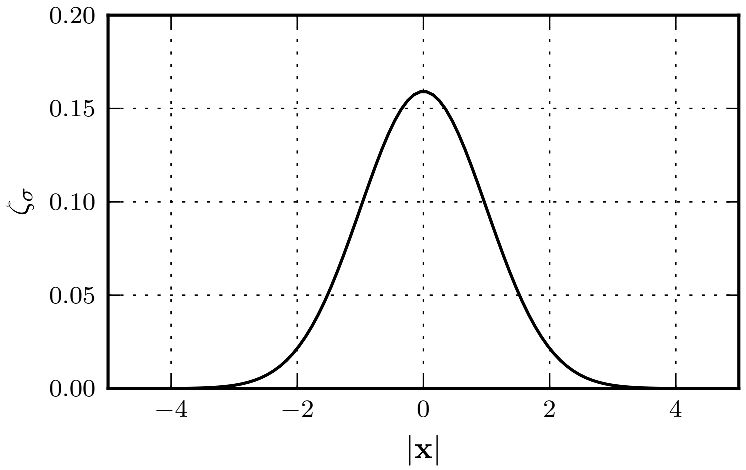

Figure 2.2: Vortex blob with Gaussian distribution: $[k = 2,\ \sigma = 1.0]$

Gaussian kernels satisfy the requirement for smooth distributon ad decays quickly and is defined as

$$\zeta_\sigma = \frac{1}{k\pi\sigma^2} \exp\left(\frac{-\left|\mathbf{x}\right|}{k\sigma^2}\right), \tag{2.14}$$

where k is 1, 2 or 4 and determines the width of the kernel, σ is core-size of the blob. So, using a smooth cut-off function ζ_σ, the mollied kernel $\mathbf{K}_\sigma$ is given as

$$\mathbf{K}_\sigma = \mathbf{K} \star \zeta_\sigma. \tag{2.15}$$

2.3 Convection of vortex blobs

2.4 Diffusion of Vortex Methods

Chorin initially employed a random walk method, however this method suffers some limitations in accuracy and since then several methods have been introduced that can be used to simulate the diffusion. Particle Strength Exchange (PSE) method [4], is an algorithm to treat diffusion by exchange of vortex element strengths. Vortex Redistribution Method (VRM) [6] models diffusion by distributing the fraction of circulation of the vortex elements to the neighbouring vortices.

2.5 Boundary conditions at solid boundary

Chapter 3

Conclusion and Recommendation

3.1 Conclusion

3.1.1 Lagrangian domain

3.1.2 Eulerian domain

3.1.3 Hybrid method

3.2 Recommendations

3.2.1 Lagrangian domain

3.2.2 Eulerian domain

3.2.3 Hybrid method

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