

NOMENCLATURE

LATIN SYMBOLS

a	leaf area density	$\text{m}^2 \text{m}^{-3}$
A	area	m^2
A_f	frontal area	m^2
A_L	net leaf area	m^2
A_n	assimilation rate	$\text{mol m}^{-2} \text{s}^{-1}$
b	Vogel exponent	-
c_d	leaf drag coefficient	-
C_d	drag coefficient	-
c_p	specific heat capacity (at constant pressure)	$\text{J kg}^{-1} \text{K}^{-1}$
c_{pa}	specific heat capacity of dry air (= 1006.43)	$\text{J kg}^{-1} \text{K}^{-1}$
c_{pl}	specific heat capacity of liquid (= 4182)	$\text{J kg}^{-1} \text{K}^{-1}$
c_{pv}	specific heat capacity of water vapor (= 1880)	$\text{J kg}^{-1} \text{K}^{-1}$
c_a^*	reference CO ₂ concentration	mol mol^{-1}
C	aerodynamic resistance coefficient	$\text{s}^{0.5} \text{m}^{-1}$
$C_{o,a}$	oxygen concentration	mol mol^{-1}
D	diameter	m
D	vapor pressure deficit	kPa
D	vapor pressure deficit	$\text{Pa Pa}^{-1}, \text{mol mol}^{-1}$
D_{va}	diffusion coefficient of vapor to dry air	$\text{m}^2 \text{s}^{-1}$
D_{ca}	diffusion coefficient of CO ₂ to dry air	$\text{m}^2 \text{s}^{-1}$
e_m	maximum quantum efficiency of the leaf	-
E	energy	J
E	energy	J
E	leaf transpiration flux	kg s^{-1}
f_c	CO ₂ assimilation (photosynthesis) rate	$\text{mol m}^{-2} \text{s}^{-1}$
f_v	transpiration rate	$\text{mol m}^{-2} \text{s}^{-1}$

F	force	N
F_d	drag force	N
g	gravitational acceleration	$\text{m}^2 \text{s}^{-1}$
g	mass flux	$\text{kg m}^{-2} \text{s}^{-1}$
$g_{c,i}$	convective mass flux of species i	$\text{kg m}^{-2} \text{s}^{-1}$
$g_{d,i}$	diffusive mass flux of species i	$\text{kg m}^{-2} \text{s}^{-1}$
g_l	liquid water flux	$\text{kg m}^{-2} \text{s}^{-1}$
g_v	water vapor flux	$\text{kg m}^{-2} \text{s}^{-1}$
$g_{v,leaf}$	water vapor mass flux	$\text{kg m}^{-2} \text{s}^{-1}$
$g_{v,root}$	root water uptake	$\text{kg m}^{-2} \text{s}^{-1}$
$G_{v,root}$	net root water uptake	kg s^{-1}
$G_{v,xylem}$	net xylem water flux	kg s^{-1}
h	enthalpy	J kg^{-1}
$h_{c,h}$	convective heat transfer coefficient	$\text{W m}^{-2} \text{K}^{-1}$
$h_{c,m}$	convective mass transfer coefficient	m s^{-1}
H	height	m
HU	Hounsfield units	-
I	turbulence intensity	%
k	turbulent kinetic energy	$\text{m}^2 \text{s}^{-2}$
k_{st}	stomatal conductance	$\text{mol m}^{-2} \text{s}^{-1}$
k_{st}^*	effective stomatal conductance	$\text{mol m}^{-2} \text{s}^{-1}$
K	permeability	m^2
K	hydraulic conductivity	m s^{-1}
K_{lp}	liquid water permeability	s
K_{vp}	water vapor permeability	s
K_{vT}	water vapor permeability due to temperature	s
K_c	Michaelis constant for CO_2	mol mol^{-1}
K_o	Michaelis constant for O_2	mol mol^{-1}
l	length	m
l	characteristic leaf size	m
\mathcal{L}	Lagrangian	-
L_v	latent heat of vaporization for water ($= 2.5 \times 10^6$)	J kg^{-1}

m	mass	kg
M	molar mass	kg mol ⁻¹
M_a	molar mass of dry air (= 0.028 966)	kg mol ⁻¹
M_v	molar mass of water vapor (= 0.018 015 34)	kg mol ⁻¹
n	Van Genuchten parameter	-
p_c	capillary pressure	Pa
p_l	liquid pressure	Pa
p_l	gas phase pressure	Pa
p_v	partial vapor pressure	Pa
$p_{v,i}$	intercellular vapor pressure	Pa
p_{vsat}	saturation vapor pressure	Pa
P	pressure	Pa
Pr	Prandtl number	-
Pr_t	turbulent Prandtl number	-
q	heat flux	W m ⁻²
$q_{lat,lat}$	latent heat flux from leaf	W m ⁻²
q_r	net radiative heat flux	W m ⁻²
$q_{r,sw}$	short-wave radiative heat flux	W m ⁻²
$q_{r,lw}$	long-wave radiative heat flux	W m ⁻²
$q_{rad,lat}$	radiative heat flux into leaf	W m ⁻²
$q_{sen,lat}$	sensible heat flux from leaf	W m ⁻²
Q_p	flux of incoming PAR	mol m ⁻² s ⁻¹
r	root area density	m ² m ⁻³
r_a	aerodynamic resistance	s m ⁻¹
r_s	stomatal resistance	s m ⁻¹
\mathcal{R}	universal gas constant (= 8.314 459 8)	J mol ⁻¹ K ⁻¹
RAI	root area index	m ² m ⁻²
Re	Reynolds number	-
R_v	specific gas constant of dry air (= 287.042)	J kg ⁻¹ K ⁻¹
R_v	specific gas constant of water vapor (= 461.524)	J kg ⁻¹ K ⁻¹
Sc_t	turbulent Schmidt number	-
s_ε	volumetric TDR source	W m ⁻³ s ⁻¹

s_ρ	volumetric mass source	$\text{kg m}^{-3} \text{s}^{-1}$
s_k	volumetric TKE source	W m^{-3}
$s_{q,r}$	volumetric radiative source	W m^{-3}
s_r	volumetric root water uptake source	$\text{kg m}^{-3} \text{s}^{-1}$
s_T	volumetric temperature source	K m^{-3}
s_u	volumetric momentum source	N m^{-3}
s_w	volumetric humidity source	$\text{kg kg}^{-1} \text{s}^{-1}$
S_l	liquid saturation	-
t	time	s
T	temperature	K
T_g	ground temperature	K
T_L	leaf temperature	K
T_{sky}	sky temperature	K
TR	net hourly transpiration rate	g h^{-1}
u	velocity	m s^{-1}
u_*	friction velocity	m s^{-1}
U	mean wind speed	m s^{-1}
U_{ref}	reference velocity	m s^{-1}
$UTCI$	universal thermal climate index	$^{\circ}\text{C}$
V_{cmax}	maximum carboxylation capacity	$\text{mol m}^{-2} \text{s}^{-1}$
w	moisture content	kg m^{-3}
w_a	dry air content	kg m^{-3}
w_{cap}	capillary moisture content	kg m^{-3}
w_l	liquid water content	kg m^{-3}
w_s	solid matrix water content	kg m^{-3}
w_v	water vapor content	kg m^{-3}
x_i	mass concentration of species i	$\text{kg}_i \text{kg}^{-1}$
z	vertical height	m
z_0	aerodynamic roughness height	m

GREEK SYMBOLS

α	aerodynamic porosity	-
α^{2D}	2D aerodynamic porosity	-
α_p	leaf absorptivity of PAR	-
β	optical porosity	-
β	volumetric thermal expansion coefficient	K^{-1}
β_p	fraction of MKE converted to TKE	-
β_d	fraction of TKE shortcut to TDR	-
γ	apparent quantum yield	-
δ_v	water vapor diffusion coefficient	s
ε	turbulent kinetic energy dissipation rate	$m^2 s^{-3}$
κ	von Kármán constant (= 0.41)	-
λ	thermal conductivity	$W m^{-1} K^{-1}$
λ	Lagrange multiplier	$mol mol^{-1}$
μ	attenuation coefficient	m^{-1}
μ	chemical potential	$J mol^{-1}$
μ	dynamic viscosity	$kg m^{-1} s^{-1}$
ν	kinematic viscosity	$m^2 s^{-1}$
ν_t	turbulent viscosity	$m^2 s^{-1}$
ρ	density	$kg m^{-3}$
ρ_a	density of air	$kg m^{-3}$
ρ_l	density of liquid water (= 1000)	$kg m^{-3}$
ρ_s	density of solid matrix	$kg m^{-3}$
σ_v	Schmidt number	-
σ_{v_t}	turbulent Schmidt number	-
ϕ_o	open porosity	$m^3 m^{-3}$
ψ	shelter parameter	-
ψ_g	gravitational potential	Pa
ψ_L	leaf water potential	Pa
ψ_R	root water potential	Pa
ψ_s	soil water potential	Pa
Ω	domain	-
Ω_a	air domain	-

Ω_s soil domain -

SUBSCRIPTS

eff effective
g gas
g ground
i intercellular
l liquid (water)
l leaf
leaf leaf
lw long-wave
L leaf (net property)
o open
pore pore
ref reference
root root (net property)
sat saturated
R root
s solid
s soil
sky sky
sw short-wave
t time
v vapor
xylem xylem

ACRONYMS

ABL atmospheric boundary layer
 CFD computational fluid dynamics

CHTC	convective heat transfer coefficient
CMTC	convective mass transfer coefficient
CT	computational tomography
DEHS	Di-Ethyl-Hexyl-Sebacat
DIG	diagonal-based incomplete Cholesky
DNS	direct numerical simulation
ETHZ	Eidgenössische Technische Hochschule Zürich
FOV	field of view
FFT	fast Fourier transformation
FVM	finite volume method
HPC	high performance computing
LAI	leaf area index
LAD	leaf area density
LDPE	low-density Polyethylene
LES	large eddy simulation
LHS	left hand side
MOE	modulus of elasticity
PAR	photosynthetically active radiation
PCG	preconditioned conjugate gradient
PIV	particle image velocimetry
PPM	parts per million
RAI	root area index
REV	representative elementary volume
RANS	Reynolds-averaged Navier-Stokes

RHS	right hand side
ROI	region of interest
SPIV	stereoscopic particle image velocimetry
TDR	turbulent dissipation rate
TKE	turbulent kinetic energy
UHI	urban heat island
UTCI	universal thermal climate index
WUE	water use efficiency

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THERMODYNAMICS OF MOIST AIR

IDEAL GAS LAW

The ideal gas law is defined as:

$$p = \rho \frac{\mathcal{R}}{M} T \quad (\text{A.1})$$

where p is the pressure (Pa), ρ is density (kg m^{-3}), $\mathcal{R} = 8.314\,559\,8 \text{ J mol}^{-1}\text{K}^{-1}$ is universal gas constant, M is molar mass (kg mol^{-1}) and T is temperature (K). The specific gas constant R ($\text{J kg}^{-1}\text{K}^{-1}$) is defined as:

$$R = \frac{\mathcal{R}}{M} \quad (\text{A.2})$$

and substituting [A.2](#) into [A.1](#)

$$p = \rho R T \quad (\text{A.3})$$

and similarly

$$p_i = \rho_i R_i T_i \quad (\text{A.4})$$

PARTIAL PRESSURES

Using Dalton's law of partial pressures, the moist air is consisted of partial pressure of dry air p_a (Pa) and water vapor p_v (Pa):

$$p = \sum_i p_i = p_v + p_a \quad (\text{A.5})$$

and we assume $p_o \ll p_v + p_a$ and $p_c \ll p_v + p_a$, the partial pressure of oxygen O_2 and CO_2 is negligible. We assuming that all the species are in thermal equilibrium ($T = T_v = T_a$) (K) and so:

$$p = (\rho_a R_a + \rho_v R_v) T \quad (A.6)$$

The molar mass of dry air and water vapor are $M_a = 18.0149 \text{ g mol}^{-1}$ and $M_a = 28.964 \text{ g mol}^{-1}$, and $R_a = 287.055 \text{ J kg}^{-1} \text{ K}^{-1}$ and $R_v = 461.5 \text{ J kg}^{-1} \text{ K}^{-1}$.

Vapor pressure p_v is related to relative humidity ϕ (-) as:

$$\frac{p_v}{p_{vsat}} = \phi \quad (A.7)$$

where p_{vsat} is the saturation vapor pressure. The relative humidity is also written as $RH = \phi \times 100$. The saturation vapor pressure, p_{vsat} (Pa), can be determined directly from temperature:

$$p_{vsat} = 610.78 \exp \left\{ \frac{17.269 (T - 273.15)}{T - 35.85} \right\} \quad (A.8)$$

MASS FRACTIONS

The mass of moist gas mixture is :

$$m = m_v + m_a \quad (A.9)$$

where the species of moist gas mixture is water vapor m_v (kg) and dry air m_a (kg).

MASS CONCENTRATION

The mass concentration x_i ($\text{kg}_i \text{ kg}^{-1}$) of a given species i is defined as:

$$x_i \equiv \frac{m_i}{m} = \frac{\rho_i}{\rho} \quad (A.10)$$

where m_i (kg_i) is the mass of species i , and m (kg) is the mass of the gas mixture. The mass fractions are related as:

$$\sum_i x_i = x_v + x_a = 1 \quad (A.11)$$

SPECIFIC HUMIDITY

The specific humidity or mixing ratio q (kg kg^{-1}) is defined as:

$$q \equiv \frac{m_v}{m} \quad (\text{A.12})$$

where it the mass ratio of water vapor to the total mass. It also satisfies:

$$q = \frac{\rho_v}{\rho} = \frac{\rho - \rho_a}{\rho} \quad (\text{A.13})$$

and so solving for total density, we obtain:

$$\rho = \frac{\rho_a}{1 - q} \quad (\text{A.14})$$

It is related to the humidity ratio w (kg kg^{-1}) as follows:

$$q = \frac{w}{1 + w} \quad (\text{A.15})$$

HUMIDITY RATIO

The humidity ratio w (kg kg^{-1}) (i.e., or moisture content or mixing ratio) is the ratio of mass of water vapor to dry air (ASHRAE 2013):

$$w \equiv \frac{m_v}{m_a} = \frac{x_v}{x_a} \quad (\text{A.16})$$

Substituting, [A.11](#) into [A.16](#) we get

$$w = \frac{x_v}{1 - x_v} \quad (\text{A.17})$$

We can rewrite [A.17](#) to determine x_v from w :

$$x_v = \frac{w}{1 + w} \quad (\text{A.18})$$

We can determine w from p_v :

$$w = \frac{p_v / R_v T}{p_a / R_a T} = \frac{p_v}{p_a} \frac{R_a}{R_v} \quad (\text{A.19})$$

and substituting A.5 into A.19, we get:

$$w = \frac{p_v}{p - p_v} \frac{R_a}{R_v} \quad (\text{A.20})$$

and rewriting A.20 for p_v , we get:

$$p_v = \frac{pw}{R_a/R_v + w} \quad (\text{A.21})$$

ENTHALPY

The specific enthalpy h (J kg^{-1}) is defined as:

$$h = e + \frac{p}{\rho} \quad (\text{A.22})$$

where e is the specific internal energy. For perfect gas, we have:

$$de = c_v dT \quad (\text{A.23})$$

$$dh = c_p dT \quad (\text{A.24})$$

where c_v and c_p are the specific heats at constant volume and pressure, respectively.

Assumption: We assume that the moist air is calorically perfect gas, i.e. $c_v = \text{constant}$, $c_p = \text{constant}$

The specific enthalpy is therefore:

$$h = c_p \int_{T_{ref}}^T dT = c_p (T - T_{ref}) \quad (\text{A.25})$$

where we take T_{ref} at (0°C).

Assumption: We assume thermal equilibrium between all the mixtures, i.e., $T = T_a = T_v$ (K) (Defraeye 2011).

Assumption: We assume binary gas mixture of dry air and water vapor and assume CO_2 does not play a role in conservation of energy.

Therefore, the specific enthalpy of moist air is

$$h = \sum_i x_i h_i = x_a h_a + x_v h_v \quad (\text{A.26})$$

where x_a and x_v (kg kg^{-1}) are dry air and vapor mass concentration, and h_a and h_v (J kg^{-1}) are:

$$h_a = c_{pa} (T - T_{ref}) \quad (\text{A.27})$$

and

$$h_v = c_{pv} (T - T_{ref}) + L_v \quad (\text{A.28})$$

where L_v (J kg^{-1}) is the latent heat of vaporization (or more accurately referenced at a reference temperature). The reference temperature T_{ref} (K) is usually taken to be 0°C , i.e., $T_{ref} = 273.15$ K and the latent heat of vaporization taken at the same temperature $L_v = 2.5 \times 10^6$ J kg^{-1} at 0°C . Thus the specific heat of moist air h (J kg^{-1}) becomes:

$$h = (x_a c_{pa} + x_v c_{pv}) (T - T_{ref}) + x_v L_v \quad (\text{A.29})$$

where the total specific heat capacity of gas c_p (J kg^{-1}) is defined as:

$$c_p = x_a c_{pa} + x_v c_{pv} \quad (\text{A.30})$$

Assumption: Typically, for atmospheric flows with relatively low water vapor concentration, we can assume $x_v c_{pv} \ll x_a c_{pa}$ and so:

$$h \approx x_a c_{pa} (T - T_{ref}) + x_v L_v \quad (\text{A.31})$$

B

CONSERVATION EQUATIONS OF MOIST FLUID FLOW

This appendix consists of detailed derivation of conservation of mass, momentum, energy and species of moist fluid flow. Additional details are also provided in the thesis of Defraeye (2011). The following chapter derives the conservation equation as is in the conservative form.

B.1 CONSERVATION PRINCIPLE

Let us consider a fluid flow in Euclidean vector space $(\mathbb{R}^3, || \cdot ||)$. The volume of fluid of interest (or control volume CV) is $\Omega \in \mathbb{R}^3$, bounded a surface $\partial\Omega$, with coordinate vector $x = (x, y, z) \in \mathbb{R}^3$. An extensive property in Ω is defined as an integral of the intensive property:

$$\Phi(x, t) \equiv \int_{\Omega} \phi(x, t) dV \quad (\text{B.1})$$

where Φ is the extensive property of interest, and ϕ is the intensive property. For example, the extensive property mass m (kg), is the integral of density ρ (kg m^{-3}), an intensive property of the fluid.

The Reynolds transport theorem is:

$$\begin{aligned} & \text{Rate of increase of } \Phi \text{ in domain } \Omega \\ &= \\ & \text{Net rate of transfer of } \phi \text{ due to } \textit{advection} \text{ at boundary } \partial\Omega \\ &+ \\ & \text{Net rate of transfer of } \phi \text{ due to } \textit{diffusion} \text{ at boundary } \partial\Omega \\ &+ \\ & \text{Net rate of transfer of } \phi \text{ within domain } \Omega \text{ (source/sink)} \end{aligned}$$

In simpler terms, the rate of increase of an extensive property Φ in the domain Ω over time is simply due to rate of transfer of the intensive property ϕ at the boundary of the domain Ω , i.e., $\partial\Omega$ due to convection and the net rate

of transfer of intensive property ϕ within the domain Ω . The rate of transfer of ϕ at the boundary is also commonly referred to as a *flux* of ϕ at the boundary. Also note that a *convection* is sum of *advection* (transport due to velocity) and *diffusion* (transport due to molecular transfer). The rate of transfer of intensive property ϕ within the domain Ω is the source or sinks of ϕ in the domain. The mathematical formulation of the Reynolds transport theorem is given as:

$$\frac{d}{dt}\Phi(\mathbf{x}, t) = \frac{d}{dt} \int_{\Omega} \phi \, dV = - \int_{\partial\Omega} \phi \, \mathbf{u} \cdot \hat{\mathbf{n}} \, dA + \int_{\Omega} s \, dV \quad (\text{B.2})$$

where \mathbf{u} is velocity, $\hat{\mathbf{n}}$ is the normal vector (*pointing outward*), and s is the positive source term in Ω . Using the divergence theorem, surface integral can be converted into volume integral

$$\int_{\partial\Omega} \phi \, \mathbf{u} \cdot \hat{\mathbf{n}} \, dA = \int_{\Omega} \nabla \cdot (\phi \mathbf{u}) \, dV \quad (\text{B.3})$$

and so B.2 becomes:

$$\frac{d}{dt} \int_{\Omega} \phi \, dV = - \int_{\Omega} \nabla \cdot (\phi \mathbf{u}) \, dV + \int_{\Omega} s \, dV \quad (\text{B.4})$$

Using the Leibniz's rules, the integral and derivative is switched assuming the ϕ is continuous in time and space:

$$\int_{\Omega} \frac{\partial}{\partial t} \phi \, dV = - \int_{\Omega} \nabla \cdot (\phi \mathbf{u}) \, dV + \int_{\Omega} s \, dV \quad (\text{B.5})$$

and by combining the terms under the same integral simplifies to:

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \mathbf{u}) = s \quad (\text{B.6})$$

resulting in the well-known conservation form (or divergence form). Note that the second term in the LHS is a dyad or outer product, i.e.:

$$\phi \mathbf{u} \equiv \phi_i \mathbf{u}_j = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \begin{pmatrix} u_1 & u_2 & u_3 \end{pmatrix} = \begin{pmatrix} \phi_1 u_1 & \phi_1 u_2 & \phi_1 u_3 \\ \phi_2 u_1 & \phi_2 u_2 & \phi_2 u_3 \\ \phi_3 u_1 & \phi_3 u_2 & \phi_3 u_3 \end{pmatrix} \quad (\text{B.7})$$

where the product of two rank-1 tensor (*vector*) results in a rank-2 tensor (*tensor*).

B.2 CONSERVATION OF MASS

The mass of moist air m (kg) is assumed to consist of dry air m_a (kg), water vapor m_v (kg) and carbon-dioxide (i.e., CO₂) m_c (kg).

$$m = m_a + m_v + m_c \quad (\text{B.8})$$

Assumption: *We do not model variation in oxygen concentration from photosynthesis, $m_o = \text{constant}$.*

B.2.1 Deriving conservation of mass

Applying the Reynolds transport theorem for mass, we can derive the conservation of mass in the fluid domain:

$$\begin{aligned} & \text{Rate of increase of mass in domain } \Omega \\ & = \\ & \text{Net rate of transfer of density by advection at boundary } \partial\Omega \\ & + \\ & \text{Net rate of transfer of density by diffusion at boundary } \partial\Omega \\ & + \\ & \text{Net rate of transfer of density within the domain } \Omega \text{ (source/sink)} \end{aligned}$$

The net mass of gas mixture in the domain is given as:

$$m = \int_{\Omega} \rho \, dV \quad (\text{B.9})$$

where ρ (kg m⁻³) is density of the gas mixture. The conservation principle also applies for each individual species i is given as:

$$m_i = \int_{\Omega} \rho_i \, dV \quad (\text{B.10})$$

The conservation of mass of individual species is given as:

$$\frac{d}{dt} m_i = \frac{d}{dt} \int_{\Omega} \rho_i \, dV = - \int_{\partial\Omega} \mathbf{g}_i \cdot \hat{\mathbf{n}} \, dA + \int_{\Omega} s_{\rho,i} \, dV \quad (\text{B.11})$$

where \mathbf{g}_i (kg m⁻² s⁻¹) is the mass flux of species i at the boundary, and $s_{\rho,i}$ (kg m⁻³ s⁻¹) is the source of mass in domain Ω . The rate of loss or gain of gas mixture at the boundary of the domain is sum of convection due bulk

fluid motion and diffusion resulted by the concentration gradient. The net flux of density from the control volume Ω is a combined convection-diffusion equation:

$$\int_{\partial\Omega} \mathbf{g}_i \cdot \hat{\mathbf{n}} \, dA = \int_{\partial\Omega} \left(-\rho D \nabla \frac{\rho_i}{\rho} + \rho_i \mathbf{u} \right) \cdot \hat{\mathbf{n}} \, dA \quad (\text{B.12})$$

where \mathbf{g}_i ($\text{kg m}^{-2} \text{s}^{-1}$) is the mass flux of species i , $x_i \equiv \rho_i / \rho$ (kg kg^{-1}) is the mass concentration of species i , D is the mass diffusivity ($\text{m}^2 \text{s}$) of species i and \mathbf{u} is the bulk / mass-averaged velocity:

$$\mathbf{u} = \frac{\sum_i \rho_i \mathbf{u}_i}{\sum_i \rho_i} \quad (\text{B.13})$$

The divergence theorem transforms B.12 into:

$$\int_{\partial\Omega} \mathbf{g}_i \cdot \hat{\mathbf{n}} \, dA = \int_{\Omega} \nabla \cdot \mathbf{g}_i \, dV = \int_{\Omega} \nabla \cdot \left(-\rho D \nabla \frac{\rho_i}{\rho} + \rho_i \mathbf{u} \right) \, dV \quad (\text{B.14})$$

The resulting conservation of mass for individual species is given as:

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot \left(\rho_i \mathbf{u} - \rho D \nabla \frac{\rho_i}{\rho} \right) = s_{\rho,i} \quad (\text{B.15})$$

We have following system of equation for quaternary mixture of dry air, water vapor and CO_2 :

$$\frac{\partial \rho_a}{\partial t} + \nabla \cdot \left(\rho_a \mathbf{u} - \rho D \nabla \frac{\rho_a}{\rho} \right) = s_{\rho,a} \quad (\text{B.16})$$

$$\frac{\partial \rho_v}{\partial t} + \nabla \cdot \left(\rho_v \mathbf{u} - \rho D \nabla \frac{\rho_v}{\rho} \right) = s_{\rho,v} \quad (\text{B.17})$$

$$\frac{\partial \rho_c}{\partial t} + \nabla \cdot \left(\rho_c \mathbf{u} - \rho D \nabla \frac{\rho_c}{\rho} \right) = s_{\rho,c} \quad (\text{B.18})$$

where $s_{\rho,a}$, $s_{\rho,v}$ and $s_{\rho,c}$ are the mass source terms ($\text{kg m}^{-3} \text{s}^{-1}$).

B.2.2 Source of mass

Assumption: We assume there is no dry air generated in the fluid, $s_{\rho,a} = 0$.

Assumption: We assume that the source of water vapor is only due to leaf transpiration, i.e., $s_{\rho,v} = g_{v,leaf}$. Water vapor condensation to water droplets or droplet evaporation and sublimation from ice is neglected.

The leaves in control volumes Ω generate water vapour (from the transpiration process) and extract CO_2 during photosynthesis. Therefore, the source of water vapour and CO_2 in control volume is:

$$s_{\rho,v} = a g_{v,leaf} \quad (\text{B.19})$$

$$s_{\rho,c} = a g_{c,leaf} \quad (\text{B.20})$$

where a is the leaf area density (m^2m^{-3}), and $g_{v,leaf}$ ($\text{kg m}^{-2}\text{s}^{-1}$) and $g_{c,leaf}$ ($\text{kg m}^{-2}\text{s}^{-1}$) are the water vapour and CO_2 mass flux from the surface of the leaf.

Thus the conservation of mass of gas mixture is written as:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = s_{\rho,v} + s_{\rho,c} \quad (\text{B.21})$$

Assumption: The order of magnitude of water vapor mass source $\mathcal{O}(s_{\rho,v}) \approx 10^{-4}$ and CO_2 mass source $\mathcal{O}(s_{\rho,c}) \approx 10^{-6}$ (Hiraoka 2005). Therefore the mass source of CO_2 is negligible compared to water vapour.

Assumption: The continuity equation is used to solve the momentum equation. In the momentum equation, the momentum contribution due to the mass source can be assumed to be negligible w.r.t to the drag force terms.

Therefore, when solving the Navier-Stokes equations, the conservation of mass can be simplified to

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (\text{B.22})$$

B.3 CONSERVATION OF MOMENTUM

B.3.1 Newton's second law of motion

Newton's second law of motion is given as:

$$\mathbf{F} = \frac{d}{dt} m \mathbf{u} = \frac{d}{dt} \int_{\Omega} \rho \mathbf{u} dV \quad (\text{B.23})$$

where F (N) is the net force. The forces that act on the fluid are body forces (directly on volumetric mass) such as gravitational, electric, magnetic and if we have vegetation if modelled as porous media (source/sink). The vegetation is not directly modeled, but their contribution to the fluid is taken as source and sink terms in conservation equations. The surface forces are acting on the surface of fluid, such as pressure force, normal and shear stresses on the surface. These contribution can be regarded influences from neighboring fluid parcels.

B.3.2 Deriving conservation of momentum

Conservation of momentum is given as:

$$\begin{aligned}
 & \text{Rate of increase of momentum in } \Omega \\
 & = \\
 & \text{Net rate of transfer of momentum by advection at the boundary } \partial\Omega \\
 & + \\
 & \text{Net rate of transfer of momentum by diffusion at the boundary } \partial\Omega \\
 & + \\
 & \text{Net rate of transfer of momentum by external forces on domain } \Omega \\
 & + \\
 & \text{Net rate of transfer of momentum within } \Omega \text{ (source/sinks)}
 \end{aligned}$$

In addition to the convective loss of momentum at the boundary, in Reynolds transport theorem, stress at the boundary of the domain results in loss of momentum. The Cauchy stress tensor $\bar{\sigma}$ (Pa or N m^{-2} or $\text{kg m}^{-1} \text{s}^{-2}$) is defined as:

$$\bar{\sigma} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix} \quad (\text{B.24})$$

where the diagonal terms are normal stresses and the non-diagonal terms are the shear stresses.

The resulting Reynolds transport theorem for momentum equation of a gas mixture,

$$F = \frac{d}{dt} \int_{\Omega} \rho u \, dV = - \int_{\partial\Omega} (\rho \mathbf{u} u - \bar{\sigma}) \cdot \hat{\mathbf{n}} \, dA + \int_{\Omega} \mathbf{f} \, dV + \int_{\Omega} \mathbf{s}_u \, dV \quad (\text{B.25})$$

Applying the divergence theorem, we get

$$\frac{d}{dt} \int_{\Omega} \rho \mathbf{u} \, dV = - \int_{\Omega} \nabla \cdot (\rho \mathbf{u} \mathbf{u}) \, dV + \int_{\Omega} \nabla \cdot \bar{\bar{\sigma}} \, dV + \int_{\Omega} \mathbf{f} \, dV + \int_{\Omega} \mathbf{s}_u \, dV \quad (\text{B.26})$$

And combining under same integral, assuming momentum is continuous in space and time:

$$\int_{\Omega} \left[\frac{\partial}{\partial t} (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) - \nabla \cdot \bar{\bar{\sigma}} - \mathbf{f} - \mathbf{s}_u \right] dV = 0 \quad (\text{B.27})$$

Therefore, the following relationship also satisfies (N m^{-3}):

$$\frac{\partial}{\partial t} (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = \nabla \cdot \bar{\bar{\sigma}} + \mathbf{f} + \mathbf{s}_u \quad (\text{B.28})$$

The cauchy stress tensor $\bar{\bar{\sigma}}$ (N m^{-2}) can be decomposed into the deviatoric and the hydrostatic component:

$$\bar{\bar{\sigma}} = \underbrace{\frac{1}{3} \text{tr}(\bar{\bar{\sigma}}) \mathbf{I}}_{\text{hydrostatic}} + \underbrace{\bar{\bar{\sigma}} - \frac{1}{3} \text{tr}(\bar{\bar{\sigma}}) \mathbf{I}}_{\text{deviatoric}} \quad (\text{B.29})$$

The hydrostatic stress component is the isotropic pressure, $\text{tr}(\bar{\bar{\sigma}})/3 = -p$ and the deviatoric component is the shear-stress tensor $\bar{\bar{\tau}}$. Thus, the Cauchy stress tensor becomes:

$$\bar{\bar{\sigma}} = -p \mathbf{I} + \bar{\bar{\tau}} \quad (\text{B.30})$$

Assumption: We assume Newtonian fluid, and therefore the viscous shear stress is symmetric and assumed to be linearly proportional to the local strain rate and equivalently the velocity gradient, i.e., $\tau_{ij} \propto \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$.

Applying the Newtonian fluid hypothesis:

$$\bar{\bar{\tau}} = \mu \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T - \frac{2}{3} (\nabla \cdot \mathbf{u}) \mathbf{I} \right) + \lambda (\nabla \cdot \mathbf{u}) \mathbf{I} \quad (\text{B.31})$$

where μ (N s m^{-2} or $\text{kg m}^{-1} \text{s}^{-1}$) is the first coefficient of viscosity (i.e. dynamic viscosity), and λ (N s m^{-2}) is the second coefficient of viscosity. Thus:

$$\begin{aligned} \frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = \\ \nabla \cdot \left[-p \mathbf{I} + \mu \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T - \frac{2}{3} (\nabla \cdot \mathbf{u}) \mathbf{I} \right) + \lambda (\nabla \cdot \mathbf{u}) \mathbf{I} \right] \\ + \mathbf{f} + \mathbf{s}_u \end{aligned} \quad (\text{B.32})$$

and so:

$$\begin{aligned} \frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot \left[\mu \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) \right] \\ + \nabla \cdot \left[\left(\lambda - \frac{2}{3} \mu \right) (\nabla \cdot \mathbf{u}) \mathbf{I} \right] + \mathbf{f} + \mathbf{s}_u \end{aligned} \quad (\text{B.33})$$

Assumption: We assume the only body forces are due to gravitational acceleration.

$$\begin{aligned} \frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot \left[\mu \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) \right] \\ + \nabla \cdot \left[\left(\lambda - \frac{2}{3} \mu \right) (\nabla \cdot \mathbf{u}) \mathbf{I} \right] + \rho \mathbf{g} + \mathbf{s}_u \end{aligned} \quad (\text{B.34})$$

Assumption: If we assume divergence-free velocity, i.e., $\nabla \cdot \mathbf{u} = 0$, the shear-stress tensor is simply: $\bar{\tau} = \mu \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right)$.

B.3.3 Source of momentum

The source term due to vegetation, i.e. \mathbf{s}_u ($\text{kg m}^{-2} \text{s}^{-1}$), the volumetric force exerted by vegetation is modeled through Darcy-Forchheimer law for the flow through porous medium:

$$\mathbf{s}_u = - \left(\left[\frac{\mu}{K} \right] \mathbf{u} + \rho \left[\frac{C_F}{\sqrt{K}} \right] |\mathbf{u}| \mathbf{u} \right) \quad (\text{B.35})$$

where K (m^2) is permeability, C_F is the Forchheimer coefficient for non-linear momentum loss (**Boulard2008**; Verboven et al. 2006).

Assumption: For high wind speeds, that the Forcheimer term becomes the dominant influence due to the quadratic relation. Furthermore, the flow in vegetation is this regime. Therefore, we assume the linear Darcy term is negligible.

The Forchheimer term for flow past porous screens (or windbreaks) with thickness δt (m) is:

$$\frac{C_F}{\sqrt{k_p}} = \frac{C_D}{\delta t} \quad (\text{B.36})$$

For vegetation the non-linear term is expressed in terms of leaf area density a ($\text{m}_{leaf}^2 \text{m}^{-3} \rightarrow \text{m}^{-1}$) and leaf drag coefficient c_d :

$$\frac{C_F}{\sqrt{k_p}} = c_d a \quad (\text{B.37})$$

and the source term of vegetation as porous medium becomes (Hiraoka; Wilson and Shaw 1977; Liu et al. 1996; Kenjereš and ter Kuile 2013):

$$s_u = -\rho c_d a |\mathbf{u}| \mathbf{u} \quad (\text{B.38})$$

Thus, the momentum transport equation becomes:

$$\frac{\partial}{\partial t} (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot \left[\mu \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) \right] + \rho \mathbf{g} - \rho c_d a |\mathbf{u}| \mathbf{u} \quad (\text{B.39})$$

B.3.4 Deriving drag coefficient

When performing measurements, the drag coefficient is characterized w.r.t to the frontal surface area of vegetation $A_{frontal}$. The measured drag coefficient C_d is related to the leaf drag coefficient c_d from the following relationship: The total force on vegetation F_d (N) is given as:

$$F_d = \frac{1}{2} \rho U_\infty^2 C_d A_{frontal} \quad (\text{B.40})$$

and should be equal to the volumetric force, i.e.:

$$F_d = - \int_{\Omega} s_u \, dV = \int_{\Omega} \rho c_d a |\mathbf{u}| \mathbf{u} \, dV \quad (\text{B.41})$$

Assumption: We assume $\rho = \text{constant}$ and $c_d = \text{constant}$.

So, equation Eq. (B.40) to Eq. (B.40) and substituting U_∞ into \mathbf{u} , we get:

$$\frac{1}{2}\rho U_\infty^2 C_d A_{frontal} = \rho c_d U_\infty^2 \int_{\Omega} a \, dV \quad (\text{B.42})$$

and so, the leaf drag coefficient becomes:

$$c_d = C_d \frac{A_{frontal}}{2A_{leaf}} \quad (\text{B.43})$$

as the integral of leaf area density a ($\text{m}^2 \text{m}^{-2}$) is:

$$\int_{\Omega} a \, dV = A_{leaf} \quad (\text{B.44})$$

where A_{leaf} (m) is the total leaf surface area.

B.4 CONSERVATION OF ENERGY

B.4.1 First law of thermodynamics

First law of thermodynamics:

$$dE = \delta Q + \delta W \quad (\text{B.45})$$

where dE (J or $\text{kg m}^2 \text{s}^{-2}$) is the change in total internal energy of the system, δQ (J) is the heat added to the system, and δW (J) is the work done on the system. The total energy per unit mass of the mixture \hat{E} is defined as:

$$E = \int_{\Omega} \rho \hat{E} dV \quad (\text{B.46})$$

where \hat{E} (J kg^{-1}) (an intensive property) is related total energy E (J) (an extensive property). The total energy per unit mass of the mixture \hat{E} is given as:

$$\hat{E} = e + \frac{|\mathbf{u}|^2}{2} + gz \quad (\text{B.47})$$

where e (J kg^{-1}) is the internal energy per unit mass, $|\mathbf{u}|^2/2$ (J kg^{-1}) is the kinetic energy per unit mass, and gz (J kg^{-1}) is the potential energy per unit mass. The internal energy of a gas mixture can be related to enthalpy and kinetic theory of gas:

$$e = h - RT = h - \frac{p}{\rho} \quad (\text{B.48})$$

where R ($\text{J kg}^{-1} \text{K}^{-1}$) is the gas constant. The total enthalpy h (J kg^{-1}) of the gas mixture is defined in Appendix A, i.e., Eq. (A.29) and given as:

$$h = \sum_i x_i h_i = (x_a c_{pa} + x_v c_{pv}) (T - T_{ref}) + x_v L_v \quad (\text{B.49})$$

Therefore, total energy (per unit mass) of the gas mixture \hat{E} (J kg^{-1}) is:

$$\hat{E} = \sum_i x_i h_i - \frac{p}{\rho} + \frac{|\mathbf{u}|^2}{2} + gz \quad (\text{B.50})$$

and substituting Eq. (B.49) into Eq. (B.50):

$$\hat{E} = (x_a c_{pa} + x_v c_{pv}) (T - T_{ref}) + x_v L_v - \frac{p}{\rho} + \frac{|\mathbf{u}|^2}{2} + gz \quad (\text{B.51})$$

B.4.2 Deriving conservation of energy

The conservation of energy is given as:

$$\begin{aligned} & \text{Rate of change of total energy in domain } \Omega \\ & = \\ & \text{Rate of transfer of energy by advection at boundary } \partial\Omega \\ & + \\ & \text{Rate of transfer of heat by heat diffusion at boundary } \partial\Omega \\ & + \\ & \text{Rate of transfer of heat by mass diffusion at boundary } \partial\Omega \\ & + \\ & \text{Rate at which work done by boundary } \Omega \\ & + \\ & \text{Rate of transfer of heat by within } \Omega \text{ (source/sink)} \end{aligned}$$

Therefore, the conservation of energy (J s^{-1} or W) is given as:

$$\frac{d}{dt} E = \frac{d}{dt} \int_{\Omega} \rho \hat{E} dV \quad (\text{B.52})$$

and applying the Reynolds transport theorem, detailed in Appendix B.1, we attain conservation of energy (W m^{-3}) as:

$$\frac{\partial}{\partial t} (\rho \hat{E}) + \nabla \cdot (\rho \hat{E} \mathbf{u}) = -\nabla \cdot \mathbf{q} - \nabla \cdot (p \mathbf{u}) - \nabla \cdot (\bar{\boldsymbol{\tau}} \cdot \mathbf{u}) + s_h \quad (\text{B.53})$$

where \mathbf{q} is the total heat flux due to heat diffusion and mass diffusion, $\nabla \cdot (p \mathbf{u})$ (W m^{-3} or $\text{J m}^{-3} \text{s}^{-1}$ or $\text{kg m}^{-3} \text{s}^{-1}$) is the work done due to pressure force, $\nabla \cdot (\bar{\boldsymbol{\tau}} \cdot \mathbf{u})$ (W m^{-3}) is the work done due to viscous force, and s_h (W m^{-3}) rate of energy added into the system, i.e. the energy source.

Assumption: We assume pressure work is negligible due to incompressible flow approximation, and viscous heat is neglected as well (Defraeye 2011). Furthermore, the source of energy is simply due to radiation (at walls) and from vegetation.

The total heat flux \mathbf{q} (W m^{-2}) is the total heat flux due to heat diffusion (i.e., conduction) and mass diffusion is given as:

$$\mathbf{q} = \mathbf{q}_c + \mathbf{q}_d \quad (\text{B.54})$$

where \mathbf{q}_c (W m^{-2}) is the heat flux due to heat diffusion (i.e., conduction) and \mathbf{q}_d (W m^{-2}) is the heat flux due to mass diffusion. The conduction term is defined by the Fourier law of heat conduction:

$$\mathbf{q}_c = -\lambda \nabla T \quad (\text{B.55})$$

where $\lambda = c_p \mu / \text{Pr}$ ($\text{W m}^{-1} \text{K}^{-1}$) is the thermal conductivity, where c_p ($\text{J kg}^{-1} \text{K}^{-1}$) is the specific heat capacity of gas mixture, μ ($\text{kg m}^{-1} \text{s}^{-1}$) is the dynamic viscosity of gas mixture. The heat flux due to mass flux \mathbf{q}_d (W m^{-2}) is given as:

$$\mathbf{q}_d = \sum_i h_i \mathbf{g}_{d,i} \quad (\text{B.56})$$

where $\mathbf{g}_{d,i}$ ($\text{kg m}^{-2} \text{s}^{-1}$) diffusive flux of species i contributing to the heat transport. So:

$$\mathbf{q} = -\lambda \nabla T + \sum_i h_i \mathbf{g}_{d,i} \quad (\text{B.57})$$

Simplifying Eq. (B.53), we obtain:

$$\frac{\partial}{\partial t} (\rho \hat{E}) + \nabla \cdot (\rho \hat{E} \mathbf{u}) = -\nabla \cdot \mathbf{q} + s_h \quad (\text{B.58})$$

We expand B.58 by substituting total energy per unit mass \hat{E} (J kg^{-1}), B.50, giving:

$$\begin{aligned} & \frac{\partial}{\partial t} \left\{ \rho \left(\sum_i x_i h_i - \frac{p}{\rho} + \frac{|\mathbf{u}|^2}{2} + gz \right) \right\} \\ & + \nabla \cdot \left\{ \rho \left(\sum_i x_i h_i - \frac{p}{\rho} + \frac{|\mathbf{u}|^2}{2} + gz \right) \mathbf{u} \right\} \\ & = -\nabla \cdot \mathbf{q} + s_h \end{aligned} \quad (\text{B.59})$$

and taking ρ inside and substituting $\rho_i = \rho x_i$ gives:

$$\begin{aligned} \frac{\partial}{\partial t} \left(\sum_i \rho_i h_i - p + \frac{\rho |\mathbf{u}|^2}{2} + \rho g z \right) \\ + \nabla \cdot \left\{ \left(\sum_i \rho_i h_i - p + \frac{\rho |\mathbf{u}|^2}{2} + \rho g z \right) \mathbf{u} \right\} \\ = -\nabla \cdot \mathbf{q} + s_h \end{aligned} \quad (\text{B.60})$$

Assumption: We assume pressure, potential energy and kinetic energy variation is small.

Therefore, Eq. (B.60) further simplifies to:

$$\frac{\partial}{\partial t} \left(\sum_i \rho_i h_i \right) + \nabla \cdot \left(\left(\sum_i \rho_i h_i \right) \mathbf{u} \right) = -\nabla \cdot \mathbf{q} + s_h \quad (\text{B.61})$$

Substituting enthalpies into Eq. (B.61), the conservation of moist air becomes:

$$\begin{aligned} \frac{\partial}{\partial t} (\rho_a c_{pa} (T - T_{ref}) + \rho_v c_{pv} (T - T_{ref}) + \rho_v L_v) \\ + \nabla \cdot ((\rho_a c_{pa} (T - T_{ref}) + \rho_v c_{pv} (T - T_{ref}) + \rho_v L_v) \mathbf{u}) \\ = -\nabla \cdot \mathbf{q} + s_h \end{aligned} \quad (\text{B.62})$$

Substituting, the total heat flux \mathbf{q} (W m^{-2}) Eq. (B.54), into Eq. (B.62), we obtain conservation equation of the form:

$$\begin{aligned} \frac{\partial}{\partial t} (\rho_a c_{pa} (T - T_{ref}) + \rho_v c_{pv} (T - T_{ref}) + \rho_v L_v) \\ + \nabla \cdot ((\rho_a c_{pa} (T - T_{ref}) + \rho_v c_{pv} (T - T_{ref}) + \rho_v L_v) \mathbf{u}) \\ = -\nabla \cdot \left(-\lambda \nabla T + \sum_i h_i \mathbf{g}_{d,i} \right) + s_h \end{aligned} \quad (\text{B.63})$$

and as $\rho_i \mathbf{u}_i = \rho_i \mathbf{u} + \mathbf{g}_{d,i}$ (see Appendix B.2), Eq. (B.63) can be simplified to:

$$\begin{aligned} \frac{\partial}{\partial t} (\rho_a c_{pa} (T - T_{ref}) + \rho_v c_{pv} (T - T_{ref}) + \rho_v L_v) \\ + \nabla \cdot (\rho_a c_{pa} (T - T_{ref}) \mathbf{u}_a + \rho_v c_{pv} (T - T_{ref}) \mathbf{u}_v + \rho_v L_v \mathbf{u}_v) \\ = \nabla \cdot (\lambda \nabla T) + s_h \end{aligned} \quad (\text{B.64})$$

Therefore, the conservation of energy Eq. (B.64) can be rewritten for separate species as follows:

$$\begin{aligned}
 & c_{pa} \left(\frac{\partial}{\partial t} (\rho_a T) + \nabla \cdot (\rho_a T \mathbf{u}_a) \right) + c_{pv} \left(\frac{\partial}{\partial t} (\rho_v T) + \nabla \cdot (\rho_v T \mathbf{u}_v) \right) \\
 & - c_{pa} T_{ref} \left(\frac{\partial}{\partial t} (\rho_a) + \nabla \cdot (\rho_a \mathbf{u}_a) \right) - c_{pv} T_{ref} \left(\frac{\partial}{\partial t} (\rho_v) + \nabla \cdot (\rho_v \mathbf{u}_v) \right) \\
 & + L_v \left(\frac{\partial}{\partial t} (\rho_v) + \nabla \cdot (\rho_v \mathbf{u}_v) \right) = \nabla \cdot (\lambda \nabla T) + s_h
 \end{aligned} \tag{B.65}$$

B.4.3 Source of energy

The source of energy s_h (W m^{-3}) is given defined as:

$$s_h = a (q_{c,leaf} + q_{d,leaf}) \tag{B.66}$$

where a ($\text{m}^2 \text{m}^{-3}$) is leaf area density, $q_{c,leaf}$ (W m^{-2}) is heat flux from leaf surface due to conduction (i.e., heat diffusion), and $q_{d,leaf}$ (W m^{-2}) is the heat flux due to mass diffusion from the leaf surface. Expanding, the mass diffusion term in Eq. (B.66) gives:

$$s_h = a \left(q_{c,leaf} + \sum_i h_i g_{i,leaf} \right) \tag{B.67}$$

where $g_{i,leaf}$ is the mass flux of species i from the leaf surface. And so Eq. (B.67) becomes:

$$s_h = a (q_{c,leaf} + h_v g_{v,leaf} + h_c g_{c,leaf}) \tag{B.68}$$

where $g_{v,leaf}$ ($\text{kg m}^{-2} \text{s}^{-1}$) and $g_{c,leaf}$ ($\text{kg m}^{-2} \text{s}^{-1}$) are mass fluxes of water vapor and CO_2 , respectively.

Assumption: Note that, we assumed there is no flux of oxygen O_2 .

The conductive heat flux $q_{c,leaf}$ (W m^{-2}) is given as:

$$q_{c,leaf} = h_{c,h} (T_l - T) \tag{B.69}$$

where $h_{c,h}$ ($\text{W m}^{-2} \text{K}^{-1}$) is the convective heat transfer coefficient of the leaf and T_l (K) is the leaf temperature. The source of energy due to water vapor flux $g_{v,leaf}$ is given as:

$$h_v g_{v,leaf} = c_{pv} (T_l - T) g_{v,leaf} + L_v g_{v,leaf} \tag{B.70}$$

where $g_{v,leaf}$ ($\text{kg m}^{-2} \text{s}^{-1}$) is the leaf transpiration rate. Similarly, the source of energy due to CO_2 flux $g_{c,leaf}$ is given as:

$$h_c g_{c,leaf} = c_{pc} (T_l - T) g_{c,leaf} \quad (\text{B.71})$$

where $g_{v,leaf}$ ($\text{kg m}^{-2} \text{s}^{-1}$) is the leaf CO_2 assimilation rate (i.e., mass flux of CO_2 due to the photosynthetic process).

Thus, substituting Eqs. (B.69) and (B.71) into Eq. (B.68), and decompose to sensible (i.e., temperature dependent) and latent component, we arrive at:

$$s_h = a \left(\underbrace{[h_{c,h} + c_{pv} g_{v,leaf} + c_{pc} g_{c,leaf}] (T_l - T)}_{\text{sensible}} + \underbrace{L_v g_{v,leaf}}_{\text{latent}} \right) \quad (\text{B.72})$$

Assumption: We assume the sensible heat flux contribution of water vapor and CO_2 is negligible compared to the conductive heat flux (Hiraoka 2005), i.e., $(c_{pv} g_{v,leaf} + c_{pc} g_{c,leaf}) \ll h_{c,h}$.

Thus, the source of energy s_h (W m^{-3}), Eq. (B.72), simplifies to:

$$s_h = a (h_{c,h} (T_l - T) + L_v g_{v,leaf}) \quad (\text{B.73})$$

where:

$$q_{sen,leaf} = h_{c,h} (T_l - T) \quad (\text{B.74})$$

$$q_{lat,leaf} = L_v g_{v,leaf} \quad (\text{B.75})$$

$$(\text{B.76})$$

$q_{sen,leaf}$ (W m^{-2}) and $q_{lat,leaf}$ (W m^{-2}) is typically known as the sensible and latent heat flux from leaf in literature (Bruse and Fleer 1998; Hiraoka 2005; Manickathan et al. 2018).

Substituting Eq. (B.73) into conservation of energy becomes:

$$\begin{aligned} & c_{pa} \left(\frac{\partial}{\partial t} (\rho_a T) + \nabla \cdot (\rho_a T \mathbf{u}_a) \right) + c_{pv} \left(\frac{\partial}{\partial t} (\rho_v T) + \nabla \cdot (\rho_v T \mathbf{u}_v) \right) \\ & - c_{pa} T_{ref} \left(\frac{\partial}{\partial t} (\rho_a) + \nabla \cdot (\rho_a \mathbf{u}_a) \right) - c_{pv} T_{ref} \left(\frac{\partial}{\partial t} (\rho_v) + \nabla \cdot (\rho_v \mathbf{u}_v) \right) \\ & + L_v \left(\frac{\partial}{\partial t} (\rho_v) + \nabla \cdot (\rho_v \mathbf{u}_v) \right) = \nabla \cdot (\lambda \nabla T) + a (h_{c,h} (T_l - T) + L_v g_{v,leaf}) \end{aligned} \quad (\text{B.77})$$

Substituting conservation of mass of air Eq. (B.16) and conservation of mass of water vapor, Eq. (B.77) simplifies as:

$$\begin{aligned}
 c_{pa} \left(\frac{\partial}{\partial t} (\rho_a T) + \nabla \cdot (\rho_a T \mathbf{u}_a) \right) + c_{pv} \left(\frac{\partial}{\partial t} (\rho_v T) + \nabla \cdot (\rho_v T \mathbf{u}_v) \right) \\
 - c_{pv} T_{ref} (a g_{v,leaf}) + L_v (a g_{v,leaf}) \\
 = \nabla \cdot (\lambda \nabla T) + a (h_{c,h} (T_l - T) + L_v g_{v,leaf})
 \end{aligned} \tag{B.78}$$

Assumption: We assume the term $c_{pv} T_{ref}$ is negligible, as $c_{pv} T_{ref} \ll L_v$.

$$\begin{aligned}
 c_{pa} \left(\frac{\partial}{\partial t} (\rho_a T) + \nabla \cdot (\rho_a T \mathbf{u}_a) \right) + c_{pv} \left(\frac{\partial}{\partial t} (\rho_v T) + \nabla \cdot (\rho_v T \mathbf{u}_v) \right) \\
 + L_v (a g_{v,leaf}) = \nabla \cdot (\lambda \nabla T) + a (h_{c,h} (T_l - T) + L_v g_{v,leaf})
 \end{aligned} \tag{B.79}$$

Thus, we can cancel the latent component terms on both sides, giving:

$$\begin{aligned}
 c_{pa} \left(\frac{\partial}{\partial t} (\rho_a T) + \nabla \cdot (\rho_a T \mathbf{u}_a) \right) + c_{pv} \left(\frac{\partial}{\partial t} (\rho_v T) + \nabla \cdot (\rho_v T \mathbf{u}_v) \right) \\
 = \nabla \cdot (\lambda \nabla T) + a (h_{c,h} (T_l - T))
 \end{aligned} \tag{B.80}$$

So, we see that following the assumption, the source of (sensible) energy (i.e., energy equation based on only the temperature) is simply due to sensible heat flux from the leaf surface. Thus, for such equations, the source of energy s_h (W m^{-3}) is:

$$s_h = a q_{sen,leaf} = a h_{c,h} (T_l - T) \tag{B.81}$$

The energy equation (i.e., now the transport equation of temperature), Eq. (B.80) can be further simplified, as density of moist air $\rho = \rho_a + \rho_v$ (kg m^{-3}) and heat capacity $c_p = c_{pa} + c_{pv}$ ($\text{J kg}^{-1} \text{K}^{-1}$):

$$c_p \left(\frac{\partial}{\partial t} (\rho T) + \nabla \cdot (\rho T \mathbf{u}) \right) = \nabla \cdot (\lambda \nabla T) + \underbrace{a q_{sen,leaf}}_{s_h} \tag{B.82}$$



STATISTICS OF TURBULENT FLOW

PROBABILITY

We define X a random (stochastic) variable of the turbulent flow with a set of outcomes $X = \{x_1, x_2, \dots, x_n\}$. The mean of a random variable X is defined as:

$$\langle X \rangle \equiv \mathbb{E} [X] = \frac{1}{N} \sum_{n=1}^N x_n \quad (\text{C.1})$$

where $\langle X \rangle$ is the stochastic mean, or statistical average or expected value, or ensemble average (Pope 2000; Sagaut 2006) of N realization assuming that each realization x_n is independent and are of the same distribution, i.e. *independent and identically distributed* (i.i.d) The *central-limit theorem* states that as N approaches to infinity, the distribution of X become normal (Gaussian) with a probability density function (PDF) of:

$$f(x_n; \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left\{ -\frac{1}{2} \left(\frac{x_n - \mu}{\sigma} \right)^2 \right\} \quad (\text{C.2})$$

where μ is the mean. The r -th centered moment of X is defined as:

$$\mu^r = \frac{1}{N} \sum_{n=1}^N (x_n - \langle X \rangle)^r \quad (\text{C.3})$$

with variance of X denoted as $\text{Var}(X)$ and is the second order centered moment:

$$\text{Var}(X) \equiv \mathbb{E} [(X - \mu)^2] = \frac{1}{N} \sum_{n=1}^N (x_n - \langle X \rangle)^2 \quad (\text{C.4})$$

and the standard deviation is simply:

$$\sigma = \sqrt{\text{Var}(X)} \quad (\text{C.5})$$

The higher-order moments also provide additional information such as *skewness* (3rd-order) which measures the asymmetry of the PDF and the *kurtosis* (4th-order) which measures the peakedness of the PDF.

The covariance of two random variables X and Y is defined as:

$$\text{Cov}(X, Y) \equiv \mathbb{E}[(X - \mu_X)(Y - \mu_Y)] = \frac{1}{N} \sum_{n=1}^N (x_n - \langle X \rangle)(y_n - \langle Y \rangle) \quad (\text{C.6})$$

REYNOLDS DECOMPOSITION

The stochastic variable of the turbulent flow such as the velocity u can be decomposed into mean $\langle u \rangle$ and turbulent component u' . The turbulent fluctuation u' of the random variable is defined as:

$$u' = u - \langle u \rangle \quad (\text{C.7})$$

and by definition $\langle u' \rangle \equiv 0$. So, the flow field is decomposed into:

$$u = \langle \phi \rangle + \phi' = \bar{\phi} + \phi' \quad (\text{C.8})$$

Assuming an ergodic process, we have that:

$$\langle \phi \rangle = \bar{\phi} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \phi(t) dt \quad (\text{C.9})$$

the time average is the same as average in the probability space.

TIME-AVERAGED NAVIER-STOKES

The mean velocity is defined as:

$$\bar{u} = \frac{1}{N} \sum_n^N u_n \quad (\text{C.10})$$

of N realizations assuming i.i.d. The variance is defined as:

$$\overline{u'^2} = \frac{1}{N-1} \sum_n^N (u_n - \bar{u})^2 \quad (\text{C.11})$$

with the Bessel's correction and $\sqrt{\overline{u'^2}} = \sigma_u$ is the standard deviation. The turbulent intensity I_u of u is defined as:

$$I_u = \frac{\sigma_u}{\bar{u}} \quad (\text{C.12})$$

The covariance is defined as:

$$\overline{u'v'} = \frac{1}{N-1} \sum_n^N (u_n - \bar{u}) (v_n - \bar{v}) \quad (\text{C.13})$$

For a vector velocity field $\mathbf{u} = (u, v, w)$, the covariance matrix is given as:

$$\overline{\mathbf{u}'\mathbf{u}'} = \begin{bmatrix} \overline{u'u'} & \overline{u'v'} & \overline{u'w'} \\ & \overline{v'v'} & \overline{v'w'} \\ & & \overline{w'w'} \end{bmatrix} \quad (\text{C.14})$$

and is symmetric and positive semi-definite. The covariance matrix in the context of turbulent flow is known as the *Reynolds stress tensor* \mathbf{R} . The turbulent kinetic energy (TKE) is defined as:

$$k \equiv \frac{1}{2} \text{tr}(\mathbf{R}) = \frac{1}{2} (\overline{u'u'} + \overline{v'v'} + \overline{w'w'}) \quad (\text{C.15})$$

SAMPLE UNCERTAINTY

The standard error (SE) of the sample mean velocity \bar{u} is given as:

$$\text{SE}_{\bar{u}} \equiv \sqrt{\frac{\text{Var}(u)}{N}} = \sqrt{\frac{\sigma_u^2}{N}} = \frac{\sigma_u}{\sqrt{N}} \quad (\text{C.16})$$

and quantifies the uncertainty of the mean velocity (**Wieneke2017**). The standard error of sample variance of u is:

$$\text{SE}_{\sigma_u} \equiv \sqrt{\frac{\text{Var}(\sigma_u^2)}{N}} = \sqrt{\frac{2\sigma_u^4}{N-1}} = \sigma_u^2 \sqrt{\frac{2}{N-1}} \quad (\text{C.17})$$

We assume uncorrelated samples N , however if the population is correlated, the effective number of independent samples N_{eff} is defined as:

$$N_{eff} = \frac{N}{\sum_{-\infty}^{\infty} \rho(n\Delta t)} \cong \frac{T}{2T_{int}} \quad (C.18)$$

where ρ is the auto-correlation coefficient, Δt is the sample frequency, T_{int} is the integral time scale.

where $Z_{\alpha/2}$ is the coefficient of confident ($Z_{\alpha/2} = 1.96$ for confidence level of 95%) and the relative standard error on the standard deviation (i.e. $\sqrt{R_{ii}}$) is:

$$\epsilon_{\sigma_{u_i}} = \frac{Z_{\alpha/2}}{\sqrt{N}} \quad (C.19)$$

The statistics of the flow field can be determined by performing multiple measurements of size N and performing statistics of measurement such as mean, standard deviation and further higher-moments. The primary requirements of constructing statistics is that measurement samples are *independent and identically distributed* (or iid). To obtain iid. sample, we require the samples to be uncorrelated (i.e. independent) from each other. This can be ensured by enforcing acquisition frequency of the samples are larger than the integral time scale, satisfying the Nyquist sampling theorem:

$$f_{acq} \leq \frac{1}{2\mathcal{T}} \quad (C.20)$$

where f_{acq} is acquisition frequency and T_I is the integral time scale. A crude estimate of the T_I is from the characteristics length and speed:

$$\mathcal{T} = \frac{\mathcal{L}}{U} \quad (C.21)$$

and so:

$$f_{acq} \leq \frac{U}{2\mathcal{L}} \quad (C.22)$$

Ensuring the sampling rate below f_{acq} , the obtained sample population can be ensured to be a normal distribution. Thus, the probabilistic and statistical methods can be applied to the sample population, such as calculating the mean, std. deviation and so on.

C.1 DIMENSIONAL ANALYSIS

Reynolds number

$$Re = \frac{UL}{\nu} \quad (C.23)$$

Turbulent cascade

The turbulent cascade proposed by Richardson consists of three scales: Injection scale, Inertial scale and dissipative scale. The injection scale corresponds to the scale where the driving energy of the flow resides and is where the energy cascade starts. The dissipative scale is where the energy cascade stops and is dominated by dissipation to thermal energy. The inertial scale is in between the energy scale and the dissipative scale.

Kolmogorov scales

Length scale:

$$\eta \sim \left(\frac{\nu^3}{\varepsilon} \right)^{1/4} \quad (C.24)$$

Time scale:

$$\tau \sim \left(\frac{\nu}{\varepsilon} \right)^{1/2} \quad (C.25)$$

TURBULENT KINETIC ENERGY PRODUCTION

TKE or k is

$$k = \frac{1}{2} \text{tr}(\overline{\mathbf{u}'\mathbf{u}'}) = \frac{1}{2} \overline{u'_i u'_i} \quad (C.26)$$

Transport of TKE:

$$\frac{\partial k}{\partial t} + \bar{\mathbf{u}} \cdot \nabla k = \nabla \cdot \left(-\frac{\overline{\mathbf{u}'p'}}{\rho} - \overline{(\mathbf{u}' \cdot \mathbf{u}')\mathbf{u}'} + \nu \nabla k \right) \\ \underbrace{-\overline{\mathbf{u}'\mathbf{u}'} : \nabla \bar{\mathbf{u}}}_{\mathcal{P}} - \underbrace{\nu \overline{\nabla \mathbf{u}' \nabla \mathbf{u}'}}_{\varepsilon} \quad (C.27)$$

where $\nabla \cdot (\overline{\mathbf{u}'p'}/\rho)$ is pressure diffusion, $\nabla \cdot (\overline{(\mathbf{u}' \cdot \mathbf{u}')\mathbf{u}'})$ is turbulence transport, $\nu \nabla^2 k$ is molecular viscous transport, $-\overline{\mathbf{u}'\mathbf{u}'} : \nabla \bar{\mathbf{u}}$ is TKE production and $-\nu \overline{\nabla \mathbf{u}' \cdot \nabla \mathbf{u}'}$ is TKE dissipation. The closure for TKE is based on gradient diffusion hypothesis

$$\frac{\overline{\mathbf{u}'p'}}{\rho} + \overline{(\mathbf{u}' \cdot \mathbf{u}')\mathbf{u}'} = -\frac{\nu_t}{\sigma_k} \nabla k \quad (\text{C.28})$$

and so:

$$\frac{\partial k}{\partial t} + \bar{\mathbf{u}} \cdot \nabla k = \nabla \cdot \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \nabla k \right] + \mathcal{P}_k - \varepsilon \quad (\text{C.29})$$

or in Einstein notation:

$$\frac{\partial k}{\partial t} + \bar{u}_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + \mathcal{P}_k - \varepsilon \quad (\text{C.30})$$

The TKE production \mathcal{P}_k is defined as:

$$\begin{aligned} \mathcal{P}_k \equiv -\overline{\mathbf{u}'\mathbf{u}'} : \nabla \bar{\mathbf{u}} = & \overline{u'u'} \frac{\partial \bar{u}}{\partial x} + \overline{v'v'} \frac{\partial \bar{v}}{\partial y} + \overline{w'w'} \frac{\partial \bar{w}}{\partial z} \\ & + \overline{u'v'} \left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right) \\ & + \overline{u'w'} \left(\frac{\partial \bar{u}}{\partial z} + \frac{\partial \bar{w}}{\partial x} \right) \\ & + \overline{v'w'} \left(\frac{\partial \bar{v}}{\partial z} + \frac{\partial \bar{w}}{\partial y} \right) \end{aligned} \quad (\text{C.31})$$

and in Einstein notation:

$$\mathcal{P}_k \equiv -\overline{u'_i u'_j} \frac{\partial \bar{u}_i}{\partial x_j} \quad (\text{C.32})$$

TURBULENT DISSIPATION RATE

The TKE dissipation rate or (TDR) ε is defined as:

$$\varepsilon \equiv 2\nu \overline{\mathbf{s}' : \mathbf{s}'} \quad (\text{C.33})$$

where

$$\mathbf{s}' = \frac{1}{2} \left(\nabla \mathbf{u}' + (\nabla \mathbf{u}')^T \right) \quad (\text{C.34})$$

and in Einstein notation is defined as:

$$\varepsilon \equiv 2\nu \overline{s'_{ij}s'_{ij}} = \nu \overline{\left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i}\right) \cdot \left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i}\right)} \quad (\text{C.35})$$

BOUSSINESQ HYPOTHESIS

$$-\overline{\mathbf{u}'\mathbf{u}'} + \frac{2}{3}k\mathbf{I} = \nu_t \left(\nabla \bar{\mathbf{u}} + (\nabla \bar{\mathbf{u}})^T \right) \quad (\text{C.36})$$

where

$$\bar{\mathbf{s}} = \frac{1}{2} \left(\nabla \bar{\mathbf{u}} + (\nabla \bar{\mathbf{u}})^T \right) \quad (\text{C.37})$$

and the error of Boussinesq hypothesis:

$$\epsilon_t = \|\mathbf{a} - 2\nu_t \bar{\mathbf{s}}\| \quad (\text{C.38})$$

The eddy-viscosity is closed using $k - \varepsilon$ equations:

$$\nu_t = C_\mu \frac{k^2}{\varepsilon} \quad (\text{C.39})$$

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