Physics

The Handbook

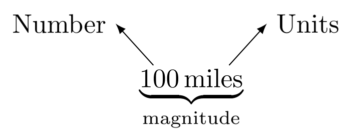
Updated on February 1, 2026

Contents

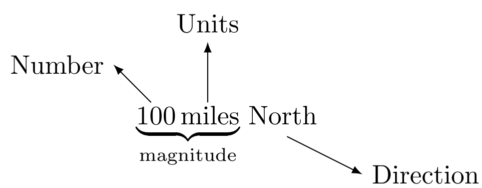
1 Constant Motion

1.1 Scalars and Vectors

A **scalar** is a quantity that has magnitude (and possibly sign) but no direction. **Magnitude** means size or amount. Examples of scalars include a length of 100 miles, a mass of 4 kilograms, and a speed of 35 mph. Scalar notation is shown below.



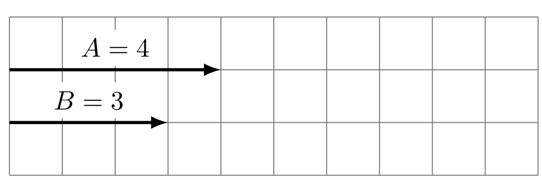
A **vector** is a quantity that has both magnitude and direction. Directions include north (N), south (S), east (E), west (W), up, left, forward, etc. Examples of vectors include a displacement of 5 m N, a velocity of 14 m/s to the right, and a momentum of 6 kg · m/s forward. Vector notation is shown below.



A vector is drawn as an arrow. The length of the arrow is the magnitude, and the arrowhead is the direction. The **tail** is the the starting point of a vector; the point opposite to the head or tip of the arrow, and the **head** is the the end point of a vector; the location of the vector's arrow; also referred to as the tip.

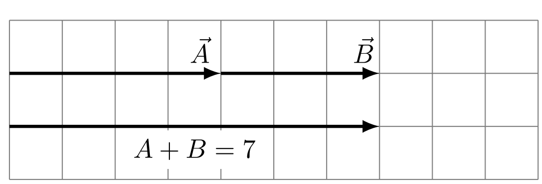
The **head-to-tail method** is a method of adding vectors in which the tail of each vector is placed at the head of the previous vector.

Example 1. Vector $\vec{A}$ is 4 units to the right. Vector $\vec{B}$ is 3 units to the right, as shown below. Add vectors $\vec{A}$ and $\vec{B}$.



Solution:

First, the vectors are added graphically using the head-to-tail method:



Therefore, the sum is

$$A + B = 4 + 3 = 7$$

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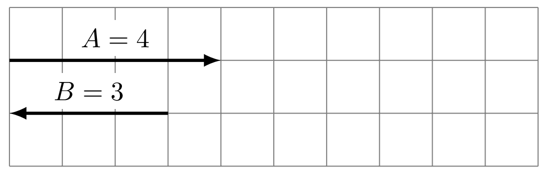
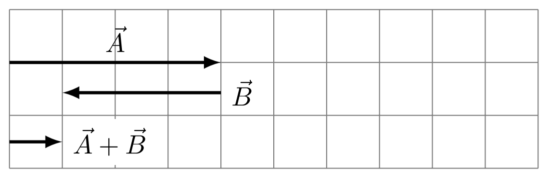
Example 2. Vector $\vec{A}$ is 4 units to the right. Vector $\vec{B}$ is 3 units to the left, as shown below. Add vectors $\vec{A}$ and $\vec{B}$.



Solution:

The vectors are added graphically using the head-to-tail method:



Since $\vec{B}$ points to the left, we subtract its magnitude to find the vector addtion:

$$A - B = 4 - 3 = 1$$

Therefore the vector sum is 1 unit to the right.

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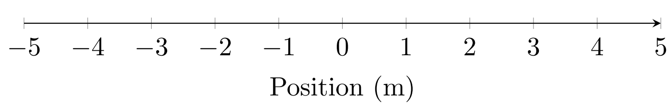
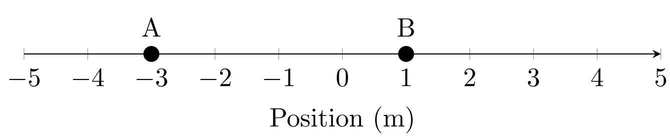
1.2 Position and Displacement

Position (x) is the location of an object at any particular time. The mathematical symbol for position is x, and the base SI unit of position is the meter (m). The basic coordinate system for position is a number line called the position axis:



The position of an object may be stated, for example, as $x = 2$ meters or $x = -3$ m. A **reference point** is a known location used to describe the position of objects, like the zero of a number line.

Example 3. A man starts at a reference point, labeled as Point A below. Then he walks to Point B.



(a) In what direction did his position change? (b) What is the magnitude of the change in his position?

Solution:

(a) The change in position is rightward, or in the positive (+) direction.

(b) Recall that magnitude means size or amount. We can find change in position by subtracting the first position from the second, exactly the way we do on a number line:

$$\begin{aligned}\text{change in position} &= 1\,\text{m} - (-3\,\text{m}) \\ &= 1\,\text{m} + 3\,\text{m} \\ &= \boxed{4\,\text{m}}\end{aligned}$$

The man's change in position has a magnitude (size) of 4 meters. That is the same value we would get if we count the tick marks from Point A to Point B.

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The "change in position" in the previous example has a special name in physics. The Greek letter Δ (Delta) means "change in." **Displacement** (Δx) is the change in position of an object against a fixed axis. Mathematically, it's

$$\Delta x = x_f - x_i \tag{1}$$

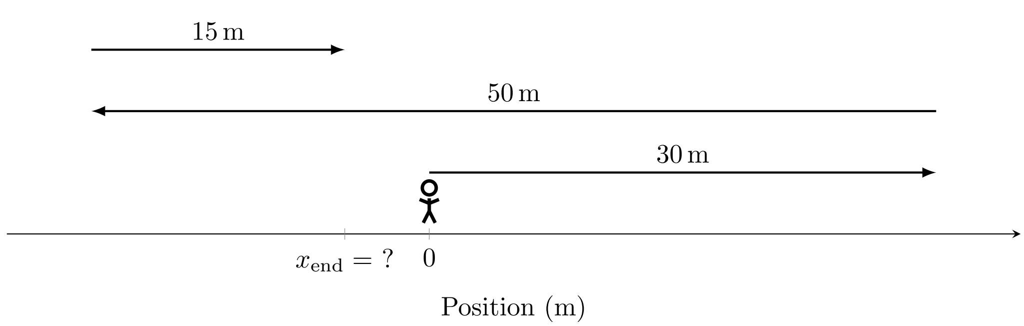
where x_f is the final position, and x_i is the initial position.

Distance (d) is the length of the path actually traveled between an initial and a final position.

Example 4. Ron Farr runs the following route in 10 seconds: 30 meters in the positive direction, 50 meters in the negative direction, and 15 meters in the positive direction. (a) What is the total distance Ron ran? (b) What is his total displacement?

Solution:

A sketch is useful.



(a) The distance is

$$d = 30\,\text{m} + 50\,\text{m} + 15\,\text{m} = \boxed{95\,\text{m}}$$

(b) For total displacement, take the sum of all displacements:

$$\Delta x = 30\,\text{m} + (-50\,\text{m}) + 15\,\text{m} = \boxed{-5\,\text{m}}$$

So, his displacement is -5 meters, or 5 meters to the left of his origin.

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1.3 Speed and Velocity

Speed is the rate at which an object changes its location. It's how fast an object is moving. Mathematically, **average speed** is distance traveled divided by the time during which the motion occurs. In equation form,

$$\text{average speed} = \frac{\text{distance}}{\text{time}} \tag{2}$$

The SI units of distance and time are meters and seconds, so the SI units of speed are meters per second (m/s).

Velocity (v) is the speed and direction of an object, or the rate at which an object changes position. **Average velocity** ($\bar{v}$) is displacement divided by the time during which the displacement occurs. Mathematically,

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{\text{displacement}}{\text{time}} \tag{3}$$

Example 5. Ron Farr (from Example 4) runs the following route in 10 seconds: 30 meters in the positive direction, 50 meters in the negative direction, and 15 meters in the positive direction. (a) What is his average speed? (b) What is his average velocity?

Solution:

In Example 4, we found the total distance traveled is $d = 95\,\text{m}$, and his displacement is $-5\,\text{m}$.

(a) His average speed is

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{95\,\text{m}}{10\,\text{s}} = \boxed{9.5\,\text{m/s}}$$

(b) Average velocity is

$$v = \frac{\text{displacement}}{\text{time}} = \frac{-5\,\text{m}}{10\,\text{s}} = \boxed{-0.5\,\text{m/s}}$$

Average speed and average velocity are not the same thing! Moreover, these answers suggest that if Ron walked at a slow pace of $0.5\,\text{m/s}$ in the negative direction, he would end up in the same final location has he did in Example 4.

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Example 6. Find a formula for displacement in terms of velocity and time.

Solution:

Average velocity (Eq. 3) is

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

We can write

$$\frac{\bar{v}}{1} = \frac{\Delta x}{\Delta t}$$

and cross-multiply:

$$(1)(\Delta x) = (\bar{v})(\Delta t)$$

Therefore,

$$\Delta x = \bar{v}\Delta t \tag{4}$$

That is, displacement equals average velocity multiplied by the time interval.

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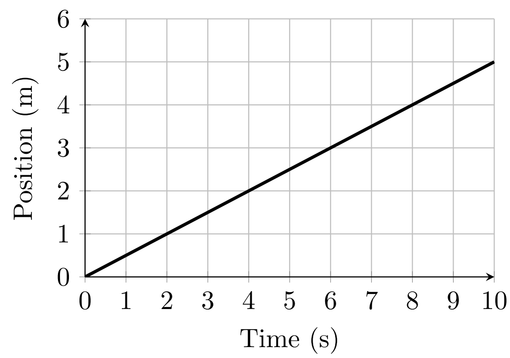
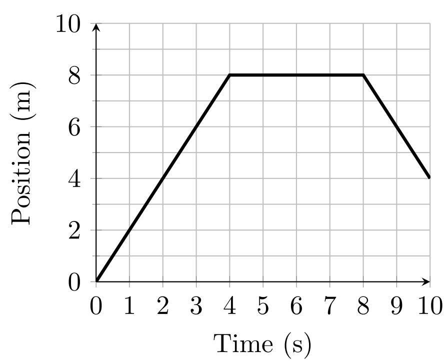
1.4 Position vs. Time Graphs

A **position vs. time graph** is a graph in which position is plotted on the vertical axis and time is plotted on the horizontal axis. An example is shown below:



Example 7. The graph below shows the position of an object as a function of time. What is the object's displacement between $t = 2\,\text{s}$ and $t = 5\,\text{s}$?



Solution:

According to the graph, the position of the object at 2 seconds is 4 meters; this is the initial position: $x_i = 4\,\text{m}$. The position at 5 seconds, the final position, is 8 meters: $x_f = 8\,\text{m}$. Displacement is

$$\Delta x = x_f - x_i = 8\,\text{m} - 4\,\text{m} = \boxed{4\,\text{m}}$$

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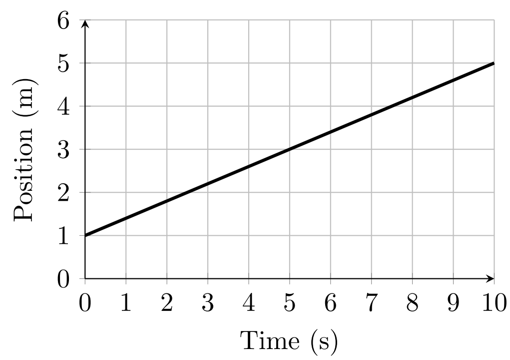
Example 8. The figure below shows the position of puck on a frictionless surface. Find the puck's average velocity.

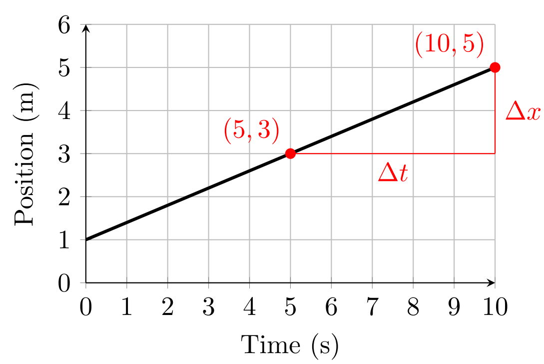


Solution:

Average velocity is displacement over time:

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

Graphically, Δx is the "rise" and Δt is the "run" between any two points on the graph, as shown below.



But, rise over run is the same thing as the slope of the line! Therefore, **in a position vs. time graph, the slope of the line is the velocity**. Using the two coordinates from the figure, we get

$$\bar{v} = \text{slope} = \frac{\Delta x}{\Delta t} = \frac{4\,\text{m/s} - 2\,\text{m/s}}{10\,\text{s} - 5\,\text{s}} = \frac{2\,\text{m/s}}{5\,\text{s}} = \frac{2}{5}\,\text{m/s} = \boxed{0.4\,\text{m/s}}$$

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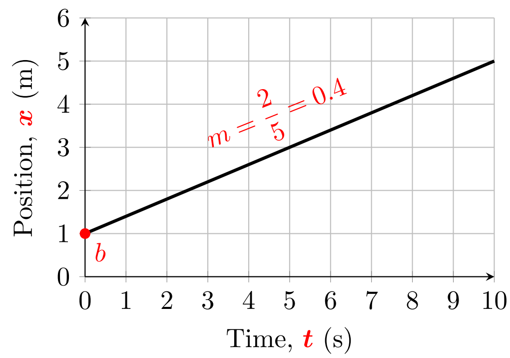
Example 9. Write the linear equation of motion for the line in the graph from Example 7.

Solution:

In algebra, we learned that the equation of a line on the xy plane is

$$y = mx + b$$

In a position vs. time graph, position (x) is plotted on the y-axis, and time (t) is plotted on the x-axis. Also, we found the slope in Example 7, and we can see below that the y-intercept b is 1 meter:



Therefore, the equation is

$$x = (0.4\,\mathrm{m/s})\,t + 1\,\mathrm{m}$$

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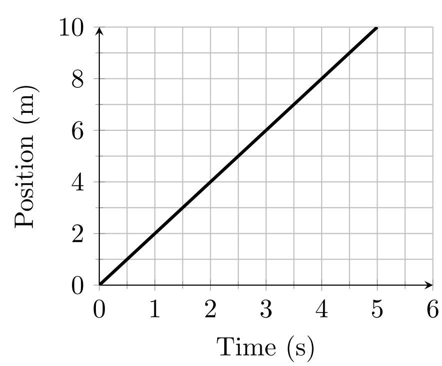
1.5 Velocity vs. Time Graphs

A **velocity vs. time graph** is a graph in which velocity is plotted on the vertical axis and time is plotted on the horizontal axis. Beside velocity information, velocity vs. time graphs contain implicit position and displacement data.

Example 10. Make a velocity vs. time graph corresponding to the position vs. time graph below.

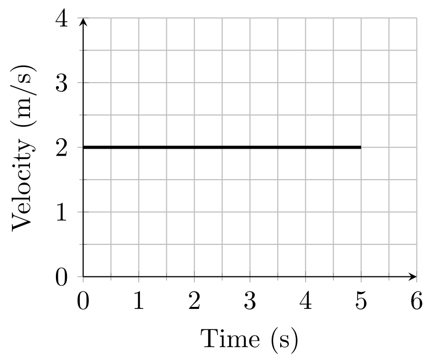


Solution:

In Example 8 we recalled how to find the slope of a line and learned that slope in a position vs. time graph is the velocity. The slope, or velocity, in the graph above is

$$v = \text{slope} = 2.0\,\mathrm{m/s}$$

and this value is constant throughout the 5-second time interval. So, the velocity-time graph is as follows:



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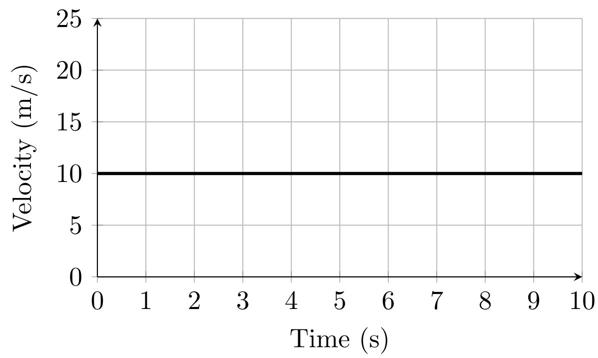
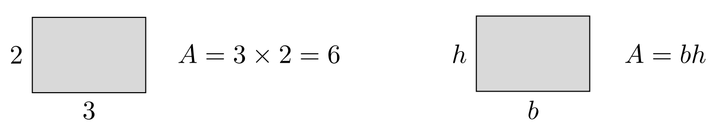
Example 11. The following graph[1] shows the constant velocity of a car traveling in one direction as a function of time. What is the car's displacement in the first 8 seconds of motion?

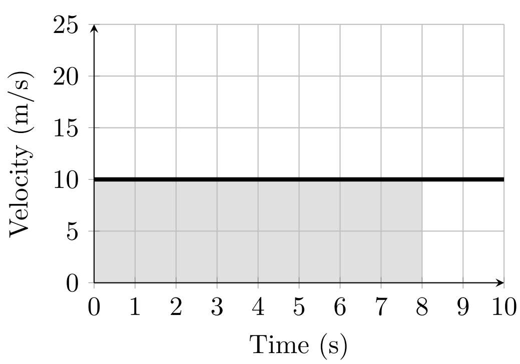


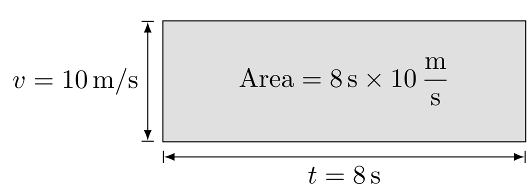
Solution:

Consider an analogy. The area of a 3- by 2-unit rectangle is 6. In general, the area of a rectangle is $A = bh$ where b is the base and h is the height.



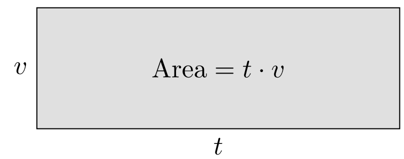
In this car example, the area bounded by the velocity line is a rectangle with a base of 8 seconds and a height of 10 meters per second.





[1] See 3Blue1Brown, "Integration and the fundamental theorem of calculus."

In general, the area bounded by a velocity vs. time graph is found by multiplying time (the base) by velocity (the height):



But, wait! Equation (4) states that the product of velocity and time is displacement:

$$\Delta x = vt$$

Therefore,

$$\Delta x = \text{displacement} = \text{area under line in a velocity vs. time graph}$$

In a velocity vs. time graph, the area bounded under the velocity line equals the displacement of the object. Therefore, by the figures above, the area bounded is

$$\text{area} = \Delta x = 8\,\text{s} \times 10\,\frac{\text{m}}{\text{s}} = \boxed{80\,\text{m}}$$

Therefore, the car's displacement is 80 meters. This problem encourages you to think of displacement geometrically in terms of the area under a velocity vs. time curve.

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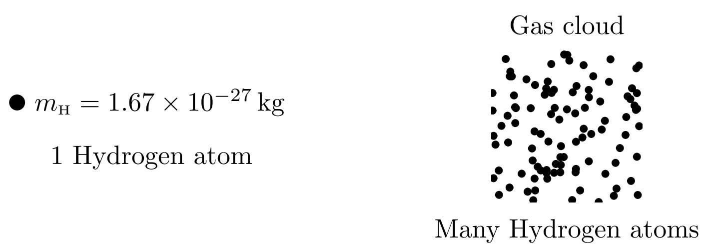
1.6 Mass and Inertia

Mass is the quantity of matter in a substance; the SI unit of mass is the kilogram (kg).



For example, in the figure above, the mass of the gas cloud is the sum of the total mass of all its constituent Hydrogen atoms.

Example 12. Suppose the gas cloud in the figure above contains 6000 moles of hydrogen. By Avogadro's number, 1 mole equals 6.02×10^{23} atoms. Find the mass of the gas cloud.

Solution:

The number of hydrogen atoms that constitute the gas cloud is

$$
\begin{aligned}
N &= 6000\,\text{mol} \\
&= 6000\,\text{mol} \times \frac{6.02 \times 10^{23}\,\text{atoms}}{1\,\text{mol}} \\
&= 3.612 \times 10^{27}\,\text{atoms}
\end{aligned}
$$

The total mass of the system is

$$m = Nm_{\text{H}} = \left(3.612 \times 10^{27}\,\text{atoms}\right)\left(1.67 \times 10^{-27}\,\text{kg}\right) = \boxed{6.0\,\text{kg}}$$

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Inertia is the tendency of an object at rest to remain at rest, or for a moving object to remain in motion in a straight line and at a constant speed.

Inertia is proportional to mass: the more mass an object has, the greater its inertia.

Newton's first law of motion states that a body at rest remains at rest or, if in motion, remains in motion at a constant speed in a straight line, unless acted on by a net external force; also known as the law of inertia.

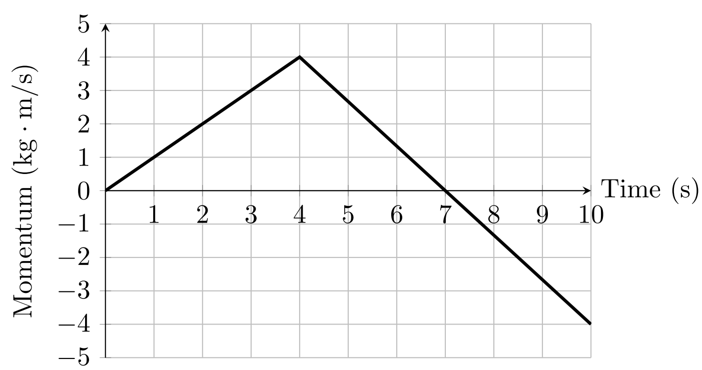
1.7 Momentum

Momentum (p) is the product of a system's mass and velocity. Mathematically,

$$p = mv \tag{5}$$

The SI unit of momentum is the combination of the units of mass and velocity: kilograms multiplied by meters per second, or kg · m/s. Objects usually don't change in mass but can change momentum with change in velocity.

A **momentum vs. time graph** is a graph in which momentum is plotted on the vertical axis and time is plotted on the horizontal axis.



1.8 Kinetic Energy

Kinetic energy (K) is energy of motion. Mathematically, it's defined as

$$K = \frac{1}{2}mv^2$$

where m is mass in kilograms and v is speed in meters per second. According to this equation, since the SI base units of mass and velocity are the kilogram and the meter per second, respectively, kinetic energy has units of

$$\mathrm{kg} \cdot \left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 = \frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^2}$$

This combination of kg, m, and s has a special name: the joule. A **joule** (J) is the metric unit for work and energy; equal to 1 newton meter ($\mathrm{N} \cdot \mathrm{m}$):

$$1\,\mathrm{J} = 1\,\frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^2}$$

"Work" is a physics concept that will be introduced in Unit 4.

Example 13. A 250-gram cart travels at 3.2 meters per second. Calculate its kinetic energy.

Solution: In base SI units, mass is $m = 0.250\,\mathrm{kg}$.

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.25\,\mathrm{kg})(3.2\,\mathrm{m/s})^2 = \boxed{1.28\,\mathrm{J}}$$

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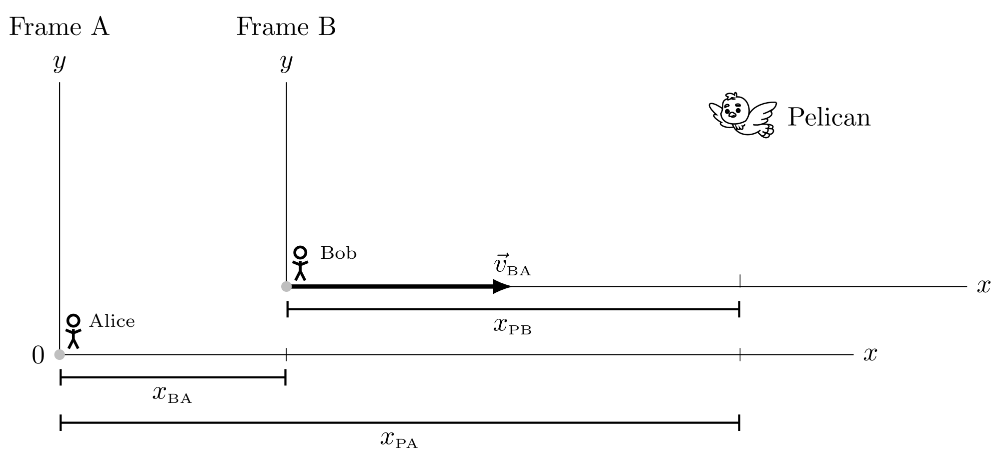
1.9 Relative Motion

Relative speed is how fast or slow an object appears to be moving to another object.

In the figure below, Alice is in frame A standing at rest on the street. She observes Bob, who is in Frame B, moving at speed v_{BA} relative to her frame. A pelican P moves relative to both frames.



Variable	What it means
x_{BA}	position of Bob relative to Alice
x_{PA}	position of the Pelican relative to Alice
x_{PB}	position of the Pelican relative to Bob
v_{BA}	velocity of Bob as measured from Alice's reference frame

Alice and Bob disagree on the Pelican's position. According to Bob, its position is x_{PB}. However, according to Alice, it's

$$x_{\mathrm{PA}} = x_{\mathrm{BA}} + x_{\mathrm{PB}}$$

Similarly, Alice and Bob disagree on the Pelican's velocity. Because velocity is the rate of change in position, the relative velocities follow the same form as above:

$$v_{\mathrm{PA}} = v_{\mathrm{BA}} + v_{\mathrm{PB}} \tag{6}$$

where each variable is defined in the table below:

Variable	What it means
v_{PA}	velocity of the Pelican relative to Alice
v_{BA}	velocity of Bob relative to Alice
v_{PB}	velocity of the Pelican relative to Bob

Example 14. If Bob, from the figure above, moves at 32 m/s away from Alice, and Alice claims the Pelican is moving at 45 m/s in the $-x$-direction, what is the Pelican's velocity relative to Bob?

Solution: Equation (6) for relative velocity is

$$v_{\mathrm{PA}} = v_{\mathrm{BA}} + v_{\mathrm{PB}}$$

Substituting given values leads to

$$-45\,\mathrm{m/s} = 32\,\mathrm{m/s} + v_{\mathrm{PB}}$$

Solving for the Pelican's velocity from Bob's frame results in

$$v_{\mathrm{PB}} = -45\,\mathrm{m/s} - 32\,\mathrm{m/s} = \boxed{-77\,\mathrm{m/s}}$$

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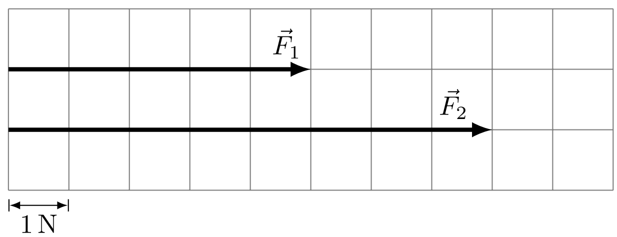
2 Force Interactions

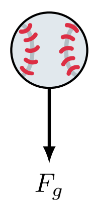
2.1 Types of Force

A **force** is a push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force; the SI unit of force is the Newton (N). Being vectors, forces are represented by arrows. The length of the arrow is the magnitude of the force. For example, force $\vec{F}_1$ below has a magnitude of 5 newtons, and the magnitude of $\vec{F}_2$ is 8 N.



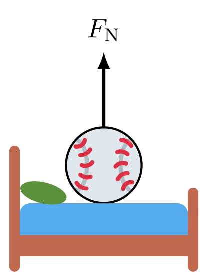
In this unit, there are six forces you should know: the gravitational force, the normal force, the frictional force, the applied force, the spring force, and the tension force.

The **gravitational force** (F_g) is the downward force on an object due to the attraction by the Earth or other massive body. In the case below, there is a force of gravity on the ball.

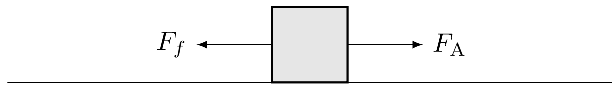


Where does the force F_{g} come from? It comes from Earth, which gravitationally pulls the ball down. The gravitational force will be explored in more detail in Unit 5: Force Analysis.

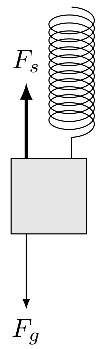
A **normal force** (F_{N}) is that component of the contact force between two objects, which acts perpendicularly to and away from their plane of contact. In the case of a baseball, if the ball is rested on a flat surface like a bed, the normal force is the upward force exerted by the bed on the ball:



The **frictional force** (F_f) is an external force that acts opposite to the direction of motion or, for when there is no relative motion, in the direction needed to prevent slipping. An **applied force** (F_{A}) is a contact force intentionally exerted by a person on an object. In the example below, if a box is resting on a rough surface and an applied force moves it to the right with constant motion, then there is a frictional force in the opposite direction.

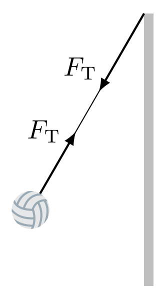


The **spring force** (F_s) is a force applied from a spring when it is either compressed or stretched. If an object is suspended from a spring, the spring applies an upward force to sustain the object's weight.



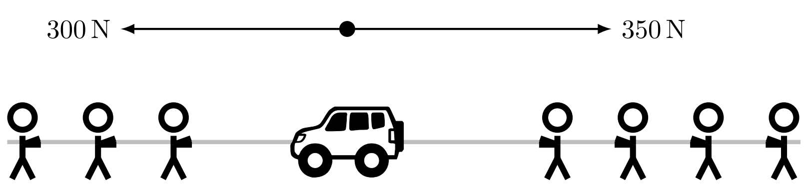
Tension (F_{T}) is a pulling force that acts along a connecting medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force exerted on the object by the rope is called tension.

One example of tension is tetherball, the sport where a ball is connect by a rope to a stationary pole:



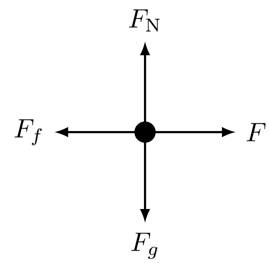
When the rope is taught, there is a tension force on the ball, and another tension force of equal magnitude is exerted on the pole, to keep the ball in motion.

Another example involving tension is a game of tug-of-war, shown below.



2.2 Free-Body Diagrams

A **free body diagram** is a diagram showing all external forces acting on a body. They are a tool for analyzing net force and predicting motion. A free body diagram is focused on one object only and shows a moment in time, as shown below.



The free-body diagram includes the forces acting *on* the object but no forces *by* the object. It consists of a dot with arrows (force vectors) coming off the object for every force interaction, as shown below.

Example 1. Name all the forces and their directions shown in the free-body diagram above.

Solution:

The free-body diagram includes the normal force (F_N) pointing up, the applied force (F) pointing right, the gravitational force (F_g) pointing down, and the frictional force (F_f) pointing left.

■

On a flat surface, the normal force F_N equals the gravity force F_g in magnitude. But F_N is affected by inclining the surface on which the object rests, as shown below.

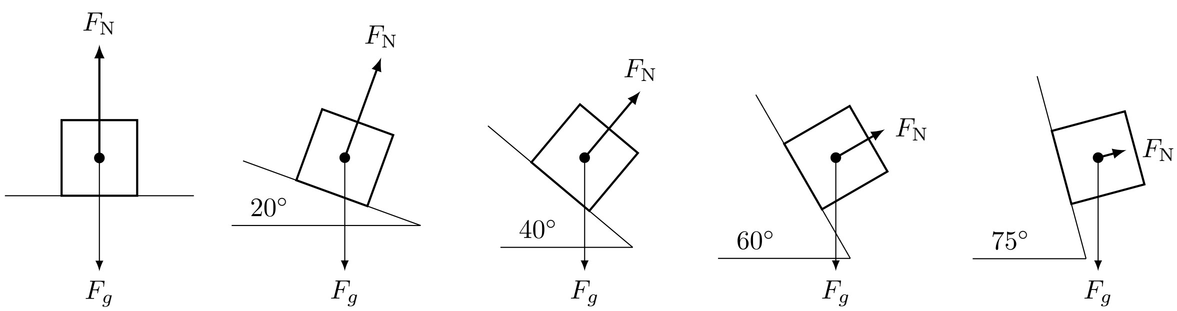


Figure 2.1: The normal force on a box changes as the angle of incline changes.

Example 2. According to Figure 2.1, how does the magnitude and direction of the normal force vector F_N change as the surface becomes more inclined?

Solution: The magnitude of the normal force decreases as the angle of incline decreases. The direction of the normal changes relative to the observer but remains perpendicular to the surface.

■

For a free-body diagram on an inclined surface, it is useful to break the gravity force vector F_g in two mutually perpendicular components: one that is parallel to the surface, and one that is perpendicular to the surface.

Let $F_{g,\parallel}$ be the parallel component of the gravity force F_g, and $F_{g,\perp}$ be the perpendicular component. These components of gravity, for the cases displayed in Figure 2.1, are shown below:

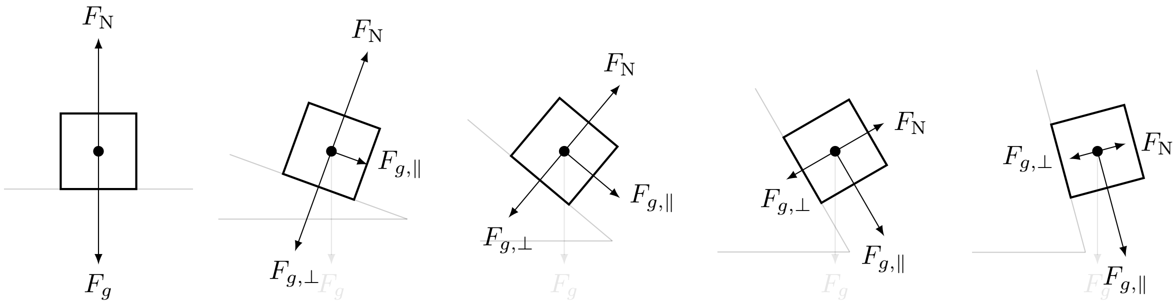


Figure 2.2: The components $F_{g,\parallel}$ and $F_{g,\perp}$ of the gravity force change as the angle incline changes.

Example 3. In Figure 2.2, how do the parallel and perpendicular components of the gravitational force on the box change as the surface becomes more inclined?

Solution: As the angle of incline increases, the parallel component $F_{g,\parallel}$ increases in magnitude, and the perpendicular component $F_{g,\perp}$ decreases in magnitude.

■

Example 4. According to Figure 2.2, what appears to be the relationship between the normal force and the perpendicular component of gravity?

Solution: The figure shows that F_N are $F_{g,\perp}$ are always equal in magnitude and opposite and direction.

■

2.3 Newton's Third Law and Interacting Force Pairs

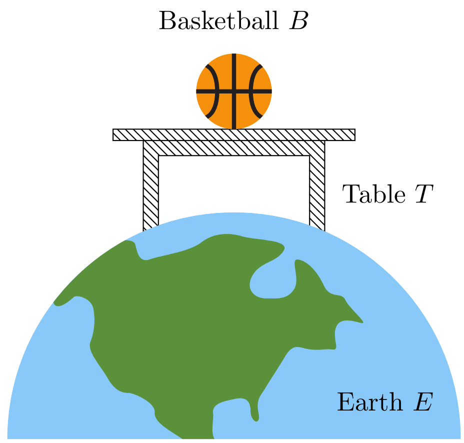
Newton's third law of motion states that whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts. "For every action, there is an equal but opposite reaction." This law represents a certain symmetry in nature: Forces always occur in pairs, and one body cannot exert a force on another without experiencing a force itself.

For example, in 2022 Astros pitcher Phil Maton exerted a large force by punching a locker, and the locker instantaneously returned a force of equal magnitude, resulting in Maton fracturing a bone in his right hand. There was a force pair interaction between Maton's hand and the locker.

The following are rules for interacting force pairs:

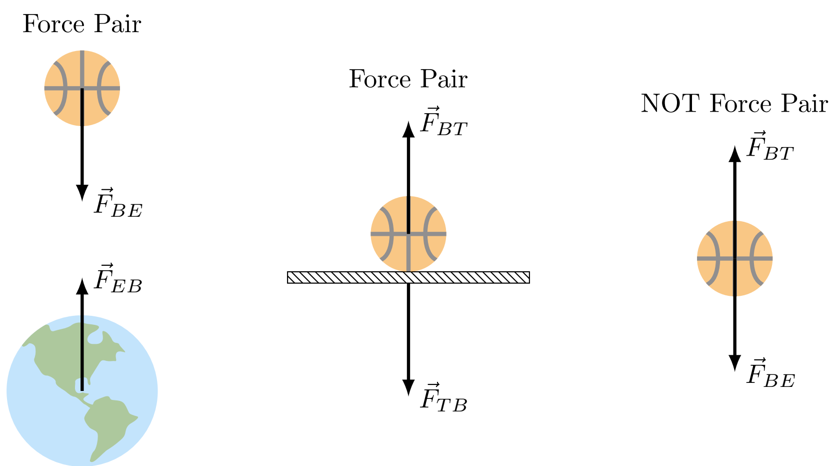
- An object can be involved in multiple force interactions.
- Any one interaction is between two objects and involves two forces.
- Forces always come in pairs.
- Each force in a force pair is the same magnitude (size or strength) but acts in opposite directions.

Example 5. A basketball rests on a table, and the table rests on the Earth. Identify the third-law force pairs and non-force-pairs in this scenario.



Solution:

Force pairs occur only between two interacting objects.



2.4 Balanced and Unbalanced Forces

3 Changing Motion I: Acceleration

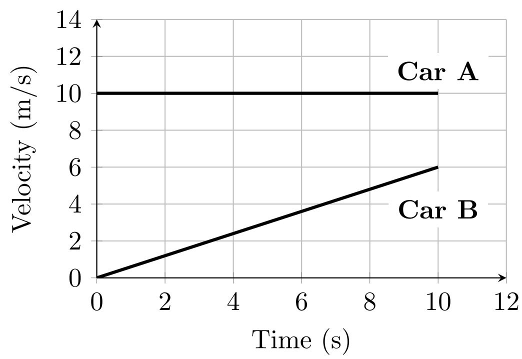
3.1 Acceleration

Acceleration (a) is a change in velocity over time. Mathematically, **average acceleration** is average acceleration:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}$$

One of the best ways to study acceleration is to look at data, both quantitative and qualitative.

Example 1. The graph below shows the velocities of Car A and Car B as a function of time. Describe the acceleration of each car.



Solution:

Car A has a constant velocity of 10 m/s. Since acceleration is a change in velocity over time, this car has an acceleration of zero. In other words, it's not accelerating.

The velocity of Car B starts at zero and increases to 6 m/s in 10 seconds. By definition, since velocity is changing, this car *is* accelerating.

■

Example 2. A toy car in a lab starts from rest and travels 1 meter in the first second. However, it travels a greater distance in the 2nd second, and increasingly longer distances with each passing second, as shown in the table below.

Time (s)	**Position** (m)
0	0
1	1
2	4
3	9
4	16
5	25

A student claims that the car is traveling at a constant velocity. Is the student correct?

Solution:

The car's displacement between $t = 0$ s and $t = 1$ s is 1 m. The displacement between $t = 1$ s and $t = 2$ s is 3 m. The displacement between $t = 2$ s and $t = 3$ s is 5 m. And, the last two displacements are 7 m and 9 m.

Because the car is not moving with equal displacements across equal time intervals, the student is wrong, the object's velocity is *not* constant but changing, and therefore the car is accelerating.

■

3.2 Newton's Second Law of Motion

Net force (F_{net}) is related to the sum of all forces acting on an object or system. Mathematically, it can be related to acceleration as

$$F_{\text{net}} = ma$$

Often, it's pronounced "F equals M A."

Example 3. In a game of tug of war, Blue Team applies a 300-newton force to the left, and Red Team applies a 350 N force to the right. Calculate the net force. Assume the positive direction is to the right.

Solution:

Taking rightward to be the positive direction, the net force is given by

$$F_{\text{net}} = F_{\text{red}} - F_{\text{blue}} = 350\,\text{N} - 300\,\text{N} = \boxed{+50\,\text{N}}$$

or 50 newtons to the right.

■

Example 4. From the previous example, Team Blue acquires a new strong player who applies an additional 150 N force. What is the net force now?

Solution:

$$F_{\text{net}} = F_{\text{red}} - F_{\text{blue}} = 350\,\text{N} - 450\,\text{N} = \boxed{-100\,\text{N}}$$

or 100 newtons to the left.

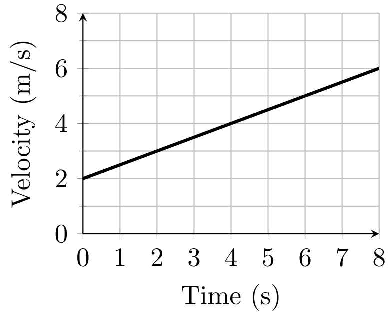
■

3.3 Change in Momentum

Momentum is $p = mv$. Momentum changes if there is a change in velocity, Δv. Change in momentum is given by $\Delta p = p_f - p_i$ and by

$$\begin{aligned} \Delta p &= m\Delta v \qquad (7) \\ &= m\left(v_f - v_i\right) \end{aligned}$$

Example 5. The velocity of a 15 kg object changes according to the function $v(t) = \left(0.5\,\text{m/s}^2\right)t + 2\,\text{m/s}$. What is the change in momentum during the entire time shown in the graph below?



Solution:

According to the graph, the initial and final velocities are $v_i = 20\,\text{m/s}$ and $v_f = 60\,\text{m/s}$. By Equation (7),

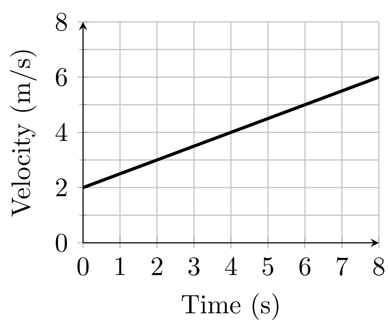
$$\begin{aligned}\Delta p &= m\Delta v \\ &= m(v_f - v_i) \\ &= (15\,\text{kg})(6\,\text{m/s} - 2\,\text{m/s}) \\ &= \boxed{60\,\text{kg}\cdot\text{m/s}}\end{aligned}$$

3.4 Change in Kinetic Energy

Kinetic energy is $K = \frac{1}{2}mv^2$. Kinetic energy changes if velocity changes. Change in kinetic energy is given by $\Delta K = K_f - K_i$ and by

$$\begin{aligned}\Delta K &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \\ &= \frac{1}{2}m\left(v_f^2 - v_i^2\right)\end{aligned} \tag{8}$$

Example 6. The graph representing the motion of the same 15 kg object from Example 5 is repeated below.



Now calculate the change in kinetic energy of the object.

Solution:

By Equation (8),

$$
\begin{aligned}
\Delta K &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \\
&= \frac{1}{2}(15\,\mathrm{kg})\,(6\,\mathrm{m/s})^2 - \frac{1}{2}(15\,\mathrm{kg})\,(2\,\mathrm{m/s})^2 \\
&= \boxed{240\,\mathrm{J}}
\end{aligned}
$$

4 Changing Motion II: Impulse and Work

4.1 Impulse

Impulse (J) is average net external force multiplied by the time the force acts; equal to the change in momentum. Mathematically, it is

$$J = F_{\text{net}}\Delta t$$

The **impulse-momentum theorem** states that the impulse equals change in momentum.

$$F_{\text{net}}\Delta t = \Delta p$$

Therefore, impulse-momentum theorem is also written as

$$F_{\text{net}}\Delta t = m\Delta v = m(v_f - v_i)$$

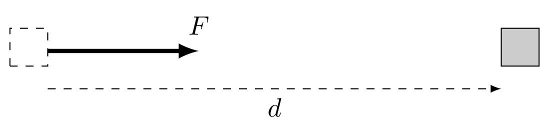
4.2 Work

Work is force multiplied by distance.

■

More specifically, work is the component of the force in the direction of the displacement, multiplied by the displacement.

Example 1. Jacob applies a force of 72.0 N on a box and pushes it a distance of 25.0 m. How much work is done on the box?



Solution:

We are given the quantities of force and distance: $F = 72.0\,\text{N}$ and $d = 25.0\,\text{m}$. The unknown quantity we want to find is work.

$$W = Fd = (72.0\,\text{N})\,(25.0\,\text{m}) = 1800\,\text{J}$$

Jacob does 1800 joules of work on the box in applying the force to move the box.

■

Example 2. You see a box with a mass of 10 kg resting on the floor. What is the work you have to do on the box to lift it to a height of 1.5 meters?

Solution: We are given the box's mass and distance lifted: $m = 10\,\text{kg}$ and $d = 1.5\,\text{m}$. The unknown we are looking for is work: $W =$? By Eq. (**??**), work is

$$W = Fd$$

However, force (F) is *not* one of the given quantities in this problem. Can you of a way to calculate how much force is necessary to lift this box? To lift the box, you must supply an upward force that is (at least) equal to the magnitude of the downward force of gravity on the box. That is, your applied force must equal the box's weight (w), which, in Unit 4 on Forces, we specifically defined as

$$w = mg \ ,$$

where $\mathrm{g} = 9.8\,\mathrm{m/s^2}$. So, if your applied force is equal to the box's weight, then your force is

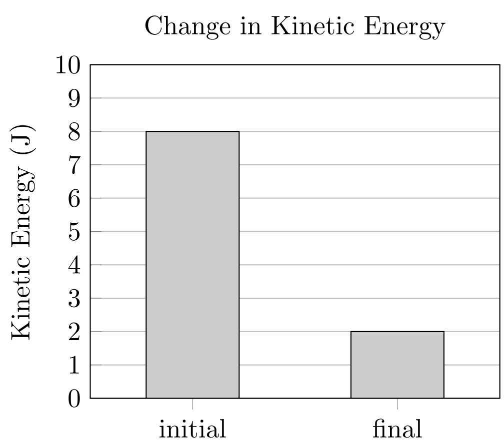
$$F = w = mg = (10)(9.8) = 98\,\mathrm{N}$$

Therefore, the work done is

$$W = Fd = (98\,\mathrm{N})(1.5\,\mathrm{m}) = 147\,\mathrm{J} \ .$$

Note: Since, in this example, the force was of the form $F = mg$ and since work is defined as $W = Fd$, the work done against gravity was of the form $W = mgd$. Compare this result with Equation (**??**) below.

■



4.3 Power

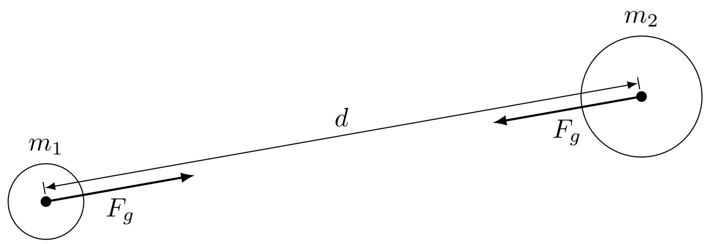
Power is work divided by time.

$$P = \frac{W}{t} \tag{9}$$

5 Force Analysis

5.1 Newton's Law of Universal Gravitation

Newton's universal law of gravitation states that gravitational force between two objects is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.



The magnitude of the gravitational force is

$$F_g = \frac{Gm_1m_2}{d^2}$$

where G is the **gravitational constant**, the proportionality constant in Newton's law of universal gravitation, and is equal to

$$G = 6.67 \times 10^{-11}\,\frac{\mathrm{N\,m^2}}{\mathrm{kg}^2}$$

Example 1. Consider two spheres of mass 100.0 kg and 400.0 kg whose centers are separated by 4.00 m. What is the magnitude of the gravitational force between them?

Solution: We are given two masses and the distance between them: $m_1 = 100.0\,\mathrm{kg}$, $m_2 = 400\,\mathrm{kg}$, $r = 4.00\,\mathrm{m}$. Also, remember that $G = 6.67\times10^{-11}\,\mathrm{N\,m^2/kg^2}$. By Newton's law of universal gravitation, the gravitational force is

$$F = \frac{Gm_1m_2}{d^2} = \frac{\left(6.67\times10^{-11}\,\frac{\mathrm{N\,m^2}}{\mathrm{kg^2}}\right)(100\,\mathrm{kg})(400\,\mathrm{kg})}{(4.00\,\mathrm{m})^2} = \boxed{1.67\times10^{-7}\,\mathrm{N}}$$

■

Example 2. When a small asteroid of mass m is a distance D from a larger asteroid of mass M, the gravitational force on m by M is F. What is the gravitational force when they are a distance $2D$ apart? State your answer in terms of the original force F.

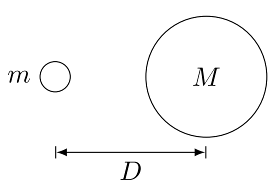


Figure 1

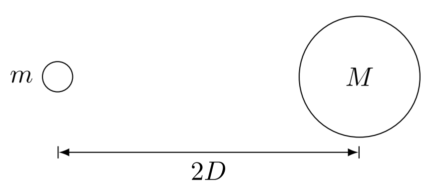


Figure 2

Solution:

In Figure 1, by Newton's law of gravitational, the gravitational force is

$$F_1 = \frac{GmM}{D^2}$$

Let F_2 be the force in Figure 2, when the distance has doubled. Then,

$$\begin{aligned} F_2 &= \frac{GmM}{(2D)^2} \\ &= \frac{GmM}{4D^2} \\ &= \frac{1}{4} \cdot \frac{GmM}{D^2} \\ &= \frac{1}{4} F_1 \end{aligned}$$

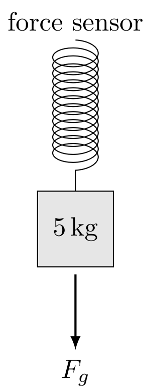
So, F_2 is 4 times weaker than F_1. That is, in Figure 2, the gravitational force decreases by a factor of 4 compared to Figure 1, when the distance is doubled. ■

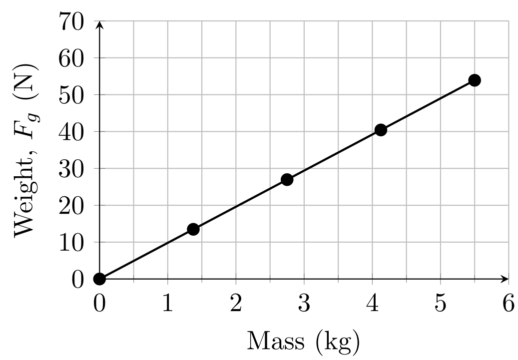
5.2 Mass and Weight

Recall that **mass** is the quantity of matter in a substance; the SI unit of mass is the kilogram. It does not change based on location. In equations it is represented as m.

Weight is the force of gravity, F_g, acting on an object of mass m.

Using a force sensor, you can plot the weight (force of gravity) as a function of mass, using a variety of masses, as shown below.





The slope of this graph is about 10 N/kg and represents the strength of the gravitational field near the surface of the Earth. Therefore, weight is defined mathematically as

$$F_g = mg \tag{10}$$

where $g = 10\,\text{N/kg}$ is the gravitational field strength. Being a force, weight is measured in newtons. It changes depending on location.

Below is a table comparing and contrasting weight and mass.

Weight	Mass
force of gravity on an object	amount of matter
measured in newtons (N)	measured in kilograms (kg)
variable: F_g	variable: m
vector (magnitude and direction)	scalar (magnitude only)
e.g., $F_g = 120\,\text{N}$ down	e.g., $m = 5\,\text{kg}$
subject to change	does not change (constant)

Example 3. What is the weight of a 12 kg box?

Solution:

$$F_g = mg = (12\,\text{kg})(10\,\text{m/s}^2) = \boxed{120\,\text{N}}$$

■

5.3 The Gravitational Field

In general, the strength of the gravitational field on the surface of a massive planetary body is given by

$$g = \frac{GM}{R^2}$$

where $G = 6.67 \times 10^{-11}\,\text{N} \cdot \text{m}^2/\text{kg}^2$ is the gravitational constant, M is the planet's mass, and R is the planet's radius.

Example 4. Find the *exact* value, rounded to one decimal place, of the gravitational field strength on Earth. The mass and radius of Earth are $M = 5.97 \times 10^{24}\,\text{kg}$ and $R = 6{,}378{,}000\,\text{m}$.

Solution:

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11}\,\text{N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\,\text{kg})}{(6.378 \times 10^6\,\text{m})^2} = \boxed{9.8\,\text{N/kg}}$$

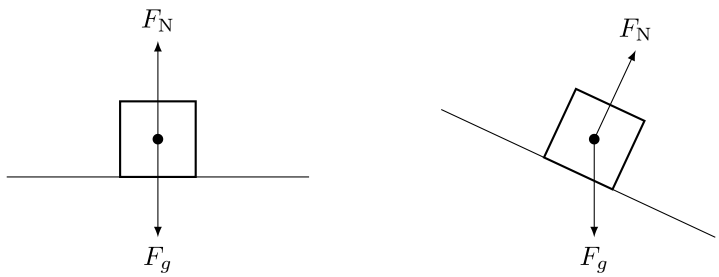
■

5.4 Forces from Surfaces

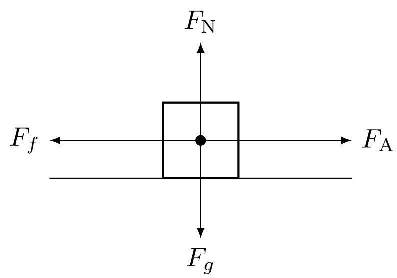
A **contact force** is a type of force that occurs when objects are physically in contact with each other. Examples of contact forces include:

- applied force (F_A): e.g., kicking a ball
- tension force (F_T): e.g., pulling a box with a rope
- spring force (F_s): e.g., jumping on a pogo stick

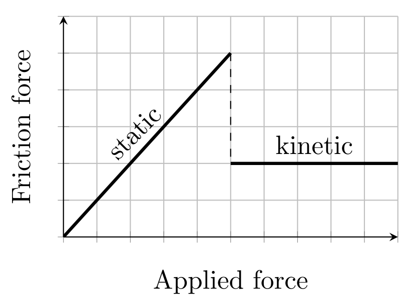
Forces that come from a surface are called normal forces (F_N). A **normal force** is that component of the contact force between two objects, which acts perpendicularly to and away from their plane of contact. Normal forces are contact forces (due to objects touching a surface), are perpendicular to the surface, and are not always vertical.



The **frictional force** (F_f) is an external force that acts opposite to the direction of motion or, for when there is no relative motion, in the direction needed to prevent slipping. Friction is parallel to the surface.



There are two types of friction. **Static friction** occurs between the surfaces of two objects that are not moving and will oppose the tendency of one object to move across the other. Static friction can change magnitude to prevent motion up to a maximum value. When this maximum value is overcome, the object starts moving. **Kinetic friction** occurs between the surfaces of two objects that *are* moving relative to one another. Kinetic friction is a constant value.

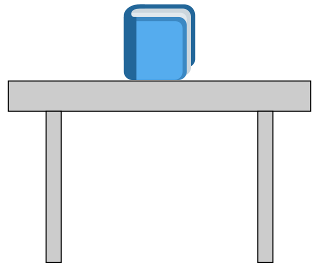


The amount of friction acting on an object is determined by the roughness of the surface and how "pushed together" the object and surface are. Friction is related to the coefficient of friction (μ).

- The coefficient μ is greater for rougher surfaces
- The coefficient μ is zero for frictionless surfaces

Friction is also related to the normal force. If the normal force increases due to a change in the scenario, then friction will increase too.

Example 5. A 3 kg book rests on a table. Find the normal force.



Recall: Normal force is the perpendicular force that an surface exerts to support the weight of an object.

Normal force is equal in magnitude to weight, opposite in direction

Weight is

$$w = mg = (3\,\mathrm{kg})(10\,\mathrm{m/s^2}) = 30\,\mathrm{N} \quad \text{down}$$

Magnitude of the normal force is

$$F_\mathrm{n} = |w| = 30\,\mathrm{N} \tag{11}$$

and the direction is up.

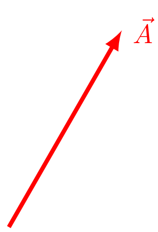
■

5.5 Analyzing Forces using Vectors

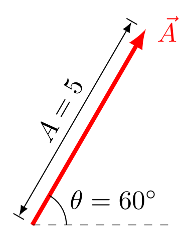
A **vector** is a quantity that has both magnitude and direction. Vectors are represented by arrows. Consider vector $\vec{A}$ with a magnitude of 5 units at 60° above the horizontal, shown below. The magnitude of this vector is

$$A = \left|\vec{A}\right| = 5\,\text{units}$$

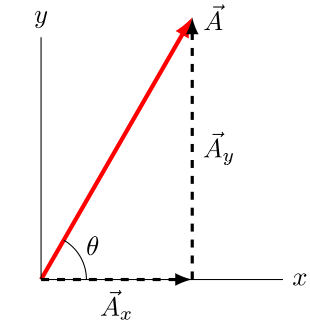
Vector $\vec{A}$:



Vector $\vec{A}$ annotated with magnitude (A) and direction (θ):

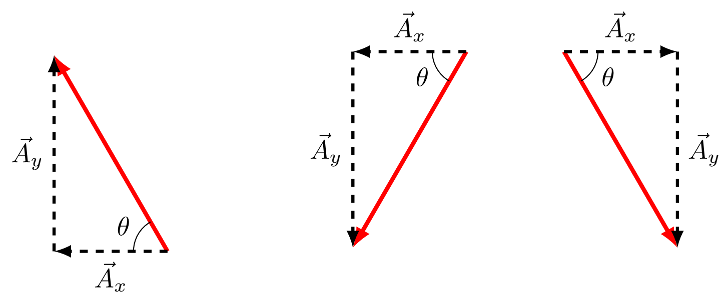


By placing the vector on the xy-coordinate plane, we can visualize and calculate the horizontal and vertical components vector $\vec{A}$. The horizontal component is called the x-component ($\vec{A}_x$), and the vertical is called the y-component ($\vec{A}_y$). Because the components $\vec{A}_x$ and $\vec{A}_y$ form a right triangle with vector $\vec{A}$, we can use trigonometry—specifically, the SOH CAH TOA method—to find magnitudes of the components. Also, we can relate the magnitudes of the components to the magnitude of the vector with the Pythagorean theorem. These relationships are summarized below.

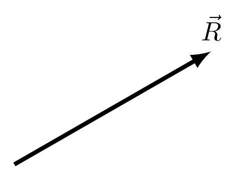


$$A_x = A\cos\theta$$
$$A_y = A\sin\theta$$
$$A_x^2 + A_y^2 = A^2$$
$$\tan\theta = \frac{A_y}{A_x}$$

Notice that θ is measured as the angle between the vector and the adjacent x-axis. These relationships hold in any quadrant of the xy-plane, calculators will give negative values if the horizontal or vertical components point left or right, respectively, as shown below:

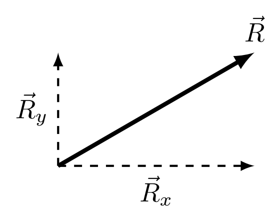


Example 6. Given vector $\vec{R}$ below, draw its horizontal and vertical components.



Solution:

Various examples of vector and their components are shown above, but it's often better to draw the components from the tail of the vector, like this:



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6 One-Dimensional Motion

6.1 Kinematic Equations

The **kinematic equations** are the equations that describe constant acceleration motion in terms of time, displacement, velocity, and acceleration. They may be derived from the work-energy theorem and impulse-momentum theorem as follows.

Example 7. Use the work-energy theorem to derive a kinematic equation.

Solution:

The work-energy theorem is

$$W = \Delta K$$

where

$$W = F\Delta x$$

and

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

By Newton's second law,

$$F = ma$$

so that

$$ma\Delta x = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Solving this equation for the term v_f^2 results in one of the kinematic equations:

$$\boxed{v_f^2 = v_i^2 + 2a\Delta x}$$

Note that this equation does not depend on mass.

■

Example 8. Use the impulse-momentum theorem to derive a kinematic equation.

Solution:

The impulse-momentum theorem states

$$Ft = \Delta p$$

where

$$\Delta p = mv_f - m_i$$

By Newton's second law,

$$F = ma$$

so that

$$mat = mv_f - mv_i$$

Solving this equation for final velocity leads to the kinematic equation

$$\boxed{v_f = v_i + at}$$

Note that this is a re-arrangement of the law of acceleration,

$$a = \frac{\Delta v}{t}$$

■

The two kinematic equations derived in the examples above may be combined to synthesize the four equations for uniformly accelerated motion. These equations are summarized below:

$$v_f = v_i + at \tag{12}$$

$$\Delta x = v_i t + \frac{1}{2}at^2 \tag{13}$$

$$v_f^2 = v_i^2 + 2a\Delta x \tag{14}$$

$$\Delta x = \frac{1}{2}(v_i + v_f)\,t \tag{15}$$

where v_f is final velocity, v_i is initial velocity, a is acceleration, Δt is the time interval during which the acceleration occurred, and Δx is displacement. All units are in meters, seconds, meters per second, or $\mathrm{m/s^2}$.

6.2 Accelerated Horizontal Motion

Example 9. A car coasts into a hill at $17.0\,\mathrm{m/s}$. It slows down with a uniform acceleration of $-1.0\,\mathrm{m/s^2}$. (a) What is the car's speed after $5.0\,\mathrm{s}$? (b) What is the car's displacement after $5.0\,\mathrm{s}$? (c) What is the car's speed after $10.0\,\mathrm{s}$? (d) What is the car's displacement after $10.0\,\mathrm{s}$?

Solution: First, we list the known variables, identify the unknown, and evaluate which equation is needed to solve the problem.

(a) The known and unknown variables are:

$$\begin{aligned} v_i &= 17\,\mathrm{m/s} \\ a &= -1.0\,\mathrm{m/s^2} \\ \Delta t &= 5.0\,\mathrm{s} \\ v_f &= \ ? \end{aligned}$$

These are related by Equation (12), which gives the answer:

$$
\begin{aligned}
v_f &= v_i + a\Delta t \\
&= 17\,\mathrm{m/s} + \left(-1.0\,\mathrm{m/s^2}\right)(5.0\,\mathrm{s}) \\
&= \boxed{12\,\mathrm{m/s}}
\end{aligned}
$$

(b) By Equation (13), the displacement is

$$
\begin{aligned}
\Delta x &= v_i \Delta t + \frac{1}{2} a \Delta t^2 \\
&= (17\,\mathrm{m/s})(5.0\,\mathrm{s}) + \frac{1}{2}\left(-1.0\,\mathrm{m/s^2}\right)(5.0\,\mathrm{s})^2 \\
&= \boxed{72.5\,\mathrm{m}}
\end{aligned}
$$

(c) If now $\Delta t = 10\,\mathrm{s}$, then the velocity is

$$
\begin{aligned}
v_f &= v_i + a\Delta t \\
&= 17\,\mathrm{m/s} + \left(-1.0\,\mathrm{m/s^2}\right)(10.0\,\mathrm{s}) \\
&= \boxed{7\,\mathrm{m/s}}
\end{aligned}
$$

(d) And with $\Delta t = 10\,\mathrm{s}$, displacement is now

$$
\begin{aligned}
\Delta x &= v_i \Delta t + \frac{1}{2} a \Delta t^2 \\
&= (17\,\mathrm{m/s})(10\,\mathrm{s}) + \frac{1}{2}\left(-1.0\,\mathrm{m/s^2}\right)(10\,\mathrm{s})^2 \\
&= \boxed{120\,\mathrm{m}}
\end{aligned}
$$

■

6.3 Free-fall: Accelerated Vertical Motion

Free fall is a situation in which the only force acting on an object is the force of gravity. By setting $a = -g$, the kinematic equations for one-dimensional free fall motion are

$$
v_f = v_i - gt \tag{16}
$$

$$
\Delta y = v_i t - \frac{1}{2} g t^2 \tag{17}
$$

$$
v_f^2 = v_i^2 - 2g\Delta y \tag{18}
$$

$$
\Delta y = \frac{1}{2}\left(v_i + v_f\right) t \tag{19}
$$

where $g = 10\,\mathrm{m/s^2}$ is the strength of the gravitational field near the surface of the Earth, also known as the acceleration due to gravity.

Example 10. The JP Morgan Chase Tower is the tallest building in Houston. If you drop a bowling ball from rest from the top of the building, it takes 7.81 seconds for it to strike the ground below. How tall is the building?

Solution:

We know time is $t = 7.81\,\mathrm{s}$ and the magnitude of gravitational acceleration is $g = 10\,\mathrm{m/s^2}$. Since the object is dropped from rest, initial velocity is $v_i = 0$. The ball's vertical displacement is

$$\begin{aligned}\Delta y &= v_i t - \frac{1}{2}gt^2 \\ &= (0\,\mathrm{m/s})(7.81\,\mathrm{s}) - \frac{1}{2}(10\,\mathrm{m/s^2})(7.81\,\mathrm{s})^2 \\ &= -305\,\mathrm{m}\end{aligned}$$

This vertical displacement must be equal in magnitude to the height of the building. Therefore, the building's height is 305 m.

■

Example 11. The Burj Kalifa, the tallest building in the world, is 830 meters tall. How long would it take a bowling ball that is released from rest from the top of the building to strike the ground?

Solution:

The displacement of the ball has the same magnitude as the height of the building, but it's negative since the ball moves downward:

$$\Delta y = -830\,\mathrm{m}$$

We are given that the ball is dropped from rest, so the initial velocity is $v_i = 0\,\mathrm{m/s}$. And gravitational acceleration has a magnitude of $g = 10\,\mathrm{m/s^2}$. These quantities are related by the kinematic equation

$$\Delta y = v_i t - \frac{1}{2}gt^2$$

Plugging in numbers leads to

$$\begin{aligned}-830\,\mathrm{m} &= (0\,\mathrm{m/s})t - \frac{1}{2}\left(10\,\mathrm{m/s^2}\right)t^2 \\ &= -\left(5\,\mathrm{m/s^2}\right)t^2\end{aligned}$$

To solve for time, we divide both sides by 5 and take the square root:

$$t = \sqrt{\frac{830\,\mathrm{m}}{5\,\mathrm{m/s^2}}} = \boxed{12.9\,\mathrm{s}}$$

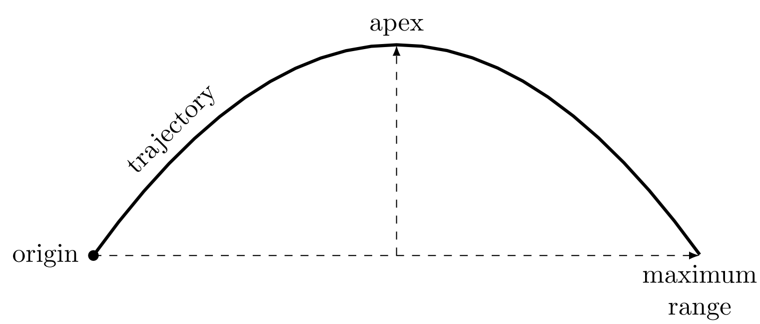
Therefore, it take 12.9 seconds for the ball to strike the ground.

■

7 Motion in Two Dimensions

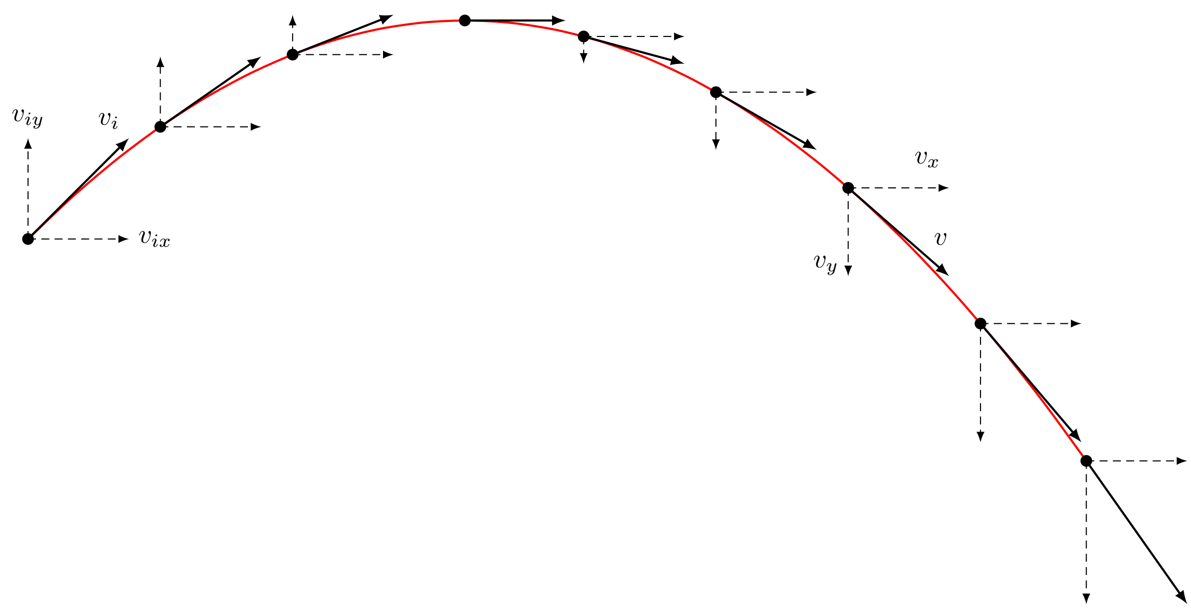
7.1 Projectile Motion

An object that travels through the air and experiences only acceleration due to gravity is called a **projectile**. Thus, **projectile motion** is the motion of an object that is subject only to the acceleration of gravity. The **trajectory** is the path of a projectile through the air. The shape of a trajectory is a parabola, shown below:



There's a special name for the point at which a projectile reaches peak vertical position: The **apex** is the location on the trajectory at which the projectile reaches maximum height.

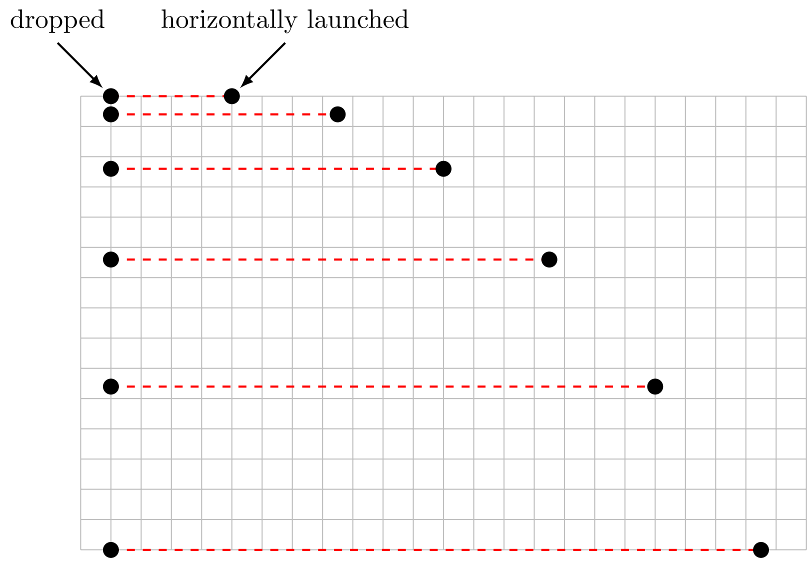
A projectile may be launched at different velocities, from different heights, and at different angles, and these changes could produce changes in the trajectory.



A projectile has horizontal and vertical motions. These are independent, meaning they don't influence one another. Vertical and horizontal motions are analyzed them separately, along perpendicular axes.

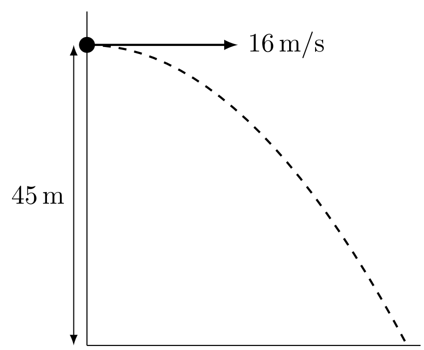
7.2 Horizontally Launched Projectiles

A **horizontally launched projectile** is, as the name suggests, a projectile whose initial velocity is entirely in the horizontal direction. The figure below is a motion map showing similarities and differences between a dropped object and a horizontally launched one.



The red dashed lines indicate that the object dropped into free fall is falling at the *same rate* as the horizontally launched projectile. Therefore, we should separate projectile motion into two components, one along horizontal axis and the other along the vertical axis. The following worked examples show how to do this.

Example 12. A cannonball is launched horizontally from the Statue of Liberty's base pedestal, which is 45 m above ground, at a speed of 16 m/s. How much time will it take the projectile to strike the ground?



Solution:

From the pedestal to the ground, the projectile's displacement is $\Delta y = -45\,\text{m}$ and during the fall its motion was governed by the kinematic equation

$$\Delta y = v_{iy}t - \frac{1}{2}gt^2$$

For a horizontally, the y-component of the initial velocity is zero: $v_{iy} = 0$. So, the equation reduces to

$$\Delta y = -\frac{1}{2}gt^2$$

Solving for time leads to

$$
\begin{aligned}
t &= \sqrt{-\frac{2\Delta y}{g}} \\
&= \sqrt{-\frac{2(-45\,\mathrm{m})}{10\,\mathrm{m/s^2}}} \\
&= \boxed{3.0\,\mathrm{s}}
\end{aligned}
$$

■

Example 13. How does the answer to Example 12, $t = 3\,\mathrm{s}$, change if the projectile's initial speed is doubled?

Solution:

The time doesn't change, since both projectiles fall from the same height at the same rate.

■

Example 14. When a ball is horizontally launched from a height h, it takes the ball $4.0\,\mathrm{s}$ to strike the ground. How long will take the same ball to strike ground if it is horizontally launched from a height $4h$?

Solution:

In Example 12 we saw that the kinematic equation may be solved for time as

$$t = \sqrt{-\frac{2\Delta y}{g}}$$

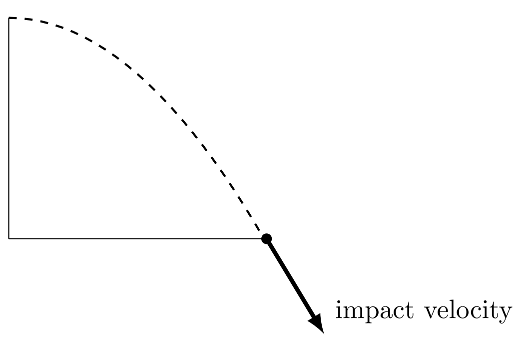
This equation shows that time is proportional to the square root of displacement:

$$t \propto \sqrt{|\Delta y|}$$

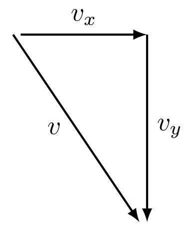
So, increasing the height by a factor of 4 increases the time by a factor of $\sqrt{4}$, or 2. Therefore the new elapsed time is $8.0\,\mathrm{s}$.

■

Impact speed is the speed at which a projectile strikes the ground after being launched. The impact velocity is the vector representing the impact speed and its direction.



The impact speed v is the magnitude of the vector addition of the horizontal and vertical components of velocity at impact.



Thus, it is calculated by the Pythagorean theorem as

$$v = \sqrt{v_x^2 + v_y^2} \tag{20}$$

where v_x and v_y are the horizontal and vertical components of velocity at the moment of impact.

Example 15. In Example 12, a cannonball was launched near the State of Liberty from a height of 45 m at 16 m/s. Find the speed of the projectile the instant it strikes the ground. (Find the impact speed.)

Solution:

Impact speed is given by Equation (20):

$$v = \sqrt{v_x^2 + v_y^2}$$

The horizontal component of speed is the same as the initial speed, since horizontal acceleration is zero in projectile motion:

$$v_x = v_i = 16\,\mathrm{m/s}$$

The vertical component of impact velocity is given by kinematic equation (16) with an initial vertical speed zero and an elapsed time of 3 seconds that was calculated in Example 12:

$$\begin{aligned} v_y &= v_i - gt \\ &= 0\,\mathrm{m/s} - (10\,\mathrm{m/s^2})(3.0\,\mathrm{s}) \\ &= -30\,\mathrm{m/s} \end{aligned}$$

The negative indicates the direction of the y-component of velocity is downward. Therefore,

$$\begin{aligned} v &= \sqrt{v_x^2 + v_y^2} \\ &= \sqrt{(16\,\mathrm{m/s})^2 + (-30\,\mathrm{m/s})^2} \\ &= \boxed{34\,\mathrm{m/s}} \end{aligned}$$

■

7.3 Circular Motion

Example 16. A car moves at a constant linear speed of 24 m/s in a circular road of radius 18 m. Calculate the car's centripetal acceleration.

Solution:

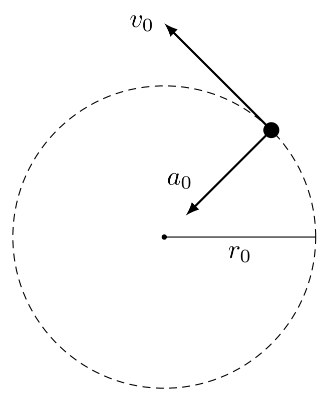
We are given linear speed $v = 24\,\mathrm{m/s}$ and radius of curvature $r = 18\,\mathrm{m}$. The centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{(24\,\mathrm{m/s})^2}{18\,\mathrm{m}} = \boxed{32\,\mathrm{m/s^2}}$$

This acceleration points towards the center of the circle.

■

Example 17. An object moves with constant speed v_0 in a circle of radius r_0 and experiences a centripetal acceleration a_0.



If the object's speed changes to $\frac{1}{2}v_0$ and the radius stays constant, what is the new centripetal acceleration?

Solution:

Let a be the new centripetal acceleration, $v = \frac{1}{2}v_0$ be the new speed, and $r = r_0$ be the constant radius. The original acceleration is

$$a_0 = \frac{v_0^2}{r_0}$$

The new centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{\left(\frac{1}{2}v_0\right)^2}{r_0} = \frac{\left(\frac{1}{2}\right)^2 v_0^2}{r_0} = \frac{1}{4}\left(\frac{v_0^2}{r_0}\right) = \frac{1}{4}a_0$$

Therefore, when the speed is halved, the acceleration decreases to $\frac{1}{4}$ of the original acceleration.

■

8 Conservation in Mechanical Systems

8.1 Gravitational and Elastic Potential Energy

A **system** is one or more objects of interest for which only the forces acting on them from the outside are considered, but not the forces acting between them or inside them.

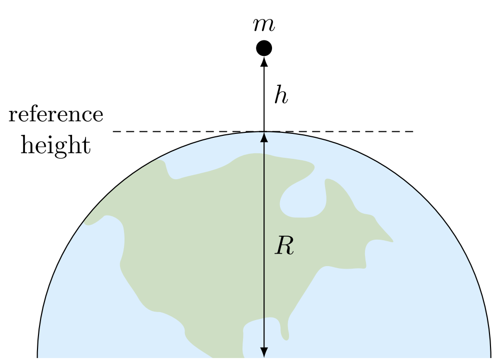
Gravitational potential energy (E_g) is energy acquired by doing work against gravity.

The gravitational potential energy between two objects of mass m_1 and m_2 whose centers of mass are separated by distance r is

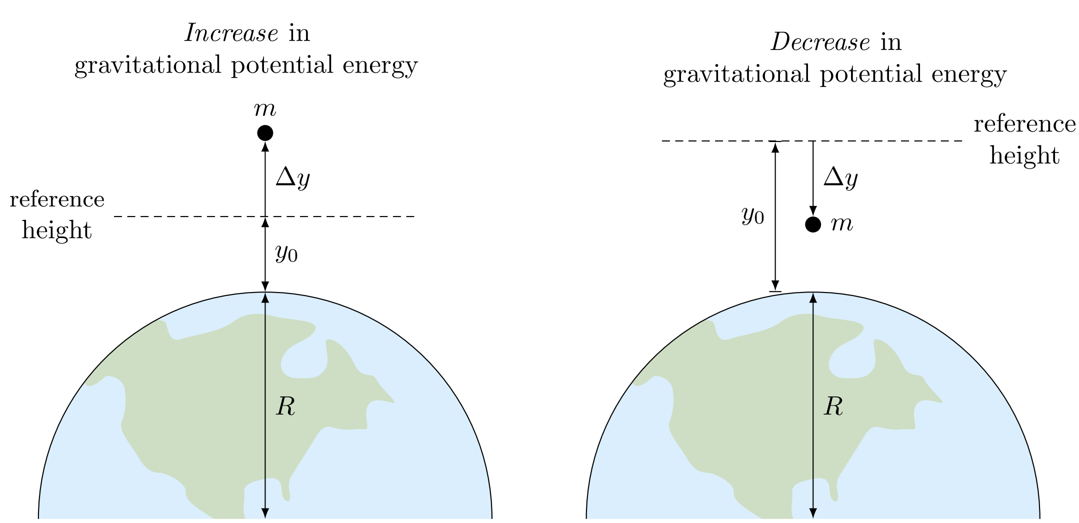
$$E_g = -\frac{Gm_1m_2}{r}$$

When an object is raised from the surface of the Earth to a height h, there is an gain in gravitational potential energy of magnitude

$$\Delta E_g = mgh$$



In general, the reference height y_0 doesn't need to be at the surface (where $y_0 = 0$). It can be set at the top of a 6-meter ramp ($y_0 = 6\,\mathrm{m}$) or on top of a tall building ($y_0 = 100\,\mathrm{m}$). Furthermore, a vertical displacement Δy above or below the reference height yields an increase or a decrease in gravitational potential energy, respectively.



Note: Lengths in figures are not to scale.

Therefore, the general formula for the change in gravitational potential energy is

$$\Delta E_g = mg\Delta y \tag{21}$$

Example 18. A 2.0-kg ball gets kicked in the air to a maximum height of 15 meters. At the peak height, what is the change in gravitational potential energy of the ball-Earth system relative to the ground?

Solution:

We are given mass and height: $m = 2.0\,\mathrm{kg}$, $h = 15\,\mathrm{m}$, and gravity is $10\,\mathrm{m/s^2}$. The unknown we are finding in gravitational potential energy: $\mathrm{PE_g} = ?$ By Eq. (**??**),

$$\Delta E_g = mg\Delta y = (2.0\,\mathrm{kg})(10\,\mathrm{m/s^2})(15\,\mathrm{m}) = \boxed{300\,\mathrm{J}}$$

■

8.2 Conservation of Mechanical Energy

Mechanical energy is the sum of kinetic energy and potential energy. The law of conservation of mechanical energy says that for a closed system energy is conserved. This law is expressed as

$$\mathrm{ME} = K + \mathrm{PE} = \text{constant} \tag{22}$$

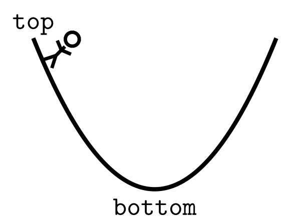
but is more usefully written as

$$\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f} \tag{23}$$

If we replace each K and PE with $\frac{1}{2}mv^2$ and mgh, respectively, the law of conservation of energy becomes

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f \tag{24}$$

Example 19. Tony Hawk starts from rest from the top of a 6-meter tall ramp. His mass is 77 kg. What will be his kinetic energy at the bottom of the ramp? Assume no energy is lost to friction.



Solution:

By the law of conservation of mechanical energy (Eq. 23), Hawk's mechanical energy is governed by the equation

$$\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}$$

At the top, Hawk has no motion, so his initial kinetic energy is zero: $\mathrm{KE_i} = 0$. At the bottom, he loses all height above the ramp, so his gravitational potential energy is zero: $\mathrm{PE_f} = 0$. Therefore, conservation of energy reduces from

$$\cancel{\mathrm{KE_i}} + \mathrm{PE_i} = \mathrm{KE_f} + \cancel{\mathrm{PE_f}}$$

to

$$\mathrm{PE_i} = \mathrm{KE_f} \tag{25}$$

This result means that all of Hawk's potential (stored) energy at the top of the ramp will be converted to kinetic (motion) energy when he reaches the bottom.

We are given Hawk's mass and his initial height above the ramp: $m = 77\,\mathrm{kg}$ and $h_i = 6\,\mathrm{m}$. The unknown we're looking for is his final kinetic energy, at the bottom of the ramp: $\mathrm{KE_f} = ?$ The given quantities are related by Eq. (**??**), which provides Hawk's initial gravitational potential energy:

$$\mathrm{PE_i} = mgh_i = (77)(9.8)(6) = 4527.6\,\mathrm{J}$$

This energy, by Eq. (25), gets converted to Hawk's kinetic energy, so

$$\mathrm{KE_f} = 4527.6\,\mathrm{J}$$

This example is to show that all of Tony Hawk's gravitational potential energy at the top of the ramp was entirely converted to his kinetic energy at the bottom.

■

Example 20. When Tony Hawk reaches the bottom, in Example 19, what is his speed?

Solution:

At the end of Example 19 we concluded that his kinetic energy at the bottom is

$$\mathrm{KE_f} = 4527.6\,\mathrm{J}$$

But his kinetic energy is

$$K = \frac{1}{2}mv^2$$

Substituting the given values leads to

$$4527.6 = \frac{1}{2} \cdot 77 \cdot v^2$$

If we solve this equation for speed (v) then his speed is

$$v = 10.8\,\mathrm{m/s}$$

■

8.3 Conservation of Momentum

Example 21. A 0.057-kg tennis ball moves at $-12\,\mathrm{m/s}$ when it gets struck to a new velocity of $+48\,\mathrm{m/s}$. What is the ball's change in momentum?

Solution:

We know the tennis ball's mass, initial velocity, and final velocity: $m = 0.057\,\mathrm{kg}$, $v_i = -12\,\mathrm{m/s}$, and $v_f = +48\,\mathrm{m/s}$. We want to find change in momentum: $\Delta p = ?$ Change in momentum is

$$\Delta p = m\,(v_f - v_i) = 0.057\,(48 - (-12)) = 0.057\,(48 + 12) = 3.42\,\mathrm{kg\,m/s}$$

■

Example 22. During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match. The ball started at rest and reached a speed of 58 m/s (130 mph). What was the average force exerted on the 0.057-kg tennis ball by Williams' racquet? Assume that the ball remained in contact with the racquet for 5 ms (milliseconds) and accelerated horizontally.

Solution:

We are given final velocity, initial velocity, mass, and elapsed time: $v_f = 58\,\mathrm{m/s}$, $v_i = 0\,\mathrm{m/s}$, $m = 0.057\,\mathrm{kg}$, and $\Delta t = 5\,\mathrm{ms} = 0.005\,\mathrm{s}$. (Note: $1\,\mathrm{ms} = 1\,\mathrm{millisecond} = 0.001\,\mathrm{s} = 1 \times 10^{-3}\,\mathrm{s}$.) We want to find the average net force: $F_{\mathrm{net}} = ?$ Equation (**??**) relates change in momentum to change in time as

$$F_{\mathrm{net}} = \frac{\Delta p}{\Delta t}$$

The tennis ball's change in momentum is

$$\Delta p = m(v_f - v_i) = 0.057(58 - 0) = 3.306\,\mathrm{kg\,m/s}$$

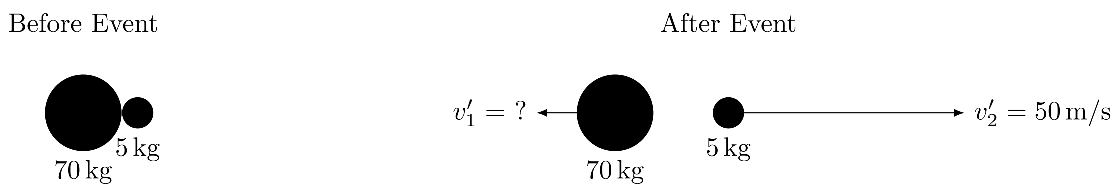
Therefore, the net force supplied by the racquet is

$$F_{\mathrm{net}} = \frac{\Delta p}{\Delta t} = \frac{3.306}{0.005} = 661\,\mathrm{N}$$

A force of 661 N is about the weight of a 150-pound person!

■

Example 23. Nancy the ice skater, whose mass is 70 kg, stands at rest with ice skates on an icy (frictionless) surface. She uses superhero strength to throw a 5 kg sack of potatoes horizontally away from her body at 50 m/s (112 mph). As a result of both the frictionless surface and the law of conservation of momentum, she will accelerate in the direction opposite the sack. The speed she gains as a result throwing the object is known as the **recoil velocity**. Calculate Nancy's recoil velocity.



Solution:

We are given Nancy's mass, the sack's mass, and the final velocity of the sack: $m_1 = 70\,\mathrm{kg}$, $m_2 = 5\,\mathrm{kg}$, and $v_2' = 50\,\mathrm{m/s}$. Furthermore, the initial velocities of both objects is zero: $v_1 = 0$ and $v_2 = 0$. The unknown quantity we're looking for is Nancy's final (recoil) velocity: $v_1' = ?$ All these quantities are related by the law of conservation of momentum:

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

Substituting the known values leads to

$$0 = 70\, v_1' + (5)(50)$$

which simplifies to

$$0 = 70\,v_1' + 250$$

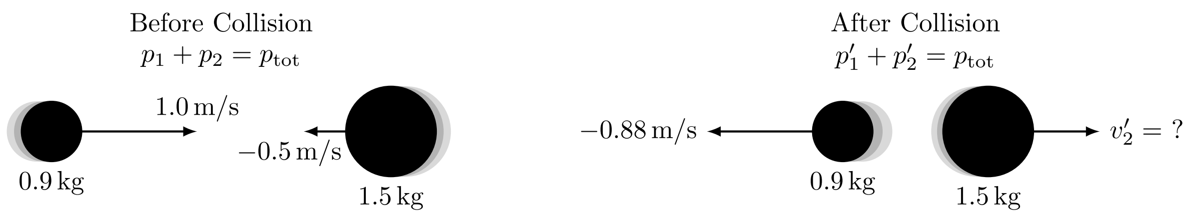
Solve for the recoil velocity (v_1') as follows:

Write v_1' on left	$70\,v_1' + 250 = 0$
Subtract 250	$70\,v_1' + \cancel{250} - \cancel{250} = -250$
Simplify	$70\,v_1' = -250$
Divide by 70	$\dfrac{\cancel{70}\,v_1'}{\cancel{70}} = \dfrac{-250}{70}$
Simplify	$v_1' = -3.57$

Therefore, Nancy's recoil velocity is $-3.57\,\mathrm{m/s}$. She slides backwards as a result of throwing the object forward, all while the momentum of the system remains conserved.

■

Example 24. An object with a mass of 0.9 kg moving at 1.0 m/s experiences an elastic collision with an object of mass 1.5 kg moving at −0.5 m/s. If the first object's velocity after collision is −0.88 m/s, what is the second object's velocity?



Solution:

We are given the masses and initial velocities of both objects and the velocity after collision of the first object: $m_1 = 0.9\,\mathrm{kg}$, $v_1 = 1.0\,\mathrm{m/s}$, $m_2 = 1.5\,\mathrm{kg}$, $v_2 = -0.5\,\mathrm{m/s}$, and $v_1' = -0.88\,\mathrm{m/s}$. The unknown we want to find is the velocity of the second object after collision: $v_2' = ?$. These variables are all related by Eq. (**??**), the conservation of momentum equation:

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

Substituting the given values leads to

$$(0.9)(1) + (1.5)(-0.5) = (0.9)(-0.88) + (1.5)v_2'\ ,$$

which reduces to

$$0.15 = -0.792 + 1.5\,v_2'$$

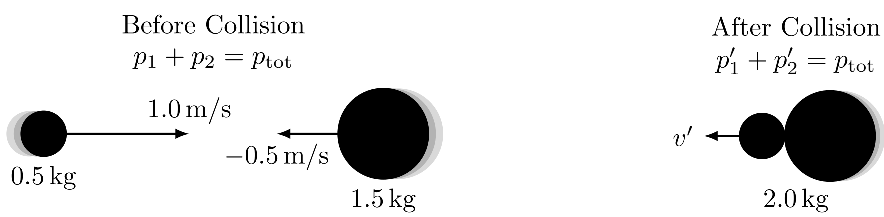
We solve this equation for v_2' via the following steps:

$$
\begin{aligned}
\textbf{Write } v_2' \textbf{ on left} \quad & 1.5\,v_2' - 0.792 = 0.15 \\
\textbf{Add 0.792} \quad & 1.5\,v_2' - \cancel{0.792} + \cancel{0.792} = 0.15 + 0.792 \\
\textbf{Reduce} \quad & 1.5\,v_2' = 0.942 \\
\textbf{Divide by 1.5} \quad & \frac{\cancel{1.5}\,v_2'}{\cancel{1.5}} = \frac{0.942}{1.5} \\
\textbf{Reduce} \quad & v_2' = 0.628
\end{aligned}
$$

Therefore, the velocity of the second object after collision is (rounding up) $v_2' = +0.63\,\mathrm{m/s}$.

■

Example 25. A 0.500 kg object that is moving at a velocity of 1.00 m/s inelastically collides with a 1.50 kg object that is moving at −0.500 m/s. What is the final velocity of the combined system after the collision?



Solution:

We are given two masses and two initial velocities: $m_1 = 0.5\,\mathrm{kg}$, $m_2 = 1.5\,\mathrm{kg}$, $v_1 = 1.0\,\mathrm{m/s}$, and $v_2 = -0.5\,\mathrm{m/s}$. We want to find the final velocity of the stuck system: $v' = ?$. These quantities are related by Equation (**??**) for inelastic collisions:

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)\, v'$$

To solve this equation for v', we take the following steps:

$$
\begin{aligned}
\textbf{Substitute given values} \quad & (0.5)(1.0) + (1.5)(-0.5) = (0.5 + 1.5)\, v' \\
\textbf{Simplify} \quad & -0.25 = 2\, v' \\
\textbf{Write } v' \textbf{ on left} \quad & 2\, v' = -0.25 \\
\textbf{Divide by 2} \quad & \frac{\cancel{2}\, v'}{\cancel{2}} = \frac{-0.25}{\cancel{2}} \\
\textbf{Simplify} \quad & v' = -0.125\,\mathrm{m/s}
\end{aligned}
$$

Therefore, the final velocity of the system is (rounding up) $-0.13\,\mathrm{m/s}$; the system moves leftward.

■

1. *YouTube*: "Collisions V2: Physics Concept Trailer" by `OpenStax` (click here)
2. *YouTube*: "Home Run Swing Slow Motion" by `Baseball Swingpedia` (click here)
3. *YouTube*: "You Can't Run From Momentum!" by `Flipping Physics` (click here). Funny dramatization to introduce momentum.
4. *YouTube*: "Baseball Swing in Slow Motion" by `PasetimeAthletics` (click here)

5. *YouTube*: "Calculating the Force of Impact when Stepping off a Wall" by `Flipping Physics` (click here). Great way to show how changes in elapsed time influence magnitude of net force.
6. *YouTube*: "Airbags - Toyota Crash Tests" by `ToyotaSubaruCrashTests` (click here). More applications on how increasing elapsed time decreases net force.
7. *YouTube*: "Venus Williams serve record 209 Km/h" by `SuperTennisTv` (click here)
8. *PhET Simulation*: "Collision Lab" (click here)

9 Oscillatory Motion

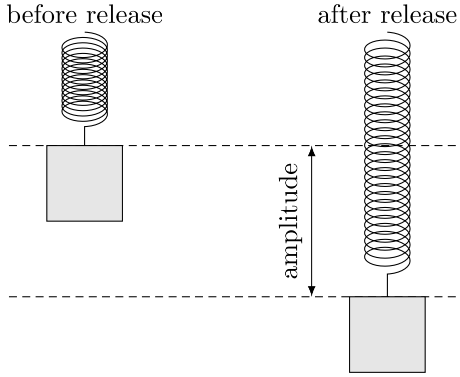
9.1 Oscillating Motion

To **oscillate** means to move back and forth regularly between two points. A **medium** is the solid, liquid, or gas material through which a wave propagates.

9.2 Amplitude, Frequency, and Period in Simple Harmonic Motion (SHM)

Simple harmonic motion (SHM) is the oscillatory motion in a system where the net force can be described by Hooke's law.

A **simple harmonic oscillator** is a device that oscillates in SHM, such as a mass that is attached to a spring, where the restoring force is proportional to the displacement and acts in the direction opposite to the displacement. The figure below shows such a oscillator, a mass attached to a spring. Before the mass is released, it is held in place without any compression or stretching on the string. After the mass is released, gravity does work on the mass, and the spring is stretched until the downward force of gravity is balanced by the upward force by the spring. The amplitude is shown below.



The **amplitude** is the maximum displacement from the equilibrium position of an object oscillating around the equilibrium position. A ruler may be used to measure the oscillator's amplitude. A stopwatch is used to measure the oscillator's period and frequency. The **period** (T) is the time it takes to complete one oscillation, and **frequency** (f) is number of wave cycles per unit of time. They are inversely related:

$$f = \frac{1}{T} \qquad T = \frac{1}{f} \tag{26}$$

As period increases, frequency decreases, and vice-versa. Period is measured in seconds. Frequency is measured in 1 over seconds, or Hertz (Hz). Thus, 1 Hz is equivalent is 1/s.

Example 26. A transverse wave completes 1 wave cycle every 0.25 seconds. What is the frequency of the wave?

Solution:

We are given the period of the wave: $T = 0.25\,\mathrm{s}$. By Equation 26, its frequency is

$$f = \frac{1}{T} = \frac{1}{0.25} = 4\,\mathrm{Hz}\ .$$

Therefore, this wave completes 4 wave cycles every second.

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Example 27. If a wave completes exactly $3\frac{1}{3}$ wave cycles every second, what is the period of oscillation of the wave?

Solution:

We are given the wave's frequency as 3 and one-third hertz, or $f = 3\frac{1}{3}\,\text{Hz}$. This mixed number is written as an improper[2] fraction as

$$3\frac{1}{3} = \frac{10}{3} = 3.\overline{3}$$

So, the frequency is $f = \frac{10}{3}\,\text{Hz}$. The period, by Equation (26), is

$$T = \frac{1}{f} = \frac{1}{\frac{10}{3}} = \frac{3}{10} = 0.3\,\text{s}$$

Therefore, it takes this wave 0.3 seconds to complete one wave cycle.

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Pendulums?

[2] See Section 4.1 of *OpenStax: Prealgebra 2e* for a review on mixed numbers and improper fractions.

10 Mechanical Waves

11 Conservation of Charge

12 Electromagnetic Induction

13 Electromagnetic Waves

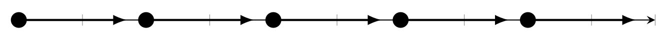
14 Quantum Physics

Multiple Representations in the Physics Classroom

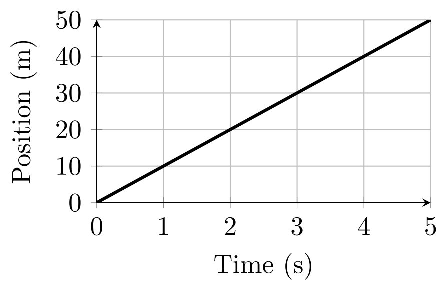
When students can interpret and use any of these representations interchangeably, they have a deeper understanding of the topics being discussed. In addition, it helps the students to have multiple ways to approach a problem rather than just memorizing a set of steps. Let's break down what representations are appropriate for physics instruction:

Words
Students should be able to use words to describe a scientific observation or principle. They should also be able to obtain information and use information provided in words to analyze a particular situation.

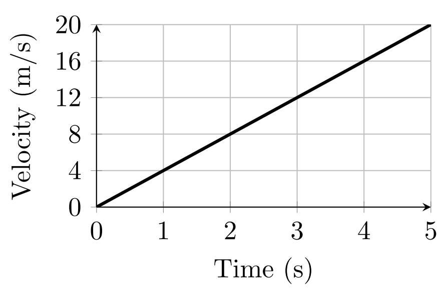
Motion Map
Using a motion map can help students compare the motion of an object from second to second. It is helpful for looking at what is happening to the distance an object travels each second.



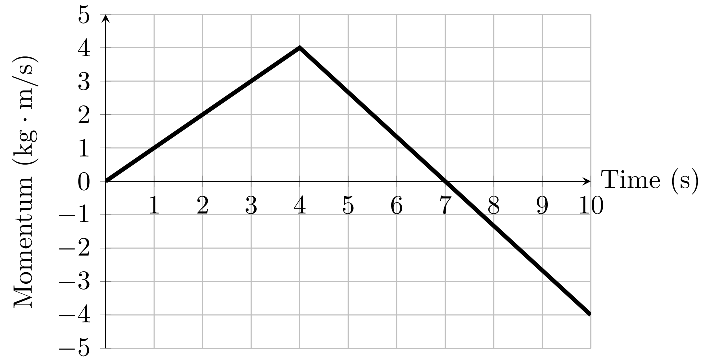
Position vs Time Graph
Students should be able to use Position vs Time graphs to describe initial position, direction of motion, and any changes in motion that might occur.



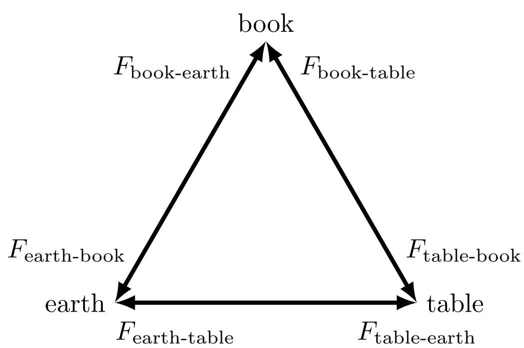
Velocity vs Time Graph
Students should be able to use Velocity vs Time graphs to describe initial speed, direction of motion, and any changes in motion that occur.



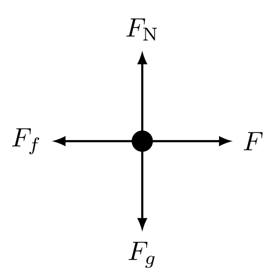
Momentum vs Time Graph
Students should be able to use Momentum vs Time graphs to describe direction of motion, any changes in motion that occur in motion, and discuss causes for the changes in motion.



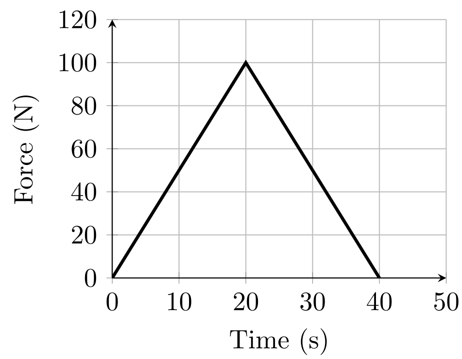
Force Schema
Students should be able to use Force Schema to describe a system, forces acting within a system, external forces that do not affect the system, and external forces that do affect the system.



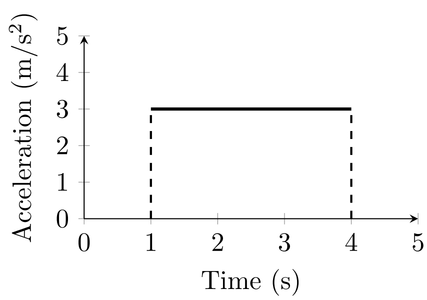
Free Body Diagrams
Students should be able to use Free Body Diagrams to determine whether forces are balanced or unbalanced, the direction of any net force present, and the direction of any acceleration present.



Force vs Time Graph
Students should be able to use Force Vs Time graphs to describe the motion of an object, any changes in motion of an object, and any impulse acting on the object.



Acceleration vs Time Graph
Students should be able to use an acceleration vs time graph to describe any change in motion in an object and the direction of any net force acting on the object.



Mathematical Models
Students should be able to use equations to help solve for an unknown or to support a conclusion. Number sense skills can also be used when writing problems to be addressed with a mathematical model.

$$v_f = v_i + at$$

$$\Delta x = v_i t + \frac{1}{2} a t^2$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$\Delta x = \frac{1}{2}\left(v_i + v_f\right) t$$

Equation Sheet

Gravitational acceleration	Gravitational constant	Coulomb constant	Speed of light
$g = 10\,\mathrm{m/s^2}$	$G = 6.67 \times 10^{-11}\ \frac{\mathrm{N\,m^2}}{\mathrm{kg^2}}$	$k = 9.0 \times 10^{9}\ \frac{\mathrm{N\,m^2}}{\mathrm{C^2}}$	$c = 3.0 \times 10^{8}\,\mathrm{m/s}$

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\Delta x = x_f - x_i$$

$$v = \frac{\Delta x}{\Delta t}$$

$$p = mv$$

$$E_\mathrm{K} = \frac{1}{2}mv^2$$

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{F_\mathrm{net}}{m}$$

$$\Delta p = m\Delta v$$

$$J = F_\mathrm{net}\Delta t$$

$$F_\mathrm{net}\Delta t = \Delta p$$

$$W = Fd$$

$$W_\mathrm{net} = \Delta E_\mathrm{K}$$

$$P = \frac{W}{\Delta t}$$

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$F_g = mg$$

$$F_f = \mu F_\mathrm{N}$$

$$F_s = -kx$$

$$v_x = v_{ix} + a_x t$$

$$\Delta x = v_{ix}t + \frac{1}{2}a_x t^2$$

$$v_x^2 = v_{ix}^2 + 2a_x\Delta x$$

$$\Delta x = \frac{1}{2}\left(v_i + v_f\right)\Delta t$$

$$v_y = v_{iy} - gt$$

$$\Delta y = v_{iy}t - \frac{1}{2}gt^2$$

$$v_y^2 = v_{iy}^2 - 2g\Delta y$$

x = position
v = speed or velocity
Δx = displacement
Δt = time interval
p = momentum
m = mass
E_K = kinetic energy
a = acceleration
F_net = net force
Δp = change in p
Δv = change in v
J = impulse
W = work
F = force
d = distance
W_net = net work
P = power
F_g = gravitational force
r = distance or radius
F_f = frictional force
μ = coefficient of friction
F_N = normal force
F_s = spring force
k = spring constant
v_x = horizontal component of velocity
v_{ix} = horizontal component of initial velocity
a_x = horizontal component of acceleration
t = time
y = vertical position
Δy = vertical displacement

$$v_t = \frac{2\pi r}{t}$$

$$a_c = \frac{v_t^2}{r}$$

$$F_c = ma_c = \frac{mv^2}{r}$$

$$\Delta E_g = mg\Delta y$$

$$E_s = \frac{1}{2}kx^2$$

$$E_i + W_\mathrm{ext} = E_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$F_\mathrm{E} = k\frac{q_1 q_2}{r}$$

$$I = \frac{V}{R}$$

$$P = IV$$

$$R_{eq,series} = R_1 + R_2 + R_3 + \cdots$$

$$\frac{1}{R_\mathrm{eq,parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

$$f = \frac{\text{cycles}}{\text{second}}$$

$$f = \frac{1}{T}$$

$$T = \frac{\text{seconds}}{\text{cycle}}$$

$$v = f\lambda$$

$$M = \frac{h_i}{h_o} = \frac{d_i}{d_o}$$

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

$$E = mc^2$$

v_t = tangential velocity
r = radius
a_c = centripetal acceleration
F_c = centripetal force
ΔE_g = change in gravitational potential energy
E_s = spring or elastic energy
k = spring constant or Coulomb constant
W_ext = work from outside system
F_E = electric force
I = current
V = voltage
R = resistance
P = power
f = frequency or focal length
T = period
λ = wavelength
M = magnification
h_i = image height
h_o = object height
d_i = image distance
d_o = object distance
E = energy

Glossary

acceleration a change in velocity over time. 20

amplitude the maximum displacement from the equilibrium position of an object oscillating around the equilibrium position. 49

apex the location on the trajectory at which the projectile reaches maximum height. 36

applied force a contact force intentionally exerted by a person on an object. 15

average acceleration change in velocity divided by the time interval over which it changed. 20

average velocity displacement divided by the time during which the displacement occurs. 6

average speed distance traveled divided by the time during which the motion occurs. 6

contact force a type of force that occurs when objects are physically in contact with each other. 28

displacement the change in position of an object against a fixed axis. 5

distance the length of the path actually traveled between an initial and a final position. 5

force a push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force; the SI unit of force is the Newton (N). 15

free fall a situation in which the only force acting on an object is the force of gravity. 34

free body diagram a diagram showing all external forces acting on a body. 16

frequency number of wave cycles per unit of time. 49

frictional force an external force that acts opposite to the direction of motion or, for when there is no relative motion, in the direction needed to prevent slipping. 15, 29

gravitational potential energy energy acquired by doing work against gravity. 41

gravitational constant the proportionality constant in Newton's law of universal gravitation. 26

gravitational force the downward force on an object due to the attraction by the Earth or other massive body. 15

head the end point of a vector; the location of the vector's arrow; also referred to as the tip. 3

head-to-tail method a method of adding vectors in which the tail of each vector is placed at the head of the previous vector. 3

horizontally launched projectile a projectile whose initial velocity is entirely in the horizontal direction. 36

impact speed the speed at which a projectile strikes the ground after being launched. 38

impulse average net external force multiplied by the time the force acts; equal to the change in momentum. 24

impulse-momentum theorem the impulse equals change in momentum. 24

inertia the tendency of an object at rest to remain at rest, or for a moving object to remain in motion in a straight line and at a constant speed. 12

joule the metric unit for work and energy; equal to 1 newton meter (N · m). 13

kinematic equations the equations that describe constant acceleration motion in terms of time, displacement, velocity, and acceleration. 32

kinetic energy energy of motion. 12

magnitude size or amount. 3, 5

mass the quantity of matter in a substance; the SI unit of mass is the kilogram. 11, 27

medium the solid, liquid, or gas material through which a wave propagates. 49

momentum the product of a system's mass and velocity. 12

momentum vs. time graph a graph in which momentum is plotted on the vertical axis and time is plotted on the horizontal axis. 12

net force the sum of all forces acting on an object or system. 21

Newton's universal law of gravitation states that gravitational force between two objects is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. 26

Newton's third law of motion whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts. 18

Newton's first law of motion a body at rest remains at rest or, if in motion, remains in motion at a constant speed in a straight line, unless acted on by a net external force; also known as the law of inertia. 12

normal force that component of the contact force between two objects, which acts perpendicularly to and away from their plane of contact. 15, 28

oscillate to move back and forth regularly between two points. 49

period the time it takes to complete one oscillation. 49

position the location of an object at any particular time. 4

position vs. time graph a graph in which position is plotted on the vertical axis and time is plotted on the horizontal axis. 7

projectile an object that travels through the air and experiences only acceleration due to gravity. 36

projectile motion the motion of an object that is subject only to the acceleration of gravity. 36

scalar a quantity that has magnitude (and possibly sign) but no direction. 3

simple harmonic oscillator a device that oscillates in SHM, such as a mass that is attached to a spring, where the restoring force is proportional to the displacement and acts in the direction opposite to the displacement. 49

simple harmonic motion the oscillatory motion in a system where the net force can be described by Hooke's law. 49

speed rate at which an object changes its location. 6

spring force a force applied from a spring when it is either compressed or stretched. 16

system one or more objects of interest for which only the forces acting on them from the outside are considered, but not the forces acting between them or inside them. 41

tail the starting point of a vector; the point opposite to the head or tip of the arrow. 3

tension a pulling force that acts along a connecting medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force exerted on the object by the rope is called tension. 16

trajectory the path of a projectile through the air. 36

vector a quantity that has both magnitude and direction. 3, 30

velocity the speed and direction of an object. 6

velocity vs. time graph a graph in which velocity is plotted on the vertical axis and time is plotted on the horizontal axis. 9

weight the force of gravity, F_g, acting on an object of mass m. 27

work force multiplied by distance. 24