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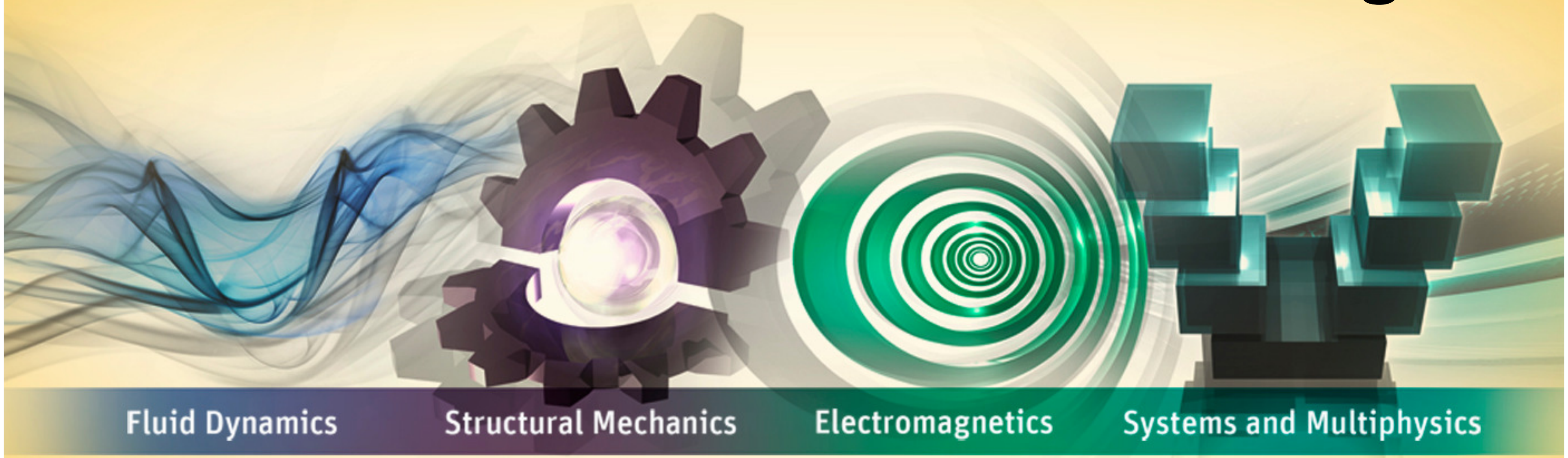
Large Eddy Simulation and Hybrid RANS-LES Turbulence Modeling



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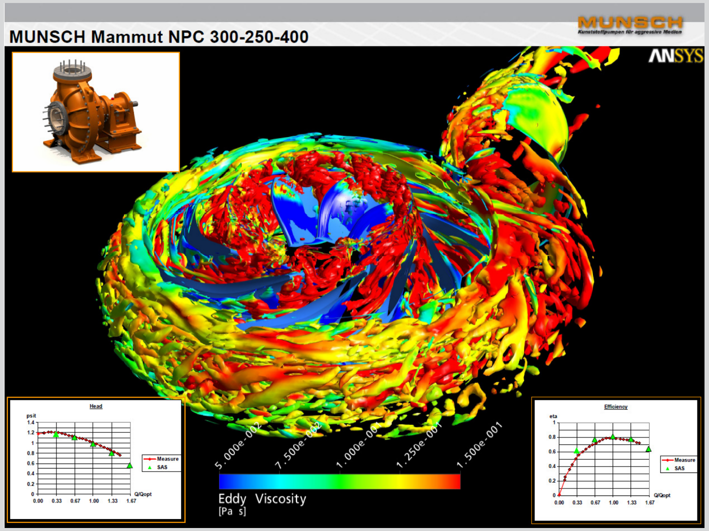
Motivation for Scale-Resolving Simulations (SRS)

- Accuracy Improvements over RANS
  – Flows with large separation zones (stalled airfoils/wings, flow past buildings, flows with swirl instabilities, etc.)
- Additional information required
  – Acoustics - Information on acoustic spectrum not reliable from RANS
  – Vortex cavitation – low pressure inside vortex causes cavitation – resolution of vortex required
  – Fluid-Structure Interaction (FSI) – unsteady forces determine frequency response of solid.



Scale-Resolving Simulation (SRS)

- SRS refers to all turbulence models, which resolve at least a portion of the turbulence spectrum in at least a part of the domain
  - Scale-Adaptive Simulation (SAS)
  - Detached Eddy Simulation (DES)
  - Large Eddy Simulation (LES)
  - Wall-modelled LES (WMLES)
  - Embedded and Zonal LES (ELES, ZFLES)
  - Other RANS-LES hybrids
- SRS is a field of intense research and many new model formulations/combinations are explored
- In ANSYS CFD R14, the most promising new approaches were selected and implemented

Eddy Viscosity Models

- Averaged Navier-Stokes (RANS/LES) Equations with Eddy Viscosity:

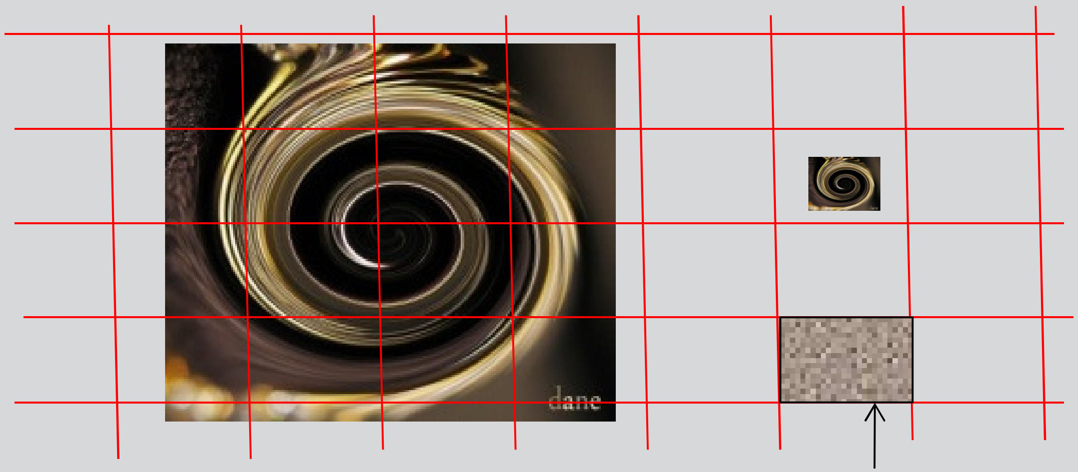
$$\frac{\partial(\overline{U}_i)}{\partial t}+\frac{\partial(\overline{U}_j\overline{U}_i)}{\partial x_j}=-\frac{1}{\rho}\frac{\partial\overline{P}}{\partial x_i}+\frac{\partial}{\partial x_j}\left[\left(\nu+\nu_t\right)\left(\frac{\partial\overline{U}_i}{\partial x_j}+\frac{\partial\overline{U}_j}{\partial x_i}\right)\right]$$

- Formally (U)RANS and LES equations are derived differently:
  - (U)RANS – Reynolds averaging
  - LES – Filtering of equations in space
- Practically the equations are modeled the same way – using EVM
- The practical difference between (U)RANS and LES is the size of the eddy viscosity
- Only for this reason are “hybrid” models (DES etc.) possible.

Classical Derivation of LES

- Average Navier Stokes equations over grid cell size
- Just like in RANS, averaging leads to additional stress terms in NS equations
- Resolve for $L_t > \Delta$ and model for $L_t < \Delta$

resolved modelled



Cell size $\Delta = \sqrt[3]{Vol}$

$$\hat{u}_i = \int u_i dVol \quad \Longrightarrow \quad \frac{\partial \rho \hat{u}_i}{\partial t} + \frac{\partial \rho \hat{u}_i \hat{u}_j}{\partial x_j} = -\frac{\partial \hat{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\hat{\tau}_{ij}^{lam} + \hat{\tau}_{ij}^{LES}\right)$$

$$\hat{\tau}_{ij}^{LES} = \rho \hat{u}_i \hat{u}_j - \rho \widehat{u_i u_j} \approx \mu_t \left(\frac{\partial \hat{u}_i}{\partial x_j} + \frac{\partial \hat{u}_j}{\partial x_i} \right)$$

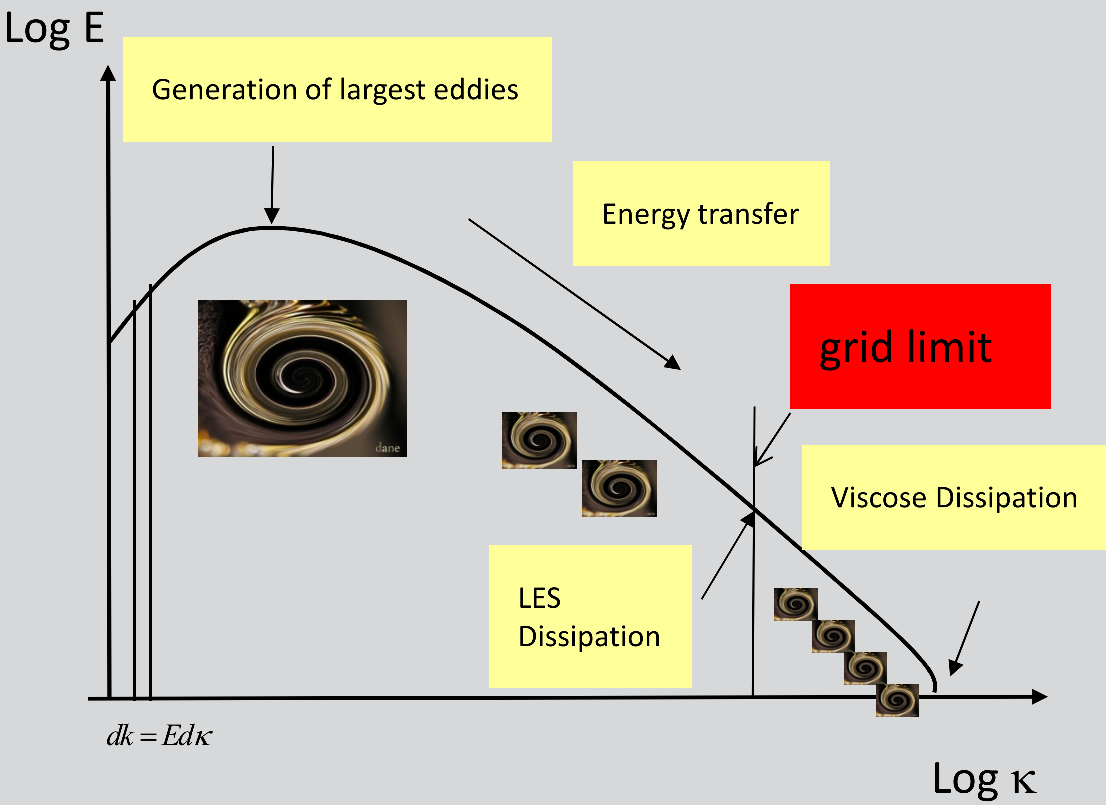
Smagorinsky model $\mu_t = (c\Delta)^2 \hat{S}$

$$\frac{\partial \rho \hat{u}_i}{\partial t} + \frac{\partial \rho \hat{u}_i \hat{u}_j}{\partial x_j} = -\frac{\partial \hat{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[(\mu + \mu_t) \left(\frac{\partial \hat{u}_i}{\partial x_j} + \frac{\partial \hat{u}_j}{\partial x_i} \right) \right]$$

Same as RANS!

Large Eddy Simulation (LES)

- **Role of LES:**
  - Turbulent spectrum cannot be resolved down to the dissipative scales (Kolmogorov scales)
  - Energy has to be dissipated from the spectrum at grid limit
  - LES Eddy Viscosity provides required damping
  - LES does not model the small scales – it just dissipates them
  - Everything of importance has to be resolved!



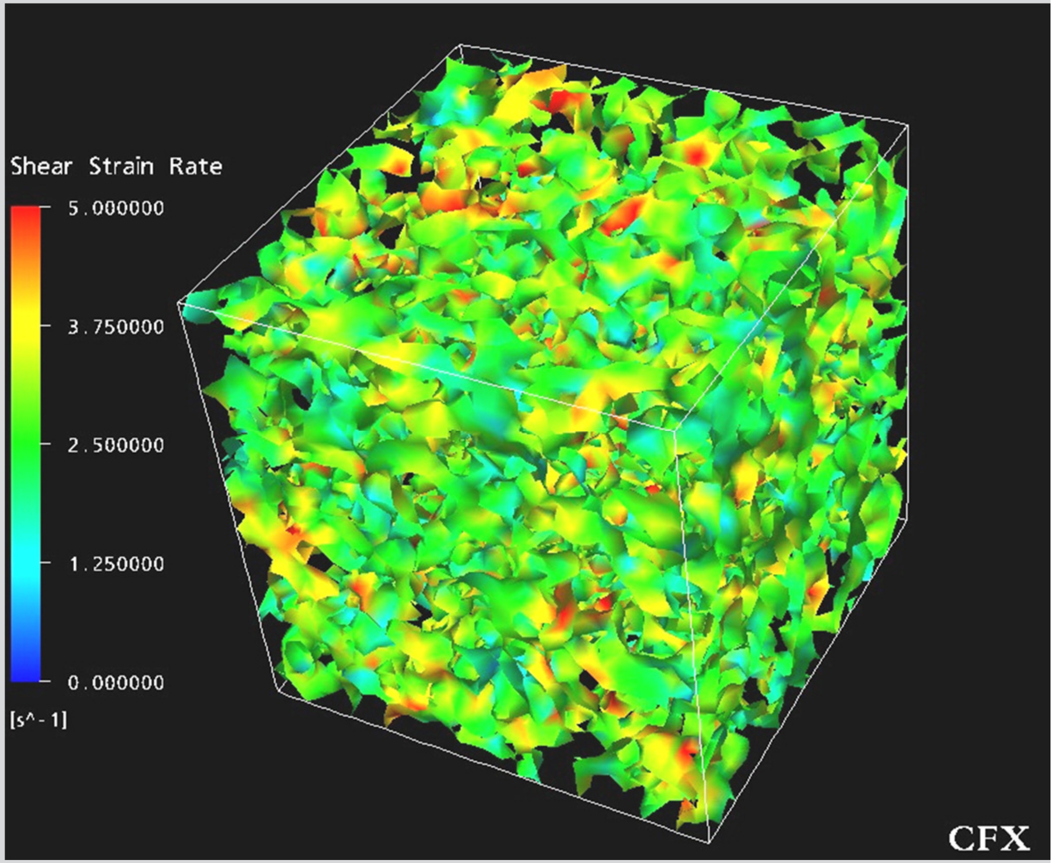
$$\varepsilon_{LES} = \varepsilon_{DNS} \longrightarrow \varepsilon_{DNS} = \nu \frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_j} \qquad \varepsilon_{LES} = \nu_t^{LES} \frac{\partial \hat{u}_i}{\partial x_j}\frac{\partial \hat{u}_i}{\partial x_j}$$

LES – Smagorinsky Modell

$$\nu_t^{LES} = (c\Delta)^2 S$$

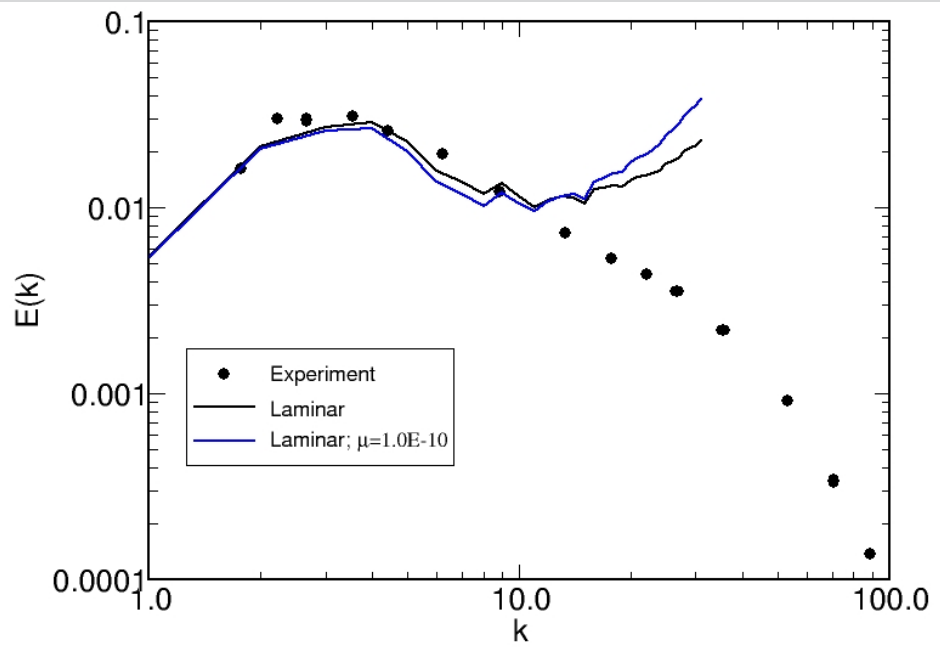
Decaying Isotropic Turbulence (DIT)

- Standard LES test case
- Artificial turbulence is generated as initial condition (see picture)
- Turbulence is then allowed to decay – integrating the Navier-Stokes equations (LES form)
- Compare turbulent spectrum at different times against experiment of Compte-Bellot

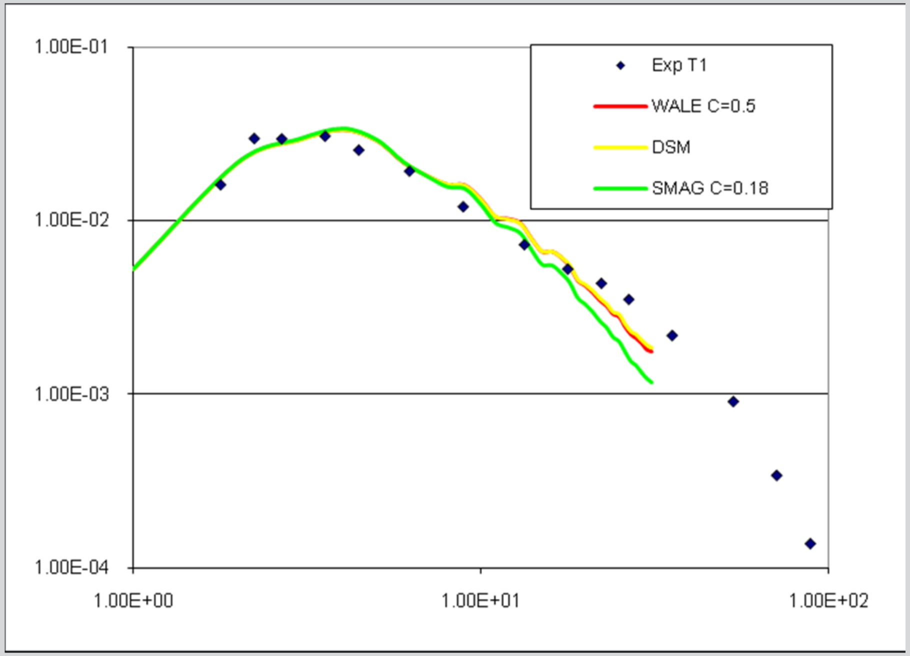


DIT Spectrum t=0.87

No LES Model



With LES Model



- Without LES model energy is accumulated at small scales (large wave number *k*)
- With LES models, energy is dissipated at grid resolution limit

Smagorinsky's Model

- **Simple algebraic (0 - equation) model**

$$\tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij} = -2\left(C_s \overline{\Delta}\right)^2 \left|\overline{S}\right| \overline{S}_{ij}$$

with $$\overline{\Delta} = V^{1/3}, \quad \overline{S} \equiv \sqrt{2 S_{ij} S_{ij}}$$

- ***C_s = 0.1 ~ 0.2***
- **The major shortcoming is that there is no *C_s* universally applicable to different types of flow**
- **Difficulty with transitional (laminar) flows**
- **An *ad hoc* damping is needed in near-wall region**

WALE Model

- Wall-Adapting Local Eddy-Viscosity model
- Algebraic (0 - equation) model – retains the simplicity of Smagorinsky's model

$$\nu_t = \left(C_s \overline{\Delta}\right)^2 \frac{\left(S_{ij}^d S_{ij}^d\right)^{3/2}}{\left(\overline{S}_{ij}\overline{S}_{ij}\right)^{5/2} + \left(S_{ij}^d S_{ij}^d\right)^{5/4}}$$

- The WALE SGS model adapts to local near-wall flow structure
  - Wall damping effects are accounted for without using the damping function explicitly

Dynamic Smagorinsky's Model

- Based on the similarity concept and Germano's identity (Germano *et al*., 1991; Lilly, 1992).
- The model parameter (C_s) is automatically adjusted using the resolved velocity field.
- Implementation:
  - Locally dynamic model.
  - Adapted for unstructured meshes (test-filter).
- Overcomes the shortcomings of the Smagorinsky's model:

SGS Models: Summary

Sub-grid stress : turbulent viscosity

$$\tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij} = -2\rho\nu_t \bar{S}_{ij}$$

- Smagorinsky model (*Smagorinsky, 1963*)
  – Need ad-hoc near wall damping

$$\nu_t = \left(C_s\bar{\Delta}\right)^2 \left|\bar{S}\right|$$

- WALE model (*Nicoud & Ducros 1999*)
  – Correct asymptotic near wall behaviour

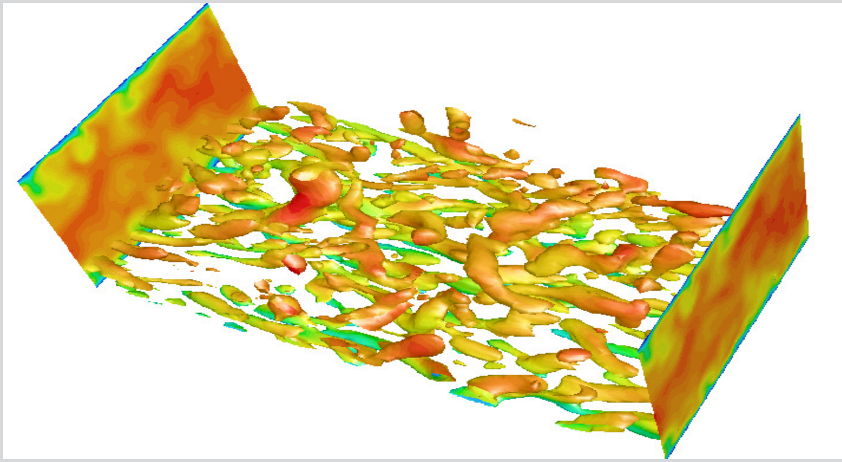
$$\nu_t = \left(C_s\bar{\Delta}\right)^2 \frac{\left(S_{ij}^d S_{ij}^d\right)^{3/2}}{\left(\bar{S}_{ij}\bar{S}_{ij}\right)^{5/2} + \left(S_{ij}^d S_{ij}^d\right)^{5/4}}$$

- Dynamic model (*Germano et al., 1991*)
  – Local adaptation of the Smagorinsky constant

$$\nu_t = \left(C_D\bar{\Delta}\right)^2 \left|\bar{S}\right|$$

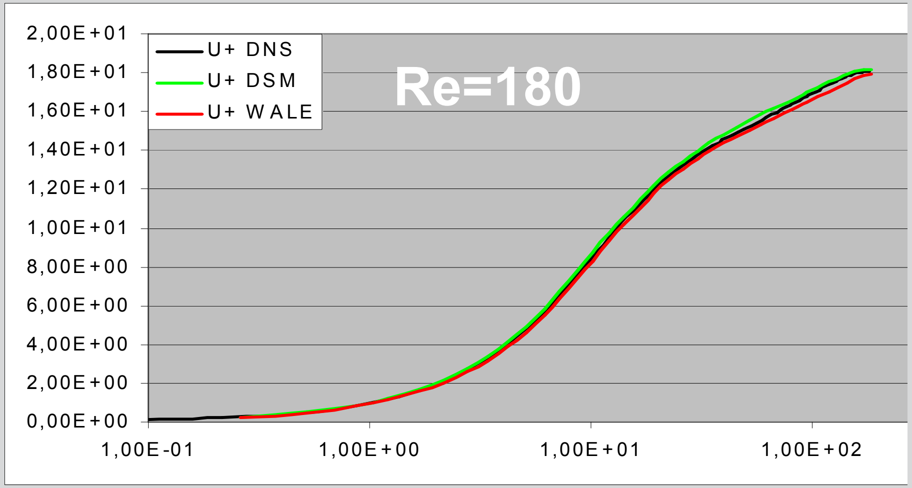
Periodic Channel Flow

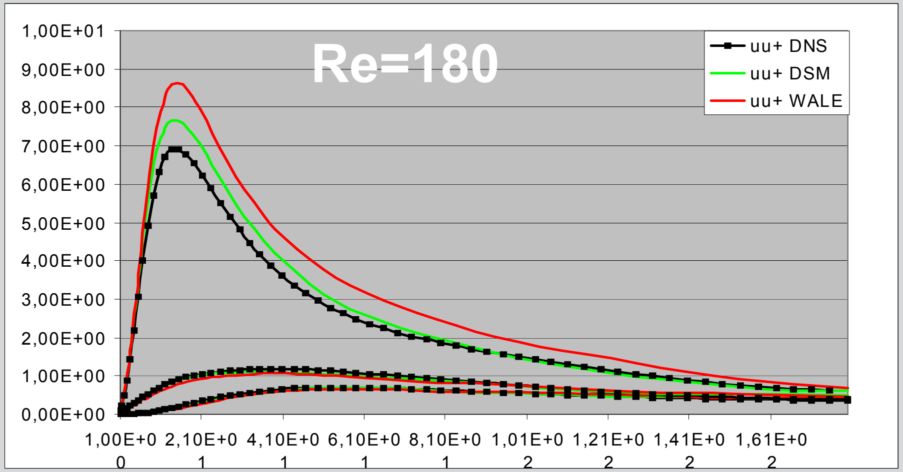
- Near the wall turbulent streaks appear
- They have to be resolved to obtain correct wall shear stress and heat transfer
- Scaling of these structures is ~ Re^2
- Wall flows can only be computed with LES for small domains and low Re numbers

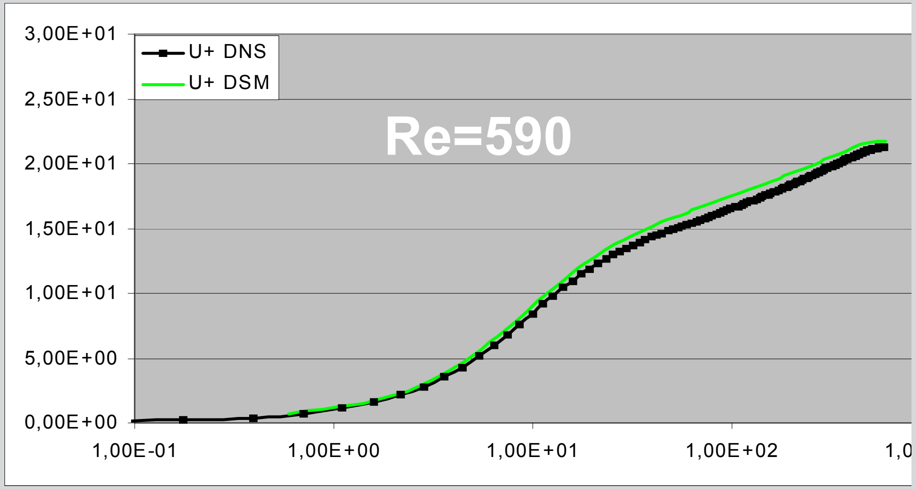


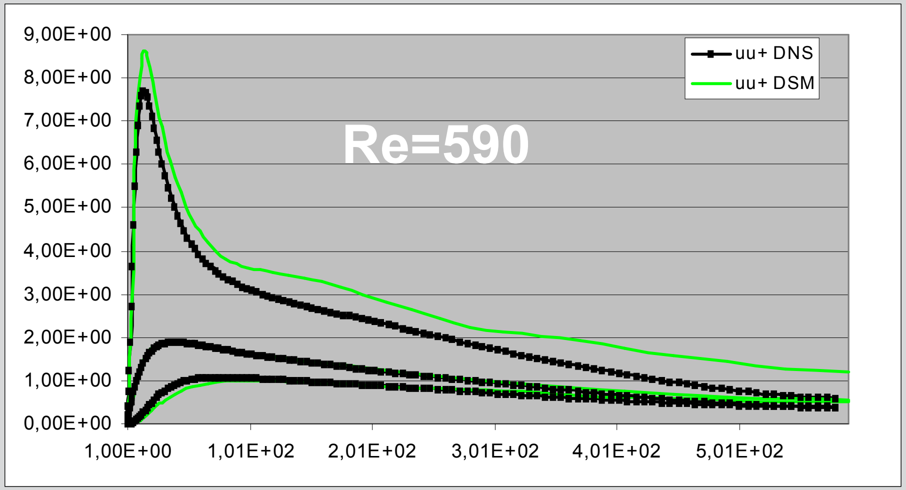
Re_T		N_x	N_y	N_z	L_x	L_y	L_z	Δx^+	Δy_1^+	Δy_c^+	Δz^+
180	Mesh 1	73	75	73	$2\pi h$	2	πh	15.7	0.254	13.47	7.85
590	Mesh 2	120	100	110	$2\pi h$	2	πh	30	1	50	15

Periodic Channel Flow





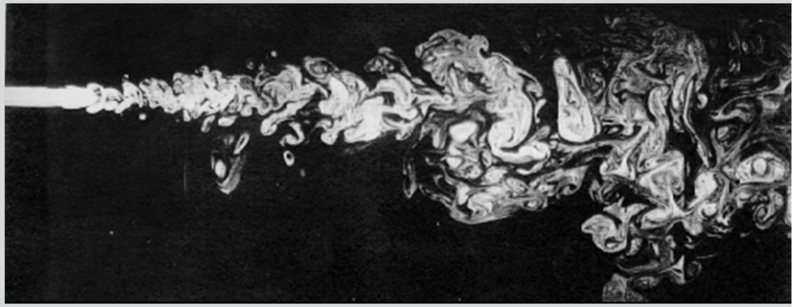




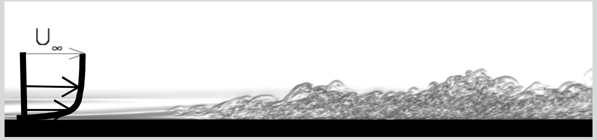
LES - Limitations

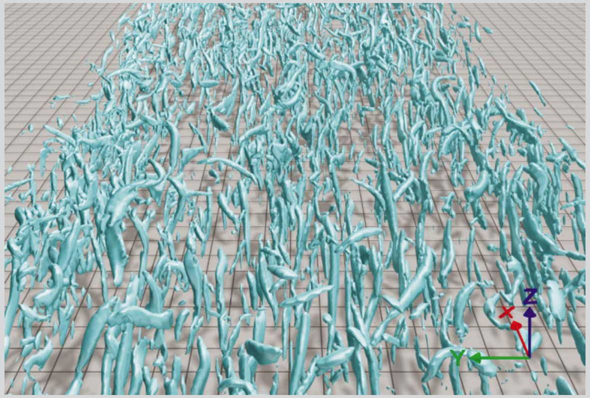
- With LES all relevant scales have to be resolved
- Free Shear Flows
  - Turbulent scales are large and can easily be resolved in time and space
  - Combustion chamber (if walls are not relevant)
- Wall Boundary Layers
  - Turbulent structures near the wall are very small (much smaller than BL thickness)
  - Need to be resolved in time and space
  - Excessive CPU costs even for small domain
  - Minimum resolution per boundary layer volume $\delta x \delta x \delta$ – 10x20x40=8000 (10^4) cells even with wall model (WMLES)

Free Shear Flow



Wall Boundary Layer





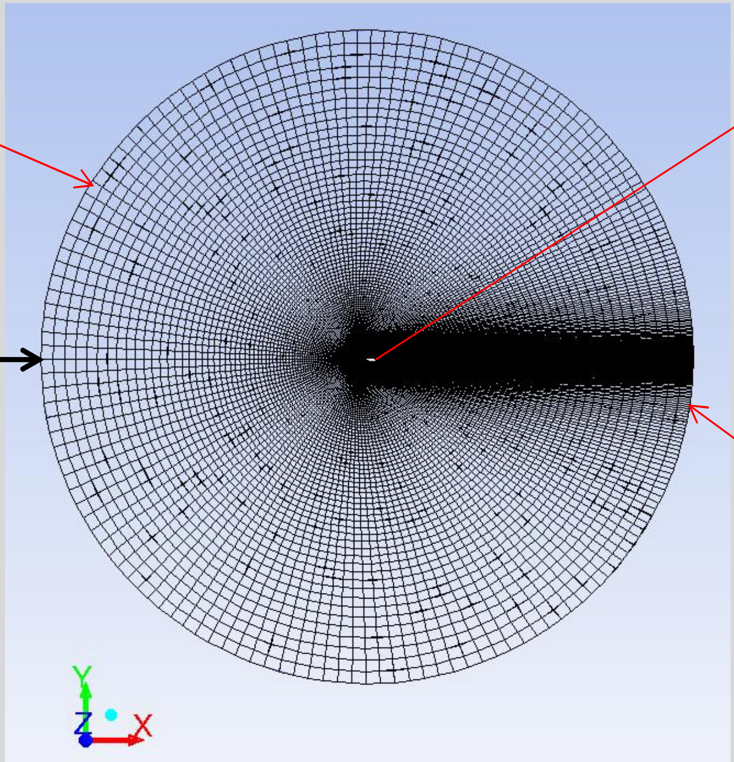
Ferrante et al
Phys. Fluids **16**, No 9, (2004)

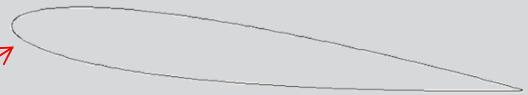
NACA 0012 Airfoil Noise

- NACA 0012: $Re_{chord} = 1.1 \cdot 10^{6}$

Velocity inlet

71.3 m/s

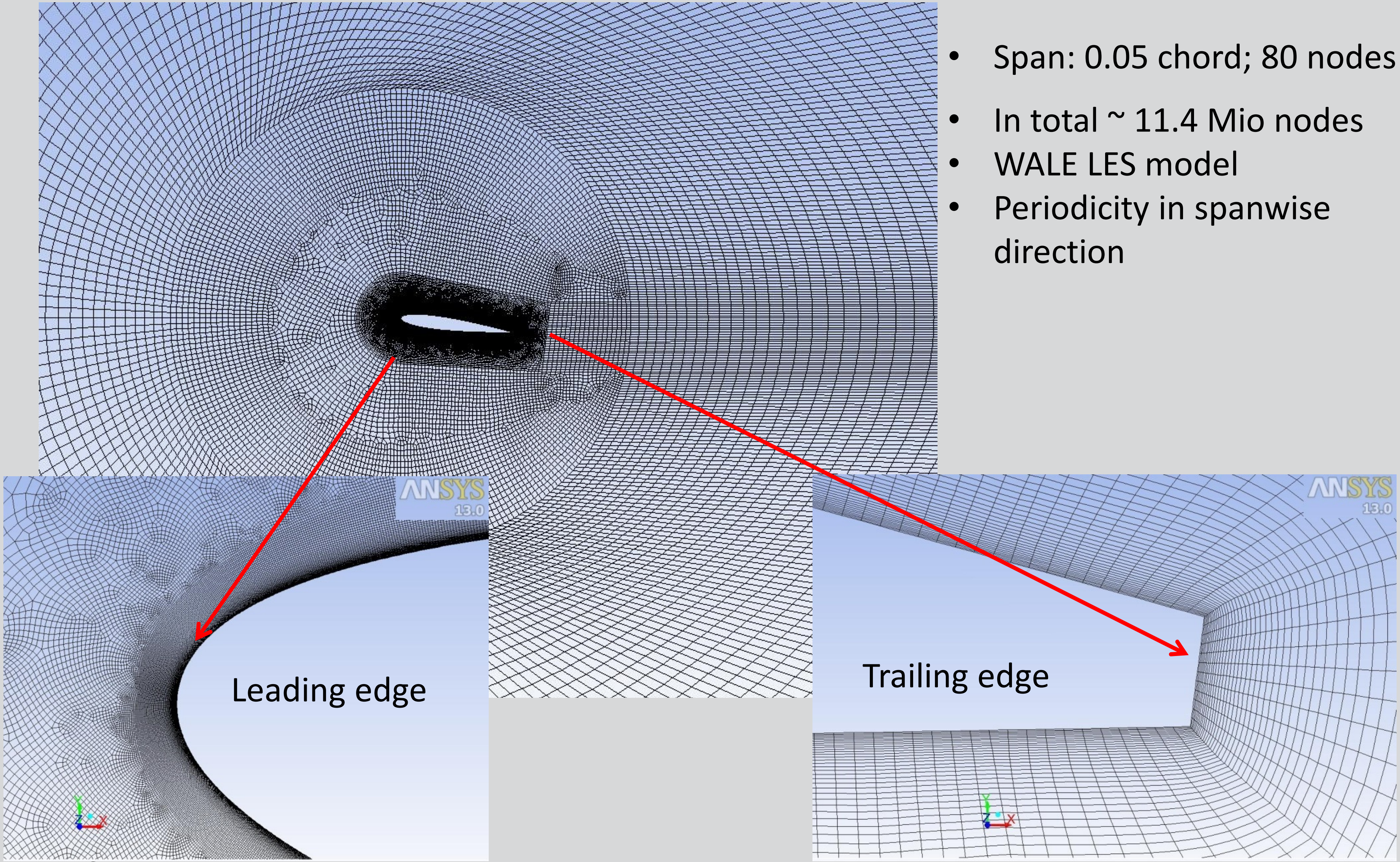




Airfoil rotated by 7.3 degree

Pressure outlet

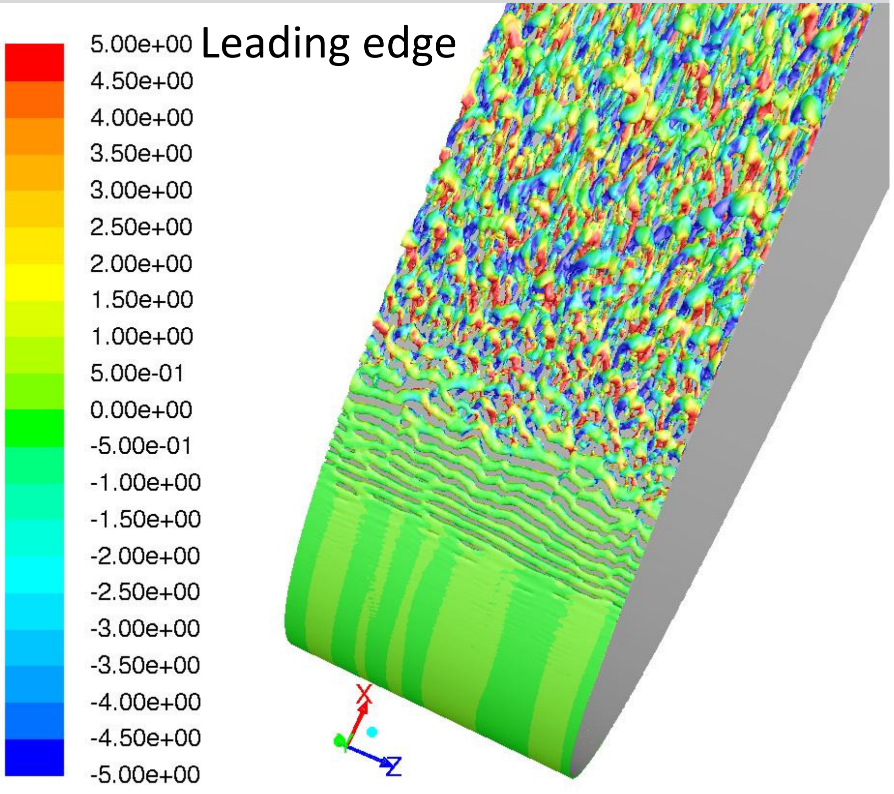
WB Unstructured Hex Mesh



- Span: 0.05 chord; 80 nodes
- In total ~ 11.4 Mio nodes
- WALE LES model
- Periodicity in spanwise direction

Q-criterion

Q-criterion (Ω^2-S^2): Q=10^9 , cc



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An objective definition of a vortex

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The most widely used definitions of a vortex are not objective: they identify different structures as vortices in frames that rotate relative to each other. Yet a frame-independent vortex definition is essential for rotating flows and for flows with interacting vortices. Here we define a vortex as a set of fluid trajectories along which the strain acceleration tensor is indefinite over directions of zero strain. Physically, this objective criterion identifies vortices as material tubes in which material elements do not align with directions suggested by the strain eigenvectors. We show using examples how this vortex criterion outperforms earlier frame-dependent criteria. As a side result, we also obtain an objective criterion for hyperbolic Lagrangian structures.

1. Introduction

The notion of a *vortex* is so widely used in fluid dynamics that few pause to examine what the word strictly means. Those who do take a closer look quickly realize the difficulty of defining vortices unambiguously.

Vortices are often thought of as regions of high vorticity, but there is no universal threshold over which vorticity is to be considered high. More alarmingly, vorticity may also be high in parallel shear flows where no vortices are present.

Definitions requiring closed or spiralling streamlines for a vortex are also ambiguous, because streamline topology changes even under simple Galilean transformations such as constant speed translations. Other definitions postulating pressure minima at vortex centres are readily refutable by counterexamples. Problems with all these definitions have been exposed by several authors, including Lugt (1979), Jeong & Hussain (1995), and Cucitore, Quadrio & Baron (1999).

1.1. *Galilean invariant vortex definitions*

Jeong & Hussain (1995) stress the need for Galilean-invariant vortex criteria, i.e. criteria that remain invariant under coordinate changes of the form $\boldsymbol{y} = \boldsymbol{Qx} + \boldsymbol{a}t$, where $\boldsymbol{Q}$ is a proper orthogonal tensor and $\boldsymbol{a}$ is a constant velocity vector. For a three-dimensional smooth velocity field $\boldsymbol{v}(\boldsymbol{x}, t)$, available Galilean-invariant vortex criteria use the velocity gradient decomposition

$$\nabla \boldsymbol{v} = \boldsymbol{S} + \boldsymbol{\Omega}, \tag{1.1}$$

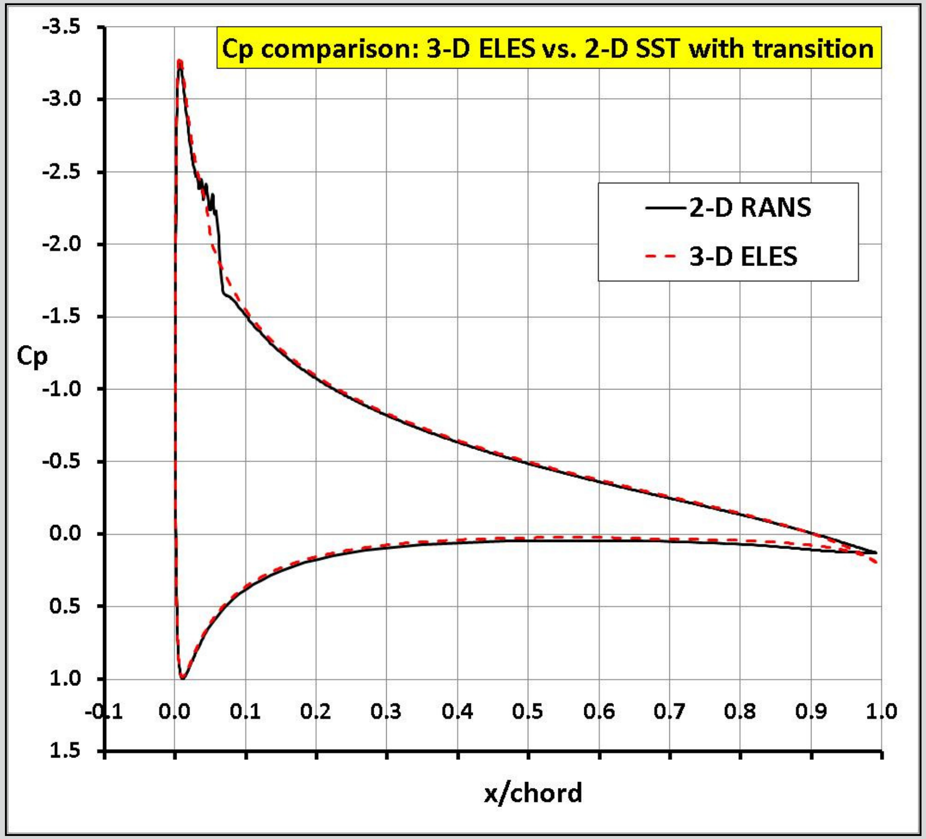
where $\boldsymbol{S} = \frac{1}{2}[\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^T]$ is the rate-of-strain tensor, and $\boldsymbol{\Omega} = \frac{1}{2}[\nabla \boldsymbol{v} - (\nabla \boldsymbol{v})^T]$ is the vorticity tensor.

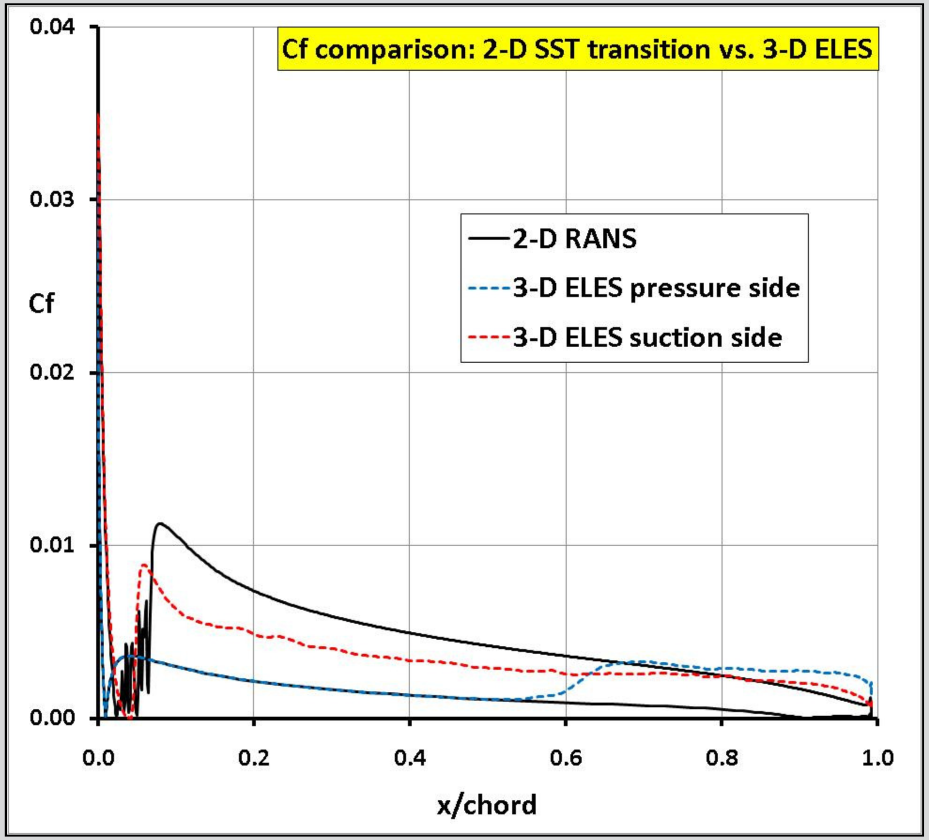
In historical order, the first three-dimensional vortex criterion using (1.1) is the *Q-criterion* of Hunt, Wray & Moin (1988) which defines a vortex as a spatial region

5%chord, 11M cells, Δt=1.5 μs

Pressure and skin friction coefficients

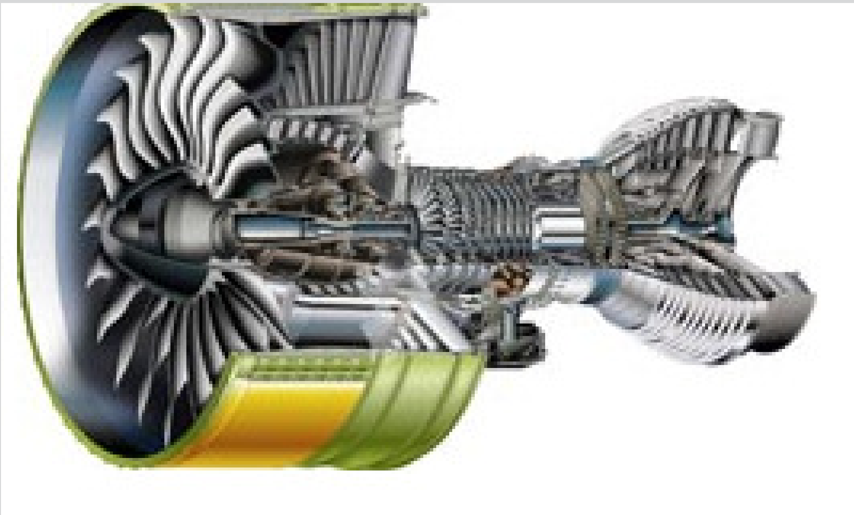
Even on this grid cf is too low -> WMLES (see later)





LES - Wall Bounded Flows

- A single Turbine (Compressor) Blade ($Re=10^5$-10^6) with hub and shroud section
- Need to resolve turbulence in boundary layers
- Need to resolve laminar-turbulent transition



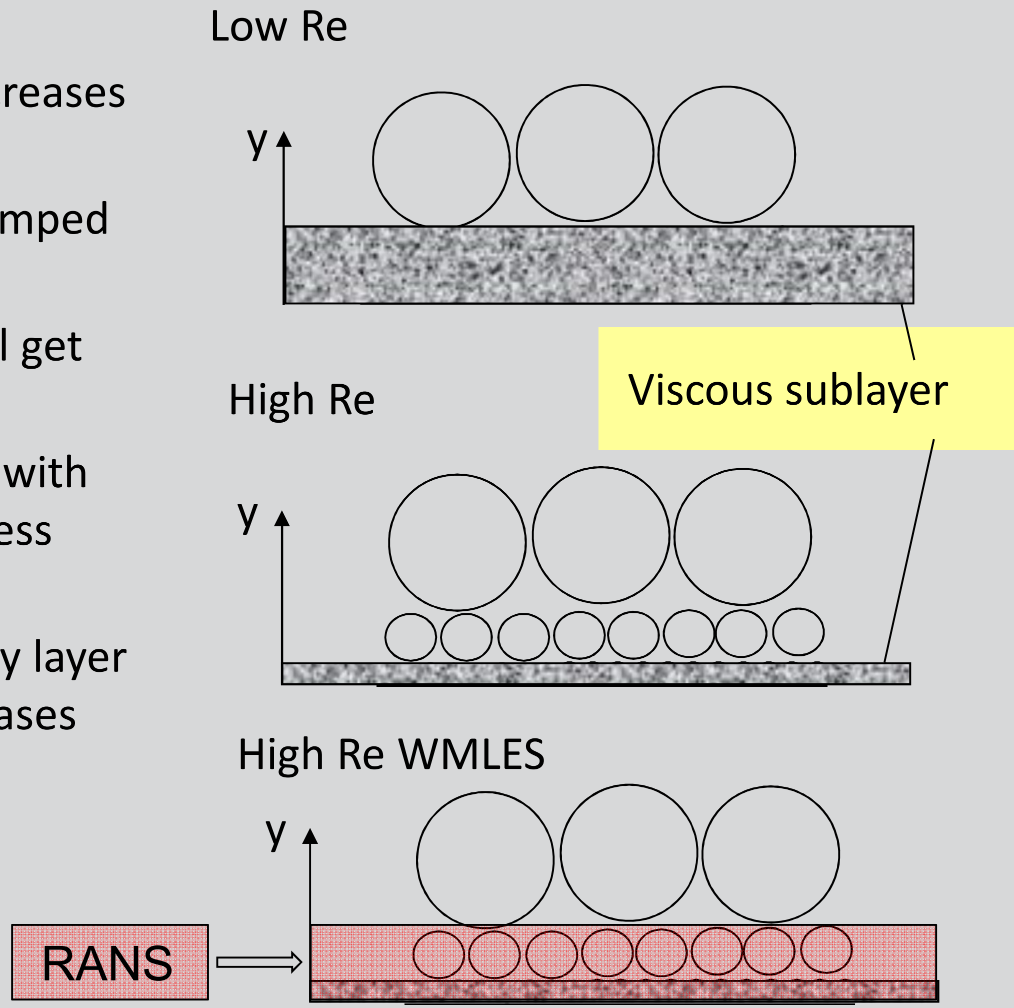
Method	Number of Cells	Number of time steps	Inner loops per Δt.	CPU Ratio
RANS	$\sim 10^6$	$\sim 10^2$	1	1
LES	$\sim 10^9$	$\sim 10^4$	10	10^6

Therefore Hybrid RANS-LES Methods

WMLES: Near Wall Scaling

- Turbulent length scale is independent of Re number
- However thickness of viscous sub layer decreases with increasing Re number
- Turbulent structures inside sublayer are damped out
- Smaller turbulence structures near the wall get "exposed" as Re increases
- WMLES: models small near wall structures with RANS and only resolve larger structures – less dependent on Re number
- Some Re number dependence for boundary layer remains as boundary layer thickness decreases with Re number

$$L_t = \kappa y$$



WMLES-Concept – Algebraic Model

- **RANS/LES blending:**

$$\nu_t = f_d \min\left\{ \underbrace{\boxed{(\kappa d_W)^2}}_{\text{RANS}}, \underbrace{\boxed{(C_{SMAG}\Delta)^2}}_{\text{LES}} \right\} S$$

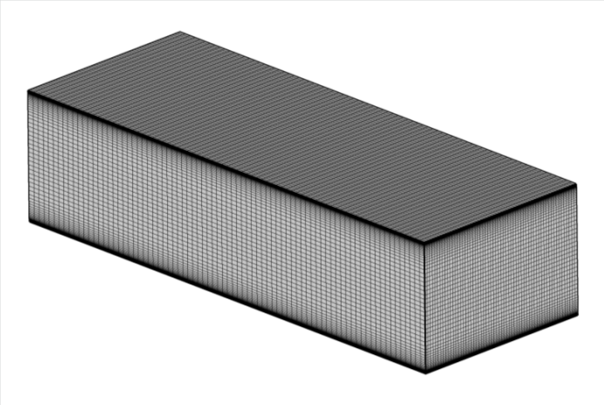
S	-	Shear Strain Rate
C_{SMAG}	-	Smagorinsky constant C_{SMAG}=0.2
Δ	-	Modified grid spacing
Fd	-	damping function

- The model has been modified and optimized for usage in ANSYS-Fluent and ANSYS-CFX

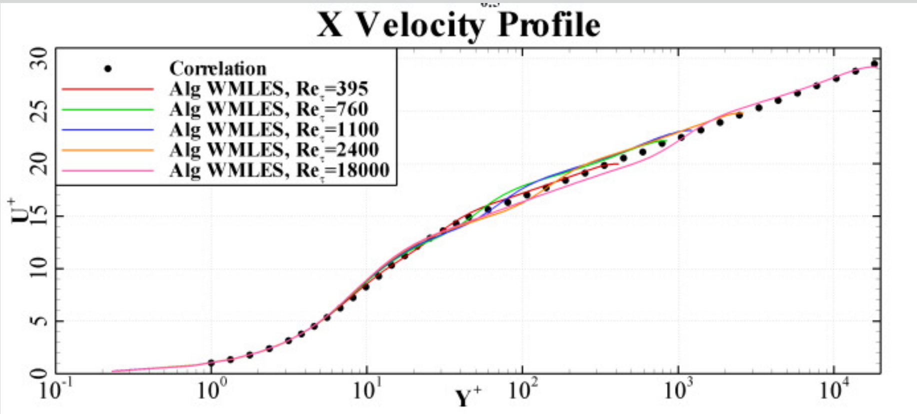
A hybrid RANS-LES approach with delayed-DES and wall-modelled LES capabilities
Mikhail L. Shur , Philippe R. Spalart , Mikhail K. Strelets , Andrey K. Travin , Int. J of Heat and Fluid Flow 29, 2008

WMLES – Channel Flow Tests

Re_τ	Cells Number	LES Cells Number	Nodes Number	ΔX^+	ΔZ^+
395	384 000	384 000	81×81×61	40.0	20.0
760	480 000	1 500 000	81×101×61	76.9	38.5
1100	480 000	4 000 000	81×101×61	111.4	55.7
2400	528 000	19 000 000	81×111×61	243.0	121.5
18000	624 000	1 294 676 760	81×131×61	1822.7	911.4



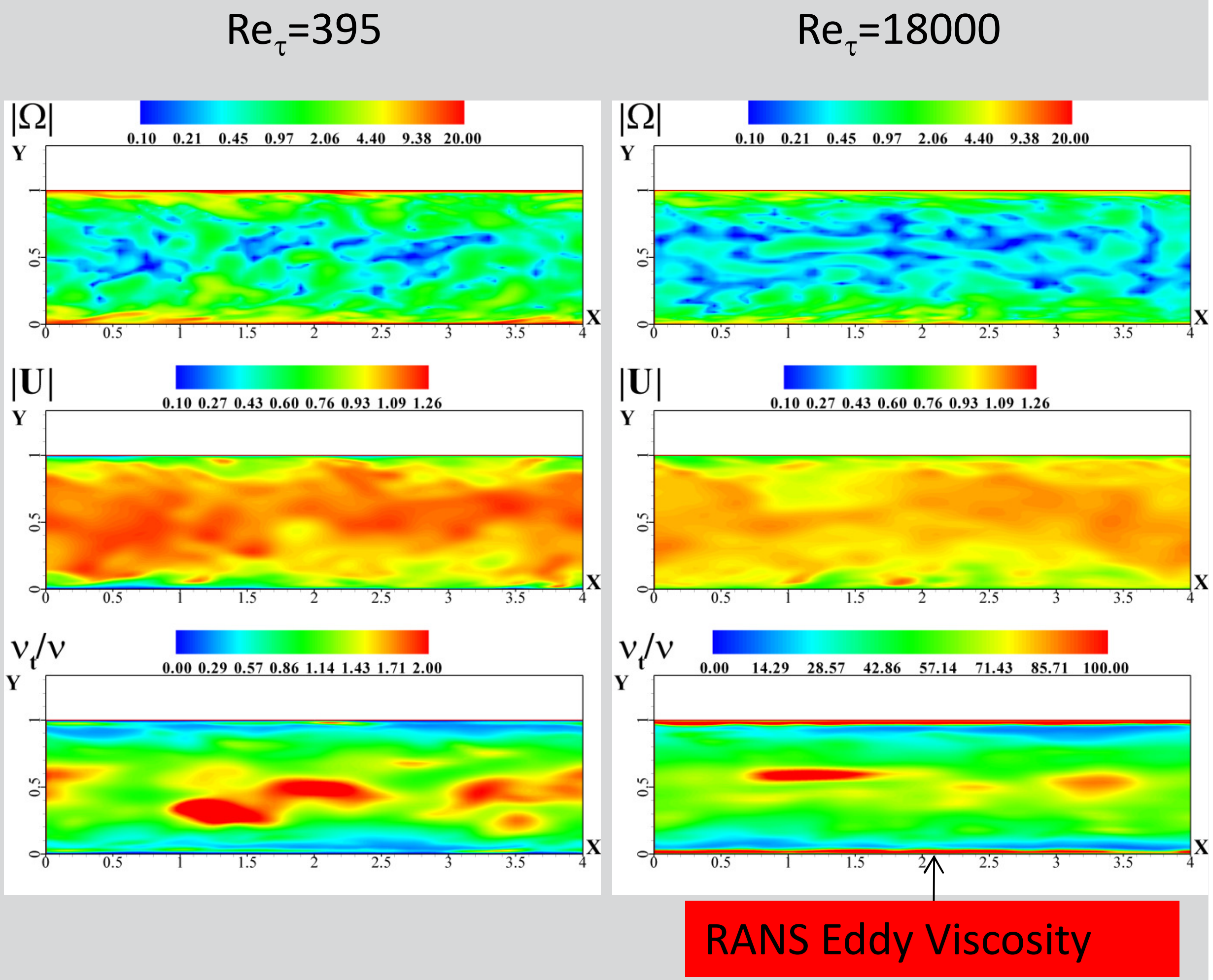
- Very large savings between WMLES and wall-resolved LES



  May 9, 2014

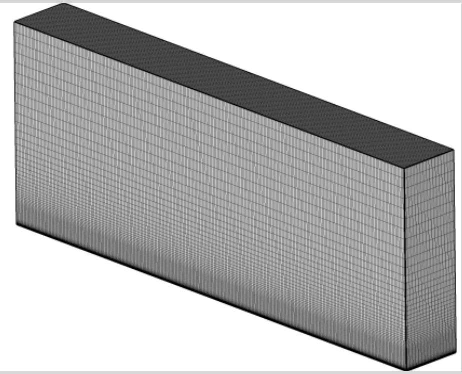
WMLES – Channel Flow at Different Re Numbers

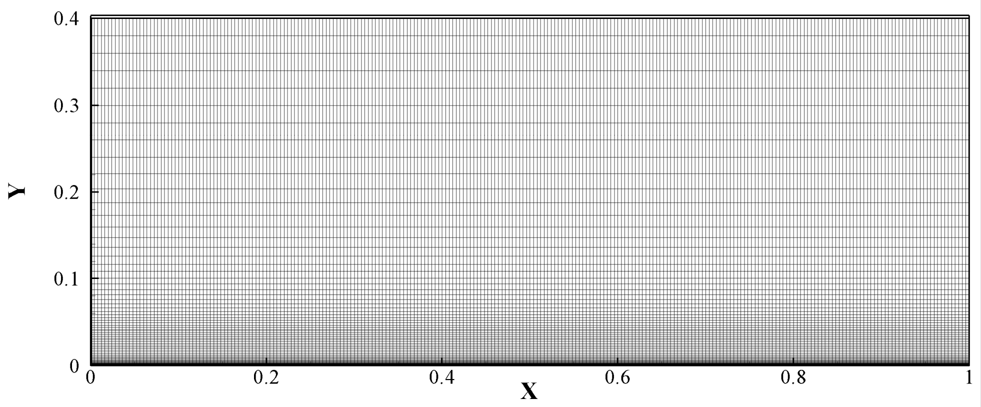
- Solutions at very different Re numbers look essentially identical
- Differences can only be seen near the wall.
- Visible is higher Eddy-Viscosity for higher Re number close to wall



WMLES – Flat Plate Grid

- Geometry and Grid
  - Inlet boundary layer thickness δ
  - L x 0.4 L x 0.1 L (Streamwise, Normal, Spanwise)
  - Approximately 3 δ spanwise (δ_0=0.032)
  - Grid ~ 1Million cells (see table)
  - Y+~0.05 (to allow for higher Re numbers)
  - Expansion factor 1.15
  - For each boundary layer thickness δ one needs ~10x40x20 cells

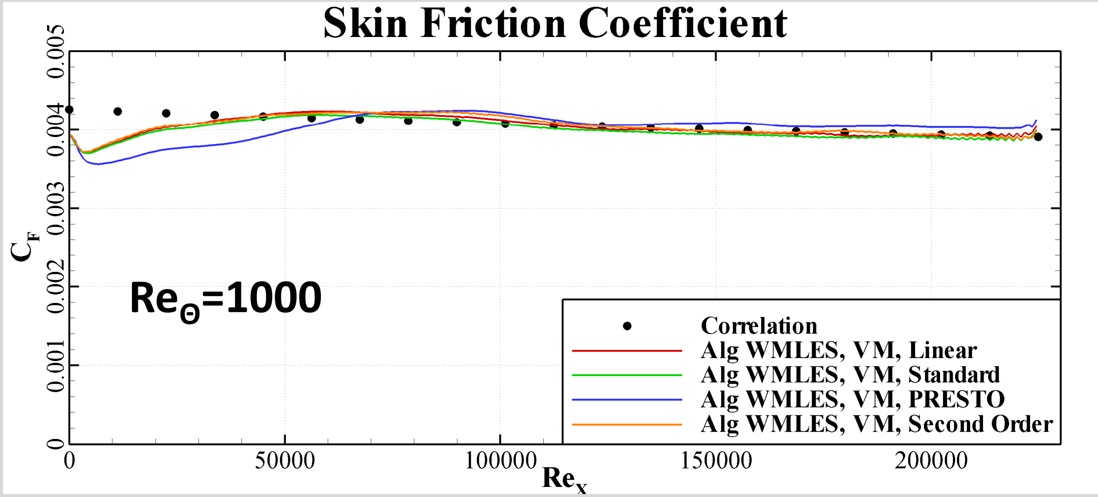


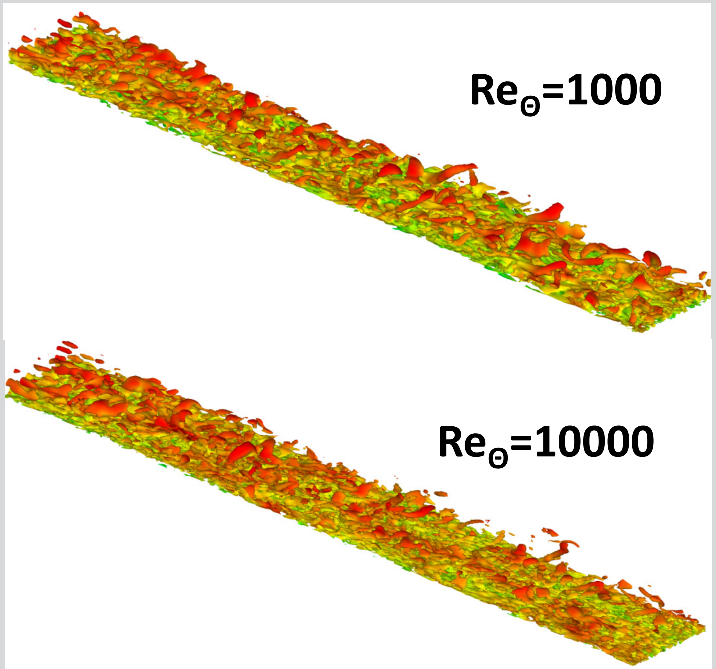


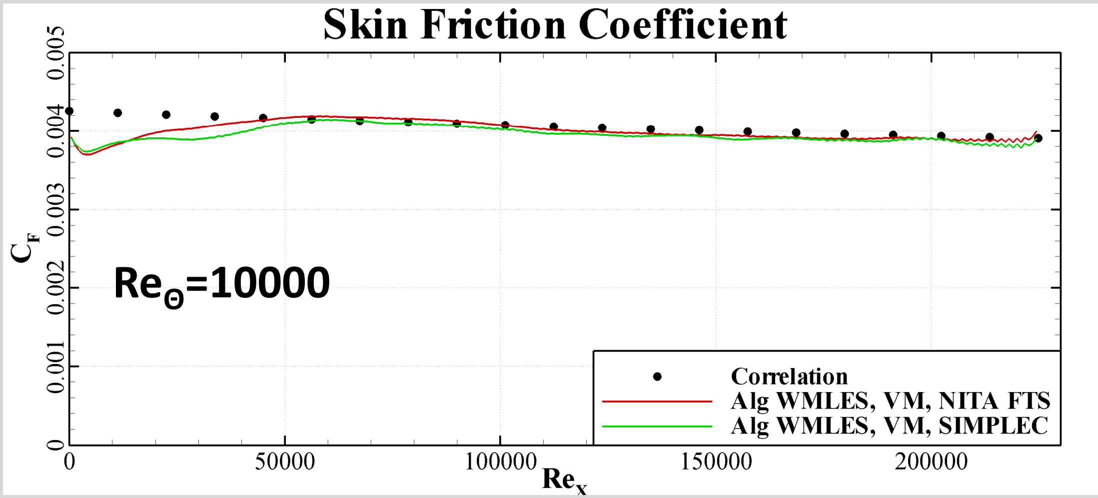
Re_Θ	Cells Number	Nodes Number	ΔX^+	ΔY^+	ΔZ^+
1000	1 085 000	251×71×63	68	0.05 ÷ 300	34
10000	1 085 000	251×71×63	520	0.4 ÷ 2300	307

WMLES – Boundary Layer

- **Boundary layer simulation:**
  - WMLES
  - Inlet: synthetic turbulence Vortex Method
  - 2 different Reynolds numbers

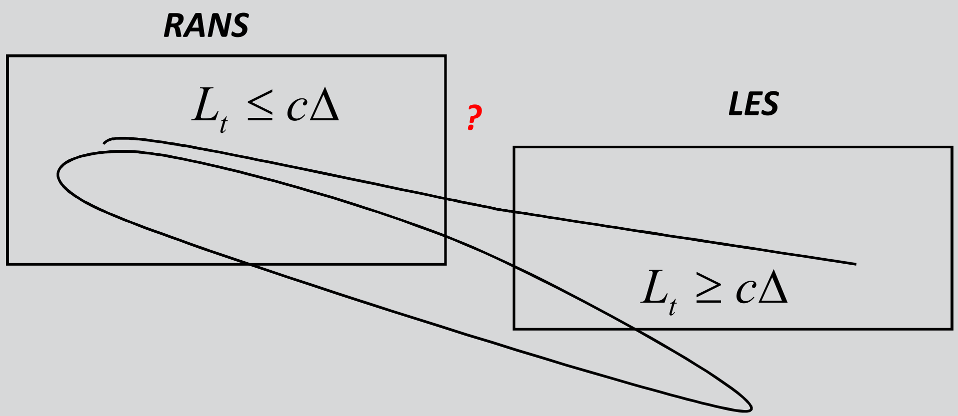






Detached Eddy Simulation (DES)

- Hybrid Model:
  - RANS equations in boundary layer.
  - LES „ detached “ regions.
- Switch of model:
  - Based on ratio of turbulent length-scale to grid size.
  - Different numerical treatment in RANS and LES regions.



- Overcomes threshold limit of LES
- Explicit grid sensitivity in RANS region
- Open question concerning transition region between RANS and LES

DES for SST – Strelets (2000)

- **k-equation RANS**

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho \bar{U}_j k)}{\partial x_j}=P_k-\rho\frac{k^{3/2}}{L_t}+\frac{\partial}{\partial x_j}\left[(\mu+\frac{\mu_t}{\tilde{\sigma}_\kappa})\frac{\partial k}{\partial x_j}\right] \qquad L_t=\frac{\sqrt{k}}{\beta^*\omega}$$

- **k-equation LES**

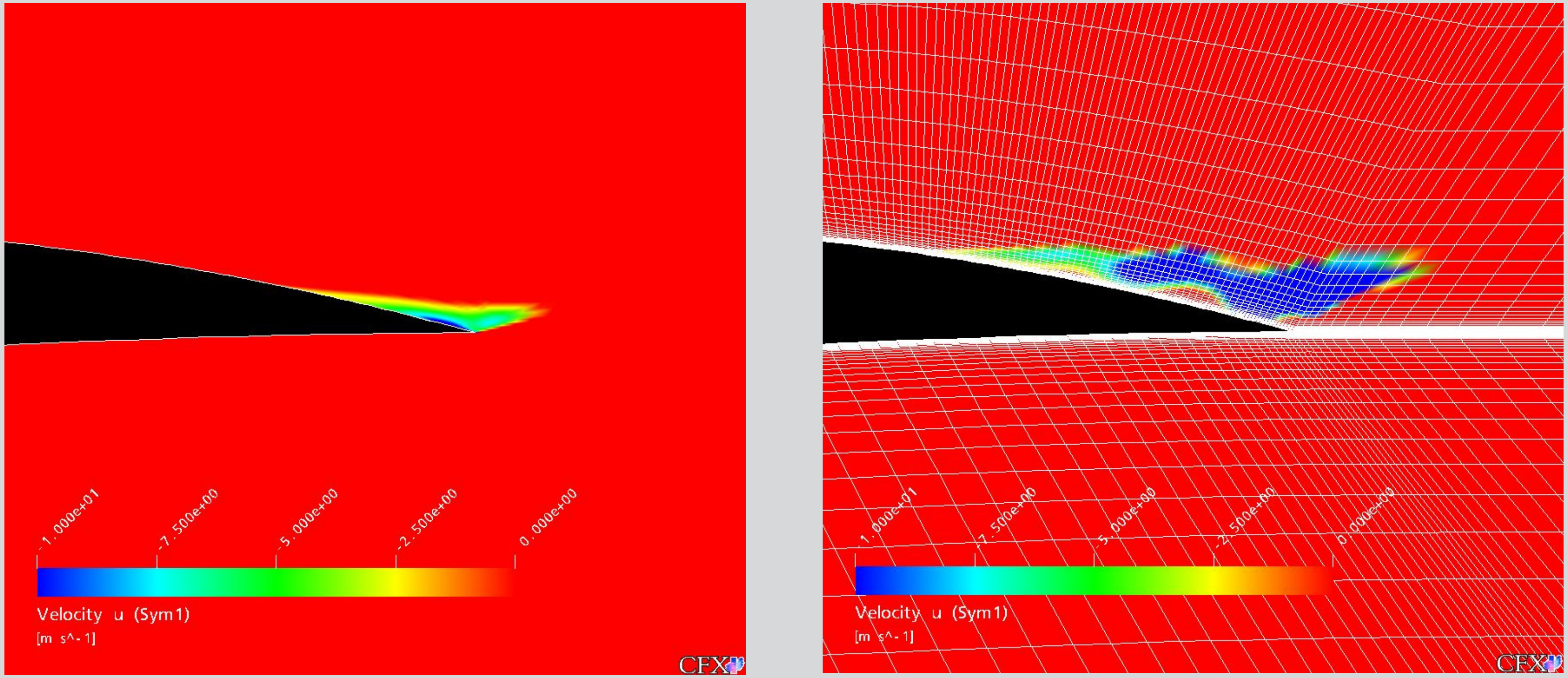
$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho \bar{U}_j k)}{\partial x_j}=P_k-\rho\frac{k^{3/2}}{C_{DES}\Delta}+\frac{\partial}{\partial x_j}\left[(\mu+\frac{\mu_t}{\tilde{\sigma}_\kappa})\frac{\partial k}{\partial x_j}\right] \qquad \Delta=\max(\Delta x,\Delta y,\Delta z)$$

- **k-equation DES**

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho \bar{U}_j k)}{\partial x_j}=P_k-\rho\frac{k^{3/2}}{\min\left(L_t;C_{DES}\Delta\right)}+\frac{\partial}{\partial x_j}\left[(\mu+\frac{\mu_t}{\tilde{\sigma}_\kappa})\frac{\partial k}{\partial x_j}\right]$$

Grid Sensitivity with DES Model

SST model Separation Zone **SST-DES-SPTU model**



Requirement: $\Delta x > \delta$

Alternative – Shielding functions – Delayed DES (DDES)

DES for SST – Delayed DES (DDES)

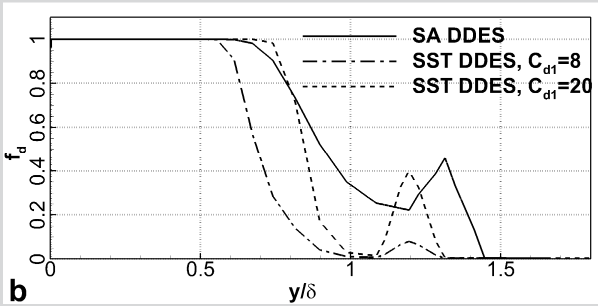
- DDES – provides shielding functions which keep DES in RANS mode in attached boundary layers even for fine grids:

- Destruction term original DES-SST model :

$$E = \rho \frac{k^{3/2}}{\min\left(L_t; C_{DES}\Delta\right)} = \rho \frac{k^{3/2}}{L_t \min\left(1; C_{DES}\Delta / L_t\right)} = \rho \frac{k^{3/2}}{L_t} \max\left(1; \frac{L_t}{C_{DES}\Delta}\right)$$

- DES function used for SST model to shield boundary layer from DES impact (Delayed DES – DDES)

$$F_{DES-CFX} = \max\left(\frac{L_t}{C_{DES}\Delta} \cdot \left(1 - F_{DDES}\right), 1\right); \quad F_{SST} = 0, F_1 \ or \ F_2, F_{DDES}$$



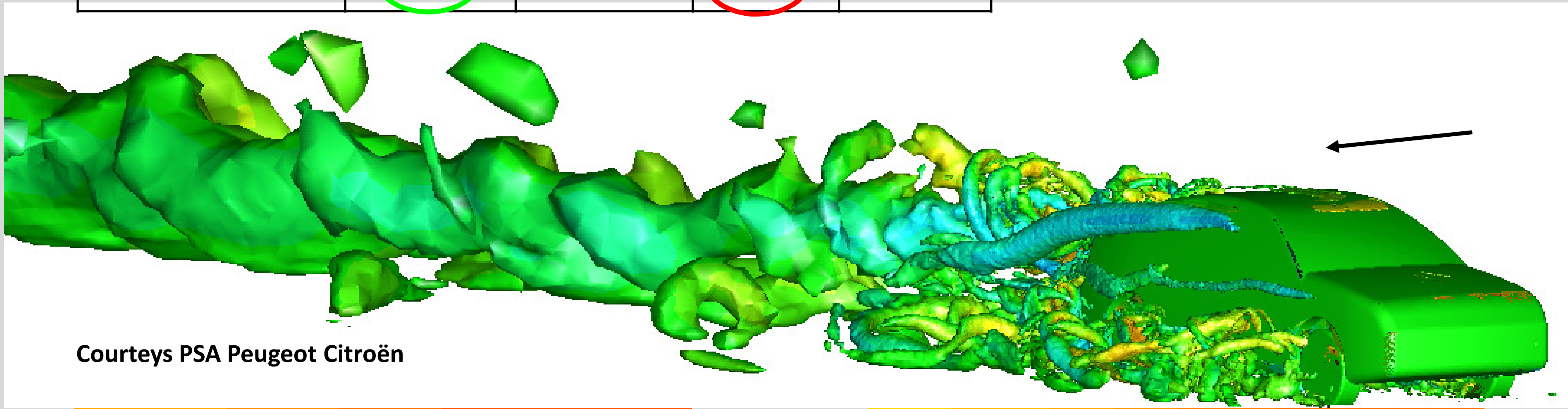
- Shielding up to: $\Delta_{max} > 0.1 \cdot \delta_{BL}$

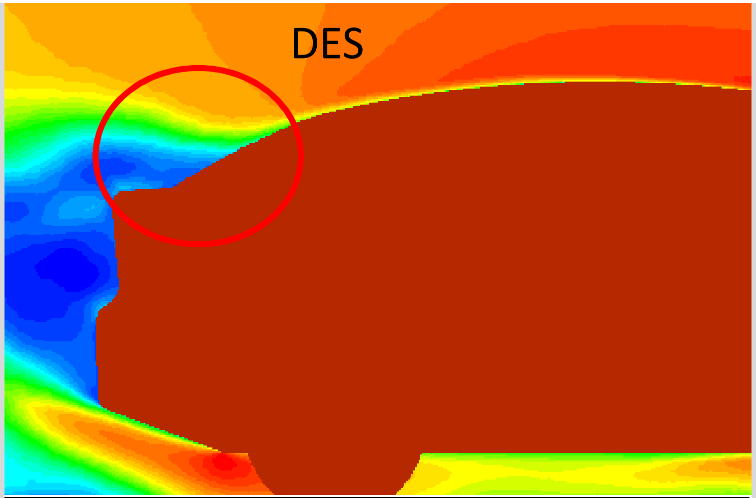
DES/DDES of Separated Flow around a Car model exposed to Crosswind

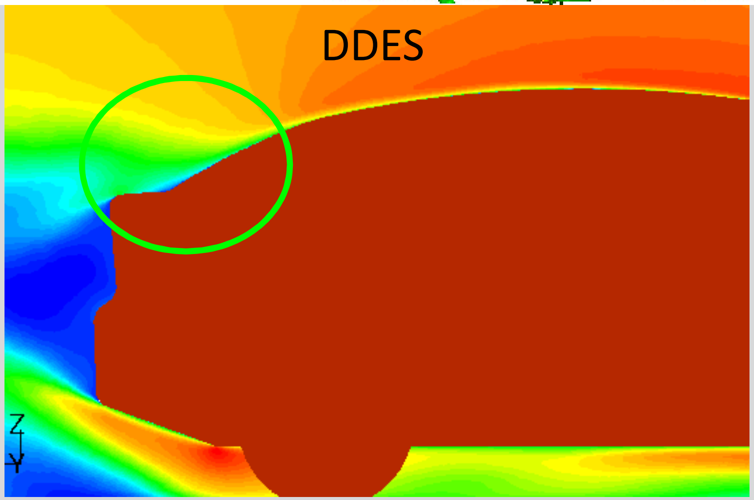
Model	Exp.	DDES	DES	LES
Drag (SCx)	0.70	0.71	0.75	0.69

U=40 m/s   Yaw angle 20°

$Re_H \sim 10^6$







DES Models

Courtesy The Boeing Company

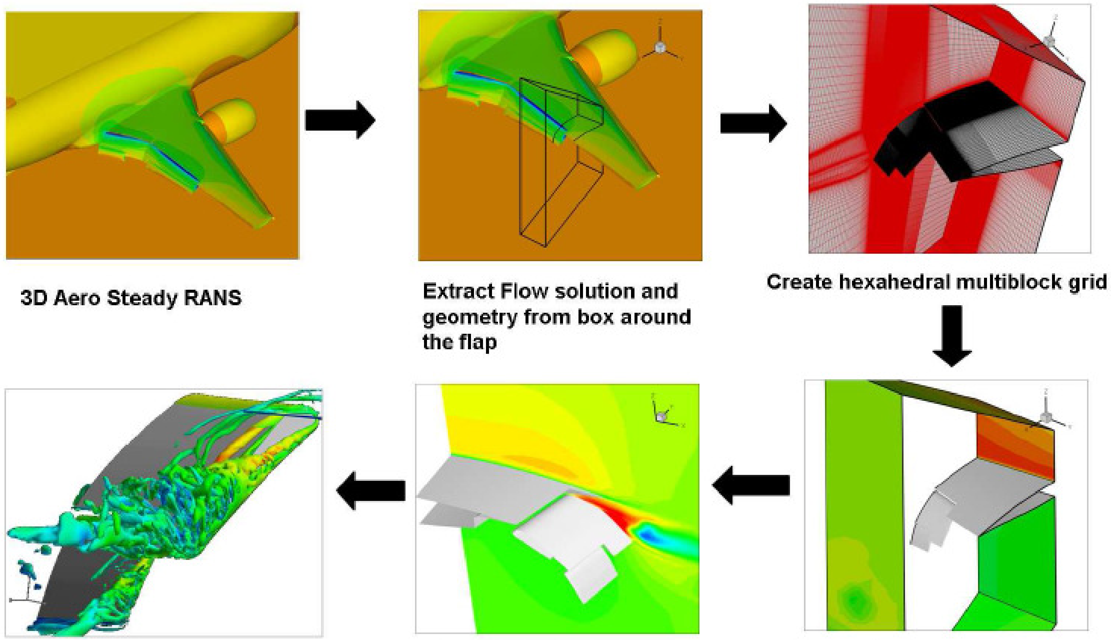


Figure 4: Process for running a local DES around the flap edge based on the full 3D steady RANS solution around an airplane.

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DES Models

- Fluent 6.3
- 300 time steps with 7 inner iterations to converge by 2 orders each time step
- 2nd order bounded central difference scheme
- Grid: 14 million nodes
- CPU: 20 nodes (40 CPUs) require ~1,5 weeks.

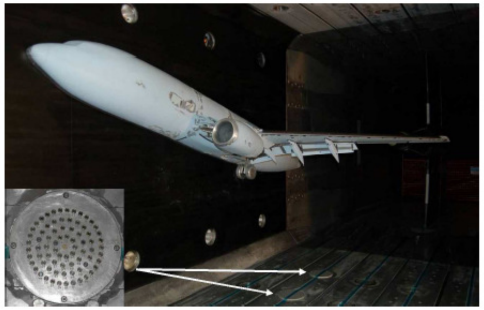

Figure 3: Boeing wind tunnel test using an Acoustic Phased Array mounted on the tunnel wall (Reproduced from Stoker et al., 2008).

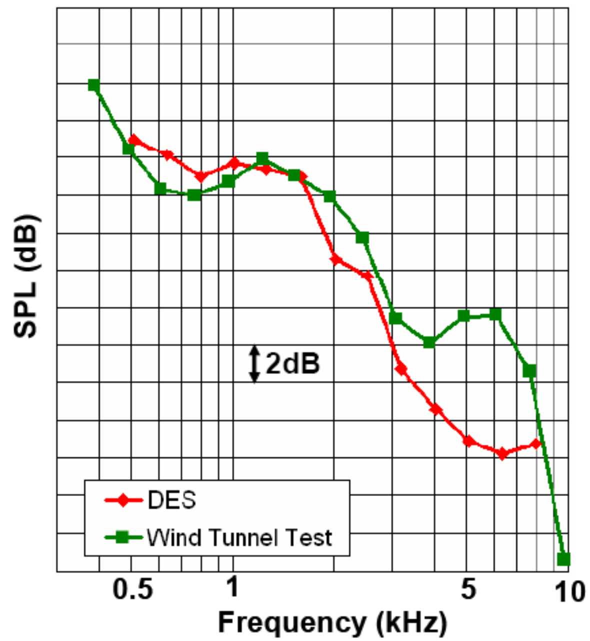

Figure 9: Relative Sound Pressure Level (SPL) vs Frequency measured in a wind tunnel test and predicted by DES for the flap edge.

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Discussion

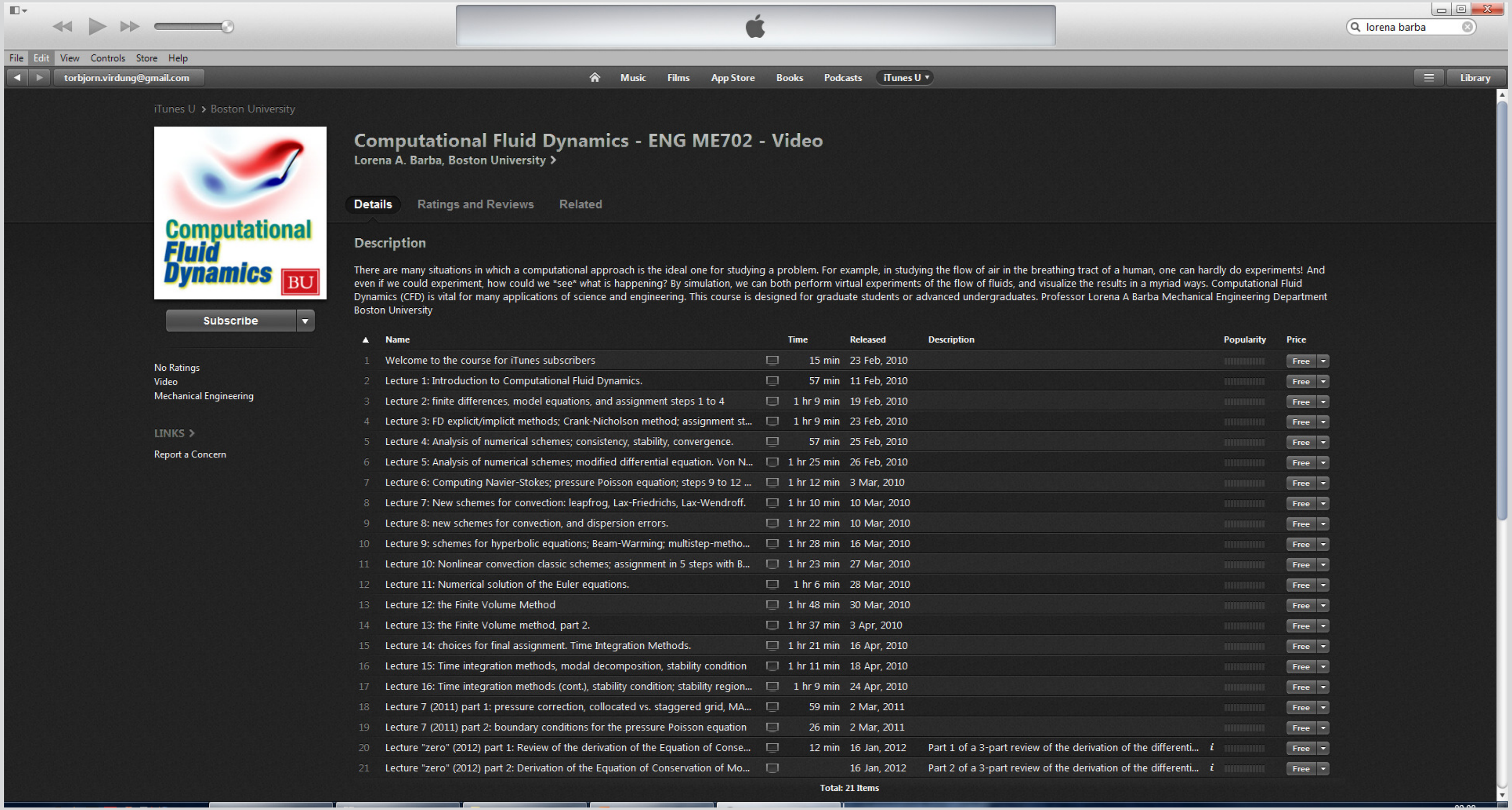
- **LES:**
  - \+ Involves least amount of modelling
  - \+ Can handle laminar domains (some models)
  - – Requires "LES-resolution" in all of the domain including boundary layers – meaning mostly simple geometries
- **(D)DES:**
  - \+ Avoids resolution of all boundary layers
  - \+ Can be used in complex geometries
  - – Requires shielding
  - – Can generate grey areas
- **SAS:**
  - \+ Often similar to DES but less dangerous – fallback is RANS
  - – Can stay steady state if flow instability is too weak

Discussion

- Unsteady simulations are the future for many CFD applications.
- Wide spectrum of models available (LES, WMLES, DES, DDES, SAS, URANS) and needed
- Question is – which closure is best suited for which type of flows
  - Best ratio of cost vs. performance
  - Safest environment for user (limited sensitivity to mesh, time step, …)
- User feed-back required

Literature

- Spalart P. R.: Strategies for turbulence modelling and simulations, Int. J. Heat Fluid Flow, 21, pp. 2, (2000).
- Strelets, M.: Detached Eddy Simulation of massively separated flows. AIAA Paper 2001-879, (2001).
- Menter, F.R., Egorov, Y, (2010): The Scale-Adaptive Simulation Method for Unsteady Turbulent Flow Predictions. Part 1: Theory and Model Description, J. Flow Turbulence and Combustion, Vol. 85, No. 1
- Egorov, Y, Menter, F.R. and Cokljat D.: Scale-Adaptive Simulation Method for Unsteady Flow Predictions. Part 2: Application to Aerodynamic Flows, companion paper, J. Flow Turbulence and Combustion, Vol. 85, No. 1.
- Rotta J. C.: *Turbulente Strömumgen*. BG Teubner Stuttgart, (1972).
- Fröhlich J. and von Terzi, D.: Hybrid LES/RANS methods for simulation of turbulent flows, Progress in Aerospace Sciences, Vol. 44, Issue 5, pp 349-377, 2008.



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