PhysRevD.67.084023 equation (20)

```
from shared import *
import cdblib

jsonfile = 'momentum.json'
cdblib.create (jsonfile)

defG2GBar = cdblib.get ('defG2GBar','gamma.json')

# ----------------------------------------------------------------------
# Momentum constraint pt.1

Mom := D_{j}{K^{i j} - g^{i j} trK}.                                    # cdb(Mom.101,Mom)

defDgD := D_{a}{g_{b c}} -> 0.
defDgU := D_{a}{g^{b c}} -> 0.

defDtrK   := D_{a}{trK} -> \partial_{a}{trK}.
defDexp   := D_{a}{\exp(-4\phi)} -> -4\exp(-4\phi) \partial_{a}{\phi}.

distribute   (Mom)                                                      # cdb(Mom.102,Mom)
product_rule (Mom)                                                      # cdb(Mom.103,Mom)
substitute   (Mom, defDgU)                                              # cdb(Mom.104,Mom)

defK2ABarU := K^{i j} -> \exp(-4\phi) ABar^{i j} + (1/3) g^{i j} trK.

substitute   (Mom, defK2ABarU)                                          # cdb(Mom.105,Mom)
distribute   (Mom)                                                      # cdb(Mom.106,Mom)
product_rule (Mom)                                                      # cdb(Mom.107,Mom)
substitute   (Mom, defDtrK)                                             # cdb(Mom.108,Mom)
substitute   (Mom, defDgU)                                              # cdb(Mom.109,Mom)
substitute   (Mom, defDexp)                                             # cdb(Mom.110,Mom)
```

$$\mathcal{D}^j = D_j \left(K^{ij} - g^{ij} \mathrm{tr} K \right) \tag{Mom.101}$$

$$= D_j K^{ij} - D_j \left(g^{ij} \mathrm{tr} K \right) \tag{Mom.102}$$

$$= D_j K^{ij} - D_j g^{ij} \mathrm{tr} K - g^{ij} D_j \mathrm{tr} K \tag{Mom.103}$$

$$= D_j K^{ij} - g^{ij} D_j \mathrm{tr} K \tag{Mom.104}$$

$$= D_j \left(\exp\left(-4\phi\right) \bar{A}^{ij} + \frac{1}{3} g^{ij} \mathrm{tr} K \right) - g^{ij} D_j \mathrm{tr} K \tag{Mom.105}$$

$$= D_j \left(\exp\left(-4\phi\right) \bar{A}^{ij} \right) + \frac{1}{3} D_j \left(g^{ij} \mathrm{tr} K \right) - g^{ij} D_j \mathrm{tr} K \tag{Mom.106}$$

$$= D_j \left(\exp\left(-4\phi\right) \right) \bar{A}^{ij} + \exp\left(-4\phi\right) D_j \bar{A}^{ij} + \frac{1}{3} D_j g^{ij} \mathrm{tr} K - \frac{2}{3} g^{ij} D_j \mathrm{tr} K \tag{Mom.107}$$

$$= D_j \left(\exp\left(-4\phi\right) \right) \bar{A}^{ij} + \exp\left(-4\phi\right) D_j \bar{A}^{ij} + \frac{1}{3} D_j g^{ij} \mathrm{tr} K - \frac{2}{3} g^{ij} \partial_j \mathrm{tr} K \tag{Mom.108}$$

$$= D_j \left(\exp\left(-4\phi\right) \right) \bar{A}^{ij} + \exp\left(-4\phi\right) D_j \bar{A}^{ij} - \frac{2}{3} g^{ij} \partial_j \mathrm{tr} K \tag{Mom.109}$$

$$= -4 \exp\left(-4\phi\right) \partial_j \phi \bar{A}^{ij} + \exp\left(-4\phi\right) D_j \bar{A}^{ij} - \frac{2}{3} g^{ij} \partial_j \mathrm{tr} K \tag{Mom.110}$$

```
# ------------------------------------------------------------------------
# Momentum constraint pt.2

confMom := \exp(4\phi) @(Mom).

defG2GBarU := g^{i j} -> \exp(-4\phi) gBar^{i j}.

distribute   (confMom)                                          # cdb(confMom.101,confMom)
substitute   (confMom, defG2GBarU)                              # cdb(confMom.102,confMom)
map_sympy    (confMom, "simplify")                              # cdb(confMom.103,confMom)

defDAabU  := D_{a}{ABar^{b c}} ->  \partial_{a}{ABar^{b c}}
                                 + \Gamma^{b}_{i a} ABar^{i c}
                                 + \Gamma^{c}_{i a} ABar^{b i}.

substitute     (confMom, defDAabU)                              # cdb(confMom.104,confMom)
substitute     (confMom, defG2GBar)                             # cdb(confMom.105,confMom)
distribute     (confMom)                                        # cdb(confMom.106,confMom)
confMom = product_sort (confMom)                                # cdb(confMom.107,confMom)
rename_dummies (confMom)                                        # cdb(confMom.108,confMom)
canonicalise   (confMom)                                        # cdb(confMom.109,confMom)
substitute     (confMom, $gBar^{i}_{i} -> 3$)                   # cdb(confMom.110,confMom)
substitute     (confMom, $gBar_{i j} ABar^{i j} -> 0$)          # cdb(confMom.111,confMom)
substitute     (confMom, $gBar_{a i} gBar^{i b} -> gBar_{a}^{b}$)  # cdb(confMom.112,confMom)
substitute     (confMom, $GammaBar^{b}_{a b} -> 0$)             # cdb(confMom.113,confMom) # follows from det gBar = 1
eliminate_kronecker (confMom)                                   # cdb(confMom.114,confMom)
rename_dummies (confMom)
canonicalise   (confMom)                                        # cdb(confMom.115,confMom)

cdblib.put ('confMom',confMom,jsonfile)
```

$$\exp(4\phi)\mathcal{D}^j = -4\exp(4\phi)\exp(-4\phi)\,\partial_j\phi\bar{A}^{ij} + \exp(4\phi)\exp(-4\phi)\,D_j\bar{A}^{ij} - \frac{2}{3}\exp(4\phi)\,g^{ij}\partial_j\mathrm{tr}K \tag{confMom.101}$$

$$= -4\exp(4\phi)\exp(-4\phi)\,\partial_j\phi\bar{A}^{ij} + \exp(4\phi)\exp(-4\phi)\,D_j\bar{A}^{ij} - \frac{2}{3}\exp(4\phi)\exp(-4\phi)\,\bar{g}^{ij}\partial_j\mathrm{tr}K \tag{confMom.102}$$

$$= -4\partial_j\phi\bar{A}^{ij} + D_j\bar{A}^{ij} - \frac{2}{3}\bar{g}^{ij}\partial_j\mathrm{tr}K \tag{confMom.103}$$

$$= -4\partial_j\phi\bar{A}^{ij} + \partial_j\bar{A}^{ij} + \Gamma^i{}_{aj}\bar{A}^{aj} + \Gamma^j{}_{aj}\bar{A}^{ia} - \frac{2}{3}\bar{g}^{ij}\partial_j\mathrm{tr}K \tag{confMom.104}$$

$$= -4\partial_j\phi\bar{A}^{ij} + \partial_j\bar{A}^{ij} + \left(2\bar{g}^i{}_j\partial_a\phi + 2\bar{g}^i{}_a\partial_j\phi - 2\bar{g}^{ie}\partial_e\phi\bar{g}_{aj} + \bar{\Gamma}^i{}_{aj}\right)\bar{A}^{aj} + \left(2\bar{g}^j{}_j\partial_a\phi + 2\bar{g}^j{}_a\partial_j\phi - 2\bar{g}^{je}\partial_e\phi\bar{g}_{aj} + \bar{\Gamma}^j{}_{aj}\right)\bar{A}^{ia}$$
$$- \frac{2}{3}\bar{g}^{ij}\partial_j\mathrm{tr}K \tag{confMom.105}$$

$$= -4\partial_j\phi\bar{A}^{ij} + \partial_j\bar{A}^{ij} + 2\bar{g}^i{}_j\partial_a\phi\bar{A}^{aj} + 2\bar{g}^i{}_a\partial_j\phi\bar{A}^{aj} - 2\bar{g}^{ie}\partial_e\phi\bar{g}_{aj}\bar{A}^{aj} + \bar{\Gamma}^i{}_{aj}\bar{A}^{aj} + 2\bar{g}^j{}_j\partial_a\phi\bar{A}^{ia} + 2\bar{g}^j{}_a\partial_j\phi\bar{A}^{ia} - 2\bar{g}^{je}\partial_e\phi\bar{g}_{aj}\bar{A}^{ia} + \bar{\Gamma}^j{}_{aj}\bar{A}^{ia}$$
$$- \frac{2}{3}\bar{g}^{ij}\partial_j\mathrm{tr}K \tag{confMom.106}$$

$$= -4\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + 2\bar{A}^{ab}\partial_a\phi\bar{g}^i{}_b + 2\bar{A}^{ab}\partial_b\phi\bar{g}^i{}_a - 2\bar{A}^{ab}\bar{g}_{ab}\bar{g}^{ic}\partial_c\phi + \bar{A}^{ab}\bar{\Gamma}^i{}_{ab} + 2\bar{A}^{ia}\partial_a\phi\bar{g}^b{}_b + 2\bar{A}^{ia}\partial_b\phi\bar{g}^b{}_a - 2\bar{A}^{ia}\bar{g}_{ab}\bar{g}^{bc}\partial_c\phi + \bar{A}^{ia}\bar{\Gamma}^b{}_{ab}$$
$$- \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \tag{confMom.107}$$

$$= -4\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + 2\bar{A}^{ab}\partial_a\phi\bar{g}^i{}_b + 2\bar{A}^{ab}\partial_b\phi\bar{g}^i{}_a - 2\bar{A}^{ab}\bar{g}_{ab}\bar{g}^{ic}\partial_c\phi + \bar{A}^{ab}\bar{\Gamma}^i{}_{ab} + 2\bar{A}^{ia}\partial_a\phi\bar{g}^b{}_b + 2\bar{A}^{ia}\partial_b\phi\bar{g}^b{}_a - 2\bar{A}^{ia}\bar{g}_{ac}\bar{g}^{cb}\partial_b\phi + \bar{A}^{ia}\bar{\Gamma}^b{}_{ab}$$
$$- \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \tag{confMom.108}$$

$$= -4\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + 4\bar{A}^{ab}\partial_a\phi\bar{g}^i{}_b - 2\bar{A}^{ab}\bar{g}_{ab}\bar{g}^{ic}\partial_c\phi + \bar{A}^{ab}\bar{\Gamma}^i{}_{ab} + 2\bar{A}^{ia}\partial_a\phi\bar{g}^b{}_b + 2\bar{A}^{ia}\partial_b\phi\bar{g}_a{}^b - 2\bar{A}^{ia}\bar{g}_{ab}\bar{g}^{bc}\partial_c\phi + \bar{A}^{ia}\bar{\Gamma}^b{}_{ab}$$
$$- \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \tag{confMom.109}$$

$$= 2\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + 4\bar{A}^{ab}\partial_a\phi\bar{g}^i{}_b - 2\bar{A}^{ab}\bar{g}_{ab}\bar{g}^{ic}\partial_c\phi + \bar{A}^{ab}\bar{\Gamma}^i{}_{ab} + 2\bar{A}^{ia}\partial_b\phi\bar{g}_a{}^b - 2\bar{A}^{ia}\bar{g}_{ab}\bar{g}^{bc}\partial_c\phi + \bar{A}^{ia}\bar{\Gamma}^b{}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \tag{confMom.110}$$

$$= 2\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + 4\bar{A}^{ab}\partial_a\phi\bar{g}^i{}_b + \bar{A}^{ab}\bar{\Gamma}^i{}_{ab} + 2\bar{A}^{ia}\partial_b\phi\bar{g}_a{}^b - 2\bar{A}^{ia}\bar{g}_{ab}\bar{g}^{bc}\partial_c\phi + \bar{A}^{ia}\bar{\Gamma}^b{}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \tag{confMom.111}$$

$$\exp(4\phi)\mathcal{D}^j = 2\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + 4\bar{A}^{ab}\partial_a\phi\bar{g}^i{}_b + \bar{A}^{ab}\bar{\Gamma}^i{}_{ab} + 2\bar{A}^{ia}\partial_b\phi\bar{g}_a{}^b - 2\bar{A}^{ia}\bar{g}_a{}^c\partial_c\phi + \bar{A}^{ia}\bar{\Gamma}^b{}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \qquad (\texttt{confMom.112})$$

$$= 2\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + 4\bar{A}^{ab}\partial_a\phi\bar{g}^i{}_b + \bar{A}^{ab}\bar{\Gamma}^i{}_{ab} + 2\bar{A}^{ia}\partial_b\phi\bar{g}_a{}^b - 2\bar{A}^{ia}\bar{g}_a{}^c\partial_c\phi - \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \qquad (\texttt{confMom.113})$$

$$= 2\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + 4\bar{A}^{ai}\partial_a\phi + \bar{A}^{ab}\bar{\Gamma}^i{}_{ab} + 2\bar{A}^{ib}\partial_b\phi - 2\bar{A}^{ic}\partial_c\phi - \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \qquad (\texttt{confMom.114})$$

$$= 6\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + \bar{A}^{ab}\bar{\Gamma}^i{}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \qquad (\texttt{confMom.115})$$

```
tmpA := @(confMom).                                          # cdb(confMom.201,tmpA)
tmpB := @(confMom).

X^{b c}_{a}::Weight(label=numX).

Xbca := \partial_{a}{ABar^{b c}}.                            # cdb(confMom.202,Xbca)

foo := \partial_{a}{ABar^{b c}} -> X^{b c}_{a}.
bah := X^{b c}_{a} -> \partial_{a}{ABar^{b c}}.

substitute  (tmpA, foo)                                      # cdb(confMom.203,tmpA)
substitute  (tmpB, foo)                                      # cdb(confMom.204,tmpB)
drop_weight (tmpA, $numX=1$)                                 # cdb(confMom.205,tmpA)
keep_weight (tmpB, $numX=1$)                                 # cdb(confMom.206,tmpB)
substitute  (tmpB, bah)                                      # cdb(confMom.207,tmpB)

tmpC := - @(tmpA).                                           # cdb(confMom.208,tmpC)

defMomSub := @(tmpB) -> @(tmpC).                             # cdb(confMom.209,defMomSub)

cdblib.put ('defMomSub',defMomSub,jsonfile)
```

$$0 = 6\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + \bar{A}^{ab}\bar{\Gamma}^{i}{}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a \mathrm{tr}K \tag{confMom.201}$$

$$0 = 6\bar{A}^{ia}\partial_a\phi + X^{ia}{}_{a} + \bar{A}^{ab}\bar{\Gamma}^{i}{}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a \mathrm{tr}K \tag{confMom.203}$$

$$\partial_a\bar{A}^{bc} = X^{ia}{}_{a} \tag{confMom.206}$$

$$= -6\bar{A}^{ia}\partial_a\phi - \bar{A}^{ab}\bar{\Gamma}^{i}{}_{ab} + \frac{2}{3}\bar{g}^{ia}\partial_a \mathrm{tr}K \tag{confMom.208}$$

$$\partial_a\bar{A}^{ia} \to -6\bar{A}^{ia}\partial_a\phi - \bar{A}^{ab}\bar{\Gamma}^{i}{}_{ab} + \frac{2}{3}\bar{g}^{ia}\partial_a \mathrm{tr}K \tag{confMom.209}$$

```
# ------------------------------------------------------------------------
# Check against prd67.

foo := @(confMom).                                  # cdb(prd67.eq20.lcb,foo)
bah  = cdblib.get('prd67.eq20.rhs','prd67.json')    # cdb(prd67.eq20.prd,bah)

diff := @(foo) - @(bah).

distribute     (diff)
diff = product_sort (diff)
rename_dummies (diff)
map_sympy      (diff, "simplify")
canonicalise   (diff)                               # cdb(prd67.eq20.chk,diff)
```

$$\texttt{prd67.eq20.lcb} := 6\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + \bar{A}^{ab}\bar{\Gamma}^{i}{}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a \mathrm{tr}K$$

$$\texttt{prd67.eq20.prd} := \partial_a\bar{A}^{ia} + 6\bar{A}^{ia}\partial_a\phi + \bar{A}^{ab}\bar{\Gamma}^{i}{}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a \mathrm{tr}K$$

$$\texttt{prd67.eq20.chk} := 0$$