PhysRevD.62.044034 equation (15)

```
from shared import *
import cdblib

jsonfile = 'eqtn15.json'
cdblib.create (jsonfile)

defG2GBar = cdblib.get ('defG2GBar','gamma.json')

# ----------------------------------------------------------------------
# Rphi = the part of Rab from the conformal factor

Rab := R_{a b}.                                                  # cdb (eq15.101,Rab)

substitute     (Rab, defRab)                                     # cdb (eq15.102,Rab)
substitute     (Rab, defRiem)                                    # cdb (eq15.103,Rab)
substitute     (Rab, defG2GBar)                                  # cdb (eq15.104,Rab)
distribute     (Rab)                                             # cdb (eq15.105,Rab)
product_rule   (Rab)                                             # cdb (eq15.106,Rab)
Rab = product_sort (Rab)                                         # cdb (eq15.107,Rab)
rename_dummies (Rab)                                             # cdb (eq15.108,Rab)
canonicalise   (Rab)                                             # cdb (eq15.109,Rab)
substitute     (Rab, $gBar_{b c} gBar^{c a} -> gBar^{a}_{b}$)
substitute     (Rab, $\partial_{a}{gBar^{a}_{b}} -> 0$)
substitute     (Rab, $\partial_{a}{gBar_{b}^{c}} -> 0$)
substitute     (Rab, $gBar^{a}_{a} -> 3$)
eliminate_kronecker (Rab)                                        # cdb (eq15.110,Rab)
Rab = product_sort (Rab)                                         # cdb (eq15.111,Rab)
rename_dummies (Rab)                                             # cdb (eq15.112,Rab)
canonicalise   (Rab)                                             # cdb (eq15.113,Rab)
substitute     (Rab, $gBar_{b c} gBar^{c a} -> gBar^{a}_{b}$)    # cdb (eq15.114,Rab)
substitute     (Rab, $gBar^{a}_{a} -> 3$)                        # cdb (eq15.115,Rab)
eliminate_kronecker (Rab)                                        # cdb (eq15.116,Rab)

#
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```
# isolate Rphi from Rab by switching to local RNC

Rphi := @(Rab).

substitute (Rphi, $GammaBar^{a}_{b c}->0$)                          # cdb (eq15.117,Rphi)
substitute (Rphi, $\partial_{a}{gBar_{b c}}->0$)                    # cdb (eq15.118,Rphi)
substitute (Rphi, $\partial_{a}{gBar^{b c}}->0$)                    # cdb (eq15.119,Rphi)

substitute (Rphi, $\partial_{a b}{\phi} -> DBar_{a b}{\phi}$)      # cdb (eq15.120,Rphi)
substitute (Rphi, $\partial_{a}{\phi} -> DBar_{a}{\phi}$)          # cdb (eq15.121,Rphi)

defRphi := Rphi_{a b} -> @(Rphi).

cdblib.put ('defRphi',defRphi,jsonfile)
```

$$R_{ab} = R^{c}{}_{acb} \tag{eq15.102}$$

$$= \partial_c \Gamma^{c}{}_{ab} + \Gamma^{c}{}_{ec}\Gamma^{e}{}_{ab} - \partial_b \Gamma^{c}{}_{ac} - \Gamma^{c}{}_{eb}\Gamma^{e}{}_{ac} \tag{eq15.103}$$

$$= \partial_c \left(2\bar{g}^{c}{}_{b}\partial_a\phi + 2\bar{g}^{c}{}_{a}\partial_b\phi - 2\bar{g}^{ce}\partial_e\phi\bar{g}_{ab} + \bar{\Gamma}^{c}{}_{ab}\right) + \left(2\bar{g}^{c}{}_{c}\partial_e\phi + 2\bar{g}^{c}{}_{e}\partial_c\phi - 2\bar{g}^{cd}\partial_d\phi\bar{g}_{ec} + \bar{\Gamma}^{c}{}_{ec}\right)\left(2\bar{g}^{e}{}_{b}\partial_a\phi + 2\bar{g}^{e}{}_{a}\partial_b\phi - 2\bar{g}^{ef}\partial_f\phi\bar{g}_{ab} + \bar{\Gamma}^{e}{}_{ab}\right)$$
$$- \partial_b \left(2\bar{g}^{c}{}_{c}\partial_a\phi + 2\bar{g}^{c}{}_{a}\partial_c\phi - 2\bar{g}^{ce}\partial_e\phi\bar{g}_{ac} + \bar{\Gamma}^{c}{}_{ac}\right) - \left(2\bar{g}^{c}{}_{b}\partial_e\phi + 2\bar{g}^{c}{}_{e}\partial_b\phi - 2\bar{g}^{cd}\partial_d\phi\bar{g}_{eb} + \bar{\Gamma}^{c}{}_{eb}\right)\left(2\bar{g}^{e}{}_{c}\partial_a\phi + 2\bar{g}^{e}{}_{a}\partial_c\phi - 2\bar{g}^{ef}\partial_f\phi\bar{g}_{ac} + \bar{\Gamma}^{e}{}_{ac}\right) \tag{eq15.104}$$

$$= 2\partial_c\left(\bar{g}^{c}{}_{b}\partial_a\phi\right) + 2\partial_c\left(\bar{g}^{c}{}_{a}\partial_b\phi\right) - 2\partial_c\left(\bar{g}^{ce}\partial_e\phi\bar{g}_{ab}\right) + \partial_c\bar{\Gamma}^{c}{}_{ab} + 4\bar{g}^{c}{}_{c}\partial_e\phi\bar{g}^{e}{}_{b}\partial_a\phi + 4\bar{g}^{c}{}_{c}\partial_e\phi\bar{g}^{e}{}_{a}\partial_b\phi - 4\bar{g}^{c}{}_{c}\partial_e\phi\bar{g}^{ef}\partial_f\phi\bar{g}_{ab} + 2\bar{g}^{c}{}_{c}\partial_e\phi\bar{\Gamma}^{e}{}_{ab} + 4\bar{g}^{c}{}_{e}\partial_c\phi\bar{g}^{e}{}_{b}\partial_a\phi$$
$$+ 4\bar{g}^{c}{}_{e}\partial_c\phi\bar{g}^{e}{}_{a}\partial_b\phi - 4\bar{g}^{c}{}_{e}\partial_c\phi\bar{g}^{ef}\partial_f\phi\bar{g}_{ab} + 2\bar{g}^{c}{}_{e}\partial_c\phi\bar{\Gamma}^{e}{}_{ab} - 4\bar{g}^{cd}\partial_d\phi\bar{g}_{ec}\bar{g}^{e}{}_{b}\partial_a\phi - 4\bar{g}^{cd}\partial_d\phi\bar{g}_{ec}\bar{g}^{e}{}_{a}\partial_b\phi + 4\bar{g}^{cd}\partial_d\phi\bar{g}_{ec}\bar{g}^{ef}\partial_f\phi\bar{g}_{ab} - 2\bar{g}^{cd}\partial_d\phi\bar{g}_{ec}\bar{\Gamma}^{e}{}_{ab}$$
$$+ 2\bar{\Gamma}^{c}{}_{ec}\bar{g}^{e}{}_{b}\partial_a\phi + 2\bar{\Gamma}^{c}{}_{ec}\bar{g}^{e}{}_{a}\partial_b\phi - 2\bar{\Gamma}^{c}{}_{ec}\bar{g}^{ef}\partial_f\phi\bar{g}_{ab} + \bar{\Gamma}^{c}{}_{ec}\bar{\Gamma}^{e}{}_{ab} - 2\partial_b\left(\bar{g}^{c}{}_{c}\partial_a\phi\right) - 2\partial_b\left(\bar{g}^{c}{}_{a}\partial_c\phi\right) + 2\partial_b\left(\bar{g}^{ce}\partial_e\phi\bar{g}_{ac}\right) - \partial_b\bar{\Gamma}^{c}{}_{ac} - 4\bar{g}^{c}{}_{b}\partial_e\phi\bar{g}^{e}{}_{c}\partial_a\phi$$
$$- 4\bar{g}^{c}{}_{b}\partial_e\phi\bar{g}^{e}{}_{a}\partial_c\phi + 4\bar{g}^{c}{}_{b}\partial_e\phi\bar{g}^{ef}\partial_f\phi\bar{g}_{ac} - 2\bar{g}^{c}{}_{b}\partial_e\phi\bar{\Gamma}^{e}{}_{ac} - 4\bar{g}^{c}{}_{e}\partial_b\phi\bar{g}^{e}{}_{c}\partial_a\phi - 4\bar{g}^{c}{}_{e}\partial_b\phi\bar{g}^{e}{}_{a}\partial_c\phi + 4\bar{g}^{c}{}_{e}\partial_b\phi\bar{g}^{ef}\partial_f\phi\bar{g}_{ac} - 2\bar{g}^{c}{}_{e}\partial_b\phi\bar{\Gamma}^{e}{}_{ac} + 4\bar{g}^{cd}\partial_d\phi\bar{g}_{eb}\bar{g}^{e}{}_{c}\partial_a\phi$$
$$+ 4\bar{g}^{cd}\partial_d\phi\bar{g}_{eb}\bar{g}^{e}{}_{a}\partial_c\phi - 4\bar{g}^{cd}\partial_d\phi\bar{g}_{eb}\bar{g}^{ef}\partial_f\phi\bar{g}_{ac} + 2\bar{g}^{cd}\partial_d\phi\bar{g}_{eb}\bar{\Gamma}^{e}{}_{ac} - 2\bar{\Gamma}^{c}{}_{eb}\bar{g}^{e}{}_{c}\partial_a\phi - 2\bar{\Gamma}^{c}{}_{eb}\bar{g}^{e}{}_{a}\partial_c\phi + 2\bar{\Gamma}^{c}{}_{eb}\bar{g}^{ef}\partial_f\phi\bar{g}_{ac} - \bar{\Gamma}^{c}{}_{eb}\bar{\Gamma}^{e}{}_{ac} \tag{eq15.105}$$

$$= 2\partial_c\bar{g}^{c}{}_{b}\partial_a\phi + 2\bar{g}^{c}{}_{b}\partial_{ca}\phi + 2\partial_c\bar{g}^{c}{}_{a}\partial_b\phi + 2\bar{g}^{c}{}_{a}\partial_{cb}\phi - 2\partial_c\bar{g}^{ce}\partial_e\phi\bar{g}_{ab} - 2\bar{g}^{ce}\partial_{ce}\phi\bar{g}_{ab} - 2\bar{g}^{ce}\partial_e\phi\partial_c\bar{g}_{ab} + \partial_c\bar{\Gamma}^{c}{}_{ab} + 4\bar{g}^{c}{}_{c}\partial_e\phi\bar{g}^{e}{}_{b}\partial_a\phi + 4\bar{g}^{c}{}_{c}\partial_e\phi\bar{g}^{e}{}_{a}\partial_b\phi$$
$$- 4\bar{g}^{c}{}_{c}\partial_e\phi\bar{g}^{ef}\partial_f\phi\bar{g}_{ab} + 2\bar{g}^{c}{}_{c}\partial_e\phi\bar{\Gamma}^{e}{}_{ab} + 4\bar{g}^{c}{}_{e}\partial_c\phi\bar{g}^{e}{}_{b}\partial_a\phi + 4\bar{g}^{c}{}_{e}\partial_c\phi\bar{g}^{e}{}_{a}\partial_b\phi - 4\bar{g}^{c}{}_{e}\partial_c\phi\bar{g}^{ef}\partial_f\phi\bar{g}_{ab} + 2\bar{g}^{c}{}_{e}\partial_c\phi\bar{\Gamma}^{e}{}_{ab} - 4\bar{g}^{cd}\partial_d\phi\bar{g}_{ec}\bar{g}^{e}{}_{b}\partial_a\phi$$
$$- 4\bar{g}^{cd}\partial_d\phi\bar{g}_{ec}\bar{g}^{e}{}_{a}\partial_b\phi + 4\bar{g}^{cd}\partial_d\phi\bar{g}_{ec}\bar{g}^{ef}\partial_f\phi\bar{g}_{ab} - 2\bar{g}^{cd}\partial_d\phi\bar{g}_{ec}\bar{\Gamma}^{e}{}_{ab} + 2\bar{\Gamma}^{c}{}_{ec}\bar{g}^{e}{}_{b}\partial_a\phi + 2\bar{\Gamma}^{c}{}_{ec}\bar{g}^{e}{}_{a}\partial_b\phi - 2\bar{\Gamma}^{c}{}_{ec}\bar{g}^{ef}\partial_f\phi\bar{g}_{ab} + \bar{\Gamma}^{c}{}_{ec}\bar{\Gamma}^{e}{}_{ab} - 2\partial_b\bar{g}^{c}{}_{c}\partial_a\phi$$
$$- 2\bar{g}^{c}{}_{c}\partial_{ba}\phi - 2\partial_b\bar{g}^{c}{}_{a}\partial_c\phi - 2\bar{g}^{c}{}_{a}\partial_{bc}\phi + 2\partial_b\bar{g}^{ce}\partial_e\phi\bar{g}_{ac} + 2\bar{g}^{ce}\partial_{be}\phi\bar{g}_{ac} + 2\bar{g}^{ce}\partial_e\phi\partial_b\bar{g}_{ac} - \partial_b\bar{\Gamma}^{c}{}_{ac} - 4\bar{g}^{c}{}_{b}\partial_e\phi\bar{g}^{e}{}_{c}\partial_a\phi - 4\bar{g}^{c}{}_{b}\partial_e\phi\bar{g}^{e}{}_{a}\partial_c\phi$$
$$+ 4\bar{g}^{c}{}_{b}\partial_e\phi\bar{g}^{ef}\partial_f\phi\bar{g}_{ac} - 2\bar{g}^{c}{}_{b}\partial_e\phi\bar{\Gamma}^{e}{}_{ac} - 4\bar{g}^{c}{}_{e}\partial_b\phi\bar{g}^{e}{}_{c}\partial_a\phi - 4\bar{g}^{c}{}_{e}\partial_b\phi\bar{g}^{e}{}_{a}\partial_c\phi + 4\bar{g}^{c}{}_{e}\partial_b\phi\bar{g}^{ef}\partial_f\phi\bar{g}_{ac} - 2\bar{g}^{c}{}_{e}\partial_b\phi\bar{\Gamma}^{e}{}_{ac} + 4\bar{g}^{cd}\partial_d\phi\bar{g}_{eb}\bar{g}^{e}{}_{c}\partial_a\phi$$
$$+ 4\bar{g}^{cd}\partial_d\phi\bar{g}_{eb}\bar{g}^{e}{}_{a}\partial_c\phi - 4\bar{g}^{cd}\partial_d\phi\bar{g}_{eb}\bar{g}^{ef}\partial_f\phi\bar{g}_{ac} + 2\bar{g}^{cd}\partial_d\phi\bar{g}_{eb}\bar{\Gamma}^{e}{}_{ac} - 2\bar{\Gamma}^{c}{}_{eb}\bar{g}^{e}{}_{c}\partial_a\phi - 2\bar{\Gamma}^{c}{}_{eb}\bar{g}^{e}{}_{a}\partial_c\phi + 2\bar{\Gamma}^{c}{}_{eb}\bar{g}^{ef}\partial_f\phi\bar{g}_{ac} - \bar{\Gamma}^{c}{}_{eb}\bar{\Gamma}^{e}{}_{ac} \tag{eq15.106}$$

$$= 2\partial_a\phi\partial_c\bar{g}^{c}{}_{b} + 2\partial_{ca}\phi\bar{g}^{c}{}_{b} + 2\partial_b\phi\partial_c\bar{g}^{c}{}_{a} + 2\partial_{cb}\phi\bar{g}^{c}{}_{a} - 2\bar{g}_{ab}\partial_d\phi\partial_c\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{cd}\partial_d\phi\partial_c\bar{g}_{ab} + \partial_c\bar{\Gamma}^{c}{}_{ab} + 4\partial_a\phi\partial_c\phi\bar{g}^{c}{}_{b}\bar{g}^{d}{}_{d} + 4\partial_b\phi\partial_c\phi\bar{g}^{c}{}_{a}\bar{g}^{d}{}_{d}$$
$$- 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi\bar{g}^{e}{}_{e} + 2\bar{\Gamma}^{c}{}_{ab}\partial_c\phi\bar{g}^{d}{}_{d} + 4\partial_a\phi\partial_c\phi\bar{g}^{c}{}_{d}\bar{g}^{d}{}_{b} - 4\bar{g}_{ab}\bar{g}^{cd}\partial_e\phi\partial_d\phi\bar{g}^{e}{}_{c} + 2\bar{\Gamma}^{c}{}_{ab}\partial_d\phi\bar{g}^{d}{}_{c} - 4\bar{g}_{cd}\bar{g}^{de}\partial_a\phi\partial_e\phi\bar{g}^{c}{}_{b} - 4\bar{g}_{cd}\bar{g}^{de}\partial_b\phi\partial_e\phi\bar{g}^{c}{}_{a}$$
$$+ 4\bar{g}_{ab}\bar{g}_{cd}\bar{g}^{de}\bar{g}^{cf}\partial_e\phi\partial_f\phi - 2\bar{g}_{cd}\bar{g}^{de}\bar{\Gamma}^{c}{}_{ab}\partial_e\phi + 2\bar{\Gamma}^{c}{}_{dc}\partial_a\phi\bar{g}^{d}{}_{b} + 2\bar{\Gamma}^{c}{}_{dc}\partial_b\phi\bar{g}^{d}{}_{a} - 2\bar{g}_{ab}\bar{g}^{cd}\bar{\Gamma}^{e}{}_{ce}\partial_d\phi + \bar{\Gamma}^{c}{}_{ab}\bar{\Gamma}^{d}{}_{cd} - 2\partial_a\phi\partial_b\bar{g}^{c}{}_{c} - 2\partial_{ba}\phi\bar{g}^{c}{}_{c} - 2\partial_c\phi\partial_b\bar{g}^{c}{}_{a}$$
$$- 2\partial_{bc}\phi\bar{g}^{c}{}_{a} + 2\bar{g}_{ac}\partial_d\phi\partial_b\bar{g}^{cd} + 2\bar{g}_{ac}\bar{g}^{cd}\partial_{bd}\phi + 2\bar{g}^{cd}\partial_d\phi\partial_b\bar{g}_{ac} - \partial_b\bar{\Gamma}^{c}{}_{ac} - 4\partial_a\phi\partial_c\phi\bar{g}^{d}{}_{b}\bar{g}^{c}{}_{d} - 4\partial_c\phi\partial_d\phi\bar{g}^{c}{}_{b}\bar{g}^{d}{}_{a} + 4\bar{g}_{ac}\bar{g}^{de}\partial_d\phi\partial_e\phi\bar{g}^{c}{}_{b} - 2\bar{\Gamma}^{c}{}_{ad}\partial_c\phi\bar{g}^{d}{}_{b}$$
$$- 4\partial_a\phi\partial_b\phi\bar{g}^{c}{}_{d}\bar{g}^{d}{}_{c} + 4\bar{g}_{ac}\bar{g}^{de}\partial_b\phi\partial_e\phi\bar{g}^{c}{}_{d} - 2\bar{\Gamma}^{c}{}_{ad}\partial_b\phi\bar{g}^{d}{}_{c} + 4\bar{g}_{cb}\bar{g}^{de}\partial_a\phi\partial_e\phi\bar{g}^{c}{}_{d} + 4\bar{g}_{cb}\bar{g}^{de}\partial_d\phi\partial_e\phi\bar{g}^{c}{}_{a} - 4\bar{g}_{ac}\bar{g}_{db}\bar{g}^{ce}\bar{g}^{df}\partial_e\phi\partial_f\phi + 2\bar{g}_{cb}\bar{g}^{de}\bar{\Gamma}^{c}{}_{ad}\partial_e\phi$$
$$- 2\bar{\Gamma}^{c}{}_{db}\partial_a\phi\bar{g}^{d}{}_{c} - 2\bar{\Gamma}^{c}{}_{db}\partial_c\phi\bar{g}^{d}{}_{a} + 2\bar{g}_{ac}\bar{g}^{de}\bar{\Gamma}^{c}{}_{db}\partial_e\phi - \bar{\Gamma}^{c}{}_{db}\bar{\Gamma}^{d}{}_{ac} \tag{eq15.107}$$

$$
\begin{aligned}
R_{ab} &= 2\partial_a\phi\partial_c\bar{g}^c{}_b + 2\partial_{ca}\phi\bar{g}^c{}_b + 2\partial_b\phi\partial_c\bar{g}^c{}_a + 2\partial_{cb}\phi\bar{g}^c{}_a - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{dc} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{dc}\partial_c\phi\partial_d\bar{g}_{ab} + \partial_c\bar{\Gamma}^c{}_{ab} + 4\partial_a\phi\partial_c\phi\bar{g}^c{}_b\bar{g}^d{}_d + 4\partial_b\phi\partial_c\phi\bar{g}^c{}_a\bar{g}^d{}_d \\
&\quad - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi\bar{g}^e{}_e + 2\bar{\Gamma}^c{}_{ab}\partial_c\phi\bar{g}^d{}_d + 4\partial_a\phi\partial_c\phi\bar{g}^c{}_d\bar{g}^d{}_b - 4\bar{g}_{ab}\bar{g}^{ed}\partial_c\phi\partial_d\phi\bar{g}^c{}_e + 2\bar{\Gamma}^c{}_{ab}\partial_d\phi\bar{g}^d{}_c - 4\bar{g}_{de}\bar{g}^{ec}\partial_a\phi\partial_c\phi\bar{g}^d{}_b - 4\bar{g}_{de}\bar{g}^{ec}\partial_b\phi\partial_c\phi\bar{g}^d{}_a \\
&\quad + 4\bar{g}_{ab}\bar{g}_{ef}\bar{g}^{fc}\bar{g}^{ed}\partial_c\phi\partial_d\phi - 2\bar{g}_{ce}\bar{g}^{ed}\bar{\Gamma}^c{}_{ab}\partial_d\phi + 2\bar{\Gamma}^c{}_{dc}\partial_a\phi\bar{g}^d{}_b + 2\bar{\Gamma}^c{}_{dc}\partial_b\phi\bar{g}^d{}_a - 2\bar{g}_{ab}\bar{g}^{de}\bar{\Gamma}^c{}_{dc}\partial_e\phi + \bar{\Gamma}^c{}_{ab}\bar{\Gamma}^d{}_{cd} - 2\partial_a\phi\partial_b\bar{g}^c{}_c - 2\partial_{ba}\phi\bar{g}^c{}_c - 2\partial_c\phi\partial_b\bar{g}^c{}_a \\
&\quad - 2\partial_{bc}\phi\bar{g}^c{}_a + 2\bar{g}_{ad}\partial_c\phi\partial_b\bar{g}^{dc} + 2\bar{g}_{ad}\bar{g}^{dc}\partial_{bc}\phi + 2\bar{g}^{dc}\partial_c\phi\partial_b\bar{g}_{ad} - \partial_b\bar{\Gamma}^c{}_{ac} - 4\partial_a\phi\partial_c\phi\bar{g}^d{}_b\bar{g}^c{}_d - 4\partial_c\phi\partial_d\phi\bar{g}^c{}_b\bar{g}^d{}_a + 4\bar{g}_{ae}\bar{g}^{cd}\partial_c\phi\partial_d\phi\bar{g}^e{}_b - 2\bar{\Gamma}^c{}_{ad}\partial_c\phi\bar{g}^d{}_b \\
&\quad - 4\partial_a\phi\partial_b\phi\bar{g}^c{}_d\bar{g}^d{}_c + 4\bar{g}_{ad}\bar{g}^{ec}\partial_b\phi\partial_c\phi\bar{g}^d{}_e - 2\bar{\Gamma}^c{}_{ad}\partial_b\phi\bar{g}^d{}_c + 4\bar{g}_{db}\bar{g}^{ec}\partial_a\phi\partial_c\phi\bar{g}^d{}_e + 4\bar{g}_{eb}\bar{g}^{cd}\partial_c\phi\partial_d\phi\bar{g}^e{}_a - 4\bar{g}_{ae}\bar{g}_{fb}\bar{g}^{ec}\bar{g}^{fd}\partial_c\phi\partial_d\phi + 2\bar{g}_{cb}\bar{g}^{de}\bar{\Gamma}^c{}_{ad}\partial_e\phi \\
&\quad - 2\bar{\Gamma}^c{}_{db}\partial_a\phi\bar{g}^d{}_c - 2\bar{\Gamma}^c{}_{db}\partial_c\phi\bar{g}^d{}_a + 2\bar{g}_{ac}\bar{g}^{de}\bar{\Gamma}^c{}_{db}\partial_e\phi - \bar{\Gamma}^c{}_{db}\bar{\Gamma}^d{}_{ac} \qquad \texttt{(eq15.108)} \\
&= 2\partial_a\phi\partial_c\bar{g}_b{}^c + 2\partial_{ac}\phi\bar{g}_b{}^c + 2\partial_b\phi\partial_c\bar{g}_a{}^c - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{cd}\partial_c\phi\partial_d\bar{g}_{ab} + \partial_c\bar{\Gamma}^c{}_{ab} + 4\partial_a\phi\partial_c\phi\bar{g}_b{}^c\bar{g}^d{}_d + 4\partial_b\phi\partial_c\phi\bar{g}_a{}^c\bar{g}^d{}_d - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi\bar{g}^e{}_e \\
&\quad + 2\bar{\Gamma}^c{}_{ab}\partial_c\phi\bar{g}^d{}_d - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_e\phi\bar{g}_d{}^e + 2\bar{\Gamma}^c{}_{ab}\partial_d\phi\bar{g}_c{}^d - 4\bar{g}_{cd}\bar{g}^{ce}\partial_a\phi\partial_e\phi\bar{g}_b{}^d - 4\bar{g}_{cd}\bar{g}^{ce}\partial_b\phi\partial_e\phi\bar{g}_a{}^d + 4\bar{g}_{ab}\bar{g}_{cd}\bar{g}^{ce}\bar{g}^{df}\partial_e\phi\partial_f\phi - 2\bar{g}_{cd}\bar{g}^{ce}\bar{\Gamma}^d{}_{ab}\partial_e\phi \\
&\quad + 2\bar{\Gamma}^c{}_{cd}\partial_a\phi\bar{g}_b{}^d + 2\bar{\Gamma}^c{}_{cd}\partial_b\phi\bar{g}_a{}^d - 2\bar{g}_{ab}\bar{g}^{cd}\bar{\Gamma}^e{}_{ce}\partial_d\phi + \bar{\Gamma}^c{}_{ab}\bar{\Gamma}^d{}_{cd} - 2\partial_a\phi\partial_b\bar{g}^c{}_c - 2\partial_{ab}\phi\bar{g}^c{}_c - 2\partial_c\phi\partial_b\bar{g}_a{}^c + 2\bar{g}_{ac}\partial_d\phi\partial_b\bar{g}^{cd} + 2\bar{g}_{ac}\bar{g}^{cd}\partial_{bd}\phi + 2\bar{g}^{cd}\partial_c\phi\partial_b\bar{g}_{ad} \\
&\quad - \partial_b\bar{\Gamma}^c{}_{ac} - 4\partial_c\phi\partial_d\phi\bar{g}_a{}^c\bar{g}_b{}^d + 4\bar{g}_{ac}\bar{g}^{de}\partial_d\phi\partial_e\phi\bar{g}_b{}^c - 2\bar{\Gamma}^c{}_{ad}\partial_c\phi\bar{g}_b{}^d - 4\partial_a\phi\partial_b\phi\bar{g}^c{}_d\bar{g}_c{}^d + 4\bar{g}_{ac}\bar{g}^{de}\partial_b\phi\partial_d\phi\bar{g}^c{}_e - 2\bar{\Gamma}^c{}_{ad}\partial_b\phi\bar{g}_c{}^d + 4\bar{g}_{bc}\bar{g}^{de}\partial_a\phi\partial_d\phi\bar{g}^c{}_e \\
&\quad + 4\bar{g}_{bc}\bar{g}^{de}\partial_d\phi\partial_e\phi\bar{g}_a{}^c - 4\bar{g}_{ac}\bar{g}_{bd}\bar{g}^{ce}\bar{g}^{df}\partial_e\phi\partial_f\phi + 2\bar{g}_{bc}\bar{g}^{de}\bar{\Gamma}^c{}_{ad}\partial_e\phi - 2\bar{\Gamma}^c{}_{bd}\partial_a\phi\bar{g}_c{}^d - 2\bar{\Gamma}^c{}_{bd}\partial_c\phi\bar{g}_a{}^d + 2\bar{g}_{ac}\bar{g}^{de}\bar{\Gamma}^c{}_{bd}\partial_e\phi - \bar{\Gamma}^c{}_{ad}\bar{\Gamma}^d{}_{bc} \qquad \texttt{(eq15.109)} \\
&= -4\partial_{ab}\phi - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{cd}\partial_c\phi\partial_d\bar{g}_{ab} + \partial_c\bar{\Gamma}^c{}_{ab} + 8\partial_a\phi\partial_b\phi + 12\partial_b\phi\partial_a\phi - 12\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi + 4\bar{\Gamma}^c{}_{ab}\partial_c\phi - 4\bar{g}_{ab}\bar{g}^{ce}\partial_c\phi\partial_e\phi \\
&\quad + 2\bar{\Gamma}^d{}_{ab}\partial_d\phi - 4\bar{g}_{cb}\bar{g}^{ce}\partial_a\phi\partial_e\phi - 4\bar{g}_{ca}\bar{g}^{ce}\partial_b\phi\partial_e\phi + 4\bar{g}_{ab}\bar{g}^{fe}\partial_e\phi\partial_f\phi - 2\bar{g}_{cd}\bar{g}^{ce}\bar{\Gamma}^d{}_{ab}\partial_e\phi + 2\bar{\Gamma}^c{}_{cb}\partial_a\phi + 2\bar{\Gamma}^c{}_{ca}\partial_b\phi - 2\bar{g}_{ab}\bar{g}^{cd}\bar{\Gamma}^e{}_{ce}\partial_d\phi + \bar{\Gamma}^c{}_{ab}\bar{\Gamma}^d{}_{cd} \\
&\quad + 2\bar{g}_{ac}\partial_d\phi\partial_b\bar{g}^{cd} + 2\partial_{ba}\phi + 2\bar{g}^{cd}\partial_c\phi\partial_b\bar{g}_{ad} - \partial_b\bar{\Gamma}^c{}_{ac} + 4\bar{g}_{ab}\bar{g}^{de}\partial_d\phi\partial_e\phi - 4\partial_a\phi\partial_b\phi\bar{g}_d{}^d + 4\bar{g}_{ae}\bar{g}^{de}\partial_b\phi\partial_d\phi - 2\bar{\Gamma}^d{}_{ad}\partial_b\phi + 4\bar{g}_{be}\bar{g}^{de}\partial_a\phi\partial_d\phi \\
&\quad + 4\bar{g}_{ba}\bar{g}^{de}\partial_d\phi\partial_e\phi - 4\bar{g}_{bd}\bar{g}^{df}\partial_a\phi\partial_f\phi + 2\bar{g}_{bc}\bar{g}^{de}\bar{\Gamma}^c{}_{ad}\partial_e\phi - 2\bar{\Gamma}^d{}_{bd}\partial_a\phi - 2\bar{\Gamma}^c{}_{ba}\partial_c\phi + 2\bar{g}_{ac}\bar{g}^{de}\bar{\Gamma}^c{}_{bd}\partial_e\phi - \bar{\Gamma}^c{}_{ad}\bar{\Gamma}^d{}_{bc} \qquad \texttt{(eq15.110)} \\
&= -4\partial_{ab}\phi - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{cd}\partial_c\phi\partial_d\bar{g}_{ab} + \partial_c\bar{\Gamma}^c{}_{ab} + 20\partial_a\phi\partial_b\phi - 12\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi + 6\bar{\Gamma}^c{}_{ab}\partial_c\phi - 4\bar{g}_{cb}\bar{g}^{cd}\partial_a\phi\partial_d\phi - 4\bar{g}_{ca}\bar{g}^{cd}\partial_b\phi\partial_d\phi \\
&\quad + 4\bar{g}_{ab}\bar{g}^{cd}\partial_d\phi\partial_c\phi - 2\bar{g}_{cd}\bar{g}^{ce}\bar{\Gamma}^d{}_{ab}\partial_e\phi + 2\bar{\Gamma}^c{}_{cb}\partial_a\phi + 2\bar{\Gamma}^c{}_{ca}\partial_b\phi - 2\bar{g}_{ab}\bar{g}^{cd}\bar{\Gamma}^e{}_{ce}\partial_d\phi + \bar{\Gamma}^c{}_{ab}\bar{\Gamma}^d{}_{cd} + 2\bar{g}_{ac}\partial_d\phi\partial_b\bar{g}^{cd} + 2\partial_{ba}\phi + 2\bar{g}^{dc}\partial_d\phi\partial_b\bar{g}_{ac} - \partial_b\bar{\Gamma}^c{}_{ac} \\
&\quad - 4\partial_a\phi\partial_b\phi\bar{g}_c{}^c + 4\bar{g}_{ac}\bar{g}^{dc}\partial_b\phi\partial_d\phi - 2\bar{\Gamma}^c{}_{ac}\partial_b\phi + 4\bar{g}_{bc}\bar{g}^{dc}\partial_a\phi\partial_d\phi + 4\bar{g}_{ba}\bar{g}^{cd}\partial_c\phi\partial_d\phi - 4\bar{g}_{bc}\bar{g}^{cd}\partial_a\phi\partial_d\phi + 2\bar{g}_{bc}\bar{g}^{de}\bar{\Gamma}^c{}_{ad}\partial_e\phi - 2\bar{\Gamma}^c{}_{bc}\partial_a\phi - 2\bar{\Gamma}^c{}_{ba}\partial_c\phi \\
&\quad + 2\bar{g}_{ac}\bar{g}^{de}\bar{\Gamma}^c{}_{bd}\partial_e\phi - \bar{\Gamma}^c{}_{ad}\bar{\Gamma}^d{}_{bc} \qquad \texttt{(eq15.111)} \\
&= -4\partial_{ab}\phi - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{cd}\partial_c\phi\partial_d\bar{g}_{ab} + \partial_c\bar{\Gamma}^c{}_{ab} + 20\partial_a\phi\partial_b\phi - 12\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi + 6\bar{\Gamma}^c{}_{ab}\partial_c\phi - 4\bar{g}_{db}\bar{g}^{dc}\partial_a\phi\partial_c\phi - 4\bar{g}_{da}\bar{g}^{dc}\partial_b\phi\partial_c\phi \\
&\quad + 4\bar{g}_{ab}\bar{g}^{dc}\partial_c\phi\partial_d\phi - 2\bar{g}_{ec}\bar{g}^{ed}\bar{\Gamma}^c{}_{ab}\partial_d\phi + 2\bar{\Gamma}^c{}_{cb}\partial_a\phi + 2\bar{\Gamma}^c{}_{ca}\partial_b\phi - 2\bar{g}_{ab}\bar{g}^{de}\bar{\Gamma}^c{}_{dc}\partial_e\phi + \bar{\Gamma}^c{}_{ab}\bar{\Gamma}^d{}_{cd} + 2\bar{g}_{ad}\partial_c\phi\partial_b\bar{g}^{dc} + 2\partial_{ba}\phi + 2\bar{g}^{cd}\partial_c\phi\partial_b\bar{g}_{ad} - \partial_b\bar{\Gamma}^c{}_{ac} \\
&\quad - 4\partial_a\phi\partial_b\phi\bar{g}_c{}^c + 4\bar{g}_{ad}\bar{g}^{cd}\partial_b\phi\partial_c\phi - 2\bar{\Gamma}^c{}_{ac}\partial_b\phi + 4\bar{g}_{bd}\bar{g}^{cd}\partial_a\phi\partial_c\phi + 4\bar{g}_{ba}\bar{g}^{cd}\partial_c\phi\partial_d\phi - 4\bar{g}_{bd}\bar{g}^{dc}\partial_a\phi\partial_c\phi + 2\bar{g}_{bc}\bar{g}^{de}\bar{\Gamma}^c{}_{ad}\partial_e\phi - 2\bar{\Gamma}^c{}_{bc}\partial_a\phi - 2\bar{\Gamma}^c{}_{ba}\partial_c\phi \\
&\quad + 2\bar{g}_{ac}\bar{g}^{de}\bar{\Gamma}^c{}_{bd}\partial_e\phi - \bar{\Gamma}^c{}_{ad}\bar{\Gamma}^d{}_{bc} \qquad \texttt{(eq15.112)}
\end{aligned}
$$

$$
\begin{aligned}
R_{ab} &= -2\partial_{ab}\phi - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{cd}\partial_c\phi\partial_d\bar{g}_{ab} + \partial_c\bar{\Gamma}^c{}_{ab} + 20\partial_a\phi\partial_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi + 4\bar{\Gamma}^c{}_{ab}\partial_c\phi - 4\bar{g}_{bc}\bar{g}^{cd}\partial_a\phi\partial_d\phi - 2\bar{g}_{cd}\bar{g}^{ce}\bar{\Gamma}^d{}_{ab}\partial_e\phi \\
&\quad - 2\bar{g}_{ab}\bar{g}^{cd}\bar{\Gamma}^e{}_{ce}\partial_d\phi + \bar{\Gamma}^c{}_{ab}\bar{\Gamma}^d{}_{cd} + 2\bar{g}_{ac}\partial_d\phi\partial_b\bar{g}^{cd} + 2\bar{g}^{cd}\partial_c\phi\partial_b\bar{g}_{ad} - \partial_b\bar{\Gamma}^c{}_{ac} - 4\partial_a\phi\partial_b\phi\bar{g}^c{}_c + 2\bar{g}_{bc}\bar{g}^{de}\bar{\Gamma}^c{}_{ad}\partial_e\phi + 2\bar{g}_{ac}\bar{g}^{de}\bar{\Gamma}^c{}_{bd}\partial_e\phi - \bar{\Gamma}^c{}_{ad}\bar{\Gamma}^d{}_{bc} \qquad \texttt{(eq15.113)} \\
&= -2\partial_{ab}\phi - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{cd}\partial_c\phi\partial_d\bar{g}_{ab} + \partial_c\bar{\Gamma}^c{}_{ab} + 20\partial_a\phi\partial_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi + 4\bar{\Gamma}^c{}_{ab}\partial_c\phi - 4\bar{g}^d{}_b\partial_a\phi\partial_d\phi - 2\bar{g}_{cd}\bar{g}^{ce}\bar{\Gamma}^d{}_{ab}\partial_e\phi \\
&\quad - 2\bar{g}_{ab}\bar{g}^{cd}\bar{\Gamma}^e{}_{ce}\partial_d\phi + \bar{\Gamma}^c{}_{ab}\bar{\Gamma}^d{}_{cd} + 2\bar{g}_{ac}\partial_d\phi\partial_b\bar{g}^{cd} + 2\bar{g}^{cd}\partial_c\phi\partial_b\bar{g}_{ad} - \partial_b\bar{\Gamma}^c{}_{ac} - 4\partial_a\phi\partial_b\phi\bar{g}^c{}_c + 2\bar{g}_{bc}\bar{g}^{de}\bar{\Gamma}^c{}_{ad}\partial_e\phi + 2\bar{g}_{ac}\bar{g}^{de}\bar{\Gamma}^c{}_{bd}\partial_e\phi - \bar{\Gamma}^c{}_{ad}\bar{\Gamma}^d{}_{bc} \qquad \texttt{(eq15.114)} \\
&= -2\partial_{ab}\phi - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{cd}\partial_c\phi\partial_d\bar{g}_{ab} + \partial_c\bar{\Gamma}^c{}_{ab} + 8\partial_a\phi\partial_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi + 4\bar{\Gamma}^c{}_{ab}\partial_c\phi - 4\bar{g}^d{}_b\partial_a\phi\partial_d\phi - 2\bar{g}_{cd}\bar{g}^{ce}\bar{\Gamma}^d{}_{ab}\partial_e\phi \\
&\quad - 2\bar{g}_{ab}\bar{g}^{cd}\bar{\Gamma}^e{}_{ce}\partial_d\phi + \bar{\Gamma}^c{}_{ab}\bar{\Gamma}^d{}_{cd} + 2\bar{g}_{ac}\partial_d\phi\partial_b\bar{g}^{cd} + 2\bar{g}^{cd}\partial_c\phi\partial_b\bar{g}_{ad} - \partial_b\bar{\Gamma}^c{}_{ac} + 2\bar{g}_{bc}\bar{g}^{de}\bar{\Gamma}^c{}_{ad}\partial_e\phi + 2\bar{g}_{ac}\bar{g}^{de}\bar{\Gamma}^c{}_{bd}\partial_e\phi - \bar{\Gamma}^c{}_{ad}\bar{\Gamma}^d{}_{bc} \qquad \texttt{(eq15.115)} \\
&= -2\partial_{ab}\phi - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{cd}\partial_c\phi\partial_d\bar{g}_{ab} + \partial_c\bar{\Gamma}^c{}_{ab} + 4\partial_a\phi\partial_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi + 4\bar{\Gamma}^c{}_{ab}\partial_c\phi - 2\bar{g}_{cd}\bar{g}^{ce}\bar{\Gamma}^d{}_{ab}\partial_e\phi - 2\bar{g}_{ab}\bar{g}^{cd}\bar{\Gamma}^e{}_{ce}\partial_d\phi \\
&\quad + \bar{\Gamma}^c{}_{ab}\bar{\Gamma}^d{}_{cd} + 2\bar{g}_{ac}\partial_d\phi\partial_b\bar{g}^{cd} + 2\bar{g}^{cd}\partial_c\phi\partial_b\bar{g}_{ad} - \partial_b\bar{\Gamma}^c{}_{ac} + 2\bar{g}_{bc}\bar{g}^{de}\bar{\Gamma}^c{}_{ad}\partial_e\phi + 2\bar{g}_{ac}\bar{g}^{de}\bar{\Gamma}^c{}_{bd}\partial_e\phi - \bar{\Gamma}^c{}_{ad}\bar{\Gamma}^d{}_{bc} \qquad \texttt{(eq15.116)}
\end{aligned}
$$

The above doesn't look much like equation (15). So, what do we do? First note that `(eq15.116)` represents the full R_{ab}, that is, equation (14). To isolate the contributions from ϕ we can first set $\bar{\Gamma}$ and its derivatives to zero (which in turn requires setting $\partial_a\bar{g}_{bc} = 0$). The result is equation `(eq15.119)` below. Having set $\bar{\Gamma}$ to zero means that we can replace ∂ with $\bar{D}$ leading to equation `(eq15.121)`. But that is clearly a tensor equation and so by the usual arguments it must be true in all frames (not just this frame with $\bar{\Gamma} = 0$). It's a standard argument and I've probably overdone the discussion. Anyway, equation `(eq15.121)` is exactly equation (15) from the paper. Yeah.

$$
\begin{aligned}
R^\phi_{ab} &= -2\partial_{ab}\phi - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{cd}\partial_c\phi\partial_d\bar{g}_{ab} + 4\partial_a\phi\partial_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi + 2\bar{g}_{ac}\partial_d\phi\partial_b\bar{g}^{cd} + 2\bar{g}^{cd}\partial_c\phi\partial_b\bar{g}_{ad} \qquad \texttt{(eq15.117)} \\
&= -2\partial_{ab}\phi - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi + 4\partial_a\phi\partial_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi + 2\bar{g}_{ac}\partial_d\phi\partial_b\bar{g}^{cd} \qquad \texttt{(eq15.118)} \\
&= -2\partial_{ab}\phi - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi + 4\partial_a\phi\partial_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi \qquad \texttt{(eq15.119)} \\
&= -2\bar{D}_{ab}\phi - 2\bar{g}_{ab}\bar{g}^{cd}\bar{D}_{cd}\phi + 4\partial_a\phi\partial_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi \qquad \texttt{(eq15.120)} \\
&= -2\bar{D}_{ab}\phi - 2\bar{g}_{ab}\bar{g}^{cd}\bar{D}_{cd}\phi + 4\bar{D}_a\phi\bar{D}_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\bar{D}_c\phi\bar{D}_d\phi \qquad \texttt{(eq15.121)}
\end{aligned}
$$

```
# ------------------------------------------------------------------------
# Check against prd62.

foo := @(Rphi).                                          # cdb(eq15.lcb,foo)
bah  = cdblib.get('prd62.eq15.rhs','prd62.json')         # cdb(eq15.prd,bah)

diff := @(foo) - @(bah).

distribute     (diff)
diff = product_sort (diff)
rename_dummies (diff)
map_sympy      (diff, "simplify")
canonicalise   (diff)                                    # cdb(eq15.chk,diff)
```

$$\texttt{eq15.lcb} := -2\bar{D}_{ab}\phi - 2\bar{g}_{ab}\bar{g}^{cd}\bar{D}_{cd}\phi + 4\bar{D}_a\phi\bar{D}_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\bar{D}_c\phi\bar{D}_d\phi$$

$$\texttt{eq15.prd} := -2\bar{D}_{ab}\phi - 2\bar{g}_{ab}\bar{g}^{cd}\bar{D}_{cd}\phi + 4\bar{D}_a\phi\bar{D}_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\bar{D}_c\phi\bar{D}_d\phi$$

$$\texttt{eq15.chk} := 0$$