

PhysRevD.67.084023 equation (19)

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1  from shared import *
2  import cdblib
3
4  jsonfile = 'hamiltonian.json'
5  cdblib.create (jsonfile)
6
7  # -----
8  # Hamiltonian constraint
9
10 Ham := R + K_{a b} g^{a b} K_{c d} g^{c d} - K_{a b} K_{c d} g^{a c} g^{b d}. # cdb (Ham.101,Ham)
11
12 defK2ABarD := K_{i j} -> \exp(4\phi) ABar_{i j} + (1/3) g_{i j} trK.
13 defG2GBarD := g_{a b} -> \exp(4\phi) gBar_{a b}.
14 defG2GBarU := g^{a b} -> \exp(-4\phi) gBar^{a b}.
15
16 substitute      (Ham, defK2ABarD)      # cdb (Ham.102,Ham)
17 substitute      (Ham, defG2GBarD)      # cdb (Ham.103,Ham)
18 substitute      (Ham, defG2GBarU)      # cdb (Ham.104,Ham)
19 distribute      (Ham)                  # cdb (Ham.105,Ham)
20 Ham = product_sort (Ham)                # cdb (Ham.106,Ham)
21 rename_dummies  (Ham)                  # cdb (Ham.107,Ham)
22 canonicalise    (Ham)                  # cdb (Ham.108,Ham)
23 map_sympy       (Ham, "simplify")      # cdb (Ham.109,Ham)
24
25 foo := gBar_{a b} gBar^{a b} -> 3.
26 bah := gBar_{a c} gBar^{b c} -> gBar_{a}^{b}.
27
28 substitute (Ham, foo)                  # cdb (Ham.110,Ham)
29 substitute (Ham, bah)                  # cdb (Ham.111,Ham)
30 eliminate_kronecker (Ham)              # cdb (Ham.112,Ham)
31
32 foo := gBar_{a b} gBar^{a b} -> 3.
33 bah := gBar_{a}^{a} -> 3.
34 moo := ABar_{a b} gBar^{a b} -> 0.
35
36 substitute      (Ham, foo)              # cdb (Ham.113,Ham)

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37 substitute      (Ham, bah)          # cdb (Ham.114,Ham)
38 substitute      (Ham, moo)          # cdb (Ham.115,Ham)
39
40 foo := ABar_{c d} gBar^{c a} gBar^{d b} -> ABar^{a b}.
41
42 substitute      (Ham, foo)          # cdb (Ham.116,Ham)
43 rename_dummies  (Ham)              # cdb (Ham.117,Ham)
44
45 cdblib.put ('Ham',Ham,jsonfile)
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$$\mathcal{H} = R + K_{ab}g^{ab}K_{cd}g^{cd} - K_{ab}K_{cd}g^{ac}g^{bd} \quad (\text{Ham.101})$$

$$= R + \left(\exp(4\phi) \bar{A}_{ab} + \frac{1}{3}g_{ab}\text{tr}K \right) g^{ab} \left(\exp(4\phi) \bar{A}_{cd} + \frac{1}{3}g_{cd}\text{tr}K \right) g^{cd} - \left(\exp(4\phi) \bar{A}_{ab} + \frac{1}{3}g_{ab}\text{tr}K \right) \left(\exp(4\phi) \bar{A}_{cd} + \frac{1}{3}g_{cd}\text{tr}K \right) g^{ac}g^{bd} \quad (\text{Ham.102})$$

$$= R + \left(\exp(4\phi) \bar{A}_{ab} + \frac{1}{3}\exp(4\phi) \bar{g}_{ab}\text{tr}K \right) g^{ab} \left(\exp(4\phi) \bar{A}_{cd} + \frac{1}{3}\exp(4\phi) \bar{g}_{cd}\text{tr}K \right) g^{cd} \\ - \left(\exp(4\phi) \bar{A}_{ab} + \frac{1}{3}\exp(4\phi) \bar{g}_{ab}\text{tr}K \right) \left(\exp(4\phi) \bar{A}_{cd} + \frac{1}{3}\exp(4\phi) \bar{g}_{cd}\text{tr}K \right) g^{ac}g^{bd} \quad (\text{Ham.103})$$

$$= R + \left(\exp(4\phi) \bar{A}_{ab} + \frac{1}{3}\exp(4\phi) \bar{g}_{ab}\text{tr}K \right) \exp(-4\phi) \bar{g}^{ab} \left(\exp(4\phi) \bar{A}_{cd} + \frac{1}{3}\exp(4\phi) \bar{g}_{cd}\text{tr}K \right) \exp(-4\phi) \bar{g}^{cd} \\ - \left(\exp(4\phi) \bar{A}_{ab} + \frac{1}{3}\exp(4\phi) \bar{g}_{ab}\text{tr}K \right) \left(\exp(4\phi) \bar{A}_{cd} + \frac{1}{3}\exp(4\phi) \bar{g}_{cd}\text{tr}K \right) \exp(-4\phi) \bar{g}^{ac} \exp(-4\phi) \bar{g}^{bd} \quad (\text{Ham.104})$$

$$= R + \exp(4\phi) \bar{A}_{ab} \exp(-4\phi) \bar{g}^{ab} \exp(4\phi) \bar{A}_{cd} \exp(-4\phi) \bar{g}^{cd} + \frac{1}{3} \exp(4\phi) \bar{A}_{ab} \exp(-4\phi) \bar{g}^{ab} \exp(4\phi) \bar{g}_{cd}\text{tr}K \exp(-4\phi) \bar{g}^{cd} \\ + \frac{1}{3} \exp(4\phi) \bar{g}_{ab}\text{tr}K \exp(-4\phi) \bar{g}^{ab} \exp(4\phi) \bar{A}_{cd} \exp(-4\phi) \bar{g}^{cd} + \frac{1}{9} \exp(4\phi) \bar{g}_{ab}\text{tr}K \exp(-4\phi) \bar{g}^{ab} \exp(4\phi) \bar{g}_{cd}\text{tr}K \exp(-4\phi) \bar{g}^{cd} \\ - \exp(4\phi) \bar{A}_{ab} \exp(4\phi) \bar{A}_{cd} \exp(-4\phi) \bar{g}^{ac} \exp(-4\phi) \bar{g}^{bd} - \frac{1}{3} \exp(4\phi) \bar{A}_{ab} \exp(4\phi) \bar{g}_{cd}\text{tr}K \exp(-4\phi) \bar{g}^{ac} \exp(-4\phi) \bar{g}^{bd} \\ - \frac{1}{3} \exp(4\phi) \bar{g}_{ab}\text{tr}K \exp(4\phi) \bar{A}_{cd} \exp(-4\phi) \bar{g}^{ac} \exp(-4\phi) \bar{g}^{bd} - \frac{1}{9} \exp(4\phi) \bar{g}_{ab}\text{tr}K \exp(4\phi) \bar{g}_{cd}\text{tr}K \exp(-4\phi) \bar{g}^{ac} \exp(-4\phi) \bar{g}^{bd} \quad (\text{Ham.105})$$

$$= R + \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ab}\bar{g}^{cd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) + \frac{1}{3} \text{tr}K \bar{A}_{ab}\bar{g}_{cd}\bar{g}^{ab}\bar{g}^{cd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) \\ + \frac{1}{3} \text{tr}K \bar{A}_{ab}\bar{g}_{cd}\bar{g}^{cd}\bar{g}^{ab} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) + \frac{1}{9} \text{tr}K \text{tr}K \bar{g}_{ab}\bar{g}_{cd}\bar{g}^{ab}\bar{g}^{cd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) \\ - \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ac}\bar{g}^{bd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) - \frac{1}{3} \text{tr}K \bar{A}_{ab}\bar{g}_{cd}\bar{g}^{ac}\bar{g}^{bd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) \\ - \frac{1}{3} \text{tr}K \bar{A}_{ab}\bar{g}_{cd}\bar{g}^{ca}\bar{g}^{db} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) - \frac{1}{9} \text{tr}K \text{tr}K \bar{g}_{ab}\bar{g}_{cd}\bar{g}^{ac}\bar{g}^{bd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) \quad (\text{Ham.106})$$

$$\begin{aligned} \mathcal{H} = & R + \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ab}\bar{g}^{cd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) + \frac{1}{3} \text{tr} K \bar{A}_{ab}\bar{g}_{cd}\bar{g}^{ab}\bar{g}^{cd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) \\ & + \frac{1}{3} \text{tr} K \bar{A}_{ab}\bar{g}_{cd}\bar{g}^{cd}\bar{g}^{ab} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) + \frac{1}{9} \text{tr} K \text{tr} K \bar{g}_{ab}\bar{g}_{cd}\bar{g}^{ab}\bar{g}^{cd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) \\ & - \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ac}\bar{g}^{bd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) - \frac{1}{3} \text{tr} K \bar{A}_{ab}\bar{g}_{cd}\bar{g}^{ac}\bar{g}^{bd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) \\ & - \frac{1}{3} \text{tr} K \bar{A}_{ab}\bar{g}_{cd}\bar{g}^{ca}\bar{g}^{db} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) - \frac{1}{9} \text{tr} K \text{tr} K \bar{g}_{ab}\bar{g}_{cd}\bar{g}^{ac}\bar{g}^{bd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) \quad (\text{Ham.107}) \end{aligned}$$

$$\begin{aligned}
&= R + \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ab}\bar{g}^{cd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) + \frac{2}{3} \text{tr} K \bar{A}_{ab}\bar{g}_{cd}\bar{g}^{ab}\bar{g}^{cd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) \\
&\quad + \frac{1}{9} \text{tr} K \text{tr} K \bar{g}_{ab}\bar{g}_{cd}\bar{g}^{ab}\bar{g}^{cd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) - \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ac}\bar{g}^{bd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) \\
&\quad - \frac{2}{3} \text{tr} K \bar{A}_{ab}\bar{g}_{cd}\bar{g}^{ac}\bar{g}^{bd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) - \frac{1}{9} \text{tr} K \text{tr} K \bar{g}_{ab}\bar{g}_{cd}\bar{g}^{ac}\bar{g}^{bd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) \quad (\text{Ham.108})
\end{aligned}$$

$$= R + \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ab}\bar{g}^{cd} + \frac{2}{3}\text{tr}K\bar{A}_{ab}\bar{g}_{cd}\bar{g}^{ab}\bar{g}^{cd} + \frac{1}{9}\text{tr}K^2\bar{g}_{ab}\bar{g}_{cd}\bar{g}^{ab}\bar{g}^{cd} - \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ac}\bar{g}^{bd} - \frac{2}{3}\text{tr}K\bar{A}_{ab}\bar{g}_{cd}\bar{g}^{ac}\bar{g}^{bd} - \frac{1}{9}\text{tr}K^2\bar{g}_{ab}\bar{g}_{cd}\bar{g}^{ac}\bar{g}^{bd} \quad (\text{Ham.109})$$

$$= R + \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ab}\bar{g}^{cd} + 2\text{tr}K\bar{A}_{ab}\bar{g}^{ab} + \frac{1}{3}\text{tr}K^2\bar{g}_{cd}\bar{g}^{cd} - \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ac}\bar{g}^{bd} - \frac{2}{3}\text{tr}K\bar{A}_{ab}\bar{g}_{cd}\bar{g}^{ac}\bar{g}^{bd} - \frac{1}{9}\text{tr}K^2\bar{g}_{ab}\bar{g}_{cd}\bar{g}^{ac}\bar{g}^{bd} \quad (\text{Ham.110})$$

$$= R + \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ab}\bar{g}^{cd} + 2\text{tr}K\bar{A}_{ab}\bar{g}^{ab} + \frac{1}{3}\text{tr}K^2\bar{g}^c_c - \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ac}\bar{g}^{bd} - \frac{2}{3}\text{tr}K\bar{A}_{ab}\bar{g}_c^b\bar{g}^{ac} - \frac{1}{9}\text{tr}K^2\bar{g}_{ab}\bar{g}_c^b\bar{g}^{ac} \quad (\text{Ham.111})$$

$$= R + \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ab}\bar{g}^{cd} + 2\text{tr}K\bar{A}_{ab}\bar{g}^{ab} + \frac{1}{3}\text{tr}K^2\bar{g}^c_c - \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ac}\bar{g}^{bd} - \frac{2}{3}\text{tr}K\bar{A}_{ac}\bar{g}^{ac} - \frac{1}{9}\text{tr}K^2\bar{g}_{ac}\bar{g}^{ac} \quad (\text{Ham.112})$$

$$= R + \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ab}\bar{g}^{cd} + 2\text{tr}K\bar{A}_{ab}\bar{g}^{ab} + \frac{1}{3}\text{tr}K^2\bar{g}^c_c - \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ac}\bar{g}^{bd} - \frac{2}{3}\text{tr}K\bar{A}_{ac}\bar{g}^{ac} - \frac{1}{3}\text{tr}K^2 \quad (\text{Ham.113})$$

$$= R + \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ab}\bar{g}^{cd} + 2\text{tr}K\bar{A}_{ab}\bar{g}^{ab} + \frac{2}{3}\text{tr}K^2 - \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ac}\bar{g}^{bd} - \frac{2}{3}\text{tr}K\bar{A}_{ac}\bar{g}^{ac} \quad (\text{Ham.114})$$

$$= R + \frac{2}{3}\text{tr}K^2 - \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ac}\bar{g}^{bd} \quad (\text{Ham.115})$$

$$= R + \frac{2}{3} \text{tr} K^2 - \bar{A}^{cd} \bar{A}_{cd} \quad (\text{Ham.116})$$

$$= R + \frac{2}{3} \text{tr} K^2 - \bar{A}^{ab} \bar{A}_{ab} \quad (\text{Ham.117})$$

```

1  # -----
2  # Check against prd67.
3
4  foo := @(Ham).                                # cdb(prd67.eq19.lcb,foo)
5  bah  = cdblib.get('prd67.eq19.rhs','prd67.json') # cdb(prd67.eq19.prd,bah)
6
7  diff := @(foo) - @(bah).
8
9  distribute      (diff)
10 diff = product_sort (diff)
11 rename_dummies (diff)
12 map_sympy      (diff, "simplify")
13 canonicalise   (diff)                        # cdb(prd67.eq19.chk,diff)

```

$$\text{prd67.eq19.lcb} := R + \frac{2}{3}\text{tr}K^2 - \bar{A}^{ab}\bar{A}_{ab}$$

$$\text{prd67.eq19.prd} := R - \bar{A}_{ab}\bar{A}^{ab} + \frac{2}{3}\text{tr}K^2$$

$$\text{prd67.eq19.chk} := 0$$