PhysRevD.62.044034 equation (12)

```
from shared import *
import cdblib

jsonfile = 'eqtn12.json'
cdblib.create (jsonfile)

DgijDt = cdblib.get ('adm.DgijDt','adm.json')
DKijDt = cdblib.get ('adm.DKijDt','adm.json')

DphiDt = cdblib.get ('DphiDt','eqtn10.json')
DKDt = cdblib.get ('DKDt','eqtn11.json')

# ----------------------------------------------------------------------
ABar2A := ABar_{i j} -> \exp(-4\phi) A_{i j}.         # prd62 eqn 08
A2ABar := A_{i j} -> \exp(4\phi) ABar_{i j}.          # prd62 eqn 08

Aij    := A_{i j} -> K_{i j} - (1/3) g_{i j} trK.     # prd62 eqn 07
Kij    := K_{i j} -> A_{i j} + (1/3) g_{i j} trK.     # prd62 eqn 07

gginv := {g_{i a} g^{a j} -> g_{i}^{j},
          g_{i a} g^{j a} -> g_{i}^{j}}.

ABarUp := ABar_{i j} g^{j k} -> \exp(-4\phi) ABar_{i}^{k}.

ABardotABar := ABar_{i j} ABar^{i j} ->
               (K_{i j}-(1/3)g_{i j} trK) (K^{i j}-(1/3)g^{i j} trK).

trg := g_{i j} g^{i j} -> 3.

trK := {K_{i j} g^{i j} -> trK,
        K^{i j} g_{i j} -> trK}.

Ham := trK**2 -> K_{i j} K^{i j} - g^{i j} R_{i j}.

# ----------------------------------------------------------------------
# dABarij/dt
```

```
37
38  dotABarij := \partial_{t}{ABar_{i j}}.          # cdb (eq12.101,dotABarij)
39
40  substitute      (dotABarij, ABar2A)             # cdb (eq12.102,dotABarij)
41  product_rule    (dotABarij)                     # cdb (eq12.103,dotABarij)
42  map_sympy       (dotABarij, "simplify")         # cdb (eq12.104,dotABarij)
43  substitute      (dotABarij, DphiDt)             # cdb (eq12.105,dotABarij)
44  substitute      (dotABarij, Aij)                # cdb (eq12.106,dotABarij)
45  distribute      (dotABarij)                     # cdb (eq12.107,dotABarij)
46  substitute      (dotABarij, DKijDt)             # cdb (eq12.108,dotABarij)
47  product_rule    (dotABarij)                     # cdb (eq12.109,dotABarij)
48  distribute      (dotABarij)                     # cdb (eq12.110,dotABarij)
49  substitute      (dotABarij, DKDt)               # cdb (eq12.111,dotABarij)
50  substitute      (dotABarij, DgijDt)             # cdb (eq12.112,dotABarij)
51  distribute      (dotABarij)                     # cdb (eq12.113,dotABarij)
52  substitute      (dotABarij, Kij)                # cdb (eq12.114,dotABarij)
53  distribute      (dotABarij)                     # cdb (eq12.115,dotABarij)
54  substitute      (dotABarij, gginv)              # cdb (eq12.116,dotABarij)
55  eliminate_kronecker (dotABarij)                 # cdb (eq12.117,dotABarij)
56  substitute      (dotABarij, A2ABar)             # cdb (eq12.118,dotABarij)
57  canonicalise    (dotABarij)                     # cdb (eq12.119,dotABarij)
58  substitute      (dotABarij, ABardotABar)        # cdb (eq12.120,dotABarij)
59  distribute      (dotABarij)                     # cdb (eq12.121,dotABarij)
60  substitute      (dotABarij, trg)                # cdb (eq12.122,dotABarij)
61  substitute      (dotABarij, trK)                # cdb (eq12.123,dotABarij)
62  map_sympy       (dotABarij, "simplify")         # cdb (eq12.124,dotABarij)
63  substitute      (dotABarij, Ham)                # cdb (eq12.125,dotABarij)
64  distribute      (dotABarij)                     # cdb (eq12.126,dotABarij)
65  dotABarij = product_sort (dotABarij)            # cdb (eq12.127,dotABarij)
66  substitute      (dotABarij, ABarUp)             # cdb (eq12.128,dotABarij)
67  map_sympy       (dotABarij, "simplify")         # cdb (eq12.129,dotABarij)
68  factor_out      (dotABarij,$\exp(-4\phi)$)      # cdb (eq12.130,dotABarij)
69
70  DABarijDt := \partial_{t}{ABar_{ij}} -> @(dotABarij).
71
72  cdblib.put ('DABarijDt',DABarijDt,jsonfile)
```

$$\partial_t \bar{A}_{ij} = \partial_t \left(\exp\left(-4\phi\right) A_{ij} \right) \tag{eq12.102}$$

$$= \partial_t \left(\exp\left(-4\phi\right) \right) A_{ij} + \exp\left(-4\phi\right) \partial_t A_{ij} \tag{eq12.103}$$

$$= -4 \exp\left(-4\phi\right) \partial_t \phi A_{ij} + \exp\left(-4\phi\right) \partial_t A_{ij} \tag{eq12.104}$$

$$= \frac{2}{3} \exp\left(-4\phi\right) \mathrm{tr}KNA_{ij} + \exp\left(-4\phi\right) \partial_t A_{ij} \tag{eq12.105}$$

$$= \frac{2}{3} \exp\left(-4\phi\right) \mathrm{tr}KN \left(K_{ij} - \frac{1}{3} g_{ij} \mathrm{tr}K \right) + \exp\left(-4\phi\right) \partial_t \left(K_{ij} - \frac{1}{3} g_{ij} \mathrm{tr}K \right) \tag{eq12.106}$$

$$= \frac{2}{3} \exp\left(-4\phi\right) \mathrm{tr}KNK_{ij} - \frac{2}{9} \exp\left(-4\phi\right) \mathrm{tr}KNg_{ij}\mathrm{tr}K + \exp\left(-4\phi\right) \partial_t K_{ij} - \frac{1}{3} \exp\left(-4\phi\right) \partial_t \left(g_{ij}\mathrm{tr}K \right) \tag{eq12.107}$$

$$= \frac{2}{3} \exp\left(-4\phi\right) \mathrm{tr}KNK_{ij} - \frac{2}{9} \exp\left(-4\phi\right) \mathrm{tr}KNg_{ij}\mathrm{tr}K + \exp\left(-4\phi\right) \left(-D_{ij}N + N \left(R_{ij} + \mathrm{tr}KK_{ij} - 2K_{ic}K_{jd}g^{cd} \right) \right)$$
$$- \frac{1}{3} \exp\left(-4\phi\right) \partial_t \left(g_{ij}\mathrm{tr}K \right) \tag{eq12.108}$$

$$= \frac{2}{3} \exp\left(-4\phi\right) \mathrm{tr}KNK_{ij} - \frac{2}{9} \exp\left(-4\phi\right) \mathrm{tr}KNg_{ij}\mathrm{tr}K + \exp\left(-4\phi\right) \left(-D_{ij}N + N \left(R_{ij} + \mathrm{tr}KK_{ij} - 2K_{ic}K_{jd}g^{cd} \right) \right)$$
$$- \frac{1}{3} \exp\left(-4\phi\right) \left(\partial_t g_{ij}\mathrm{tr}K + g_{ij}\partial_t \mathrm{tr}K \right) \tag{eq12.109}$$

$$= \frac{2}{3} \exp\left(-4\phi\right) \mathrm{tr}KNK_{ij} - \frac{2}{9} \exp\left(-4\phi\right) \mathrm{tr}KNg_{ij}\mathrm{tr}K - \exp\left(-4\phi\right) D_{ij}N + \exp\left(-4\phi\right) NR_{ij} + \exp\left(-4\phi\right) N\mathrm{tr}KK_{ij}$$
$$- 2 \exp\left(-4\phi\right) NK_{ic}K_{jd}g^{cd} - \frac{1}{3} \exp\left(-4\phi\right) \partial_t g_{ij}\mathrm{tr}K - \frac{1}{3} \exp\left(-4\phi\right) g_{ij}\partial_t \mathrm{tr}K \tag{eq12.110}$$

$$= \frac{2}{3} \exp\left(-4\phi\right) \mathrm{tr}KNK_{ij} - \frac{2}{9} \exp\left(-4\phi\right) \mathrm{tr}KNg_{ij}\mathrm{tr}K - \exp\left(-4\phi\right) D_{ij}N + \exp\left(-4\phi\right) NR_{ij} + \exp\left(-4\phi\right) N\mathrm{tr}KK_{ij}$$
$$- 2 \exp\left(-4\phi\right) NK_{ic}K_{jd}g^{cd} - \frac{1}{3} \exp\left(-4\phi\right) \partial_t g_{ij}\mathrm{tr}K - \frac{1}{3} \exp\left(-4\phi\right) g_{ij} \left(-g^{ab}D_{ab}N + N\bar{A}_{ab}\bar{A}^{ab} + \frac{1}{3}\mathrm{tr}K^2 N \right) \tag{eq12.111}$$

$$= \frac{2}{3} \exp\left(-4\phi\right) \mathrm{tr}KNK_{ij} - \frac{2}{9} \exp\left(-4\phi\right) \mathrm{tr}KNg_{ij}\mathrm{tr}K - \exp\left(-4\phi\right) D_{ij}N + \exp\left(-4\phi\right) NR_{ij} + \exp\left(-4\phi\right) N\mathrm{tr}KK_{ij}$$
$$- 2 \exp\left(-4\phi\right) NK_{ic}K_{jd}g^{cd} + \frac{2}{3} \exp\left(-4\phi\right) NK_{ij}\mathrm{tr}K - \frac{1}{3} \exp\left(-4\phi\right) g_{ij} \left(-g^{ab}D_{ab}N + N\bar{A}_{ab}\bar{A}^{ab} + \frac{1}{3}\mathrm{tr}K^2 N \right) \tag{eq12.112}$$

$$\partial_t \bar{A}_{ij} = \frac{2}{3}\exp\left(-4\phi\right)\mathrm{tr}KNK_{ij} - \frac{2}{9}\exp\left(-4\phi\right)\mathrm{tr}KNg_{ij}\mathrm{tr}K - \exp\left(-4\phi\right)D_{ij}N + \exp\left(-4\phi\right)NR_{ij} + \exp\left(-4\phi\right)N\mathrm{tr}KK_{ij} - 2\exp\left(-4\phi\right)NK_{ic}K_{jd}g^{cd}$$
$$+ \frac{2}{3}\exp\left(-4\phi\right)NK_{ij}\mathrm{tr}K + \frac{1}{3}\exp\left(-4\phi\right)g_{ij}g^{ab}D_{ab}N - \frac{1}{3}\exp\left(-4\phi\right)g_{ij}N\bar{A}_{ab}\bar{A}^{ab} - \frac{1}{9}\exp\left(-4\phi\right)g_{ij}\mathrm{tr}K^2N \qquad \texttt{(eq12.113)}$$
$$= \frac{2}{3}\exp\left(-4\phi\right)\mathrm{tr}KN\left(A_{ij} + \frac{1}{3}g_{ij}\mathrm{tr}K\right) - \frac{2}{9}\exp\left(-4\phi\right)\mathrm{tr}KNg_{ij}\mathrm{tr}K - \exp\left(-4\phi\right)D_{ij}N + \exp\left(-4\phi\right)NR_{ij}$$
$$+ \exp\left(-4\phi\right)N\mathrm{tr}K\left(A_{ij} + \frac{1}{3}g_{ij}\mathrm{tr}K\right) - 2\exp\left(-4\phi\right)N\left(A_{ic} + \frac{1}{3}g_{ic}\mathrm{tr}K\right)\left(A_{jd} + \frac{1}{3}g_{jd}\mathrm{tr}K\right)g^{cd} + \frac{2}{3}\exp\left(-4\phi\right)N\left(A_{ij} + \frac{1}{3}g_{ij}\mathrm{tr}K\right)\mathrm{tr}K$$
$$+ \frac{1}{3}\exp\left(-4\phi\right)g_{ij}g^{ab}D_{ab}N - \frac{1}{3}\exp\left(-4\phi\right)g_{ij}N\bar{A}_{ab}\bar{A}^{ab} - \frac{1}{9}\exp\left(-4\phi\right)g_{ij}\mathrm{tr}K^2N \qquad \texttt{(eq12.114)}$$
$$= \frac{2}{3}\exp\left(-4\phi\right)\mathrm{tr}KNA_{ij} - \exp\left(-4\phi\right)D_{ij}N + \exp\left(-4\phi\right)NR_{ij} + \exp\left(-4\phi\right)N\mathrm{tr}KA_{ij} + \frac{1}{3}\exp\left(-4\phi\right)N\mathrm{tr}Kg_{ij}\mathrm{tr}K - 2\exp\left(-4\phi\right)NA_{ic}A_{jd}g^{cd}$$
$$- \frac{2}{3}\exp\left(-4\phi\right)NA_{ic}g_{jd}\mathrm{tr}Kg^{cd} - \frac{2}{3}\exp\left(-4\phi\right)Ng_{ic}\mathrm{tr}KA_{jd}g^{cd} - \frac{2}{9}\exp\left(-4\phi\right)Ng_{ic}\mathrm{tr}Kg_{jd}\mathrm{tr}Kg^{cd} + \frac{2}{3}\exp\left(-4\phi\right)NA_{ij}\mathrm{tr}K$$
$$+ \frac{2}{9}\exp\left(-4\phi\right)Ng_{ij}\mathrm{tr}K\mathrm{tr}K + \frac{1}{3}\exp\left(-4\phi\right)g_{ij}g^{ab}D_{ab}N - \frac{1}{3}\exp\left(-4\phi\right)g_{ij}N\bar{A}_{ab}\bar{A}^{ab} - \frac{1}{9}\exp\left(-4\phi\right)g_{ij}\mathrm{tr}K^2N \qquad \texttt{(eq12.115)}$$
$$= \frac{2}{3}\exp\left(-4\phi\right)\mathrm{tr}KNA_{ij} - \exp\left(-4\phi\right)D_{ij}N + \exp\left(-4\phi\right)NR_{ij} + \exp\left(-4\phi\right)N\mathrm{tr}KA_{ij} + \frac{1}{3}\exp\left(-4\phi\right)N\mathrm{tr}Kg_{ij}\mathrm{tr}K - 2\exp\left(-4\phi\right)NA_{ic}A_{jd}g^{cd}$$
$$- \frac{2}{3}\exp\left(-4\phi\right)NA_{ic}g_{j}{}^{c}\mathrm{tr}K - \frac{2}{3}\exp\left(-4\phi\right)Ng_{i}{}^{d}\mathrm{tr}KA_{jd} - \frac{2}{9}\exp\left(-4\phi\right)Ng_{i}{}^{d}\mathrm{tr}Kg_{jd}\mathrm{tr}K + \frac{2}{3}\exp\left(-4\phi\right)NA_{ij}\mathrm{tr}K$$
$$+ \frac{2}{9}\exp\left(-4\phi\right)Ng_{ij}\mathrm{tr}K\mathrm{tr}K + \frac{1}{3}\exp\left(-4\phi\right)g_{ij}g^{ab}D_{ab}N - \frac{1}{3}\exp\left(-4\phi\right)g_{ij}N\bar{A}_{ab}\bar{A}^{ab} - \frac{1}{9}\exp\left(-4\phi\right)g_{ij}\mathrm{tr}K^2N \qquad \texttt{(eq12.116)}$$
$$= \frac{2}{3}\exp\left(-4\phi\right)\mathrm{tr}KNA_{ij} - \exp\left(-4\phi\right)D_{ij}N + \exp\left(-4\phi\right)NR_{ij} + \exp\left(-4\phi\right)N\mathrm{tr}KA_{ij} + \frac{1}{3}\exp\left(-4\phi\right)N\mathrm{tr}Kg_{ij}\mathrm{tr}K - 2\exp\left(-4\phi\right)NA_{ic}A_{jd}g^{cd}$$
$$- \frac{2}{3}\exp\left(-4\phi\right)N\mathrm{tr}KA_{ji} - \frac{2}{9}\exp\left(-4\phi\right)N\mathrm{tr}Kg_{ji}\mathrm{tr}K + \frac{2}{9}\exp\left(-4\phi\right)Ng_{ij}\mathrm{tr}K\mathrm{tr}K + \frac{1}{3}\exp\left(-4\phi\right)g_{ij}g^{ab}D_{ab}N - \frac{1}{3}\exp\left(-4\phi\right)g_{ij}N\bar{A}_{ab}\bar{A}^{ab}$$
$$- \frac{1}{9}\exp\left(-4\phi\right)g_{ij}\mathrm{tr}K^2N \qquad \texttt{(eq12.117)}$$

$$\partial_t \bar{A}_{ij} = \frac{2}{3}\exp\left(-4\phi\right)\mathrm{tr}KN\exp\left(4\phi\right)\bar{A}_{ij} - \exp\left(-4\phi\right)D_{ij}N + \exp\left(-4\phi\right)NR_{ij} + \exp\left(-4\phi\right)N\mathrm{tr}K\exp\left(4\phi\right)\bar{A}_{ij} + \frac{1}{3}\exp\left(-4\phi\right)N\mathrm{tr}Kg_{ij}\mathrm{tr}K$$
$$- 2\exp\left(-4\phi\right)N\exp\left(4\phi\right)\bar{A}_{ic}\exp\left(4\phi\right)\bar{A}_{jd}g^{cd} - \frac{2}{3}\exp\left(-4\phi\right)N\mathrm{tr}K\exp\left(4\phi\right)\bar{A}_{ji} - \frac{2}{9}\exp\left(-4\phi\right)N\mathrm{tr}Kg_{ji}\mathrm{tr}K + \frac{2}{9}\exp\left(-4\phi\right)Ng_{ij}\mathrm{tr}K\mathrm{tr}K$$
$$+ \frac{1}{3}\exp\left(-4\phi\right)g_{ij}g^{ab}D_{ab}N - \frac{1}{3}\exp\left(-4\phi\right)g_{ij}N\bar{A}_{ab}\bar{A}^{ab} - \frac{1}{9}\exp\left(-4\phi\right)g_{ij}\mathrm{tr}K^2N \qquad \texttt{(eq12.118)}$$
$$= \frac{2}{3}\exp\left(-4\phi\right)\mathrm{tr}KN\exp\left(4\phi\right)\bar{A}_{ij} - \exp\left(-4\phi\right)D_{ij}N + \exp\left(-4\phi\right)NR_{ij} + \frac{1}{3}\exp\left(-4\phi\right)N\mathrm{tr}K\exp\left(4\phi\right)\bar{A}_{ij} + \frac{1}{9}\exp\left(-4\phi\right)N\mathrm{tr}Kg_{ij}\mathrm{tr}K$$
$$- 2\exp\left(-4\phi\right)N\exp\left(4\phi\right)\bar{A}_{ic}\exp\left(4\phi\right)\bar{A}_{jd}g^{cd} + \frac{2}{9}\exp\left(-4\phi\right)Ng_{ij}\mathrm{tr}K\mathrm{tr}K + \frac{1}{3}\exp\left(-4\phi\right)g_{ij}g^{ab}D_{ab}N - \frac{1}{3}\exp\left(-4\phi\right)g_{ij}N\bar{A}_{ab}\bar{A}^{ab}$$
$$- \frac{1}{9}\exp\left(-4\phi\right)g_{ij}\mathrm{tr}K^2N \qquad \texttt{(eq12.119)}$$
$$= \frac{2}{3}\exp\left(-4\phi\right)\mathrm{tr}KN\exp\left(4\phi\right)\bar{A}_{ij} - \exp\left(-4\phi\right)D_{ij}N + \exp\left(-4\phi\right)NR_{ij} + \frac{1}{3}\exp\left(-4\phi\right)N\mathrm{tr}K\exp\left(4\phi\right)\bar{A}_{ij} + \frac{1}{9}\exp\left(-4\phi\right)N\mathrm{tr}Kg_{ij}\mathrm{tr}K$$
$$- 2\exp\left(-4\phi\right)N\exp\left(4\phi\right)\bar{A}_{ic}\exp\left(4\phi\right)\bar{A}_{jd}g^{cd} + \frac{2}{9}\exp\left(-4\phi\right)Ng_{ij}\mathrm{tr}K\mathrm{tr}K + \frac{1}{3}\exp\left(-4\phi\right)g_{ij}g^{ab}D_{ab}N$$
$$- \frac{1}{3}\exp\left(-4\phi\right)g_{ij}N\left(K_{ab} - \frac{1}{3}g_{ab}\mathrm{tr}K\right)\left(K^{ab} - \frac{1}{3}g^{ab}\mathrm{tr}K\right) - \frac{1}{9}\exp\left(-4\phi\right)g_{ij}\mathrm{tr}K^2N \qquad \texttt{(eq12.120)}$$
$$= \frac{2}{3}\exp\left(-4\phi\right)\mathrm{tr}KN\exp\left(4\phi\right)\bar{A}_{ij} - \exp\left(-4\phi\right)D_{ij}N + \exp\left(-4\phi\right)NR_{ij} + \frac{1}{3}\exp\left(-4\phi\right)N\mathrm{tr}K\exp\left(4\phi\right)\bar{A}_{ij} + \frac{1}{9}\exp\left(-4\phi\right)N\mathrm{tr}Kg_{ij}\mathrm{tr}K$$
$$- 2\exp\left(-4\phi\right)N\exp\left(4\phi\right)\bar{A}_{ic}\exp\left(4\phi\right)\bar{A}_{jd}g^{cd} + \frac{2}{9}\exp\left(-4\phi\right)Ng_{ij}\mathrm{tr}K\mathrm{tr}K + \frac{1}{3}\exp\left(-4\phi\right)g_{ij}g^{ab}D_{ab}N - \frac{1}{3}\exp\left(-4\phi\right)g_{ij}NK_{ab}K^{ab}$$
$$+ \frac{1}{9}\exp\left(-4\phi\right)g_{ij}NK_{ab}g^{ab}\mathrm{tr}K + \frac{1}{9}\exp\left(-4\phi\right)g_{ij}Ng_{ab}\mathrm{tr}KK^{ab} - \frac{1}{27}\exp\left(-4\phi\right)g_{ij}Ng_{ab}\mathrm{tr}Kg^{ab}\mathrm{tr}K - \frac{1}{9}\exp\left(-4\phi\right)g_{ij}\mathrm{tr}K^2N \qquad \texttt{(eq12.121)}$$

$$\partial_t \bar{A}_{ij} = \frac{2}{3}\exp\left(-4\phi\right)\mathrm{tr}KN\exp\left(4\phi\right)\bar{A}_{ij} - \exp\left(-4\phi\right)D_{ij}N + \exp\left(-4\phi\right)NR_{ij} + \frac{1}{3}\exp\left(-4\phi\right)N\mathrm{tr}K\exp\left(4\phi\right)\bar{A}_{ij} + \frac{1}{9}\exp\left(-4\phi\right)N\mathrm{tr}Kg_{ij}\mathrm{tr}K$$
$$- 2\exp\left(-4\phi\right)N\exp\left(4\phi\right)\bar{A}_{ic}\exp\left(4\phi\right)\bar{A}_{jd}g^{cd} + \frac{2}{9}\exp\left(-4\phi\right)Ng_{ij}\mathrm{tr}K\mathrm{tr}K + \frac{1}{3}\exp\left(-4\phi\right)g_{ij}g^{ab}D_{ab}N - \frac{1}{3}\exp\left(-4\phi\right)g_{ij}NK_{ab}K^{ab}$$
$$+ \frac{1}{9}\exp\left(-4\phi\right)g_{ij}NK_{ab}g^{ab}\mathrm{tr}K + \frac{1}{9}\exp\left(-4\phi\right)g_{ij}Ng_{ab}\mathrm{tr}KK^{ab} - \frac{1}{9}\exp\left(-4\phi\right)g_{ij}N\mathrm{tr}K\mathrm{tr}K - \frac{1}{9}\exp\left(-4\phi\right)g_{ij}\mathrm{tr}K^2N \quad \texttt{(eq12.122)}$$
$$= \frac{2}{3}\exp\left(-4\phi\right)\mathrm{tr}KN\exp\left(4\phi\right)\bar{A}_{ij} - \exp\left(-4\phi\right)D_{ij}N + \exp\left(-4\phi\right)NR_{ij} + \frac{1}{3}\exp\left(-4\phi\right)N\mathrm{tr}K\exp\left(4\phi\right)\bar{A}_{ij} + \frac{1}{9}\exp\left(-4\phi\right)N\mathrm{tr}Kg_{ij}\mathrm{tr}K$$
$$- 2\exp\left(-4\phi\right)N\exp\left(4\phi\right)\bar{A}_{ic}\exp\left(4\phi\right)\bar{A}_{jd}g^{cd} + \frac{2}{9}\exp\left(-4\phi\right)Ng_{ij}\mathrm{tr}K\mathrm{tr}K + \frac{1}{3}\exp\left(-4\phi\right)g_{ij}g^{ab}D_{ab}N - \frac{1}{3}\exp\left(-4\phi\right)g_{ij}NK_{ab}K^{ab}$$
$$+ \frac{1}{9}\exp\left(-4\phi\right)g_{ij}N\mathrm{tr}K\mathrm{tr}K - \frac{1}{9}\exp\left(-4\phi\right)g_{ij}\mathrm{tr}K^2N \quad \texttt{(eq12.123)}$$
$$= \mathrm{tr}KN\bar{A}_{ij} - \exp\left(-4\phi\right)D_{ij}N + N\exp\left(-4\phi\right)R_{ij} + \frac{1}{3}\mathrm{tr}K^2N\exp\left(-4\phi\right)g_{ij} - 2N\exp\left(4\phi\right)\bar{A}_{ic}\bar{A}_{jd}g^{cd} + \frac{1}{3}\exp\left(-4\phi\right)g_{ij}g^{ab}D_{ab}N$$
$$- \frac{1}{3}N\exp\left(-4\phi\right)g_{ij}K_{ab}K^{ab} \quad \texttt{(eq12.124)}$$
$$= \mathrm{tr}KN\bar{A}_{ij} - \exp\left(-4\phi\right)D_{ij}N + N\exp\left(-4\phi\right)R_{ij} + \frac{1}{3}\left(K_{ab}K^{ab} - g^{ab}R_{ab}\right)N\exp\left(-4\phi\right)g_{ij} - 2N\exp\left(4\phi\right)\bar{A}_{ic}\bar{A}_{jd}g^{cd}$$
$$+ \frac{1}{3}\exp\left(-4\phi\right)g_{ij}g^{ab}D_{ab}N - \frac{1}{3}N\exp\left(-4\phi\right)g_{ij}K_{ab}K^{ab} \quad \texttt{(eq12.125)}$$
$$= \mathrm{tr}KN\bar{A}_{ij} - \exp\left(-4\phi\right)D_{ij}N + N\exp\left(-4\phi\right)R_{ij} + \frac{1}{3}K_{ab}K^{ab}N\exp\left(-4\phi\right)g_{ij} - \frac{1}{3}g^{ab}R_{ab}N\exp\left(-4\phi\right)g_{ij} - 2N\exp\left(4\phi\right)\bar{A}_{ic}\bar{A}_{jd}g^{cd}$$
$$+ \frac{1}{3}\exp\left(-4\phi\right)g_{ij}g^{ab}D_{ab}N - \frac{1}{3}N\exp\left(-4\phi\right)g_{ij}K_{ab}K^{ab} \quad \texttt{(eq12.126)}$$
$$= N\mathrm{tr}K\bar{A}_{ij} - D_{ij}N\exp\left(-4\phi\right) + NR_{ij}\exp\left(-4\phi\right) - \frac{1}{3}Ng_{ij}g^{ab}R_{ab}\exp\left(-4\phi\right) - 2N\bar{A}_{ia}\bar{A}_{jb}g^{ab}\exp\left(4\phi\right) + \frac{1}{3}g_{ij}g^{ab}D_{ab}N\exp\left(-4\phi\right) \quad \texttt{(eq12.127)}$$

$$\partial_t \bar{A}_{ij} = N \mathrm{tr} K \bar{A}_{ij} - D_{ij} N \exp(-4\phi) + N R_{ij} \exp(-4\phi) - \frac{1}{3} N g_{ij} g^{ab} R_{ab} \exp(-4\phi) - 2N \exp(-4\phi) \bar{A}_i{}^b \bar{A}_{jb} \exp(4\phi)$$
$$+ \frac{1}{3} g_{ij} g^{ab} D_{ab} N \exp(-4\phi) \qquad \texttt{(eq12.128)}$$
$$= \mathrm{tr} K N \bar{A}_{ij} - D_{ij} N \exp(-4\phi) + N \exp(-4\phi) R_{ij} - \frac{1}{3} N \exp(-4\phi) g_{ij} g^{ab} R_{ab} - 2N \bar{A}_i{}^b \bar{A}_{jb} + \frac{1}{3} g_{ij} g^{ab} D_{ab} N \exp(-4\phi) \qquad \texttt{(eq12.129)}$$
$$= \mathrm{tr} K N \bar{A}_{ij} - 2N \bar{A}_i{}^b \bar{A}_{jb} + \exp(-4\phi) \left(-D_{ij} N + N R_{ij} - \frac{1}{3} N g_{ij} g^{ab} R_{ab} + \frac{1}{3} g_{ij} g^{ab} D_{ab} N \right) \qquad \texttt{(eq12.130)}$$

```
# ------------------------------------------------------------------------
# Check against prd62.

foo := @(dotABarij).                                  # cdb(eq12.lcb,foo)
bah  = cdblib.get('prd62.eq12.rhs','prd62.json')      # cdb(eq12.prd,bah)

diff := @(foo) - @(bah).

foo := ABar_{a}^{b} -> gBar^{b c} ABar_{a c}.
bah := ABar^{a}_{b} -> gBar^{a c} ABar_{c b}.

substitute     (diff, foo)
substitute     (diff, bah)
distribute     (diff)
diff = product_sort (diff)
rename_dummies (diff)
map_sympy      (diff, "simplify")
canonicalise   (diff)                                 # cdb(eq12.chk,diff)
```

$$\texttt{eq12.lcb} := \mathrm{tr}K N\bar{A}_{ij} - 2N\bar{A}_i{}^b\bar{A}_{jb} + \exp\left(-4\phi\right)\left(-D_{ij}N + NR_{ij} - \frac{1}{3}Ng_{ij}g^{ab}R_{ab} + \frac{1}{3}g_{ij}g^{ab}D_{ab}N\right)$$

$$\texttt{eq12.prd} := N\left(\mathrm{tr}K\bar{A}_{ij} - 2\bar{A}_{ia}\bar{A}^a{}_j\right) + \exp\left(-4\phi\right)\left(NR_{ij} - D_{ij}N - \frac{1}{3}g_{ij}\left(NR_{ab} - D_{ab}N\right)g^{ab}\right)$$

$$\texttt{eq12.chk} := 0$$