

## PhysRevD.67.084023 equation (20)

```
1  from shared import *
2  import cdblib
3
4  jsonfile = 'momentum.json'
5  cdblib.create (jsonfile)
6
7  defG2GBar = cdblib.get ('defG2GBar','gamma.json')
8
9  # -----
10 # Momentum constraint pt.1
11
12 Mom := D_{j}{K^{i j} - g^{i j} trK}. # cdb(Mom.101,Mom)
13
14 defDgD := D_{a}{g_{b c}} -> 0.
15 defDgU := D_{a}{g^{b c}} -> 0.
16
17 defDtrK := D_{a}{trK} -> \partial_{a}{trK}.
18 defDexp := D_{a}{\exp(-4\phi)} -> -4\exp(-4\phi) \partial_{a}{\phi}.
19
20 distribute (Mom) # cdb(Mom.102,Mom)
21 product_rule (Mom) # cdb(Mom.103,Mom)
22 substitute (Mom, defDgU) # cdb(Mom.104,Mom)
23
24 defK2ABarU := K^{i j} -> \exp(-4\phi) ABar^{i j} + (1/3) g^{i j} trK.
25
26 substitute (Mom, defK2ABarU) # cdb(Mom.105,Mom)
27 distribute (Mom) # cdb(Mom.106,Mom)
28 product_rule (Mom) # cdb(Mom.107,Mom)
29 substitute (Mom, defDtrK) # cdb(Mom.108,Mom)
30 substitute (Mom, defDgU) # cdb(Mom.109,Mom)
31 substitute (Mom, defDexp) # cdb(Mom.110,Mom)
```

$$\mathcal{D}^j = D_j (K^{ij} - g^{ij} \text{tr} K) \quad (\text{Mom. 101})$$

$$= D_j K^{ij} - D_j (g^{ij} \text{tr} K) \quad (\text{Mom. 102})$$

$$= D_j K^{ij} - D_j g^{ij} \text{tr} K - g^{ij} D_j \text{tr} K \quad (\text{Mom. 103})$$

$$= D_j K^{ij} - g^{ij} D_j \text{tr} K \quad (\text{Mom. 104})$$

$$= D_j \left( \exp(-4\phi) \bar{A}^{ij} + \frac{1}{3} g^{ij} \text{tr} K \right) - g^{ij} D_j \text{tr} K \quad (\text{Mom. 105})$$

$$= D_j (\exp(-4\phi) \bar{A}^{ij}) + \frac{1}{3} D_j (g^{ij} \text{tr} K) - g^{ij} D_j \text{tr} K \quad (\text{Mom. 106})$$

$$= D_j (\exp(-4\phi)) \bar{A}^{ij} + \exp(-4\phi) D_j \bar{A}^{ij} + \frac{1}{3} D_j g^{ij} \text{tr} K - \frac{2}{3} g^{ij} D_j \text{tr} K \quad (\text{Mom. 107})$$

$$= D_j (\exp(-4\phi)) \bar{A}^{ij} + \exp(-4\phi) D_j \bar{A}^{ij} + \frac{1}{3} D_j g^{ij} \text{tr} K - \frac{2}{3} g^{ij} \partial_j \text{tr} K \quad (\text{Mom. 108})$$

$$= D_j (\exp(-4\phi)) \bar{A}^{ij} + \exp(-4\phi) D_j \bar{A}^{ij} - \frac{2}{3} g^{ij} \partial_j \text{tr} K \quad (\text{Mom. 109})$$

$$= -4 \exp(-4\phi) \partial_j \phi \bar{A}^{ij} + \exp(-4\phi) D_j \bar{A}^{ij} - \frac{2}{3} g^{ij} \partial_j \text{tr} K \quad (\text{Mom. 110})$$

```

1  # -----
2  # Momentum constraint pt.2
3
4  confMom := \exp(4\phi) @ (Mom).
5
6  defG2GBarU := g^{i j} -> \exp(-4\phi) gBar^{i j}.
7
8  distribute      (confMom)                # cdb(confMom.101,confMom)
9  substitute      (confMom, defG2GBarU)     # cdb(confMom.102,confMom)
10 map_sympy       (confMom, "simplify")     # cdb(confMom.103,confMom)
11
12 defDAabU := D_{a}{ABar^{b c}} -> \partial_{a}{ABar^{b c}}
13           + \Gamma^{b}_{i a} ABar^{i c}
14           + \Gamma^{c}_{i a} ABar^{b i}.
15
16 substitute      (confMom, defDAabU)       # cdb(confMom.104,confMom)
17 substitute      (confMom, defG2GBar)      # cdb(confMom.105,confMom)
18 distribute      (confMom)                # cdb(confMom.106,confMom)
19 confMom = product_sort (confMom)          # cdb(confMom.107,confMom)
20 rename_dummies  (confMom)                # cdb(confMom.108,confMom)
21 canonicalise    (confMom)                # cdb(confMom.109,confMom)
22 substitute      (confMom, $gBar^{i}_{i} -> 3$) # cdb(confMom.110,confMom)
23 substitute      (confMom, $gBar_{i j} ABar^{i j} -> 0$) # cdb(confMom.111,confMom)
24 substitute      (confMom, $gBar_{a i} gBar^{i b} -> gBar_{a}^{b}$) # cdb(confMom.112,confMom)
25 substitute      (confMom, $GammaBar^{b}_{a b} -> 0$) # cdb(confMom.113,confMom) # follows from det gBar = 1
26 eliminate_kronecker (confMom)           # cdb(confMom.114,confMom)
27 rename_dummies  (confMom)
28 canonicalise    (confMom)                # cdb(confMom.115,confMom)
29
30 cdblib.put ('confMom',confMom,jsonfile)

```

$$\exp(4\phi)\mathcal{D}^j = -4\exp(4\phi)\exp(-4\phi)\partial_j\phi\bar{A}^{ij} + \exp(4\phi)\exp(-4\phi)D_j\bar{A}^{ij} - \frac{2}{3}\exp(4\phi)g^{ij}\partial_j\text{tr}K \quad (\text{confMom.101})$$

$$= -4\exp(4\phi)\exp(-4\phi)\partial_j\phi\bar{A}^{ij} + \exp(4\phi)\exp(-4\phi)D_j\bar{A}^{ij} - \frac{2}{3}\exp(4\phi)\exp(-4\phi)\bar{g}^{ij}\partial_j\text{tr}K \quad (\text{confMom.102})$$

$$= -4\partial_j\phi\bar{A}^{ij} + D_j\bar{A}^{ij} - \frac{2}{3}\bar{g}^{ij}\partial_j\text{tr}K \quad (\text{confMom.103})$$

$$= -4\partial_j\phi\bar{A}^{ij} + \partial_j\bar{A}^{ij} + \Gamma^i_{aj}\bar{A}^{aj} + \Gamma^j_{aj}\bar{A}^{ia} - \frac{2}{3}\bar{g}^{ij}\partial_j\text{tr}K \quad (\text{confMom.104})$$

$$= -4\partial_j\phi\bar{A}^{ij} + \partial_j\bar{A}^{ij} + (2\bar{g}^i_j\partial_a\phi + 2\bar{g}^i_a\partial_j\phi - 2\bar{g}^{ie}\partial_e\phi\bar{g}_{aj} + \bar{\Gamma}^i_{aj})\bar{A}^{aj} + (2\bar{g}^j_j\partial_a\phi + 2\bar{g}^j_a\partial_j\phi - 2\bar{g}^{je}\partial_e\phi\bar{g}_{aj} + \bar{\Gamma}^j_{aj})\bar{A}^{ia} - \frac{2}{3}\bar{g}^{ij}\partial_j\text{tr}K \quad (\text{confMom.105})$$

$$= -4\partial_j\phi\bar{A}^{ij} + \partial_j\bar{A}^{ij} + 2\bar{g}^i_j\partial_a\phi\bar{A}^{aj} + 2\bar{g}^i_a\partial_j\phi\bar{A}^{aj} - 2\bar{g}^{ie}\partial_e\phi\bar{g}_{aj}\bar{A}^{aj} + \bar{\Gamma}^i_{aj}\bar{A}^{aj} + 2\bar{g}^j_j\partial_a\phi\bar{A}^{ia} + 2\bar{g}^j_a\partial_j\phi\bar{A}^{ia} - 2\bar{g}^{je}\partial_e\phi\bar{g}_{aj}\bar{A}^{ia} + \bar{\Gamma}^j_{aj}\bar{A}^{ia} - \frac{2}{3}\bar{g}^{ij}\partial_j\text{tr}K \quad (\text{confMom.106})$$

$$= -4\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + 2\bar{A}^{ab}\partial_a\phi\bar{g}^i_b + 2\bar{A}^{ab}\partial_b\phi\bar{g}^i_a - 2\bar{A}^{ab}\bar{g}_{ab}\bar{g}^{ic}\partial_c\phi + \bar{A}^{ab}\bar{\Gamma}^i_{ab} + 2\bar{A}^{ia}\partial_a\phi\bar{g}^b_b + 2\bar{A}^{ia}\partial_b\phi\bar{g}^b_a - 2\bar{A}^{ia}\bar{g}_{ab}\bar{g}^{bc}\partial_c\phi + \bar{A}^{ia}\bar{\Gamma}^b_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\text{tr}K \quad (\text{confMom.107})$$

$$= -4\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + 2\bar{A}^{ab}\partial_a\phi\bar{g}^i_b + 2\bar{A}^{ab}\partial_b\phi\bar{g}^i_a - 2\bar{A}^{ab}\bar{g}_{ab}\bar{g}^{ic}\partial_c\phi + \bar{A}^{ab}\bar{\Gamma}^i_{ab} + 2\bar{A}^{ia}\partial_a\phi\bar{g}^b_b + 2\bar{A}^{ia}\partial_b\phi\bar{g}^b_a - 2\bar{A}^{ia}\bar{g}_{ac}\bar{g}^{cb}\partial_b\phi + \bar{A}^{ia}\bar{\Gamma}^b_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\text{tr}K \quad (\text{confMom.108})$$

$$= -4\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + 4\bar{A}^{ab}\partial_a\phi\bar{g}^i_b - 2\bar{A}^{ab}\bar{g}_{ab}\bar{g}^{ic}\partial_c\phi + \bar{A}^{ab}\bar{\Gamma}^i_{ab} + 2\bar{A}^{ia}\partial_a\phi\bar{g}^b_b + 2\bar{A}^{ia}\partial_b\phi\bar{g}^b_a - 2\bar{A}^{ia}\bar{g}_{ab}\bar{g}^{bc}\partial_c\phi + \bar{A}^{ia}\bar{\Gamma}^b_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\text{tr}K \quad (\text{confMom.109})$$

$$= 2\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + 4\bar{A}^{ab}\partial_a\phi\bar{g}^i_b - 2\bar{A}^{ab}\bar{g}_{ab}\bar{g}^{ic}\partial_c\phi + \bar{A}^{ab}\bar{\Gamma}^i_{ab} + 2\bar{A}^{ia}\partial_b\phi\bar{g}^b_a - 2\bar{A}^{ia}\bar{g}_{ab}\bar{g}^{bc}\partial_c\phi + \bar{A}^{ia}\bar{\Gamma}^b_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\text{tr}K \quad (\text{confMom.110})$$

$$= 2\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + 4\bar{A}^{ab}\partial_a\phi\bar{g}^i_b + \bar{A}^{ab}\bar{\Gamma}^i_{ab} + 2\bar{A}^{ia}\partial_b\phi\bar{g}^b_a - 2\bar{A}^{ia}\bar{g}_{ab}\bar{g}^{bc}\partial_c\phi + \bar{A}^{ia}\bar{\Gamma}^b_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\text{tr}K \quad (\text{confMom.111})$$

$$\exp(4\phi)\mathcal{D}^j = 2\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + 4\bar{A}^{ab}\partial_a\phi\bar{g}^i{}_b + \bar{A}^{ab}\bar{\Gamma}^i{}_{ab} + 2\bar{A}^{ia}\partial_b\phi\bar{g}_a{}^b - 2\bar{A}^{ia}\bar{g}_a{}^c\partial_c\phi + \bar{A}^{ia}\bar{\Gamma}^b{}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\text{tr}K \quad (\text{confMom.112})$$

$$= 2\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + 4\bar{A}^{ab}\partial_a\phi\bar{g}^i{}_b + \bar{A}^{ab}\bar{\Gamma}^i{}_{ab} + 2\bar{A}^{ia}\partial_b\phi\bar{g}_a{}^b - 2\bar{A}^{ia}\bar{g}_a{}^c\partial_c\phi - \frac{2}{3}\bar{g}^{ia}\partial_a\text{tr}K \quad (\text{confMom.113})$$

$$= 2\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + 4\bar{A}^{ai}\partial_a\phi + \bar{A}^{ab}\bar{\Gamma}^i{}_{ab} + 2\bar{A}^{ib}\partial_b\phi - 2\bar{A}^{ic}\partial_c\phi - \frac{2}{3}\bar{g}^{ia}\partial_a\text{tr}K \quad (\text{confMom.114})$$

$$= 6\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + \bar{A}^{ab}\bar{\Gamma}^i{}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\text{tr}K \quad (\text{confMom.115})$$

```

1  tmpA := @(confMom).                                # cdb(confMom.201,tmpA)
2  tmpB := @(confMom).
3
4  X^{b c}_{a}::Weight(label=numX).
5
6  Xbca := \partial_{a}{A\bar{b c}}.                    # cdb(confMom.202,Xbca)
7
8  foo := \partial_{a}{A\bar{b c}} -> X^{b c}_{a}.
9  bah := X^{b c}_{a} -> \partial_{a}{A\bar{b c}}.
10
11 substitute (tmpA, foo)                               # cdb(confMom.203,tmpA)
12 substitute (tmpB, foo)                               # cdb(confMom.204,tmpB)
13 drop_weight (tmpA, $numX=1$)                         # cdb(confMom.205,tmpA)
14 keep_weight (tmpB, $numX=1$)                         # cdb(confMom.206,tmpB)
15 substitute (tmpB, bah)                               # cdb(confMom.207,tmpB)
16
17 tmpC := - @(tmpA).                                   # cdb(confMom.208,tmpC)
18
19 defMomSub := @(tmpB) -> @(tmpC).                     # cdb(confMom.209,defMomSub)
20
21 cdblib.put ('defMomSub',defMomSub,jsonfile)

```

$$0 = 6\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + \bar{A}^{ab}\bar{\Gamma}^i{}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \quad (\text{confMom.201})$$

$$0 = 6\bar{A}^{ia}\partial_a\phi + X^{ia}{}_a + \bar{A}^{ab}\bar{\Gamma}^i{}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \quad (\text{confMom.203})$$

$$\partial_a\bar{A}^{bc} = X^{ia}{}_a \quad (\text{confMom.206})$$

$$= -6\bar{A}^{ia}\partial_a\phi - \bar{A}^{ab}\bar{\Gamma}^i{}_{ab} + \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \quad (\text{confMom.208})$$

$$\partial_a\bar{A}^{ia} \rightarrow -6\bar{A}^{ia}\partial_a\phi - \bar{A}^{ab}\bar{\Gamma}^i{}_{ab} + \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \quad (\text{confMom.209})$$

```

1  # -----
2  # Check against prd67.
3
4  foo := @(confMom).                # cdb(prd67.eq20.lcb,foo)
5  bah  = cdblib.get('prd67.eq20.rhs','prd67.json')  # cdb(prd67.eq20.prd,bah)
6
7  diff := @(foo) - @(bah).
8
9  distribute      (diff)
10 diff = product_sort (diff)
11 rename_dummies (diff)
12 map_sympy      (diff, "simplify")
13 canonicalise   (diff)                # cdb(prd67.eq20.chk,diff)

```

$$\text{prd67.eq20.lcb} := 6\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + \bar{A}^{ab}\bar{\Gamma}^i_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\text{tr}K$$

$$\text{prd67.eq20.prd} := \partial_a\bar{A}^{ia} + 6\bar{A}^{ia}\partial_a\phi + \bar{A}^{ab}\bar{\Gamma}^i_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\text{tr}K$$

$$\text{prd67.eq20.chk} := 0$$