

## PhysRevD.62.044034 equation (11)

```
1  from shared import *
2  import cdblib
3
4  jsonfile = 'eqtn11.json'
5  cdblib.create (jsonfile)
6
7  DhijDt = cdblib.get ('adm.DhijDt','adm.json')
8  DKijDt = cdblib.get ('adm.DKijDt','adm.json')
9
10 # -----
11
12 trK      := trK -> g^{i j} K_{i j}.
13 gdotK    := g^{i j} K_{i j} -> trK.
14
15 Kup := g^{i a} g^{j b} K_{i j} -> K^{a b}.
16
17 Ham := g^{i j} R_{i j} -> K_{i j} K^{i j} - trK trK.
18
19 Kij := K_{i j} -> A_{i j} + (1/3) g_{i j} trK.    # prd62 eqn 07
20 Lij := K^{i j} -> A^{i j} + (1/3) g^{i j} trK.    # prd62 eqn 07
21
22 trA1 := A_{i j} g^{i j} -> 0.                    # Aij is trace free
23 trA2 := A^{i j} g_{i j} -> 0.
24
25 Asq := A_{i j} A^{i j} -> ABar_{i j} ABar^{i j}.
26
27 gdotg := g_{i j} g^{i j} -> 3.
28
29 # -----
30 # dK/dt
31
32 dotK := \partial_t{trK}.                          # cdb (eq11.101,dotK)
33
34 substitute      (dotK,trK)                        # cdb (eq11.102,dotK)
35 product_rule    (dotK)                            # cdb (eq11.103,dotK)
36 substitute      (dotK,DhijDt)                     # cdb (eq11.104,dotK)
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37 substitute      (dotK,DKijDt)      # cdb (eq11.105,dotK)
38 distribute      (dotK)              # cdb (eq11.106,dotK)
39 substitute      (dotK,gdotK)        # cdb (eq11.107,dotK)
40 substitute      (dotK,Kup)          # cdb (eq11.108,dotK)
41 dotK = product_sort (dotK)          # cdb (eq11.109,dotK)
42 substitute      (dotK,Ham)          # cdb (eq11.110,dotK)
43 distribute      (dotK)              # cdb (eq11.111,dotK)
44 substitute      (dotK,Kij)          # cdb (eq11.112,dotK)
45 substitute      (dotK,Lij)          # cdb (eq11.113,dotK)
46 distribute      (dotK)              # cdb (eq11.114,dotK)
47 substitute      (dotK,trA1)         # cdb (eq11.115,dotK)
48 substitute      (dotK,trA2)         # cdb (eq11.116,dotK)
49 substitute      (dotK,Asq)          # cdb (eq11.117,dotK)
50 substitute      (dotK,gdotg)        # cdb (eq11.118,dotK)
51 map_sympy       (dotK, "simplify")  # cdb (eq11.119,dotK)
52
53 DKDt := \partial_{t}{trK} -> @(dotK).
54
55 cdblib.put ('DKDt',DKDt,jsonfile)

```

$$\partial_t \text{tr} K = \partial_t (g^{ij} K_{ij}) \quad (\text{eq11.102})$$

$$= \partial_t g^{ij} K_{ij} + g^{ij} \partial_t K_{ij} \quad (\text{eq11.103})$$

$$= 2N K^{ij} K_{ij} + g^{ij} \partial_t K_{ij} \quad (\text{eq11.104})$$

$$= 2N K^{ij} K_{ij} + g^{ij} (-D_{ij} N + N (R_{ij} + \text{tr} K K_{ij} - 2K_{ic} K_{jd} g^{cd})) \quad (\text{eq11.105})$$

$$= 2N K^{ij} K_{ij} - g^{ij} D_{ij} N + g^{ij} N R_{ij} + g^{ij} N \text{tr} K K_{ij} - 2g^{ij} N K_{ic} K_{jd} g^{cd} \quad (\text{eq11.106})$$

$$= 2N K^{ij} K_{ij} - g^{ij} D_{ij} N + g^{ij} N R_{ij} + \text{tr} K N \text{tr} K - 2g^{ij} N K_{ic} K_{jd} g^{cd} \quad (\text{eq11.107})$$

$$= 2N K^{ij} K_{ij} - g^{ij} D_{ij} N + g^{ij} N R_{ij} + \text{tr} K N \text{tr} K - 2K^{jd} N K_{jd} \quad (\text{eq11.108})$$

$$= -g^{ab} D_{ab} N + N g^{ab} R_{ab} + N \text{tr} K \text{tr} K \quad (\text{eq11.109})$$

$$= -g^{ab} D_{ab} N + N (K_{ab} K^{ab} - \text{tr} K \text{tr} K) + N \text{tr} K \text{tr} K \quad (\text{eq11.110})$$

$$= -g^{ab} D_{ab} N + N K_{ab} K^{ab} \quad (\text{eq11.111})$$

$$= -g^{ab} D_{ab} N + N \left( A_{ab} + \frac{1}{3} g_{ab} \text{tr} K \right) K^{ab} \quad (\text{eq11.112})$$

$$= -g^{ab} D_{ab} N + N \left( A_{ab} + \frac{1}{3} g_{ab} \text{tr} K \right) \left( A^{ab} + \frac{1}{3} g^{ab} \text{tr} K \right) \quad (\text{eq11.113})$$

$$= -g^{ab} D_{ab} N + N A_{ab} A^{ab} + \frac{1}{3} N A_{ab} g^{ab} \text{tr} K + \frac{1}{3} N g_{ab} \text{tr} K A^{ab} + \frac{1}{9} N g_{ab} \text{tr} K g^{ab} \text{tr} K \quad (\text{eq11.114})$$

$$= -g^{ab} D_{ab} N + N A_{ab} A^{ab} + \frac{1}{3} N g_{ab} \text{tr} K A^{ab} + \frac{1}{9} N g_{ab} \text{tr} K g^{ab} \text{tr} K \quad (\text{eq11.115})$$

$$= -g^{ab} D_{ab} N + N A_{ab} A^{ab} + \frac{1}{9} N g_{ab} \text{tr} K g^{ab} \text{tr} K \quad (\text{eq11.116})$$

$$= -g^{ab} D_{ab} N + N \bar{A}_{ab} \bar{A}^{ab} + \frac{1}{9} N g_{ab} \text{tr} K g^{ab} \text{tr} K \quad (\text{eq11.117})$$

$$= -g^{ab} D_{ab} N + N \bar{A}_{ab} \bar{A}^{ab} + \frac{1}{3} N \text{tr} K \text{tr} K \quad (\text{eq11.118})$$

$$= -g^{ab} D_{ab} N + N \bar{A}_{ab} \bar{A}^{ab} + \frac{1}{3} \text{tr} K^2 N \quad (\text{eq11.119})$$

```

1  # -----
2  # Check against prd62.
3
4  foo := @(dotK).                # cdb(eq11.lcb,foo)
5  bah  = cdblib.get('prd62.eq11.rhs','prd62.json')  # cdb(eq11.prd,bah)
6
7  diff := @(foo) - @(bah).
8
9  distribute      (diff)
10 diff = product_sort (diff)
11 rename_dummies (diff)
12 map_sympy      (diff, "simplify")
13 canonicalise   (diff)          # cdb(eq11.chk,diff)

```

$$\text{eq11.lcb} := -g^{ab}D_{ab}N + N\bar{A}_{ab}\bar{A}^{ab} + \frac{1}{3}\text{tr}K^2N$$

$$\text{eq11.prd} := -g^{ij}D_{ij}N + N\left(\bar{A}_{ij}\bar{A}^{ij} + \frac{1}{3}\text{tr}K^2\right)$$

$$\text{eq11.chk} := 0$$