

PhysRevD.62.044034 equation (15)

```
1  from shared import *
2  import cdblib
3
4  jsonfile = 'eqtn15.json'
5  cdblib.create (jsonfile)
6
7  defG2GBar = cdblib.get ('defG2GBar','gamma.json')
8
9  # -----
10 # Rphi = the part of Rab from the conformal factor
11
12 Rab := R_{a b}. # cdb (eq15.101,Rab)
13
14 substitute      (Rab, defRab) # cdb (eq15.102,Rab)
15 substitute      (Rab, defRiem) # cdb (eq15.103,Rab)
16 substitute      (Rab, defG2GBar) # cdb (eq15.104,Rab)
17 distribute      (Rab) # cdb (eq15.105,Rab)
18 product_rule    (Rab) # cdb (eq15.106,Rab)
19 Rab = product_sort (Rab) # cdb (eq15.107,Rab)
20 rename_dummies  (Rab) # cdb (eq15.108,Rab)
21 canonicalise    (Rab) # cdb (eq15.109,Rab)
22 substitute      (Rab, $gBar_{b c} gBar^{c a} -> gBar^{a}_{b}$)
23 substitute      (Rab, $\partial_{a}\{gBar^{a}_{b}\} -> 0$)
24 substitute      (Rab, $\partial_{a}\{gBar_{b}^{c}\} -> 0$)
25 substitute      (Rab, $gBar^{a}_{a} -> 3$)
26 eliminate_kronecker (Rab) # cdb (eq15.110,Rab)
27 Rab = product_sort (Rab) # cdb (eq15.111,Rab)
28 rename_dummies  (Rab) # cdb (eq15.112,Rab)
29 canonicalise    (Rab) # cdb (eq15.113,Rab)
30 substitute      (Rab, $gBar_{b c} gBar^{c a} -> gBar^{a}_{b}$) # cdb (eq15.114,Rab)
31 substitute      (Rab, $gBar^{a}_{a} -> 3$) # cdb (eq15.115,Rab)
32 eliminate_kronecker (Rab) # cdb (eq15.116,Rab)
33
34 #
```

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35  # isolate Rphi from Rab by switching to local RNC
36
37  Rphi := @(Rab).
38
39  substitute (Rphi,  $\Gamma_{a,b,c}$ ) # cdb (eq15.117,Rphi)
40  substitute (Rphi,  $\partial_a g_{b,c}$ ) # cdb (eq15.118,Rphi)
41  substitute (Rphi,  $\partial_a \Gamma_{b,c}$ ) # cdb (eq15.119,Rphi)
42
43  substitute (Rphi,  $\partial_a b_\phi \rightarrow D_{a,b}\phi$ ) # cdb (eq15.120,Rphi)
44  substitute (Rphi,  $\partial_a \phi \rightarrow D_a \phi$ ) # cdb (eq15.121,Rphi)
45
46  defRphi := Rphi_{a b} -> @(Rphi).
47
48  cdblib.put ('defRphi',defRphi,jsonfile)

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$$R_{ab} = R^c_{acb} \quad (\text{eq15.102})$$

$$= \partial_c \Gamma^c_{ab} + \Gamma^c_{ec} \Gamma^e_{ab} - \partial_b \Gamma^c_{ac} - \Gamma^c_{eb} \Gamma^e_{ac} \quad (\text{eq15.103})$$

$$= \partial_c (2\bar{g}^c_b \partial_a \phi + 2\bar{g}^c_a \partial_b \phi - 2\bar{g}^{ce} \partial_e \phi \bar{g}_{ab} + \bar{\Gamma}^c_{ab}) + (2\bar{g}^c_c \partial_e \phi + 2\bar{g}^c_e \partial_c \phi - 2\bar{g}^{cd} \partial_d \phi \bar{g}_{ec} + \bar{\Gamma}^c_{ec}) (2\bar{g}^e_b \partial_a \phi + 2\bar{g}^e_a \partial_b \phi - 2\bar{g}^{ef} \partial_f \phi \bar{g}_{ab} + \bar{\Gamma}^e_{ab}) \\ - \partial_b (2\bar{g}^c_c \partial_a \phi + 2\bar{g}^c_a \partial_c \phi - 2\bar{g}^{ce} \partial_e \phi \bar{g}_{ac} + \bar{\Gamma}^c_{ac}) - (2\bar{g}^c_b \partial_e \phi + 2\bar{g}^c_e \partial_b \phi - 2\bar{g}^{cd} \partial_d \phi \bar{g}_{eb} + \bar{\Gamma}^c_{eb}) (2\bar{g}^e_c \partial_a \phi + 2\bar{g}^e_a \partial_c \phi - 2\bar{g}^{ef} \partial_f \phi \bar{g}_{ac} + \bar{\Gamma}^e_{ac}) \quad (\text{eq15.104})$$

$$= 2\partial_c (\bar{g}^c_b \partial_a \phi) + 2\partial_c (\bar{g}^c_a \partial_b \phi) - 2\partial_c (\bar{g}^{ce} \partial_e \phi \bar{g}_{ab}) + \partial_c \bar{\Gamma}^c_{ab} + 4\bar{g}^c_c \partial_e \phi \bar{g}^e_b \partial_a \phi + 4\bar{g}^c_c \partial_e \phi \bar{g}^e_a \partial_b \phi - 4\bar{g}^c_c \partial_e \phi \bar{g}^{ef} \partial_f \phi \bar{g}_{ab} + 2\bar{g}^c_c \partial_e \phi \bar{\Gamma}^e_{ab} + 4\bar{g}^c_e \partial_c \phi \bar{g}^e_b \partial_a \phi \\ + 4\bar{g}^c_e \partial_c \phi \bar{g}^e_a \partial_b \phi - 4\bar{g}^c_e \partial_c \phi \bar{g}^{ef} \partial_f \phi \bar{g}_{ab} + 2\bar{g}^c_e \partial_c \phi \bar{\Gamma}^e_{ab} - 4\bar{g}^{cd} \partial_d \phi \bar{g}_{ec} \bar{g}^e_b \partial_a \phi - 4\bar{g}^{cd} \partial_d \phi \bar{g}_{ec} \bar{g}^e_a \partial_b \phi + 4\bar{g}^{cd} \partial_d \phi \bar{g}_{ec} \bar{g}^{ef} \partial_f \phi \bar{g}_{ab} - 2\bar{g}^{cd} \partial_d \phi \bar{g}_{ec} \bar{\Gamma}^e_{ab} \\ + 2\bar{\Gamma}^c_{ec} \bar{g}^e_b \partial_a \phi + 2\bar{\Gamma}^c_{ec} \bar{g}^e_a \partial_b \phi - 2\bar{\Gamma}^c_{ec} \bar{g}^{ef} \partial_f \phi \bar{g}_{ab} + \bar{\Gamma}^c_{ec} \bar{\Gamma}^e_{ab} - 2\partial_b (\bar{g}^c_c \partial_a \phi) - 2\partial_b (\bar{g}^c_a \partial_c \phi) + 2\partial_b (\bar{g}^{ce} \partial_e \phi \bar{g}_{ac}) - \partial_b \bar{\Gamma}^c_{ac} - 4\bar{g}^c_b \partial_e \phi \bar{g}^e_c \partial_a \phi \\ - 4\bar{g}^c_b \partial_e \phi \bar{g}^e_a \partial_c \phi + 4\bar{g}^c_b \partial_e \phi \bar{g}^{ef} \partial_f \phi \bar{g}_{ac} - 2\bar{g}^c_b \partial_e \phi \bar{\Gamma}^e_{ac} - 4\bar{g}^c_e \partial_b \phi \bar{g}^e_c \partial_a \phi - 4\bar{g}^c_e \partial_b \phi \bar{g}^e_a \partial_c \phi + 4\bar{g}^c_e \partial_b \phi \bar{g}^{ef} \partial_f \phi \bar{g}_{ac} - 2\bar{g}^c_e \partial_b \phi \bar{\Gamma}^e_{ac} + 4\bar{g}^{cd} \partial_d \phi \bar{g}_{eb} \bar{g}^e_c \partial_a \phi \\ + 4\bar{g}^{cd} \partial_d \phi \bar{g}_{eb} \bar{g}^e_a \partial_c \phi - 4\bar{g}^{cd} \partial_d \phi \bar{g}_{eb} \bar{g}^{ef} \partial_f \phi \bar{g}_{ac} + 2\bar{g}^{cd} \partial_d \phi \bar{g}_{eb} \bar{\Gamma}^e_{ac} - 2\bar{\Gamma}^c_{eb} \bar{g}^e_c \partial_a \phi - 2\bar{\Gamma}^c_{eb} \bar{g}^e_a \partial_c \phi + 2\bar{\Gamma}^c_{eb} \bar{g}^{ef} \partial_f \phi \bar{g}_{ac} - \bar{\Gamma}^c_{eb} \bar{\Gamma}^e_{ac} \quad (\text{eq15.105})$$

$$= 2\partial_c \bar{g}^c_b \partial_a \phi + 2\bar{g}^c_b \partial_{ca} \phi + 2\partial_c \bar{g}^c_a \partial_b \phi + 2\bar{g}^c_a \partial_{cb} \phi - 2\partial_c \bar{g}^{ce} \partial_e \phi \bar{g}_{ab} - 2\bar{g}^{ce} \partial_{ce} \phi \bar{g}_{ab} - 2\bar{g}^{ce} \partial_e \phi \partial_c \bar{g}_{ab} + \partial_c \bar{\Gamma}^c_{ab} + 4\bar{g}^c_c \partial_e \phi \bar{g}^e_b \partial_a \phi + 4\bar{g}^c_c \partial_e \phi \bar{g}^e_a \partial_b \phi \\ - 4\bar{g}^c_c \partial_e \phi \bar{g}^{ef} \partial_f \phi \bar{g}_{ab} + 2\bar{g}^c_c \partial_e \phi \bar{\Gamma}^e_{ab} + 4\bar{g}^c_e \partial_c \phi \bar{g}^e_b \partial_a \phi + 4\bar{g}^c_e \partial_c \phi \bar{g}^e_a \partial_b \phi - 4\bar{g}^c_e \partial_c \phi \bar{g}^{ef} \partial_f \phi \bar{g}_{ab} + 2\bar{g}^c_e \partial_c \phi \bar{\Gamma}^e_{ab} - 4\bar{g}^{cd} \partial_d \phi \bar{g}_{ec} \bar{g}^e_b \partial_a \phi \\ - 4\bar{g}^{cd} \partial_d \phi \bar{g}_{ec} \bar{g}^e_a \partial_b \phi + 4\bar{g}^{cd} \partial_d \phi \bar{g}_{ec} \bar{g}^{ef} \partial_f \phi \bar{g}_{ab} - 2\bar{g}^{cd} \partial_d \phi \bar{g}_{ec} \bar{\Gamma}^e_{ab} + 2\bar{\Gamma}^c_{ec} \bar{g}^e_b \partial_a \phi + 2\bar{\Gamma}^c_{ec} \bar{g}^e_a \partial_b \phi - 2\bar{\Gamma}^c_{ec} \bar{g}^{ef} \partial_f \phi \bar{g}_{ab} + \bar{\Gamma}^c_{ec} \bar{\Gamma}^e_{ab} - 2\partial_b \bar{g}^c_c \partial_a \phi \\ - 2\bar{g}^c_c \partial_{ba} \phi - 2\partial_b \bar{g}^c_a \partial_c \phi - 2\bar{g}^c_a \partial_{bc} \phi + 2\partial_b \bar{g}^{ce} \partial_e \phi \bar{g}_{ac} + 2\bar{g}^{ce} \partial_{be} \phi \bar{g}_{ac} + 2\bar{g}^{ce} \partial_e \phi \partial_b \bar{g}_{ac} - \partial_b \bar{\Gamma}^c_{ac} - 4\bar{g}^c_b \partial_e \phi \bar{g}^e_c \partial_a \phi - 4\bar{g}^c_b \partial_e \phi \bar{g}^e_a \partial_c \phi \\ + 4\bar{g}^c_b \partial_e \phi \bar{g}^{ef} \partial_f \phi \bar{g}_{ac} - 2\bar{g}^c_b \partial_e \phi \bar{\Gamma}^e_{ac} - 4\bar{g}^c_e \partial_b \phi \bar{g}^e_c \partial_a \phi - 4\bar{g}^c_e \partial_b \phi \bar{g}^e_a \partial_c \phi + 4\bar{g}^c_e \partial_b \phi \bar{g}^{ef} \partial_f \phi \bar{g}_{ac} - 2\bar{g}^c_e \partial_b \phi \bar{\Gamma}^e_{ac} + 4\bar{g}^{cd} \partial_d \phi \bar{g}_{eb} \bar{g}^e_c \partial_a \phi \\ + 4\bar{g}^{cd} \partial_d \phi \bar{g}_{eb} \bar{g}^e_a \partial_c \phi - 4\bar{g}^{cd} \partial_d \phi \bar{g}_{eb} \bar{g}^{ef} \partial_f \phi \bar{g}_{ac} + 2\bar{g}^{cd} \partial_d \phi \bar{g}_{eb} \bar{\Gamma}^e_{ac} - 2\bar{\Gamma}^c_{eb} \bar{g}^e_c \partial_a \phi - 2\bar{\Gamma}^c_{eb} \bar{g}^e_a \partial_c \phi + 2\bar{\Gamma}^c_{eb} \bar{g}^{ef} \partial_f \phi \bar{g}_{ac} - \bar{\Gamma}^c_{eb} \bar{\Gamma}^e_{ac} \quad (\text{eq15.106})$$

$$= 2\partial_a \phi \partial_c \bar{g}^c_b + 2\partial_{ca} \phi \bar{g}^c_b + 2\partial_b \phi \partial_c \bar{g}^c_a + 2\partial_{cb} \phi \bar{g}^c_a - 2\bar{g}_{ab} \partial_d \phi \partial_c \bar{g}^{cd} - 2\bar{g}_{ab} \bar{g}^{cd} \partial_{cd} \phi - 2\bar{g}^{cd} \partial_d \phi \partial_c \bar{g}_{ab} + \partial_c \bar{\Gamma}^c_{ab} + 4\partial_a \phi \partial_c \phi \bar{g}^c_b \bar{g}^d_d + 4\partial_b \phi \partial_c \phi \bar{g}^c_a \bar{g}^d_d \\ - 4\bar{g}_{ab} \bar{g}^{cd} \partial_c \phi \partial_d \phi \bar{g}^e_e + 2\bar{\Gamma}^c_{ab} \partial_c \phi \bar{g}^d_d + 4\partial_a \phi \partial_c \phi \bar{g}^c_d \bar{g}^d_b - 4\bar{g}_{ab} \bar{g}^{cd} \partial_e \phi \partial_d \phi \bar{g}^e_c + 2\bar{\Gamma}^c_{ab} \partial_d \phi \bar{g}^d_c - 4\bar{g}_{cd} \bar{g}^{de} \partial_a \phi \partial_e \phi \bar{g}^c_b - 4\bar{g}_{cd} \bar{g}^{de} \partial_b \phi \partial_e \phi \bar{g}^c_a \\ + 4\bar{g}_{ab} \bar{g}_{cd} \bar{g}^{de} \bar{g}^{cf} \partial_e \phi \partial_f \phi - 2\bar{g}_{cd} \bar{g}^{de} \bar{\Gamma}^c_{ab} \partial_e \phi + 2\bar{\Gamma}^c_{dc} \partial_a \phi \bar{g}^d_b + 2\bar{\Gamma}^c_{dc} \partial_b \phi \bar{g}^d_a - 2\bar{g}_{ab} \bar{g}^{cd} \bar{\Gamma}^e_{ce} \partial_d \phi + \bar{\Gamma}^c_{ab} \bar{\Gamma}^d_{cd} - 2\partial_a \phi \partial_b \bar{g}^c_c - 2\partial_{ba} \phi \bar{g}^c_c - 2\partial_c \phi \partial_b \bar{g}^c_a \\ - 2\partial_{bc} \phi \bar{g}^c_a + 2\bar{g}_{ac} \partial_d \phi \partial_b \bar{g}^{cd} + 2\bar{g}_{ac} \bar{g}^{cd} \partial_{bd} \phi + 2\bar{g}^{cd} \partial_d \phi \partial_b \bar{g}_{ac} - \partial_b \bar{\Gamma}^c_{ac} - 4\partial_a \phi \partial_c \phi \bar{g}^d_b \bar{g}^d_c - 4\partial_c \phi \partial_d \phi \bar{g}^c_b \bar{g}^d_a + 4\bar{g}_{ac} \bar{g}^{de} \partial_d \phi \partial_e \phi \bar{g}^c_b - 2\bar{\Gamma}^c_{ad} \partial_c \phi \bar{g}^d_b \\ - 4\partial_a \phi \partial_b \phi \bar{g}^c_a \bar{g}^d_c + 4\bar{g}_{ac} \bar{g}^{de} \partial_b \phi \partial_e \phi \bar{g}^c_d - 2\bar{\Gamma}^c_{ad} \partial_b \phi \bar{g}^d_c + 4\bar{g}_{cb} \bar{g}^{de} \partial_a \phi \partial_e \phi \bar{g}^c_d + 4\bar{g}_{cb} \bar{g}^{de} \partial_d \phi \partial_e \phi \bar{g}^c_a - 4\bar{g}_{ac} \bar{g}_{db} \bar{g}^{ce} \bar{g}^{df} \partial_e \phi \partial_f \phi + 2\bar{g}_{cb} \bar{g}^{de} \bar{\Gamma}^c_{ad} \partial_e \phi \\ - 2\bar{\Gamma}^c_{db} \partial_a \phi \bar{g}^d_c - 2\bar{\Gamma}^c_{db} \partial_c \phi \bar{g}^d_a + 2\bar{g}_{ac} \bar{g}^{de} \bar{\Gamma}^c_{db} \partial_e \phi - \bar{\Gamma}^c_{db} \bar{\Gamma}^d_{ac} \quad (\text{eq15.107})$$

$$\begin{aligned}
R_{ab} = & 2\partial_a\phi\partial_c\bar{g}^c{}_b + 2\partial_{ca}\phi\bar{g}^c{}_b + 2\partial_b\phi\partial_c\bar{g}^c{}_a + 2\partial_{cb}\phi\bar{g}^c{}_a - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{dc} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{dc}\partial_c\phi\partial_d\bar{g}_{ab} + \partial_c\bar{\Gamma}^c{}_{ab} + 4\partial_a\phi\partial_c\phi\bar{g}^c{}_b\bar{g}^d{}_d + 4\partial_b\phi\partial_c\phi\bar{g}^c{}_a\bar{g}^d{}_d \\
& - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi\bar{g}^e{}_e + 2\bar{\Gamma}^c{}_{ab}\partial_c\phi\bar{g}^d{}_d + 4\partial_a\phi\partial_c\phi\bar{g}^c{}_d\bar{g}^d{}_b - 4\bar{g}_{ab}\bar{g}^{ed}\partial_c\phi\partial_d\phi\bar{g}^c{}_e + 2\bar{\Gamma}^c{}_{ab}\partial_d\phi\bar{g}^d{}_c - 4\bar{g}_{de}\bar{g}^{ec}\partial_a\phi\partial_c\phi\bar{g}^d{}_b - 4\bar{g}_{de}\bar{g}^{ec}\partial_b\phi\partial_c\phi\bar{g}^d{}_a \\
& + 4\bar{g}_{ab}\bar{g}_{ef}\bar{g}^{fc}\bar{g}^{ed}\partial_c\phi\partial_d\phi - 2\bar{g}_{ce}\bar{g}^{ed}\bar{\Gamma}^c{}_{ab}\partial_d\phi + 2\bar{\Gamma}^c{}_{dc}\partial_a\phi\bar{g}^d{}_b + 2\bar{\Gamma}^c{}_{dc}\partial_b\phi\bar{g}^d{}_a - 2\bar{g}_{ab}\bar{g}^{de}\bar{\Gamma}^c{}_{dc}\partial_e\phi + \bar{\Gamma}^c{}_{ab}\bar{\Gamma}^d{}_{cd} - 2\partial_a\phi\partial_b\bar{g}^c{}_c - 2\partial_{ba}\phi\bar{g}^c{}_c - 2\partial_c\phi\partial_b\bar{g}^c{}_a \\
& - 2\partial_{bc}\phi\bar{g}^c{}_a + 2\bar{g}_{ad}\partial_c\phi\partial_b\bar{g}^{dc} + 2\bar{g}_{ad}\bar{g}^{dc}\partial_{bc}\phi + 2\bar{g}^{dc}\partial_c\phi\partial_b\bar{g}_{ad} - \partial_b\bar{\Gamma}^c{}_{ac} - 4\partial_a\phi\partial_c\phi\bar{g}^d{}_b\bar{g}^c{}_d - 4\partial_c\phi\partial_d\phi\bar{g}^c{}_b\bar{g}^d{}_a + 4\bar{g}_{ae}\bar{g}^{cd}\partial_c\phi\partial_d\phi\bar{g}^e{}_b - 2\bar{\Gamma}^c{}_{ad}\partial_c\phi\bar{g}^d{}_b \\
& - 4\partial_a\phi\partial_b\phi\bar{g}^c{}_d\bar{g}^d{}_c + 4\bar{g}_{ad}\bar{g}^{ec}\partial_b\phi\partial_c\phi\bar{g}^d{}_e - 2\bar{\Gamma}^c{}_{ad}\partial_b\phi\bar{g}^d{}_c + 4\bar{g}_{db}\bar{g}^{ec}\partial_a\phi\partial_c\phi\bar{g}^d{}_e + 4\bar{g}_{eb}\bar{g}^{cd}\partial_c\phi\partial_d\phi\bar{g}^e{}_a - 4\bar{g}_{ae}\bar{g}_{fb}\bar{g}^{ec}\bar{g}^{fd}\partial_c\phi\partial_d\phi + 2\bar{g}_{cb}\bar{g}^{de}\bar{\Gamma}^c{}_{ad}\partial_e\phi \\
& - 2\bar{\Gamma}^c{}_{db}\partial_a\phi\bar{g}^d{}_c - 2\bar{\Gamma}^c{}_{db}\partial_c\phi\bar{g}^d{}_a + 2\bar{g}_{ac}\bar{g}^{de}\bar{\Gamma}^c{}_{db}\partial_e\phi - \bar{\Gamma}^c{}_{db}\bar{\Gamma}^d{}_{ac}
\end{aligned} \tag{eq15.108}$$

$$\begin{aligned}
&= 2\partial_a\phi\partial_c\bar{g}_b{}^c + 2\partial_{ac}\phi\bar{g}_b{}^c + 2\partial_b\phi\partial_c\bar{g}_a{}^c - 2\bar{g}_{ab}\partial_c\phi\partial_a\bar{g}{}^{cd} - 2\bar{g}_{ab}\bar{g}{}^{cd}\partial_{cd}\phi - 2\bar{g}{}^{cd}\partial_c\phi\partial_d\bar{g}_{ab} + \partial_c\bar{\Gamma}{}^c{}_{ab} + 4\partial_a\phi\partial_c\phi\bar{g}_b{}^c\bar{g}{}^d{}_d + 4\partial_b\phi\partial_c\phi\bar{g}_a{}^c\bar{g}{}^d{}_d - 4\bar{g}_{ab}\bar{g}{}^{cd}\partial_c\phi\partial_d\phi\bar{g}{}^e{}_e \\
&\quad + 2\bar{\Gamma}{}^c{}_{ab}\partial_c\phi\bar{g}{}^d{}_d - 4\bar{g}_{ab}\bar{g}{}^{cd}\partial_c\phi\partial_e\phi\bar{g}{}^d{}_e + 2\bar{\Gamma}{}^c{}_{ab}\partial_d\phi\bar{g}_c{}^d - 4\bar{g}_{cd}\bar{g}{}^{ce}\partial_a\phi\partial_e\phi\bar{g}_b{}^d - 4\bar{g}_{cd}\bar{g}{}^{ce}\partial_b\phi\partial_e\phi\bar{g}_a{}^d + 4\bar{g}_{ab}\bar{g}_{cd}\bar{g}{}^{ce}\bar{g}{}^{df}\partial_e\phi\partial_f\phi - 2\bar{g}_{cd}\bar{g}{}^{ce}\bar{\Gamma}{}^d{}_{ab}\partial_e\phi \\
&\quad + 2\bar{\Gamma}{}^c{}_{cd}\partial_a\phi\bar{g}_b{}^d + 2\bar{\Gamma}{}^c{}_{cd}\partial_b\phi\bar{g}_a{}^d - 2\bar{g}_{ab}\bar{g}{}^{cd}\bar{\Gamma}{}^e{}_{ce}\partial_d\phi + \bar{\Gamma}{}^c{}_{ab}\bar{\Gamma}{}^d{}_{cd} - 2\partial_a\phi\partial_b\bar{g}{}^c{}_c - 2\partial_{ab}\phi\bar{g}{}^c{}_c - 2\partial_c\phi\partial_b\bar{g}_a{}^c + 2\bar{g}_{ac}\partial_d\phi\partial_b\bar{g}{}^{cd} + 2\bar{g}_{ac}\bar{g}{}^{cd}\partial_{bd}\phi + 2\bar{g}{}^{cd}\partial_c\phi\partial_b\bar{g}_{ad} \\
&\quad - \partial_b\bar{\Gamma}{}^c{}_{ac} - 4\partial_c\phi\partial_d\phi\bar{g}_a{}^c\bar{g}_b{}^d + 4\bar{g}_{ac}\bar{g}{}^{de}\partial_d\phi\partial_e\phi\bar{g}_b{}^c - 2\bar{\Gamma}{}^c{}_{ad}\partial_c\phi\bar{g}_b{}^d - 4\partial_a\phi\partial_b\phi\bar{g}{}^c{}_d\bar{g}_c{}^d + 4\bar{g}_{ac}\bar{g}{}^{de}\partial_b\phi\partial_d\phi\bar{g}{}^c{}_e - 2\bar{\Gamma}{}^c{}_{ad}\partial_b\phi\bar{g}_c{}^d + 4\bar{g}_{bc}\bar{g}{}^{de}\partial_a\phi\partial_d\phi\bar{g}{}^c{}_e \\
&\quad + 4\bar{g}_{bc}\bar{g}{}^{de}\partial_d\phi\partial_e\phi\bar{g}_a{}^c - 4\bar{g}_{ac}\bar{g}_{bd}\bar{g}{}^{ce}\bar{g}{}^{df}\partial_e\phi\partial_f\phi + 2\bar{g}_{bc}\bar{g}{}^{de}\bar{\Gamma}{}^c{}_{ad}\partial_e\phi - 2\bar{\Gamma}{}^c{}_{bd}\partial_a\phi\bar{g}_c{}^d - 2\bar{\Gamma}{}^c{}_{bd}\partial_c\phi\bar{g}_a{}^d + 2\bar{g}_{ac}\bar{g}{}^{de}\bar{\Gamma}{}^c{}_{bd}\partial_e\phi - \bar{\Gamma}{}^c{}_{ad}\bar{\Gamma}{}^d{}_{bc} \quad (\text{eq15.109})
\end{aligned}$$

$$\begin{aligned}
&= -4\partial_{ab}\phi - 2\bar{g}_{ab}\partial_c\phi\partial_a\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{cd}\partial_c\phi\partial_d\bar{g}_{ab} + \partial_c\bar{\Gamma}^c{}_{ab} + 8\partial_a\phi\partial_b\phi + 12\partial_b\phi\partial_a\phi - 12\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi + 4\bar{\Gamma}^c{}_{ab}\partial_c\phi - 4\bar{g}_{ab}\bar{g}^{ce}\partial_c\phi\partial_e\phi \\
&\quad + 2\bar{\Gamma}^d{}_{ab}\partial_d\phi - 4\bar{g}_{cb}\bar{g}^{ce}\partial_a\phi\partial_e\phi - 4\bar{g}_{ca}\bar{g}^{ce}\partial_b\phi\partial_e\phi + 4\bar{g}_{ab}\bar{g}^{fe}\partial_e\phi\partial_f\phi - 2\bar{g}_{cd}\bar{g}^{ce}\bar{\Gamma}^d{}_{ab}\partial_e\phi + 2\bar{\Gamma}^c{}_{cb}\partial_a\phi + 2\bar{\Gamma}^c{}_{ca}\partial_b\phi - 2\bar{g}_{ab}\bar{g}^{cd}\bar{\Gamma}^e{}_{ce}\partial_d\phi + \bar{\Gamma}^c{}_{ab}\bar{\Gamma}^d{}_{cd} \\
&\quad + 2\bar{g}_{ac}\partial_d\phi\partial_b\bar{g}^{cd} + 2\partial_{ba}\phi + 2\bar{g}^{cd}\partial_c\phi\partial_b\bar{g}_{ad} - \partial_b\bar{\Gamma}^c{}_{ac} + 4\bar{g}_{ab}\bar{g}^{de}\partial_d\phi\partial_e\phi - 4\partial_a\phi\partial_b\phi\bar{g}^d{}_d + 4\bar{g}_{ae}\bar{g}^{de}\partial_b\phi\partial_d\phi - 2\bar{\Gamma}^d{}_{ad}\partial_b\phi + 4\bar{g}_{be}\bar{g}^{de}\partial_a\phi\partial_d\phi \\
&\quad + 4\bar{g}_{ba}\bar{g}^{de}\partial_d\phi\partial_e\phi - 4\bar{g}_{bd}\bar{g}^{df}\partial_a\phi\partial_f\phi + 2\bar{g}_{bc}\bar{g}^{de}\bar{\Gamma}^c{}_{ad}\partial_e\phi - 2\bar{\Gamma}^d{}_{bd}\partial_a\phi - 2\bar{\Gamma}^c{}_{ba}\partial_c\phi + 2\bar{g}_{ac}\bar{g}^{de}\bar{\Gamma}^c{}_{bd}\partial_e\phi - \bar{\Gamma}^c{}_{ad}\bar{\Gamma}^d{}_{bc} \quad (\text{eq15.110})
\end{aligned}$$

$$\begin{aligned}
= & -4\partial_{ab}\phi - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{cd}\partial_c\phi\partial_d\bar{g}_{ab} + \partial_c\bar{\Gamma}^c{}_{ab} + 20\partial_a\phi\partial_b\phi - 12\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi + 6\bar{\Gamma}^c{}_{ab}\partial_c\phi - 4\bar{g}_{cb}\bar{g}^{cd}\partial_a\phi\partial_d\phi - 4\bar{g}_{ca}\bar{g}^{cd}\partial_b\phi\partial_d\phi \\
& + 4\bar{g}_{ab}\bar{g}^{cd}\partial_d\phi\partial_c\phi - 2\bar{g}_{cd}\bar{g}^{ce}\bar{\Gamma}^d{}_{ab}\partial_e\phi + 2\bar{\Gamma}^c{}_{cb}\partial_a\phi + 2\bar{\Gamma}^c{}_{ca}\partial_b\phi - 2\bar{g}_{ab}\bar{g}^{cd}\bar{\Gamma}^e{}_{ce}\partial_d\phi + \bar{\Gamma}^c{}_{ab}\bar{\Gamma}^d{}_{cd} + 2\bar{g}_{ac}\partial_d\phi\partial_b\bar{g}^{cd} + 2\partial_{ba}\phi + 2\bar{g}^{dc}\partial_d\phi\partial_b\bar{g}_{ac} - \partial_b\bar{\Gamma}^c{}_{ac} \\
& - 4\partial_a\phi\partial_b\phi\bar{g}^c{}_c + 4\bar{g}_{ac}\bar{g}^{dc}\partial_b\phi\partial_d\phi - 2\bar{\Gamma}^c{}_{ac}\partial_b\phi + 4\bar{g}_{bc}\bar{g}^{dc}\partial_a\phi\partial_d\phi + 4\bar{g}_{ba}\bar{g}^{cd}\partial_c\phi\partial_d\phi - 4\bar{g}_{bc}\bar{g}^{cd}\partial_a\phi\partial_d\phi + 2\bar{g}_{bc}\bar{g}^{de}\bar{\Gamma}^c{}_{ad}\partial_e\phi - 2\bar{\Gamma}^c{}_{bc}\partial_a\phi - 2\bar{\Gamma}^c{}_{ba}\partial_c\phi \\
& + 2\bar{g}_{ac}\bar{g}^{de}\bar{\Gamma}^c{}_{bd}\partial_e\phi - \bar{\Gamma}^c{}_{ad}\bar{\Gamma}^d{}_{bc}
\end{aligned} \tag{eq15.111}$$

$$\begin{aligned}
= & -4\partial_{ab}\phi - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{cd}\partial_c\phi\partial_d\bar{g}_{ab} + \partial_c\bar{\Gamma}^c{}_{ab} + 20\partial_a\phi\partial_b\phi - 12\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi + 6\bar{\Gamma}^c{}_{ab}\partial_c\phi - 4\bar{g}_{ab}\bar{g}^{dc}\partial_a\phi\partial_c\phi - 4\bar{g}_{da}\bar{g}^{dc}\partial_b\phi\partial_c\phi \\
& + 4\bar{g}_{ab}\bar{g}^{dc}\partial_c\phi\partial_d\phi - 2\bar{g}_{ec}\bar{g}^{ed}\bar{\Gamma}^c{}_{ab}\partial_d\phi + 2\bar{\Gamma}^c{}_{cb}\partial_a\phi + 2\bar{\Gamma}^c{}_{ca}\partial_b\phi - 2\bar{g}_{ab}\bar{g}^{de}\bar{\Gamma}^c{}_{dc}\partial_e\phi + \bar{\Gamma}^c{}_{ab}\bar{\Gamma}^d{}_{cd} + 2\bar{g}_{ad}\partial_c\phi\partial_b\bar{g}^{dc} + 2\partial_{ba}\phi + 2\bar{g}^{cd}\partial_c\phi\partial_b\bar{g}_{ad} - \partial_b\bar{\Gamma}^c{}_{ac} \\
& - 4\partial_a\phi\partial_b\phi\bar{g}^c{}_c + 4\bar{g}_{ad}\bar{g}^{cd}\partial_b\phi\partial_c\phi - 2\bar{\Gamma}^c{}_{ac}\partial_b\phi + 4\bar{g}_{ba}\bar{g}^{cd}\partial_a\phi\partial_c\phi + 4\bar{g}_{ba}\bar{g}^{cd}\partial_c\phi\partial_d\phi - 4\bar{g}_{ba}\bar{g}^{dc}\partial_a\phi\partial_c\phi + 2\bar{g}_{bc}\bar{g}^{de}\bar{\Gamma}^c{}_{ad}\partial_e\phi - 2\bar{\Gamma}^c{}_{bc}\partial_a\phi - 2\bar{\Gamma}^c{}_{ba}\partial_c\phi \\
& + 2\bar{g}_{ac}\bar{g}^{de}\bar{\Gamma}^c{}_{bd}\partial_e\phi - \bar{\Gamma}^c{}_{ad}\bar{\Gamma}^d{}_{bc}
\end{aligned} \tag{eq15.112}$$

$$\begin{aligned}
R_{ab} = & -2\partial_{ab}\phi - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{cd}\partial_c\phi\partial_d\bar{g}_{ab} + \partial_c\bar{\Gamma}^c_{ab} + 20\partial_a\phi\partial_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi + 4\bar{\Gamma}^c_{ab}\partial_c\phi - 4\bar{g}_{bc}\bar{g}^{cd}\partial_a\phi\partial_d\phi - 2\bar{g}_{cd}\bar{g}^{ce}\bar{\Gamma}^d_{ab}\partial_e\phi \\
& - 2\bar{g}_{ab}\bar{g}^{cd}\bar{\Gamma}^e_{ce}\partial_d\phi + \bar{\Gamma}^c_{ab}\bar{\Gamma}^d_{cd} + 2\bar{g}_{ac}\partial_d\phi\partial_b\bar{g}^{cd} + 2\bar{g}^{cd}\partial_c\phi\partial_b\bar{g}_{ad} - \partial_b\bar{\Gamma}^c_{ac} - 4\partial_a\phi\partial_b\phi\bar{g}^c_c + 2\bar{g}_{bc}\bar{g}^{de}\bar{\Gamma}^c_{ad}\partial_e\phi + 2\bar{g}_{ac}\bar{g}^{de}\bar{\Gamma}^c_{bd}\partial_e\phi - \bar{\Gamma}^c_{ad}\bar{\Gamma}^d_{bc} \quad (\text{eq15.113})
\end{aligned}$$

$$\begin{aligned}
= & -2\partial_{ab}\phi - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{cd}\partial_c\phi\partial_d\bar{g}_{ab} + \partial_c\bar{\Gamma}^c_{ab} + 20\partial_a\phi\partial_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi + 4\bar{\Gamma}^c_{ab}\partial_c\phi - 4\bar{g}^d_b\partial_a\phi\partial_d\phi - 2\bar{g}_{cd}\bar{g}^{ce}\bar{\Gamma}^d_{ab}\partial_e\phi \\
& - 2\bar{g}_{ab}\bar{g}^{cd}\bar{\Gamma}^e_{ce}\partial_d\phi + \bar{\Gamma}^c_{ab}\bar{\Gamma}^d_{cd} + 2\bar{g}_{ac}\partial_d\phi\partial_b\bar{g}^{cd} + 2\bar{g}^{cd}\partial_c\phi\partial_b\bar{g}_{ad} - \partial_b\bar{\Gamma}^c_{ac} - 4\partial_a\phi\partial_b\phi\bar{g}^c_c + 2\bar{g}_{bc}\bar{g}^{de}\bar{\Gamma}^c_{ad}\partial_e\phi + 2\bar{g}_{ac}\bar{g}^{de}\bar{\Gamma}^c_{bd}\partial_e\phi - \bar{\Gamma}^c_{ad}\bar{\Gamma}^d_{bc} \quad (\text{eq15.114})
\end{aligned}$$

$$\begin{aligned}
= & -2\partial_{ab}\phi - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{cd}\partial_c\phi\partial_d\bar{g}_{ab} + \partial_c\bar{\Gamma}^c_{ab} + 8\partial_a\phi\partial_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi + 4\bar{\Gamma}^c_{ab}\partial_c\phi - 4\bar{g}^d_b\partial_a\phi\partial_d\phi - 2\bar{g}_{cd}\bar{g}^{ce}\bar{\Gamma}^d_{ab}\partial_e\phi \\
& - 2\bar{g}_{ab}\bar{g}^{cd}\bar{\Gamma}^e_{ce}\partial_d\phi + \bar{\Gamma}^c_{ab}\bar{\Gamma}^d_{cd} + 2\bar{g}_{ac}\partial_d\phi\partial_b\bar{g}^{cd} + 2\bar{g}^{cd}\partial_c\phi\partial_b\bar{g}_{ad} - \partial_b\bar{\Gamma}^c_{ac} + 2\bar{g}_{bc}\bar{g}^{de}\bar{\Gamma}^c_{ad}\partial_e\phi + 2\bar{g}_{ac}\bar{g}^{de}\bar{\Gamma}^c_{bd}\partial_e\phi - \bar{\Gamma}^c_{ad}\bar{\Gamma}^d_{bc} \quad (\text{eq15.115})
\end{aligned}$$

$$\begin{aligned}
= & -2\partial_{ab}\phi - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{cd}\partial_c\phi\partial_d\bar{g}_{ab} + \partial_c\bar{\Gamma}^c_{ab} + 4\partial_a\phi\partial_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi + 4\bar{\Gamma}^c_{ab}\partial_c\phi - 2\bar{g}_{cd}\bar{g}^{ce}\bar{\Gamma}^d_{ab}\partial_e\phi - 2\bar{g}_{ab}\bar{g}^{cd}\bar{\Gamma}^e_{ce}\partial_d\phi \\
& + \bar{\Gamma}^c_{ab}\bar{\Gamma}^d_{cd} + 2\bar{g}_{ac}\partial_d\phi\partial_b\bar{g}^{cd} + 2\bar{g}^{cd}\partial_c\phi\partial_b\bar{g}_{ad} - \partial_b\bar{\Gamma}^c_{ac} + 2\bar{g}_{bc}\bar{g}^{de}\bar{\Gamma}^c_{ad}\partial_e\phi + 2\bar{g}_{ac}\bar{g}^{de}\bar{\Gamma}^c_{bd}\partial_e\phi - \bar{\Gamma}^c_{ad}\bar{\Gamma}^d_{bc} \quad (\text{eq15.116})
\end{aligned}$$

The above doesn't look much like equation (15). So, what do we do? First note that (eq15.116) represents the full R_{ab} , that is, equation (14). To isolate the contributions from ϕ we can first set $\bar{\Gamma}$ and its derivatives to zero (which in turn requires setting $\partial_a\bar{g}_{bc} = 0$). The result is equation (eq15.119) below. Having set $\bar{\Gamma}$ to zero means that we can replace ∂ with \bar{D} leading to equation (eq15.121). But that is clearly a tensor equation and so by the usual arguments it must be true in all frames (not just this frame with $\bar{\Gamma} = 0$). It's a standard argument and I've probably overdone the discussion. Anyway, equation (eq15.121) is exactly equation (15) from the paper. Yeah.

$$R_{ab}^\phi = -2\partial_{ab}\phi - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}^{cd}\partial_c\phi\partial_d\bar{g}_{ab} + 4\partial_a\phi\partial_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi + 2\bar{g}_{ac}\partial_d\phi\partial_b\bar{g}^{cd} + 2\bar{g}^{cd}\partial_c\phi\partial_b\bar{g}_{ad} \quad (\text{eq15.117})$$

$$= -2\partial_{ab}\phi - 2\bar{g}_{ab}\partial_c\phi\partial_d\bar{g}^{cd} - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi + 4\partial_a\phi\partial_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi + 2\bar{g}_{ac}\partial_d\phi\partial_b\bar{g}^{cd} \quad (\text{eq15.118})$$

$$= -2\partial_{ab}\phi - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi + 4\partial_a\phi\partial_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi \quad (\text{eq15.119})$$

$$= -2\bar{D}_{ab}\phi - 2\bar{g}_{ab}\bar{g}^{cd}\bar{D}_{cd}\phi + 4\partial_a\phi\partial_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi \quad (\text{eq15.120})$$

$$= -2\bar{D}_{ab}\phi - 2\bar{g}_{ab}\bar{g}^{cd}\bar{D}_{cd}\phi + 4\bar{D}_a\phi\bar{D}_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\bar{D}_c\phi\bar{D}_d\phi \quad (\text{eq15.121})$$

```

1  # -----
2  # Check against prd62.
3
4  foo := @(Rphi).                                # cdb(eq15.lcb,foo)
5  bah  = cdblib.get('prd62.eq15.rhs','prd62.json') # cdb(eq15.prd,bah)
6
7  diff := @(foo) - @(bah).
8
9  distribute      (diff)
10 diff = product_sort (diff)
11 rename_dummies (diff)
12 map_sympy      (diff, "simplify")
13 canonicalise   (diff)                        # cdb(eq15.chk,diff)

```

$$\text{eq15.lcb} := -2\bar{D}_{ab}\phi - 2\bar{g}_{ab}\bar{g}^{cd}\bar{D}_{cd}\phi + 4\bar{D}_a\phi\bar{D}_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\bar{D}_c\phi\bar{D}_d\phi$$

$$\text{eq15.prd} := -2\bar{D}_{ab}\phi - 2\bar{g}_{ab}\bar{g}^{cd}\bar{D}_{cd}\phi + 4\bar{D}_a\phi\bar{D}_b\phi - 4\bar{g}_{ab}\bar{g}^{cd}\bar{D}_c\phi\bar{D}_d\phi$$

$$\text{eq15.chk} := 0$$