PhysRevD.62.044034 equation (19)

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from shared import *
import cdblib

jsonfile = 'eqtn19.json'
cdblib.create (jsonfile)

defGiBar = cdblib.get ('defGiBar','eqtn17.json')

# ---------------------------------------------------------------------------
# DGiBarDt pt.1

dotgBar_{a b}::Symmetric.
dotgBar^{a b}::Symmetric.
dotgBar{#}::LaTeXForm("{\bar{dg}}").

dotGiBar := \partial_{t}{GammaBar^{i}}.            # cdb (eq19.101,dotGiBar)

substitute   (dotGiBar, defGiBar)                  # cdb (eq19.102,dotGiBar)
substitute   (dotGiBar, $\partial_{t a}{gBar^{i a}} -> \partial_{a}{dotgBar^{i a}}$)
                                                   # cdb (eq19.103,dotGiBar)

defdotgBarD := dotgBar_{i j} -> -2 N ABar_{i j}.
defdotgBarU := dotgBar^{i j} ->  2 N ABar^{i j}.
# defABarD2ABarU := ABar_{i j} -> ABar^{a b} gBar_{a i} gBar_{b j}.

substitute   (dotGiBar, defdotgBarU )              # cdb (eq19.104,dotGiBar)
product_rule (dotGiBar)                            # cdb (eq19.105,dotGiBar)

dotGiBar = product_sort (dotGiBar)                 # cdb (eq19.106,dotGiBar)

cdblib.put ('dotGiBar',dotGiBar,jsonfile)
```

$$\partial_t \bar{\Gamma}^i = -\partial_{tb} \bar{g}^{ib} \tag{eq19.102}$$

$$= -\partial_b \bar{dg}^{ib} \tag{eq19.103}$$

$$= -2\partial_b \left(N \bar{A}^{ib} \right) \tag{eq19.104}$$

$$= -2\partial_b N \bar{A}^{ib} - 2N \partial_b \bar{A}^{ib} \tag{eq19.105}$$

$$= -2\bar{A}^{ia} \partial_a N - 2N \partial_a \bar{A}^{ia} \tag{eq19.106}$$

```
# ------------------------------------------------------------------------
# Check against prd62.

foo := @(dotGiBar).                                    # cdb (eq19.lcb,foo)
bah  = cdblib.get('prd62.eq19.rhs','prd62.json')       # cdb (eq19.prd,bah)

diff := @(foo) - @(bah).

distribute     (diff)
product_rule   (diff)
diff = product_sort (diff)
rename_dummies (diff)
map_sympy      (diff, "simplify")
canonicalise   (diff)                                  # cdb (eq19.chk,diff)
```

$$\texttt{eq19.lcb} := -2\bar{A}^{ia}\partial_a N - 2N\partial_a \bar{A}^{ia}$$

$$\texttt{eq19.prd} := -2\partial_j \left(N\bar{A}^{ij}\right)$$

$$\texttt{eq19.chk} := 0$$