PhysRevD.67.084023 equation (19)

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from shared import *
import cdblib

jsonfile = 'hamiltonian.json'
cdblib.create (jsonfile)

# ----------------------------------------------------------------------------
# Hamiltonian constraint

Ham := R + K_{a b} g^{a b} K_{c d} g^{c d} - K_{a b} K_{c d} g^{a c} g^{b d}.  # cdb (Ham.101,Ham)

defK2ABarD := K_{i j} -> \exp(4\phi) ABar_{i j} + (1/3) g_{i j} trK.
defG2GBarD := g_{a b} -> \exp(4\phi) gBar_{a b}.
defG2GBarU := g^{a b} -> \exp(-4\phi) gBar^{a b}.

substitute     (Ham, defK2ABarD)   # cdb (Ham.102,Ham)
substitute     (Ham, defG2GBarD)   # cdb (Ham.103,Ham)
substitute     (Ham, defG2GBarU)   # cdb (Ham.104,Ham)
distribute     (Ham)               # cdb (Ham.105,Ham)
Ham = product_sort (Ham)           # cdb (Ham.106,Ham)
rename_dummies (Ham)               # cdb (Ham.107,Ham)
canonicalise   (Ham)               # cdb (Ham.108,Ham)
map_sympy      (Ham, "simplify")   # cdb (Ham.109,Ham)

foo := gBar_{a b} gBar^{a b} -> 3.
bah := gBar_{a c} gBar^{b c} -> gBar_{a}^{b}.

substitute (Ham, foo)              # cdb (Ham.110,Ham)
substitute (Ham, bah)              # cdb (Ham.111,Ham)
eliminate_kronecker (Ham)          # cdb (Ham.112,Ham)

foo := gBar_{a b} gBar^{a b} -> 3.
bah := gBar_{a}^{a} -> 3.
moo := ABar_{a b} gBar^{a b} -> 0.

substitute     (Ham, foo)          # cdb (Ham.113,Ham)
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substitute     (Ham, bah)          # cdb (Ham.114,Ham)
substitute     (Ham, moo)          # cdb (Ham.115,Ham)

foo := ABar_{c d} gBar^{c a} gBar^{d b} -> ABar^{a b}.

substitute     (Ham, foo)          # cdb (Ham.116,Ham)
rename_dummies (Ham)               # cdb (Ham.117,Ham)

cdblib.put ('Ham',Ham,jsonfile)
```

$$\mathcal{H} = R + K_{ab} g^{ab} K_{cd} g^{cd} - K_{ab} K_{cd} g^{ac} g^{bd} \tag{Ham.101}$$

$$= R + \left(\exp(4\phi)\, \bar{A}_{ab} + \frac{1}{3} g_{ab} \mathrm{tr} K \right) g^{ab} \left(\exp(4\phi)\, \bar{A}_{cd} + \frac{1}{3} g_{cd} \mathrm{tr} K \right) g^{cd} - \left(\exp(4\phi)\, \bar{A}_{ab} + \frac{1}{3} g_{ab} \mathrm{tr} K \right) \left(\exp(4\phi)\, \bar{A}_{cd} + \frac{1}{3} g_{cd} \mathrm{tr} K \right) g^{ac} g^{bd} \tag{Ham.102}$$

$$\begin{aligned} &= R + \left(\exp(4\phi)\, \bar{A}_{ab} + \frac{1}{3} \exp(4\phi)\, \bar{g}_{ab} \mathrm{tr} K \right) g^{ab} \left(\exp(4\phi)\, \bar{A}_{cd} + \frac{1}{3} \exp(4\phi)\, \bar{g}_{cd} \mathrm{tr} K \right) g^{cd} \\ &\quad - \left(\exp(4\phi)\, \bar{A}_{ab} + \frac{1}{3} \exp(4\phi)\, \bar{g}_{ab} \mathrm{tr} K \right) \left(\exp(4\phi)\, \bar{A}_{cd} + \frac{1}{3} \exp(4\phi)\, \bar{g}_{cd} \mathrm{tr} K \right) g^{ac} g^{bd} \end{aligned} \tag{Ham.103}$$

$$\begin{aligned} &= R + \left(\exp(4\phi)\, \bar{A}_{ab} + \frac{1}{3} \exp(4\phi)\, \bar{g}_{ab} \mathrm{tr} K \right) \exp(-4\phi)\, \bar{g}^{ab} \left(\exp(4\phi)\, \bar{A}_{cd} + \frac{1}{3} \exp(4\phi)\, \bar{g}_{cd} \mathrm{tr} K \right) \exp(-4\phi)\, \bar{g}^{cd} \\ &\quad - \left(\exp(4\phi)\, \bar{A}_{ab} + \frac{1}{3} \exp(4\phi)\, \bar{g}_{ab} \mathrm{tr} K \right) \left(\exp(4\phi)\, \bar{A}_{cd} + \frac{1}{3} \exp(4\phi)\, \bar{g}_{cd} \mathrm{tr} K \right) \exp(-4\phi)\, \bar{g}^{ac} \exp(-4\phi)\, \bar{g}^{bd} \end{aligned} \tag{Ham.104}$$

$$\begin{aligned} &= R + \exp(4\phi)\, \bar{A}_{ab} \exp(-4\phi)\, \bar{g}^{ab} \exp(4\phi)\, \bar{A}_{cd} \exp(-4\phi)\, \bar{g}^{cd} + \frac{1}{3} \exp(4\phi)\, \bar{A}_{ab} \exp(-4\phi)\, \bar{g}^{ab} \exp(4\phi)\, \bar{g}_{cd} \mathrm{tr} K \exp(-4\phi)\, \bar{g}^{cd} \\ &\quad + \frac{1}{3} \exp(4\phi)\, \bar{g}_{ab} \mathrm{tr} K \exp(-4\phi)\, \bar{g}^{ab} \exp(4\phi)\, \bar{A}_{cd} \exp(-4\phi)\, \bar{g}^{cd} + \frac{1}{9} \exp(4\phi)\, \bar{g}_{ab} \mathrm{tr} K \exp(-4\phi)\, \bar{g}^{ab} \exp(4\phi)\, \bar{g}_{cd} \mathrm{tr} K \exp(-4\phi)\, \bar{g}^{cd} \\ &\quad - \exp(4\phi)\, \bar{A}_{ab} \exp(4\phi)\, \bar{A}_{cd} \exp(-4\phi)\, \bar{g}^{ac} \exp(-4\phi)\, \bar{g}^{bd} - \frac{1}{3} \exp(4\phi)\, \bar{A}_{ab} \exp(4\phi)\, \bar{g}_{cd} \mathrm{tr} K \exp(-4\phi)\, \bar{g}^{ac} \exp(-4\phi)\, \bar{g}^{bd} \\ &\quad - \frac{1}{3} \exp(4\phi)\, \bar{g}_{ab} \mathrm{tr} K \exp(4\phi)\, \bar{A}_{cd} \exp(-4\phi)\, \bar{g}^{ac} \exp(-4\phi)\, \bar{g}^{bd} - \frac{1}{9} \exp(4\phi)\, \bar{g}_{ab} \mathrm{tr} K \exp(4\phi)\, \bar{g}_{cd} \mathrm{tr} K \exp(-4\phi)\, \bar{g}^{ac} \exp(-4\phi)\, \bar{g}^{bd} \end{aligned} \tag{Ham.105}$$

$$\begin{aligned} &= R + \bar{A}_{ab} \bar{A}_{cd} \bar{g}^{ab} \bar{g}^{cd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) + \frac{1}{3} \mathrm{tr} K \bar{A}_{ab} \bar{g}_{cd} \bar{g}^{ab} \bar{g}^{cd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) \\ &\quad + \frac{1}{3} \mathrm{tr} K \bar{A}_{ab} \bar{g}_{cd} \bar{g}^{cd} \bar{g}^{ab} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) + \frac{1}{9} \mathrm{tr} K \mathrm{tr} K \bar{g}_{ab} \bar{g}_{cd} \bar{g}^{ab} \bar{g}^{cd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) \\ &\quad - \bar{A}_{ab} \bar{A}_{cd} \bar{g}^{ac} \bar{g}^{bd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) - \frac{1}{3} \mathrm{tr} K \bar{A}_{ab} \bar{g}_{cd} \bar{g}^{ac} \bar{g}^{bd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) \\ &\quad - \frac{1}{3} \mathrm{tr} K \bar{A}_{ab} \bar{g}_{cd} \bar{g}^{ca} \bar{g}^{db} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) - \frac{1}{9} \mathrm{tr} K \mathrm{tr} K \bar{g}_{ab} \bar{g}_{cd} \bar{g}^{ac} \bar{g}^{bd} \exp(-4\phi) \exp(-4\phi) \exp(4\phi) \exp(4\phi) \end{aligned} \tag{Ham.106}$$

$$
\begin{aligned}
\mathcal{H} &= R + \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ab}\bar{g}^{cd}\exp(-4\,\phi)\exp(-4\,\phi)\exp(4\,\phi)\exp(4\,\phi) + \frac{1}{3}\,\mathrm{tr}K\bar{A}_{ab}\bar{g}_{cd}\bar{g}^{ab}\bar{g}^{cd}\exp(-4\,\phi)\exp(-4\,\phi)\exp(4\,\phi)\exp(4\,\phi) \\
&\quad + \frac{1}{3}\,\mathrm{tr}K\bar{A}_{ab}\bar{g}_{cd}\bar{g}^{cd}\bar{g}^{ab}\exp(-4\,\phi)\exp(-4\,\phi)\exp(4\,\phi)\exp(4\,\phi) + \frac{1}{9}\,\mathrm{tr}K\mathrm{tr}K\bar{g}_{ab}\bar{g}_{cd}\bar{g}^{ab}\bar{g}^{cd}\exp(-4\,\phi)\exp(-4\,\phi)\exp(4\,\phi)\exp(4\,\phi) \\
&\quad - \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ac}\bar{g}^{bd}\exp(-4\,\phi)\exp(-4\,\phi)\exp(4\,\phi)\exp(4\,\phi) - \frac{1}{3}\,\mathrm{tr}K\bar{A}_{ab}\bar{g}_{cd}\bar{g}^{ac}\bar{g}^{bd}\exp(-4\,\phi)\exp(-4\,\phi)\exp(4\,\phi)\exp(4\,\phi) \\
&\quad - \frac{1}{3}\,\mathrm{tr}K\bar{A}_{ab}\bar{g}_{cd}\bar{g}^{ca}\bar{g}^{db}\exp(-4\,\phi)\exp(-4\,\phi)\exp(4\,\phi)\exp(4\,\phi) - \frac{1}{9}\,\mathrm{tr}K\mathrm{tr}K\bar{g}_{ab}\bar{g}_{cd}\bar{g}^{ac}\bar{g}^{bd}\exp(-4\,\phi)\exp(-4\,\phi)\exp(4\,\phi)\exp(4\,\phi) \qquad \texttt{(Ham.107)} \\
&= R + \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ab}\bar{g}^{cd}\exp(-4\,\phi)\exp(-4\,\phi)\exp(4\,\phi)\exp(4\,\phi) + \frac{2}{3}\,\mathrm{tr}K\bar{A}_{ab}\bar{g}_{cd}\bar{g}^{ab}\bar{g}^{cd}\exp(-4\,\phi)\exp(-4\,\phi)\exp(4\,\phi)\exp(4\,\phi) \\
&\quad + \frac{1}{9}\,\mathrm{tr}K\mathrm{tr}K\bar{g}_{ab}\bar{g}_{cd}\bar{g}^{ab}\bar{g}^{cd}\exp(-4\,\phi)\exp(-4\,\phi)\exp(4\,\phi)\exp(4\,\phi) - \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ac}\bar{g}^{bd}\exp(-4\,\phi)\exp(-4\,\phi)\exp(4\,\phi)\exp(4\,\phi) \\
&\quad - \frac{2}{3}\,\mathrm{tr}K\bar{A}_{ab}\bar{g}_{cd}\bar{g}^{ac}\bar{g}^{bd}\exp(-4\,\phi)\exp(-4\,\phi)\exp(4\,\phi)\exp(4\,\phi) - \frac{1}{9}\,\mathrm{tr}K\mathrm{tr}K\bar{g}_{ab}\bar{g}_{cd}\bar{g}^{ac}\bar{g}^{bd}\exp(-4\,\phi)\exp(-4\,\phi)\exp(4\,\phi)\exp(4\,\phi) \qquad \texttt{(Ham.108)} \\
&= R + \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ab}\bar{g}^{cd} + \frac{2}{3}\,\mathrm{tr}K\bar{A}_{ab}\bar{g}_{cd}\bar{g}^{ab}\bar{g}^{cd} + \frac{1}{9}\,\mathrm{tr}K^2\,\bar{g}_{ab}\bar{g}_{cd}\bar{g}^{ab}\bar{g}^{cd} - \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ac}\bar{g}^{bd} - \frac{2}{3}\,\mathrm{tr}K\bar{A}_{ab}\bar{g}_{cd}\bar{g}^{ac}\bar{g}^{bd} - \frac{1}{9}\,\mathrm{tr}K^2\,\bar{g}_{ab}\bar{g}_{cd}\bar{g}^{ac}\bar{g}^{bd} \qquad \texttt{(Ham.109)} \\
&= R + \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ab}\bar{g}^{cd} + 2\,\mathrm{tr}K\bar{A}_{ab}\bar{g}^{ab} + \frac{1}{3}\,\mathrm{tr}K^2\,\bar{g}_{cd}\bar{g}^{cd} - \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ac}\bar{g}^{bd} - \frac{2}{3}\,\mathrm{tr}K\bar{A}_{ab}\bar{g}_{cd}\bar{g}^{ac}\bar{g}^{bd} - \frac{1}{9}\,\mathrm{tr}K^2\,\bar{g}_{ab}\bar{g}_{cd}\bar{g}^{ac}\bar{g}^{bd} \qquad \texttt{(Ham.110)} \\
&= R + \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ab}\bar{g}^{cd} + 2\,\mathrm{tr}K\bar{A}_{ab}\bar{g}^{ab} + \frac{1}{3}\,\mathrm{tr}K^2\,\bar{g}_{c}^{\ c} - \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ac}\bar{g}^{bd} - \frac{2}{3}\,\mathrm{tr}K\bar{A}_{ab}\bar{g}_{c}^{\ b}\bar{g}^{ac} - \frac{1}{9}\,\mathrm{tr}K^2\,\bar{g}_{ab}\bar{g}_{c}^{\ b}\bar{g}^{ac} \qquad \texttt{(Ham.111)} \\
&= R + \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ab}\bar{g}^{cd} + 2\,\mathrm{tr}K\bar{A}_{ab}\bar{g}^{ab} + \frac{1}{3}\,\mathrm{tr}K^2\,\bar{g}_{c}^{\ c} - \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ac}\bar{g}^{bd} - \frac{2}{3}\,\mathrm{tr}K\bar{A}_{ac}\bar{g}^{ac} - \frac{1}{9}\,\mathrm{tr}K^2\,\bar{g}_{ac}\bar{g}^{ac} \qquad \texttt{(Ham.112)} \\
&= R + \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ab}\bar{g}^{cd} + 2\,\mathrm{tr}K\bar{A}_{ab}\bar{g}^{ab} + \frac{1}{3}\,\mathrm{tr}K^2\,\bar{g}_{c}^{\ c} - \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ac}\bar{g}^{bd} - \frac{2}{3}\,\mathrm{tr}K\bar{A}_{ac}\bar{g}^{ac} - \frac{1}{3}\,\mathrm{tr}K^2 \qquad \texttt{(Ham.113)} \\
&= R + \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ab}\bar{g}^{cd} + 2\,\mathrm{tr}K\bar{A}_{ab}\bar{g}^{ab} + \frac{2}{3}\,\mathrm{tr}K^2 - \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ac}\bar{g}^{bd} - \frac{2}{3}\,\mathrm{tr}K\bar{A}_{ac}\bar{g}^{ac} \qquad \texttt{(Ham.114)} \\
&= R + \frac{2}{3}\,\mathrm{tr}K^2 - \bar{A}_{ab}\bar{A}_{cd}\bar{g}^{ac}\bar{g}^{bd} \qquad \texttt{(Ham.115)} \\
&= R + \frac{2}{3}\,\mathrm{tr}K^2 - \bar{A}^{cd}\bar{A}_{cd} \qquad \texttt{(Ham.116)} \\
&= R + \frac{2}{3}\,\mathrm{tr}K^2 - \bar{A}^{ab}\bar{A}_{ab} \qquad \texttt{(Ham.117)}
\end{aligned}
$$

```
# ------------------------------------------------------------------------
# Check against prd67.

foo := @(Ham).                                          # cdb(prd67.eq19.lcb,foo)
bah  = cdblib.get('prd67.eq19.rhs','prd67.json')        # cdb(prd67.eq19.prd,bah)

diff := @(foo) - @(bah).

distribute     (diff)
diff = product_sort (diff)
rename_dummies (diff)
map_sympy      (diff, "simplify")
canonicalise   (diff)                                   # cdb(prd67.eq19.chk,diff)
```

$$\texttt{prd67.eq19.lcb} := R + \frac{2}{3}\,\mathrm{tr}K^2 - \bar{A}^{ab}\bar{A}_{ab}$$

$$\texttt{prd67.eq19.prd} := R - \bar{A}_{ab}\bar{A}^{ab} + \frac{2}{3}\,\mathrm{tr}K^2$$

$$\texttt{prd67.eq19.chk} := 0$$