PhysRevD.62.044034 equation (12)

```
1   from shared import *
2   import cdblib
3
4   jsonfile = 'eqtn12.json'
5   cdblib.create (jsonfile)
6
7   DgijDt = cdblib.get ('adm.DgijDt','adm.json')
8   DKijDt = cdblib.get ('adm.DKijDt','adm.json')
9
10  DphiDt = cdblib.get ('DphiDt','eqtn10.json')
11  DKDt = cdblib.get ('DKDt','eqtn11.json')
12
13  # ----------------------------------------------------------------------
14  ABar2A := ABar_{i j} -> \exp(-4\phi) A_{i j}.          # prd62 eqn 08
15  A2ABar := A_{i j} -> \exp(4\phi) ABar_{i j}.          # prd62 eqn 08
16
17  Aij    := A_{i j} -> K_{i j} - (1/3) g_{i j} trK.     # prd62 eqn 07
18  Kij    := K_{i j} -> A_{i j} + (1/3) g_{i j} trK.     # prd62 eqn 07
19
20  gginv := {g_{i a} g^{a j} -> g_{i}^{j},
21           g_{i a} g^{j a} -> g_{i}^{j}}.
22
23  ABarUp := ABar_{i j} g^{j k} -> \exp(-4\phi) ABar_{i}^{k}.
24
25  ABardotABar := ABar_{i j} ABar^{i j} ->
26                 (K_{i j}-(1/3)g_{i j} trK) (K^{i j}-(1/3)g^{i j} trK).
27
28  trg := g_{i j} g^{i j} -> 3.
29
30  trK := {K_{i j} g^{i j} -> trK,
31          K^{i j} g_{i j} -> trK}.
32
33  Ham := trK**2 -> K_{i j} K^{i j} - g^{i j} R_{i j}.
34
35  # ----------------------------------------------------------------------
36  # dABarij/dt
```

```
37
38   dotABarij := \partial_{t}{ABar_{i j}}.          # cdb (eq12.101,dotABarij)
39
40   substitute       (dotABarij, ABar2A)           # cdb (eq12.102,dotABarij)
41   product_rule     (dotABarij)                   # cdb (eq12.103,dotABarij)
42   map_sympy        (dotABarij, "simplify")       # cdb (eq12.104,dotABarij)
43   substitute       (dotABarij, DphiDt)           # cdb (eq12.105,dotABarij)
44   substitute       (dotABarij, Aij)              # cdb (eq12.106,dotABarij)
45   distribute       (dotABarij)                   # cdb (eq12.107,dotABarij)
46   substitute       (dotABarij, DKijDt)           # cdb (eq12.108,dotABarij)
47   product_rule     (dotABarij)                   # cdb (eq12.109,dotABarij)
48   distribute       (dotABarij)                   # cdb (eq12.110,dotABarij)
49   substitute       (dotABarij, DKDt)             # cdb (eq12.111,dotABarij)
50   substitute       (dotABarij, DgijDt)           # cdb (eq12.112,dotABarij)
51   distribute       (dotABarij)                   # cdb (eq12.113,dotABarij)
52   substitute       (dotABarij, Kij)              # cdb (eq12.114,dotABarij)
53   distribute       (dotABarij)                   # cdb (eq12.115,dotABarij)
54   substitute       (dotABarij, gginv)            # cdb (eq12.116,dotABarij)
55   eliminate_kronecker (dotABarij)                # cdb (eq12.117,dotABarij)
56   substitute       (dotABarij, A2ABar)           # cdb (eq12.118,dotABarij)
57   canonicalise     (dotABarij)                   # cdb (eq12.119,dotABarij)
58   substitute       (dotABarij, ABardotABar)      # cdb (eq12.120,dotABarij)
59   distribute       (dotABarij)                   # cdb (eq12.121,dotABarij)
60   substitute       (dotABarij, trg)              # cdb (eq12.122,dotABarij)
61   substitute       (dotABarij, trK)              # cdb (eq12.123,dotABarij)
62   map_sympy        (dotABarij, "simplify")       # cdb (eq12.124,dotABarij)
63   substitute       (dotABarij, Ham)              # cdb (eq12.125,dotABarij)
64   distribute       (dotABarij)                   # cdb (eq12.126,dotABarij)
65   dotABarij = product_sort (dotABarij)           # cdb (eq12.127,dotABarij)
66   substitute       (dotABarij, ABarUp)           # cdb (eq12.128,dotABarij)
67   map_sympy        (dotABarij, "simplify")       # cdb (eq12.129,dotABarij)
68   factor_out       (dotABarij,$\exp(-4\phi)$)    # cdb (eq12.130,dotABarij)
69
70   DABarijDt := \partial_{t}{ABar_{ij}} -> @(dotABarij).
71
72   cdblib.put ('DABarijDt',DABarijDt,jsonfile)
```

$$\partial_t \bar{A}_{ij} = \partial_t(\exp\left(-4\,\phi\right) A_{ij}) \tag{eq12.102}$$

$$= \partial_t(\exp\left(-4\,\phi\right)) A_{ij} + \exp\left(-4\,\phi\right) \partial_t A_{ij} \tag{eq12.103}$$

$$= -4 \exp\left(-4\,\phi\right) \partial_t\phi A_{ij} + \exp\left(-4\,\phi\right) \partial_t A_{ij} \tag{eq12.104}$$

$$= \frac{2}{3} \exp\left(-4\,\phi\right) \mathrm{tr}KN A_{ij} + \exp\left(-4\,\phi\right) \partial_t A_{ij} \tag{eq12.105}$$

$$= \frac{2}{3} \exp\left(-4\,\phi\right) \mathrm{tr}KN \left(K_{ij} - \frac{1}{3}\, g_{ij}\mathrm{tr}K \right) + \exp\left(-4\,\phi\right) \partial_t \left(K_{ij} - \frac{1}{3}\, g_{ij}\mathrm{tr}K \right) \tag{eq12.106}$$

$$= \frac{2}{3} \exp\left(-4\,\phi\right) \mathrm{tr}KN K_{ij} - \frac{2}{9} \exp\left(-4\,\phi\right) \mathrm{tr}KN g_{ij}\mathrm{tr}K + \exp\left(-4\,\phi\right) \partial_t K_{ij} - \frac{1}{3} \exp\left(-4\,\phi\right) \partial_t(g_{ij}\mathrm{tr}K) \tag{eq12.107}$$

$$= \frac{2}{3} \exp\left(-4\,\phi\right) \mathrm{tr}KN K_{ij} - \frac{2}{9} \exp\left(-4\,\phi\right) \mathrm{tr}KN g_{ij}\mathrm{tr}K + \exp\left(-4\,\phi\right) \left(- D_{ij}N + N\left(R_{ij} + \mathrm{tr}K K_{ij} - 2\,K_{ic}K_{jd}g^{cd}\right) \right)$$
$$- \frac{1}{3} \exp\left(-4\,\phi\right) \partial_t(g_{ij}\mathrm{tr}K) \tag{eq12.108}$$

$$= \frac{2}{3} \exp\left(-4\,\phi\right) \mathrm{tr}KN K_{ij} - \frac{2}{9} \exp\left(-4\,\phi\right) \mathrm{tr}KN g_{ij}\mathrm{tr}K + \exp\left(-4\,\phi\right) \left(- D_{ij}N + N\left(R_{ij} + \mathrm{tr}K K_{ij} - 2\,K_{ic}K_{jd}g^{cd}\right) \right)$$
$$- \frac{1}{3} \exp\left(-4\,\phi\right) \left(\partial_t g_{ij}\mathrm{tr}K + g_{ij}\partial_t\mathrm{tr}K \right) \tag{eq12.109}$$

$$= \frac{2}{3} \exp\left(-4\,\phi\right) \mathrm{tr}KN K_{ij} - \frac{2}{9} \exp\left(-4\,\phi\right) \mathrm{tr}KN g_{ij}\mathrm{tr}K - \exp\left(-4\,\phi\right) D_{ij}N + \exp\left(-4\,\phi\right) N R_{ij} + \exp\left(-4\,\phi\right) N\mathrm{tr}K K_{ij}$$
$$- 2 \exp\left(-4\,\phi\right) N K_{ic}K_{jd}g^{cd} - \frac{1}{3} \exp\left(-4\,\phi\right) \partial_t g_{ij}\mathrm{tr}K - \frac{1}{3} \exp\left(-4\,\phi\right) g_{ij}\partial_t\mathrm{tr}K \tag{eq12.110}$$

$$= \frac{2}{3} \exp\left(-4\,\phi\right) \mathrm{tr}KN K_{ij} - \frac{2}{9} \exp\left(-4\,\phi\right) \mathrm{tr}KN g_{ij}\mathrm{tr}K - \exp\left(-4\,\phi\right) D_{ij}N + \exp\left(-4\,\phi\right) N R_{ij} + \exp\left(-4\,\phi\right) N\mathrm{tr}K K_{ij}$$
$$- 2 \exp\left(-4\,\phi\right) N K_{ic}K_{jd}g^{cd} - \frac{1}{3} \exp\left(-4\,\phi\right) \partial_t g_{ij}\mathrm{tr}K - \frac{1}{3} \exp\left(-4\,\phi\right) g_{ij} \left(- g^{ab}D_{ab}N + N\bar{A}_{ab}\bar{A}^{ab} + \frac{1}{3}\,\mathrm{tr}K^2\,N \right) \tag{eq12.111}$$

$$= \frac{2}{3} \exp\left(-4\,\phi\right) \mathrm{tr}KN K_{ij} - \frac{2}{9} \exp\left(-4\,\phi\right) \mathrm{tr}KN g_{ij}\mathrm{tr}K - \exp\left(-4\,\phi\right) D_{ij}N + \exp\left(-4\,\phi\right) N R_{ij} + \exp\left(-4\,\phi\right) N\mathrm{tr}K K_{ij}$$
$$- 2 \exp\left(-4\,\phi\right) N K_{ic}K_{jd}g^{cd} + \frac{2}{3} \exp\left(-4\,\phi\right) N K_{ij}\mathrm{tr}K - \frac{1}{3} \exp\left(-4\,\phi\right) g_{ij} \left(- g^{ab}D_{ab}N + N\bar{A}_{ab}\bar{A}^{ab} + \frac{1}{3}\,\mathrm{tr}K^2\,N \right) \tag{eq12.112}$$

$$\partial_t \bar{A}_{ij} = \frac{2}{3}\exp\left(-4\phi\right)\mathrm{tr}KNK_{ij} - \frac{2}{9}\exp\left(-4\phi\right)\mathrm{tr}KNg_{ij}\mathrm{tr}K - \exp\left(-4\phi\right)D_{ij}N + \exp\left(-4\phi\right)NR_{ij} + \exp\left(-4\phi\right)N\mathrm{tr}KK_{ij} - 2\exp\left(-4\phi\right)NK_{ic}K_{jd}g^{cd}$$

$$+ \frac{2}{3}\exp\left(-4\phi\right)NK_{ij}\mathrm{tr}K + \frac{1}{3}\exp\left(-4\phi\right)g_{ij}g^{ab}D_{ab}N - \frac{1}{3}\exp\left(-4\phi\right)g_{ij}N\bar{A}_{ab}\bar{A}^{ab} - \frac{1}{9}\exp\left(-4\phi\right)g_{ij}\mathrm{tr}K^2N \qquad (\text{eq12.113})$$

$$= \frac{2}{3}\exp\left(-4\phi\right)\mathrm{tr}KN\left(A_{ij} + \frac{1}{3}g_{ij}\mathrm{tr}K\right) - \frac{2}{9}\exp\left(-4\phi\right)\mathrm{tr}KNg_{ij}\mathrm{tr}K - \exp\left(-4\phi\right)D_{ij}N + \exp\left(-4\phi\right)NR_{ij}$$

$$+ \exp\left(-4\phi\right)N\mathrm{tr}K\left(A_{ij} + \frac{1}{3}g_{ij}\mathrm{tr}K\right) - 2\exp\left(-4\phi\right)N\left(A_{ic} + \frac{1}{3}g_{ic}\mathrm{tr}K\right)\left(A_{jd} + \frac{1}{3}g_{jd}\mathrm{tr}K\right)g^{cd} + \frac{2}{3}\exp\left(-4\phi\right)N\left(A_{ij} + \frac{1}{3}g_{ij}\mathrm{tr}K\right)\mathrm{tr}K$$

$$+ \frac{1}{3}\exp\left(-4\phi\right)g_{ij}g^{ab}D_{ab}N - \frac{1}{3}\exp\left(-4\phi\right)g_{ij}N\bar{A}_{ab}\bar{A}^{ab} - \frac{1}{9}\exp\left(-4\phi\right)g_{ij}\mathrm{tr}K^2N \qquad (\text{eq12.114})$$

$$= \frac{2}{3}\exp\left(-4\phi\right)\mathrm{tr}KNA_{ij} - \exp\left(-4\phi\right)D_{ij}N + \exp\left(-4\phi\right)NR_{ij} + \exp\left(-4\phi\right)N\mathrm{tr}KA_{ij} + \frac{1}{3}\exp\left(-4\phi\right)N\mathrm{tr}Kg_{ij}\mathrm{tr}K$$

$$- 2\exp\left(-4\phi\right)NA_{ic}A_{jd}g^{cd} - \frac{2}{3}\exp\left(-4\phi\right)NA_{ic}g_{jd}\mathrm{tr}Kg^{cd} - \frac{2}{3}\exp\left(-4\phi\right)Ng_{ic}\mathrm{tr}KA_{jd}g^{cd} - \frac{2}{9}\exp\left(-4\phi\right)Ng_{ic}\mathrm{tr}Kg_{jd}\mathrm{tr}Kg^{cd}$$

$$+ \frac{2}{3}\exp\left(-4\phi\right)NA_{ij}\mathrm{tr}K + \frac{2}{9}\exp\left(-4\phi\right)Ng_{ij}\mathrm{tr}K\mathrm{tr}K + \frac{1}{3}\exp\left(-4\phi\right)g_{ij}g^{ab}D_{ab}N - \frac{1}{3}\exp\left(-4\phi\right)g_{ij}N\bar{A}_{ab}\bar{A}^{ab} - \frac{1}{9}\exp\left(-4\phi\right)g_{ij}\mathrm{tr}K^2N \qquad (\text{eq12.115})$$

$$= \frac{2}{3}\exp\left(-4\phi\right)\mathrm{tr}KNA_{ij} - \exp\left(-4\phi\right)D_{ij}N + \exp\left(-4\phi\right)NR_{ij} + \exp\left(-4\phi\right)N\mathrm{tr}KA_{ij} + \frac{1}{3}\exp\left(-4\phi\right)N\mathrm{tr}Kg_{ij}\mathrm{tr}K$$

$$- 2\exp\left(-4\phi\right)NA_{ic}A_{jd}g^{cd} - \frac{2}{3}\exp\left(-4\phi\right)NA_{ic}g_j^c\mathrm{tr}K - \frac{2}{3}\exp\left(-4\phi\right)Ng_i^d\mathrm{tr}KA_{jd} - \frac{2}{9}\exp\left(-4\phi\right)Ng_i^d\mathrm{tr}Kg_{jd}\mathrm{tr}K$$

$$+ \frac{2}{3}\exp\left(-4\phi\right)NA_{ij}\mathrm{tr}K + \frac{2}{9}\exp\left(-4\phi\right)Ng_{ij}\mathrm{tr}K\mathrm{tr}K + \frac{1}{3}\exp\left(-4\phi\right)g_{ij}g^{ab}D_{ab}N - \frac{1}{3}\exp\left(-4\phi\right)g_{ij}N\bar{A}_{ab}\bar{A}^{ab} - \frac{1}{9}\exp\left(-4\phi\right)g_{ij}\mathrm{tr}K^2N \qquad (\text{eq12.116})$$

$$= \frac{2}{3}\exp\left(-4\phi\right)\mathrm{tr}KNA_{ij} - \exp\left(-4\phi\right)D_{ij}N + \exp\left(-4\phi\right)NR_{ij} + \exp\left(-4\phi\right)N\mathrm{tr}KA_{ij} + \frac{1}{3}\exp\left(-4\phi\right)N\mathrm{tr}Kg_{ij}\mathrm{tr}K$$

$$- 2\exp\left(-4\phi\right)NA_{ic}A_{jd}g^{cd} - \frac{2}{3}\exp\left(-4\phi\right)N\mathrm{tr}KA_{ji} - \frac{2}{9}\exp\left(-4\phi\right)N\mathrm{tr}Kg_{ji}\mathrm{tr}K + \frac{2}{9}\exp\left(-4\phi\right)Ng_{ij}\mathrm{tr}K\mathrm{tr}K + \frac{1}{3}\exp\left(-4\phi\right)g_{ij}g^{ab}D_{ab}N$$

$$- \frac{1}{3}\exp\left(-4\phi\right)g_{ij}N\bar{A}_{ab}\bar{A}^{ab} - \frac{1}{9}\exp\left(-4\phi\right)g_{ij}\mathrm{tr}K^2N \qquad (\text{eq12.117})$$

$$\partial_t \bar{A}_{ij} = \frac{2}{3} \exp\left(-4\,\phi\right) \mathrm{tr} K N \exp\left(4\,\phi\right) \bar{A}_{ij} - \exp\left(-4\,\phi\right) D_{ij} N + \exp\left(-4\,\phi\right) N R_{ij} + \exp\left(-4\,\phi\right) N \mathrm{tr} K \exp\left(4\,\phi\right) \bar{A}_{ij} + \frac{1}{3} \exp\left(-4\,\phi\right) N \mathrm{tr} K g_{ij} \mathrm{tr} K$$

$$- 2 \exp\left(-4\,\phi\right) N \exp\left(4\,\phi\right) \bar{A}_{ic} \exp\left(4\,\phi\right) \bar{A}_{jd} g^{cd} - \frac{2}{3} \exp\left(-4\,\phi\right) N \mathrm{tr} K \exp\left(4\,\phi\right) \bar{A}_{ji} - \frac{2}{9} \exp\left(-4\,\phi\right) N \mathrm{tr} K g_{ji} \mathrm{tr} K$$

$$+ \frac{2}{9} \exp\left(-4\,\phi\right) N g_{ij} \mathrm{tr} K \mathrm{tr} K + \frac{1}{3} \exp\left(-4\,\phi\right) g_{ij} g^{ab} D_{ab} N - \frac{1}{3} \exp\left(-4\,\phi\right) g_{ij} N \bar{A}_{ab} \bar{A}^{ab} - \frac{1}{9} \exp\left(-4\,\phi\right) g_{ij} \mathrm{tr} K^2 N \qquad \text{(eq12.118)}$$

$$= \frac{2}{3} \exp\left(-4\,\phi\right) \mathrm{tr} K N \exp\left(4\,\phi\right) \bar{A}_{ij} - \exp\left(-4\,\phi\right) D_{ij} N + \exp\left(-4\,\phi\right) N R_{ij} + \frac{1}{3} \exp\left(-4\,\phi\right) N \mathrm{tr} K \exp\left(4\,\phi\right) \bar{A}_{ij} + \frac{1}{9} \exp\left(-4\,\phi\right) N \mathrm{tr} K g_{ij} \mathrm{tr} K$$

$$- 2 \exp\left(-4\,\phi\right) N \exp\left(4\,\phi\right) \bar{A}_{ic} \exp\left(4\,\phi\right) \bar{A}_{jd} g^{cd} + \frac{2}{9} \exp\left(-4\,\phi\right) N g_{ij} \mathrm{tr} K \mathrm{tr} K + \frac{1}{3} \exp\left(-4\,\phi\right) g_{ij} g^{ab} D_{ab} N - \frac{1}{3} \exp\left(-4\,\phi\right) g_{ij} N \bar{A}_{ab} \bar{A}^{ab}$$

$$- \frac{1}{9} \exp\left(-4\,\phi\right) g_{ij} \mathrm{tr} K^2 N \qquad \text{(eq12.119)}$$

$$= \frac{2}{3} \exp\left(-4\,\phi\right) \mathrm{tr} K N \exp\left(4\,\phi\right) \bar{A}_{ij} - \exp\left(-4\,\phi\right) D_{ij} N + \exp\left(-4\,\phi\right) N R_{ij} + \frac{1}{3} \exp\left(-4\,\phi\right) N \mathrm{tr} K \exp\left(4\,\phi\right) \bar{A}_{ij}$$

$$+ \frac{1}{9} \exp\left(-4\,\phi\right) N \mathrm{tr} K g_{ij} \mathrm{tr} K - 2 \exp\left(-4\,\phi\right) N \exp\left(4\,\phi\right) \bar{A}_{ic} \exp\left(4\,\phi\right) \bar{A}_{jd} g^{cd} + \frac{2}{9} \exp\left(-4\,\phi\right) N g_{ij} \mathrm{tr} K \mathrm{tr} K + \frac{1}{3} \exp\left(-4\,\phi\right) g_{ij} g^{ab} D_{ab} N$$

$$- \frac{1}{3} \exp\left(-4\,\phi\right) g_{ij} N \left(K_{ab} - \frac{1}{3} g_{ab} \mathrm{tr} K \right) \left(K^{ab} - \frac{1}{3} g^{ab} \mathrm{tr} K \right) - \frac{1}{9} \exp\left(-4\,\phi\right) g_{ij} \mathrm{tr} K^2 N \qquad \text{(eq12.120)}$$

$$= \frac{2}{3} \exp\left(-4\,\phi\right) \mathrm{tr} K N \exp\left(4\,\phi\right) \bar{A}_{ij} - \exp\left(-4\,\phi\right) D_{ij} N + \exp\left(-4\,\phi\right) N R_{ij} + \frac{1}{3} \exp\left(-4\,\phi\right) N \mathrm{tr} K \exp\left(4\,\phi\right) \bar{A}_{ij} + \frac{1}{9} \exp\left(-4\,\phi\right) N \mathrm{tr} K g_{ij} \mathrm{tr} K$$

$$- 2 \exp\left(-4\,\phi\right) N \exp\left(4\,\phi\right) \bar{A}_{ic} \exp\left(4\,\phi\right) \bar{A}_{jd} g^{cd} + \frac{2}{9} \exp\left(-4\,\phi\right) N g_{ij} \mathrm{tr} K \mathrm{tr} K + \frac{1}{3} \exp\left(-4\,\phi\right) g_{ij} g^{ab} D_{ab} N - \frac{1}{3} \exp\left(-4\,\phi\right) g_{ij} N K_{ab} K^{ab}$$

$$+ \frac{1}{9} \exp\left(-4\,\phi\right) g_{ij} N K_{ab} g^{ab} \mathrm{tr} K + \frac{1}{9} \exp\left(-4\,\phi\right) g_{ij} N g_{ab} \mathrm{tr} K K^{ab} - \frac{1}{27} \exp\left(-4\,\phi\right) g_{ij} N g_{ab} \mathrm{tr} K g^{ab} \mathrm{tr} K - \frac{1}{9} \exp\left(-4\,\phi\right) g_{ij} \mathrm{tr} K^2 N \; \text{(eq12.121)}$$

$$\partial_t \bar{A}_{ij} = \frac{2}{3} \exp\left(-4\phi\right) \operatorname{tr}KN \exp\left(4\phi\right) \bar{A}_{ij} - \exp\left(-4\phi\right) D_{ij}N + \exp\left(-4\phi\right) NR_{ij} + \frac{1}{3} \exp\left(-4\phi\right) N\operatorname{tr}K \exp\left(4\phi\right) \bar{A}_{ij} + \frac{1}{9} \exp\left(-4\phi\right) N\operatorname{tr}Kg_{ij}\operatorname{tr}K$$

$$- 2 \exp\left(-4\phi\right) N \exp\left(4\phi\right) \bar{A}_{ic} \exp\left(4\phi\right) \bar{A}_{jd}g^{cd} + \frac{2}{9} \exp\left(-4\phi\right) Ng_{ij}\operatorname{tr}K\operatorname{tr}K + \frac{1}{3} \exp\left(-4\phi\right) g_{ij}g^{ab}D_{ab}N - \frac{1}{3} \exp\left(-4\phi\right) g_{ij}NK_{ab}K^{ab}$$

$$+ \frac{1}{9} \exp\left(-4\phi\right) g_{ij}NK_{ab}g^{ab}\operatorname{tr}K + \frac{1}{9} \exp\left(-4\phi\right) g_{ij}Ng_{ab}\operatorname{tr}KK^{ab} - \frac{1}{9} \exp\left(-4\phi\right) g_{ij}N\operatorname{tr}K\operatorname{tr}K - \frac{1}{9} \exp\left(-4\phi\right) g_{ij}\operatorname{tr}K^2 N \qquad \text{(eq12.122)}$$

$$= \frac{2}{3} \exp\left(-4\phi\right) \operatorname{tr}KN \exp\left(4\phi\right) \bar{A}_{ij} - \exp\left(-4\phi\right) D_{ij}N + \exp\left(-4\phi\right) NR_{ij} + \frac{1}{3} \exp\left(-4\phi\right) N\operatorname{tr}K \exp\left(4\phi\right) \bar{A}_{ij} + \frac{1}{9} \exp\left(-4\phi\right) N\operatorname{tr}Kg_{ij}\operatorname{tr}K$$

$$- 2 \exp\left(-4\phi\right) N \exp\left(4\phi\right) \bar{A}_{ic} \exp\left(4\phi\right) \bar{A}_{jd}g^{cd} + \frac{2}{9} \exp\left(-4\phi\right) Ng_{ij}\operatorname{tr}K\operatorname{tr}K + \frac{1}{3} \exp\left(-4\phi\right) g_{ij}g^{ab}D_{ab}N - \frac{1}{3} \exp\left(-4\phi\right) g_{ij}NK_{ab}K^{ab}$$

$$+ \frac{1}{9} \exp\left(-4\phi\right) g_{ij}N\operatorname{tr}K\operatorname{tr}K - \frac{1}{9} \exp\left(-4\phi\right) g_{ij}\operatorname{tr}K^2 N \qquad \text{(eq12.123)}$$

$$= \operatorname{tr}KN\bar{A}_{ij} - \exp\left(-4\phi\right) D_{ij}N + N \exp\left(-4\phi\right) R_{ij} + \frac{1}{3} \operatorname{tr}K^2 N \exp\left(-4\phi\right) g_{ij} - 2 N \exp\left(4\phi\right) \bar{A}_{ic}\bar{A}_{jd}g^{cd} + \frac{1}{3} \exp\left(-4\phi\right) g_{ij}g^{ab}D_{ab}N$$

$$- \frac{1}{3} N \exp\left(-4\phi\right) g_{ij}K_{ab}K^{ab} \qquad \text{(eq12.124)}$$

$$= \operatorname{tr}KN\bar{A}_{ij} - \exp\left(-4\phi\right) D_{ij}N + N \exp\left(-4\phi\right) R_{ij} + \frac{1}{3} \left(K_{ab}K^{ab} - g^{ab}R_{ab}\right) N \exp\left(-4\phi\right) g_{ij} - 2 N \exp\left(4\phi\right) \bar{A}_{ic}\bar{A}_{jd}g^{cd}$$

$$+ \frac{1}{3} \exp\left(-4\phi\right) g_{ij}g^{ab}D_{ab}N - \frac{1}{3} N \exp\left(-4\phi\right) g_{ij}K_{ab}K^{ab} \qquad \text{(eq12.125)}$$

$$= \operatorname{tr}KN\bar{A}_{ij} - \exp\left(-4\phi\right) D_{ij}N + N \exp\left(-4\phi\right) R_{ij} + \frac{1}{3} K_{ab}K^{ab}N \exp\left(-4\phi\right) g_{ij} - \frac{1}{3} g^{ab}R_{ab}N \exp\left(-4\phi\right) g_{ij} - 2 N \exp\left(4\phi\right) \bar{A}_{ic}\bar{A}_{jd}g^{cd}$$

$$+ \frac{1}{3} \exp\left(-4\phi\right) g_{ij}g^{ab}D_{ab}N - \frac{1}{3} N \exp\left(-4\phi\right) g_{ij}K_{ab}K^{ab} \qquad \text{(eq12.126)}$$

$$= N\operatorname{tr}K\bar{A}_{ij} - D_{ij}N \exp\left(-4\phi\right) + NR_{ij} \exp\left(-4\phi\right) - \frac{1}{3} Ng_{ij}g^{ab}R_{ab} \exp\left(-4\phi\right) - 2 N\bar{A}_{ia}\bar{A}_{jb}g^{ab} \exp\left(4\phi\right) + \frac{1}{3} g_{ij}g^{ab}D_{ab}N \exp\left(-4\phi\right) \quad \text{(eq12.127)}$$

$$\partial_t \bar{A}_{ij} = N \text{tr} K \bar{A}_{ij} - D_{ij} N \exp\left(-4\,\phi\right) + N R_{ij} \exp\left(-4\,\phi\right) - \frac{1}{3} N g_{ij} g^{ab} R_{ab} \exp\left(-4\,\phi\right) - 2\,N \exp\left(-4\,\phi\right) \bar{A}_i^{\ b} \bar{A}_{jb} \exp\left(4\,\phi\right)$$

$$+ \frac{1}{3}\, g_{ij} g^{ab} D_{ab} N \exp\left(-4\,\phi\right) \tag{eq12.128}$$

$$= \text{tr} K N \bar{A}_{ij} - D_{ij} N \exp\left(-4\,\phi\right) + N \exp\left(-4\,\phi\right) R_{ij} - \frac{1}{3} N \exp\left(-4\,\phi\right) g_{ij} g^{ab} R_{ab} - 2\,N \bar{A}_i^{\ b} \bar{A}_{jb} + \frac{1}{3}\, g_{ij} g^{ab} D_{ab} N \exp\left(-4\,\phi\right) \tag{eq12.129}$$

$$= \text{tr} K N \bar{A}_{ij} - 2\,N \bar{A}_i^{\ b} \bar{A}_{jb} + \exp\left(-4\,\phi\right) \left(- D_{ij} N + N R_{ij} - \frac{1}{3} N g_{ij} g^{ab} R_{ab} + \frac{1}{3}\, g_{ij} g^{ab} D_{ab} N \right) \tag{eq12.130}$$

```
1   # ------------------------------------------------------------------------
2   # Check against prd62.
3
4   foo := @(dotABarij).                                    # cdb(eq12.lcb,foo)
5   bah  = cdblib.get('prd62.eq12.rhs','prd62.json')        # cdb(eq12.prd,bah)
6
7   diff := @(foo) - @(bah).
8
9   foo := ABar_{a}^{b} -> gBar^{b c} ABar_{a c}.
10  bah := ABar^{a}_{b} -> gBar^{a c} ABar_{c b}.
11
12  substitute      (diff, foo)
13  substitute      (diff, bah)
14  distribute      (diff)
15  diff = product_sort (diff)
16  rename_dummies (diff)
17  map_sympy       (diff, "simplify")
18  canonicalise    (diff)                                  # cdb(eq12.chk,diff)
```

$$\mathtt{eq12.lcb} := \mathrm{tr} K N \bar{A}_{ij} - 2\, N \bar{A}_i^b \bar{A}_{jb} + \exp\left(-4\,\phi\right)\left(-\,D_{ij}N + NR_{ij} - \frac{1}{3}\,N g_{ij} g^{ab} R_{ab} + \frac{1}{3}\,g_{ij} g^{ab} D_{ab} N\right)$$

$$\mathtt{eq12.prd} := N\left(\mathrm{tr} K \bar{A}_{ij} - 2\,\bar{A}_{ia}\bar{A}_j^a\right) + \exp\left(-4\,\phi\right)\left(NR_{ij} -\,D_{ij}N - \frac{1}{3}\,g_{ij}\left(NR_{ab} -\,D_{ab}N\right)g^{ab}\right)$$

$$\mathtt{eq12.chk} := 0$$