

PhysRevD.62.044034 equation (12)

```
1  from shared import *
2  import cdblib
3
4  jsonfile = 'eqtn12.json'
5  cdblib.create (jsonfile)
6
7  DgijDt = cdblib.get ('adm.DgijDt','adm.json')
8  DKijDt = cdblib.get ('adm.DKijDt','adm.json')
9
10 DphiDt = cdblib.get ('DphiDt','eqtn10.json')
11 DKDt = cdblib.get ('DKDt','eqtn11.json')
12
13 # -----
14 ABar2A := ABar_{i j} -> \exp(-4\phi) A_{i j}.      # prd62 eqn 08
15 A2ABar := A_{i j} -> \exp(4\phi) ABar_{i j}.      # prd62 eqn 08
16
17 Aij     := A_{i j} -> K_{i j} - (1/3) g_{i j} trK.  # prd62 eqn 07
18 Kij     := K_{i j} -> A_{i j} + (1/3) g_{i j} trK.  # prd62 eqn 07
19
20 gginv := {g_{i a} g^{a j} -> g_{i}^{j},
21           g_{i a} g^{j a} -> g_{i}^{j}}.
22
23 ABarUp := ABar_{i j} g^{j k} -> \exp(-4\phi) ABar_{i}^{k}.
24
25 ABardotABar := ABar_{i j} ABar^{i j} ->
26               (K_{i j}-(1/3)g_{i j} trK) (K^{i j}-(1/3)g^{i j} trK).
27
28 trg := g_{i j} g^{i j} -> 3.
29
30 trK := {K_{i j} g^{i j} -> trK,
31         K^{i j} g_{i j} -> trK}.
32
33 Ham := trK**2 -> K_{i j} K^{i j} - g^{i j} R_{i j}.
34
35 # -----
36 # dABar_{ij}/dt
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37
38 dotABarij := \partial_{t}{ABar_{i j}}.          # cdb (eq12.101,dotABarij)
39
40 substitute      (dotABarij, ABar2A)           # cdb (eq12.102,dotABarij)
41 product_rule    (dotABarij)                   # cdb (eq12.103,dotABarij)
42 map_sympy       (dotABarij, "simplify")       # cdb (eq12.104,dotABarij)
43 substitute      (dotABarij, DphiDt)           # cdb (eq12.105,dotABarij)
44 substitute      (dotABarij, Aij)              # cdb (eq12.106,dotABarij)
45 distribute      (dotABarij)                   # cdb (eq12.107,dotABarij)
46 substitute      (dotABarij, DKijDt)           # cdb (eq12.108,dotABarij)
47 product_rule    (dotABarij)                   # cdb (eq12.109,dotABarij)
48 distribute      (dotABarij)                   # cdb (eq12.110,dotABarij)
49 substitute      (dotABarij, DKDt)             # cdb (eq12.111,dotABarij)
50 substitute      (dotABarij, DgijDt)           # cdb (eq12.112,dotABarij)
51 distribute      (dotABarij)                   # cdb (eq12.113,dotABarij)
52 substitute      (dotABarij, Kij)              # cdb (eq12.114,dotABarij)
53 distribute      (dotABarij)                   # cdb (eq12.115,dotABarij)
54 substitute      (dotABarij, gginv)            # cdb (eq12.116,dotABarij)
55 eliminate_kronecker (dotABarij)              # cdb (eq12.117,dotABarij)
56 substitute      (dotABarij, A2ABar)           # cdb (eq12.118,dotABarij)
57 canonicalise    (dotABarij)                   # cdb (eq12.119,dotABarij)
58 substitute      (dotABarij, ABardotABar)      # cdb (eq12.120,dotABarij)
59 distribute      (dotABarij)                   # cdb (eq12.121,dotABarij)
60 substitute      (dotABarij, trg)              # cdb (eq12.122,dotABarij)
61 substitute      (dotABarij, trK)              # cdb (eq12.123,dotABarij)
62 map_sympy       (dotABarij, "simplify")       # cdb (eq12.124,dotABarij)
63 substitute      (dotABarij, Ham)              # cdb (eq12.125,dotABarij)
64 distribute      (dotABarij)                   # cdb (eq12.126,dotABarij)
65 dotABarij = product_sort (dotABarij)          # cdb (eq12.127,dotABarij)
66 substitute      (dotABarij, ABarUp)           # cdb (eq12.128,dotABarij)
67 map_sympy       (dotABarij, "simplify")       # cdb (eq12.129,dotABarij)
68 factor_out      (dotABarij,$\exp(-4\phi)$)    # cdb (eq12.130,dotABarij)
69
70 DABarijDt := \partial_{t}{ABar_{ij}} -> @(dotABarij).
71
72 cdblib.put ('DABarijDt',DABarijDt,jsonfile)

```

$$\partial_t \bar{A}_{ij} = \partial_t (\exp(-4\phi) A_{ij}) \quad (\text{eq12.102})$$

$$= \partial_t (\exp(-4\phi)) A_{ij} + \exp(-4\phi) \partial_t A_{ij} \quad (\text{eq12.103})$$

$$= -4 \exp(-4\phi) \partial_t \phi A_{ij} + \exp(-4\phi) \partial_t A_{ij} \quad (\text{eq12.104})$$

$$= \frac{2}{3} \exp(-4\phi) \text{tr} K N A_{ij} + \exp(-4\phi) \partial_t A_{ij} \quad (\text{eq12.105})$$

$$= \frac{2}{3} \exp(-4\phi) \text{tr} K N \left(K_{ij} - \frac{1}{3} g_{ij} \text{tr} K \right) + \exp(-4\phi) \partial_t \left(K_{ij} - \frac{1}{3} g_{ij} \text{tr} K \right) \quad (\text{eq12.106})$$

$$= \frac{2}{3} \exp(-4\phi) \text{tr} K N K_{ij} - \frac{2}{9} \exp(-4\phi) \text{tr} K N g_{ij} \text{tr} K + \exp(-4\phi) \partial_t K_{ij} - \frac{1}{3} \exp(-4\phi) \partial_t (g_{ij} \text{tr} K) \quad (\text{eq12.107})$$

$$= \frac{2}{3} \exp(-4\phi) \text{tr} K N K_{ij} - \frac{2}{9} \exp(-4\phi) \text{tr} K N g_{ij} \text{tr} K + \exp(-4\phi) (-D_{ij} N + N (R_{ij} + \text{tr} K K_{ij} - 2K_{ic} K_{jd} g^{cd})) - \frac{1}{3} \exp(-4\phi) \partial_t (g_{ij} \text{tr} K) \quad (\text{eq12.108})$$

$$= \frac{2}{3} \exp(-4\phi) \text{tr} K N K_{ij} - \frac{2}{9} \exp(-4\phi) \text{tr} K N g_{ij} \text{tr} K + \exp(-4\phi) (-D_{ij} N + N (R_{ij} + \text{tr} K K_{ij} - 2K_{ic} K_{jd} g^{cd})) - \frac{1}{3} \exp(-4\phi) (\partial_t g_{ij} \text{tr} K + g_{ij} \partial_t \text{tr} K) \quad (\text{eq12.109})$$

$$= \frac{2}{3} \exp(-4\phi) \text{tr} K N K_{ij} - \frac{2}{9} \exp(-4\phi) \text{tr} K N g_{ij} \text{tr} K - \exp(-4\phi) D_{ij} N + \exp(-4\phi) N R_{ij} + \exp(-4\phi) N \text{tr} K K_{ij} - 2 \exp(-4\phi) N K_{ic} K_{jd} g^{cd} - \frac{1}{3} \exp(-4\phi) \partial_t g_{ij} \text{tr} K - \frac{1}{3} \exp(-4\phi) g_{ij} \partial_t \text{tr} K \quad (\text{eq12.110})$$

$$= \frac{2}{3} \exp(-4\phi) \text{tr} K N K_{ij} - \frac{2}{9} \exp(-4\phi) \text{tr} K N g_{ij} \text{tr} K - \exp(-4\phi) D_{ij} N + \exp(-4\phi) N R_{ij} + \exp(-4\phi) N \text{tr} K K_{ij} - 2 \exp(-4\phi) N K_{ic} K_{jd} g^{cd} - \frac{1}{3} \exp(-4\phi) \partial_t g_{ij} \text{tr} K - \frac{1}{3} \exp(-4\phi) g_{ij} \left(-g^{ab} D_{ab} N + N \bar{A}_{ab} \bar{A}^{ab} + \frac{1}{3} \text{tr} K^2 N \right) \quad (\text{eq12.111})$$

$$= \frac{2}{3} \exp(-4\phi) \text{tr} K N K_{ij} - \frac{2}{9} \exp(-4\phi) \text{tr} K N g_{ij} \text{tr} K - \exp(-4\phi) D_{ij} N + \exp(-4\phi) N R_{ij} + \exp(-4\phi) N \text{tr} K K_{ij} - 2 \exp(-4\phi) N K_{ic} K_{jd} g^{cd} + \frac{2}{3} \exp(-4\phi) N K_{ij} \text{tr} K - \frac{1}{3} \exp(-4\phi) g_{ij} \left(-g^{ab} D_{ab} N + N \bar{A}_{ab} \bar{A}^{ab} + \frac{1}{3} \text{tr} K^2 N \right) \quad (\text{eq12.112})$$

$$\begin{aligned}
\partial_t \bar{A}_{ij} &= \frac{2}{3} \exp(-4\phi) \text{tr} K N K_{ij} - \frac{2}{9} \exp(-4\phi) \text{tr} K N g_{ij} \text{tr} K - \exp(-4\phi) D_{ij} N + \exp(-4\phi) N R_{ij} + \exp(-4\phi) N \text{tr} K K_{ij} - 2 \exp(-4\phi) N K_{ic} K_{jd} g^{cd} \\
&\quad + \frac{2}{3} \exp(-4\phi) N K_{ij} \text{tr} K + \frac{1}{3} \exp(-4\phi) g_{ij} g^{ab} D_{ab} N - \frac{1}{3} \exp(-4\phi) g_{ij} N \bar{A}_{ab} \bar{A}^{ab} - \frac{1}{9} \exp(-4\phi) g_{ij} \text{tr} K^2 N
\end{aligned} \tag{eq12.113}$$

$$\begin{aligned}
&= \frac{2}{3} \exp(-4\phi) \text{tr} K N \left(A_{ij} + \frac{1}{3} g_{ij} \text{tr} K \right) - \frac{2}{9} \exp(-4\phi) \text{tr} K N g_{ij} \text{tr} K - \exp(-4\phi) D_{ij} N + \exp(-4\phi) N R_{ij} \\
&\quad + \exp(-4\phi) N \text{tr} K \left(A_{ij} + \frac{1}{3} g_{ij} \text{tr} K \right) - 2 \exp(-4\phi) N \left(A_{ic} + \frac{1}{3} g_{ic} \text{tr} K \right) \left(A_{jd} + \frac{1}{3} g_{jd} \text{tr} K \right) g^{cd} + \frac{2}{3} \exp(-4\phi) N \left(A_{ij} + \frac{1}{3} g_{ij} \text{tr} K \right) \text{tr} K \\
&\quad + \frac{1}{3} \exp(-4\phi) g_{ij} g^{ab} D_{ab} N - \frac{1}{3} \exp(-4\phi) g_{ij} N \bar{A}_{ab} \bar{A}^{ab} - \frac{1}{9} \exp(-4\phi) g_{ij} \text{tr} K^2 N
\end{aligned} \tag{eq12.114}$$

$$\begin{aligned}
&= \frac{2}{3} \exp(-4\phi) \text{tr} K N A_{ij} - \exp(-4\phi) D_{ij} N + \exp(-4\phi) N R_{ij} + \exp(-4\phi) N \text{tr} K A_{ij} + \frac{1}{3} \exp(-4\phi) N \text{tr} K g_{ij} \text{tr} K - 2 \exp(-4\phi) N A_{ic} A_{jd} g^{cd} \\
&\quad - \frac{2}{3} \exp(-4\phi) N A_{ic} g_{jd} \text{tr} K g^{cd} - \frac{2}{3} \exp(-4\phi) N g_{ic} \text{tr} K A_{jd} g^{cd} - \frac{2}{9} \exp(-4\phi) N g_{ic} \text{tr} K g_{jd} \text{tr} K g^{cd} + \frac{2}{3} \exp(-4\phi) N A_{ij} \text{tr} K \\
&\quad + \frac{2}{9} \exp(-4\phi) N g_{ij} \text{tr} K \text{tr} K + \frac{1}{3} \exp(-4\phi) g_{ij} g^{ab} D_{ab} N - \frac{1}{3} \exp(-4\phi) g_{ij} N \bar{A}_{ab} \bar{A}^{ab} - \frac{1}{9} \exp(-4\phi) g_{ij} \text{tr} K^2 N
\end{aligned} \tag{eq12.115}$$

$$\begin{aligned}
&= \frac{2}{3} \exp(-4\phi) \text{tr} K N A_{ij} - \exp(-4\phi) D_{ij} N + \exp(-4\phi) N R_{ij} + \exp(-4\phi) N \text{tr} K A_{ij} + \frac{1}{3} \exp(-4\phi) N \text{tr} K g_{ij} \text{tr} K - 2 \exp(-4\phi) N A_{ic} A_{jd} g^{cd} \\
&\quad - \frac{2}{3} \exp(-4\phi) N A_{ic} g_j^c \text{tr} K - \frac{2}{3} \exp(-4\phi) N g_i^d \text{tr} K A_{jd} - \frac{2}{9} \exp(-4\phi) N g_i^d \text{tr} K g_{jd} \text{tr} K + \frac{2}{3} \exp(-4\phi) N A_{ij} \text{tr} K \\
&\quad + \frac{2}{9} \exp(-4\phi) N g_{ij} \text{tr} K \text{tr} K + \frac{1}{3} \exp(-4\phi) g_{ij} g^{ab} D_{ab} N - \frac{1}{3} \exp(-4\phi) g_{ij} N \bar{A}_{ab} \bar{A}^{ab} - \frac{1}{9} \exp(-4\phi) g_{ij} \text{tr} K^2 N
\end{aligned} \tag{eq12.116}$$

$$\begin{aligned}
&= \frac{2}{3} \exp(-4\phi) \text{tr} K N A_{ij} - \exp(-4\phi) D_{ij} N + \exp(-4\phi) N R_{ij} + \exp(-4\phi) N \text{tr} K A_{ij} + \frac{1}{3} \exp(-4\phi) N \text{tr} K g_{ij} \text{tr} K - 2 \exp(-4\phi) N A_{ic} A_{jd} g^{cd} \\
&\quad - \frac{2}{3} \exp(-4\phi) N \text{tr} K A_{ji} - \frac{2}{9} \exp(-4\phi) N \text{tr} K g_{ji} \text{tr} K + \frac{2}{9} \exp(-4\phi) N g_{ij} \text{tr} K \text{tr} K + \frac{1}{3} \exp(-4\phi) g_{ij} g^{ab} D_{ab} N - \frac{1}{3} \exp(-4\phi) g_{ij} N \bar{A}_{ab} \bar{A}^{ab} \\
&\quad - \frac{1}{9} \exp(-4\phi) g_{ij} \text{tr} K^2 N
\end{aligned} \tag{eq12.117}$$

$$\begin{aligned}
\partial_t \bar{A}_{ij} = & \frac{2}{3} \exp(-4\phi) \text{tr} K N \exp(4\phi) \bar{A}_{ij} - \exp(-4\phi) D_{ij} N + \exp(-4\phi) N R_{ij} + \exp(-4\phi) N \text{tr} K \exp(4\phi) \bar{A}_{ij} + \frac{1}{3} \exp(-4\phi) N \text{tr} K g_{ij} \text{tr} K \\
& - 2 \exp(-4\phi) N \exp(4\phi) \bar{A}_{ic} \exp(4\phi) \bar{A}_{jd} g^{cd} - \frac{2}{3} \exp(-4\phi) N \text{tr} K \exp(4\phi) \bar{A}_{ji} - \frac{2}{9} \exp(-4\phi) N \text{tr} K g_{ji} \text{tr} K + \frac{2}{9} \exp(-4\phi) N g_{ij} \text{tr} K \text{tr} K \\
& + \frac{1}{3} \exp(-4\phi) g_{ij} g^{ab} D_{ab} N - \frac{1}{3} \exp(-4\phi) g_{ij} N \bar{A}_{ab} \bar{A}^{ab} - \frac{1}{9} \exp(-4\phi) g_{ij} \text{tr} K^2 N \quad (\text{eq12.118})
\end{aligned}$$

$$\begin{aligned}
= & \frac{2}{3} \exp(-4\phi) \text{tr} K N \exp(4\phi) \bar{A}_{ij} - \exp(-4\phi) D_{ij} N + \exp(-4\phi) N R_{ij} + \frac{1}{3} \exp(-4\phi) N \text{tr} K \exp(4\phi) \bar{A}_{ij} + \frac{1}{9} \exp(-4\phi) N \text{tr} K g_{ij} \text{tr} K \\
& - 2 \exp(-4\phi) N \exp(4\phi) \bar{A}_{ic} \exp(4\phi) \bar{A}_{jd} g^{cd} + \frac{2}{9} \exp(-4\phi) N g_{ij} \text{tr} K \text{tr} K + \frac{1}{3} \exp(-4\phi) g_{ij} g^{ab} D_{ab} N - \frac{1}{3} \exp(-4\phi) g_{ij} N \bar{A}_{ab} \bar{A}^{ab} \\
& - \frac{1}{9} \exp(-4\phi) g_{ij} \text{tr} K^2 N \quad (\text{eq12.119})
\end{aligned}$$

$$\begin{aligned}
= & \frac{2}{3} \exp(-4\phi) \text{tr} K N \exp(4\phi) \bar{A}_{ij} - \exp(-4\phi) D_{ij} N + \exp(-4\phi) N R_{ij} + \frac{1}{3} \exp(-4\phi) N \text{tr} K \exp(4\phi) \bar{A}_{ij} + \frac{1}{9} \exp(-4\phi) N \text{tr} K g_{ij} \text{tr} K \\
& - 2 \exp(-4\phi) N \exp(4\phi) \bar{A}_{ic} \exp(4\phi) \bar{A}_{jd} g^{cd} + \frac{2}{9} \exp(-4\phi) N g_{ij} \text{tr} K \text{tr} K + \frac{1}{3} \exp(-4\phi) g_{ij} g^{ab} D_{ab} N \\
& - \frac{1}{3} \exp(-4\phi) g_{ij} N \left(K_{ab} - \frac{1}{3} g_{ab} \text{tr} K \right) \left(K^{ab} - \frac{1}{3} g^{ab} \text{tr} K \right) - \frac{1}{9} \exp(-4\phi) g_{ij} \text{tr} K^2 N \quad (\text{eq12.120})
\end{aligned}$$

$$\begin{aligned}
= & \frac{2}{3} \exp(-4\phi) \text{tr} K N \exp(4\phi) \bar{A}_{ij} - \exp(-4\phi) D_{ij} N + \exp(-4\phi) N R_{ij} + \frac{1}{3} \exp(-4\phi) N \text{tr} K \exp(4\phi) \bar{A}_{ij} + \frac{1}{9} \exp(-4\phi) N \text{tr} K g_{ij} \text{tr} K \\
& - 2 \exp(-4\phi) N \exp(4\phi) \bar{A}_{ic} \exp(4\phi) \bar{A}_{jd} g^{cd} + \frac{2}{9} \exp(-4\phi) N g_{ij} \text{tr} K \text{tr} K + \frac{1}{3} \exp(-4\phi) g_{ij} g^{ab} D_{ab} N - \frac{1}{3} \exp(-4\phi) g_{ij} N K_{ab} K^{ab} \\
& + \frac{1}{9} \exp(-4\phi) g_{ij} N K_{ab} g^{ab} \text{tr} K + \frac{1}{9} \exp(-4\phi) g_{ij} N g_{ab} \text{tr} K K^{ab} - \frac{1}{27} \exp(-4\phi) g_{ij} N g_{ab} \text{tr} K g^{ab} \text{tr} K - \frac{1}{9} \exp(-4\phi) g_{ij} \text{tr} K^2 N \quad (\text{eq12.121})
\end{aligned}$$

$$\begin{aligned}
\partial_t \bar{A}_{ij} = & \frac{2}{3} \exp(-4\phi) \text{tr} K N \exp(4\phi) \bar{A}_{ij} - \exp(-4\phi) D_{ij} N + \exp(-4\phi) N R_{ij} + \frac{1}{3} \exp(-4\phi) N \text{tr} K \exp(4\phi) \bar{A}_{ij} + \frac{1}{9} \exp(-4\phi) N \text{tr} K g_{ij} \text{tr} K \\
& - 2 \exp(-4\phi) N \exp(4\phi) \bar{A}_{ic} \exp(4\phi) \bar{A}_{jd} g^{cd} + \frac{2}{9} \exp(-4\phi) N g_{ij} \text{tr} K \text{tr} K + \frac{1}{3} \exp(-4\phi) g_{ij} g^{ab} D_{ab} N - \frac{1}{3} \exp(-4\phi) g_{ij} N K_{ab} K^{ab} \\
& + \frac{1}{9} \exp(-4\phi) g_{ij} N K_{ab} g^{ab} \text{tr} K + \frac{1}{9} \exp(-4\phi) g_{ij} N g_{ab} \text{tr} K K^{ab} - \frac{1}{9} \exp(-4\phi) g_{ij} N \text{tr} K \text{tr} K - \frac{1}{9} \exp(-4\phi) g_{ij} \text{tr} K^2 N \quad (\text{eq12.122})
\end{aligned}$$

$$\begin{aligned}
= & \frac{2}{3} \exp(-4\phi) \text{tr} K N \exp(4\phi) \bar{A}_{ij} - \exp(-4\phi) D_{ij} N + \exp(-4\phi) N R_{ij} + \frac{1}{3} \exp(-4\phi) N \text{tr} K \exp(4\phi) \bar{A}_{ij} + \frac{1}{9} \exp(-4\phi) N \text{tr} K g_{ij} \text{tr} K \\
& - 2 \exp(-4\phi) N \exp(4\phi) \bar{A}_{ic} \exp(4\phi) \bar{A}_{jd} g^{cd} + \frac{2}{9} \exp(-4\phi) N g_{ij} \text{tr} K \text{tr} K + \frac{1}{3} \exp(-4\phi) g_{ij} g^{ab} D_{ab} N - \frac{1}{3} \exp(-4\phi) g_{ij} N K_{ab} K^{ab} \\
& + \frac{1}{9} \exp(-4\phi) g_{ij} N \text{tr} K \text{tr} K - \frac{1}{9} \exp(-4\phi) g_{ij} \text{tr} K^2 N \quad (\text{eq12.123})
\end{aligned}$$

$$\begin{aligned}
= & \text{tr} K N \bar{A}_{ij} - \exp(-4\phi) D_{ij} N + N \exp(-4\phi) R_{ij} + \frac{1}{3} \text{tr} K^2 N \exp(-4\phi) g_{ij} - 2N \exp(4\phi) \bar{A}_{ic} \bar{A}_{jd} g^{cd} + \frac{1}{3} \exp(-4\phi) g_{ij} g^{ab} D_{ab} N \\
& - \frac{1}{3} N \exp(-4\phi) g_{ij} K_{ab} K^{ab} \quad (\text{eq12.124})
\end{aligned}$$

$$\begin{aligned}
= & \text{tr} K N \bar{A}_{ij} - \exp(-4\phi) D_{ij} N + N \exp(-4\phi) R_{ij} + \frac{1}{3} (K_{ab} K^{ab} - g^{ab} R_{ab}) N \exp(-4\phi) g_{ij} - 2N \exp(4\phi) \bar{A}_{ic} \bar{A}_{jd} g^{cd} \\
& + \frac{1}{3} \exp(-4\phi) g_{ij} g^{ab} D_{ab} N - \frac{1}{3} N \exp(-4\phi) g_{ij} K_{ab} K^{ab} \quad (\text{eq12.125})
\end{aligned}$$

$$\begin{aligned}
= & \text{tr} K N \bar{A}_{ij} - \exp(-4\phi) D_{ij} N + N \exp(-4\phi) R_{ij} + \frac{1}{3} K_{ab} K^{ab} N \exp(-4\phi) g_{ij} - \frac{1}{3} g^{ab} R_{ab} N \exp(-4\phi) g_{ij} - 2N \exp(4\phi) \bar{A}_{ic} \bar{A}_{jd} g^{cd} \\
& + \frac{1}{3} \exp(-4\phi) g_{ij} g^{ab} D_{ab} N - \frac{1}{3} N \exp(-4\phi) g_{ij} K_{ab} K^{ab} \quad (\text{eq12.126})
\end{aligned}$$

$$= N \text{tr} K \bar{A}_{ij} - D_{ij} N \exp(-4\phi) + N R_{ij} \exp(-4\phi) - \frac{1}{3} N g_{ij} g^{ab} R_{ab} \exp(-4\phi) - 2N \bar{A}_{ia} \bar{A}_{jb} g^{ab} \exp(4\phi) + \frac{1}{3} g_{ij} g^{ab} D_{ab} N \exp(-4\phi) \quad (\text{eq12.127})$$

$$\begin{aligned}
\partial_t \bar{A}_{ij} = & N \text{tr} K \bar{A}_{ij} - D_{ij} N \exp(-4\phi) + N R_{ij} \exp(-4\phi) - \frac{1}{3} N g_{ij} g^{ab} R_{ab} \exp(-4\phi) - 2N \exp(-4\phi) \bar{A}_i{}^b \bar{A}_{jb} \exp(4\phi) \\
& + \frac{1}{3} g_{ij} g^{ab} D_{ab} N \exp(-4\phi)
\end{aligned} \tag{eq12.128}$$

$$= \text{tr} K N \bar{A}_{ij} - D_{ij} N \exp(-4\phi) + N \exp(-4\phi) R_{ij} - \frac{1}{3} N \exp(-4\phi) g_{ij} g^{ab} R_{ab} - 2N \bar{A}_i{}^b \bar{A}_{jb} + \frac{1}{3} g_{ij} g^{ab} D_{ab} N \exp(-4\phi) \tag{eq12.129}$$

$$= \text{tr} K N \bar{A}_{ij} - 2N \bar{A}_i{}^b \bar{A}_{jb} + \exp(-4\phi) \left(-D_{ij} N + N R_{ij} - \frac{1}{3} N g_{ij} g^{ab} R_{ab} + \frac{1}{3} g_{ij} g^{ab} D_{ab} N \right) \tag{eq12.130}$$

```

1  # -----
2  # Check against prd62.
3
4  foo := @(dotABarij).                # cdb(eq12.lcb,foo)
5  bah  = cdblib.get('prd62.eq12.rhs','prd62.json') # cdb(eq12.prd,bah)
6
7  diff := @(foo) - @(bah).
8
9  foo := ABar_{a}^{b} -> gBar^{b c} ABar_{a c}.
10 bah := ABar^{a}_{a}_{b} -> gBar^{a c} ABar_{c b}.
11
12 substitute      (diff, foo)
13 substitute      (diff, bah)
14 distribute      (diff)
15 diff = product_sort (diff)
16 rename_dummies  (diff)
17 map_sympy       (diff, "simplify")
18 canonicalise    (diff)                # cdb(eq12.chk,diff)

```

$$\text{eq12.lcb} := \text{tr} K N \bar{A}_{ij} - 2N \bar{A}_i^b \bar{A}_{jb} + \exp(-4\phi) \left(-D_{ij}N + N R_{ij} - \frac{1}{3} N g_{ij} g^{ab} R_{ab} + \frac{1}{3} g_{ij} g^{ab} D_{ab}N \right)$$

$$\text{eq12.prd} := N \left(\text{tr} K \bar{A}_{ij} - 2\bar{A}_{ia} \bar{A}^a_j \right) + \exp(-4\phi) \left(N R_{ij} - D_{ij}N - \frac{1}{3} g_{ij} (N R_{ab} - D_{ab}N) g^{ab} \right)$$

$$\text{eq12.chk} := 0$$