

PhysRevD.62.044034 equation (18)

```
1  from shared import *
2  import cdblib
3
4  jsonfile = 'eqtn18.json'
5  cdblib.create (jsonfile)
6
7  # -----
8  # RBar pt.1 = split into two terms
9
10 defGammaBar := GammaBar^{a}_{b c} ->
11             (1/2) gBar^{a e} ( \partial_{b}{gBar_{e c}}
12                               + \partial_{c}{gBar_{b e}}
13                               - \partial_{e}{gBar_{b c}}).
14
15 defRiemBar := RBar^{a}_{b c d} ->
16             \partial_{c}{GammaBar^{a}_{b d}} + GammaBar^{a}_{e c} GammaBar^{e}_{b d}
17             - \partial_{d}{GammaBar^{a}_{b c}} - GammaBar^{a}_{e d} GammaBar^{e}_{b c}.
18
19 defRBar := RBar_{a b} -> RBar^{c}_{a c b}.
20
21 RBar := RBar_{a b}.                                # cdb(eq18.000,RBar)
22
23 substitute (RBar, defRBar)                          # cdb(eq18.001,RBar)
24 substitute (RBar, defRiemBar)                       # cdb(eq18.002,RBar)
25 substitute (RBar, $GammaBar^{a}_{b a} -> 0$)        # cdb(eq18.003,RBar) # follows from det g = 1
26 canonicalise (RBar)
```

$$\bar{R}_{ab} = \bar{R}^c{}_{acb} \tag{eq18.001}$$

$$= \partial_c \bar{\Gamma}^c{}_{ab} + \bar{\Gamma}^c{}_{ec} \bar{\Gamma}^e{}_{ab} - \partial_b \bar{\Gamma}^c{}_{ac} - \bar{\Gamma}^c{}_{eb} \bar{\Gamma}^e{}_{ac} \tag{eq18.002}$$

$$= \partial_c \bar{\Gamma}^c{}_{ab} - \bar{\Gamma}^c{}_{eb} \bar{\Gamma}^e{}_{ac} \tag{eq18.003}$$

From here the computations will be splt into two threads, one for each of the two terms in the above result.

```

1  # -----
2  # get tmpA & tmpB from RBar
3
4  GammaBar^{a}_{b c}::Weight(label=numG).
5  \partial{#}::WeightInherit(label=all, type=multiplicative).
6
7  tmpA := @(RBar).                                # cdb(tmp18.101,tmpA)
8  keep_weight (tmpA, $numG=1$)                     # cdb(tmp18.102,tmpA) # the derivative terms
9
10 tmpB := @(RBar).
11 keep_weight (tmpB, $numG=2$)                     # cdb(tmp18.103,tmpB) # the quadrtaic terms

```

$$\text{tmp18.101} := \partial_c \bar{\Gamma}^c_{ab} - \bar{\Gamma}^c_{ae} \bar{\Gamma}^e_{bc}$$

$$\text{tmp18.102} := \partial_c \bar{\Gamma}^c_{ab}$$

$$\text{tmp18.103} := -\bar{\Gamma}^c_{ae} \bar{\Gamma}^e_{bc}$$

```

1  # -----
2  # tmpA pt. 1
3
4  substitute (tmpA, defGammaBar) # cdb(tmp18.201,tmpA)
5  distribute (tmpA) # cdb(tmp18.202,tmpA)
6  product_rule (tmpA) # cdb(tmp18.203,tmpA)
7  substitute (tmpA, $\partial_{a}\{\bar{g}^{b}\} \rightarrow -\Gamma^{b}_{a}$) # cdb(tmp18.204,tmpA)

```

$$\partial_c \bar{\Gamma}^c_{ab} = \frac{1}{2} \partial_c (\bar{g}^{ce} (\partial_a \bar{g}_{eb} + \partial_b \bar{g}_{ae} - \partial_e \bar{g}_{ab})) \quad (\text{tmp18.201})$$

$$= \frac{1}{2} \partial_c (\bar{g}^{ce} \partial_a \bar{g}_{eb}) + \frac{1}{2} \partial_c (\bar{g}^{ce} \partial_b \bar{g}_{ae}) - \frac{1}{2} \partial_c (\bar{g}^{ce} \partial_e \bar{g}_{ab}) \quad (\text{tmp18.202})$$

$$= \frac{1}{2} \partial_c \bar{g}^{ce} \partial_a \bar{g}_{eb} + \frac{1}{2} \bar{g}^{ce} \partial_{ca} \bar{g}_{eb} + \frac{1}{2} \partial_c \bar{g}^{ce} \partial_b \bar{g}_{ae} + \frac{1}{2} \bar{g}^{ce} \partial_{cb} \bar{g}_{ae} - \frac{1}{2} \partial_c \bar{g}^{ce} \partial_e \bar{g}_{ab} - \frac{1}{2} \bar{g}^{ce} \partial_{ce} \bar{g}_{ab} \quad (\text{tmp18.203})$$

$$= -\frac{1}{2} \bar{\Gamma}^e \partial_a \bar{g}_{eb} + \frac{1}{2} \bar{g}^{ce} \partial_{ca} \bar{g}_{eb} - \frac{1}{2} \bar{\Gamma}^e \partial_b \bar{g}_{ae} + \frac{1}{2} \bar{g}^{ce} \partial_{cb} \bar{g}_{ae} + \frac{1}{2} \bar{\Gamma}^e \partial_e \bar{g}_{ab} - \frac{1}{2} \bar{g}^{ce} \partial_{ce} \bar{g}_{ab} \quad (\text{tmp18.204})$$

Notice that this result contains two terms containing second derivatives of \bar{g}_{ij} . This pair of terms will now be replaced with an expression built from the first derivatives of Γ^i .

```

1  # -----
2  # tmpC
3
4  defGi := GammaBar{i} -> - \partial_{j}{gBar{i j}}.
5
6  # lower the indices on gBar{b c}
7
8  defLowerIndices := \partial_{a}{gBar{b c}} -> - gBar{i b} gBar{j c} \partial_{a}{gBar_{i j}}.
9
10 substitute (defGi, defLowerIndices)
11
12 tmpC := gBar_{a i} \partial_{b}{GammaBar{i}}
13        + gBar_{b i} \partial_{a}{GammaBar{i}}. # cdb(tmp18.301,tmpC)
14
15 saveC := @(tmpC).
16
17 substitute (tmpC, defGi) # cdb(tmp18.302,tmpC)
18 product_rule (tmpC) # cdb(tmp18.303,tmpC)
19 distribute (tmpC) # cdb(tmp18.304,tmpC)
20 canonicalise (tmpC) # cdb(tmp18.305,tmpC)
21 substitute (tmpC, $gBar_{a b} gBar{b c} -> gBar_{a}^{c}$) # cdb(tmp18.306,tmpC)
22 eliminate_kronecker (tmpC) # cdb(tmp18.307,tmpC)
23
24 # foo is the target expression to be moved to the lhs
25
26 foo := gBar{i j} \partial_{a i}{gBar_{b j}}
27        + gBar{i j} \partial_{b i}{gBar_{a j}} -> X_{a b}.
28
29 # bah helps when rebuilding the equation
30
31 bah := X_{a b} ->
32        gBar{i j} \partial_{a i}{gBar_{b j}}
33        + gBar{i j} \partial_{b i}{gBar_{a j}}.
34
35 substitute (tmpC, foo) # cdb(tmp18.308,tmpC)
36
37 #

```

```

38  # rearrange to move the target to the lhs
39
40  tmpE := @(tmpC).
41  tmpF := @(tmpC).
42
43  X_{a b}::Weight(label=numX).
44
45  # get the two pieces of the equation
46  keep_weight (tmpE, $numX=0$)           # cdb(tmp18.309,tmpE)
47  keep_weight (tmpF, $numX=1$)           # cdb(tmp18.310,tmpF)
48
49  substitute (tmpF, bah)                  # cdb(tmp18.311,tmpF)
50
51  # now rebuild with terms reordered
52  tmpG := @(saveC) - @(tmpE).             # cdb(tmp18.312,tmpG)
53
54  defTmpSub := @(tmpF) -> @(tmpG).         # cdb(tmp18.313,defTmpSub)

```

$$\bar{g}_{ai}\partial_b\bar{\Gamma}^i + \bar{g}_{bi}\partial_a\bar{\Gamma}^i = \bar{g}_{ai}\partial_b(\bar{g}^{ci}\bar{g}^{dj}\partial_j\bar{g}_{cd}) + \bar{g}_{bi}\partial_a(\bar{g}^{ci}\bar{g}^{dj}\partial_j\bar{g}_{cd}) \quad (\text{tmp18.302})$$

$$= \bar{g}_{ai}(\partial_b\bar{g}^{ci}\bar{g}^{dj}\partial_j\bar{g}_{cd} + \bar{g}^{ci}\partial_b\bar{g}^{dj}\partial_j\bar{g}_{cd} + \bar{g}^{ci}\bar{g}^{dj}\partial_{bj}\bar{g}_{cd}) + \bar{g}_{bi}(\partial_a\bar{g}^{ci}\bar{g}^{dj}\partial_j\bar{g}_{cd} + \bar{g}^{ci}\partial_a\bar{g}^{dj}\partial_j\bar{g}_{cd} + \bar{g}^{ci}\bar{g}^{dj}\partial_{aj}\bar{g}_{cd}) \quad (\text{tmp18.303})$$

$$= \bar{g}_{ai}\partial_b\bar{g}^{ci}\bar{g}^{dj}\partial_j\bar{g}_{cd} + \bar{g}_{ai}\bar{g}^{ci}\partial_b\bar{g}^{dj}\partial_j\bar{g}_{cd} + \bar{g}_{ai}\bar{g}^{ci}\bar{g}^{dj}\partial_{bj}\bar{g}_{cd} + \bar{g}_{bi}\partial_a\bar{g}^{ci}\bar{g}^{dj}\partial_j\bar{g}_{cd} + \bar{g}_{bi}\bar{g}^{ci}\partial_a\bar{g}^{dj}\partial_j\bar{g}_{cd} + \bar{g}_{bi}\bar{g}^{ci}\bar{g}^{dj}\partial_{aj}\bar{g}_{cd} \quad (\text{tmp18.304})$$

$$= \bar{g}_{ac}\partial_b\bar{g}^{cd}\bar{g}^{ij}\partial_i\bar{g}_{dj} + \bar{g}_{ac}\bar{g}^{cd}\partial_b\bar{g}^{ij}\partial_i\bar{g}_{dj} + \bar{g}_{ac}\bar{g}^{cd}\bar{g}^{ij}\partial_{bi}\bar{g}_{dj} + \bar{g}_{bc}\partial_a\bar{g}^{cd}\bar{g}^{ij}\partial_i\bar{g}_{dj} + \bar{g}_{bc}\bar{g}^{cd}\partial_a\bar{g}^{ij}\partial_i\bar{g}_{dj} + \bar{g}_{bc}\bar{g}^{cd}\bar{g}^{ij}\partial_{ai}\bar{g}_{dj} \quad (\text{tmp18.305})$$

$$= \bar{g}_{ac}\partial_b\bar{g}^{cd}\bar{g}^{ij}\partial_i\bar{g}_{dj} + \bar{g}_a{}^d\partial_b\bar{g}^{ij}\partial_i\bar{g}_{dj} + \bar{g}_a{}^d\bar{g}^{ij}\partial_{bi}\bar{g}_{dj} + \bar{g}_{bc}\partial_a\bar{g}^{cd}\bar{g}^{ij}\partial_i\bar{g}_{dj} + \bar{g}_b{}^d\partial_a\bar{g}^{ij}\partial_i\bar{g}_{dj} + \bar{g}_b{}^d\bar{g}^{ij}\partial_{ai}\bar{g}_{dj} \quad (\text{tmp18.306})$$

$$= \bar{g}_{ac}\partial_b\bar{g}^{cd}\bar{g}^{ij}\partial_i\bar{g}_{dj} + \partial_b\bar{g}^{ij}\partial_i\bar{g}_{aj} + \bar{g}^{ij}\partial_{bi}\bar{g}_{aj} + \bar{g}_{bc}\partial_a\bar{g}^{cd}\bar{g}^{ij}\partial_i\bar{g}_{dj} + \partial_a\bar{g}^{ij}\partial_i\bar{g}_{bj} + \bar{g}^{ij}\partial_{ai}\bar{g}_{bj} \quad (\text{tmp18.307})$$

$$= \bar{g}_{ac}\partial_b\bar{g}^{cd}\bar{g}^{ij}\partial_i\bar{g}_{dj} + \partial_b\bar{g}^{ij}\partial_i\bar{g}_{aj} + X_{ba} + \bar{g}_{bc}\partial_a\bar{g}^{cd}\bar{g}^{ij}\partial_i\bar{g}_{dj} + \partial_a\bar{g}^{ij}\partial_i\bar{g}_{bj} \quad (\text{tmp18.308})$$

$$\bar{g}^{ij}\partial_{bi}\bar{g}_{aj} + \bar{g}^{ij}\partial_{ai}\bar{g}_{bj} = X_{ba} \quad (\text{tmp18.310})$$

$$= \bar{g}_{ai}\partial_b\bar{\Gamma}^i + \bar{g}_{bi}\partial_a\bar{\Gamma}^i - \bar{g}_{ac}\partial_b\bar{g}^{cd}\bar{g}^{ij}\partial_i\bar{g}_{dj} - \partial_b\bar{g}^{ij}\partial_i\bar{g}_{aj} - \bar{g}_{bc}\partial_a\bar{g}^{cd}\bar{g}^{ij}\partial_i\bar{g}_{dj} - \partial_a\bar{g}^{ij}\partial_i\bar{g}_{bj} \quad (\text{tmp18.312})$$

This result will now be applied to the earlier equation (tmp18.204).

```

1  # -----
2  # tmpA pt.2 eliminate second partial derivatives of gBar
3
4  canonicalise      (tmpA)                                # cdb(tmp18.401,tmpA)
5
6  substitute        (tmpA, defTmpSub)                      # cdb(tmp18.402,tmpA)
7  tmpA = product_sort (tmpA)
8  rename_dummies    (tmpA)
9  canonicalise      (tmpA)                                # cdb(tmp18.403,tmpA)
10
11  foo := gBar^{d e} \partial_{c}{gBar_{e f}} -> - gBar_{e f} \partial_{c}{gBar^{d e}}.
12  bah := \partial_{d}{gBar^{d f}} -> - GammaBar^{f}.
13
14  substitute        (tmpA, foo)                            # cdb(tmp18.404,tmpA)
15  substitute        (tmpA, bah)                            # cdb(tmp18.405,tmpA)
16
17  foo := gBar_{e f} \partial_{a}{gBar^{c f}} -> - \partial_{a}{gBar_{e f}} gBar^{c f}.
18
19  substitute        (tmpA, foo)                            # cdb(tmp18.406,tmpA)
20
21  foo := gBar_{b d} gBar^{d e} -> gBar_{b}^{e}.
22
23  substitute        (tmpA, foo)                            # cdb(tmp18.407,tmpA)
24  eliminate_kronecker (tmpA)                              # cdb(tmp18.408,tmpA)
25  tmpA = product_sort (tmpA)
26  rename_dummies    (tmpA)
27  canonicalise      (tmpA)                                # cdb(tmp18.409,tmpA)

```


$$\partial_c \bar{\Gamma}^c_{ab} = -\frac{1}{2} \bar{\Gamma}^e_a \partial_a \bar{g}_{be} + \frac{1}{2} \bar{g}^{ce} \partial_{ac} \bar{g}_{be} - \frac{1}{2} \bar{\Gamma}^e_b \partial_b \bar{g}_{ae} + \frac{1}{2} \bar{g}^{ce} \partial_{bc} \bar{g}_{ae} + \frac{1}{2} \bar{\Gamma}^e_e \partial_e \bar{g}_{ab} - \frac{1}{2} \bar{g}^{ce} \partial_{ce} \bar{g}_{ab} \quad (\text{tmp18.401})$$

$$\begin{aligned}
&= -\frac{1}{2}\bar{\Gamma}^e\partial_a\bar{g}_{be} + \frac{1}{2}\bar{g}_{bc}\partial_a\bar{\Gamma}^c + \frac{1}{2}\bar{g}_{ac}\partial_b\bar{\Gamma}^c - \frac{1}{2}\bar{g}_{bf}\partial_a\bar{g}^{fd}\bar{g}^{ce}\partial_c\bar{g}_{de} - \frac{1}{2}\partial_a\bar{g}^{ce}\partial_c\bar{g}_{be} - \frac{1}{2}\bar{g}_{af}\partial_b\bar{g}^{fd}\bar{g}^{ce}\partial_c\bar{g}_{de} - \frac{1}{2}\partial_b\bar{g}^{ce}\partial_c\bar{g}_{ae} - \frac{1}{2}\bar{\Gamma}^e\partial_b\bar{g}_{ae} + \frac{1}{2}\bar{\Gamma}^e\partial_e\bar{g}_{ab} \\
&\quad - \frac{1}{2}\bar{g}^{ce}\partial_{ce}\bar{g}_{ab} \tag{tmp18.402}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{1}{2}\bar{\Gamma}^c\partial_a\bar{g}_{bc} + \frac{1}{2}\bar{g}_{bc}\partial_a\bar{\Gamma}^c + \frac{1}{2}\bar{g}_{ac}\partial_b\bar{\Gamma}^c - \frac{1}{2}\bar{g}_{bc}\bar{g}^{de}\partial_d\bar{g}_{ef}\partial_a\bar{g}^{cf} - \frac{1}{2}\partial_c\bar{g}_{bd}\partial_a\bar{g}^{cd} - \frac{1}{2}\bar{g}_{ac}\bar{g}^{de}\partial_d\bar{g}_{ef}\partial_b\bar{g}^{cf} - \frac{1}{2}\partial_c\bar{g}_{ad}\partial_b\bar{g}^{cd} - \frac{1}{2}\bar{\Gamma}^c\partial_b\bar{g}_{ac} + \frac{1}{2}\bar{\Gamma}^c\partial_c\bar{g}_{ab} \\
&\quad - \frac{1}{2}\bar{g}^{cd}\partial_{cd}\bar{g}_{ab}
\end{aligned}
\tag{tmp18.403}$$

$$\begin{aligned}
&= -\frac{1}{2}\bar{\Gamma}^c\partial_a\bar{g}_{bc} + \frac{1}{2}\bar{g}_{bc}\partial_a\bar{\Gamma}^c + \frac{1}{2}\bar{g}_{ac}\partial_b\bar{\Gamma}^c + \frac{1}{2}\bar{g}_{bc}\bar{g}_{ef}\partial_d\bar{g}^{de}\partial_a\bar{g}^{cf} - \frac{1}{2}\partial_c\bar{g}_{bd}\partial_a\bar{g}^{cd} + \frac{1}{2}\bar{g}_{ac}\bar{g}_{ef}\partial_d\bar{g}^{de}\partial_b\bar{g}^{cf} - \frac{1}{2}\partial_c\bar{g}_{ad}\partial_b\bar{g}^{cd} - \frac{1}{2}\bar{\Gamma}^c\partial_b\bar{g}_{ac} + \frac{1}{2}\bar{\Gamma}^c\partial_c\bar{g}_{ab} \\
&\quad - \frac{1}{2}\bar{g}^{cd}\partial_{cd}\bar{g}_{ab} \tag{tmp18.404}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{1}{2}\bar{\Gamma}^c\partial_a\bar{g}_{bc} + \frac{1}{2}\bar{g}_{bc}\partial_a\bar{\Gamma}^c + \frac{1}{2}\bar{g}_{ac}\partial_b\bar{\Gamma}^c - \frac{1}{2}\bar{g}_{bc}\bar{g}_{ef}\bar{\Gamma}^e\partial_a\bar{g}^{cf} - \frac{1}{2}\partial_c\bar{g}_{bd}\partial_a\bar{g}^{cd} - \frac{1}{2}\bar{g}_{ac}\bar{g}_{ef}\bar{\Gamma}^e\partial_b\bar{g}^{cf} - \frac{1}{2}\partial_c\bar{g}_{ad}\partial_b\bar{g}^{cd} - \frac{1}{2}\bar{\Gamma}^c\partial_b\bar{g}_{ac} + \frac{1}{2}\bar{\Gamma}^c\partial_c\bar{g}_{ab} \\
&\quad - \frac{1}{2}\bar{g}^{cd}\partial_{cd}\bar{g}_{ab}
\end{aligned}
\tag{tmp18.405}$$

$$\begin{aligned}
&= -\frac{1}{2}\bar{\Gamma}^c\partial_a\bar{g}_{bc} + \frac{1}{2}\bar{g}_{bc}\partial_a\bar{\Gamma}^c + \frac{1}{2}\bar{g}_{ac}\partial_b\bar{\Gamma}^c + \frac{1}{2}\bar{g}_{bc}\partial_a\bar{g}_{ef}\bar{g}^{ef}\bar{\Gamma}^e - \frac{1}{2}\partial_c\bar{g}_{bd}\partial_a\bar{g}^{cd} + \frac{1}{2}\bar{g}_{ac}\partial_b\bar{g}_{ef}\bar{g}^{ef}\bar{\Gamma}^e - \frac{1}{2}\partial_c\bar{g}_{ad}\partial_b\bar{g}^{cd} - \frac{1}{2}\bar{\Gamma}^c\partial_b\bar{g}_{ac} + \frac{1}{2}\bar{\Gamma}^c\partial_c\bar{g}_{ab} \\
&\quad - \frac{1}{2}\bar{g}^{cd}\partial_{cd}\bar{g}_{ab}
\end{aligned}
\tag{tmp18.406}$$

$$\begin{aligned}
&= -\frac{1}{2}\bar{\Gamma}^c\partial_a\bar{g}_{bc} + \frac{1}{2}\bar{g}_{bc}\partial_a\bar{\Gamma}^c + \frac{1}{2}\bar{g}_{ac}\partial_b\bar{\Gamma}^c + \frac{1}{2}\bar{g}_b{}^f\partial_a\bar{g}_{ef}\bar{\Gamma}^e - \frac{1}{2}\partial_c\bar{g}_{bd}\partial_a\bar{g}^{cd} + \frac{1}{2}\bar{g}_a{}^f\partial_b\bar{g}_{ef}\bar{\Gamma}^e - \frac{1}{2}\partial_c\bar{g}_{ad}\partial_b\bar{g}^{cd} - \frac{1}{2}\bar{\Gamma}^c\partial_b\bar{g}_{ac} + \frac{1}{2}\bar{\Gamma}^c\partial_c\bar{g}_{ab} \\
&\quad - \frac{1}{2}\bar{g}^{cd}\partial_{cd}\bar{g}_{ab} \tag{tmp18.407}
\end{aligned}$$

$$= -\frac{1}{2}\bar{\Gamma}^c\partial_a\bar{g}_{bc} + \frac{1}{2}\bar{g}_{bc}\partial_a\bar{\Gamma}^c + \frac{1}{2}\bar{g}_{ac}\partial_b\bar{\Gamma}^c + \frac{1}{2}\partial_a\bar{g}_{eb}\bar{\Gamma}^e - \frac{1}{2}\partial_c\bar{g}_{bd}\partial_a\bar{g}^{cd} + \frac{1}{2}\partial_b\bar{g}_{ea}\bar{\Gamma}^e - \frac{1}{2}\partial_c\bar{g}_{ad}\partial_b\bar{g}^{cd} - \frac{1}{2}\bar{\Gamma}^c\partial_b\bar{g}_{ac} + \frac{1}{2}\bar{\Gamma}^c\partial_c\bar{g}_{ab} - \frac{1}{2}\bar{g}^{cd}\partial_{cd}\bar{g}_{ab} \quad (\text{tmp18.408})$$

$$= \frac{1}{2} \bar{g}_{bc} \partial_a \bar{\Gamma}^c + \frac{1}{2} \bar{g}_{ac} \partial_b \bar{\Gamma}^c - \frac{1}{2} \partial_c \bar{g}_{bd} \partial_a \bar{g}^{cd} - \frac{1}{2} \partial_c \bar{g}_{ad} \partial_b \bar{g}^{cd} + \frac{1}{2} \bar{\Gamma}^c \partial_c \bar{g}_{ab} - \frac{1}{2} \bar{g}^{cd} \partial_{cd} \bar{g}_{ab} \quad (\text{tmp18.409})$$

```

1 # -----
2 # tmpB
3
4 substitute      (tmpB, defGammaBar)          # cdb(tmp18.501,tmpB)
5 distribute      (tmpB)                        # cdb(tmp18.502,tmpB)
6 tmpB = product_sort (tmpB)                    # cdb(tmp18.503,tmpB)
7 rename_dummies  (tmpB)                        # cdb(tmp18.504,tmpB)
8 canonicalise    (tmpB)                        # cdb(tmp18.505,tmpB)

```

$$-\bar{\Gamma}_{ae}^c \bar{\Gamma}_{bc}^e = -\frac{1}{4} \bar{g}^{cd} (\partial_a \bar{g}_{de} + \partial_e \bar{g}_{ad} - \partial_d \bar{g}_{ae}) \bar{g}^{ef} (\partial_b \bar{g}_{fc} + \partial_c \bar{g}_{bf} - \partial_f \bar{g}_{bc}) \quad (\text{tmp18.501})$$

$$\begin{aligned}
&= -\frac{1}{4} \bar{g}^{cd} \partial_a \bar{g}_{de} \bar{g}^{ef} \partial_b \bar{g}_{fc} - \frac{1}{4} \bar{g}^{cd} \partial_a \bar{g}_{de} \bar{g}^{ef} \partial_c \bar{g}_{bf} + \frac{1}{4} \bar{g}^{cd} \partial_a \bar{g}_{de} \bar{g}^{ef} \partial_f \bar{g}_{bc} - \frac{1}{4} \bar{g}^{cd} \partial_e \bar{g}_{ad} \bar{g}^{ef} \partial_b \bar{g}_{fc} - \frac{1}{4} \bar{g}^{cd} \partial_e \bar{g}_{ad} \bar{g}^{ef} \partial_c \bar{g}_{bf} + \frac{1}{4} \bar{g}^{cd} \partial_e \bar{g}_{ad} \bar{g}^{ef} \partial_f \bar{g}_{bc} \\
&\quad + \frac{1}{4} \bar{g}^{cd} \partial_d \bar{g}_{ae} \bar{g}^{ef} \partial_b \bar{g}_{fc} + \frac{1}{4} \bar{g}^{cd} \partial_d \bar{g}_{ae} \bar{g}^{ef} \partial_c \bar{g}_{bf} - \frac{1}{4} \bar{g}^{cd} \partial_d \bar{g}_{ae} \bar{g}^{ef} \partial_f \bar{g}_{bc}
\end{aligned} \quad (\text{tmp18.502})$$

$$\begin{aligned}
&= -\frac{1}{4} \bar{g}^{fc} \bar{g}^{de} \partial_a \bar{g}_{cd} \partial_b \bar{g}_{ef} - \frac{1}{4} \bar{g}^{fc} \bar{g}^{de} \partial_a \bar{g}_{cd} \partial_f \bar{g}_{be} + \frac{1}{4} \bar{g}^{ec} \bar{g}^{df} \partial_a \bar{g}_{cd} \partial_f \bar{g}_{be} - \frac{1}{4} \bar{g}^{de} \bar{g}^{fc} \partial_b \bar{g}_{cd} \partial_f \bar{g}_{ae} - \frac{1}{4} \bar{g}^{ed} \bar{g}^{fc} \partial_e \bar{g}_{bc} \partial_f \bar{g}_{ad} + \frac{1}{4} \bar{g}^{dc} \bar{g}^{ef} \partial_e \bar{g}_{ac} \partial_f \bar{g}_{bd} \\
&\quad + \frac{1}{4} \bar{g}^{df} \bar{g}^{ec} \partial_b \bar{g}_{cd} \partial_f \bar{g}_{ae} + \frac{1}{4} \bar{g}^{ef} \bar{g}^{dc} \partial_e \bar{g}_{bc} \partial_f \bar{g}_{ad} - \frac{1}{4} \bar{g}^{de} \bar{g}^{cf} \partial_e \bar{g}_{ac} \partial_f \bar{g}_{bd}
\end{aligned} \quad (\text{tmp18.503})$$

$$\begin{aligned}
&= -\frac{1}{4} \bar{g}^{cd} \bar{g}^{ef} \partial_a \bar{g}_{de} \partial_b \bar{g}_{fc} - \frac{1}{4} \bar{g}^{cd} \bar{g}^{ef} \partial_a \bar{g}_{de} \partial_c \bar{g}_{bf} + \frac{1}{4} \bar{g}^{de} \bar{g}^{fc} \partial_a \bar{g}_{ef} \partial_c \bar{g}_{bd} - \frac{1}{4} \bar{g}^{de} \bar{g}^{cf} \partial_b \bar{g}_{fd} \partial_c \bar{g}_{ae} - \frac{1}{4} \bar{g}^{ce} \bar{g}^{df} \partial_c \bar{g}_{bf} \partial_d \bar{g}_{ae} + \frac{1}{4} \bar{g}^{ef} \bar{g}^{cd} \partial_c \bar{g}_{af} \partial_d \bar{g}_{be} \\
&\quad + \frac{1}{4} \bar{g}^{dc} \bar{g}^{ef} \partial_b \bar{g}_{fd} \partial_c \bar{g}_{ae} + \frac{1}{4} \bar{g}^{cd} \bar{g}^{ef} \partial_c \bar{g}_{bf} \partial_d \bar{g}_{ae} - \frac{1}{4} \bar{g}^{ec} \bar{g}^{fd} \partial_c \bar{g}_{af} \partial_d \bar{g}_{be}
\end{aligned} \quad (\text{tmp18.504})$$

$$= -\frac{1}{4} \bar{g}^{cd} \bar{g}^{ef} \partial_a \bar{g}_{ce} \partial_b \bar{g}_{df} - \frac{1}{2} \bar{g}^{cd} \bar{g}^{ef} \partial_c \bar{g}_{ae} \partial_f \bar{g}_{bd} + \frac{1}{2} \bar{g}^{cd} \bar{g}^{ef} \partial_c \bar{g}_{ae} \partial_d \bar{g}_{bf} \quad (\text{tmp18.505})$$

```

1  # -----
2  # RBar pt.2 = Rebuild Rab from tmpA and tmpB
3
4  RBar := @(tmpA) + @(tmpB).
5
6  canonicalise    (RBar)                                # cdb(eq18.601,RBar)
7
8  foo := \partial_{a}{gBar^{c d}} -> - gBar^{c i} gBar^{d j} \partial_{a}{gBar_{i j}}.
9
10 substitute      (RBar, foo)
11 distribute      (RBar)
12 RBar = product_sort (RBar)
13 rename_dummies  (RBar)
14 canonicalise    (RBar)                                # cdb(eq18.602,RBar)
15
16 foo := \partial_{a}{gBar_{b c}} -> GammaBar_{b c a} + GammaBar_{c b a}.
17
18 substitute      (RBar, foo)                            # cdb(eq18.603,RBar)
19 distribute      (RBar)
20 RBar = product_sort (RBar)
21 rename_dummies  (RBar)
22 canonicalise    (RBar)                                # cdb(eq18.604,RBar)
23
24 foo := GammaBar_{d e f} gBar^{d e} -> 0.
25
26 substitute      (RBar, foo)                            # cdb(eq18.605,RBar)
27
28 defRab := RBar_{a b} -> @(RBar).

```

$$\bar{R}_{ab} = \partial_c \bar{\Gamma}^c_{ab} - \bar{\Gamma}^c_{eb} \bar{\Gamma}^e_{ac} \quad (\text{eq18.003})$$

$$\begin{aligned} &= \frac{1}{2} \bar{g}_{bc} \partial_a \bar{\Gamma}^c + \frac{1}{2} \bar{g}_{ac} \partial_b \bar{\Gamma}^c - \frac{1}{2} \partial_c \bar{g}_{bd} \partial_a \bar{g}^{cd} - \frac{1}{2} \partial_c \bar{g}_{ad} \partial_b \bar{g}^{cd} + \frac{1}{2} \bar{\Gamma}^c \partial_c \bar{g}_{ab} - \frac{1}{2} \bar{g}^{cd} \partial_{cd} \bar{g}_{ab} - \frac{1}{4} \bar{g}^{cd} \bar{g}^{ef} \partial_a \bar{g}_{ce} \partial_b \bar{g}_{df} - \frac{1}{2} \bar{g}^{cd} \bar{g}^{ef} \partial_c \bar{g}_{ae} \partial_f \bar{g}_{bd} \\ &\quad + \frac{1}{2} \bar{g}^{cd} \bar{g}^{ef} \partial_c \bar{g}_{ae} \partial_d \bar{g}_{bf} \end{aligned} \quad (\text{eq18.601})$$

$$\begin{aligned} &= \frac{1}{2} \bar{g}_{bc} \partial_a \bar{\Gamma}^c + \frac{1}{2} \bar{g}_{ac} \partial_b \bar{\Gamma}^c + \frac{1}{2} \bar{g}^{cd} \bar{g}^{ef} \partial_a \bar{g}_{ce} \partial_d \bar{g}_{bf} + \frac{1}{2} \bar{g}^{cd} \bar{g}^{ef} \partial_b \bar{g}_{ce} \partial_d \bar{g}_{af} + \frac{1}{2} \bar{\Gamma}^c \partial_c \bar{g}_{ab} - \frac{1}{2} \bar{g}^{cd} \partial_{cd} \bar{g}_{ab} - \frac{1}{4} \bar{g}^{cd} \bar{g}^{ef} \partial_a \bar{g}_{ce} \partial_b \bar{g}_{df} - \frac{1}{2} \bar{g}^{cd} \bar{g}^{ef} \partial_c \bar{g}_{ae} \partial_f \bar{g}_{bd} \\ &\quad + \frac{1}{2} \bar{g}^{cd} \bar{g}^{ef} \partial_c \bar{g}_{ae} \partial_d \bar{g}_{bf} \end{aligned} \quad (\text{eq18.602})$$

$$\begin{aligned} &= \frac{1}{2} \bar{g}_{bc} \partial_a \bar{\Gamma}^c + \frac{1}{2} \bar{g}_{ac} \partial_b \bar{\Gamma}^c + \frac{1}{2} \bar{g}^{cd} \bar{g}^{ef} (\bar{\Gamma}_{cea} + \bar{\Gamma}_{eca}) (\bar{\Gamma}_{bfd} + \bar{\Gamma}_{fbd}) + \frac{1}{2} \bar{g}^{cd} \bar{g}^{ef} (\bar{\Gamma}_{ceb} + \bar{\Gamma}_{ecb}) (\bar{\Gamma}_{afd} + \bar{\Gamma}_{fad}) + \frac{1}{2} \bar{\Gamma}^c (\bar{\Gamma}_{abc} + \bar{\Gamma}_{bac}) - \frac{1}{2} \bar{g}^{cd} \partial_{cd} \bar{g}_{ab} \\ &\quad - \frac{1}{4} \bar{g}^{cd} \bar{g}^{ef} (\bar{\Gamma}_{cea} + \bar{\Gamma}_{eca}) (\bar{\Gamma}_{dfb} + \bar{\Gamma}_{fdb}) - \frac{1}{2} \bar{g}^{cd} \bar{g}^{ef} (\bar{\Gamma}_{aec} + \bar{\Gamma}_{eac}) (\bar{\Gamma}_{bdf} + \bar{\Gamma}_{dbf}) + \frac{1}{2} \bar{g}^{cd} \bar{g}^{ef} (\bar{\Gamma}_{aec} + \bar{\Gamma}_{eac}) (\bar{\Gamma}_{bfd} + \bar{\Gamma}_{fbd}) \end{aligned} \quad (\text{eq18.603})$$

$$= \frac{1}{2} \bar{g}_{bc} \partial_a \bar{\Gamma}^c + \frac{1}{2} \bar{g}_{ac} \partial_b \bar{\Gamma}^c + \bar{g}^{cd} \bar{g}^{ef} \bar{\Gamma}_{bce} \bar{\Gamma}_{daf} + \bar{g}^{cd} \bar{g}^{ef} \bar{\Gamma}_{cae} \bar{\Gamma}_{dbf} + \bar{g}^{cd} \bar{g}^{ef} \bar{\Gamma}_{ace} \bar{\Gamma}_{dbf} + \frac{1}{2} \bar{\Gamma}^c \bar{\Gamma}_{abc} + \frac{1}{2} \bar{\Gamma}^c \bar{\Gamma}_{bac} - \frac{1}{2} \bar{g}^{cd} \partial_{cd} \bar{g}_{ab} \quad (\text{eq18.604})$$

$$= \frac{1}{2} \bar{g}_{bc} \partial_a \bar{\Gamma}^c + \frac{1}{2} \bar{g}_{ac} \partial_b \bar{\Gamma}^c + \bar{g}^{cd} \bar{g}^{ef} \bar{\Gamma}_{bce} \bar{\Gamma}_{daf} + \bar{g}^{cd} \bar{g}^{ef} \bar{\Gamma}_{cae} \bar{\Gamma}_{dbf} + \bar{g}^{cd} \bar{g}^{ef} \bar{\Gamma}_{ace} \bar{\Gamma}_{dbf} + \frac{1}{2} \bar{\Gamma}^c \bar{\Gamma}_{abc} + \frac{1}{2} \bar{\Gamma}^c \bar{\Gamma}_{bac} - \frac{1}{2} \bar{g}^{cd} \partial_{cd} \bar{g}_{ab} \quad (\text{eq18.605})$$

```

1  # -----
2  # Check against prd62.
3
4  foo := @(RBar).                                # cdb(eq18.lcb,foo)
5  bah  = cdblib.get('prd62.eq18.rhs','prd62.json') # cdb(eq18.prd,bah)
6
7  diff := @(foo) - @(bah).
8
9  distribute      (diff)
10 diff = product_sort (diff)
11 rename_dummies (diff)
12 map_sympy      (diff, "simplify")
13 canonicalise   (diff)                        # cdb(eq18.chk,diff)

```

$$\text{eq18.lcb} := \frac{1}{2}\bar{g}_{bc}\partial_a\bar{\Gamma}^c + \frac{1}{2}\bar{g}_{ac}\partial_b\bar{\Gamma}^c + \bar{g}^{cd}\bar{g}^{ef}\bar{\Gamma}_{bce}\bar{\Gamma}_{daf} + \bar{g}^{cd}\bar{g}^{ef}\bar{\Gamma}_{cae}\bar{\Gamma}_{dbf} + \bar{g}^{cd}\bar{g}^{ef}\bar{\Gamma}_{ace}\bar{\Gamma}_{dbf} + \frac{1}{2}\bar{\Gamma}^c\bar{\Gamma}_{abc} + \frac{1}{2}\bar{\Gamma}^c\bar{\Gamma}_{bac} - \frac{1}{2}\bar{g}^{cd}\partial_{cd}\bar{g}_{ab}$$

$$\text{eq18.prd} := -\frac{1}{2}\bar{g}^{lm}\partial_{lm}\bar{g}_{ab} + \frac{1}{2}\bar{g}_{ka}\partial_b\bar{\Gamma}^k + \frac{1}{2}\bar{g}_{kb}\partial_a\bar{\Gamma}^k + \frac{1}{2}\bar{\Gamma}^k\bar{\Gamma}_{abk} + \frac{1}{2}\bar{\Gamma}^k\bar{\Gamma}_{bak} + \bar{g}^{lm}\bar{g}^{ke}(\bar{\Gamma}_{ela}\bar{\Gamma}_{bkm} + \bar{\Gamma}_{elb}\bar{\Gamma}_{akm} + \bar{\Gamma}_{kam}\bar{\Gamma}_{elb})$$

$$\text{eq18.chk} := 0$$