PhysRevD.67.084023 equation (19)

```
from shared import *
import cdblib

jsonfile = 'bssn-constraints.json'
cdblib.create (jsonfile)

# ----------------------------------------------------------------------------
# Hamiltonian constraint

Ham := R + K_{a b} g^{a b} K_{c d} g^{c d} - K_{a b} K_{c d} g^{a c} g^{b d}.
                                                        # cdb(Ham.101,Ham)

Ham := R + (2/3) (trK)**2 - ABar_{a b} ABar^{a b}.      # cdb(Ham.102,Ham)
```

$$\mathcal{H} = R + K_{ab} g^{ab} K_{cd} g^{cd} - K_{ab} K_{cd} g^{ac} g^{bd} \tag{Ham.101}$$

$$= R + \frac{2}{3} \mathrm{tr} K^2 - \bar{A}_{ab} \bar{A}^{ab} \tag{Ham.102}$$

PhysRevD.67.084023 equation (20)

```
# ------------------------------------------------------------------------
# Momentum constraint

confMom := 6 ABar^{i a} \partial_{a}{\phi}
           + \partial_{a}{ABar^{i a}}
           + ABar^{a b} GammaBar^{i}_{a b}
           - (2/3) gBar^{i a} \partial_{a}{trK}.

defGammaBar := GammaBar^{a}_{b c} ->
               (1/2) gBar^{a e} (   \partial_{b}{gBar_{e c}}
                                  + \partial_{c}{gBar_{b e}}
                                  - \partial_{e}{gBar_{b c}}).

substitute (confMom, defGammaBar)                       # cdb(confMom.101,confMom)
distribute (confMom)                                    # cdb(confMom.102,confMom)

confMom = product_sort (confMom)                        # cdb(confMom.103,confMom)

rename_dummies (confMom)                                # cdb(confMom.104,confMom)
canonicalise   (confMom)                                # cdb(confMom.105,confMom)

foo := \partial_{a}{ABar^{i a}} -> \partial_{a}{gBar^{i c} gBar^{a d} ABar_{c d}}.

substitute   (confMom, foo)                             # cdb(confMom.106,confMom)
product_rule (confMom)                                  # cdb(confMom.107,confMom)

confMom = product_sort (confMom)                        # cdb(confMom.108,confMom)

rename_dummies (confMom)                                # cdb(confMom.109,confMom)
canonicalise   (confMom)                                # cdb(confMom.110,confMom)

cdblib.put ('Ham',Ham,jsonfile)
cdblib.put ('confMom',confMom,jsonfile)
```

$$\exp(4\phi)\mathcal{D}^j = 6\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + \frac{1}{2}\bar{A}^{ab}\bar{g}^{ie}\left(\partial_a\bar{g}_{eb} + \partial_b\bar{g}_{ae} - \partial_e\bar{g}_{ab}\right) - \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \tag{confMom.101}$$

$$= 6\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + \frac{1}{2}\bar{A}^{ab}\bar{g}^{ie}\partial_a\bar{g}_{eb} + \frac{1}{2}\bar{A}^{ab}\bar{g}^{ie}\partial_b\bar{g}_{ae} - \frac{1}{2}\bar{A}^{ab}\bar{g}^{ie}\partial_e\bar{g}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \tag{confMom.102}$$

$$= 6\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + \frac{1}{2}\bar{A}^{ab}\bar{g}^{ic}\partial_a\bar{g}_{cb} + \frac{1}{2}\bar{A}^{ab}\bar{g}^{ic}\partial_b\bar{g}_{ac} - \frac{1}{2}\bar{A}^{ab}\bar{g}^{ic}\partial_c\bar{g}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \tag{confMom.103}$$

$$= 6\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + \frac{1}{2}\bar{A}^{ab}\bar{g}^{ic}\partial_a\bar{g}_{cb} + \frac{1}{2}\bar{A}^{ab}\bar{g}^{ic}\partial_b\bar{g}_{ac} - \frac{1}{2}\bar{A}^{ab}\bar{g}^{ic}\partial_c\bar{g}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \tag{confMom.104}$$

$$= 6\bar{A}^{ia}\partial_a\phi + \partial_a\bar{A}^{ia} + \bar{A}^{ab}\bar{g}^{ic}\partial_a\bar{g}_{bc} - \frac{1}{2}\bar{A}^{ab}\bar{g}^{ic}\partial_c\bar{g}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \tag{confMom.105}$$

$$= 6\bar{A}^{ia}\partial_a\phi + \partial_a\left(\bar{g}^{ic}\bar{g}^{ad}\bar{A}_{cd}\right) + \bar{A}^{ab}\bar{g}^{ic}\partial_a\bar{g}_{bc} - \frac{1}{2}\bar{A}^{ab}\bar{g}^{ic}\partial_c\bar{g}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \tag{confMom.106}$$

$$= 6\bar{A}^{ia}\partial_a\phi + \partial_a\bar{g}^{ic}\bar{g}^{ad}\bar{A}_{cd} + \bar{g}^{ic}\partial_a\bar{g}^{ad}\bar{A}_{cd} + \bar{g}^{ic}\bar{g}^{ad}\partial_a\bar{A}_{cd} + \bar{A}^{ab}\bar{g}^{ic}\partial_a\bar{g}_{bc} - \frac{1}{2}\bar{A}^{ab}\bar{g}^{ic}\partial_c\bar{g}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \tag{confMom.107}$$

$$= 6\bar{A}^{ia}\partial_a\phi + \bar{A}_{ab}\bar{g}^{cb}\partial_c\bar{g}^{ia} + \bar{A}_{ab}\bar{g}^{ia}\partial_c\bar{g}^{cb} + \bar{g}^{cb}\bar{g}^{ia}\partial_c\bar{A}_{ab} + \bar{A}^{ab}\bar{g}^{ic}\partial_a\bar{g}_{bc} - \frac{1}{2}\bar{A}^{ab}\bar{g}^{ic}\partial_c\bar{g}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \tag{confMom.108}$$

$$= 6\bar{A}^{ia}\partial_a\phi + \bar{A}_{ab}\bar{g}^{cb}\partial_c\bar{g}^{ia} + \bar{A}_{ab}\bar{g}^{ia}\partial_c\bar{g}^{cb} + \bar{g}^{cb}\bar{g}^{ia}\partial_c\bar{A}_{ab} + \bar{A}^{ab}\bar{g}^{ic}\partial_a\bar{g}_{bc} - \frac{1}{2}\bar{A}^{ab}\bar{g}^{ic}\partial_c\bar{g}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \tag{confMom.109}$$

$$= 6\bar{A}^{ia}\partial_a\phi + \bar{A}_{ab}\bar{g}^{ac}\partial_c\bar{g}^{ib} + \bar{A}_{ab}\bar{g}^{ia}\partial_c\bar{g}^{bc} + \bar{g}^{ia}\bar{g}^{bc}\partial_b\bar{A}_{ac} + \bar{A}^{ab}\bar{g}^{ic}\partial_a\bar{g}_{bc} - \frac{1}{2}\bar{A}^{ab}\bar{g}^{ic}\partial_c\bar{g}_{ab} - \frac{2}{3}\bar{g}^{ia}\partial_a\mathrm{tr}K \tag{confMom.110}$$