PhysRevD.62.044034 equation (11)

```
1    from shared import *
2    import cdblib
3
4    jsonfile = 'bssn-eqtns-11.json'
5    cdblib.create (jsonfile)
6
7    # ----------------------------------------------------------------------
8
9    DtrKDt  := \partial_{t}{trK}.                  # cdb(eq11.00,DtrKDt)
10   DtrKDt  := - g^{i j} D_{i j}{N}
11            + N (ABar_{i j} ABar^{i j} + (1/3) trK**2).
12
13                                                  # cdb(eq11.01,DtrKDt)
14
15   substitute (DtrKDt, defD2)                     # cdb(eq11.02,DtrKDt)
16   substitute (DtrKDt, defGamma2GammaBar)         # cdb(eq11.03,DtrKDt)
17
18   foo := g^{a b} -> \exp(-4\phi) gBar^{a b}.
19
20   substitute (DtrKDt, foo)                       # cdb(eq11.04,DtrKDt)
21   distribute (DtrKDt)                            # cdb(eq11.05,DtrKDt)
22   eliminate_kronecker (DtrKDt)                   # cdb(eq11.06,DtrKDt)
23   canonicalise (DtrKDt)                          # cdb(eq11.07,DtrKDt)
24   substitute   (DtrKDt, defGBarSq)               # cdb(eq11.08,DtrKDt)
25
26   DtrKDt = product_sort (DtrKDt)
27
28   factor_out (DtrKDt, $\exp(-4\phi)$)            # cdb(eq11.08,DtrKDt)
29   substitute (DtrKDt, defGammaBarU)              # cdb(eq11.09,DtrKDt)
30   distribute (DtrKDt)                            # cdb(eq11.10,DtrKDt)
31
32   DtrKDt = product_sort (DtrKDt)                 # cdb(eq11.11,DtrKDt)
33
34   canonicalise (DtrKDt)                          # cdb(eq11.12,DtrKDt)
35
36   foo := gBar^{b c} \partial_{a}{gBar_{b c}} -> 0.   # follows from det(g) = 1
```

```
37    bah := gBar^{e b} gBar^{f c} \partial_{a}{gBar_{b c}} -> - \partial_{a}{gBar^{e f}}.

38

39    substitute (DtrKDt, foo)                        # cdb(eq11.13,DtrKDt)
40    substitute (DtrKDt, bah)                        # cdb(eq11.14,DtrKDt)

41

42    DtrKDt = product_sort (DtrKDt)

43

44    canonicalise (DtrKDt)                           # cdb(eq11.15,DtrKDt)
45    factor_out   (DtrKDt, $\exp(-4\phi)$)           # cdb(eq11.16,DtrKDt)
46                                                    # cdb(eq11.99,DtrKDt)

47

48    cdblib.put ('DtrKDt',DtrKDt,jsonfile)
```

$$\partial_t \mathrm{tr} K = -g^{ij} D_{ij} N + N\left(\bar{A}_{ij}\bar{A}^{ij} + \frac{1}{3}\mathrm{tr} K^2\right) \tag{eq11.01}$$

$$= -g^{ij}\left(\partial_{ij} N - \Gamma^c{}_{ij}\partial_c N\right) + N\left(\bar{A}_{ij}\bar{A}^{ij} + \frac{1}{3}\mathrm{tr} K^2\right) \tag{eq11.02}$$

$$= -g^{ij}\left(\partial_{ij} N - \left(\bar{\Gamma}^c{}_{ij} + 2\bar{g}^c{}_j\partial_i\phi + 2\bar{g}^c{}_i\partial_j\phi - 2\bar{g}^{ce}\bar{g}_{ij}\partial_e\phi\right)\partial_c N\right) + N\left(\bar{A}_{ij}\bar{A}^{ij} + \frac{1}{3}\mathrm{tr} K^2\right) \tag{eq11.03}$$

$$= -\exp\left(-4\phi\right)\bar{g}^{ij}\left(\partial_{ij} N - \left(\bar{\Gamma}^c{}_{ij} + 2\bar{g}^c{}_j\partial_i\phi + 2\bar{g}^c{}_i\partial_j\phi - 2\bar{g}^{ce}\bar{g}_{ij}\partial_e\phi\right)\partial_c N\right) + N\left(\bar{A}_{ij}\bar{A}^{ij} + \frac{1}{3}\mathrm{tr} K^2\right) \tag{eq11.04}$$

$$= -\exp\left(-4\phi\right)\bar{g}^{ij}\partial_{ij} N + \exp\left(-4\phi\right)\bar{g}^{ij}\bar{\Gamma}^c{}_{ij}\partial_c N + 2\exp\left(-4\phi\right)\bar{g}^{ij}\bar{g}^c{}_j\partial_i\phi\partial_c N + 2\exp\left(-4\phi\right)\bar{g}^{ij}\bar{g}^c{}_i\partial_j\phi\partial_c N - 2\exp\left(-4\phi\right)\bar{g}^{ij}\bar{g}^{ce}\bar{g}_{ij}\partial_e\phi\partial_c N$$
$$+ N\bar{A}_{ij}\bar{A}^{ij} + \frac{1}{3}N\mathrm{tr} K^2 \tag{eq11.05}$$

$$= -\exp\left(-4\phi\right)\bar{g}^{ij}\partial_{ij} N + \exp\left(-4\phi\right)\bar{g}^{ij}\bar{\Gamma}^c{}_{ij}\partial_c N + 2\exp\left(-4\phi\right)\bar{g}^{ic}\partial_i\phi\partial_c N + 2\exp\left(-4\phi\right)\bar{g}^{cj}\partial_j\phi\partial_c N - 2\exp\left(-4\phi\right)\bar{g}^{ij}\bar{g}^{ce}\bar{g}_{ij}\partial_e\phi\partial_c N$$
$$+ N\bar{A}_{ij}\bar{A}^{ij} + \frac{1}{3}N\mathrm{tr} K^2 \tag{eq11.06}$$

$$= -\exp\left(-4\phi\right)\bar{g}^{ij}\partial_{ij} N + \exp\left(-4\phi\right)\bar{g}^{ci}\bar{\Gamma}^j{}_{ci}\partial_j N + 2\exp\left(-4\phi\right)\bar{g}^{ci}\partial_c\phi\partial_i N + 2\exp\left(-4\phi\right)\bar{g}^{cj}\partial_c\phi\partial_j N - 2\exp\left(-4\phi\right)\bar{g}^{ce}\bar{g}^{ij}\bar{g}_{ce}\partial_i\phi\partial_j N$$
$$+ N\bar{A}_{ij}\bar{A}^{ij} + \frac{1}{3}N\mathrm{tr} K^2 \tag{eq11.07}$$

$$= N\bar{A}_{ab}\bar{A}^{ab} + \frac{1}{3}N\mathrm{tr} K^2 + \exp\left(-4\phi\right)\left(-\bar{g}^{ab}\partial_{ab} N + \bar{g}^{ab}\bar{\Gamma}^c{}_{ab}\partial_c N - 2\bar{g}^{ab}\partial_a\phi\partial_b N\right) \tag{eq11.08}$$

$$= N\bar{A}_{ab}\bar{A}^{ab} + \frac{1}{3}N\mathrm{tr} K^2 + \exp\left(-4\phi\right)\left(-\bar{g}^{ab}\partial_{ab} N + \frac{1}{2}\bar{g}^{ab}\bar{g}^{ce}\left(\partial_a\bar{g}_{eb} + \partial_b\bar{g}_{ae} - \partial_e\bar{g}_{ab}\right)\partial_c N - 2\bar{g}^{ab}\partial_a\phi\partial_b N\right) \tag{eq11.09}$$

$$= N\bar{A}_{ab}\bar{A}^{ab} + \frac{1}{3}N\mathrm{tr} K^2 - \exp\left(-4\phi\right)\bar{g}^{ab}\partial_{ab} N + \frac{1}{2}\exp\left(-4\phi\right)\bar{g}^{ab}\bar{g}^{ce}\partial_a\bar{g}_{eb}\partial_c N + \frac{1}{2}\exp\left(-4\phi\right)\bar{g}^{ab}\bar{g}^{ce}\partial_b\bar{g}_{ae}\partial_c N - \frac{1}{2}\exp\left(-4\phi\right)\bar{g}^{ab}\bar{g}^{ce}\partial_e\bar{g}_{ab}\partial_c N$$
$$- 2\exp\left(-4\phi\right)\bar{g}^{ab}\partial_a\phi\partial_b N \tag{eq11.10}$$

$$= N\bar{A}_{ab}\bar{A}^{ab} + \frac{1}{3}N\mathrm{tr} K^2 - \bar{g}^{ab}\exp\left(-4\phi\right)\partial_{ab} N + \frac{1}{2}\bar{g}^{cb}\bar{g}^{da}\exp\left(-4\phi\right)\partial_d N\partial_c\bar{g}_{ab} + \frac{1}{2}\bar{g}^{ac}\bar{g}^{db}\exp\left(-4\phi\right)\partial_d N\partial_c\bar{g}_{ab} - \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\exp\left(-4\phi\right)\partial_c N\partial_d\bar{g}_{ab}$$
$$- 2\bar{g}^{ab}\partial_a\phi\exp\left(-4\phi\right)\partial_b N \tag{eq11.11}$$

3

$$\partial_t \mathrm{tr} K = N \bar{A}_{ab} \bar{A}^{ab} + \frac{1}{3} N \mathrm{tr} K^2 - \bar{g}^{ab} \exp\left(-4\phi\right) \partial_{ab} N + \bar{g}^{ab} \bar{g}^{cd} \exp\left(-4\phi\right) \partial_a N \partial_c \bar{g}_{bd} - \frac{1}{2} \bar{g}^{ab} \bar{g}^{cd} \exp\left(-4\phi\right) \partial_a N \partial_b \bar{g}_{cd} - 2 \bar{g}^{ab} \partial_a \phi \exp\left(-4\phi\right) \partial_b N \quad \text{(eq11.12)}$$

$$= N \bar{A}_{ab} \bar{A}^{ab} + \frac{1}{3} N \mathrm{tr} K^2 - \bar{g}^{ab} \exp\left(-4\phi\right) \partial_{ab} N + \bar{g}^{ab} \bar{g}^{cd} \exp\left(-4\phi\right) \partial_a N \partial_c \bar{g}_{bd} - 2 \bar{g}^{ab} \partial_a \phi \exp\left(-4\phi\right) \partial_b N \quad \text{(eq11.13)}$$

$$= N \bar{A}_{ab} \bar{A}^{ab} + \frac{1}{3} N \mathrm{tr} K^2 - \bar{g}^{ab} \exp\left(-4\phi\right) \partial_{ab} N - \partial_c \bar{g}^{ac} \exp\left(-4\phi\right) \partial_a N - 2 \bar{g}^{ab} \partial_a \phi \exp\left(-4\phi\right) \partial_b N \quad \text{(eq11.14)}$$

$$= N \bar{A}_{ab} \bar{A}^{ab} + \frac{1}{3} N \mathrm{tr} K^2 - \bar{g}^{ab} \exp\left(-4\phi\right) \partial_{ab} N - \exp\left(-4\phi\right) \partial_a N \partial_b \bar{g}^{ab} - 2 \bar{g}^{ab} \partial_a \phi \exp\left(-4\phi\right) \partial_b N \quad \text{(eq11.15)}$$

$$= N \bar{A}_{ab} \bar{A}^{ab} + \frac{1}{3} N \mathrm{tr} K^2 + \exp\left(-4\phi\right) \left(-\bar{g}^{ab} \partial_{ab} N - \partial_a N \partial_b \bar{g}^{ab} - 2 \bar{g}^{ab} \partial_a \phi \partial_b N\right) \quad \text{(eq11.16)}$$