

The BSSN evolution equations. Code generation.

Finally, the actual BSSN equations used in the computer code. Yeah.

$$\partial_t \bar{g}_{ij} = -2N \bar{A}_{ij} \quad (\text{eq09.99})$$

$$\partial_t \phi = -\frac{1}{6} N \text{tr} K \quad (\text{eq10.99})$$

$$\partial_t \text{tr} K = N \bar{A}_{ab} \bar{A}^{ab} + \frac{1}{3} N \text{tr} K^2 + \exp(-4\phi) (-\bar{g}^{ab} \partial_{ab} N - \partial_a N \partial_b \bar{g}^{ab} - 2\bar{g}^{ab} \partial_a \phi \partial_b N) \quad (\text{eq11.99})$$

$$\begin{aligned} \partial_t \bar{A}_{ij} = N \text{tr} K \bar{A}_{ij} - 2N \bar{A}_{ia} \bar{A}_{jb} \bar{g}^{ab} + \exp(-4\phi) & \left(N R_{ij} - \partial_{ij} N + \frac{1}{2} \bar{g}^{ab} \partial_a N \partial_i \bar{g}_{jb} + \frac{1}{2} \bar{g}^{ab} \partial_a N \partial_j \bar{g}_{ib} - \frac{1}{2} \bar{g}^{ab} \partial_a N \partial_b \bar{g}_{ij} + 2\partial_i \phi \partial_j N + 2\partial_j \phi \partial_i N \right. \\ & \left. - \frac{4}{3} \bar{g}_{ij} \bar{g}^{ab} \partial_a \phi \partial_b N - \frac{1}{3} N \bar{g}_{ij} \bar{g}^{ab} R_{ab} + \frac{1}{3} \bar{g}_{ij} \bar{g}^{ab} \partial_{ab} N + \frac{1}{3} \bar{g}_{ij} \partial_a N \partial_b \bar{g}^{ab} \right) \end{aligned} \quad (\text{eq12.99})$$

$$\partial_t \bar{\Gamma}^i = -2\bar{A}^{ia} \partial_a N + 2N \bar{A}^{ab} \bar{g}^{ic} \partial_a \bar{g}_{bc} - N \bar{A}^{ab} \bar{g}^{ic} \partial_c \bar{g}_{ab} - \frac{4}{3} N \bar{g}^{ia} \partial_a \text{tr} K + 12N \bar{A}^{ia} \partial_a \phi \quad (\text{eq20.99})$$

$$\begin{aligned} R_{ab} = -2\partial_{ab} \phi + \bar{g}^{cd} \partial_a \phi \partial_a \bar{g}_{bc} + \bar{g}^{cd} \partial_d \phi \partial_b \bar{g}_{ac} - \bar{g}^{cd} \partial_d \phi \partial_c \bar{g}_{ab} + 12\partial_a \phi \partial_b \phi - 2\bar{g}_{ab} \bar{g}^{cd} \partial_{cd} \phi - 2\bar{g}_{ab} \partial_d \phi \partial_c \bar{g}^{cd} - 12\bar{g}_{ab} \bar{g}^{cd} \partial_c \phi \partial_d \phi - \frac{1}{2} \bar{g}^{cd} \partial_{cd} \bar{g}_{ab} + \frac{1}{2} \bar{g}_{ac} \partial_b \bar{\Gamma}^c \\ + \frac{1}{2} \bar{g}_{bc} \partial_a \bar{\Gamma}^c - \frac{1}{2} \partial_c \bar{g}_{ab} \partial_d \bar{g}^{cd} - \frac{1}{2} \partial_d \bar{g}_{bc} \partial_a \bar{g}^{dc} - \frac{1}{2} \bar{g}^{ed} \bar{g}^{cf} \partial_e \bar{g}_{ac} \partial_f \bar{g}_{bd} + \frac{1}{2} \bar{g}^{ef} \bar{g}^{cd} \partial_e \bar{g}_{ac} \partial_f \bar{g}_{bd} + \frac{1}{4} \partial_b \bar{g}_{cd} \partial_a \bar{g}^{cd} - \frac{1}{2} \partial_d \bar{g}_{ac} \partial_b \bar{g}^{dc} \end{aligned} \quad (\text{eq14.99})$$

The last job for Cadabra is to convert the right hand sides into C-code. And then it's goodnight from Cadabra and time to send the hounds out for Ada and friends (last seen lingering in a quiet corner of GitHub).

```

1 from shared import *
2 import cdblib
3
4 Rscalar = cdblib.get ('Rscalar', 'bssn-ricci-scalar.json')
5 Rab     = cdblib.get ('Rab',     'bssn-eqtns-14.json')
6 DphiDt  = cdblib.get ('DphiDt',  'bssn-eqtns-10.json')
7 DtrKDt  = cdblib.get ('DtrKDt',  'bssn-eqtns-11.json')
8 DgBarDt = cdblib.get ('DgBarDt', 'bssn-eqtns-09.json')
9 DABarDt = cdblib.get ('DABarDt', 'bssn-eqtns-12.json')
10 DGiBarDt = cdblib.get ('DGiBarDt', 'bssn-eqtns-20.json')
11 DNDt    = cdblib.get ('DNDt',    'bssn-lapse.json')
12 Ham     = cdblib.get ('Ham',     'bssn-constraints.json')
13 confMom  = cdblib.get ('confMom', 'bssn-constraints.json')
14
15 # -----
16 # Replace partial derivs with indices
17 # why? because evaluate() on \partial produces zero
18
19 def substitute_deriv (ex):
20     substitute (ex, $ \partial_{a b}{gBar_{c d}} -> dgBar_{c d a b} $)
21     substitute (ex, $ \partial_a{gBar_{b c}} -> dgBar_{b c a} $)
22     substitute (ex, $ \partial_a{gBar^{b c}} -> dgBar^{b c}_a $)
23     substitute (ex, $ \partial_a{ABar_{b c}} -> dABar_{b c a} $)
24     substitute (ex, $ \partial_a{\phi} -> dphi_a $)
25     substitute (ex, $ \partial_a{\phi} -> dphi_a $)
26     substitute (ex, $ \partial_a{\Gamma^{b}} -> dGi^{b}_a $)
27     substitute (ex, $ \partial_a{N} -> dN_a $)
28     substitute (ex, $ \partial_a{N} -> dN_a $)
29     substitute (ex, $ \partial_a{trK} -> dtrK_a $)
30     return ex
31
32 Rscalar = substitute_deriv (Rscalar)
33 Rab     = substitute_deriv (Rab)
34 DphiDt  = substitute_deriv (DphiDt)
35 DtrKDt  = substitute_deriv (DtrKDt)
36 DgBarDt = substitute_deriv (DgBarDt)
37 DABarDt = substitute_deriv (DABarDt)
38 DGiBarDt = substitute_deriv (DGiBarDt)

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39 DNDt      = substitute_deriv (DNDt)
40 Ham       = substitute_deriv (Ham)
41 confMom    = substitute_deriv (confMom)
42
43 # -----
44 # build rules to export Cadabra expressions to Python
45 # use known symmetries for g_{ab}, dg_{ab,c,d} etc.
46 # note: replacements must not contain underscores (reserved for subscripts),
47 #       so gBar_{x x} -> gBar_xx is not allowed
48
49 gBarRule := {gBar_{x x} -> gBarxx, gBar_{x y} -> gBarxy, gBar_{x z} -> gBarxz,
50             gBar_{y x} -> gBarxy, gBar_{y y} -> gBaryy, gBar_{y z} -> gBaryz,
51             gBar_{z x} -> gBarxz, gBar_{z y} -> gBaryz, gBar_{z z} -> gBarzz}.
52
53 iBarRule := {gBar^{x x} -> iBarxx, gBar^{x y} -> iBarxy, gBar^{x z} -> iBarxz,
54             gBar^{y x} -> iBarxy, gBar^{y y} -> iBaryy, gBar^{y z} -> iBaryz,
55             gBar^{z x} -> iBarxz, gBar^{z y} -> iBaryz, gBar^{z z} -> iBarzz}.
56
57 ABarRule := {ABar_{x x} -> ABarxx, ABar_{x y} -> ABarxy, ABar_{x z} -> ABarxz,
58             ABar_{y x} -> ABarxy, ABar_{y y} -> ABaryy, ABar_{y z} -> ABaryz,
59             ABar_{z x} -> ABarxz, ABar_{z y} -> ABaryz, ABar_{z z} -> ABarzz}.
60
61 BBarRule := {ABar^{x x} -> BBarxx, ABar^{x y} -> BBarxy, ABar^{x z} -> BBarxz,
62             ABar^{y x} -> BBarxy, ABar^{y y} -> BBaryy, ABar^{y z} -> BBaryz,
63             ABar^{z x} -> BBarxz, ABar^{z y} -> BBaryz, ABar^{z z} -> BBarzz}.
64
65 RRule := {R_{x x} -> Rxx, R_{x y} -> Rxy, R_{x z} -> Rxz,
66           R_{y x} -> Rxy, R_{y y} -> Ryy, R_{y z} -> Ryz,
67           R_{z x} -> Rxz, R_{z y} -> Ryz, R_{z z} -> Rzz}.
68
69 trKRule := {dtrK_{x} -> trKx, dtrK_{y} -> trKy, dtrK_{z} -> trKz}.
70
71 GiRule := {GammaBar^{x} -> Gix, GammaBar^{y} -> Giy, GammaBar^{z} -> Giz}.
72
73 d1GiRule := {dGi^{x}_{x} -> Gixx, dGi^{x}_{y} -> Gixy, dGi^{x}_{z} -> Gixz,
74             dGi^{y}_{x} -> Giyx, dGi^{y}_{y} -> Giyy, dGi^{y}_{z} -> Giyz,
75             dGi^{z}_{x} -> Gizx, dGi^{z}_{y} -> Gizy, dGi^{z}_{z} -> Gizz}.
76

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77 d1NRule := {dN_{x} -> Nx, dN_{y} -> Ny, dN_{z} -> Nz}.
78
79 d2NRule := {dN_{x x} -> Nxx, dN_{x y} -> Nxy, dN_{x z} -> Nxz,
80             dN_{y x} -> Nxy, dN_{y y} -> Nyy, dN_{y z} -> Nyz,
81             dN_{z x} -> Nxz, dN_{z y} -> Nyz, dN_{z z} -> Nzz}.
82
83 d1phiRule := {dphi_{x} -> phix, dphi_{y} -> phiy, dphi_{z} -> phiz}.
84
85 d2phiRule := {dphi_{x x} -> phixx, dphi_{x y} -> phixy, dphi_{x z} -> phixz,
86             dphi_{y x} -> phixy, dphi_{y y} -> phiyy, dphi_{y z} -> phiyz,
87             dphi_{z x} -> phixz, dphi_{z y} -> phiyz, dphi_{z z} -> phizz}.
88
89 d1ABarRule := {dABar_{x x x} -> ABarxxx, dABar_{x y x} -> ABarxyx, dABar_{x z x} -> ABarxxz,
90              dABar_{y x x} -> ABarxyx, dABar_{y y x} -> ABaryyx, dABar_{y z x} -> ABaryzx,
91              dABar_{z x x} -> ABarxxz, dABar_{z y x} -> ABaryzx, dABar_{z z x} -> ABarzzx,
92
93              dABar_{x x y} -> ABarxxy, dABar_{x y y} -> ABarxyy, dABar_{x z y} -> ABarxzy,
94              dABar_{y x y} -> ABarxyy, dABar_{y y y} -> ABaryyy, dABar_{y z y} -> ABaryzy,
95              dABar_{z x y} -> ABarxzy, dABar_{z y y} -> ABaryzy, dABar_{z z y} -> ABarzzy,
96
97              dABar_{x x z} -> ABarxxz, dABar_{x y z} -> ABarxyz, dABar_{x z z} -> ABarxzz,
98              dABar_{y x z} -> ABarxyz, dABar_{y y z} -> ABaryyz, dABar_{y z z} -> ABaryzz,
99              dABar_{z x z} -> ABarxzz, dABar_{z y z} -> ABaryzz, dABar_{z z z} -> ABarzzz}.
100
101 d1gBarRule := {dgBar_{x x x} -> gBarxxx, dgBar_{x y x} -> gBarxyx, dgBar_{x z x} -> gBarxxz,
102              dgBar_{y x x} -> gBarxyx, dgBar_{y y x} -> gBaryyx, dgBar_{y z x} -> gBaryzx,
103              dgBar_{z x x} -> gBarxxz, dgBar_{z y x} -> gBaryzx, dgBar_{z z x} -> gBarzzx,
104
105              dgBar_{x x y} -> gBarxxy, dgBar_{x y y} -> gBarxyy, dgBar_{x z y} -> gBarxzy,
106              dgBar_{y x y} -> gBarxyy, dgBar_{y y y} -> gBaryyy, dgBar_{y z y} -> gBaryzy,
107              dgBar_{z x y} -> gBarxzy, dgBar_{z y y} -> gBaryzy, dgBar_{z z y} -> gBarzzy,
108
109              dgBar_{x x z} -> gBarxxz, dgBar_{x y z} -> gBarxyz, dgBar_{x z z} -> gBarxzz,
110              dgBar_{y x z} -> gBarxyz, dgBar_{y y z} -> gBaryyz, dgBar_{y z z} -> gBaryzz,
111              dgBar_{z x z} -> gBarxzz, dgBar_{z y z} -> gBaryzz, dgBar_{z z z} -> gBarzzz}.
112
113 d1iBarRule := {dgBar^{x x}_{x} -> iBarxxx, dgBar^{x y}_{x} -> iBarxyx, dgBar^{x z}_{x} -> iBarxxz,
114              dgBar^{y x}_{x} -> iBarxyx, dgBar^{y y}_{x} -> iBaryyx, dgBar^{y z}_{x} -> iBaryzx,

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115     dgBar^{z x}_{x} -> iBarxxz, dgBar^{z y}_{x} -> iBaryzx, dgBar^{z z}_{x} -> iBarzzx,
116
117     dgBar^{x x}_{y} -> iBarxxy, dgBar^{x y}_{y} -> iBarxyy, dgBar^{x z}_{y} -> iBarxzy,
118     dgBar^{y x}_{y} -> iBarxyy, dgBar^{y y}_{y} -> iBaryyy, dgBar^{y z}_{y} -> iBaryzy,
119     dgBar^{z x}_{y} -> iBarxzy, dgBar^{z y}_{y} -> iBaryzy, dgBar^{z z}_{y} -> iBarzzy,
120
121     dgBar^{x x}_{z} -> iBarxxz, dgBar^{x y}_{z} -> iBarxyz, dgBar^{x z}_{z} -> iBarxzz,
122     dgBar^{y x}_{z} -> iBarxyz, dgBar^{y y}_{z} -> iBaryyz, dgBar^{y z}_{z} -> iBaryzz,
123     dgBar^{z x}_{z} -> iBarxzz, dgBar^{z y}_{z} -> iBaryzz, dgBar^{z z}_{z} -> iBarzzz}.
124
125 d2gBarRule := {dgBar_{x x x x} -> gBarxxxx, dgBar_{x y x x} -> gBarxyxx, dgBar_{x z x x} -> gBarxzx,
126                dgBar_{y x x x} -> gBarxyxx, dgBar_{y y x x} -> gBaryyxx, dgBar_{y z x x} -> gBaryzxx,
127                dgBar_{z x x x} -> gBarxzx, dgBar_{z y x x} -> gBaryzxx, dgBar_{z z x x} -> gBarzzxx,
128                dgBar_{x x y x} -> gBarxxxy, dgBar_{x y y x} -> gBarxyxy, dgBar_{x z y x} -> gBarxzxy,
129                dgBar_{y x y x} -> gBarxyxy, dgBar_{y y y x} -> gBaryyxy, dgBar_{y z y x} -> gBaryzxy,
130                dgBar_{z x y x} -> gBarxzxy, dgBar_{z y y x} -> gBaryzxy, dgBar_{z z y x} -> gBarzzxy,
131                dgBar_{x x z x} -> gBarxxxz, dgBar_{x y z x} -> gBarxyxz, dgBar_{x z z x} -> gBarxzzx,
132                dgBar_{y x z x} -> gBarxyxz, dgBar_{y y z x} -> gBaryyxz, dgBar_{y z z x} -> gBaryzxx,
133                dgBar_{z x z x} -> gBarxzzx, dgBar_{z y z x} -> gBaryzxx, dgBar_{z z z x} -> gBarzzxx,
134
135                dgBar_{x x x y} -> gBarxxxy, dgBar_{x y x y} -> gBarxyxy, dgBar_{x z x y} -> gBarxzxy,
136                dgBar_{y x x y} -> gBarxyxy, dgBar_{y y x y} -> gBaryyxy, dgBar_{y z x y} -> gBaryzxy,
137                dgBar_{z x x y} -> gBarxzxy, dgBar_{z y x y} -> gBaryzxy, dgBar_{z z x y} -> gBarzzxy,
138                dgBar_{x x y y} -> gBarxxyy, dgBar_{x y y y} -> gBarxyyy, dgBar_{x z y y} -> gBarxzyy,
139                dgBar_{y x y y} -> gBarxyyy, dgBar_{y y y y} -> gBaryyyy, dgBar_{y z y y} -> gBaryzyy,
140                dgBar_{z x y y} -> gBarxzyy, dgBar_{z y y y} -> gBaryzyy, dgBar_{z z y y} -> gBarzzyy,
141                dgBar_{x x z y} -> gBarxxyz, dgBar_{x y z y} -> gBarxyyz, dgBar_{x z z y} -> gBarxzyz,
142                dgBar_{y x z y} -> gBarxyyz, dgBar_{y y z y} -> gBaryyyz, dgBar_{y z z y} -> gBaryzyz,
143                dgBar_{z x z y} -> gBarxzyz, dgBar_{z y z y} -> gBaryzyz, dgBar_{z z z y} -> gBarzzyz,
144
145                dgBar_{x x x z} -> gBarxxxz, dgBar_{x y x z} -> gBarxyxz, dgBar_{x z x z} -> gBarxzzz,
146                dgBar_{y x x z} -> gBarxyxz, dgBar_{y y x z} -> gBaryyxz, dgBar_{y z x z} -> gBaryzxx,
147                dgBar_{z x x z} -> gBarxzzz, dgBar_{z y x z} -> gBaryzxx, dgBar_{z z x z} -> gBarzzxx,
148                dgBar_{x x y z} -> gBarxxyz, dgBar_{x y y z} -> gBarxyyz, dgBar_{x z y z} -> gBarxzyz,
149                dgBar_{y x y z} -> gBarxyyz, dgBar_{y y y z} -> gBaryyyz, dgBar_{y z y z} -> gBaryzyz,
150                dgBar_{z x y z} -> gBarxzyz, dgBar_{z y y z} -> gBaryzyz, dgBar_{z z y z} -> gBarzzyz,
151                dgBar_{x x z z} -> gBarxxxz, dgBar_{x y z z} -> gBarxyzz, dgBar_{x z z z} -> gBarxzzz,
152                dgBar_{y x z z} -> gBarxyzz, dgBar_{y y z z} -> gBaryyzz, dgBar_{y z z z} -> gBaryzzz,

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153         dgBar_{z x z z} -> gBarxxxx, dgBar_{z y z z} -> gBaryzzz, dgBar_{z z z z} -> gBarzzzz}.
154
155 allRules = gBarRule + d1gBarRule + d2gBarRule \
156           + iBarRule + d1iBarRule \
157           + ABarRule + d1ABarRule \
158           + BBarRule \
159           + GiRule + d1GiRule \
160           + RRule \
161           + trKRule \
162           + d1NRule + d2NRule \
163           + d1phiRule + d2phiRule
164
165 evaluate (Rscalar, allRules, simplify=False)
166 evaluate (Rab, allRules, simplify=False)
167 evaluate (DphiDt, allRules, simplify=False)
168 evaluate (DtrKDt, allRules, simplify=False)
169 evaluate (DgBarDt, allRules, simplify=False)
170 evaluate (DABarDt, allRules, simplify=False)
171 evaluate (DGiBarDt, allRules, simplify=False)
172 evaluate (DNDt, allRules, simplify=False)
173 evaluate (Ham, allRules, simplify=False)
174 evaluate (confMom, allRules, simplify=False)
175
176 # -----
177 # export to C-code
178
179 import writecode
180
181 writecode.cdb_write_code (Rscalar, 'R', 'ricci-scalar.c', 0)
182 writecode.cdb_write_code (Rab, 'Rab', 'ricci.c', 2)
183 writecode.cdb_write_code (DphiDt, 'dot_phi', 'dot-phi.c', 0)
184 writecode.cdb_write_code (DtrKDt, 'dot_trK', 'dot-trK.c', 0)
185 writecode.cdb_write_code (DgBarDt, 'dot_gBar', 'dot-gBar.c', 2)
186 writecode.cdb_write_code (DABarDt, 'dot_ABar', 'dot-ABar.c', 2)
187 writecode.cdb_write_code (DGiBarDt, 'dot_Gi', 'dot-Gi.c', 1)
188 writecode.cdb_write_code (DNDt, 'dot_N', 'dot-N.c', 0)
189
190 writecode.cdb_write_code (Ham, 'Ham', 'hamiltonian.c', 0)

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191 writecode.cdb_write_code (confMom, 'Mom', 'momentum.c', 1)
```