Ricci scalar

Here we compute the Ricci scalar R in terms of the BSSN data.

Note that this expression for R will only be used when evaluating the constraints. It will *not* be used in the evolution equations so the advice that the evolved $\bar{\Gamma}^i$ should be expressed in terms of $\bar{g}_{ij}$ does not apply here.

```
from shared import *
import cdblib

jsonfile = 'bssn-ricci-scalar.json'
cdblib.create (jsonfile)

defRab = cdblib.get ('defRab','bssn-eqtns-14.json')

# -------------------------------------------------------------------------

defG2GBarU := g^{a b} -> \exp(-4\phi) gBar^{a b}.

Rscalar := R.                                  # cdb(Rscalar.00,Rscalar)
Rscalar := g^{a b} R_{a b}.                    # cdb(Rscalar.01,Rscalar)

substitute (Rscalar, defRab)                   # cdb(Rscalar.02,Rscalar)
substitute (Rscalar, defG2GBarU)               # cdb(Rscalar.03,Rscalar)
distribute (Rscalar)                           # cdb(Rscalar.04,Rscalar)

Rscalar = product_sort (Rscalar)               # cdb(Rscalar.05,Rscalar)

rename_dummies (Rscalar)                        # cdb(Rscalar.06,Rscalar)
canonicalise   (Rscalar)                        # cdb(Rscalar.07,Rscalar)

foo := gBar^{b c} \partial_{a}{gBar_{b c}} -> 0.   # follows from det(g) = 1

substitute (Rscalar, foo)                       # cdb(Rscalar.08,Rscalar)

foo := gBar_{a b} gBar^{a b} -> 3.
bah := gBar_{a b} gBar^{a c} -> gBar_{b}^{c}.
moo := gBar^{c d} gBar^{e f} \partial_{a}{gBar_{c e}} -> - \partial_{a}{gBar^{d f}}.
```

```
32
33    substitute (Rscalar, foo)                          # cdb(Rscalar.09,Rscalar)
34    substitute (Rscalar, bah)                          # cdb(Rscalar.10,Rscalar)
35    substitute (Rscalar, moo)                          # cdb(Rscalar.11,Rscalar)
36    eliminate_kronecker (Rscalar)                      # cdb(Rscalar.12,Rscalar)
37    rename_dummies (Rscalar)                           # cdb(Rscalar.13,Rscalar)
38    canonicalise    (Rscalar)                          # cdb(Rscalar.14,Rscalar)
39
40    foo := gBar^{a b} gBar^{c d} \partial_{c}{gBar_{b d}} -> - \partial_{c}{gBar^{a c}}.
41    bah := \partial_{b}{gBar^{a b}} -> - GammaBar^{a}.  # prd62.eqn17
42
43    substitute (Rscalar, foo)                          # cdb(Rscalar.15,Rscalar)
44    substitute (Rscalar, bah)                          # cdb(Rscalar.16,Rscalar)
45
46    Rscalar = product_sort (Rscalar)                   # cdb(Rscalar.17,Rscalar)
47
48    rename_dummies (Rscalar)                           # cdb(Rscalar.18,Rscalar)
49    canonicalise    (Rscalar)                          # cdb(Rscalar.19,Rscalar)
50
51    foo := gBar^{a b} gBar^{c d} \partial_{a b}{gBar_{c d}} ->
52         - gBar^{a b} \partial_{a}{gBar_{c d}} \partial_{b}{gBar^{c d}}. # follows from det(g) = 1
53
54    substitute (Rscalar, foo)                          # cdb(Rscalar.20,Rscalar)
55    factor_out (Rscalar, $\exp(-4\phi)$)               # cdb(Rscalar.21,Rscalar)
56
57    cdblib.put ('Rscalar',Rscalar,jsonfile)
```

$$R = g^{ab} R_{ab} \tag{Rscalar.01}$$

$$
\begin{aligned}
= g^{ab} \bigg(&-2\partial_{ab}\phi + \bar{g}^{cd}\partial_d\phi\partial_a\bar{g}_{bc} + \bar{g}^{cd}\partial_d\phi\partial_b\bar{g}_{ac} - \bar{g}^{cd}\partial_d\phi\partial_c\bar{g}_{ab} + 12\partial_a\phi\partial_b\phi - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}_{ab}\partial_d\phi\partial_c\bar{g}^{cd} - 12\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi - \frac{1}{2}\bar{g}^{cd}\partial_{cd}\bar{g}_{ab} + \frac{1}{2}\bar{g}_{ac}\partial_b\bar{\Gamma}^c \\
&+ \frac{1}{2}\bar{g}_{bc}\partial_a\bar{\Gamma}^c - \frac{1}{2}\partial_c\bar{g}_{ab}\partial_d\bar{g}^{cd} - \frac{1}{2}\partial_d\bar{g}_{bc}\partial_a\bar{g}^{dc} - \frac{1}{2}\bar{g}^{ed}\bar{g}^{cf}\partial_e\bar{g}_{ac}\partial_f\bar{g}_{bd} + \frac{1}{2}\bar{g}^{ef}\bar{g}^{cd}\partial_e\bar{g}_{ac}\partial_f\bar{g}_{bd} + \frac{1}{4}\partial_b\bar{g}_{cd}\partial_a\bar{g}^{cd} - \frac{1}{2}\partial_d\bar{g}_{ac}\partial_b\bar{g}^{dc} \bigg) \tag{Rscalar.02}
\end{aligned}
$$

$$
\begin{aligned}
= \exp\left(-4\phi\right)\bar{g}^{ab}\bigg(&-2\partial_{ab}\phi + \bar{g}^{cd}\partial_d\phi\partial_a\bar{g}_{bc} + \bar{g}^{cd}\partial_d\phi\partial_b\bar{g}_{ac} - \bar{g}^{cd}\partial_d\phi\partial_c\bar{g}_{ab} + 12\partial_a\phi\partial_b\phi - 2\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\bar{g}_{ab}\partial_d\phi\partial_c\bar{g}^{cd} - 12\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi - \frac{1}{2}\bar{g}^{cd}\partial_{cd}\bar{g}_{ab} \\
&+ \frac{1}{2}\bar{g}_{ac}\partial_b\bar{\Gamma}^c + \frac{1}{2}\bar{g}_{bc}\partial_a\bar{\Gamma}^c - \frac{1}{2}\partial_c\bar{g}_{ab}\partial_d\bar{g}^{cd} - \frac{1}{2}\partial_d\bar{g}_{bc}\partial_a\bar{g}^{dc} - \frac{1}{2}\bar{g}^{ed}\bar{g}^{cf}\partial_e\bar{g}_{ac}\partial_f\bar{g}_{bd} + \frac{1}{2}\bar{g}^{ef}\bar{g}^{cd}\partial_e\bar{g}_{ac}\partial_f\bar{g}_{bd} + \frac{1}{4}\partial_b\bar{g}_{cd}\partial_a\bar{g}^{cd} - \frac{1}{2}\partial_d\bar{g}_{ac}\partial_b\bar{g}^{dc} \bigg) \tag{Rscalar.03}
\end{aligned}
$$

$$
\begin{aligned}
= &-2\exp\left(-4\phi\right)\bar{g}^{ab}\partial_{ab}\phi + \exp\left(-4\phi\right)\bar{g}^{ab}\bar{g}^{cd}\partial_d\phi\partial_a\bar{g}_{bc} + \exp\left(-4\phi\right)\bar{g}^{ab}\bar{g}^{cd}\partial_d\phi\partial_b\bar{g}_{ac} - \exp\left(-4\phi\right)\bar{g}^{ab}\bar{g}^{cd}\partial_d\phi\partial_c\bar{g}_{ab} + 12\exp\left(-4\phi\right)\bar{g}^{ab}\partial_a\phi\partial_b\phi \\
&- 2\exp\left(-4\phi\right)\bar{g}^{ab}\bar{g}_{ab}\bar{g}^{cd}\partial_{cd}\phi - 2\exp\left(-4\phi\right)\bar{g}^{ab}\bar{g}_{ab}\partial_d\phi\partial_c\bar{g}^{cd} - 12\exp\left(-4\phi\right)\bar{g}^{ab}\bar{g}_{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi - \frac{1}{2}\exp\left(-4\phi\right)\bar{g}^{ab}\bar{g}^{cd}\partial_{cd}\bar{g}_{ab} \\
&+ \frac{1}{2}\exp\left(-4\phi\right)\bar{g}^{ab}\bar{g}_{ac}\partial_b\bar{\Gamma}^c + \frac{1}{2}\exp\left(-4\phi\right)\bar{g}^{ab}\bar{g}_{bc}\partial_a\bar{\Gamma}^c - \frac{1}{2}\exp\left(-4\phi\right)\bar{g}^{ab}\partial_c\bar{g}_{ab}\partial_d\bar{g}^{cd} - \frac{1}{2}\exp\left(-4\phi\right)\bar{g}^{ab}\partial_d\bar{g}_{bc}\partial_a\bar{g}^{dc} \\
&- \frac{1}{2}\exp\left(-4\phi\right)\bar{g}^{ab}\bar{g}^{ed}\bar{g}^{cf}\partial_e\bar{g}_{ac}\partial_f\bar{g}_{bd} + \frac{1}{2}\exp\left(-4\phi\right)\bar{g}^{ab}\bar{g}^{ef}\bar{g}^{cd}\partial_e\bar{g}_{ac}\partial_f\bar{g}_{bd} + \frac{1}{4}\exp\left(-4\phi\right)\bar{g}^{ab}\partial_b\bar{g}_{cd}\partial_a\bar{g}^{cd} - \frac{1}{2}\exp\left(-4\phi\right)\bar{g}^{ab}\partial_d\bar{g}_{ac}\partial_b\bar{g}^{dc} \tag{Rscalar.04}
\end{aligned}
$$

$$
\begin{aligned}
= &-2\bar{g}^{ab}\partial_{ab}\phi\exp\left(-4\phi\right) + \bar{g}^{ca}\bar{g}^{bd}\partial_d\phi\exp\left(-4\phi\right)\partial_c\bar{g}_{ab} + \bar{g}^{ac}\bar{g}^{bd}\partial_d\phi\exp\left(-4\phi\right)\partial_c\bar{g}_{ab} - \bar{g}^{ab}\bar{g}^{cd}\partial_d\phi\exp\left(-4\phi\right)\partial_c\bar{g}_{ab} + 12\bar{g}^{ab}\partial_a\phi\partial_b\phi\exp\left(-4\phi\right) \\
&- 2\bar{g}_{ab}\bar{g}^{ab}\bar{g}^{cd}\partial_{cd}\phi\exp\left(-4\phi\right) - 2\bar{g}_{ab}\bar{g}^{ab}\partial_d\phi\exp\left(-4\phi\right)\partial_c\bar{g}^{cd} - 12\bar{g}_{ab}\bar{g}^{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi\exp\left(-4\phi\right) - \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\exp\left(-4\phi\right)\partial_{cd}\bar{g}_{ab} + \frac{1}{2}\bar{g}_{ab}\bar{g}^{ac}\exp\left(-4\phi\right)\partial_c\bar{\Gamma}^b \\
&+ \frac{1}{2}\bar{g}_{ab}\bar{g}^{ca}\exp\left(-4\phi\right)\partial_c\bar{\Gamma}^b - \frac{1}{2}\bar{g}^{ab}\exp\left(-4\phi\right)\partial_c\bar{g}_{ab}\partial_d\bar{g}^{cd} - \frac{1}{2}\bar{g}^{ca}\exp\left(-4\phi\right)\partial_d\bar{g}_{ab}\partial_c\bar{g}^{db} - \frac{1}{2}\bar{g}^{ac}\bar{g}^{be}\bar{g}^{fd}\exp\left(-4\phi\right)\partial_f\bar{g}_{ab}\partial_e\bar{g}_{cd} \\
&+ \frac{1}{2}\bar{g}^{ac}\bar{g}^{bd}\bar{g}^{ef}\exp\left(-4\phi\right)\partial_e\bar{g}_{ab}\partial_f\bar{g}_{cd} + \frac{1}{4}\bar{g}^{cd}\exp\left(-4\phi\right)\partial_d\bar{g}_{ab}\partial_c\bar{g}^{ab} - \frac{1}{2}\bar{g}^{ac}\exp\left(-4\phi\right)\partial_d\bar{g}_{ab}\partial_c\bar{g}^{db} \tag{Rscalar.05}
\end{aligned}
$$

$$
\begin{aligned}
= &-2\bar{g}^{ab}\partial_{ab}\phi\exp\left(-4\phi\right) + \bar{g}^{bc}\bar{g}^{da}\partial_a\phi\exp\left(-4\phi\right)\partial_b\bar{g}_{cd} + \bar{g}^{cb}\bar{g}^{da}\partial_a\phi\exp\left(-4\phi\right)\partial_b\bar{g}_{cd} - \bar{g}^{cd}\bar{g}^{ba}\partial_a\phi\exp\left(-4\phi\right)\partial_b\bar{g}_{cd} + 12\bar{g}^{ab}\partial_a\phi\partial_b\phi\exp\left(-4\phi\right) \\
&- 2\bar{g}_{cd}\bar{g}^{cd}\bar{g}^{ab}\partial_{ab}\phi\exp\left(-4\phi\right) - 2\bar{g}_{cd}\bar{g}^{cd}\partial_a\phi\exp\left(-4\phi\right)\partial_b\bar{g}^{ba} - 12\bar{g}_{cd}\bar{g}^{cd}\bar{g}^{ab}\partial_a\phi\partial_b\phi\exp\left(-4\phi\right) - \frac{1}{2}\bar{g}^{cd}\bar{g}^{ab}\exp\left(-4\phi\right)\partial_{ab}\bar{g}_{cd} + \frac{1}{2}\bar{g}_{ca}\bar{g}^{cb}\exp\left(-4\phi\right)\partial_b\bar{\Gamma}^a \\
&+ \frac{1}{2}\bar{g}_{ca}\bar{g}^{bc}\exp\left(-4\phi\right)\partial_b\bar{\Gamma}^a - \frac{1}{2}\bar{g}^{cd}\exp\left(-4\phi\right)\partial_a\bar{g}_{cd}\partial_b\bar{g}^{ab} - \frac{1}{2}\bar{g}^{bc}\exp\left(-4\phi\right)\partial_a\bar{g}_{cd}\partial_b\bar{g}^{ad} - \frac{1}{2}\bar{g}^{cd}\bar{g}^{eb}\bar{g}^{af}\exp\left(-4\phi\right)\partial_a\bar{g}_{ce}\partial_b\bar{g}_{df} \\
&+ \frac{1}{2}\bar{g}^{cd}\bar{g}^{ef}\bar{g}^{ab}\exp\left(-4\phi\right)\partial_a\bar{g}_{ce}\partial_b\bar{g}_{df} + \frac{1}{4}\bar{g}^{ba}\exp\left(-4\phi\right)\partial_a\bar{g}_{cd}\partial_b\bar{g}^{cd} - \frac{1}{2}\bar{g}^{cb}\exp\left(-4\phi\right)\partial_a\bar{g}_{cd}\partial_b\bar{g}^{ad} \tag{Rscalar.06}
\end{aligned}
$$

$$R = -2\bar{g}^{ab}\partial_{ab}\phi\exp(-4\phi) + 2\bar{g}^{ab}\bar{g}^{cd}\partial_a\phi\exp(-4\phi)\,\partial_c\bar{g}_{bd} - \bar{g}^{ab}\bar{g}^{cd}\partial_a\phi\exp(-4\phi)\,\partial_b\bar{g}_{cd} + 12\bar{g}^{ab}\partial_a\phi\partial_b\phi\exp(-4\phi) - 2\bar{g}_{ab}\bar{g}^{ab}\bar{g}^{cd}\partial_{cd}\phi\exp(-4\phi)$$

$$\quad - 2\bar{g}_{ab}\bar{g}^{ab}\partial_c\phi\exp(-4\phi)\,\partial_d\bar{g}^{cd} - 12\bar{g}_{ab}\bar{g}^{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi\exp(-4\phi) - \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\exp(-4\phi)\,\partial_{ab}\bar{g}_{cd} + \bar{g}_{ab}\bar{g}^{ac}\exp(-4\phi)\,\partial_c\bar{\Gamma}^b - \frac{1}{2}\bar{g}^{ab}\exp(-4\phi)\,\partial_c\bar{g}_{ab}\partial_d\bar{g}^{cd}$$

$$\quad - \bar{g}^{ab}\exp(-4\phi)\,\partial_c\bar{g}_{ad}\partial_b\bar{g}^{cd} - \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\bar{g}^{ef}\exp(-4\phi)\,\partial_a\bar{g}_{ce}\partial_d\bar{g}_{bf} + \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\bar{g}^{ef}\exp(-4\phi)\,\partial_a\bar{g}_{ce}\partial_b\bar{g}_{df} + \frac{1}{4}\bar{g}^{ab}\exp(-4\phi)\,\partial_a\bar{g}_{cd}\partial_b\bar{g}^{cd} \qquad \text{(Rscalar.07)}$$

$$= -2\bar{g}^{ab}\partial_{ab}\phi\exp(-4\phi) + 2\bar{g}^{ab}\bar{g}^{cd}\partial_a\phi\exp(-4\phi)\,\partial_c\bar{g}_{bd} + 12\bar{g}^{ab}\partial_a\phi\partial_b\phi\exp(-4\phi) - 2\bar{g}_{ab}\bar{g}^{ab}\bar{g}^{cd}\partial_{cd}\phi\exp(-4\phi) - 2\bar{g}_{ab}\bar{g}^{ab}\partial_c\phi\exp(-4\phi)\,\partial_d\bar{g}^{cd}$$

$$\quad - 12\bar{g}_{ab}\bar{g}^{ab}\bar{g}^{cd}\partial_c\phi\partial_d\phi\exp(-4\phi) - \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\exp(-4\phi)\,\partial_{ab}\bar{g}_{cd} + \bar{g}_{ab}\bar{g}^{ac}\exp(-4\phi)\,\partial_c\bar{\Gamma}^b - \bar{g}^{ab}\exp(-4\phi)\,\partial_c\bar{g}_{ad}\partial_b\bar{g}^{cd}$$

$$\quad - \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\bar{g}^{ef}\exp(-4\phi)\,\partial_a\bar{g}_{ce}\partial_d\bar{g}_{bf} + \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\bar{g}^{ef}\exp(-4\phi)\,\partial_a\bar{g}_{ce}\partial_b\bar{g}_{df} + \frac{1}{4}\bar{g}^{ab}\exp(-4\phi)\,\partial_a\bar{g}_{cd}\partial_b\bar{g}^{cd} \qquad \text{(Rscalar.08)}$$

$$= -2\bar{g}^{ab}\partial_{ab}\phi\exp(-4\phi) + 2\bar{g}^{ab}\bar{g}^{cd}\partial_a\phi\exp(-4\phi)\,\partial_c\bar{g}_{bd} + 12\bar{g}^{ab}\partial_a\phi\partial_b\phi\exp(-4\phi) - 6\bar{g}^{cd}\partial_{cd}\phi\exp(-4\phi) - 6\partial_c\phi\exp(-4\phi)\,\partial_d\bar{g}^{cd}$$

$$\quad - 36\bar{g}^{cd}\partial_c\phi\partial_d\phi\exp(-4\phi) - \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\exp(-4\phi)\,\partial_{ab}\bar{g}_{cd} + \bar{g}_{ab}\bar{g}^{ac}\exp(-4\phi)\,\partial_c\bar{\Gamma}^b - \bar{g}^{ab}\exp(-4\phi)\,\partial_c\bar{g}_{ad}\partial_b\bar{g}^{cd} - \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\bar{g}^{ef}\exp(-4\phi)\,\partial_a\bar{g}_{ce}\partial_d\bar{g}_{bf}$$

$$\quad + \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\bar{g}^{ef}\exp(-4\phi)\,\partial_a\bar{g}_{ce}\partial_b\bar{g}_{df} + \frac{1}{4}\bar{g}^{ab}\exp(-4\phi)\,\partial_a\bar{g}_{cd}\partial_b\bar{g}^{cd} \qquad \text{(Rscalar.09)}$$

$$= -2\bar{g}^{ab}\partial_{ab}\phi\exp(-4\phi) + 2\bar{g}^{ab}\bar{g}^{cd}\partial_a\phi\exp(-4\phi)\,\partial_c\bar{g}_{bd} + 12\bar{g}^{ab}\partial_a\phi\partial_b\phi\exp(-4\phi) - 6\bar{g}^{cd}\partial_{cd}\phi\exp(-4\phi) - 6\partial_c\phi\exp(-4\phi)\,\partial_d\bar{g}^{cd}$$

$$\quad - 36\bar{g}^{cd}\partial_c\phi\partial_d\phi\exp(-4\phi) - \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\exp(-4\phi)\,\partial_{ab}\bar{g}_{cd} + \bar{g}_b{}^c\exp(-4\phi)\,\partial_c\bar{\Gamma}^b - \bar{g}^{ab}\exp(-4\phi)\,\partial_c\bar{g}_{ad}\partial_b\bar{g}^{cd} - \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\bar{g}^{ef}\exp(-4\phi)\,\partial_a\bar{g}_{ce}\partial_d\bar{g}_{bf}$$

$$\quad + \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\bar{g}^{ef}\exp(-4\phi)\,\partial_a\bar{g}_{ce}\partial_b\bar{g}_{df} + \frac{1}{4}\bar{g}^{ab}\exp(-4\phi)\,\partial_a\bar{g}_{cd}\partial_b\bar{g}^{cd} \qquad \text{(Rscalar.10)}$$

$$= -2\bar{g}^{ab}\partial_{ab}\phi\exp(-4\phi) + 2\bar{g}^{ab}\bar{g}^{cd}\partial_a\phi\exp(-4\phi)\,\partial_c\bar{g}_{bd} + 12\bar{g}^{ab}\partial_a\phi\partial_b\phi\exp(-4\phi) - 6\bar{g}^{cd}\partial_{cd}\phi\exp(-4\phi) - 6\partial_c\phi\exp(-4\phi)\,\partial_d\bar{g}^{cd}$$

$$\quad - 36\bar{g}^{cd}\partial_c\phi\partial_d\phi\exp(-4\phi) - \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\exp(-4\phi)\,\partial_{ab}\bar{g}_{cd} + \bar{g}_b{}^c\exp(-4\phi)\,\partial_c\bar{\Gamma}^b - \bar{g}^{ab}\exp(-4\phi)\,\partial_c\bar{g}_{ad}\partial_b\bar{g}^{cd} + \frac{1}{2}\bar{g}^{ab}\partial_a\bar{g}^{df}\exp(-4\phi)\,\partial_d\bar{g}_{bf}$$

$$\quad - \frac{1}{2}\bar{g}^{ab}\partial_a\bar{g}^{df}\exp(-4\phi)\,\partial_b\bar{g}_{df} + \frac{1}{4}\bar{g}^{ab}\exp(-4\phi)\,\partial_a\bar{g}_{cd}\partial_b\bar{g}^{cd} \qquad \text{(Rscalar.11)}$$

$$= -2\bar{g}^{ab}\partial_{ab}\phi\exp(-4\phi) + 2\bar{g}^{ab}\bar{g}^{cd}\partial_a\phi\exp(-4\phi)\,\partial_c\bar{g}_{bd} + 12\bar{g}^{ab}\partial_a\phi\partial_b\phi\exp(-4\phi) - 6\bar{g}^{cd}\partial_{cd}\phi\exp(-4\phi) - 6\partial_c\phi\exp(-4\phi)\,\partial_d\bar{g}^{cd}$$

$$\quad - 36\bar{g}^{cd}\partial_c\phi\partial_d\phi\exp(-4\phi) - \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\exp(-4\phi)\,\partial_{ab}\bar{g}_{cd} + \exp(-4\phi)\,\partial_b\bar{\Gamma}^b - \bar{g}^{ab}\exp(-4\phi)\,\partial_c\bar{g}_{ad}\partial_b\bar{g}^{cd} + \frac{1}{2}\bar{g}^{ab}\partial_a\bar{g}^{df}\exp(-4\phi)\,\partial_d\bar{g}_{bf}$$

$$\quad - \frac{1}{2}\bar{g}^{ab}\partial_a\bar{g}^{df}\exp(-4\phi)\,\partial_b\bar{g}_{df} + \frac{1}{4}\bar{g}^{ab}\exp(-4\phi)\,\partial_a\bar{g}_{cd}\partial_b\bar{g}^{cd} \qquad \text{(Rscalar.12)}$$

$$R = -8\bar{g}^{ab}\partial_{ab}\phi\exp\left(-4\phi\right) + 2\bar{g}^{ac}\bar{g}^{bd}\partial_a\phi\exp\left(-4\phi\right)\partial_b\bar{g}_{cd} - 24\bar{g}^{ab}\partial_a\phi\partial_b\phi\exp\left(-4\phi\right) - 6\partial_a\phi\exp\left(-4\phi\right)\partial_b\bar{g}^{ab} - \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\exp\left(-4\phi\right)\partial_{ab}\bar{g}_{cd}$$

$$+\exp\left(-4\phi\right)\partial_a\bar{\Gamma}^a - \bar{g}^{cb}\exp\left(-4\phi\right)\partial_a\bar{g}_{cd}\partial_b\bar{g}^{ad} + \frac{1}{2}\bar{g}^{ac}\partial_a\bar{g}^{bd}\exp\left(-4\phi\right)\partial_b\bar{g}_{cd} - \frac{1}{2}\bar{g}^{ab}\partial_a\bar{g}^{cd}\exp\left(-4\phi\right)\partial_b\bar{g}_{cd} + \frac{1}{4}\bar{g}^{ab}\exp\left(-4\phi\right)\partial_a\bar{g}_{cd}\partial_b\bar{g}^{cd} \quad \text{(Rscalar.13)}$$

$$= -8\bar{g}^{ab}\partial_{ab}\phi\exp\left(-4\phi\right) + 2\bar{g}^{ab}\bar{g}^{cd}\partial_a\phi\exp\left(-4\phi\right)\partial_c\bar{g}_{bd} - 24\bar{g}^{ab}\partial_a\phi\partial_b\phi\exp\left(-4\phi\right) - 6\partial_a\phi\exp\left(-4\phi\right)\partial_b\bar{g}^{ab} - \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\exp\left(-4\phi\right)\partial_{ab}\bar{g}_{cd}$$

$$+\exp\left(-4\phi\right)\partial_a\bar{\Gamma}^a - \bar{g}^{ab}\exp\left(-4\phi\right)\partial_c\bar{g}_{ad}\partial_b\bar{g}^{cd} + \frac{1}{2}\bar{g}^{ab}\partial_a\bar{g}^{cd}\exp\left(-4\phi\right)\partial_c\bar{g}_{bd} - \frac{1}{2}\bar{g}^{ab}\partial_a\bar{g}^{cd}\exp\left(-4\phi\right)\partial_b\bar{g}_{cd} + \frac{1}{4}\bar{g}^{ab}\exp\left(-4\phi\right)\partial_a\bar{g}_{cd}\partial_b\bar{g}^{cd} \quad \text{(Rscalar.14)}$$

$$= -8\bar{g}^{ab}\partial_{ab}\phi\exp\left(-4\phi\right) - 2\partial_c\bar{g}^{ac}\partial_a\phi\exp\left(-4\phi\right) - 24\bar{g}^{ab}\partial_a\phi\partial_b\phi\exp\left(-4\phi\right) - 6\partial_a\phi\exp\left(-4\phi\right)\partial_b\bar{g}^{ab} - \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\exp\left(-4\phi\right)\partial_{ab}\bar{g}_{cd} + \exp\left(-4\phi\right)\partial_a\bar{\Gamma}^a$$

$$- \bar{g}^{ab}\exp\left(-4\phi\right)\partial_c\bar{g}_{ad}\partial_b\bar{g}^{cd} + \frac{1}{2}\bar{g}^{ab}\partial_a\bar{g}^{cd}\exp\left(-4\phi\right)\partial_c\bar{g}_{bd} - \frac{1}{2}\bar{g}^{ab}\partial_a\bar{g}^{cd}\exp\left(-4\phi\right)\partial_b\bar{g}_{cd} + \frac{1}{4}\bar{g}^{ab}\exp\left(-4\phi\right)\partial_a\bar{g}_{cd}\partial_b\bar{g}^{cd} \quad \text{(Rscalar.15)}$$

$$= -8\bar{g}^{ab}\partial_{ab}\phi\exp\left(-4\phi\right) + 2\bar{\Gamma}^a\partial_a\phi\exp\left(-4\phi\right) - 24\bar{g}^{ab}\partial_a\phi\partial_b\phi\exp\left(-4\phi\right) + 6\partial_a\phi\exp\left(-4\phi\right)\bar{\Gamma}^a - \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\exp\left(-4\phi\right)\partial_{ab}\bar{g}_{cd} + \exp\left(-4\phi\right)\partial_a\bar{\Gamma}^a$$

$$- \bar{g}^{ab}\exp\left(-4\phi\right)\partial_c\bar{g}_{ad}\partial_b\bar{g}^{cd} + \frac{1}{2}\bar{g}^{ab}\partial_a\bar{g}^{cd}\exp\left(-4\phi\right)\partial_c\bar{g}_{bd} - \frac{1}{2}\bar{g}^{ab}\partial_a\bar{g}^{cd}\exp\left(-4\phi\right)\partial_b\bar{g}_{cd} + \frac{1}{4}\bar{g}^{ab}\exp\left(-4\phi\right)\partial_a\bar{g}_{cd}\partial_b\bar{g}^{cd} \quad \text{(Rscalar.16)}$$

$$= -8\bar{g}^{ab}\partial_{ab}\phi\exp\left(-4\phi\right) + 8\bar{\Gamma}^a\partial_a\phi\exp\left(-4\phi\right) - 24\bar{g}^{ab}\partial_a\phi\partial_b\phi\exp\left(-4\phi\right) - \frac{1}{2}\bar{g}^{cd}\bar{g}^{ab}\exp\left(-4\phi\right)\partial_{cd}\bar{g}_{ab} + \exp\left(-4\phi\right)\partial_a\bar{\Gamma}^a - \bar{g}^{ac}\exp\left(-4\phi\right)\partial_d\bar{g}_{ab}\partial_c\bar{g}^{db}$$

$$+ \frac{1}{2}\bar{g}^{ca}\exp\left(-4\phi\right)\partial_d\bar{g}_{ab}\partial_c\bar{g}^{db} - \frac{1}{2}\bar{g}^{cd}\exp\left(-4\phi\right)\partial_d\bar{g}_{ab}\partial_c\bar{g}^{ab} + \frac{1}{4}\bar{g}^{cd}\exp\left(-4\phi\right)\partial_c\bar{g}_{ab}\partial_d\bar{g}^{ab} \quad \text{(Rscalar.17)}$$

$$= -8\bar{g}^{ab}\partial_{ab}\phi\exp\left(-4\phi\right) + 8\bar{\Gamma}^a\partial_a\phi\exp\left(-4\phi\right) - 24\bar{g}^{ab}\partial_a\phi\partial_b\phi\exp\left(-4\phi\right) - \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\exp\left(-4\phi\right)\partial_{ab}\bar{g}_{cd} + \exp\left(-4\phi\right)\partial_a\bar{\Gamma}^a - \bar{g}^{cb}\exp\left(-4\phi\right)\partial_a\bar{g}_{cd}\partial_b\bar{g}^{ad}$$

$$+ \frac{1}{2}\bar{g}^{bc}\exp\left(-4\phi\right)\partial_a\bar{g}_{cd}\partial_b\bar{g}^{ad} - \frac{1}{2}\bar{g}^{ba}\exp\left(-4\phi\right)\partial_a\bar{g}_{cd}\partial_b\bar{g}^{cd} + \frac{1}{4}\bar{g}^{ab}\exp\left(-4\phi\right)\partial_a\bar{g}_{cd}\partial_b\bar{g}^{cd} \quad \text{(Rscalar.18)}$$

$$= -8\bar{g}^{ab}\partial_{ab}\phi\exp\left(-4\phi\right) + 8\bar{\Gamma}^a\partial_a\phi\exp\left(-4\phi\right) - 24\bar{g}^{ab}\partial_a\phi\partial_b\phi\exp\left(-4\phi\right) - \frac{1}{2}\bar{g}^{ab}\bar{g}^{cd}\exp\left(-4\phi\right)\partial_{ab}\bar{g}_{cd} + \exp\left(-4\phi\right)\partial_a\bar{\Gamma}^a - \frac{1}{2}\bar{g}^{ab}\exp\left(-4\phi\right)\partial_c\bar{g}_{ad}\partial_b\bar{g}^{cd}$$

$$- \frac{1}{4}\bar{g}^{ab}\exp\left(-4\phi\right)\partial_a\bar{g}_{cd}\partial_b\bar{g}^{cd} \quad \text{(Rscalar.19)}$$

$$= -8\bar{g}^{ab}\partial_{ab}\phi\exp\left(-4\phi\right) + 8\bar{\Gamma}^a\partial_a\phi\exp\left(-4\phi\right) - 24\bar{g}^{ab}\partial_a\phi\partial_b\phi\exp\left(-4\phi\right) + \frac{1}{2}\bar{g}^{ab}\partial_a\bar{g}_{cd}\partial_b\bar{g}^{cd}\exp\left(-4\phi\right) + \exp\left(-4\phi\right)\partial_a\bar{\Gamma}^a$$

$$- \frac{1}{2}\bar{g}^{ab}\exp\left(-4\phi\right)\partial_c\bar{g}_{ad}\partial_b\bar{g}^{cd} - \frac{1}{4}\bar{g}^{ab}\exp\left(-4\phi\right)\partial_a\bar{g}_{cd}\partial_b\bar{g}^{cd} \quad \text{(Rscalar.20)}$$

$$= \exp\left(-4\phi\right)\left(-8\bar{g}^{ab}\partial_{ab}\phi + 8\bar{\Gamma}^a\partial_a\phi - 24\bar{g}^{ab}\partial_a\phi\partial_b\phi + \frac{1}{4}\bar{g}^{ab}\partial_a\bar{g}_{cd}\partial_b\bar{g}^{cd} + \partial_a\bar{\Gamma}^a - \frac{1}{2}\bar{g}^{ab}\partial_c\bar{g}_{ad}\partial_b\bar{g}^{cd}\right) \quad \text{(Rscalar.21)}$$