Exercise 3.6 Commutation of ∇ on the Riemann tensor – simple computation

```
{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w#}::Indices(position=independent).

DD{#}::Derivative.
\nabla{#}::Derivative.

RabcdF := R_{a b c d} -> A_{a} B_{b} C_{c} D_{d}.       # cdb(RabcdF.000,RabcdF)
RabcdB := A_{a} B_{b} C_{c} D_{d} -> R_{a b c d}.       # cdb(RabcdB.000,RabcdB)

derivDD := DD_{b c}{V?_{a}} -> R^{d}_{a b c} V?_{d}.  # cdb(derivDD.000,derivDD)

nablaDD := \nabla_{f}{\nabla_{e}{R_{a b c d}}}
         - \nabla_{e}{\nabla_{f}{R_{a b c d}}} -> DD_{e f}{R_{a b c d}}.

# product rule for DD acting on A_{a} B_{b} C_{c} D_{d}
pruleDD := DD_{e f}{A_{a} B_{b} C_{c} D_{d}} -> DD_{e f}{A_{a}} B_{b} C_{c} D_{d}
                                              + A_{a} DD_{e f}{B_{b}} C_{c} D_{d}
                                              + A_{a} B_{b} DD_{e f}{C_{c}} D_{d}
                                              + A_{a} B_{b} C_{c} DD_{e f}{D_{d}}.
                                                       # cdb(pruleDD.000,pruleDD)

expr :=   \nabla_{f}{\nabla_{e}{R_{a b c d}}}
        - \nabla_{e}{\nabla_{f}{R_{a b c d}}}.         # cdb (ex-0306.100, expr)

substitute   (expr,nablaDD)                            # cdb (ex-0306.101, expr)
substitute   (expr,RabcdF)                             # cdb (ex-0306.102, expr)
substitute   (expr,pruleDD)                            # cdb (ex-0306.103, expr)
substitute   (expr,derivDD)                            # cdb (ex-0306.104, expr)
sort_product (expr)                                    # cdb (ex-0306.105, expr)
substitute   (expr,RabcdB)                             # cdb (ex-0306.106, expr)
```

$$
\begin{aligned}
\nabla_f\left(\nabla_e R_{abcd}\right)-\nabla_e\left(\nabla_f R_{abcd}\right) &= DD_{ef}R_{abcd} && \texttt{(ex-0306.101)}\\
&= DD_{ef}\left(A_aB_bC_cD_d\right) && \texttt{(ex-0306.102)}\\
&= DD_{ef}A_aB_bC_cD_d + A_aDD_{ef}B_bC_cD_d + A_aB_bDD_{ef}C_cD_d + A_aB_bC_cDD_{ef}D_d && \texttt{(ex-0306.103)}\\
&= R^g{}_{aef}A_gB_bC_cD_d + A_aR^g{}_{bef}B_gC_cD_d + A_aB_bR^g{}_{cef}C_gD_d + A_aB_bC_cR^g{}_{def}D_g && \texttt{(ex-0306.104)}\\
&= A_gB_bC_cD_dR^g{}_{aef} + A_aB_gC_cD_dR^g{}_{bef} + A_aB_bC_gD_dR^g{}_{cef} + A_aB_bC_cD_gR^g{}_{def} && \texttt{(ex-0306.105)}\\
&= R_{gbcd}R^g{}_{aef} + R_{agcd}R^g{}_{bef} + R_{abgd}R^g{}_{cef} + R_{abcg}R^g{}_{def} && \texttt{(ex-0306.106)}
\end{aligned}
$$