Exercise 2.3 Covariant derivative of $v^a{}_b$

```
1   {a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices(position=independent).
2
3   \nabla{#}::Derivative.
4   \partial{#}::PartialDerivative.
5
6   # template for covariant derivative of a vector
7
8   derivU := \nabla_{a}{A?^{b}} -> \partial_{a}{A?^{b}} + \Gamma^{b}_{c a} A?^{c}.
9   derivD := \nabla_{a}{A?_{b}} -> \partial_{a}{A?_{b}} - \Gamma^{c}_{b a} A?_{c}.
10
11  vab := v^{a}_{b} -> A^{a} B_{b}.
12  iab := A^{a} B_{b} -> v^{a}_{b}.
13
14  pab := \partial_{a}{A^{b}} B_{c} -> \partial_{a}{A^{b} B_{c}} - A^{b} \partial_{a}{B_{c}}.
15
16  # create an object
17
18  Dvab := \nabla_{a}{v^{b}_{c}}.    # cdb (ex-0203.101,Dvab)
19
20  # apply the rule, then simplify
21
22  substitute      (Dvab,vab)         # cdb (ex-0203.102,Dvab)
23  product_rule    (Dvab)             # cdb (ex-0203.103,Dvab)
24  substitute      (Dvab,derivD)      # cdb (ex-0203.104,Dvab)
25  substitute      (Dvab,derivU)      # cdb (ex-0203.105,Dvab)
26  distribute      (Dvab)             # cdb (ex-0203.106,Dvab)
27  substitute      (Dvab,pab)         # cdb (ex-0203.107,Dvab)
28  canonicalise    (Dvab)             # cdb (ex-0203.108,Dvab)
29  substitute      (Dvab,iab)         # cdb (ex-0203.109,Dvab)
30  sort_product    (Dvab)             # cdb (ex-0203.110,Dvab)
```

$$\nabla_a v^b{}_c = \nabla_a \left(A^b B_c \right) \tag{ex-0203.102}$$

$$= \nabla_a A^b B_c + A^b \nabla_a B_c \tag{ex-0203.103}$$

$$= \nabla_a A^b B_c + A^b \left(\partial_a B_c - \Gamma^d{}_{ca} B_d \right) \tag{ex-0203.104}$$

$$= \left(\partial_a A^b + \Gamma^b{}_{da} A^d \right) B_c + A^b \left(\partial_a B_c - \Gamma^d{}_{ca} B_d \right) \tag{ex-0203.105}$$

$$= \partial_a A^b B_c + \Gamma^b{}_{da} A^d B_c + A^b \partial_a B_c - A^b \Gamma^d{}_{ca} B_d \tag{ex-0203.106}$$

$$= \partial_a \left(A^b B_c \right) + \Gamma^b{}_{da} A^d B_c - A^b \Gamma^d{}_{ca} B_d \tag{ex-0203.107}$$

$$= \partial_a \left(A^b B_c \right) + \Gamma^b{}_{da} A^d B_c - A^b \Gamma^d{}_{ca} B_d \tag{ex-0203.108}$$

$$= \partial_a v^b{}_c + \Gamma^b{}_{da} v^d{}_c - v^b{}_d \Gamma^d{}_{ca} \tag{ex-0203.109}$$

$$= \partial_a v^b{}_c + \Gamma^b{}_{da} v^d{}_c - \Gamma^d{}_{ca} v^b{}_d \tag{ex-0203.110}$$