

Exercise 1.3 Christoffel symbol and dg with a single rule

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1  {a,b,c,d,e,f,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices(position=independent).
2
3  g_{a b}::Metric.
4  g_{a}^{b}::KroneckerDelta.
5
6  \partial{#}::PartialDerivative.
7
8  GammaU := \Gamma^{a}_{b c} -> (1/2) g^{a d} ( \partial_{b}{g_{d c}}
9                                     + \partial_{c}{g_{b d}}
10                                    - \partial_{d}{g_{b c}} ).
11
12  GammaD := \Gamma_{a b c} -> g_{a d} \Gamma^{d}_{b c}.           # cdb (ex-0103.101,GammaD)
13
14  substitute      (GammaD, GammaU)                               # cdb (ex-0103.102,GammaD) # requires Indices(position=independent)
15  distribute      (GammaD)                                       # cdb (ex-0103.103,GammaD)
16  eliminate_metric (GammaD)                                       # cdb (ex-0103.104,GammaD)
17  eliminate_kronecker (GammaD)                                    # cdb (ex-0103.105,GammaD)
18
19  expr := \Gamma_{a b c} + \Gamma_{b a c} - \partial_{c}{g_{a b}}.   # cdb (ex-0103.201,expr)
20
21  substitute      (expr, GammaD)                                   # cdb (ex-0103.202,expr)
22  canonicalise    (expr)                                           # cdb (ex-0103.203,expr)

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$$\Gamma_{abc} \rightarrow g_{ad}\Gamma^d_{bc} \quad (\text{ex-0103.101})$$

$$\Gamma_{abc} \rightarrow \frac{1}{2}g_{ad}g^{de}(\partial_b g_{ec} + \partial_c g_{be} - \partial_e g_{bc}) \quad (\text{ex-0103.102})$$

$$\Gamma_{abc} \rightarrow \frac{1}{2}g_{ad}g^{de}\partial_b g_{ec} + \frac{1}{2}g_{ad}g^{de}\partial_c g_{be} - \frac{1}{2}g_{ad}g^{de}\partial_e g_{bc} \quad (\text{ex-0103.103})$$

$$\Gamma_{abc} \rightarrow \frac{1}{2}g_a{}^e\partial_b g_{ec} + \frac{1}{2}g_a{}^e\partial_c g_{be} - \frac{1}{2}g_a{}^e\partial_e g_{bc} \quad (\text{ex-0103.104})$$

$$\Gamma_{abc} \rightarrow \frac{1}{2}\partial_b g_{ac} + \frac{1}{2}\partial_c g_{ba} - \frac{1}{2}\partial_a g_{bc} \quad (\text{ex-0103.105})$$

$$\Gamma_{abc} + \Gamma_{bac} - \partial_c g_{ab} = \frac{1}{2}\partial_c g_{ba} - \frac{1}{2}\partial_c g_{ab} \quad (\text{ex-0103.202})$$

$$= 0 \quad (\text{ex-0103.203})$$

Exercise 1.3 Repeat but without position=independent

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1  {a,b,c,d,e,f,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices.
2
3  g_{a b}::Metric.
4  g_{a}^{b}::KroneckerDelta.
5
6  \partial{#}::PartialDerivative.
7
8  GammaU := \Gamma^{a}_{b c} -> (1/2) g^{a d} ( \partial_{b}{g_{d c}}
9                                     + \partial_{c}{g_{b d}}
10                                    - \partial_{d}{g_{b c}} ).
11
12  GammaD := \Gamma_{a b c} -> g_{a d} \Gamma^{d}_{b c}.           # cdb (ex-0103.301,GammaD)
13
14  substitute      (GammaD, GammaU)                               # cdb (ex-0103.302,GammaD)
15  distribute      (GammaD)                                       # cdb (ex-0103.303,GammaD)
16  eliminate_metric (GammaD)                                       # cdb (ex-0103.304,GammaD)
17  eliminate_kronecker (GammaD)                                    # cdb (ex-0103.305,GammaD)
18
19  expr := \Gamma_{a b c} + \Gamma_{b a c} - \partial_{c}{g_{a b}}.   # cdb (ex-0103.401,expr)
20
21  substitute      (expr, GammaD)                                   # cdb (ex-0103.402,expr)
22  canonicalise    (expr)                                           # cdb (ex-0103.403,expr)

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$$\Gamma_{abc} \rightarrow g_{ad}\Gamma^d_{bc} \quad (\text{ex-0103.301})$$

$$\frac{1}{2}g_a{}^d(\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc}) \rightarrow \frac{1}{2}g_{ad}g^{de}(\partial_b g_{ec} + \partial_c g_{be} - \partial_e g_{bc}) \quad (\text{ex-0103.302})$$

$$\frac{1}{2}g_a{}^d\partial_b g_{dc} + \frac{1}{2}g_a{}^d\partial_c g_{bd} - \frac{1}{2}g_a{}^d\partial_d g_{bc} \rightarrow \frac{1}{2}g_{ad}g^{de}\partial_b g_{ec} + \frac{1}{2}g_{ad}g^{de}\partial_c g_{be} - \frac{1}{2}g_{ad}g^{de}\partial_e g_{bc} \quad (\text{ex-0103.303})$$

$$\frac{1}{2}\partial_b g_{ac} + \frac{1}{2}\partial_c g_{ba} - \frac{1}{2}\partial_a g_{bc} \rightarrow \frac{1}{2}g_a{}^e\partial_b g_{ec} + \frac{1}{2}g_a{}^e\partial_c g_{be} - \frac{1}{2}g_a{}^e\partial_e g_{bc} \quad (\text{ex-0103.304})$$

$$\frac{1}{2}\partial_b g_{ac} + \frac{1}{2}\partial_c g_{ba} - \frac{1}{2}\partial_a g_{bc} \rightarrow \frac{1}{2}\partial_b g_{ac} + \frac{1}{2}\partial_c g_{ba} - \frac{1}{2}\partial_a g_{bc} \quad (\text{ex-0103.305})$$

$$\Gamma_{abc} + \Gamma_{bac} - \partial_c g_{ab} = \Gamma_{abc} + \Gamma_{bac} - \partial_c g_{ab} \quad (\text{ex-0103.402})$$

$$= \Gamma_{abc} + \Gamma_{bac} - \partial_c g_{ab} \quad (\text{ex-0103.403})$$