Exercise 6.8 The Kasner cosmology

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{t, x, y, z}::Coordinate.
{a,b,c,d,e,f,g,h#}::Indices(values={t, x, y, z}, position=independent).

\partial{#}::PartialDerivative.

p1::LaTeXForm("p_1").
p2::LaTeXForm("p_2").
p3::LaTeXForm("p_3").

g^{a b}::InverseMetric.  # essential when using complete (gab, $g^{a b}$)

Gamma := \Gamma^{a}_{f g} -> 1/2 g^{a b} (   \partial_{g}{g_{b f}}
                                           + \partial_{f}{g_{b g}}
                                           - \partial_{b}{g_{f g}} ).

Rabcd := R^{d}_{e f g} ->   \partial_{f}{\Gamma^{d}_{e g}}
                          - \partial_{g}{\Gamma^{d}_{e f}}
                          + \Gamma^{d}_{b f} \Gamma^{b}_{e g}
                          - \Gamma^{d}_{b g} \Gamma^{b}_{e f}.

Rab := R_{a b} -> R^{c}_{a c b}.

gab := { g_{t t} = -1,
         g_{x x} = t**(2*p1),
         g_{y y} = t**(2*p2),
         g_{z z} = t**(2*p3)}.                          # cdb(ex-0608.101,gab)

complete   (gab, $g^{a b}$)                             # cdb(ex-0608.102,gab)

substitute (Rabcd, Gamma)
substitute (Rab, Rabcd)

evaluate   (Gamma, gab, rhsonly=True)                   # cdb(ex-0608.103,Gamma)
evaluate   (Rabcd, gab, rhsonly=True)                   # cdb(ex-0608.104,Rabcd)
evaluate   (Rab,   gab, rhsonly=True)                   # cdb(ex-0608.105,Rab)
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$$\left[g_{tt} = -1,\ g_{xx} = t^{2p_1},\ g_{yy} = t^{2p_2},\ g_{zz} = t^{2p_3}\right] \tag{ex-0608.101}$$

$$\left[g_{tt} = -1,\ g_{xx} = t^{2p_1},\ g_{yy} = t^{2p_2},\ g_{zz} = t^{2p_3},\ g^{tt} = -1,\ g^{xx} = t^{-2p_1},\ g^{yy} = t^{-2p_2},\ g^{zz} = t^{-2p_3}\right] \tag{ex-0608.102}$$

$$\Gamma^{a}{}_{fg} \rightarrow \Box_{fg}{}^{a} \begin{cases} \Box_{zt}{}^{z} = p_3 t^{-1} \\ \Box_{yt}{}^{y} = p_2 t^{-1} \\ \Box_{xt}{}^{x} = p_1 t^{-1} \\ \Box_{tz}{}^{z} = p_3 t^{-1} \\ \Box_{ty}{}^{y} = p_2 t^{-1} \\ \Box_{tx}{}^{x} = p_1 t^{-1} \\ \Box_{zz}{}^{t} = p_3 t^{(2p_3-1)} \\ \Box_{yy}{}^{t} = p_2 t^{(2p_2-1)} \\ \Box_{xx}{}^{t} = p_1 t^{(2p_1-1)} \end{cases} \tag{ex-0608.103}$$

$$
R^{d}{}_{efg} \to \Box_{eg}{}^{d}{}_{f} \begin{cases}
\Box_{xx}{}^{t}{}_{t} = p_1 t^{(2p_1-2)} (p_1 - 1) \\
\Box_{yy}{}^{t}{}_{t} = p_2 t^{(2p_2-2)} (p_2 - 1) \\
\Box_{zz}{}^{t}{}_{t} = p_3 t^{(2p_3-2)} (p_3 - 1) \\
\Box_{tx}{}^{x}{}_{t} = p_1 (p_1 - 1) t^{-2} \\
\Box_{ty}{}^{y}{}_{t} = p_2 (p_2 - 1) t^{-2} \\
\Box_{tz}{}^{z}{}_{t} = p_3 (p_3 - 1) t^{-2} \\
\Box_{xt}{}^{t}{}_{x} = -p_1 t^{(2p_1-2)} (p_1 - 1) \\
\Box_{yt}{}^{t}{}_{y} = -p_2 t^{(2p_2-2)} (p_2 - 1) \\
\Box_{zt}{}^{t}{}_{z} = -p_3 t^{(2p_3-2)} (p_3 - 1) \\
\Box_{tt}{}^{x}{}_{x} = p_1 (-p_1 + 1) t^{-2} \\
\Box_{tt}{}^{y}{}_{y} = p_2 (-p_2 + 1) t^{-2} \\
\Box_{tt}{}^{z}{}_{z} = p_3 (-p_3 + 1) t^{-2} \\
\Box_{zz}{}^{y}{}_{y} = p_2 p_3 t^{(2p_3-2)} \\
\Box_{zz}{}^{x}{}_{x} = p_1 p_3 t^{(2p_3-2)} \\
\Box_{yy}{}^{z}{}_{z} = p_2 p_3 t^{(2p_2-2)} \\
\Box_{yy}{}^{x}{}_{x} = p_1 p_2 t^{(2p_2-2)} \\
\Box_{xx}{}^{z}{}_{z} = p_1 p_3 t^{(2p_1-2)} \\
\Box_{xx}{}^{y}{}_{y} = p_1 p_2 t^{(2p_1-2)} \\
\Box_{zy}{}^{y}{}_{z} = -p_2 p_3 t^{(2p_3-2)} \\
\Box_{zx}{}^{x}{}_{z} = -p_1 p_3 t^{(2p_3-2)} \\
\Box_{yz}{}^{z}{}_{y} = -p_2 p_3 t^{(2p_2-2)} \\
\Box_{yx}{}^{x}{}_{y} = -p_1 p_2 t^{(2p_2-2)} \\
\Box_{xz}{}^{z}{}_{x} = -p_1 p_3 t^{(2p_1-2)} \\
\Box_{xy}{}^{y}{}_{x} = -p_1 p_2 t^{(2p_1-2)}
\end{cases} \tag{ex-0608.104}
$$

$$
R_{ab} \to \Box_{ab} \begin{cases}
\Box_{xx} = p_1 t^{(2p_1-2)} (p_1 + p_2 + p_3 - 1) \\
\Box_{yy} = p_2 t^{(2p_2-2)} (p_1 + p_2 + p_3 - 1) \\
\Box_{zz} = p_3 t^{(2p_3-2)} (p_1 + p_2 + p_3 - 1) \\
\Box_{tt} = \left(-{p_1}^2 + p_1 - {p_2}^2 + p_2 - {p_3}^2 + p_3\right) t^{-2}
\end{cases} \tag{ex-0608.105}
$$