

Example 1 The metric connection

```
1  # Define some properties
2
3  {a,b,c,d,e,f,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices.
4
5  g_{a b}::Metric.
6  g_{a}^{b}::KroneckerDelta.
7
8  \nabla{#}::Derivative.
9  \partial{#}::PartialDerivative.
10
11 # Define rules for covariant derivative and the Christoffel symbol
12
13 nabla := \nabla_{c}{g_{a b}} -> \partial_{c}{g_{a b}} - g_{a d}\Gamma^{d}_{b c}
14                                     - g_{d b}\Gamma^{d}_{a c}.    # cdb (nabla.100,nabla)
15
16 Gamma := \Gamma^{a}_{b c} -> (1/2) g^{a d} ( \partial_{b}{g_{d c}}
17                                     + \partial_{c}{g_{b d}}
18                                     - \partial_{d}{g_{b c}} ).    # cdb (Gamma.100,Gamma)
19
20 # Start with a simple expression
21
22 cderiv := \nabla_{c}{g_{a b}}.    # cdb (ex-01.100,cderiv)
23
24 # Do the computations
25
26 substitute      (cderiv, nabla)    # cdb (ex-01.101,cderiv)
27 substitute      (cderiv, Gamma)    # cdb (ex-01.102,cderiv)
28 distribute      (cderiv)           # cdb (ex-01.103,cderiv)
29 eliminate_metric (cderiv)           # cdb (ex-01.104,cderiv)
30 eliminate_kronecker (cderiv)        # cdb (ex-01.105,cderiv)
31 canonicalise     (cderiv)           # cdb (ex-01.106,cderiv)
32
33 checkpoint.append (cderiv)
```

$$\nabla_c g_{ab} \rightarrow \partial_c g_{ab} - g_{ad} \Gamma_{bc}^d - g_{db} \Gamma_{ac}^d \quad (\text{nabla.100})$$

$$\Gamma_{bc}^a \rightarrow \frac{1}{2} g^{ad} (\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc}) \quad (\text{Gamma.100})$$

$$\nabla_c g_{ab} = \partial_c g_{ab} - g_{ad} \Gamma_{bc}^d - g_{db} \Gamma_{ac}^d \quad (\text{ex-01.101})$$

$$= \partial_c g_{ab} - \frac{1}{2} g_{ad} g^{de} (\partial_b g_{ec} + \partial_c g_{be} - \partial_e g_{bc}) - \frac{1}{2} g_{db} g^{de} (\partial_a g_{ec} + \partial_c g_{ae} - \partial_e g_{ac}) \quad (\text{ex-01.102})$$

$$= \partial_c g_{ab} - \frac{1}{2} g_{ad} g^{de} \partial_b g_{ec} - \frac{1}{2} g_{ad} g^{de} \partial_c g_{be} + \frac{1}{2} g_{ad} g^{de} \partial_e g_{bc} - \frac{1}{2} g_{db} g^{de} \partial_a g_{ec} - \frac{1}{2} g_{db} g^{de} \partial_c g_{ae} + \frac{1}{2} g_{db} g^{de} \partial_e g_{ac} \quad (\text{ex-01.103})$$

$$= \partial_c g_{ab} - \frac{1}{2} g_a^e \partial_b g_{ec} - \frac{1}{2} g_a^e \partial_c g_{be} + \frac{1}{2} g_a^e \partial_e g_{bc} - \frac{1}{2} g_b^e \partial_a g_{ec} - \frac{1}{2} g_b^e \partial_c g_{ae} + \frac{1}{2} g_b^e \partial_e g_{ac} \quad (\text{ex-01.104})$$

$$= \frac{1}{2} \partial_c g_{ab} - \frac{1}{2} \partial_c g_{ba} \quad (\text{ex-01.105})$$

$$= 0 \quad (\text{ex-01.106})$$