Example 2 Covariant derivatives

```
{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u\#}::Indices.
     \nabla{#}::Derivative.
     \partial{#}::PartialDerivative.
     # rule for covariant derivative of v^{a}
     deriv := \nabla_{a}{v^{b}} \rightarrow \partial_{a}{v^{b}} + \Gamma^{b}_{c} \ a} \ v^{c}.
     # create an expression
10
11
     foo := \\nabla_{a}{v^{b}}.
                                                     # cdb (ex-02.101,foo)
12
13
     # apply the rule, then simplify
14
15
                  (foo,deriv)
     substitute
                                                     # cdb (ex-02.102,foo)
16
                                                     # cdb (ex-02.103,foo)
     canonicalise (foo)
17
     checkpoint.append (foo)
```

$$\nabla_a v^b = \partial_a v^b + \Gamma^b{}_{ca} v^c$$

$$= \partial_a v^b + \Gamma^{bc}{}_a v_c$$
(ex-02.102)
$$= (ex-02.103)$$

Example 2 Covariant derivatives using "position=independent"

```
{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u\#}::Indices(position=independent).
     \nabla{#}::Derivative.
     \partial{#}::PartialDerivative.
     # rule for covariant derivative of v^{a}
     deriv := \nabla_{a}{v^{b}} \rightarrow \partial_{a}{v^{b}} + \Gamma^{b}_{c} \ a} \ v^{c}.
     # create an expression
10
11
     foo := \\nabla_{a}{v^{b}}.
                                                     # cdb (ex-02.201,foo)
12
13
     # apply the rule, then simplify
14
15
     substitute
                  (foo,deriv)
                                                     # cdb (ex-02.202,foo)
16
                                                     # cdb (ex-02.203,foo)
     canonicalise (foo)
17
18
     checkpoint.append (foo)
```

$$\nabla_a v^b = \partial_a v^b + \Gamma^b{}_{ca} v^c$$

$$= \partial_a v^b + \Gamma^b{}_{ca} v^c$$
(ex-02.202)
$$= (ex-02.203)$$

Example 2 Covariant derivatives using generic rule for deriv

```
\{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u\#\}::Indices(position=independent).
     \nabla{#}::Derivative.
     \partial{#}::PartialDerivative.
     # template for covariant derivative of a vector
     deriv := \mathbb{A}^{a}_{A,^{b}} -> \operatorname{lal}_{a}^{A,^{b}} + \operatorname{lal}_{a}^{b}_{c} = A,^{c}.
     # create an expression
10
11
     foo := \frac{a}{u^{b}} + \frac{a}{v^{b}}. # cdb (ex-02.301,foo)
12
13
     # apply the rule, then simplify
14
15
     substitute
                   (foo,deriv)
                                                       # cdb (ex-02.302,foo)
16
     canonicalise (foo)
                                                       # cdb (ex-02.303,foo)
17
18
     checkpoint.append (foo)
```

$$\nabla_a u^b + \nabla_a v^b = \partial_a u^b + \Gamma^b{}_{ca} u^c + \partial_a v^b + \Gamma^b{}_{ca} v^c$$

$$= \partial_a u^b + \Gamma^b{}_{ca} u^c + \partial_a v^b + \Gamma^b{}_{ca} v^c$$
(ex-02.302)
$$(ex-02.303)$$