## Exercise 1.4 Experiments with sorting

```
 \{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w,x,y,z\#\}:: \underline{Indices} (position=independent). 
     \partial{#}::PartialDerivative.
     expr := C^{f}
              w^{e}
             B^{d}
             v^{c}
             A^{b}
             u^{a}.
                                              # cdb (ex-0104.100,expr)
10
11
     sort_product (expr)
                                              # cdb (ex-0104.101,expr)
12
13
     expr := \Omega_{f}
14
              \gamma_{e}
15
              \Pi_{d}
16
             \beta_{c}
17
              \Gamma_{b}
18
             \alpha_{a}.
                                              # cdb (ex-0104.200,expr)
19
20
     sort_product (expr)
                                              # cdb (ex-0104.201,expr)
21
22
     expr := C^{f}
23
              w^{e}
24
             B^{d}
             v^{c}
             A^{b}
27
             u^{a}
28
             \Omega_{f}
29
              \gamma_{e}
30
             \Pi_{d}
31
              \beta_{c}
32
              \Gamma_{b}
33
              \alpha_{a}.
                                              # cdb (ex-0104.300,expr)
34
35
                                              # cdb (ex-0104.301,expr)
     sort_product (expr)
```

```
37
     expr := \partial_{f}{C^{f}}
38
             w^{1}
39
             \partial_{d}{B^{d}}
40
             v^{k}
41
             \partial_{b}{A^{b}}
42
             u^{j}
43
             \Omega_{i}
44
             \partial^{e}{ \gamma_{e}}}
45
             \Pi_{h}
46
             \partial^{c}{\beta_{c}}
47
             \Gamma_{g}
48
             \partial^{a}{\alpha_{a}}.
                                              # cdb (ex-0104.400,expr)
49
50
     sort_product (expr)
                                              # cdb (ex-0104.401,expr)
51
52
     expr := \partial{C}
53
54
             \partial{B}
55
56
             \partial{A}
             u
58
             \Omega
59
             \partial{ \gamma}
60
              \Pi
61
             \partial{\beta}
62
             \Gamma
63
             \partial{\alpha}.
                                              # cdb (ex-0104.500,expr)
64
65
     sort_product (expr)
                                              # cdb (ex-0104.501,expr)
66
67
     expr := A_{b}
68
             A_{a}
69
             A_{cde}
70
             A_{f} g}.
                                              # cdb (ex-0104.600,expr)
71
72
     sort_product (expr)
                                              # cdb (ex-0104.601,expr)
73
74
```

```
expr := A_{a} A^{a} 

+ A^{a} A_{a}.  # cdb (ex-0104.700, expr)

sort_product (expr)  # cdb (ex-0104.701, expr)

ex-0104.100 := C^f w^e B^d v^c A^b u^a
```

$$\begin{split} &\operatorname{ex-0104.100} := C^f w^e B^d v^c A^b u^a \\ &\operatorname{ex-0104.101} := A^b B^d C^f u^a v^c w^e \\ &\operatorname{ex-0104.200} := \Omega_f \gamma_e \Pi_d \beta_c \Gamma_b \alpha_a \\ &\operatorname{ex-0104.201} := \Gamma_b \Omega_f \Pi_d \alpha_a \beta_c \gamma_e \\ &\operatorname{ex-0104.300} := C^f w^e B^d v^c A^b u^a \Omega_f \gamma_e \Pi_d \beta_c \Gamma_b \alpha_a \\ &\operatorname{ex-0104.301} := A^b B^d C^f \Gamma_b \Omega_f \Pi_d \alpha_a \beta_c \gamma_e u^a v^c w^e \\ &\operatorname{ex-0104.400} := \partial_f C^f w^l \partial_d B^d v^k \partial_b A^b u^j \Omega_i \partial^e \gamma_e \Pi_h \partial^c \beta_c \Gamma_g \partial^a \alpha_a \\ &\operatorname{ex-0104.401} := \Gamma_g \Omega_i \Pi_h \partial_b A^b \partial_d B^d \partial_f C^f \partial^a \alpha_a \partial^c \beta_c \partial^e \gamma_e u^j v^k w^l \\ &\operatorname{ex-0104.500} := \partial C w \partial B v \partial A u \Omega \partial \gamma \Pi \partial \beta \Gamma \partial \alpha \\ &\operatorname{ex-0104.501} := \Gamma \Omega \Pi \partial A \partial B \partial C \partial \alpha \partial \beta \partial \gamma u v w \\ &\operatorname{ex-0104.600} := A_b A_a A_{cde} A_{fg} \\ &\operatorname{ex-0104.601} := A_a A_b A_{fg} A_{cde} \\ &\operatorname{ex-0104.700} := A_a A^a + A^a A_a \\ &\operatorname{ex-0104.701} := 2 A_a A^a \end{split}$$