Exercise 1.3 Christoffel symbol and dg with a single rule

```
{a,b,c,d,e,f,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices(position=independent).

g_{a b}::Metric.
g_{a}^{b}::KroneckerDelta.

\partial{#}::PartialDerivative.

GammaU := \Gamma^{a}_{b c} -> (1/2) g^{a d} (  \partial_{b}{g_{d c}}
                                             + \partial_{c}{g_{b d}}
                                             - \partial_{d}{g_{b c}} ).

GammaD := \Gamma_{a b c} -> g_{a d} \Gamma^{d}_{b c}.              # cdb (ex-0103.101,GammaD)

substitute          (GammaD, GammaU)                               # cdb (ex-0103.102,GammaD)  # requires Indices(position=independent)
distribute          (GammaD)                                       # cdb (ex-0103.103,GammaD)
eliminate_metric    (GammaD)                                       # cdb (ex-0103.104,GammaD)
eliminate_kronecker (GammaD)                                       # cdb (ex-0103.105,GammaD)

expr := \Gamma_{a b c} + \Gamma_{b a c} - \partial_{c}{g_{a b}}.   # cdb (ex-0103.201,expr)

substitute          (expr, GammaD)                                 # cdb (ex-0103.202,expr)
canonicalise        (expr)                                         # cdb (ex-0103.203,expr)
```

$$\Gamma_{abc} \to g_{ad}\Gamma^{d}{}_{bc} \tag{ex-0103.101}$$

$$\Gamma_{abc} \to \frac{1}{2} g_{ad} g^{de} \left(\partial_b g_{ec} + \partial_c g_{be} - \partial_e g_{bc}\right) \tag{ex-0103.102}$$

$$\Gamma_{abc} \to \frac{1}{2} g_{ad} g^{de} \partial_b g_{ec} + \frac{1}{2} g_{ad} g^{de} \partial_c g_{be} - \frac{1}{2} g_{ad} g^{de} \partial_e g_{bc} \tag{ex-0103.103}$$

$$\Gamma_{abc} \to \frac{1}{2} g_a{}^e \partial_b g_{ec} + \frac{1}{2} g_a{}^e \partial_c g_{be} - \frac{1}{2} g_a{}^e \partial_e g_{bc} \tag{ex-0103.104}$$

$$\Gamma_{abc} \to \frac{1}{2} \partial_b g_{ac} + \frac{1}{2} \partial_c g_{ba} - \frac{1}{2} \partial_a g_{bc} \tag{ex-0103.105}$$

$$\Gamma_{abc} + \Gamma_{bac} - \partial_c g_{ab} = \frac{1}{2} \partial_c g_{ba} - \frac{1}{2} \partial_c g_{ab} \tag{ex-0103.202}$$

$$= 0 \tag{ex-0103.203}$$

Exercise 1.3 Repeat but without `position=independent`

```
{a,b,c,d,e,f,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices.

g_{a b}::Metric.
g_{a}^{b}::KroneckerDelta.

\partial{#}::PartialDerivative.

GammaU := \Gamma^{a}_{b c} -> (1/2) g^{a d} (  \partial_{b}{g_{d c}}
                                             + \partial_{c}{g_{b d}}
                                             - \partial_{d}{g_{b c}} ).

GammaD := \Gamma_{a b c} -> g_{a d} \Gamma^{d}_{b c}.             # cdb (ex-0103.301,GammaD)

substitute          (GammaD, GammaU)                              # cdb (ex-0103.302,GammaD)
distribute          (GammaD)                                      # cdb (ex-0103.303,GammaD)
eliminate_metric    (GammaD)                                      # cdb (ex-0103.304,GammaD)
eliminate_kronecker (GammaD)                                      # cdb (ex-0103.305,GammaD)

expr := \Gamma_{a b c} + \Gamma_{b a c} - \partial_{c}{g_{a b}}.   # cdb (ex-0103.401,expr)

substitute          (expr, GammaD)                                # cdb (ex-0103.402,expr)
canonicalise        (expr)                                        # cdb (ex-0103.403,expr)
```

$$\Gamma_{abc} \to g_{ad}\Gamma^{d}{}_{bc} \quad \texttt{(ex-0103.301)}$$

$$\frac{1}{2}g_a{}^d\left(\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc}\right) \to \frac{1}{2}g_{ad}g^{de}\left(\partial_b g_{ec} + \partial_c g_{be} - \partial_e g_{bc}\right) \quad \texttt{(ex-0103.302)}$$

$$\frac{1}{2}g_a{}^d\partial_b g_{dc} + \frac{1}{2}g_a{}^d\partial_c g_{bd} - \frac{1}{2}g_a{}^d\partial_d g_{bc} \to \frac{1}{2}g_{ad}g^{de}\partial_b g_{ec} + \frac{1}{2}g_{ad}g^{de}\partial_c g_{be} - \frac{1}{2}g_{ad}g^{de}\partial_e g_{bc} \quad \texttt{(ex-0103.303)}$$

$$\frac{1}{2}\partial_b g_{ac} + \frac{1}{2}\partial_c g_{ba} - \frac{1}{2}\partial_a g_{bc} \to \frac{1}{2}g_a{}^e\partial_b g_{ec} + \frac{1}{2}g_a{}^e\partial_c g_{be} - \frac{1}{2}g_a{}^e\partial_e g_{bc} \quad \texttt{(ex-0103.304)}$$

$$\frac{1}{2}\partial_b g_{ac} + \frac{1}{2}\partial_c g_{ba} - \frac{1}{2}\partial_a g_{bc} \to \frac{1}{2}\partial_b g_{ac} + \frac{1}{2}\partial_c g_{ba} - \frac{1}{2}\partial_a g_{bc} \quad \texttt{(ex-0103.305)}$$

$$\begin{aligned}\Gamma_{abc} + \Gamma_{bac} - \partial_c g_{ab} &= \Gamma_{abc} + \Gamma_{bac} - \partial_c g_{ab} && \texttt{(ex-0103.402)}\\ &= \Gamma_{abc} + \Gamma_{bac} - \partial_c g_{ab} && \texttt{(ex-0103.403)}\end{aligned}$$