

```
{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w#}::Indices(position=independent).
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$\nabla\{\# \}::\text{Derivative}.$

 $\partial_{\#}::\text{PartialDerivative}.$

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g_{a b}::Metric.
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$$g^{a\ b}::\text{InverseMetric}.$$
$$g_{\{a\}}^{\{b\}}::\text{KroneckerDelta}.$$
$$g^{\{a\}}_{\{b\}}::\text{KroneckerDelta}.$$
$$R_{\{a \ b \ c \ d\}}::\text{RiemannTensor}.$$
$$R^{\{a\}}_{\{b\ c\ d\}}::\text{RiemannTensor}.$$
$$\backslash \text{Gamma}^{\{a\}}_{\{b\}c}::\text{TableauSymmetry}(\text{shape}=\{2\}, \text{indices}=\{1,2\}).$$

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\Gamma_{a\ b\ c}::TableuSymmetry(shape={2}, indices={1,2}).
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$$g_{\{a \ b\}} :: \text{Depends}(\backslash \text{partial}\{\#\}).$$
$$g^{a\ b}::\text{Depends}(\backslash\text{partial}\{\#\}).$$
$$\Gamma_{a_{bc}}::\text{Depends}(\partial\{ \# \}).$$
$$\Gamma^{a\ b\ c}::\text{Depends}(\partial^{\#}).$$

```
dgab := \partial_{c\{g_{a\ b}\}} -> \Gamma^d_{\{a\ c\}} g_{\{d\ b\}
+ \Gamma^d_{\{b\ c\}} g_{\{a\ d\}}. # cdb(dgab.000,dgab)
```

```
diab := \partial_{c}\{g^{a b}\} \rightarrow - \Gamma^{a}_{d c} g^{d b}
      - \Gamma^{b}_{d c} g^{a d}. # cdb(diab.000,diab)
```

$$\Gamma_U := \Gamma_{a,b,c} \rightarrow \frac{1}{2} g_{a,d} \left(\frac{\partial}{\partial g_{d,c}} + \frac{\partial}{\partial g_{b,d}} - \frac{\partial}{\partial g_{b,c}} \right). \quad \# \text{ cdb}(\Gamma_{000}, \Gamma_U)$$

```
GammaD := \Gamma_{a b c} -> 1/2 ( \partial_b \{g_{a c}\}
+ \partial_c \{g_{b a}\}
- \partial_a \{g_{b c}\}). # cdb(Gamma.010, GammaD)
```

$$\begin{aligned} \text{RabcdU} := & R^{\{a\}}_{\{b\ c\ d\}} \rightarrow \quad \partial_c \{ \Gamma^{\{a\}}_{\{b\ d\}} \\ & - \partial_d \{ \Gamma^{\{a\}}_{\{b\ c\}} \} \\ & + \Gamma^{\{e\}}_{\{b\ d\}} \Gamma^{\{a\}}_{\{c\ e\}} \end{aligned}$$

```

- \Gamma^{e}_{b c} \Gamma^{a}_{d e}. # cdb(Rabcd.000,RabcdU)

RabcdD := R_{a b c d} -> \partial_c \{ \Gamma_{a b d} \}
- \partial_d \{ \Gamma_{a b c} \}
+ \Gamma_{e a d} \Gamma^{e}_{b c}
- \Gamma_{e a c} \Gamma^{e}_{b d}. # cdb(Rabcd.010,RabcdD)

Rab := R_{a b} -> R^{c}_{c} \{ a c b \}. # cdb(Rab.000,Rab)

Rscalar := R -> g^{a b} R_{a b}. # cdb(Rscalar.000,Rscalar)

# Weyl in 4 dimensions
Cabcd := C_{a b c d} -> R_{a b c d} - (1/2) (R_{a c} g_{b d} - R_{a d} g_{b c})
- (1/2) (g_{a c} R_{b d} - g_{a d} R_{b c})
+ (R/6) (g_{a c} g_{b d} - g_{a d} g_{b c}). # cdb(Cabcd.000,Cabcd)

expr := \partial_c \{ g_{a b} \}. # cdb(libdg.dgab.000,expr)
substitute (expr,dgab) # cdb(libdg.dgab.001,expr)

expr := \partial_c \{ g^{a b} \}. # cdb(libdg.diab.000,expr)
substitute (expr,diab) # cdb(libdg.diab.001,expr)

expr := \Gamma^{a}_{b c}. # cdb(libdg.Gamma.000,expr)
substitute (expr,GammaU) # cdb(libdg.Gamma.001,expr)

expr := \Gamma_{a b c}. # cdb(libdg.Gamma.010,expr)
substitute (expr,GammaD) # cdb(libdg.Gamma.011,expr)

expr := R^{a}_{a} \{ b c d \}. # cdb(libdg.Rabcd.000,expr)
substitute (expr,RabcdU) # cdb(libdg.Rabcd.001,expr)

expr := R_{a b c d}. # cdb(libdg.Rabcd.010,expr)
substitute (expr,RabcdD) # cdb(libdg.Rabcd.011,expr)

expr := R_{a b}. # cdb(libdg.Rab.000,expr)
substitute (expr,Rab) # cdb(libdg.Rab.001,expr)

expr := R. # cdb(libdg.Rscalar.000,expr)

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```
substitute (expr,Rscalar)      # cdb(libdg.Rscalar.001,expr)

expr := C_{a b c d}.          # cdb(libdg.Cabcd.000,expr)
substitute (expr,Cabcd)       # cdb(libdg.Cabcd.001,expr)
```

$$\partial_c g_{ab} \rightarrow \Gamma^d_{ac} g_{db} + \Gamma^d_{bc} g_{ad} \quad (1)$$

$$\partial_c g^{ab} \rightarrow -\Gamma^a_{dc} g^{db} - \Gamma^b_{dc} g^{ad} \quad (2)$$

$$\Gamma^a_{bc} \rightarrow \frac{1}{2} g^{ad} (\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc}) \quad (3)$$

$$\Gamma_{abc} \rightarrow \frac{1}{2} \partial_b g_{ac} + \frac{1}{2} \partial_c g_{ba} - \frac{1}{2} \partial_a g_{bc} \quad (4)$$

$$R^a_{bcd} \rightarrow \partial_c \Gamma^a_{bd} - \partial_d \Gamma^a_{bc} + \Gamma^e_{bd} \Gamma^a_{ce} - \Gamma^e_{bc} \Gamma^a_{de} \quad (5)$$

$$R_{abcd} \rightarrow \partial_c \Gamma_{abd} - \partial_d \Gamma_{abc} + \Gamma_{ead} \Gamma^e_{bc} - \Gamma_{eac} \Gamma^e_{bd} \quad (6)$$

$$R_{ab} \rightarrow R^c_{acb} \quad (7)$$

$$R \rightarrow g^{ab} R_{ab} \quad (8)$$

$$C_{abcd} \rightarrow R_{abcd} - \frac{1}{2} R_{ac} g_{bd} + \frac{1}{2} R_{ad} g_{bc} - \frac{1}{2} g_{ac} R_{bd} + \frac{1}{2} g_{ad} R_{bc} + \frac{1}{6} R (g_{ac} g_{bd} - g_{ad} g_{bc}) \quad (9)$$

$$\partial_c g_{ab} = \Gamma^d_{ac} g_{db} + \Gamma^d_{bc} g_{ad} \quad (10)$$

$$\partial_c g^{ab} = -\Gamma^a_{dc} g^{db} - \Gamma^b_{dc} g^{ad} \quad (11)$$

$$\Gamma^a_{bc} = \frac{1}{2} g^{ad} (\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc}) \quad (12)$$

$$\Gamma_{abc} = \frac{1}{2} \partial_b g_{ac} + \frac{1}{2} \partial_c g_{ba} - \frac{1}{2} \partial_a g_{bc} \quad (13)$$

$$R^a_{bcd} = \partial_c \Gamma^a_{bd} - \partial_d \Gamma^a_{bc} + \Gamma^e_{bd} \Gamma^a_{ce} - \Gamma^e_{bc} \Gamma^a_{de} \quad (14)$$

$$R_{abcd} = \partial_c \Gamma_{abd} - \partial_d \Gamma_{abc} + \Gamma_{ead} \Gamma^e_{bc} - \Gamma_{eac} \Gamma^e_{bd} \quad (15)$$

$$R_{ab} = R^c_{acb} \quad (16)$$

$$R = g^{ab} R_{ab} \quad (17)$$

$$C_{abcd} = R_{abcd} - \frac{1}{2} R_{ac} g_{bd} + \frac{1}{2} R_{ad} g_{bc} - \frac{1}{2} g_{ac} R_{bd} + \frac{1}{2} g_{ad} R_{bc} + \frac{1}{6} R (g_{ac} g_{bd} - g_{ad} g_{bc}) \quad (18)$$

```
import cdblib

cdblib.create ('dgeom.json')

cdblib.put ('dgab',      dgab,      'dgeom.json')
cdblib.put ('diab',      diab,      'dgeom.json')
cdblib.put ('GammaU',    GammaU,    'dgeom.json')
cdblib.put ('GammaD',    GammaD,    'dgeom.json')
cdblib.put ('RabcdU',    RabcdU,    'dgeom.json')
cdblib.put ('RabcdD',    RabcdD,    'dgeom.json')
cdblib.put ('Rab',       Rab,       'dgeom.json')
cdblib.put ('Rscalara',  Rscalar,   'dgeom.json')
cdblib.put ('Cabcd',     Cabcd,     'dgeom.json')
```