

Example 2 Covariant derivatives

```
1 {a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices.
2
3 \nabla{#}::Derivative.
4 \partial{#}::PartialDerivative.
5
6 # rule for covariant derivative of v^{a}
7
8 deriv := \nabla_{a}{v^{b}} -> \partial_{a}{v^{b}} + \Gamma^{b}_{c a} v^{c}.
9
10 # create an expression
11
12 foo := \nabla_{a}{v^{b}}. # cdb (ex-02.101,foo)
13
14 # apply the rule, then simplify
15
16 substitute (foo,deriv) # cdb (ex-02.102,foo)
17 canonicalise (foo) # cdb (ex-02.103,foo)
18
19 checkpoint.append (foo)
```

$$\nabla_a v^b = \partial_a v^b + \Gamma^b_{ca} v^c \quad (\text{ex-02.102})$$

$$= \partial_a v^b + \Gamma^{bc}_a v_c \quad (\text{ex-02.103})$$

Example 2 Covariant derivatives using “position=independent”

```
1 {a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices(position=independent).
2
3 \nabla{#}::Derivative.
4 \partial{#}::PartialDerivative.
5
6 # rule for covariant derivative of v^{a}
7
8 deriv := \nabla_{a}{v^{b}} -> \partial_{a}{v^{b}} + \Gamma^{b}_{c a} v^{c}.
9
10 # create an expression
11
12 foo := \nabla_{a}{v^{b}}. # cdb (ex-02.201,foo)
13
14 # apply the rule, then simplify
15
16 substitute (foo,deriv) # cdb (ex-02.202,foo)
17 canonicalise (foo) # cdb (ex-02.203,foo)
18
19 checkpoint.append (foo)
```

$$\nabla_a v^b = \partial_a v^b + \Gamma^b_{ca} v^c \quad (\text{ex-02.202})$$

$$= \partial_a v^b + \Gamma^b_{ca} v^c \quad (\text{ex-02.203})$$

Example 2 Covariant derivatives using generic rule for deriv

```

1 {a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices(position=independent).
2
3 \nabla{#}::Derivative.
4 \partial{#}::PartialDerivative.
5
6 # template for covariant derivative of a vector
7
8 deriv := \nabla_{a}{A^{b}} -> \partial_{a}{A^{b}} + \Gamma^{b}_{c a} A^{c}.
9
10 # create an expression
11
12 foo := \nabla_{a}{u^{b}} + \nabla_{a}{v^{b}}. # cdb (ex-02.301,foo)
13
14 # apply the rule, then simplify
15
16 substitute (foo,deriv) # cdb (ex-02.302,foo)
17 canonicalise (foo) # cdb (ex-02.303,foo)
18
19 checkpoint.append (foo)

```

$$\nabla_a u^b + \nabla_a v^b = \partial_a u^b + \Gamma^b_{ca} u^c + \partial_a v^b + \Gamma^b_{ca} v^c \quad (\text{ex-02.302})$$

$$= \partial_a u^b + \Gamma^b_{ca} u^c + \partial_a v^b + \Gamma^b_{ca} v^c \quad (\text{ex-02.303})$$