Exercise 6.4 Scalar curavture of a 2-sphere

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{\theta, \varphi}::Coordinate.
{a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).

\partial{#}::PartialDerivative.

g^{a b}::InverseMetric.  # essential when using complete (gab, $g^{a b}$)

Gamma := \Gamma^{a}_{b c} -> 1/2 g^{a d} (   \partial_{b}{g_{d c}}
                                           + \partial_{c}{g_{b d}}
                                           - \partial_{d}{g_{b c}}).

Rabcd := R^{a}_{b c d} ->   \partial_{c}{\Gamma^{a}_{b d}}
                          - \partial_{d}{\Gamma^{a}_{b c}}
                          + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
                          - \Gamma^{e}_{b c} \Gamma^{a}_{d e}.

Rab := R_{a b} -> R^{c}_{a c b}.

R := R -> R_{a b} g^{a b}.

gab := { g_{\theta\theta}   = r**2,
         g_{\varphi\varphi} = r**2 \sin(\theta)**2 }.      # cdb(ex-0604.101,gab)

complete   (gab, $g^{a b}$)                                # cdb(ex-0604.102,gab)

substitute (Rabcd, Gamma)
substitute (Rab, Rabcd)
substitute (R, Rab)

evaluate   (Gamma, gab, rhsonly=True)                      # cdb(ex-0604.103,Gamma)
evaluate   (Rabcd, gab, rhsonly=True)                      # cdb(ex-0604.104,Rabcd)
evaluate   (Rab,   gab, rhsonly=True)                      # cdb(ex-0604.105,Rab)
evaluate   (R,     gab, rhsonly=True)                      # cdb(ex-0604.106,R)
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$$\left[g_{\theta\theta} = r^2,\ g_{\varphi\varphi} = r^2(\sin\theta)^2\right] \tag{ex-0604.101}$$

$$\left[g_{\theta\theta} = r^2,\ g_{\varphi\varphi} = r^2(\sin\theta)^2,\ g^{\theta\theta} = r^{-2},\ g^{\varphi\varphi} = \left(r^2(\sin\theta)^2\right)^{-1}\right] \tag{ex-0604.102}$$

$$\Gamma^{a}{}_{bc} \to \square_{cb}{}^{a} \begin{cases} \square_{\varphi\theta}{}^{\varphi} = (\tan\theta)^{-1} \\ \square_{\theta\varphi}{}^{\varphi} = (\tan\theta)^{-1} \\ \square_{\varphi\varphi}{}^{\theta} = -\dfrac{1}{2}\sin(2\theta) \end{cases} \tag{ex-0604.103}$$

$$R^{a}{}_{bcd} \to \square_{db}{}^{a}{}_{c} \begin{cases} \square_{\varphi\varphi}{}^{\theta}{}_{\theta} = (\sin\theta)^2 \\ \square_{\varphi\theta}{}^{\varphi}{}_{\theta} = -1 \\ \square_{\theta\varphi}{}^{\theta}{}_{\varphi} = -(\sin\theta)^2 \\ \square_{\theta\theta}{}^{\varphi}{}_{\varphi} = 1 \end{cases} \tag{ex-0604.104}$$

$$R_{ab} \to \square_{ba} \begin{cases} \square_{\varphi\varphi} = (\sin\theta)^2 \\ \square_{\theta\theta} = 1 \end{cases} \tag{ex-0604.105}$$

$$R \to 2r^{-2} \tag{ex-0604.106}$$