Example 1 The metric connection

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# Define some properties

{a,b,c,d,e,f,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices.

g_{a b}::Metric.
g_{a}^{b}::KroneckerDelta.

\nabla{#}::Derivative.
\partial{#}::PartialDerivative.

# Define rules for covariant derivative and the Christoffel symbol

nabla := \nabla_{c}{g_{a b}} -> \partial_{c}{g_{a b}} - g_{a d}\Gamma^{d}_{b c}
                                                      - g_{d b}\Gamma^{d}_{a c}.   # cdb (nabla.100,nabla)

Gamma := \Gamma^{a}_{b c} -> (1/2) g^{a d} (  \partial_{b}{g_{d c}}
                                            + \partial_{c}{g_{b d}}
                                            - \partial_{d}{g_{b c}} ).   # cdb (Gamma.100,Gamma)

# Start with a simple expression

cderiv := \nabla_{c}{g_{a b}}.                                           # cdb (ex-01.100,cderiv)

# Do the computations

substitute          (cderiv, nabla)                                      # cdb (ex-01.101,cderiv)
substitute          (cderiv, Gamma)                                      # cdb (ex-01.102,cderiv)
distribute          (cderiv)                                             # cdb (ex-01.103,cderiv)
eliminate_metric    (cderiv)                                             # cdb (ex-01.104,cderiv)
eliminate_kronecker (cderiv)                                             # cdb (ex-01.105,cderiv)
canonicalise        (cderiv)                                             # cdb (ex-01.106,cderiv)

checkpoint.append (cderiv)
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$$\nabla_c g_{ab} \to \partial_c g_{ab} - g_{ad}\Gamma^d{}_{bc} - g_{db}\Gamma^d{}_{ac} \tag{nabla.100}$$

$$\Gamma^a{}_{bc} \to \frac{1}{2} g^{ad} \left(\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc}\right) \tag{Gamma.100}$$

$$\begin{aligned}
\nabla_c g_{ab} &= \partial_c g_{ab} - g_{ad}\Gamma^d{}_{bc} - g_{db}\Gamma^d{}_{ac} && \text{(ex-01.101)}\\
&= \partial_c g_{ab} - \frac{1}{2} g_{ad} g^{de} \left(\partial_b g_{ec} + \partial_c g_{be} - \partial_e g_{bc}\right) - \frac{1}{2} g_{db} g^{de} \left(\partial_a g_{ec} + \partial_c g_{ae} - \partial_e g_{ac}\right) && \text{(ex-01.102)}\\
&= \partial_c g_{ab} - \frac{1}{2} g_{ad} g^{de} \partial_b g_{ec} - \frac{1}{2} g_{ad} g^{de} \partial_c g_{be} + \frac{1}{2} g_{ad} g^{de} \partial_e g_{bc} - \frac{1}{2} g_{db} g^{de} \partial_a g_{ec} - \frac{1}{2} g_{db} g^{de} \partial_c g_{ae} + \frac{1}{2} g_{db} g^{de} \partial_e g_{ac} && \text{(ex-01.103)}\\
&= \partial_c g_{ab} - \frac{1}{2} g_a{}^e \partial_b g_{ec} - \frac{1}{2} g_a{}^e \partial_c g_{be} + \frac{1}{2} g_a{}^e \partial_e g_{bc} - \frac{1}{2} g_b{}^e \partial_a g_{ec} - \frac{1}{2} g_b{}^e \partial_c g_{ae} + \frac{1}{2} g_b{}^e \partial_e g_{ac} && \text{(ex-01.104)}\\
&= \frac{1}{2} \partial_c g_{ab} - \frac{1}{2} \partial_c g_{ba} && \text{(ex-01.105)}\\
&= 0 && \text{(ex-01.106)}
\end{aligned}$$