

## Exercise 2.2 Covariant derivative of $v_{ab}$

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1 {a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices(position=independent).
2
3 \nabla{#}::Derivative.
4 \partial{#}::PartialDerivative.
5
6 # template for covariant derivative of a vector
7
8 derivU := \nabla_{a}{A^{b}} -> \partial_{a}{A^{b}} + \Gamma^{b}_{c a} A^{c}.
9 derivD := \nabla_{a}{A_{b}} -> \partial_{a}{A_{b}} - \Gamma^{c}_{c b} A_{c}.
10
11 vab := v_{a b} -> A_{a} B_{b}.
12 iab := A_{a} B_{b} -> v_{a b}.
13
14 pab := \partial_{a}{A_{b}} B_{c} -> \partial_{a}{A_{b} B_{c}} - A_{b} \partial_{a}{B_{c}}.
15
16 # create an object
17
18 Dvab := \nabla_{a}{v_{b c}}. # cdb (ex-0202.101,Dvab)
19
20 # apply the rule, then simplify
21
22 substitute (Dvab,vab) # cdb (ex-0202.102,Dvab)
23 product_rule (Dvab) # cdb (ex-0202.103,Dvab)
24 substitute (Dvab,derivD) # cdb (ex-0202.104,Dvab)
25 substitute (Dvab,derivU) # cdb (ex-0202.105,Dvab)
26 distribute (Dvab) # cdb (ex-0202.106,Dvab)
27 substitute (Dvab,pab) # cdb (ex-0202.107,Dvab)
28 canonicalise (Dvab) # cdb (ex-0202.108,Dvab)
29 substitute (Dvab,iab) # cdb (ex-0202.109,Dvab)
30 sort_product (Dvab) # cdb (ex-0202.110,Dvab)
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$$\begin{aligned}\nabla_a v_{bc} &= \nabla_a (A_b B_c) && (\text{ex-0202.102}) \\ &= \nabla_a A_b B_c + A_b \nabla_a B_c && (\text{ex-0202.103}) \\ &= (\partial_a A_b - \Gamma^d_{ba} A_d) B_c + A_b (\partial_a B_c - \Gamma^d_{ca} B_d) && (\text{ex-0202.104}) \\ &= (\partial_a A_b - \Gamma^d_{ba} A_d) B_c + A_b (\partial_a B_c - \Gamma^d_{ca} B_d) && (\text{ex-0202.105}) \\ &= \partial_a A_b B_c - \Gamma^d_{ba} A_d B_c + A_b \partial_a B_c - A_b \Gamma^d_{ca} B_d && (\text{ex-0202.106}) \\ &= \partial_a (A_b B_c) - \Gamma^d_{ba} A_d B_c - A_b \Gamma^d_{ca} B_d && (\text{ex-0202.107}) \\ &= \partial_a (A_b B_c) - \Gamma^d_{ba} A_d B_c - A_b \Gamma^d_{ca} B_d && (\text{ex-0202.108}) \\ &= \partial_a v_{bc} - \Gamma^d_{ba} v_{dc} - v_{bd} \Gamma^d_{ca} && (\text{ex-0202.109}) \\ &= \partial_a v_{bc} - \Gamma^d_{ba} v_{dc} - \Gamma^d_{ca} v_{bd} && (\text{ex-0202.110})\end{aligned}$$