## Exercise 3.4 More symmetries of Riemann

```
\{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w\#\}::Indices(position=independent).
     \partial{#}::PartialDerivative.
     g_{a b}::Symmetric.
     g^{a b}::Symmetric.
     \Gamma^{a}_{b c}::TableauSymmetry(shape={2}, indices={1,2}).
     \Gamma_{a b c}::TableauSymmetry(shape={2}, indices={1,2}).
10
     GammaU := Gamma^{a}_{b c} \rightarrow 1/2 g^{a d} ( partial_{b}_{g_{d c}})
11
                                                  + \partial_{c}{g_{b d}}
12
                                                   - \partial_{d}{g_{b c}}). # cdb(Gamma.000,GammaU)
13
14
     GammaD := \Gamma_{a b c} -> 1/2 ( \partial_{b}_{g_{a c}})
15
                                        + \partial_{c}{g_{b a}}
16
                                        - \partial_{a}{g_{b c}}).
                                                                             # cdb(Gamma.010,GammaD)
17
18
     Rabcd := R_{a b c d} \rightarrow \beta_{c d} 
19
                             - \partial_{d}{\Gamma_{a b c}}
20
                             + \Gamma_{e a d} \Gamma^{e}_{b c}
21
                             - \Gamma_{e a c} \Gamma^{e}_{b d}.
                                                                              # cdb(Rabcd.000,Rabcd)
22
```

## Exercise 3.4 Antisymmetry on first pair of indices

```
expr := R_{a b c d} + R_{b a c d}.
                                    # cdb(ex-0304.101,expr)
               (expr, Rabcd)
                                     # cdb(ex-0304.102,expr)
substitute
               (expr, GammaU)
                                     # cdb(ex-0304.103,expr)
substitute
               (expr, GammaD)
substitute
                                     # cdb(ex-0304.104,expr)
               (expr)
                                     # cdb(ex-0304.105,expr)
distribute
                                     # cdb(ex-0304.106,expr)
               (expr)
product_rule
                                     # cdb(ex-0304.107,expr)
sort_product
               (expr)
                                     # cdb(ex-0304.108,expr)
rename_dummies (expr)
canonicalise
               (expr)
                                     # cdb(ex-0304.109,expr)
```

$$\begin{split} R_{abcd} + R_{bacd} &= \partial_{c} \Gamma_{abd} - \partial_{d} \Gamma_{abc} + \Gamma_{cad} \Gamma^{c}_{bc} - \Gamma_{cac} \Gamma^{c}_{bd} + \partial_{c} \Gamma_{bad} - \partial_{d} \Gamma_{bac} + \Gamma_{cbd} \Gamma^{c}_{ca} - \Gamma_{cbc} \Gamma^{c}_{ca} \\ &= \partial_{c} \Gamma_{abd} - \partial_{d} \Gamma_{abc} + \frac{1}{2} \Gamma_{cad} g^{ef} \left( \partial_{b} g_{fc} + \partial_{c} g_{bf} - \partial_{f} g_{bc} \right) - \frac{1}{2} \Gamma_{cac} g^{ef} \left( \partial_{b} g_{fd} + \partial_{d} g_{bf} - \partial_{f} g_{bd} \right) + \partial_{c} \Gamma_{bad} - \partial_{d} \Gamma_{bac} \\ &+ \frac{1}{2} \Gamma_{cbd} g^{ef} \left( \partial_{a} g_{fc} + \partial_{c} g_{af} - \partial_{f} g_{ac} \right) - \frac{1}{2} \Gamma_{cbc} g^{ef} \left( \partial_{a} g_{fd} + \partial_{d} g_{af} - \partial_{f} g_{ad} \right) \\ &= \partial_{c} \left( \frac{1}{2} \partial_{b} g_{ad} + \frac{1}{2} \partial_{d} g_{ba} - \frac{1}{2} \partial_{a} g_{bd} \right) - \partial_{d} \left( \frac{1}{2} \partial_{b} g_{ac} + \frac{1}{2} \partial_{c} g_{ba} - \frac{1}{2} \partial_{a} g_{bc} \right) + \frac{1}{2} \left( \frac{1}{2} \partial_{a} g_{cd} + \frac{1}{2} \partial_{d} g_{ac} - \frac{1}{2} \partial_{c} g_{ad} \right) g^{ef} \left( \partial_{b} g_{fc} + \partial_{c} g_{bf} - \partial_{f} g_{bc} \right) \\ &- \frac{1}{2} \left( \frac{1}{2} \partial_{a} g_{cc} + \frac{1}{2} \partial_{c} g_{ac} - \frac{1}{2} \partial_{c} g_{ac} \right) g^{ef} \left( \partial_{b} g_{fd} + \partial_{d} g_{bf} - \partial_{f} g_{bd} \right) + \partial_{c} \left( \frac{1}{2} \partial_{a} g_{bd} + \frac{1}{2} \partial_{d} g_{ac} - \frac{1}{2} \partial_{c} g_{ad} \right) g^{ef} \left( \partial_{b} g_{fc} + \partial_{c} g_{bf} - \partial_{f} g_{bc} \right) \\ &- \frac{1}{2} \left( \frac{1}{2} \partial_{a} g_{bc} + \frac{1}{2} \partial_{c} g_{ac} - \frac{1}{2} \partial_{c} g_{ac} \right) g^{ef} \left( \partial_{b} g_{fd} + \partial_{d} g_{bf} - \partial_{f} g_{bd} \right) + \partial_{c} \left( \frac{1}{2} \partial_{a} g_{bd} + \frac{1}{2} \partial_{d} g_{ac} - \frac{1}{2} \partial_{c} g_{ad} \right) \\ &- \partial_{d} \left( \frac{1}{2} \partial_{a} g_{bc} - \frac{1}{2} \partial_{c} g_{ac} \right) g^{ef} \left( \partial_{a} g_{fd} + \partial_{d} g_{bf} - \partial_{f} g_{bc} \right) g^{ef} \left( \partial_{a} g_{fc} + \partial_{c} g_{af} - \partial_{f} g_{ac} \right) \\ &- \frac{1}{2} \left( \frac{1}{2} \partial_{b} g_{cc} + \frac{1}{2} \partial_{c} g_{bc} \right) g^{ef} \left( \partial_{a} g_{fd} + \partial_{d} g_{bc} - \frac{1}{2} \partial_{e} g_{bd} \right) g^{ef} \left( \partial_{a} g_{fc} + \partial_{c} g_{af} - \partial_{f} g_{ac} \right) \\ &- \frac{1}{2} \partial_{a} g_{bc} - \frac{1}{2} \partial_{c} g_{bc} - \frac{1}{2} \partial_{e} g_{bc} \right) g^{ef} \left( \partial_{a} g_{fd} + \partial_{d} g_{bc} - \partial_{f} g_{ad} \right) \\ &- \frac{1}{2} \partial_{a} g_{bc} - \frac{1}{2} \partial_{c} g_{bc} + \frac{1}{4} \partial_{a} g_{ac} g^{ef} \partial_{e} g_{bc} - \frac{1}{4} \partial_{a} g_{ac} g^{ef} \partial_{f} g_{bc} - \frac{1}{4} \partial_{a} g_{ac} g^{ef} \partial_{f} g_{bc} - \frac{1}{4} \partial_{a} g_{ac} g^{ef} \partial_{f}$$

$$R_{abcd} + R_{bacd} = \frac{1}{2} \partial_{cd}g_{ba} - \frac{1}{2} \partial_{dc}g_{ba} + \frac{1}{4} \partial_{a}g_{cd}g^{ef} \partial_{b}g_{fc} + \frac{1}{4} \partial_{a}g_{cd}g^{ef} \partial_{c}g_{bf} - \frac{1}{4} \partial_{a}g_{cd}g^{ef} \partial_{c}g_{bf} - \frac{1}{4} \partial_{a}g_{ac}g^{ef} \partial_{c}g_{bf} - \frac{1}{4} \partial_{a}g_{ac}g^{ef} \partial_{b}g_{fc} - \frac{1}{4} \partial_{a}g_{ac}g^{ef} \partial_{b}g_{fc} - \frac{1}{4} \partial_{a}g_{ac}g^{ef} \partial_{b}g_{fd} - \frac{1}{4} \partial_{a}g_{ac}g^{ef} \partial_{b}g_{fd} - \frac{1}{4} \partial_{a}g_{ac}g^{ef} \partial_{b}g_{fd} - \frac{1}{4} \partial_{a}g_{ac}g^{ef} \partial_{b}g_{bf} + \frac{1}{4} \partial_{a}g_{ac}g^{ef} \partial_{b}g_{fd} - \frac{1}{4} \partial_{a}g_{ac}g^{ef} \partial_{a}g_{bf} + \frac{1}{4} \partial_{a}g_{ac}g^{ef} \partial_{b}g_{fd} - \frac{1}{4} \partial_{a}g_{ac}g^{ef} \partial_{a}g_{bf} + \frac{1}{4} \partial_{a}g_{ac}g^{ef} \partial_{b}g_{fd} + \frac{1}{4} \partial_{a}g_{ac}g^{ef} \partial_{a}g_{bf} - \frac{1}{4} \partial_{a}g_{bc}g^{ef} \partial_{a}g_{bf} - \frac{1}{4} \partial_{a}g_{$$

$$R_{abcd} + R_{bacd} = \frac{1}{2} \partial_{cd}g_{ba} - \frac{1}{2} \partial_{dc}g_{ba} + \frac{1}{4} \partial_{a}g_{ed}\partial_{b}g_{fc}g^{ef} + \frac{1}{4} \partial_{a}g_{ed}\partial_{c}g_{bf}g^{ef} - \frac{1}{4} \partial_{a}g_{fd}\partial_{e}g_{bc}g^{fe} + \frac{1}{4} \partial_{b}g_{ec}\partial_{d}g_{af}g^{fe} + \frac{1}{4} \partial_{c}g_{be}\partial_{d}g_{af}g^{fe} - \frac{1}{4} \partial_{d}g_{af}\partial_{e}g_{bc}g^{fe}$$

$$- \frac{1}{4} \partial_{b}g_{fc}\partial_{e}g_{ad}g^{ef} - \frac{1}{4} \partial_{c}g_{bf}\partial_{e}g_{ad}g^{ef} + \frac{1}{4} \partial_{e}g_{ad}\partial_{f}g_{bc}g^{ef} - \frac{1}{4} \partial_{a}g_{ec}\partial_{b}g_{fd}g^{ef} - \frac{1}{4} \partial_{a}g_{ec}\partial_{d}g_{bf}g^{ef} + \frac{1}{4} \partial_{a}g_{fc}\partial_{e}g_{bd}g^{fe} - \frac{1}{4} \partial_{b}g_{ed}\partial_{c}g_{af}g^{fe}$$

$$- \frac{1}{4} \partial_{c}g_{ae}\partial_{d}g_{bf}g^{ef} + \frac{1}{4} \partial_{c}g_{af}\partial_{e}g_{bd}g^{fe} + \frac{1}{4} \partial_{b}g_{fd}\partial_{e}g_{ac}g^{ef} + \frac{1}{4} \partial_{d}g_{bf}\partial_{e}g_{ac}g^{ef} - \frac{1}{4} \partial_{e}g_{ac}\partial_{f}g_{bd}g^{ef} + \frac{1}{2} \partial_{c}g_{ab} - \frac{1}{2} \partial_{c}g_{ab} + \frac{1}{4} \partial_{a}g_{ec}\partial_{b}g_{fd}g^{fe}$$

$$+ \frac{1}{4} \partial_{b}g_{ed}\partial_{c}g_{af}g^{ef} - \frac{1}{4} \partial_{b}g_{fd}\partial_{e}g_{ac}g^{fe} + \frac{1}{4} \partial_{a}g_{ec}\partial_{d}g_{bf}g^{fe} + \frac{1}{4} \partial_{c}g_{ae}\partial_{d}g_{bf}g^{fe} - \frac{1}{4} \partial_{d}g_{bf}\partial_{e}g_{ac}g^{fe} - \frac{1}{4} \partial_{a}g_{ec}\partial_{e}g_{bd}g^{ef}$$

$$+ \frac{1}{4} \partial_{e}g_{bd}\partial_{c}g_{af}g^{ef} - \frac{1}{4} \partial_{a}g_{ed}\partial_{b}g_{fc}g^{fe} - \frac{1}{4} \partial_{b}g_{ec}\partial_{d}g_{af}g^{ef} + \frac{1}{4} \partial_{c}g_{ae}\partial_{d}g_{bf}g^{fe} - \frac{1}{4} \partial_{a}g_{ed}\partial_{c}g_{bf}g^{fe} - \frac{1}{4} \partial_{a}g_{ed}\partial_{e}g_{bd}g^{ef}$$

$$+ \frac{1}{4} \partial_{e}g_{bd}\partial_{f}g_{ac}g^{ef} - \frac{1}{4} \partial_{a}g_{ed}\partial_{b}g_{fc}g^{fe} - \frac{1}{4} \partial_{b}g_{ec}\partial_{d}g_{af}g^{ef} + \frac{1}{4} \partial_{b}g_{fc}\partial_{e}g_{ad}g^{fe}$$

$$+ \frac{1}{4} \partial_{a}g_{fd}\partial_{e}g_{bc}g^{ef} + \frac{1}{4} \partial_{d}g_{af}\partial_{e}g_{bc}g^{ef} - \frac{1}{4} \partial_{e}g_{bc}\partial_{f}g_{ad}g^{ef}$$

$$+ \frac{1}{4} \partial_{a}g_{fd}\partial_{e}g_{bc}g^{ef} + \frac{1}{4} \partial_{e}g_{bc}\partial_{f}g_{ad}g^{ef}$$

$$+ \frac{1}{4} \partial_{a}g_{fd}\partial_{e}g_{bc}g^{ef} + \frac{1}{4} \partial_{e}g_{bc}\partial_{f}g_{ad}g^{ef}$$

$$+ \frac{1}{4} \partial_{a}g_{fd}\partial_{e}g_{bc}g^{ef} - \frac{1}{4} \partial_{e}g_{bc}\partial_{f}g_{ad}g^{ef}$$

$$+ \frac{1}{4} \partial_{a}g_{fd}\partial_{e}g_{bc}g^{ef} - \frac{1}{4} \partial_{e}g_{bc}\partial_{f}g_{ad}g^{ef}$$

$$+ \frac{1}{4} \partial_{e}g_{fd}\partial_{e}g_{bc}g^{ef} - \frac{1}{4} \partial_{e}g_{fd}\partial_{e}g_{bc}g^{ef}$$

$$+ \frac{1}{4} \partial_{e}g_{fd}\partial_{e}g_{fd}\partial_{e}g$$

## Exercise 3.4 Symmetric on swapping first and second pair of indices

```
expr := R_{a b c d} - R_{c d a b}.
                                     # cdb(ex-0304.201,expr)
               (expr, Rabcd)
                                     # cdb(ex-0304.202,expr)
substitute
               (expr, GammaU)
                                     # cdb(ex-0304.203,expr)
substitute
substitute
               (expr, GammaD)
                                     # cdb(ex-0304.204,expr)
               (expr)
                                     # cdb(ex-0304.205,expr)
distribute
                                     # cdb(ex-0304.206,expr)
               (expr)
product_rule
sort_product
               (expr)
                                     # cdb(ex-0304.207,expr)
rename_dummies (expr)
                                     # cdb(ex-0304.208,expr)
canonicalise
               (expr)
                                     # cdb(ex-0304.209,expr)
```

$$\begin{split} R_{abcd} - R_{cdab} &= \partial_c \Gamma_{abd} - \partial_d \Gamma_{abc} + \Gamma_{cad} \Gamma^e_{bd} - \Gamma_{cac} \Gamma^e_{bd} - \partial_a \Gamma_{cdb} + \partial_b \Gamma_{cda} - \Gamma_{ccb} \Gamma^e_{da} + \Gamma_{cca} \Gamma^e_{db} \\ &= \partial_c \Gamma_{abd} - \partial_d \Gamma_{abc} + \frac{1}{2} \Gamma_{cad} g^{ef} \left( \partial_b g_{fc} + \partial_c g_{bf} - \partial_f g_{bc} \right) - \frac{1}{2} \Gamma_{cac} g^{ef} \left( \partial_b g_{fd} + \partial_d g_{bf} - \partial_f g_{bd} \right) - \partial_a \Gamma_{cdb} + \partial_b \Gamma_{cda} \\ &= -\frac{1}{2} \Gamma_{ccb} g^{ef} \left( \partial_d g_{fa} + \partial_a g_{df} - \partial_f g_{da} \right) + \frac{1}{2} \Gamma_{ccc} g^{ef} \left( \partial_d g_{fb} + \partial_b g_{df} - \partial_f g_{db} \right) \\ &= \partial_c \left( \frac{1}{2} \partial_b g_{bd} + \frac{1}{2} \partial_d g_{ba} - \frac{1}{2} \partial_a g_{bc} \right) - \partial_d \left( \frac{1}{2} \partial_b g_{ac} + \frac{1}{2} \partial_c g_{ba} - \frac{1}{2} \partial_a g_{bc} \right) + \frac{1}{2} \left( \frac{1}{2} \partial_a g_{cd} + \frac{1}{2} \partial_d g_{ac} - \frac{1}{2} \partial_c g_{bf} - \partial_f g_{bc} \right) \\ &= -\frac{1}{2} \left( \frac{1}{2} \partial_a g_{cc} + \frac{1}{2} \partial_c g_{ac} - \frac{1}{2} \partial_c g_{bd} \right) - \partial_d \left( \frac{1}{2} \partial_b g_{ac} + \frac{1}{2} \partial_a g_{bc} \right) - \partial_a \left( \frac{1}{2} \partial_a g_{cd} + \frac{1}{2} \partial_a g_{bc} - \frac{1}{2} \partial_c g_{bh} \right) \\ &= -\frac{1}{2} \left( \frac{1}{2} \partial_a g_{cc} + \frac{1}{2} \partial_a g_{bc} - \frac{1}{2} \partial_c g_{bd} \right) - \frac{1}{2} \left( \frac{1}{2} \partial_a g_{cc} - \frac{1}{2} \partial_c g_{bd} \right) - \partial_a \left( \frac{1}{2} \partial_a g_{cc} - \frac{1}{2} \partial_c g_{bb} \right) \\ &+ \partial_b \left( \frac{1}{2} \partial_d g_{ca} + \frac{1}{2} \partial_a g_{cc} - \frac{1}{2} \partial_c g_{bd} \right) - \frac{1}{2} \left( \frac{1}{2} \partial_c g_{cb} + \frac{1}{2} \partial_a g_{cc} - \frac{1}{2} \partial_c g_{bb} \right) \\ &+ \left( \frac{1}{2} \partial_a g_{cc} + \frac{1}{2} \partial_a g_{cc} - \frac{1}{2} \partial_c g_{cd} \right) - \frac{1}{2} \left( \frac{1}{2} \partial_c g_{cb} + \frac{1}{2} \partial_a g_{cc} - \frac{1}{2} \partial_c g_{cb} \right) g^{ef} \left( \partial_d g_{fb} + \partial_a g_{df} - \partial_f g_{df} \right) \\ &+ \frac{1}{2} \partial_{cd} g_{cd} + \frac{1}{2} \partial_a g_{bc} - \frac{1}{2} \partial_{cd} g_{cd} \right) g^{ef} \left( \partial_d g_{fb} + \partial_a g_{df} - \partial_f g_{db} \right) \\ &= \frac{1}{2} \partial_{cd} g_{cd} + \frac{1}{2} \partial_a g_{bc} - \frac{1}{2} \partial_{cd} g_{ac} - \frac{1}{2} \partial_{cd} g_{bf} - \partial_f g_{db} \right) \\ &+ \frac{1}{4} \partial_a g_{cd} g^{ef} \partial_c g_{bf} - \frac{1}{4} \partial_a g_{ac} g^{ef} \partial_d g_{bc} + \frac{1}{4} \partial_a g_{ac} g^{ef} \partial_b g_{fc} + \frac{1}{4} \partial_a g_{ac} g^{ef} \partial_b g_{ff} - \frac{1}{4} \partial_a g_{ac} g^{ef} \partial_d g_{fb} + \frac{1}{4} \partial_a g_{ac} g^{ef} \partial_d g_{fb} + \frac{1}{4} \partial_a g_{ac} g^{ef} \partial_d g_{ff} - \frac{1}{4} \partial_a g_{ac} g^{ef} \partial_d g_{ff} - \frac{1}{4} \partial_a g_$$

$$\begin{split} R_{abcd} - R_{cdab} &= \frac{1}{2} \partial_{cb} g_{ad} + \frac{1}{2} \partial_{cd} g_{ba} - \frac{1}{2} \partial_{ab} g_{ac} - \frac{1}{2} \partial_{dc} g_{ba} + \frac{1}{4} \partial_{a} g_{bc} g^{ef} \partial_{b} g_{fc} + \frac{1}{4} \partial_{a} g_{cd} g^{ef} \partial_{c} g_{bf} - \frac{1}{4} \partial_{a} g_{cd} g^{ef} \partial_{f} g_{bc} + \frac{1}{4} \partial_{d} g_{ac} g^{ef} \partial_{b} g_{fc} \\ &+ \frac{1}{4} \partial_{d} g_{ac} g^{ef} \partial_{c} g_{bf} - \frac{1}{4} \partial_{d} g_{ac} g^{ef} \partial_{f} g_{bc} - \frac{1}{4} \partial_{c} g_{ad} g^{ef} \partial_{b} g_{fc} - \frac{1}{4} \partial_{c} g_{ad} g^{ef} \partial_{f} g_{bc} - \frac{1}{4} \partial_{a} g_{cc} g^{ef} \partial_{g} g_{bf} \\ &+ \frac{1}{4} \partial_{a} g_{ac} g^{ef} \partial_{f} g_{bd} - \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{b} g_{fd} - \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{g} g_{ff} \\ &+ \frac{1}{4} \partial_{a} g_{ac} g^{ef} \partial_{f} g_{bd} - \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{b} g_{fd} - \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{g} g_{ff} \\ &+ \frac{1}{4} \partial_{a} g_{ac} g^{ef} \partial_{g} g_{ff} - \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{g} g_{ff} \\ &+ \frac{1}{4} \partial_{a} g_{ac} g^{ef} \partial_{g} g_{ff} - \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{g} g_{ff} \\ &+ \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{g} g_{ff} - \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{g} g_{ff} \\ &+ \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{g} g_{ff} - \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{g} g_{ff} \\ &+ \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{g} g_{ff} - \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{g} g_{ff} \\ &+ \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{g} g_{ff} - \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{g} g_{ff} \\ &+ \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{g} g_{ff} - \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{g} g_{ff} \\ &+ \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{g} g_{ff} - \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{g} g_{ff} \\ &+ \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{g} g_{ff} + \frac{1}{4} \partial_{c} g_{ac} g^{ef} \partial_{g} g_{ff} \\ &+ \frac{1}{$$

$$\begin{split} R_{abcd} - R_{cdab} &= \frac{1}{2} \partial_{cb} g_{ad} + \frac{1}{2} \partial_{ca} g_{ba} - \frac{1}{2} \partial_{ca} g_{bd} - \frac{1}{2} \partial_{dc} g_{ba} + \frac{1}{2} \partial_{da} g_{bc} + \frac{1}{4} \partial_{a} g_{ed} \partial_{b} g_{fc} g^{ef} + \frac{1}{4} \partial_{a} g_{ed} \partial_{c} g_{bf} g^{ef} - \frac{1}{4} \partial_{a} g_{fd} \partial_{e} g_{bc} g^{fe} + \frac{1}{4} \partial_{b} g_{ec} \partial_{d} g_{af} g^{fe} \\ &+ \frac{1}{4} \partial_{c} g_{be} \partial_{d} g_{af} g^{fe} - \frac{1}{4} \partial_{d} g_{af} \partial_{e} g_{bc} g^{fe} - \frac{1}{4} \partial_{b} g_{fc} \partial_{e} g_{ad} g^{ef} - \frac{1}{4} \partial_{c} g_{bf} \partial_{e} g_{ad} g^{ef} + \frac{1}{4} \partial_{e} g_{ad} \partial_{f} g_{bc} g^{ef} - \frac{1}{4} \partial_{a} g_{ec} \partial_{b} g_{fd} g^{ef} \\ &+ \frac{1}{4} \partial_{a} g_{fc} \partial_{e} g_{bd} g^{fe} - \frac{1}{4} \partial_{b} g_{ed} \partial_{c} g_{af} g^{fe} - \frac{1}{4} \partial_{c} g_{ae} \partial_{d} g_{bf} g^{ef} + \frac{1}{4} \partial_{c} g_{af} \partial_{e} g_{bd} g^{fe} + \frac{1}{4} \partial_{b} g_{fd} \partial_{e} g_{ac} g^{ef} + \frac{1}{4} \partial_{d} g_{fd} \partial_{e} g_{ac} g^{ef} - \frac{1}{4} \partial_{a} g_{ec} \partial_{d} g_{bf} g^{ef} \\ &- \frac{1}{2} \partial_{ad} g_{cb} - \frac{1}{2} \partial_{ab} g_{dc} + \frac{1}{2} \partial_{bc} g_{da} + \frac{1}{2} \partial_{ba} g_{dc} - \frac{1}{2} \partial_{bc} g_{da} - \frac{1}{4} \partial_{c} g_{eb} \partial_{d} g_{fa} g^{ef} - \frac{1}{4} \partial_{a} g_{de} \partial_{c} g_{fb} g^{fe} \\ &- \frac{1}{4} \partial_{b} g_{ce} \partial_{d} g_{fa} g^{ef} - \frac{1}{4} \partial_{a} g_{de} \partial_{b} g_{ff} g^{fe} + \frac{1}{4} \partial_{b} g_{ce} \partial_{d} g_{fb} g^{fe} + \frac{1}{4} \partial_{d} g_{fa} \partial_{e} g_{cb} g^{ef} + \frac{1}{4} \partial_{a} g_{de} \partial_{c} g_{fb} g^{fe} + \frac{1}{4} \partial_{a} g_{de} \partial_{c} g_{fb} g^{fe} \\ &- \frac{1}{4} \partial_{b} g_{ce} \partial_{d} g_{fa} g^{ef} - \frac{1}{4} \partial_{a} g_{de} \partial_{b} g_{ff} g^{fe} + \frac{1}{4} \partial_{b} g_{ce} \partial_{d} g_{fb} g^{fe} + \frac{1}{4} \partial_{d} g_{fa} \partial_{e} g_{cb} g^{ef} - \frac{1}{4} \partial_{a} g_{de} \partial_{c} g_{fb} g^{ef} \\ &+ \frac{1}{4} \partial_{b} g_{de} \partial_{c} g_{fa} g^{fe} - \frac{1}{4} \partial_{a} g_{de} \partial_{b} g_{ff} g^{fe} + \frac{1}{4} \partial_{a} g_{ce} \partial_{d} g_{fb} g^{ef} - \frac{1}{4} \partial_{a} g_{de} \partial_{e} g_{db} g^{fe} - \frac{1}{4} \partial_{a} g_{fb} \partial_{e} g_{ca} g^{ef} - \frac{1}{4} \partial_{a} g_{fb} \partial_{e} g_{ca} g^{ef} - \frac{1}{4} \partial_{a} g_{fb} \partial_{e} g_{de} g^{ef} - \frac{1}{4} \partial_{a} g_{fb} \partial_{e} g_{de} g^{ef} - \frac{1}{4} \partial_{a} g_{fb} \partial_{e} g_{de} g^{ef} - \frac{1}{4} \partial_{a} g_{fb} \partial_{e} g_{fb} \partial_{e} g_{fb} \partial_{e} g_{fb} \partial_{e} g_{fb} \partial_{e} g_{fb$$