Exercise 6.6 The Kasner cosmology

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{t, x, y, z}::Coordinate.
{a,b,c,d,e,f,g,h#}::Indices(values={t, x, y, z}, position=independent).

\partial{#}::PartialDerivative.

p1::LaTeXForm("p_1").
p2::LaTeXForm("p_2").
p3::LaTeXForm("p_3").

g^{a b}::InverseMetric.  # essential when using complete (gab, $g^{a b}$)

Gamma := \Gamma^{a}_{b c} -> 1/2 g^{a d} (   \partial_{b}{g_{d c}}
                                           + \partial_{c}{g_{b d}}
                                           - \partial_{d}{g_{b c}}).

Rabcd := R^{a}_{b c d} ->   \partial_{c}{\Gamma^{a}_{b d}}
                          - \partial_{d}{\Gamma^{a}_{b c}}
                          + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
                          - \Gamma^{e}_{b c} \Gamma^{a}_{d e}.

Rab := R_{a b} -> R^{c}_{a c b}.

gab := { g_{t t} = -1,
         g_{x x} = t**(2*p1),
         g_{y y} = t**(2*p2),
         g_{z z} = t**(2*p3)}.                            # cdb(ex-0606.101,gab)

complete   (gab, $g^{a b}$)                               # cdb(ex-0606.102,gab)

substitute (Rabcd, Gamma)
substitute (Rab, Rabcd)

evaluate   (Gamma, gab, rhsonly=True)                     # cdb(ex-0606.103,Gamma)
evaluate   (Rabcd, gab, rhsonly=True)                     # cdb(ex-0606.104,Rabcd)
evaluate   (Rab,   gab, rhsonly=True)                     # cdb(ex-0606.105,Rab)
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$$\left[g_{tt} = -1,\ g_{xx} = t^{2p_1},\ g_{yy} = t^{2p_2},\ g_{zz} = t^{2p_3}\right] \tag{ex-0606.101}$$

$$\left[g_{tt} = -1,\ g_{xx} = t^{2p_1},\ g_{yy} = t^{2p_2},\ g_{zz} = t^{2p_3},\ g^{tt} = -1,\ g^{xx} = t^{-2p_1},\ g^{yy} = t^{-2p_2},\ g^{zz} = t^{-2p_3}\right] \tag{ex-0606.102}$$

$$\Gamma^a{}_{bc} \to \Box_{cb}{}^a \begin{cases} \Box_{zt}{}^z = p_3 t^{-1} \\ \Box_{yt}{}^y = p_2 t^{-1} \\ \Box_{xt}{}^x = p_1 t^{-1} \\ \Box_{tz}{}^z = p_3 t^{-1} \\ \Box_{ty}{}^y = p_2 t^{-1} \\ \Box_{tx}{}^x = p_1 t^{-1} \\ \Box_{zz}{}^t = p_3 t^{(2p_3-1)} \\ \Box_{yy}{}^t = p_2 t^{(2p_2-1)} \\ \Box_{xx}{}^t = p_1 t^{(2p_1-1)} \end{cases} \tag{ex-0606.103}$$

$$R^{a}{}_{bcd} \to \square_{db}{}^{a}{}_{c} \begin{cases} \square_{xx}{}^{t}{}_{t} = p_1 t^{(2p_1-2)}\left(p_1-1\right) \\ \square_{yy}{}^{t}{}_{t} = p_2 t^{(2p_2-2)}\left(p_2-1\right) \\ \square_{zz}{}^{t}{}_{t} = p_3 t^{(2p_3-2)}\left(p_3-1\right) \\ \square_{xt}{}^{x}{}_{t} = p_1\left(p_1-1\right) t^{-2} \\ \square_{yt}{}^{y}{}_{t} = p_2\left(p_2-1\right) t^{-2} \\ \square_{zt}{}^{z}{}_{t} = p_3\left(p_3-1\right) t^{-2} \\ \square_{tx}{}^{t}{}_{x} = -p_1 t^{(2p_1-2)}\left(p_1-1\right) \\ \square_{ty}{}^{t}{}_{y} = -p_2 t^{(2p_2-2)}\left(p_2-1\right) \\ \square_{tz}{}^{t}{}_{z} = -p_3 t^{(2p_3-2)}\left(p_3-1\right) \\ \square_{tt}{}^{x}{}_{x} = p_1\left(-p_1+1\right) t^{-2} \\ \square_{tt}{}^{y}{}_{y} = p_2\left(-p_2+1\right) t^{-2} \\ \square_{tt}{}^{z}{}_{z} = p_3\left(-p_3+1\right) t^{-2} \\ \square_{zz}{}^{y}{}_{y} = p_2 p_3 t^{(2p_3-2)} \\ \square_{zz}{}^{x}{}_{x} = p_1 p_3 t^{(2p_3-2)} \\ \square_{yy}{}^{z}{}_{z} = p_2 p_3 t^{(2p_2-2)} \\ \square_{yy}{}^{x}{}_{x} = p_1 p_2 t^{(2p_2-2)} \\ \square_{xx}{}^{z}{}_{z} = p_1 p_3 t^{(2p_1-2)} \\ \square_{xx}{}^{y}{}_{y} = p_1 p_2 t^{(2p_1-2)} \\ \square_{yz}{}^{y}{}_{z} = -p_2 p_3 t^{(2p_3-2)} \\ \square_{xz}{}^{x}{}_{z} = -p_1 p_3 t^{(2p_3-2)} \\ \square_{zy}{}^{z}{}_{y} = -p_2 p_3 t^{(2p_2-2)} \\ \square_{xy}{}^{x}{}_{y} = -p_1 p_2 t^{(2p_2-2)} \\ \square_{zx}{}^{z}{}_{x} = -p_1 p_3 t^{(2p_1-2)} \\ \square_{yx}{}^{y}{}_{x} = -p_1 p_2 t^{(2p_1-2)} \end{cases} \qquad (\texttt{ex-0606.104})$$

$$R_{ab} \to \square_{ba} \begin{cases} \square_{xx} = p_1 t^{(2p_1-2)}\left(p_1+p_2+p_3-1\right) \\ \square_{yy} = p_2 t^{(2p_2-2)}\left(p_1+p_2+p_3-1\right) \\ \square_{zz} = p_3 t^{(2p_3-2)}\left(p_1+p_2+p_3-1\right) \\ \square_{tt} = \left(-p_1{}^2+p_1-p_2{}^2+p_2-p_3{}^2+p_3\right) t^{-2} \end{cases} \qquad (\texttt{ex-0606.105})$$