Example 99 The second Bianchi identity

```
{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices(position=independent).

def add_tags (obj,tag):
   n = 0
   ans = Ex('0')
   for i in obj.top().terms():
      foo = obj[i]
      bah = Ex(tag+'_{'+str(n)+'}')
      ans := @(ans) + @(bah) @(foo).
      n = n + 1
   return ans

def clear_tags (obj,tag):
   ans := @(obj).
   foo  = Ex(tag+'_{a?} -> 1')
   substitute (ans,foo)
   return ans

;::Symbol;  # Suggsted by Kasper as a way to make use of ; legal
            # see https://cadabra.science/qa/473/is-this-legal-syntax
            # this code works with and without this trick

# rules for the first two covariant derivs of V^a

# deriv1 = commutator for 2nd derivatives of v^a
# unused here
deriv1 := v^{a}_{; c ; d} - v^{a}_{; d ; c} -> R^{a}_{b c d} v^{b}.   # cdb (ex-99.100,deriv1)

# deriv2 = covariant derivative of deriv1
deriv2 := A^{a}_{c d e} -> v^{a}_{; c ; d ; e} - v^{a}_{; d ; c ; e} - R^{a}_{b c d ; e} v^{b}
                                                                     - R^{a}_{b c d} v^{b}_{; e}.   # cdb (ex-99.101,deriv2)

# deriv3 = commutator for 3rd derivatives of v^a
deriv3 := v^{a}_{; c ; d ; e} -> v^{a}_{; c ; e ; d}
                               + R^{a}_{f d e} v^{f}_{; c}
                               - R^{f}_{c d e} v^{a}_{; f}.     # cdb (ex-99.102,deriv3)
```

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37
38 cycle := A^{a}_{c d e} + A^{a}_{d e c} + A^{a}_{e c d}.         # cdb (ex-99.103,cycle)
39
40 substitute (cycle,deriv2)                          # cdb (ex-99.104, cycle)
41
42 cycle = add_tags (cycle,'\\mu')                    # cdb (ex-99.105, cycle)
43
44 # sub on the first pair
45 zoom (cycle, $\mu_{0} Q??$)
46 substitute (cycle,deriv3)                          # cdb (ex-99.106, cycle)
47 unzoom (cycle)
48
49 # sub on the second pair
50 zoom (cycle, $\mu_{4} Q??$)
51 substitute (cycle,deriv3,)                         # cdb (ex-99.107, cycle)
52 unzoom (cycle)
53
54 # sub on the third pair
55 zoom (cycle, $\mu_{8} Q??$)
56 substitute (cycle,deriv3,)                         # cdb (ex-99.108, cycle)
57 unzoom (cycle)
58
59 cycle = clear_tags (cycle,'\\mu')
60
61 sort_sum (cycle)
62 rename_dummies (cycle)
63
64 factor_out (cycle,$v^{a?}_{; b?},v^{a?}$)          # cdb (ex-99.109, cycle)
```

$$v^a{}_{;c;d} - v^a{}_{;d;c} \to R^a{}_{bcd} v^b \tag{ex-99.100}$$

$$A^a{}_{cde} \to v^a{}_{;c;d;e} - v^a{}_{;d;c;e} - R^a{}_{bcd;e} v^b - R^a{}_{bcd} v^b{}_{;e} \tag{ex-99.101}$$

$$v^a{}_{;c;d;e} \to v^a{}_{;c;e;d} + R^a{}_{fde} v^f{}_{;c} - R^f{}_{cde} v^a{}_{;f} \tag{ex-99.102}$$

$$A^a{}_{cde} + A^a{}_{dec} + A^a{}_{ecd} \tag{ex-99.103}$$

$$v^a{}_{;c;d;e} - v^a{}_{;d;c;e} - R^a{}_{bcd;e} v^b - R^a{}_{bcd} v^b{}_{;e} + v^a{}_{;d;e;c} - v^a{}_{;e;d;c} - R^a{}_{bde;c} v^b - R^a{}_{bde} v^b{}_{;c} + v^a{}_{;e;c;d} - v^a{}_{;c;e;d} - R^a{}_{bec;d} v^b - R^a{}_{bec} v^b{}_{;d} \tag{ex-99.104}$$

$$\mu_0 v^a{}_{;c;d;e} - \mu_1 v^a{}_{;d;c;e} - \mu_2 R^a{}_{bcd;e} v^b - \mu_3 R^a{}_{bcd} v^b{}_{;e} + \mu_4 v^a{}_{;d;e;c} - \mu_5 v^a{}_{;e;d;c} - \mu_6 R^a{}_{bde;c} v^b - \mu_7 R^a{}_{bde} v^b{}_{;c} + \mu_8 v^a{}_{;e;c;d} - \mu_9 v^a{}_{;c;e;d} - \mu_{10} R^a{}_{bec;d} v^b - \mu_{11} R^a{}_{bec} v^b{}_{;d} \tag{ex-99.105}$$

$$\mu_0 \left(v^a{}_{;c;e;d} + R^a{}_{fde} v^f{}_{;c} - R^f{}_{cde} v^a{}_{;f} \right) + \ldots \tag{ex-99.106}$$

$$\ldots + \mu_4 \left(v^a{}_{;d;c;e} + R^a{}_{fec} v^f{}_{;d} - R^f{}_{dec} v^a{}_{;f} \right) + \ldots \tag{ex-99.107}$$

$$\ldots + \mu_8 \left(v^a{}_{;e;d;c} + R^a{}_{fcd} v^f{}_{;e} - R^f{}_{ecd} v^a{}_{;f} \right) + \ldots \tag{ex-99.108}$$

$$v^a{}_{;b} \left(-R^b{}_{cde} - R^b{}_{dec} - R^b{}_{ecd} \right) + v^b \left(-R^a{}_{bcd;e} - R^a{}_{bde;c} - R^a{}_{bec;d} \right) \tag{ex-99.109}$$