Example 2 Covariant derivatives

```
{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices.

\nabla{#}::Derivative.
\partial{#}::PartialDerivative.

# rule for covariant derivative of v^{a}

deriv := \nabla_{a}{v^{b}} -> \partial_{a}{v^{b}} + \Gamma^{b}_{c a} v^{c}.

# create an expression

foo := \nabla_{a}{v^{b}}.                    # cdb (ex-02.101,foo)

# apply the rule, then simplify

substitute   (foo,deriv)                     # cdb (ex-02.102,foo)
canonicalise (foo)                           # cdb (ex-02.103,foo)

checkpoint.append (foo)
```

$$\nabla_{a} v^{b} = \partial_{a} v^{b} + \Gamma^{b}{}_{ca} v^{c} \tag{ex-02.102}$$

$$= \partial_{a} v^{b} + \Gamma^{bc}{}_{a} v_{c} \tag{ex-02.103}$$

Example 2 Covariant derivatives using "position=independent"

```
{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices(position=independent).

\nabla{#}::Derivative.
\partial{#}::PartialDerivative.

# rule for covariant derivative of v^{a}

deriv := \nabla_{a}{v^{b}} -> \partial_{a}{v^{b}} + \Gamma^{b}_{c a} v^{c}.

# create an expression

foo := \nabla_{a}{v^{b}}.                    # cdb (ex-02.201,foo)

# apply the rule, then simplify

substitute   (foo,deriv)                     # cdb (ex-02.202,foo)
canonicalise (foo)                           # cdb (ex-02.203,foo)

checkpoint.append (foo)
```

$$\nabla_{a}v^{b} = \partial_{a}v^{b} + \Gamma^{b}{}_{ca}v^{c} \tag{ex-02.202}$$

$$= \partial_{a}v^{b} + \Gamma^{b}{}_{ca}v^{c} \tag{ex-02.203}$$

Example 2 Covariant derivatives using generic rule for deriv

```
{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices(position=independent).

\nabla{#}::Derivative.
\partial{#}::PartialDerivative.

# template for covariant derivative of a vector

deriv := \nabla_{a}{A?^{b}} -> \partial_{a}{A?^{b}} + \Gamma^{b}_{c a} A?^{c}.

# create an expression

foo := \nabla_{a}{u^{b}} + \nabla_{a}{v^{b}}. # cdb (ex-02.301,foo)

# apply the rule, then simplify

substitute   (foo,deriv)                      # cdb (ex-02.302,foo)
canonicalise (foo)                            # cdb (ex-02.303,foo)

checkpoint.append (foo)
```

$$\nabla_{a}u^{b} + \nabla_{a}v^{b} = \partial_{a}u^{b} + \Gamma^{b}{}_{ca}u^{c} + \partial_{a}v^{b} + \Gamma^{b}{}_{ca}v^{c} \tag{ex-02.302}$$

$$= \partial_{a}u^{b} + \Gamma^{b}{}_{ca}u^{c} + \partial_{a}v^{b} + \Gamma^{b}{}_{ca}v^{c} \tag{ex-02.303}$$