

## Exercise 1.1 Verify symmetry of $\Gamma^a_{bc}$

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1 {a,b,c,d,e,f,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices.
2
3 g_{a b}::Metric.
4
5 \partial{#}::PartialDerivative.
6
7 Gamma := \Gamma^a_{b c} -> (1/2) g^a d ( \partial_b{g_d c}
8                                     + \partial_c{g_b d}
9                                     - \partial_d{g_b c} ).
10
11 diff := \Gamma^a_{b c} - \Gamma^a_{c b}.    # cdb (ex-0101.101,diff)
12
13 substitute (diff, Gamma)                  # cdb (ex-0101.102,diff)
14 distribute (diff)                         # cdb (ex-0101.103,diff)
15 canonicalise (diff)                       # cdb (ex-0101.104,diff)

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$$\begin{aligned}
 \Gamma^a_{bc} - \Gamma^a_{cb} &= \frac{1}{2}g^{ad}(\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc}) - \frac{1}{2}g^{ad}(\partial_c g_{db} + \partial_b g_{cd} - \partial_d g_{cb}) \\
 &= \frac{1}{2}g^{ad}\partial_b g_{dc} + \frac{1}{2}g^{ad}\partial_c g_{bd} - \frac{1}{2}g^{ad}\partial_d g_{bc} - \frac{1}{2}g^{ad}\partial_c g_{db} - \frac{1}{2}g^{ad}\partial_b g_{cd} + \frac{1}{2}g^{ad}\partial_d g_{cb} \\
 &= 0
 \end{aligned}$$