Exercise 6.6 The Kasner cosmology

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{t, x, y, z}::Coordinate.
{a,b,c,d,e,f,g,h#}::Indices(values={t, x, y, z}, position=independent).

\partial{#}::PartialDerivative.

p1::LaTeXForm("p_1").
p2::LaTeXForm("p_2").
p3::LaTeXForm("p_3").

g^{a b}::InverseMetric.  # essential when using complete (gab, $g^{a b}$)

Gamma := \Gamma^{a}_{b c} -> 1/2 g^{a d} (   \partial_{b}{g_{d c}}
                                          + \partial_{c}{g_{b d}}
                                          - \partial_{d}{g_{b c}}).

Rabcd := R^{a}_{b c d} ->   \partial_{c}{\Gamma^{a}_{b d}}
                          - \partial_{d}{\Gamma^{a}_{b c}}
                          + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
                          - \Gamma^{e}_{b c} \Gamma^{a}_{d e}.

Rab := R_{a b} -> R^{c}_{a c b}.

gab := { g_{t t} = -1,
         g_{x x} = t**(2*p1),
         g_{y y} = t**(2*p2),
         g_{z z} = t**(2*p3)}.                              # cdb(ex-0606.101,gab)

complete   (gab, $g^{a b}$)                                 # cdb(ex-0606.102,gab)

substitute (Rabcd, Gamma)
substitute (Rab, Rabcd)

evaluate   (Gamma, gab, rhsonly=True)                       # cdb(ex-0606.103,Gamma)
evaluate   (Rabcd, gab, rhsonly=True)                       # cdb(ex-0606.104,Rabcd)
evaluate   (Rab,   gab, rhsonly=True)                       # cdb(ex-0606.105,Rab)
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$$\left[g_{tt}=-1, g_{xx}=t^{2\,p_1}, g_{yy}=t^{2\,p_2}, g_{zz}=t^{2\,p_3}\right] \tag{ex-0606.101}$$

$$\left[g_{tt}=-1, g_{xx}=t^{2\,p_1}, g_{yy}=t^{2\,p_2}, g_{zz}=t^{2\,p_3}, g^{tt}=-1, g^{xx}=t^{-2\,p_1}, g^{yy}=t^{-2\,p_2}, g^{zz}=t^{-2\,p_3}\right] \tag{ex-0606.102}$$

$$\Gamma^{a}_{\ bc} \to \square_{cb}{}^{a} \begin{cases} \square_{zt}{}^{z} = p_3 t^{-1} \\ \square_{yt}{}^{y} = p_2 t^{-1} \\ \square_{xt}{}^{x} = p_1 t^{-1} \\ \square_{tz}{}^{z} = p_3 t^{-1} \\ \square_{ty}{}^{y} = p_2 t^{-1} \\ \square_{tx}{}^{x} = p_1 t^{-1} \\ \square_{zz}{}^{t} = p_3 t^{(2\,p_3-1)} \\ \square_{yy}{}^{t} = p_2 t^{(2\,p_2-1)} \\ \square_{xx}{}^{t} = p_1 t^{(2\,p_1-1)} \end{cases} \tag{ex-0606.103}$$

$$
R^{a}{}_{bcd} \to \Box_{db}{}^{a}{}_{c}
\begin{cases}
\Box_{xx}{}^{t}{}_{t} = p_1 t^{(2\,p_1-2\,)}\,(p_1-\,1) \\
\Box_{yy}{}^{t}{}_{t} = p_2 t^{(2\,p_2-2\,)}\,(p_2-\,1) \\
\Box_{zz}{}^{t}{}_{t} = p_3 t^{(2\,p_3-2\,)}\,(p_3-\,1) \\
\Box_{xt}{}^{x}{}_{t} = p_1\,(p_1-\,1)\,t^{-2} \\
\Box_{yt}{}^{y}{}_{t} = p_2\,(p_2-\,1)\,t^{-2} \\
\Box_{zt}{}^{z}{}_{t} = p_3\,(p_3-\,1)\,t^{-2} \\
\Box_{tx}{}^{t}{}_{x} = p_1 t^{(2\,p_1-2\,)}\,(1-\,p_1) \\
\Box_{ty}{}^{t}{}_{y} = p_2 t^{(2\,p_2-2\,)}\,(1-\,p_2) \\
\Box_{tz}{}^{t}{}_{z} = p_3 t^{(2\,p_3-2\,)}\,(1-\,p_3) \\
\Box_{tt}{}^{x}{}_{x} = p_1\,(1-\,p_1)\,t^{-2} \\
\Box_{tt}{}^{y}{}_{y} = p_2\,(1-\,p_2)\,t^{-2} \\
\Box_{tt}{}^{z}{}_{z} = p_3\,(1-\,p_3)\,t^{-2} \\
\Box_{zz}{}^{y}{}_{y} = p_2 p_3 t^{(2\,p_3-2\,)} \\
\Box_{zz}{}^{x}{}_{x} = p_1 p_3 t^{(2\,p_3-2\,)} \\
\Box_{yy}{}^{z}{}_{z} = p_2 p_3 t^{(2\,p_2-2\,)} \\
\Box_{yy}{}^{x}{}_{x} = p_1 p_2 t^{(2\,p_2-2\,)} \\
\Box_{xx}{}^{z}{}_{z} = p_1 p_3 t^{(2\,p_1-2\,)} \\
\Box_{xx}{}^{y}{}_{y} = p_1 p_2 t^{(2\,p_1-2\,)} \\
\Box_{yz}{}^{y}{}_{z} = -\,p_2 p_3 t^{(2\,p_3-2\,)} \\
\Box_{xz}{}^{x}{}_{z} = -\,p_1 p_3 t^{(2\,p_3-2\,)} \\
\Box_{zy}{}^{z}{}_{y} = -\,p_2 p_3 t^{(2\,p_2-2\,)} \\
\Box_{xy}{}^{x}{}_{y} = -\,p_1 p_2 t^{(2\,p_2-2\,)} \\
\Box_{zx}{}^{z}{}_{x} = -\,p_1 p_3 t^{(2\,p_1-2\,)} \\
\Box_{yx}{}^{y}{}_{x} = -\,p_1 p_2 t^{(2\,p_1-2\,)}
\end{cases}
\tag{ex-0606.104}
$$

$$
R_{ab} \to \Box_{ba}
\begin{cases}
\Box_{xx} = p_1 t^{(2\,p_1-2\,)}\,(p_1+p_2+p_3-\,1) \\
\Box_{yy} = p_2 t^{(2\,p_2-2\,)}\,(p_1+p_2+p_3-\,1) \\
\Box_{zz} = p_3 t^{(2\,p_3-2\,)}\,(p_1+p_2+p_3-\,1) \\
\Box_{tt} = \left(-\,{p_1}^2+p_1-\,{p_2}^2+p_2-\,{p_3}^2+p_3\right) t^{-2}
\end{cases}
\tag{ex-0606.105}
$$