Exercise 6.6 More digging around in Cadabra's datastructure

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{\theta, \varphi}::Coordinate.
{a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).

\partial{#}::PartialDerivative.

g^{a b}::InverseMetric.  # essential when using complete (gab, $g^{a b}$)

Gamma := \Gamma^{a}_{f g} -> 1/2 g^{a b} (   \partial_{g}{g_{b f}}
                                          + \partial_{f}{g_{b g}}
                                          - \partial_{b}{g_{f g}} ).

Rabcd := R^{d}_{e f g} ->   \partial_{f}{\Gamma^{d}_{e g}}
                          - \partial_{g}{\Gamma^{d}_{e f}}
                          + \Gamma^{d}_{b f} \Gamma^{b}_{e g}
                          - \Gamma^{d}_{b g} \Gamma^{b}_{e f}.

Rab := R_{a b} -> R^{c}_{a c b}.

gab := { g_{\theta \theta}   = r**2,
         g_{\varphi \varphi} = r**2 \sin(\theta)**2 }.   # cdb(ex-0606.101,gab)

complete   (gab, $g^{a b}$)                              # cdb(ex-0606.102,gab)

substitute (Rabcd, Gamma)
substitute (Rab, Rabcd)

evaluate   (Gamma, gab, rhsonly=True)                    # cdb(ex-0606.103,Gamma)
evaluate   (Rabcd, gab, rhsonly=True)                    # cdb(ex-0606.104,Rabcd)
evaluate   (Rab,   gab, rhsonly=True)                    # cdb(ex-0606.105,Rab)

indcs = Rab[1][2][0][0]                                  # cdb(ex-0606.106,indcs)
compt = Rab[1][2][0][1]                                  # cdb(ex-0606.107,compt)

# cdbBeg(print.0606)
print ('Rab = ' + str(Rab.input_form())+'\n')  # reveals Cadabra's internal structure for storing Rab

```

```
37 print ('Rab[0] = ' + str(Rab[0]))
38 print ('Rab[1] = ' + str(Rab[1])+'\n')
39
40 print ('Rab[1][0] = ' + str(Rab[1][0]))
41 print ('Rab[1][1] = ' + str(Rab[1][1]))
42 print ('Rab[1][2] = ' + str(Rab[1][2])+'\n')
43
44 print ('Rab[1][2][0] = ' + str(Rab[1][2][0]))
45 print ('Rab[1][2][0][0] = ' + str(Rab[1][2][0][0]))
46 print ('Rab[1][2][0][1] = ' + str(Rab[1][2][0][1]))
47 # cdbEnd(print.0606)
```

$$\left[g_{\theta\theta} = r^2,\ g_{\varphi\varphi} = r^2(\sin\theta)^2\right] \tag{ex-0606.101}$$

$$\left[g_{\theta\theta} = r^2,\ g_{\varphi\varphi} = r^2(\sin\theta)^2,\ g^{\theta\theta} = r^{-2},\ g^{\varphi\varphi} = \left(r^2(\sin\theta)^2\right)^{-1}\right] \tag{ex-0606.102}$$

$$\Gamma^{a}{}_{fg} \to \square_{fg}{}^{a} \begin{cases} \square_{\varphi\theta}{}^{\varphi} = (\tan\theta)^{-1} \\ \square_{\theta\varphi}{}^{\varphi} = (\tan\theta)^{-1} \\ \square_{\varphi\varphi}{}^{\theta} = -\dfrac{1}{2}\sin(2\theta) \end{cases} \tag{ex-0606.103}$$

$$R^{d}{}_{efg} \to \square_{eg}{}^{d}{}_{f} \begin{cases} \square_{\varphi\varphi}{}^{\theta}{}_{\theta} = (\sin\theta)^2 \\ \square_{\theta\varphi}{}^{\varphi}{}_{\theta} = -1 \\ \square_{\varphi\theta}{}^{\theta}{}_{\varphi} = -(\sin\theta)^2 \\ \square_{\theta\theta}{}^{\varphi}{}_{\varphi} = 1 \end{cases} \tag{ex-0606.104}$$

$$R_{ab} \to \square_{ab} \begin{cases} \square_{\varphi\varphi} = (\sin\theta)^2 \\ \square_{\theta\theta} = 1 \end{cases} \tag{ex-0606.105}$$

$$R_{\varphi\varphi} = R_{[\varphi,\ \varphi]} \tag{ex-0606.106}$$

$$= (\sin\theta)^2 \tag{ex-0606.107}$$

```
Rab = R_{a b} -> \components_{a b}({{\varphi, \varphi} = (\sin(\theta))**2, {\theta, \theta} = 1})

Rab[0] = R_{a b}
Rab[1] = \components_{a b}({{\varphi, \varphi} = (\sin(\theta))**2, {\theta, \theta} = 1})

Rab[1][0] = a
Rab[1][1] = b
Rab[1][2] = {{\varphi, \varphi} = (\sin(\theta))**2, {\theta, \theta} = 1}

Rab[1][2][0] = {\varphi, \varphi} = (\sin(\theta))**2
Rab[1][2][0][0] = {\varphi, \varphi}
Rab[1][2][0][1] = (\sin(\theta))**2
```