

Exercise 2.4 Combining rules – a problem

```
1 {a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices(position=independent).
2
3 \nabla{#}::Derivative.
4 \partial{#}::PartialDerivative.
5
6 # rules for covariant derivatives of v
7
8 deriv1 := \nabla_{a}{v^{b}} -> \partial_{a}{v^{b}}
9          + \Gamma^{b}_{d a} v^{d}.
10
11 deriv2 := \nabla_{a}{\nabla_{b}{v^{c}}} -> \partial_{a}{\nabla_{b}{v^{c}}}
12          + \Gamma^{c}_{d a} \nabla_{b}{v^{d}}
13          - \Gamma^{d}_{d b a} \nabla_{d}{v^{c}}.
14
15 # attempt to combine both rules for second covariant derivative of v
16
17 substitute (deriv2,deriv1)      # cdb (ex-0204.101,deriv2)
```

Note that the call to `substitute` has made changes to both sides of the rule for `deriv2`. This is not ideal and a better method is developed in the following exercise.

$$\nabla_a (\partial_b v^c + \Gamma^c_{db} v^d) \rightarrow \partial_a (\partial_b v^c + \Gamma^c_{db} v^d) + \Gamma^c_{da} (\partial_b v^d + \Gamma^d_{eb} v^e) - \Gamma^d_{ba} (\partial_d v^c + \Gamma^c_{ed} v^e) \quad (\text{ex-0204.101})$$