

## Exercise 1.1 Verify symmetry of $\Gamma^a_{bc}$

```

1 {a,b,c,d,e,f,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices.
2
3 g_{a b}::Metric.
4
5 \partialial{#}::PartialDerivative.
6
7 Gamma := \Gamma^{\{a\}_{\{b\} c\}} -> (1/2) g^{\{a\} d\} ( \partialial_{\{b\}}\{g_{\{d\} c\}}
8                                     + \partialial_{\{c\}}\{g_{\{b\} d\}}
9                                     - \partialial_{\{d\}}\{g_{\{b\} c\}} ).
10
11 diff := \Gamma^{\{a\}_{\{b\} c\}} - \Gamma^{\{a\}_{\{c\} b\}}. # cdb (ex-0101.101,diff)
12
13 substitute (diff, Gamma) # cdb (ex-0101.102,diff)
14 distribute (diff) # cdb (ex-0101.103,diff)
15 canonicalise (diff) # cdb (ex-0101.104,diff)

```

$$\begin{aligned}
 \Gamma^a_{bc} - \Gamma^a_{cb} &= \frac{1}{2}g^{ad}(\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc}) - \frac{1}{2}g^{ad}(\partial_c g_{db} + \partial_b g_{cd} - \partial_d g_{cb}) \\
 &= \frac{1}{2}g^{ad}\partial_b g_{dc} + \frac{1}{2}g^{ad}\partial_c g_{bd} - \frac{1}{2}g^{ad}\partial_d g_{bc} - \frac{1}{2}g^{ad}\partial_c g_{db} - \frac{1}{2}g^{ad}\partial_b g_{cd} + \frac{1}{2}g^{ad}\partial_d g_{cb} \\
 &= 0
 \end{aligned}$$