# Exercise 3.5 Commutation of covariant derivatives

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{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w#}::Indices(position=independent).

\nabla{#}::Derivative.

expr :=   \nabla_{d}{\nabla_{c}{A_{a} B_{b}}}
        - \nabla_{c}{\nabla_{d}{A_{a} B_{b}}}.  # cdb(ex-0305.100,expr)

product_rule (expr)                             # cdb(ex-0305.101,expr)
distribute   (expr)                             # cdb(ex-0305.102,expr)
product_rule (expr)                             # cdb(ex-0305.103,expr)
factor_out   (expr,$A_{a?},B_{b?}$)             # cdb(ex-0305.104,expr)
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$$\nabla_d(\nabla_c(A_a B_b)) - \nabla_c(\nabla_d(A_a B_b)) = \nabla_d(\nabla_c A_a B_b + A_a \nabla_c B_b) - \nabla_c(\nabla_d A_a B_b + A_a \nabla_d B_b) \tag{ex-0305.101}$$

$$= \nabla_d(\nabla_c A_a B_b) + \nabla_d(A_a \nabla_c B_b) - \nabla_c(\nabla_d A_a B_b) - \nabla_c(A_a \nabla_d B_b) \tag{ex-0305.102}$$

$$= \nabla_d(\nabla_c A_a)\, B_b + A_a \nabla_d(\nabla_c B_b) - \nabla_c(\nabla_d A_a)\, B_b - A_a \nabla_c(\nabla_d B_b) \tag{ex-0305.103}$$

$$= B_b\,(\nabla_d(\nabla_c A_a) - \nabla_c(\nabla_d A_a)) + A_a\,(\nabla_d(\nabla_c B_b) - \nabla_c(\nabla_d B_b)) \tag{ex-0305.104}$$