Exercise 1.2 Christoffel symbol and dg

```
{a,b,c,d,e,f,h,i,j,k,l,m,n,o,p,q,r,s,t,u\#}::Indices.
    g_{a b}::Metric.
    g_{a}^{b}::KroneckerDelta.
    \partial{#}::PartialDerivative.
    + \partial_{c}{g_{b d}}
                                           - \partial_{d}{g_{b c}} ).
10
11
    12
13
    expr := \Gamma_{a b c} + \Gamma_{b a c} - \Gamma_{c}\{g_{a b}\}.
                                                              # cdb (ex-0102.101,expr)
14
15
                     (expr, GammaD)
                                                              # cdb (ex-0102.102,expr)
    substitute
16
                     (expr, GammaU)
                                                              # cdb (ex-0102.103,expr)
    substitute
17
                                                              # cdb (ex-0102.104,expr)
    distribute
                     (expr)
    eliminate_metric
                     (expr)
                                                              # cdb (ex-0102.105,expr)
19
    eliminate_kronecker (expr)
                                                              # cdb (ex-0102.106,expr)
20
                     (expr)
                                                              # cdb (ex-0102.107,expr)
    canonicalise
21
```

$$\begin{split} \Gamma_{abc} + \Gamma_{bac} - \partial_c g_{ab} &= g_{ad} \Gamma^d_{\ bc} + g_{bd} \Gamma^d_{\ ac} - \partial_c g_{ab} \\ &= \frac{1}{2} g_{ad} g^{de} \left(\partial_b g_{ec} + \partial_c g_{be} - \partial_e g_{bc} \right) + \frac{1}{2} g_{bd} g^{de} \left(\partial_a g_{ec} + \partial_c g_{ae} - \partial_e g_{ac} \right) - \partial_c g_{ab} \\ &= \frac{1}{2} g_{ad} g^{de} \partial_b g_{ec} + \frac{1}{2} g_{ad} g^{de} \partial_c g_{be} - \frac{1}{2} g_{ad} g^{de} \partial_e g_{bc} + \frac{1}{2} g_{bd} g^{de} \partial_a g_{ec} + \frac{1}{2} g_{bd} g^{de} \partial_c g_{ae} - \frac{1}{2} g_{bd} g^{de} \partial_e g_{ac} - \partial_c g_{ab} \\ &= \frac{1}{2} g_a^{\ e} \partial_b g_{ec} + \frac{1}{2} g_a^{\ e} \partial_c g_{be} - \frac{1}{2} g_a^{\ e} \partial_e g_{bc} + \frac{1}{2} g_b^{\ e} \partial_a g_{ec} + \frac{1}{2} g_b^{\ e} \partial_c g_{ae} - \frac{1}{2} g_b^{\ e} \partial_e g_{ac} - \partial_c g_{ab} \\ &= \frac{1}{2} \partial_c g_{ba} - \frac{1}{2} \partial_c g_{ab} \\ &= 0 \end{split}$$