

Exercise 1.2 Christoffel symbol and dg

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1  {a,b,c,d,e,f,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices.
2
3  g_{a b}::Metric.
4  g_{a}^{b}::KroneckerDelta.
5
6  \partial{#}::PartialDerivative.
7
8  GammaU := \Gamma^{a}_{b c} -> (1/2) g^{a d} ( \partial_{b}{g_{d c}}
9              + \partial_{c}{g_{b d}}
10             - \partial_{d}{g_{b c}} ).
11
12  GammaD := \Gamma_{a b c} -> g_{a d} \Gamma^{d}_{b c}.
13
14  expr := \Gamma_{a b c} + \Gamma_{b a c} - \partial_{c}{g_{a b}}. # cdb (ex-0102.101,expr)
15
16  substitute      (expr, GammaD) # cdb (ex-0102.102,expr)
17  substitute      (expr, GammaU) # cdb (ex-0102.103,expr)
18  distribute      (expr) # cdb (ex-0102.104,expr)
19  eliminate_metric (expr) # cdb (ex-0102.105,expr)
20  eliminate_kronecker (expr) # cdb (ex-0102.106,expr)
21  canonicalise     (expr) # cdb (ex-0102.107,expr)

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$$\begin{aligned}
 \Gamma_{abc} + \Gamma_{bac} - \partial_c g_{ab} &= g_{ad} \Gamma^d_{bc} + g_{bd} \Gamma^d_{ac} - \partial_c g_{ab} \\
 &= \frac{1}{2} g_{ad} g^{de} (\partial_b g_{ec} + \partial_c g_{be} - \partial_e g_{bc}) + \frac{1}{2} g_{bd} g^{de} (\partial_a g_{ec} + \partial_c g_{ae} - \partial_e g_{ac}) - \partial_c g_{ab} \\
 &= \frac{1}{2} g_{ad} g^{de} \partial_b g_{ec} + \frac{1}{2} g_{ad} g^{de} \partial_c g_{be} - \frac{1}{2} g_{ad} g^{de} \partial_e g_{bc} + \frac{1}{2} g_{bd} g^{de} \partial_a g_{ec} + \frac{1}{2} g_{bd} g^{de} \partial_c g_{ae} - \frac{1}{2} g_{bd} g^{de} \partial_e g_{ac} - \partial_c g_{ab} \\
 &= \frac{1}{2} g_a^e \partial_b g_{ec} + \frac{1}{2} g_a^e \partial_c g_{be} - \frac{1}{2} g_a^e \partial_e g_{bc} + \frac{1}{2} g_b^e \partial_a g_{ec} + \frac{1}{2} g_b^e \partial_c g_{ae} - \frac{1}{2} g_b^e \partial_e g_{ac} - \partial_c g_{ab} \\
 &= \frac{1}{2} \partial_c g_{ba} - \frac{1}{2} \partial_c g_{ab} \\
 &= 0
 \end{aligned}$$