

Exercise 6.6 The Kasner cosmology

```
1 {t, x, y, z}::Coordinate.
2 {a,b,c,d,e,f,g,h#}::Indices(values={t, x, y, z}, position=independent).
3
4 \partial{#}::PartialDerivative.
5
6 p1::LaTeXForm("p_1").
7 p2::LaTeXForm("p_2").
8 p3::LaTeXForm("p_3").
9
10 g^{a b}::InverseMetric. # essential when using complete (gab, $g^{a b}$)
11
12 Gamma := \Gamma^{a}_{b c} -> 1/2 g^{a d} ( \partial_{b}{g_{d c}}
13                                     + \partial_{c}{g_{b d}}
14                                     - \partial_{d}{g_{b c}}).
15
16 Rabcd := R^{a}_{b c d} -> \partial_{c}{\Gamma^{a}_{b d}}
17                             - \partial_{d}{\Gamma^{a}_{b c}}
18                             + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
19                             - \Gamma^{e}_{b c} \Gamma^{a}_{d e}.
20
21 Rab := R_{a b} -> R^{c}_{c}_{a b}.
22
23 gab := { g_{t t} = -1,
24          g_{x x} = t**(2*p1),
25          g_{y y} = t**(2*p2),
26          g_{z z} = t**(2*p3)}. # cdb(ex-0606.101,gab)
27
28 complete (gab, $g^{a b}$) # cdb(ex-0606.102,gab)
29
30 substitute (Rabcd, Gamma)
31 substitute (Rab, Rabcd)
32
33 evaluate (Gamma, gab, rhsonly=True) # cdb(ex-0606.103,Gamma)
34 evaluate (Rabcd, gab, rhsonly=True) # cdb(ex-0606.104,Rabcd)
35 evaluate (Rab, gab, rhsonly=True) # cdb(ex-0606.105,Rab)
```

$$[g_{tt} = -1, g_{xx} = t^{2p_1}, g_{yy} = t^{2p_2}, g_{zz} = t^{2p_3}] \quad (\text{ex-0606.101})$$

$$[g_{tt} = -1, g_{xx} = t^{2p_1}, g_{yy} = t^{2p_2}, g_{zz} = t^{2p_3}, g^{tt} = -1, g^{xx} = t^{-2p_1}, g^{yy} = t^{-2p_2}, g^{zz} = t^{-2p_3}] \quad (\text{ex-0606.102})$$

$$\Gamma^a_{bc} \rightarrow \square_{cb}^a \left\{ \begin{array}{l} \square_{zt}^z = p_3 t^{-1} \\ \square_{yt}^y = p_2 t^{-1} \\ \square_{xt}^x = p_1 t^{-1} \\ \square_{tz}^z = p_3 t^{-1} \\ \square_{ty}^y = p_2 t^{-1} \\ \square_{tx}^x = p_1 t^{-1} \\ \square_{zz}^t = p_3 t^{(2p_3-1)} \\ \square_{yy}^t = p_2 t^{(2p_2-1)} \\ \square_{xx}^t = p_1 t^{(2p_1-1)} \end{array} \right. \quad (\text{ex-0606.103})$$

$$R^a{}_{bcd} \rightarrow \square_{db}{}^a{}_c \left\{ \begin{array}{l} \square_{xx}{}^t{}_t = p_1 t^{(2p_1-2)} (p_1 - 1) \\ \square_{yy}{}^t{}_t = p_2 t^{(2p_2-2)} (p_2 - 1) \\ \square_{zz}{}^t{}_t = p_3 t^{(2p_3-2)} (p_3 - 1) \\ \square_{xt}{}^x{}_t = p_1 (p_1 - 1) t^{-2} \\ \square_{yt}{}^y{}_t = p_2 (p_2 - 1) t^{-2} \\ \square_{zt}{}^z{}_t = p_3 (p_3 - 1) t^{-2} \\ \square_{tx}{}^t{}_x = -p_1 t^{(2p_1-2)} (p_1 - 1) \\ \square_{ty}{}^t{}_y = -p_2 t^{(2p_2-2)} (p_2 - 1) \\ \square_{tz}{}^t{}_z = -p_3 t^{(2p_3-2)} (p_3 - 1) \\ \square_{tt}{}^x{}_x = p_1 (1 - p_1) t^{-2} \\ \square_{tt}{}^y{}_y = p_2 (1 - p_2) t^{-2} \\ \square_{tt}{}^z{}_z = p_3 (1 - p_3) t^{-2} \\ \square_{zz}{}^y{}_y = p_2 p_3 t^{(2p_3-2)} \\ \square_{zz}{}^x{}_x = p_1 p_3 t^{(2p_3-2)} \\ \square_{yy}{}^z{}_z = p_2 p_3 t^{(2p_2-2)} \\ \square_{yy}{}^x{}_x = p_1 p_2 t^{(2p_2-2)} \\ \square_{xx}{}^z{}_z = p_1 p_3 t^{(2p_1-2)} \\ \square_{xx}{}^y{}_y = p_1 p_2 t^{(2p_1-2)} \\ \square_{yz}{}^y{}_z = -p_2 p_3 t^{(2p_3-2)} \\ \square_{xz}{}^x{}_z = -p_1 p_3 t^{(2p_3-2)} \\ \square_{zy}{}^z{}_y = -p_2 p_3 t^{(2p_2-2)} \\ \square_{xy}{}^x{}_y = -p_1 p_2 t^{(2p_2-2)} \\ \square_{zx}{}^z{}_x = -p_1 p_3 t^{(2p_1-2)} \\ \square_{yx}{}^y{}_x = -p_1 p_2 t^{(2p_1-2)} \end{array} \right. \quad (\text{ex-0606.104})$$

$$R_{ab} \rightarrow \square_{ba} \left\{ \begin{array}{l} \square_{xx} = p_1 t^{(2p_1-2)} (p_1 + p_2 + p_3 - 1) \\ \square_{yy} = p_2 t^{(2p_2-2)} (p_1 + p_2 + p_3 - 1) \\ \square_{zz} = p_3 t^{(2p_3-2)} (p_1 + p_2 + p_3 - 1) \\ \square_{tt} = (-p_1^2 + p_1 - p_2^2 + p_2 - p_3^2 + p_3) t^{-2} \end{array} \right. \quad (\text{ex-0606.105})$$