

Exercise 1.1 Verify symmetry of Γ^a_{bc}

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1 {a,b,c,d,e,f,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices.
2
3 g_{a b}::Metric.
4
5 \partial{#}::PartialDerivative.
6
7 Gamma := \Gamma^{a}_{b c} -> (1/2) g^{a d} ( \partial_{b}{g_{d c}}
8                                     + \partial_{c}{g_{b d}}
9                                     - \partial_{d}{g_{b c}} ).
10
11 diff := \Gamma^{a}_{b c} - \Gamma^{a}_{c b}.    # cdb (ex-0101.101,diff)
12
13 substitute      (diff, Gamma)                # cdb (ex-0101.102,diff)
14 distribute      (diff)                       # cdb (ex-0101.103,diff)
15 canonicalise    (diff)                       # cdb (ex-0101.104,diff)

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$$\begin{aligned}
 \Gamma^a_{bc} - \Gamma^a_{cb} &= \frac{1}{2} g^{ad} (\partial_{bg} \partial_{dc} + \partial_{bd} \partial_{gc} - \partial_{bc} \partial_{gd}) - \frac{1}{2} g^{ad} (\partial_{cb} \partial_{gd} + \partial_{cd} \partial_{gb} - \partial_{cb} \partial_{gd}) \\
 &= \frac{1}{2} g^{ad} \partial_{bg} \partial_{dc} + \frac{1}{2} g^{ad} \partial_{bd} \partial_{gc} - \frac{1}{2} g^{ad} \partial_{bc} \partial_{gd} - \frac{1}{2} g^{ad} \partial_{cb} \partial_{gd} - \frac{1}{2} g^{ad} \partial_{cd} \partial_{gb} + \frac{1}{2} g^{ad} \partial_{cb} \partial_{gd} \\
 &= 0
 \end{aligned}$$