

Exercise 5.5 Commuting covariant derivatives

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1 {a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices(position=independent).
2
3 ::Symbol.
4
5 def add_tags (obj,tag):
6     n = 0
7     ans = Ex('0')
8     for i in obj.top().terms():
9         foo = obj[i]
10        bah = Ex(tag+'_'+str(n)+'')
11        ans := @ (ans) + @ (bah) @ (foo).
12        n = n + 1
13    return ans
14
15 def clear_tags (obj,tag):
16     ans := @ (obj).
17     foo = Ex(tag+'_{a?} -> 1')
18     substitute (ans,foo)
19     return ans
20
21 rule := V^{a}_{; b ; c} -> V^{a}_{; c ; b} - R^{a}_{d b c} V^{d}.
22
23 expr := V^{a}_{; b ; c} - V^{a}_{; c ; b}.      # cdb (ex-0505.100,expr)
24
25 expr = add_tags (expr,'\mu')                  # cdb (ex-0505.101,expr)
26
27 zoom      (expr, $\mu_{0} Q??$)                # cdb (ex-0505.102,expr)
28 substitute (expr, rule)                        # cdb (ex-0505.103,expr)
29 unzoom    (expr)                              # cdb (ex-0505.104,expr)
30
31 expr = clear_tags (expr,'\mu')                 # cdb (ex-0505.105,expr)
```

$$\begin{aligned}
V^a{}_{;b;c} - V^a{}_{;c;b} &= \mu_0 V^a{}_{;b;c} - \mu_1 V^a{}_{;c;b} && (\text{ex-0505.101}) \\
&= \mu_0 V^a{}_{;b;c} - \mu_1 V^a{}_{;c;b} && (\text{ex-0505.101}) \\
&= \mu_0 V^a{}_{;b;c} + \dots && (\text{ex-0505.102}) \\
&= \mu_0 (V^a{}_{;c;b} - R^a{}_{dbc} V^d) + \dots && (\text{ex-0505.103}) \\
&= \mu_0 (V^a{}_{;c;b} - R^a{}_{dbc} V^d) - \mu_1 V^a{}_{;c;b} && (\text{ex-0505.104}) \\
&= -R^a{}_{dbc} V^d && (\text{ex-0505.105})
\end{aligned}$$