Exercise 3.7 Commutation of ∇ on the Riemann tensor – direct computation

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{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w#}::Indices(position=independent).

;::Symbol;

\partial{#}::PartialDerivative.

\Gamma^{a}_{b c}::TableauSymmetry(shape={2}, indices={1,2}).

RabcdD :=   \partial_{c}{\Gamma_{a b d}}
          - \partial_{d}{\Gamma_{a b c}}
          + \Gamma_{e a d} \Gamma^{e}_{b c}
          - \Gamma_{e a c} \Gamma^{e}_{b d} -> R_{a b c d}.            # cdb(Rabcd.010,RabcdD)

RabcdU :=   \partial_{c}{\Gamma^{a}_{b d}}
          - \partial_{d}{\Gamma^{a}_{b c}}
          + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
          - \Gamma^{e}_{b c} \Gamma^{a}_{d e} -> R^{a}_{b c d}.        # cdb(Rabcd.000,RabcdU)

d1Rabcd := R_{a b c d ; e} ->   \partial_{e}{R_{a b c d}}
                             - \Gamma^{f}_{a e} R_{f b c d}
                             - \Gamma^{f}_{b e} R_{a f c d}
                             - \Gamma^{f}_{c e} R_{a b f d}
                             - \Gamma^{f}_{d e} R_{a b c f}.          # cdb(d1Rabcd.000,d1Rabcd)

d2Rabcd := R_{a b c d ; e ; f} ->   \partial_{f}{R_{a b c d ; e}}
                                  - \Gamma^{g}_{a f} R_{g b c d ; e}
                                  - \Gamma^{g}_{b f} R_{a g c d ; e}
                                  - \Gamma^{g}_{c f} R_{a b g d ; e}
                                  - \Gamma^{g}_{d f} R_{a b c g ; e}
                                  - \Gamma^{g}_{e f} R_{a b c d ; g}.  # cdb(d2Rabcd.000,d2Rabcd)

substitute (d2Rabcd,d1Rabcd)                                          # cdb (d2Rabcd.001, d2Rabcd)

expr := R_{a b c d ; e ; f} - R_{a b c d ; f ; e}.                    # cdb (ex-0307.100, expr)

substitute     (expr,d2Rabcd)                                         # cdb (ex-0307.101, expr)
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37
38 distribute     (expr)                                          # cdb (ex-0307.102, expr)
39 product_rule   (expr)                                          # cdb (ex-0307.103, expr)
40
41 sort_product   (expr)                                          # cdb (ex-0307.104, expr)
42 rename_dummies (expr)                                          # cdb (ex-0307.105, expr)
43 canonicalise   (expr)                                          # cdb (ex-0307.106, expr)
44 factor_out     (expr,$R_{a? b? c? d?}$)                        # cdb (ex-0307.107, expr)
45
46 substitute     (expr,RabcdU)                                   # cdb (ex-0307.108, expr)
47 substitute     (expr,$R^{a}_{b c d} -> -R^{a}_{b d c}$)        # cdb (ex-0307.109, expr)
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\begin{aligned}
R_{abcd;e;f} - R_{abcd;f;e} = {} & \partial_f(\partial_e R_{abcd} - \Gamma^g_{\ ae} R_{gbcd} - \Gamma^g_{\ be} R_{agcd} - \Gamma^g_{\ ce} R_{abgd} - \Gamma^g_{\ de} R_{abcg}) - \Gamma^g_{\ af}\left(\partial_e R_{gbcd} - \Gamma^h_{\ ge} R_{hbcd} - \Gamma^h_{\ be} R_{ghcd} - \Gamma^h_{\ ce} R_{gbhd} - \Gamma^h_{\ de} R_{gbch}\right) \\
& - \Gamma^g_{\ bf}\left(\partial_e R_{agcd} - \Gamma^h_{\ ae} R_{hgcd} - \Gamma^h_{\ ge} R_{ahcd} - \Gamma^h_{\ ce} R_{aghd} - \Gamma^h_{\ de} R_{agch}\right) \\
& - \Gamma^g_{\ cf}\left(\partial_e R_{abgd} - \Gamma^h_{\ ae} R_{hbgd} - \Gamma^h_{\ be} R_{ahgd} - \Gamma^h_{\ ge} R_{abhd} - \Gamma^h_{\ de} R_{abgh}\right) \\
& - \Gamma^g_{\ df}\left(\partial_e R_{abcg} - \Gamma^h_{\ ae} R_{hbcg} - \Gamma^h_{\ be} R_{ahcg} - \Gamma^h_{\ ce} R_{abhg} - \Gamma^h_{\ ge} R_{abch}\right) \\
& - \Gamma^g_{\ ef}\left(\partial_g R_{abcd} - \Gamma^h_{\ ag} R_{hbcd} - \Gamma^h_{\ bg} R_{ahcd} - \Gamma^h_{\ cg} R_{abhd} - \Gamma^h_{\ dg} R_{abch}\right) \\
& - \partial_e(\partial_f R_{abcd} - \Gamma^g_{\ af} R_{gbcd} - \Gamma^g_{\ bf} R_{agcd} - \Gamma^g_{\ cf} R_{abgd} - \Gamma^g_{\ df} R_{abcg}) + \Gamma^g_{\ ae}\left(\partial_f R_{gbcd} - \Gamma^h_{\ gf} R_{hbcd} - \Gamma^h_{\ bf} R_{ghcd} - \Gamma^h_{\ cf} R_{gbhd} - \Gamma^h_{\ df} R_{gbch}\right) \\
& + \Gamma^g_{\ be}\left(\partial_f R_{agcd} - \Gamma^h_{\ af} R_{hgcd} - \Gamma^h_{\ gf} R_{ahcd} - \Gamma^h_{\ cf} R_{aghd} - \Gamma^h_{\ df} R_{agch}\right) \\
& + \Gamma^g_{\ ce}\left(\partial_f R_{abgd} - \Gamma^h_{\ af} R_{hbgd} - \Gamma^h_{\ bf} R_{ahgd} - \Gamma^h_{\ gf} R_{abhd} - \Gamma^h_{\ df} R_{abgh}\right) \\
& + \Gamma^g_{\ de}\left(\partial_f R_{abcg} - \Gamma^h_{\ af} R_{hbcg} - \Gamma^h_{\ bf} R_{ahcg} - \Gamma^h_{\ cf} R_{abhg} - \Gamma^h_{\ gf} R_{abch}\right) \\
& + \Gamma^g_{\ fe}\left(\partial_g R_{abcd} - \Gamma^h_{\ ag} R_{hbcd} - \Gamma^h_{\ bg} R_{ahcd} - \Gamma^h_{\ cg} R_{abhd} - \Gamma^h_{\ dg} R_{abch}\right) \qquad \texttt{(ex-0307.101)}
\end{aligned}
$$

$$
\begin{aligned}
R_{abcd;e;f} - R_{abcd;f;e} = {} & \partial_{fe} R_{abcd} - \partial_f(\Gamma^g_{\ ae} R_{gbcd}) - \partial_f(\Gamma^g_{\ be} R_{agcd}) - \partial_f(\Gamma^g_{\ ce} R_{abgd}) - \partial_f(\Gamma^g_{\ de} R_{abcg}) - \Gamma^g_{\ af}\partial_e R_{gbcd} + \Gamma^g_{\ af}\Gamma^h_{\ ge} R_{hbcd} + \Gamma^g_{\ af}\Gamma^h_{\ be} R_{ghcd} \\
& + \Gamma^g_{\ af}\Gamma^h_{\ ce} R_{gbhd} + \Gamma^g_{\ af}\Gamma^h_{\ de} R_{gbch} - \Gamma^g_{\ bf}\partial_e R_{agcd} + \Gamma^g_{\ bf}\Gamma^h_{\ ae} R_{hgcd} + \Gamma^g_{\ bf}\Gamma^h_{\ ge} R_{ahcd} + \Gamma^g_{\ bf}\Gamma^h_{\ ce} R_{aghd} + \Gamma^g_{\ bf}\Gamma^h_{\ de} R_{agch} - \Gamma^g_{\ cf}\partial_e R_{abgd} \\
& + \Gamma^g_{\ cf}\Gamma^h_{\ ae} R_{hbgd} + \Gamma^g_{\ cf}\Gamma^h_{\ be} R_{ahgd} + \Gamma^g_{\ cf}\Gamma^h_{\ ge} R_{abhd} + \Gamma^g_{\ cf}\Gamma^h_{\ de} R_{abgh} - \Gamma^g_{\ df}\partial_e R_{abcg} + \Gamma^g_{\ df}\Gamma^h_{\ ae} R_{hbcg} + \Gamma^g_{\ df}\Gamma^h_{\ be} R_{ahcg} + \Gamma^g_{\ df}\Gamma^h_{\ ce} R_{abhg} \\
& + \Gamma^g_{\ df}\Gamma^h_{\ ge} R_{abch} - \Gamma^g_{\ ef}\partial_g R_{abcd} + \Gamma^g_{\ ef}\Gamma^h_{\ ag} R_{hbcd} + \Gamma^g_{\ ef}\Gamma^h_{\ bg} R_{ahcd} + \Gamma^g_{\ ef}\Gamma^h_{\ cg} R_{abhd} + \Gamma^g_{\ ef}\Gamma^h_{\ dg} R_{abch} - \partial_{ef} R_{abcd} + \partial_e(\Gamma^g_{\ af} R_{gbcd}) \\
& + \partial_e(\Gamma^g_{\ bf} R_{agcd}) + \partial_e(\Gamma^g_{\ cf} R_{abgd}) + \partial_e(\Gamma^g_{\ df} R_{abcg}) + \Gamma^g_{\ ae}\partial_f R_{gbcd} - \Gamma^g_{\ ae}\Gamma^h_{\ gf} R_{hbcd} - \Gamma^g_{\ ae}\Gamma^h_{\ bf} R_{ghcd} - \Gamma^g_{\ ae}\Gamma^h_{\ cf} R_{gbhd} - \Gamma^g_{\ ae}\Gamma^h_{\ df} R_{gbch} \\
& + \Gamma^g_{\ be}\partial_f R_{agcd} - \Gamma^g_{\ be}\Gamma^h_{\ af} R_{hgcd} - \Gamma^g_{\ be}\Gamma^h_{\ gf} R_{ahcd} - \Gamma^g_{\ be}\Gamma^h_{\ cf} R_{aghd} - \Gamma^g_{\ be}\Gamma^h_{\ df} R_{agch} + \Gamma^g_{\ ce}\partial_f R_{abgd} - \Gamma^g_{\ ce}\Gamma^h_{\ af} R_{hbgd} - \Gamma^g_{\ ce}\Gamma^h_{\ bf} R_{ahgd} \\
& - \Gamma^g_{\ ce}\Gamma^h_{\ gf} R_{abhd} - \Gamma^g_{\ ce}\Gamma^h_{\ df} R_{abgh} + \Gamma^g_{\ de}\partial_f R_{abcg} - \Gamma^g_{\ de}\Gamma^h_{\ af} R_{hbcg} - \Gamma^g_{\ de}\Gamma^h_{\ bf} R_{ahcg} - \Gamma^g_{\ de}\Gamma^h_{\ cf} R_{abhg} - \Gamma^g_{\ de}\Gamma^h_{\ gf} R_{abch} + \Gamma^g_{\ fe}\partial_g R_{abcd} \\
& - \Gamma^g_{\ fe}\Gamma^h_{\ ag} R_{hbcd} - \Gamma^g_{\ fe}\Gamma^h_{\ bg} R_{ahcd} - \Gamma^g_{\ fe}\Gamma^h_{\ cg} R_{abhd} - \Gamma^g_{\ fe}\Gamma^h_{\ dg} R_{abch} \qquad \texttt{(ex-0307.102)}
\end{aligned}
$$

$$
\begin{aligned}
R_{abcd;e;f} - R_{abcd;f;e} = {} & \partial_{fe} R_{abcd} - \partial_f\Gamma^g_{\ ae} R_{gbcd} - \partial_f\Gamma^g_{\ be} R_{agcd} - \partial_f\Gamma^g_{\ ce} R_{abgd} - \partial_f\Gamma^g_{\ de} R_{abcg} + \Gamma^g_{\ af}\Gamma^h_{\ ge} R_{hbcd} + \Gamma^g_{\ af}\Gamma^h_{\ be} R_{ghcd} + \Gamma^g_{\ af}\Gamma^h_{\ ce} R_{gbhd} + \Gamma^g_{\ af}\Gamma^h_{\ de} R_{gbch} \\
& + \Gamma^g_{\ bf}\Gamma^h_{\ ae} R_{hgcd} + \Gamma^g_{\ bf}\Gamma^h_{\ ge} R_{ahcd} + \Gamma^g_{\ bf}\Gamma^h_{\ ce} R_{aghd} + \Gamma^g_{\ bf}\Gamma^h_{\ de} R_{agch} + \Gamma^g_{\ cf}\Gamma^h_{\ ae} R_{hbgd} + \Gamma^g_{\ cf}\Gamma^h_{\ be} R_{ahgd} + \Gamma^g_{\ cf}\Gamma^h_{\ ge} R_{abhd} + \Gamma^g_{\ cf}\Gamma^h_{\ de} R_{abgh} \\
& + \Gamma^g_{\ df}\Gamma^h_{\ ae} R_{hbcg} + \Gamma^g_{\ df}\Gamma^h_{\ be} R_{ahcg} + \Gamma^g_{\ df}\Gamma^h_{\ ce} R_{abhg} + \Gamma^g_{\ df}\Gamma^h_{\ ge} R_{abch} - \Gamma^g_{\ ef}\partial_g R_{abcd} + \Gamma^g_{\ ef}\Gamma^h_{\ ag} R_{hbcd} + \Gamma^g_{\ ef}\Gamma^h_{\ bg} R_{ahcd} + \Gamma^g_{\ ef}\Gamma^h_{\ cg} R_{abhd} \\
& + \Gamma^g_{\ ef}\Gamma^h_{\ dg} R_{abch} - \partial_{ef} R_{abcd} + \partial_e\Gamma^g_{\ af} R_{gbcd} + \partial_e\Gamma^g_{\ bf} R_{agcd} + \partial_e\Gamma^g_{\ cf} R_{abgd} + \partial_e\Gamma^g_{\ df} R_{abcg} - \Gamma^g_{\ ae}\Gamma^h_{\ gf} R_{hbcd} - \Gamma^g_{\ ae}\Gamma^h_{\ bf} R_{ghcd} \\
& - \Gamma^g_{\ ae}\Gamma^h_{\ cf} R_{gbhd} - \Gamma^g_{\ ae}\Gamma^h_{\ df} R_{gbch} - \Gamma^g_{\ be}\Gamma^h_{\ af} R_{hgcd} - \Gamma^g_{\ be}\Gamma^h_{\ gf} R_{ahcd} - \Gamma^g_{\ be}\Gamma^h_{\ cf} R_{aghd} - \Gamma^g_{\ be}\Gamma^h_{\ df} R_{agch} - \Gamma^g_{\ ce}\Gamma^h_{\ af} R_{hbgd} \\
& - \Gamma^g_{\ ce}\Gamma^h_{\ bf} R_{ahgd} - \Gamma^g_{\ ce}\Gamma^h_{\ gf} R_{abhd} - \Gamma^g_{\ ce}\Gamma^h_{\ df} R_{abgh} - \Gamma^g_{\ de}\Gamma^h_{\ af} R_{hbcg} - \Gamma^g_{\ de}\Gamma^h_{\ bf} R_{ahcg} - \Gamma^g_{\ de}\Gamma^h_{\ cf} R_{abhg} - \Gamma^g_{\ de}\Gamma^h_{\ gf} R_{abch} \\
& + \Gamma^g_{\ fe}\partial_g R_{abcd} - \Gamma^g_{\ fe}\Gamma^h_{\ ag} R_{hbcd} - \Gamma^g_{\ fe}\Gamma^h_{\ bg} R_{ahcd} - \Gamma^g_{\ fe}\Gamma^h_{\ cg} R_{abhd} - \Gamma^g_{\ fe}\Gamma^h_{\ dg} R_{abch} \qquad \texttt{(ex-0307.103)}
\end{aligned}
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$$R_{abcd;e;f} - R_{abcd;f;e} = \partial_{fe} R_{abcd} - R_{gbcd}\partial_f \Gamma^g{}_{ae} - R_{agcd}\partial_f \Gamma^g{}_{be} - R_{abgd}\partial_f \Gamma^g{}_{ce} - R_{abcg}\partial_f \Gamma^g{}_{de} + R_{hbcd}\Gamma^g{}_{af}\Gamma^h{}_{ge} + R_{ghcd}\Gamma^g{}_{af}\Gamma^h{}_{be} + R_{gbhd}\Gamma^g{}_{af}\Gamma^h{}_{ce} + R_{gbch}\Gamma^g{}_{af}\Gamma^h{}_{de}$$
$$+ R_{hgcd}\Gamma^g{}_{bf}\Gamma^h{}_{ae} + R_{ahcd}\Gamma^g{}_{bf}\Gamma^h{}_{ge} + R_{aghd}\Gamma^g{}_{bf}\Gamma^h{}_{ce} + R_{agch}\Gamma^g{}_{bf}\Gamma^h{}_{de} + R_{hbgd}\Gamma^g{}_{cf}\Gamma^h{}_{ae} + R_{ahgd}\Gamma^g{}_{cf}\Gamma^h{}_{be} + R_{abhd}\Gamma^g{}_{cf}\Gamma^h{}_{ge} + R_{abgh}\Gamma^g{}_{cf}\Gamma^h{}_{de}$$
$$+ R_{hbcg}\Gamma^g{}_{df}\Gamma^h{}_{ae} + R_{ahcg}\Gamma^g{}_{df}\Gamma^h{}_{be} + R_{abhg}\Gamma^g{}_{df}\Gamma^h{}_{ce} + R_{abch}\Gamma^g{}_{df}\Gamma^h{}_{ge} - \Gamma^g{}_{ef}\partial_g R_{abcd} + R_{hbcd}\Gamma^g{}_{ef}\Gamma^h{}_{ag} + R_{ahcd}\Gamma^g{}_{ef}\Gamma^h{}_{bg} + R_{abhd}\Gamma^g{}_{ef}\Gamma^h{}_{cg}$$
$$+ R_{abch}\Gamma^g{}_{ef}\Gamma^h{}_{dg} - \partial_{ef} R_{abcd} + R_{gbcd}\partial_e \Gamma^g{}_{af} + R_{agcd}\partial_e \Gamma^g{}_{bf} + R_{abgd}\partial_e \Gamma^g{}_{cf} + R_{abcg}\partial_e \Gamma^g{}_{df} - R_{hbcd}\Gamma^g{}_{ae}\Gamma^h{}_{gf} - R_{ghcd}\Gamma^g{}_{ae}\Gamma^h{}_{bf}$$
$$- R_{gbhd}\Gamma^g{}_{ae}\Gamma^h{}_{cf} - R_{gbch}\Gamma^g{}_{ae}\Gamma^h{}_{df} - R_{hgcd}\Gamma^g{}_{be}\Gamma^h{}_{af} - R_{ahcd}\Gamma^g{}_{be}\Gamma^h{}_{gf} - R_{aghd}\Gamma^g{}_{be}\Gamma^h{}_{cf} - R_{agch}\Gamma^g{}_{be}\Gamma^h{}_{df} - R_{hbgd}\Gamma^g{}_{ce}\Gamma^h{}_{af}$$
$$- R_{ahgd}\Gamma^g{}_{ce}\Gamma^h{}_{bf} - R_{abhd}\Gamma^g{}_{ce}\Gamma^h{}_{gf} - R_{abgh}\Gamma^g{}_{ce}\Gamma^h{}_{df} - R_{hbcg}\Gamma^g{}_{de}\Gamma^h{}_{af} - R_{ahcg}\Gamma^g{}_{de}\Gamma^h{}_{bf} - R_{abhg}\Gamma^g{}_{de}\Gamma^h{}_{cf} - R_{abch}\Gamma^g{}_{de}\Gamma^h{}_{gf}$$
$$+ \Gamma^g{}_{fe}\partial_g R_{abcd} - R_{hbcd}\Gamma^g{}_{fe}\Gamma^h{}_{ag} - R_{ahcd}\Gamma^g{}_{fe}\Gamma^h{}_{bg} - R_{abhd}\Gamma^g{}_{fe}\Gamma^h{}_{cg} - R_{abch}\Gamma^g{}_{fe}\Gamma^h{}_{dg} \tag{ex-0307.104}$$

$$R_{abcd;e;f} - R_{abcd;f;e} = \partial_{fe} R_{abcd} - R_{gbcd}\partial_f \Gamma^g{}_{ae} - R_{agcd}\partial_f \Gamma^g{}_{be} - R_{abgd}\partial_f \Gamma^g{}_{ce} - R_{abcg}\partial_f \Gamma^g{}_{de} + R_{gbcd}\Gamma^h{}_{af}\Gamma^g{}_{he} + R_{ghcd}\Gamma^g{}_{af}\Gamma^h{}_{be} + R_{gbhd}\Gamma^g{}_{af}\Gamma^h{}_{ce} + R_{gbch}\Gamma^g{}_{af}\Gamma^h{}_{de}$$
$$+ R_{ghcd}\Gamma^h{}_{bf}\Gamma^g{}_{ae} + R_{agcd}\Gamma^h{}_{bf}\Gamma^g{}_{he} + R_{aghd}\Gamma^g{}_{bf}\Gamma^h{}_{ce} + R_{agch}\Gamma^g{}_{bf}\Gamma^h{}_{de} + R_{gbhd}\Gamma^h{}_{cf}\Gamma^g{}_{ae} + R_{aghd}\Gamma^h{}_{cf}\Gamma^g{}_{be} + R_{abgd}\Gamma^h{}_{cf}\Gamma^g{}_{he} + R_{abgh}\Gamma^g{}_{cf}\Gamma^h{}_{de}$$
$$+ R_{gbch}\Gamma^h{}_{df}\Gamma^g{}_{ae} + R_{agch}\Gamma^h{}_{df}\Gamma^g{}_{be} + R_{abgh}\Gamma^h{}_{df}\Gamma^g{}_{ce} + R_{abcg}\Gamma^h{}_{df}\Gamma^g{}_{he} - \Gamma^g{}_{ef}\partial_g R_{abcd} + R_{gbcd}\Gamma^h{}_{ef}\Gamma^g{}_{ah} + R_{agcd}\Gamma^h{}_{ef}\Gamma^g{}_{bh} + R_{abgd}\Gamma^h{}_{ef}\Gamma^g{}_{ch}$$
$$+ R_{abcg}\Gamma^h{}_{ef}\Gamma^g{}_{dh} - \partial_{ef} R_{abcd} + R_{gbcd}\partial_e \Gamma^g{}_{af} + R_{agcd}\partial_e \Gamma^g{}_{bf} + R_{abgd}\partial_e \Gamma^g{}_{cf} + R_{abcg}\partial_e \Gamma^g{}_{df} - R_{gbcd}\Gamma^h{}_{ae}\Gamma^g{}_{hf} - R_{ghcd}\Gamma^g{}_{ae}\Gamma^h{}_{bf}$$
$$- R_{gbhd}\Gamma^g{}_{ae}\Gamma^h{}_{cf} - R_{gbch}\Gamma^g{}_{ae}\Gamma^h{}_{df} - R_{ghcd}\Gamma^h{}_{be}\Gamma^g{}_{af} - R_{agcd}\Gamma^h{}_{be}\Gamma^g{}_{hf} - R_{aghd}\Gamma^g{}_{be}\Gamma^h{}_{cf} - R_{agch}\Gamma^g{}_{be}\Gamma^h{}_{df} - R_{gbhd}\Gamma^h{}_{ce}\Gamma^g{}_{af}$$
$$- R_{aghd}\Gamma^h{}_{ce}\Gamma^g{}_{bf} - R_{abgd}\Gamma^h{}_{ce}\Gamma^g{}_{hf} - R_{abgh}\Gamma^g{}_{ce}\Gamma^h{}_{df} - R_{gbch}\Gamma^h{}_{de}\Gamma^g{}_{af} - R_{agch}\Gamma^h{}_{de}\Gamma^g{}_{bf} - R_{abgh}\Gamma^h{}_{de}\Gamma^g{}_{cf} - R_{abcg}\Gamma^h{}_{de}\Gamma^g{}_{hf}$$
$$+ \Gamma^g{}_{fe}\partial_g R_{abcd} - R_{gbcd}\Gamma^h{}_{fe}\Gamma^g{}_{ah} - R_{agcd}\Gamma^h{}_{fe}\Gamma^g{}_{bh} - R_{abgd}\Gamma^h{}_{fe}\Gamma^g{}_{ch} - R_{abcg}\Gamma^h{}_{fe}\Gamma^g{}_{dh} \tag{ex-0307.105}$$

$$R_{abcd;e;f} - R_{abcd;f;e} = - R_{gbcd}\partial_f \Gamma^g{}_{ae} - R_{agcd}\partial_f \Gamma^g{}_{be} - R_{abgd}\partial_f \Gamma^g{}_{ce} - R_{abcg}\partial_f \Gamma^g{}_{de} + R_{gbcd}\Gamma^h{}_{af}\Gamma^g{}_{eh} + R_{agcd}\Gamma^h{}_{bf}\Gamma^g{}_{eh} + R_{abgd}\Gamma^h{}_{cf}\Gamma^g{}_{eh} + R_{abcg}\Gamma^h{}_{df}\Gamma^g{}_{eh}$$
$$+ R_{gbcd}\partial_e \Gamma^g{}_{af} + R_{agcd}\partial_e \Gamma^g{}_{bf} + R_{abgd}\partial_e \Gamma^g{}_{cf} + R_{abcg}\partial_e \Gamma^g{}_{df} - R_{gbcd}\Gamma^h{}_{ae}\Gamma^g{}_{fh} - R_{agcd}\Gamma^h{}_{be}\Gamma^g{}_{fh} - R_{abgd}\Gamma^h{}_{ce}\Gamma^g{}_{fh}$$
$$- R_{abcg}\Gamma^h{}_{de}\Gamma^g{}_{fh} \tag{ex-0307.106}$$

$$R_{abcd;e;f} - R_{abcd;f;e} = R_{gbcd}\left(-\partial_f \Gamma^g{}_{ae} + \Gamma^h{}_{af}\Gamma^g{}_{eh} + \partial_e \Gamma^g{}_{af} - \Gamma^h{}_{ae}\Gamma^g{}_{fh}\right) + R_{agcd}\left(-\partial_f \Gamma^g{}_{be} + \Gamma^h{}_{bf}\Gamma^g{}_{eh} + \partial_e \Gamma^g{}_{bf} - \Gamma^h{}_{be}\Gamma^g{}_{fh}\right)$$
$$+ R_{abgd}\left(-\partial_f \Gamma^g{}_{ce} + \Gamma^h{}_{cf}\Gamma^g{}_{eh} + \partial_e \Gamma^g{}_{cf} - \Gamma^h{}_{ce}\Gamma^g{}_{fh}\right) + R_{abcg}\left(-\partial_f \Gamma^g{}_{de} + \Gamma^h{}_{df}\Gamma^g{}_{eh} + \partial_e \Gamma^g{}_{df} - \Gamma^h{}_{de}\Gamma^g{}_{fh}\right) \tag{ex-0307.107}$$

$$R_{abcd;e;f} - R_{abcd;f;e} = - R_{gbcd}R^g{}_{afe} - R_{agcd}R^g{}_{bfe} - R_{abgd}R^g{}_{cfe} - R_{abcg}R^g{}_{dfe} \tag{ex-0307.108}$$

$$R_{abcd;e;f} - R_{abcd;f;e} = R_{gbcd}R^g{}_{aef} + R_{agcd}R^g{}_{bef} + R_{abgd}R^g{}_{cef} + R_{abcg}R^g{}_{def} \tag{ex-0307.109}$$