```
\{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w\#\}::Indices(position=independent).
\nabla{#}::Derivative.
\partial{#}::PartialDerivative.
g_{a b}::Metric.
g^{a b}::InverseMetric.
g_{a}^{b}::KroneckerDelta.
g^{a}_{b}::KroneckerDelta.
R_{a b c d}::RiemannTensor.
R^{a}_{b c d}::RiemannTensor.
\Gamma^{a}_{b c}::TableauSymmetry(shape={2}, indices={1,2}).
\Gamma_{a b c}::TableauSymmetry(shape={2}, indices={1,2}).
g_{a b}::Depends(\partial{#}).
g^{a b}::Depends(\partial{#}).
\Gamma^{a}_{b c}::Depends(\partial{#}).
\Gamma^{a b c}::Depends(\partial{#}).
dgab := \frac{c}{g_{a b}} \rightarrow \frac{d}_{a c} g_{d b}
                                 + \Gamma^{d}_{b c} g_{a d}.
                                                                         # cdb(dgab.000,dgab)
diab := \operatorname{[c]{g^{a} b}} -> - \operatorname{[d c] g^{d} b}
                                 - \Gamma^{b}_{d c} g^{a d}.
                                                                         # cdb(diab.000,diab)
GammaU := Gamma^{a}_{b c} \rightarrow 1/2 g^{a d} ( partial_{b}_{g_{d c}})
                                              + \partial_{c}{g_{b d}}
                                              - \partial_{d}{g_{b c}}). # cdb(Gamma.000, GammaU)
GammaD := \Gamma_{a b c} -> 1/2 ( \partial_{b}_{g_{a c}})
                                   + \partial_{c}{g_{b a}}
                                   - \partial_{a}{g_{b c}}).
                                                                         # cdb(Gamma.010,GammaD)
RabcdU := R^{a}_{b c d} \rightarrow partial_{c}{Gamma^{a}_{b d}}
                           - \partial_{d}{\Gamma^{a}_{b c}}
                           + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
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- \Gamma^{e}_{b c} \Gamma^{a}_{d e}.
                                                                          # cdb(Rabcd.000,RabcdU)
RabcdD := R_{a b c d} \rightarrow partial_{c}{Gamma_{a b d}}
                          - \partial_{d}{\Gamma_{a b c}}
                         + \Gamma_{e a d} \Gamma^{e}_{b c}
                          - \Gamma_{e a c} \Gamma^{e}_{b d}.
                                                                          # cdb(Rabcd.010,RabcdD)
Rab := R_{ab} \rightarrow R^{c}_{ac}
                                                                          # cdb(Rab.000, Rab)
Rscalar := R \rightarrow g^{a} b R_{a}.
                                                                          # cdb(Rscalar.000,Rscalar)
# Weyl in 4 dimensions
Cabcd := C_{a b c d} - R_{a b c d} - (1/2) (R_{a c} g_{b d} - R_{a d} g_{b c})
                                     - (1/2) (g_{a c} R_{b d} - g_{a d} R_{b c})
                                     + (R/6) (g_{a c} g_{b d} - g_{a d} g_{b c}).
                                                                                        # cdb(Cabcd.000,Cabcd)
expr := \frac{c}{g_{a}}  # cdb(libdg.dgab.000, expr)
substitute (expr,dgab)
                                  # cdb(libdg.dgab.001,expr)
expr := \displaystyle \left\{ c\right\} \left\{ g^{a} b\right\}.
                                  # cdb(libdg.diab.000,expr)
substitute (expr,diab)
                                  # cdb(libdg.diab.001,expr)
expr := \Gamma_{a} \ c.
                                  # cdb(libdg.Gamma.000,expr)
substitute (expr,GammaU)
                                  # cdb(libdg.Gamma.001,expr)
expr := \Gamma_a b c.
                                  # cdb(libdg.Gamma.010,expr)
substitute (expr,GammaD)
                                  # cdb(libdg.Gamma.011,expr)
expr := R^{a}_{b c d}.
                                  # cdb(libdg.Rabcd.000,expr)
substitute (expr,RabcdU)
                                  # cdb(libdg.Rabcd.001,expr)
expr := R_{a} b c d.
                                  # cdb(libdg.Rabcd.010,expr)
substitute (expr,RabcdD)
                                  # cdb(libdg.Rabcd.011,expr)
expr := R_{a b}.
                                  # cdb(libdg.Rab.000,expr)
substitute (expr,Rab)
                                  # cdb(libdg.Rab.001,expr)
                                  # cdb(libdg.Rscalar.000,expr)
expr := R.
```

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substitute (expr,Rscalar) # cdb(libdg.Rscalar.001,expr)

expr := C_{a b c d}. # cdb(libdg.Cabcd.000,expr)

substitute (expr,Cabcd) # cdb(libdg.Cabcd.001,expr)
```

$$\partial_c g_{ab} \to \Gamma^d_{ac} g_{db} + \Gamma^d_{bc} g_{ad} \tag{1}$$

$$\partial_c g^{ab} \to -\Gamma^a{}_{dc} g^{db} - \Gamma^b{}_{dc} g^{ad} \tag{2}$$

$$\Gamma^{a}{}_{bc} \to \frac{1}{2} g^{ad} \left(\partial_{b} g_{dc} + \partial_{c} g_{bd} - \partial_{d} g_{bc} \right) \tag{3}$$

$$\Gamma_{abc} \to \frac{1}{2} \partial_b g_{ac} + \frac{1}{2} \partial_c g_{ba} - \frac{1}{2} \partial_a g_{bc}$$
(4)

$$R^{a}_{bcd} \to \partial_{c} \Gamma^{a}_{bd} - \partial_{d} \Gamma^{a}_{bc} + \Gamma^{e}_{bd} \Gamma^{a}_{ce} - \Gamma^{e}_{bc} \Gamma^{a}_{de}$$

$$\tag{5}$$

$$R_{abcd} \to \partial_c \Gamma_{abd} - \partial_d \Gamma_{abc} + \Gamma_{ead} \Gamma^e_{bc} - \Gamma_{eac} \Gamma^e_{bd} \tag{6}$$

$$R_{ab} \to R^c_{acb}$$
 (7)

$$R \to g^{ab} R_{ab} \tag{8}$$

$$C_{abcd} \to R_{abcd} - \frac{1}{2}R_{ac}g_{bd} + \frac{1}{2}R_{ad}g_{bc} - \frac{1}{2}g_{ac}R_{bd} + \frac{1}{2}g_{ad}R_{bc} + \frac{1}{6}R\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right) \tag{9}$$

$$\partial_c g_{ab} = \Gamma^d_{ac} g_{db} + \Gamma^d_{bc} g_{ad} \tag{10}$$

$$\partial_c g^{ab} = -\Gamma^a{}_{dc} g^{db} - \Gamma^b{}_{dc} g^{ad} \tag{11}$$

$$\Gamma^{a}{}_{bc} = \frac{1}{2}g^{ad}\left(\partial_{b}g_{dc} + \partial_{c}g_{bd} - \partial_{d}g_{bc}\right) \tag{12}$$

$$\Gamma_{abc} = \frac{1}{2}\partial_b g_{ac} + \frac{1}{2}\partial_c g_{ba} - \frac{1}{2}\partial_a g_{bc} \tag{13}$$

$$R^{a}_{bcd} = \partial_{c}\Gamma^{a}_{bd} - \partial_{d}\Gamma^{a}_{bc} + \Gamma^{e}_{bd}\Gamma^{a}_{ce} - \Gamma^{e}_{bc}\Gamma^{a}_{de}$$

$$\tag{14}$$

$$R_{abcd} = \partial_c \Gamma_{abd} - \partial_d \Gamma_{abc} + \Gamma_{ead} \Gamma^e_{bc} - \Gamma_{eac} \Gamma^e_{bd}$$
(15)

$$R_{ab} = R^c{}_{acb} \tag{16}$$

$$R = g^{ab}R_{ab} (17)$$

$$C_{abcd} = R_{abcd} - \frac{1}{2}R_{ac}g_{bd} + \frac{1}{2}R_{ad}g_{bc} - \frac{1}{2}g_{ac}R_{bd} + \frac{1}{2}g_{ad}R_{bc} + \frac{1}{6}R\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right)$$
(18)

```
import cdblib
cdblib.create ('dgeom.json')
                               'dgeom.json')
cdblib.put ('dgab',
                       dgab,
cdblib.put ('diab',
                       diab,
                               'dgeom.json')
cdblib.put ('GammaU',
                      GammaU, 'dgeom.json')
cdblib.put ('GammaD',
                       GammaD, 'dgeom.json')
cdblib.put ('RabcdU',
                       RabcdU, 'dgeom.json')
cdblib.put ('RabcdD',
                       RabcdD, 'dgeom.json')
cdblib.put ('Rab',
                       Rab,
                                'dgeom.json')
cdblib.put ('Rscalara', Rscalar, 'dgeom.json')
cdblib.put ('Cabcd',
                       Cabcd,
                               'dgeom.json')
```