Exercise 3.6 Commutation of ∇ on the Riemann tensor − simple computation

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1   {a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w#}::Indices(position=independent).
2
3   DD{#}::Derivative.
4   \nabla{#}::Derivative.
5
6   RabcdF := R_{a b c d} -> A_{a} B_{b} C_{c} D_{d}.        # cdb(RabcdF.000,RabcdF)
7   RabcdB := A_{a} B_{b} C_{c} D_{d} -> R_{a b c d}.        # cdb(RabcdB.000,RabcdB)
8
9   derivDD := DD_{b c}{V?_{a}}} -> R^{d}_{a b c} V?_{d}.  # cdb(derivDD.000,derivDD)
10
11  nablaDD := \nabla_{f}{\nabla_{e}{R_{a b c d}}}
12          - \nabla_{e}{\nabla_{f}{R_{a b c d}}} -> DD_{e f}{R_{a b c d}}.
13
14  # product rule for DD acting on A_{a} B_{b} C_{c} D_{d}
15  pruleDD := DD_{e f}{A_{a} B_{b} C_{c} D_{d}} -> DD_{e f}{A_{a}} B_{b} C_{c} D_{d}
16                                             + A_{a} DD_{e f}{B_{b}} C_{c} D_{d}
17                                             + A_{a} B_{b} DD_{e f}{C_{c}} D_{d}
18                                             + A_{a} B_{b} C_{c} DD_{e f}{D_{d}}.
19                                                      # cdb(pruleDD.000,pruleDD)
20
21  expr :=   \nabla_{f}{\nabla_{e}{R_{a b c d}}}
22          - \nabla_{e}{\nabla_{f}{R_{a b c d}}}.         # cdb (ex-0306.100, expr)
23
24  substitute   (expr,nablaDD)                            # cdb (ex-0306.101, expr)
25  substitute   (expr,RabcdF)                             # cdb (ex-0306.102, expr)
26  substitute   (expr,pruleDD)                            # cdb (ex-0306.103, expr)
27  substitute   (expr,derivDD)                            # cdb (ex-0306.104, expr)
28  sort_product (expr)                                    # cdb (ex-0306.105, expr)
29  substitute   (expr,RabcdB)                             # cdb (ex-0306.106, expr)
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$$\nabla_f \left(\nabla_e R_{abcd} \right) - \nabla_e \left(\nabla_f R_{abcd} \right) = DD_{ef} R_{abcd} \tag{ex-0306.101}$$

$$= DD_{ef} \left(A_a B_b C_c D_d \right) \tag{ex-0306.102}$$

$$= DD_{ef} A_a B_b C_c D_d + A_a DD_{ef} B_b C_c D_d + A_a B_b DD_{ef} C_c D_d + A_a B_b C_c DD_{ef} D_d \tag{ex-0306.103}$$

$$= R^g{}_{aef} A_g B_b C_c D_d + A_a R^g{}_{bef} B_g C_c D_d + A_a B_b R^g{}_{cef} C_g D_d + A_a B_b C_c R^g{}_{def} D_g \tag{ex-0306.104}$$

$$= A_g B_b C_c D_d R^g{}_{aef} + A_a B_g C_c D_d R^g{}_{bef} + A_a B_b C_g D_d R^g{}_{cef} + A_a B_b C_c D_g R^g{}_{def} \tag{ex-0306.105}$$

$$= R_{gbcd} R^g{}_{aef} + R_{agcd} R^g{}_{bef} + R_{abgd} R^g{}_{cef} + R_{abcg} R^g{}_{def} \tag{ex-0306.106}$$