## Exercise 6.5 Schwarzschild spacetime in isotropic coordinates

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{t, r, \theta, \varphi}::Coordinate.
    {a,b,c,d,e,f,g,h#}::Indices(values={t, r, \theta, \varphi}, position=independent).
    \partial{#}::PartialDerivative.
    g^{a b}::InverseMetric. # essential when using complete (gab, $g^{a b}$)
    Gamma := Gamma^{a}_{b c} -> 1/2 g^{a d} ( partial_{b}_{g_{d c}})
                                            + \partial_{c}{g_{b d}}
                                             - \partial_{d}{g_{b c}}).
10
11
    12
                             - \partial_{d}{\Gamma^{a}_{b c}}
13
                             + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
14
                             - \Gamma^{e}_{b c} \Gamma^{a}_{d e}.
15
16
    Rab := R_{a b} -> R^{c}_{a c b}.
17
18
    gab := { g_{t} = -((2*r-m)/(2*r+m))**2,
19
             g_{r} = (1+m/(2*r))**4,
20
             g_{\text{theta}} = r**2 (1+m/(2*r))**4,
21
             g_{\text{varphi}} = r**2 \sin(\theta)**2 (1+m/(2*r))**4. # cdb(ex-0605.101,gab)
22
23
    complete (gab, $g^{a b}$)
                                                                     # cdb(ex-0605.102,gab)
24
25
    substitute (Rabcd, Gamma)
26
    substitute (Rab, Rabcd)
27
28
                                                                     # cdb(ex-0605.103, Gamma)
    evaluate
               (Gamma, gab, rhsonly=True)
29
               (Rabcd, gab, rhsonly=True)
                                                                     # cdb(ex-0605.104,Rabcd)
    evaluate
               (Rab, gab, rhsonly=True)
                                                                     # cdb(ex-0605.105,Rab)
    evaluate
```

$$\left[g_{tt} = -\left((2r - m)\left(2r + m\right)^{-1}\right)^{2}, g_{rr} = \left(1 + \frac{1}{2}mr^{-1}\right)^{4}, g_{\theta\theta} = r^{2}\left(1 + \frac{1}{2}mr^{-1}\right)^{4}, g_{\varphi\varphi} = r^{2}(\sin\theta)^{2}\left(1 + \frac{1}{2}mr^{-1}\right)^{4} \right]$$

$$\left[g_{tt} = -\left((2r - m)\left(2r + m\right)^{-1}\right)^{2}, g_{rr} = \left(1 + \frac{1}{2}mr^{-1}\right)^{4}, g_{\theta\theta} = r^{2}\left(1 + \frac{1}{2}mr^{-1}\right)^{4}, g_{\theta\theta} = r^{2}\left(1$$

 $R^a_{bcd}$ 

$$\begin{array}{c} R^{o}_{bol} \\ R^{o}_{bol} \\ \\ = \begin{pmatrix} \Box_{n}r_{r} = 128mr^{3} \left(-m^{2} + 4mr - 4r^{2}\right) \left(m^{8} + 16m^{7}r + 112m^{6}r^{2} + 448m^{5}r^{3} + 1120m^{4}r^{4} + 1792m^{3}r^{5} + 1792m^{2}r^{6} + 1024mr^{7} + 256r^{8}\right)^{-1} \\ \Box_{g\varphi} = g = 8mr(\sin\theta)^{2} \left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{g\varphi} = 4mr(\sin\theta)^{2} \left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{g\varphi} = 4mr\left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{g\varphi} = 8mr\left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{r} = \frac{1}{r} = 128m^{3} \left(m^{2} - 4mr + 4r^{2}\right)^{-1} \\ \Box_{r} = \frac{1}{r} = 4mr\left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{r} = 8mr\left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{r} = \frac{1}{r} = 4mr\left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{r} = \frac{1}{r} = 4mr\left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{r} = \frac{1}{r} = 4mr\left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{r} = \frac{1}{r} = 4mr\left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{r} = \frac{1}{r} = 4mr\left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{r} = \frac{1}{r} = 4mr\left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{r} = \frac{1}{r} = 4mr\left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{r} = \frac{1}{r} = 4mr\left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{r} = \frac{1}{r} = 4mr\left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{r} = \frac{1}{r} = 4mr\left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{r} = \frac{1}{r} = 4mr\left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{r} = \frac{1}{r} = 4mr\left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{r} = \frac{1}{r} = 4mr\left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{r} = \frac{1}{r} = 4mr\left(m^{2} + 4mr + 4r^{2}\right)^{-1} \\ \Box_{r} = \frac{1}{r} = 64mr^{3}\left(m - 2r\right)^{2}\left(m^{4} + 8m^{3}r + 24m^{2}r^{2} + 32mr^{3} + 16r^{4}\right)^{-2} \\ \Box_{r} = \frac{1}{r} = 64mr^{3}\left(m - 2r\right)^{2}\left(m^{4} + 8m^{3}r + 24m^{2}r^{2} + 32mr^{3} + 16r^{4}\right)^{-2} \\ \Box_{r} = \frac{1}{r} = 64mr^{3}\left(m - 2r\right)^{2}\left(m^{4} + 8m^{3}r + 24m^{2}r^{2} + 32mr^{3}$$