

Example 1 The metric connection

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1 {a,b,c,d,e,f,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices.
2
3 g_{a b}::Metric.
4 g_{a}^{b}::KroneckerDelta.
5
6 \partial{#}::PartialDerivative.
7
8 Gamma := \Gamma^{a}_{b c} -> (1/2) g^{a d} ( \partial_{b}{g_{d c}}
9                                     + \partial_{c}{g_{b d}}
10                                    - \partial_{d}{g_{b c}} ). # cdb (ex-01.101,Gamma)
11
12 cderiv := \partial_{c}{g_{a b}} - g_{a d}\Gamma^{d}_{b c}
13          - g_{d b}\Gamma^{d}_{a c}. # cdb (ex-01.102,cderiv)
14
15 substitute      (cderiv, Gamma) # cdb (ex-01.103,cderiv)
16 distribute      (cderiv) # cdb (ex-01.104,cderiv)
17 eliminate_metric (cderiv) # cdb (ex-01.105,cderiv)
18 eliminate_kronecker (cderiv) # cdb (ex-01.106,cderiv)
19 canonicalise     (cderiv) # cdb (ex-01.107,cderiv)
20
21 checkpoint.append (cderiv)
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$$\Gamma = \Gamma^a{}_{bc} \rightarrow \frac{1}{2}g^{ad}(\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc}) \quad (\text{ex-01.101})$$

$$g_{ab;c} = \partial_c g_{ab} - g_{ad}\Gamma^d{}_{bc} - g_{db}\Gamma^d{}_{ac} \quad (\text{ex-01.102})$$

$$= \partial_c g_{ab} - \frac{1}{2}g_{ad}g^{de}(\partial_b g_{ec} + \partial_c g_{be} - \partial_e g_{bc}) - \frac{1}{2}g_{db}g^{de}(\partial_a g_{ec} + \partial_c g_{ae} - \partial_e g_{ac}) \quad (\text{ex-01.103})$$

$$= \partial_c g_{ab} - \frac{1}{2}g_{ad}g^{de}\partial_b g_{ec} - \frac{1}{2}g_{ad}g^{de}\partial_c g_{be} + \frac{1}{2}g_{ad}g^{de}\partial_e g_{bc} - \frac{1}{2}g_{db}g^{de}\partial_a g_{ec} - \frac{1}{2}g_{db}g^{de}\partial_c g_{ae} + \frac{1}{2}g_{db}g^{de}\partial_e g_{ac} \quad (\text{ex-01.104})$$

$$= \partial_c g_{ab} - \frac{1}{2}g_a{}^e\partial_b g_{ec} - \frac{1}{2}g_a{}^e\partial_c g_{be} + \frac{1}{2}g_a{}^e\partial_e g_{bc} - \frac{1}{2}g_b{}^e\partial_a g_{ec} - \frac{1}{2}g_b{}^e\partial_c g_{ae} + \frac{1}{2}g_b{}^e\partial_e g_{ac} \quad (\text{ex-01.105})$$

$$= \frac{1}{2}\partial_c g_{ab} - \frac{1}{2}\partial_c g_{ba} \quad (\text{ex-01.106})$$

$$= 0 \quad (\text{ex-01.107})$$