Exercise 5.5 Commuting covariant derivatives

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{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices(position=independent).

;::Symbol.

def add_tags (obj,tag):
   n = 0
   ans = Ex('0')
   for i in obj.top().terms():
      foo = obj[i]
      bah = Ex(tag+'_{'+str(n)+'}')
      ans := @(ans) + @(bah) @(foo).
      n = n + 1
   return ans

def clear_tags (obj,tag):
   ans := @(obj).
   foo  = Ex(tag+'_{a?} -> 1')
   substitute (ans,foo)
   return ans

rule := V^{a}_{; b ; c} -> V^{a}_{; c ; b} - R^{a}_{d b c} V^{d}.

expr := V^{a}_{; b ; c} - V^{a}_{; c ; b}.    # cdb (ex-0505.100,expr)

expr  = add_tags (expr,'\\mu')                # cdb (ex-0505.101,expr)

zoom       (expr, $\mu_{0} Q??$)              # cdb (ex-0505.102,expr)
substitute (expr, rule)                       # cdb (ex-0505.103,expr)
unzoom     (expr)                             # cdb (ex-0505.104,expr)

expr = clear_tags (expr,'\\mu')               # cdb (ex-0505.105,expr)
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$$
\begin{aligned}
V^a{}_{;b;c} - V^a{}_{;c;b} &= \mu_0 V^a{}_{;b;c} - \mu_1 V^a{}_{;c;b} && \texttt{(ex-0505.101)}\\
&= \mu_0 V^a{}_{;b;c} - \mu_1 V^a{}_{;c;b} && \texttt{(ex-0505.101)}\\
&= \mu_0 V^a{}_{;b;c} + \ldots && \texttt{(ex-0505.102)}\\
&= \mu_0 \left(V^a{}_{;c;b} - R^a{}_{dbc} V^d \right) + \ldots && \texttt{(ex-0505.103)}\\
&= \mu_0 \left(V^a{}_{;c;b} - R^a{}_{dbc} V^d \right) - \mu_1 V^a{}_{;c;b} && \texttt{(ex-0505.104)}\\
&= - R^a{}_{dbc} V^d && \texttt{(ex-0505.105)}
\end{aligned}
$$