Exercise 2.1 Using Cadabra's own product rule

```
{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices(position=independent).

\nabla{#}::Derivative.
\partial{#}::PartialDerivative.

# templates for covariant derivatives

deriv1 := \nabla_{a}{A?^{b}} -> \partial_{a}{A?^{b}}
                              + \Gamma^{b}_{c a} A?^{c}.

deriv2 := \nabla_{a}{A?_{b}} -> \partial_{a}{A?_{b}}
                              - \Gamma^{c}_{b a} A?_{c}.

# create an object

uv := \nabla_{a}{v_{b} u^{b}}
    - \partial_{a}{v_{b} u^{b}}.      # cdb (ex-0201.101,uv)

# apply the rules, then simplify

product_rule   (uv)                   # cdb (ex-0201.102,uv)
substitute     (uv,deriv1)            # cdb (ex-0201.103,uv)
substitute     (uv,deriv2)            # cdb (ex-0201.104,uv)
distribute     (uv)                   # cdb (ex-0201.105,uv)
sort_product   (uv)                   # cdb (ex-0201.106,uv)
rename_dummies (uv)                   # cdb (ex-0201.107,uv)
```

$$
\begin{aligned}
\nabla_a \left(v_b u^b\right) - \partial_a \left(v_b u^b\right) &= \nabla_a v_b u^b + v_b \nabla_a u^b - \partial_a v_b u^b - v_b \partial_a u^b && \texttt{(ex-0201.102)} \\
&= \nabla_a v_b u^b + v_b \left(\partial_a u^b + \Gamma^b{}_{ca} u^c\right) - \partial_a v_b u^b - v_b \partial_a u^b && \texttt{(ex-0201.103)} \\
&= \left(\partial_a v_b - \Gamma^c{}_{ba} v_c\right) u^b + v_b \left(\partial_a u^b + \Gamma^b{}_{ca} u^c\right) - \partial_a v_b u^b - v_b \partial_a u^b && \texttt{(ex-0201.104)} \\
&= -\Gamma^c{}_{ba} v_c u^b + v_b \Gamma^b{}_{ca} u^c && \texttt{(ex-0201.105)} \\
&= -\Gamma^c{}_{ba} u^b v_c + \Gamma^b{}_{ca} u^c v_b && \texttt{(ex-0201.106)} \\
&= 0 && \texttt{(ex-0201.107)}
\end{aligned}
$$

Exercise 2.1 Using hand crafted product rules

```
{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices(position=independent).

\nabla{#}::Derivative.
\partial{#}::PartialDerivative.

# templates for covariant derivatives

deriv1 := \nabla_{a}{A?^{b}} -> \partial_{a}{A?^{b}}
                              + \Gamma^{b}_{c a} A?^{c}.

deriv2 := \nabla_{a}{A?_{b}} -> \partial_{a}{A?_{b}}
                              - \Gamma^{c}_{b a} A?_{c}.

# tempaltes for product rules

deriv3 := \nabla_{a}{A?_{b} B?^{c}} -> B?^{c} \nabla_{a}{A?_{b}}
                                     + A?_{b} \nabla_{a}{B?^{c}}.

deriv4 := \partial_{a}{A?_{b} B?^{c}} -> B?^{c} \partial_{a}{A?_{b}}
                                       + A?_{b} \partial_{a}{B?^{c}}.

# create an object

uv := \nabla_{a}{v_{b} u^{b}}
    - \partial_{a}{v_{b} u^{b}}.      # cdb (ex-0201.201,uv)

# apply the rules, then simplify

substitute     (uv,deriv3)            # cdb (ex-0201.202,uv)
substitute     (uv,deriv4)            # cdb (ex-0201.203,uv)
substitute     (uv,deriv1)            # cdb (ex-0201.204,uv)
substitute     (uv,deriv2)            # cdb (ex-0201.205,uv)
distribute     (uv)                   # cdb (ex-0201.206,uv)
sort_product   (uv)                   # cdb (ex-0201.207,uv)
rename_dummies (uv)                   # cdb (ex-0201.208,uv)
```

$$
\begin{aligned}
\nabla_a \left(v_b u^b\right) - \partial_a \left(v_b u^b\right) &= u^b \nabla_a v_b + v_b \nabla_a u^b - \partial_a \left(v_b u^b\right) && \texttt{(ex-0201.202)} \\
&= u^b \nabla_a v_b + v_b \nabla_a u^b - u^b \partial_a v_b - v_b \partial_a u^b && \texttt{(ex-0201.203)} \\
&= u^b \nabla_a v_b + v_b \left(\partial_a u^b + \Gamma^b{}_{ca} u^c\right) - u^b \partial_a v_b - v_b \partial_a u^b && \texttt{(ex-0201.204)} \\
&= u^b \left(\partial_a v_b - \Gamma^c{}_{ba} v_c\right) + v_b \left(\partial_a u^b + \Gamma^b{}_{ca} u^c\right) - u^b \partial_a v_b - v_b \partial_a u^b && \texttt{(ex-0201.205)} \\
&= -u^b \Gamma^c{}_{ba} v_c + v_b \Gamma^b{}_{ca} u^c && \texttt{(ex-0201.206)} \\
&= -\Gamma^c{}_{ba} u^b v_c + \Gamma^b{}_{ca} u^c v_b && \texttt{(ex-0201.207)} \\
&= 0 && \texttt{(ex-0201.208)}
\end{aligned}
$$