Example 6-01 Evaluating components

```
1    {\theta, \varphi}::Coordinate.
2    {a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).
3
4    \partial{#}::PartialDerivative.
5
6    V  := { V_{\theta} = \varphi, V_{\varphi} = \sin(\theta) }.    # cdb(ex-06.100,V)
7    dV := \partial_{b}{V_{a}} - \partial_{a}{V_{b}}.              # cdb(ex-06.101,dV)
8
9    evaluate (dV, V)    # cdb(ex-06.102,dV)
```

$$V_a = [V_\theta = \varphi, V_\varphi = \sin\theta] \tag{ex-06.100}$$

$$\partial_b V_a - \partial_a V_b = \square_{ab} \begin{cases} \square_{\varphi\theta} = \cos\theta - 1 \\ \square_{\theta\varphi} = 1 - \cos\theta \end{cases} \tag{ex-06.102}$$

Example 6-02 Riemann tensor of a 2-sphere

```
1    {\theta, \varphi}::Coordinate.
2    {a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).
3
4    \partial{#}::PartialDerivative.
5
6    Gamma := \Gamma^{a}_{b c} -> 1/2 g^{a d} (   \partial_{b}{g_{d c}}
7                                              + \partial_{c}{g_{b d}}
8                                              - \partial_{d}{g_{b c}}).
9
10   Rabcd := R^{a}_{b c d} ->   \partial_{c}{\Gamma^{a}_{b d}}
11                             - \partial_{d}{\Gamma^{a}_{b c}}
12                             + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
13                             - \Gamma^{e}_{b c} \Gamma^{a}_{d e}.
14
15   gab := { g_{\theta\theta}   = r**2,
16            g_{\varphi\varphi} = r**2 \sin(\theta)**2 }.       # cdb(ex-06.201,gab)
17
18   iab := { g^{\theta\theta}   = 1/r**2,
19            g^{\varphi\varphi} = 1/(r**2 \sin(\theta)**2) }.   # cdb(ex-06.202,iab)
20
21   substitute (Rabcd, Gamma)                                   # cdb(ex-06.203,Gamma)
22
23   evaluate    (Gamma, join (gab,iab), rhsonly=True)          # cdb(ex-06.204,Gamma)
24   evaluate    (Rabcd, join (gab,iab), rhsonly=True)          # cdb(ex-06.205,Rabcd)
25
26   # convert from a rule to a simple expression
27   Riem := R^{a}_{b c d}.
28   substitute (Riem, Rabcd)                                    # cdb(ex-06.206,Riem)
29
30   from cdb.core.component import *
31
32   RiemCompt = get_component (Riem, $\theta, \varphi, \theta, \varphi$)   # cdb(ex-06.207,RiemCompt)
```

$$\left[g_{\theta\theta} = r^2 \,, g_{\varphi\varphi} = r^2 \left(\sin\theta \right)^2 \right] \tag{ex-06.201}$$

$$\left[g^{\theta\theta} = r^{-2} \,, g^{\varphi\varphi} = \left(r^2 \left(\sin\theta \right)^2 \right)^{-1} \right] \tag{ex-06.202}$$

$$\Gamma^a{}_{bc} \to \square_{cb}{}^a \begin{cases} \square_{\varphi\theta}{}^\varphi = \left(\tan\theta \right)^{-1} \\ \square_{\theta\varphi}{}^\varphi = \left(\tan\theta \right)^{-1} \\ \square_{\varphi\varphi}{}^\theta = -\dfrac{1}{2} \sin\left(2\theta \right) \end{cases} \tag{ex-06.204}$$

$$R^a{}_{bcd} \to \square_{db}{}^a{}_c \begin{cases} \square_{\varphi\varphi}{}^\theta{}_\theta = \left(\sin\theta \right)^2 \\ \square_{\varphi\theta}{}^\varphi{}_\theta = -1 \\ \square_{\theta\varphi}{}^\theta{}_\varphi = -\left(\sin\theta \right)^2 \\ \square_{\theta\theta}{}^\varphi{}_\varphi = 1 \end{cases} \tag{ex-06.205}$$

$$\square_{db}{}^a{}_c \begin{cases} \square_{\varphi\varphi}{}^\theta{}_\theta = \left(\sin\theta \right)^2 \\ \square_{\varphi\theta}{}^\varphi{}_\theta = -1 \\ \square_{\theta\varphi}{}^\theta{}_\varphi = -\left(\sin\theta \right)^2 \\ \square_{\theta\theta}{}^\varphi{}_\varphi = 1 \end{cases} \tag{ex-06.206}$$

$$R^\theta{}_{\varphi\varphi\theta} = -\left(\sin\theta \right)^2 \tag{ex-06.207}$$

Example 6-03 Using `complete` to compute the inverse metric

This version uses `complete` to compute the inverse metric.

```
1   {\theta, \varphi}::Coordinate.
2   {a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).
3
4   \partial{#}::PartialDerivative.
5
6   g^{a b}::InverseMetric.  # essential when using complete (gab, $g^{a b}$)
7
8   Gamma := \Gamma^{a}_{b c} -> 1/2 g^{a d} (   \partial_{b}{g_{d c}}
9                                             + \partial_{c}{g_{b d}}
10                                            - \partial_{d}{g_{b c}}).
11
12  Rabcd := R^{a}_{b c d} ->   \partial_{c}{\Gamma^{a}_{b d}}
13                            - \partial_{d}{\Gamma^{a}_{b c}}
14                            + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
15                            - \Gamma^{e}_{b c} \Gamma^{a}_{d e}.
16
17  gab := { g_{\theta\theta}   = r**2,
18           g_{\varphi\varphi} = r**2 \sin(\theta)**2 }.      # cdb(ex-06.301,gab)
19
20  complete   (gab, $g^{a b}$)                               # cdb(ex-06.302,gab)
21
22  substitute (Rabcd, Gamma)
23
24  evaluate   (Gamma, gab, rhsonly=True)                     # cdb(ex-06.303,Gamma)
25  evaluate   (Rabcd, gab, rhsonly=True)                     # cdb(ex-06.304,Rabcd)
```

$$\left[g_{\theta\theta} = r^2 , g_{\varphi\varphi} = r^2 \left(\sin\theta \right)^2 \right] \tag{ex-06.301}$$

$$\left[g_{\theta\theta} = r^2 , g_{\varphi\varphi} = r^2 \left(\sin\theta \right)^2 , g^{\theta\theta} = r^{-2} , g^{\varphi\varphi} = \left(r^2 \left(\sin\theta \right)^2 \right)^{-1} \right] \tag{ex-06.302}$$

$$\Gamma^a{}_{bc} \to \Box_{cb}{}^a \begin{cases} \Box_{\varphi\theta}{}^{\varphi} = \left(\tan\theta \right)^{-1} \\ \Box_{\theta\varphi}{}^{\varphi} = \left(\tan\theta \right)^{-1} \\ \Box_{\varphi\varphi}{}^{\theta} = -\dfrac{1}{2} \sin\left(2\,\theta \right) \end{cases} \tag{ex-06.303}$$

$$R^a{}_{bcd} \to \Box_{db}{}^a{}_c \begin{cases} \Box_{\varphi\varphi}{}^{\theta}{}_{\theta} = \left(\sin\theta \right)^2 \\ \Box_{\varphi\theta}{}^{\varphi}{}_{\theta} = -1 \\ \Box_{\theta\varphi}{}^{\theta}{}_{\varphi} = -\left(\sin\theta \right)^2 \\ \Box_{\theta\theta}{}^{\varphi}{}_{\varphi} = 1 \end{cases} \tag{ex-06.304}$$

Example 6-04 Components by scalar projection

This example shows how one component of the Riemann tensor can be computed using a scalar projection.

```
1    {\theta, \varphi}::Coordinate.
2    {a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).
3
4    theta{#}::LaTeXForm("\theta").
5    varphi{#}::LaTeXForm("\varphi").
6
7    # usual definitions for the connection and Riemann tensor
8
9    Gamma := \Gamma^{a}_{b c} -> 1/2 g^{a d} (   \partial_{b}{g_{d c}}
10                                          + \partial_{c}{g_{b d}}
11                                          - \partial_{d}{g_{b c}}).
12
13   Rabcd := R^{a}_{b c d} ->   \partial_{c}{\Gamma^{a}_{b d}}
14                             - \partial_{d}{\Gamma^{a}_{b c}}
15                             + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
16                             - \Gamma^{e}_{b c} \Gamma^{a}_{d e}.
17
18   gab := { g_{\theta \theta}   = r**2,
19            g_{\varphi \varphi} = r**2 \sin(\theta)**2 }.   # cdb(ex-06.400,gab)
20
21   iab := { g^{\theta\theta}   = 1/r**2,
22            g^{\varphi\varphi} = 1/(r**2 \sin(\theta)**2) }.
23
24   substitute (Rabcd, Gamma)
25   evaluate   (Rabcd, join (gab,iab), rhsonly=True)
26
27   # above code just to compute Rabcd
28   # following code is all that is needed for the scalar projection method
29
30   # define the basis for vectors and dual vectors
31
32   basis := {theta^{\theta} = 1, varphi^{\varphi} = 1}.
33   dual   := {theta_{\theta} = 1, varphi_{\varphi} = 1}.
34
```

```
35    # obtain components by contracting with basis
36
37    compt := R^{a}_{b c d} theta_{a} varphi^{b} theta^{c} varphi^{d}.    # cdb(ex-06.401,compt)
38    substitute (compt,Rabcd)
39
40    evaluate (compt, join (basis,dual))                                  # cdb(ex-06.402,compt)
41
42    compt_sympy = compt._sympy_()
43
44    # cdbBeg(print.ex-06.04)
45    print ('type compt  = ' + str(type(compt)))         # shows that compt is a Cadabra object
46    print ('type gphiphi = ' + str(type(compt_sympy)))  # shows that gphiphi is a Python object
47    print ('      compt  = ' + str(compt))              # will contain LaTeX markup
48    print ('      gphiphi = ' + str(compt_sympy))       # will be pure Python/SymPy
49    # cdbEnd(print.ex-06.04)
50
51    checkpoint.append (compt)
```

$$R^{\theta}{}_{\varphi\theta\varphi} = R^{a}{}_{bcd}\theta_{a}\varphi^{b}\theta^{c}\varphi^{d} \qquad\qquad \text{(ex-06.401)}$$
$$= (\sin\theta)^{2} \qquad\qquad \text{(ex-06.402)}$$

```
1    type compt  = <class 'cadabra2.Ex'>
2    type gphiphi = <class 'sympy.core.power.Pow'>
3         compt  = (\sin(\theta))**2
4         gphiphi = sin(theta)**2
```

Example 6-05 Components by selection

This example shows how one component of the metric tensor can be computed by indexing the result of a call to `evaluate`.

```
1    {\theta, \varphi}::Coordinate.
2    {a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).
3
4    theta{#}::LaTeXForm("\theta").
5    varphi{#}::LaTeXForm("\varphi").
6
7    gab := { g_{\theta \theta}   = r**2,
8             g_{\varphi \varphi} = r**2 \sin(\theta)**2 }.   # cdb(ex-06.500,gab)
9
10   metric := g_{a b}.
11
12   evaluate (metric,gab)
13
14   indcs = metric[2][1][0]                                 # cdb(ex-06.501,indcs)
15   compt = metric[2][1][1]                                 # cdb(ex-06.502,compt)
16
17   # cdbBeg(print.ex-06.05)
18   print ('metric = ' + str(metric.input_form())+'\n')  # reveals Cadabra's internal structure for storing metric
19
20   print ('metric[0] = ' + str(metric[0]))
21   print ('metric[1] = ' + str(metric[1]))
22   print ('metric[2] = ' + str(metric[2])+'\n')
23
24   print ('metric[2][1] = '+ str(metric[2][1]))
25   print ('metric[2][1][0] = '+ str(metric[2][1][0]))
26   print ('metric[2][1][1] = '+ str(metric[2][1][1]))
27   # cdbEnd(print.ex-06.05)
28
29   checkpoint.append (indcs)
30   checkpoint.append (compt)
```

$$g_{\varphi\varphi} = g_{[\varphi,\varphi]} \tag{ex-06.501}$$
$$= r^2 \left(\sin\theta\right)^2 \tag{ex-06.502}$$

```
1   metric = \components_{a b}({{\theta, \theta} = (r)**2, {\varphi, \varphi} = (r)**2 (\sin(\theta))**2})
2
3   metric[0] = a
4   metric[1] = b
5   metric[2] = {{\theta, \theta} = (r)**2, {\varphi, \varphi} = (r)**2 (\sin(\theta))**2}
6
7   metric[2][1] = {\varphi, \varphi} = (r)**2 (\sin(\theta))**2
8   metric[2][1][0] = {\varphi, \varphi}
9   metric[2][1][1] = (r)**2 (\sin(\theta))**2
```