Exercise 3.2 Riemann tensor from commutation of ∇

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{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices(position=independent).

\nabla{#}::Derivative.
\partial{#}::PartialDerivative.

\Gamma^{a}_{b c}::TableauSymmetry(shape={2}, indices={1,2});

# rules for the first two covariant derivs of V^a

deriv1 := \nabla_{a}{V^{b}} -> \partial_{a}{V^{b}}
                             + \Gamma^{b}_{d a} V^{d}.          # cdb (ex-0302.101,deriv1)

deriv2 := \nabla_{a}{\nabla_{b}{V^{c}}} -> \partial_{a}{\nabla_{b}{V^{c}}}
                                         + \Gamma^{c}_{d a} \nabla_{b}{V^{d}}
                                         - \Gamma^{d}_{b a} \nabla_{d}{V^{c}}.
                                                                # cdb (ex-0302.102,deriv2)

Vabc :=   \nabla_{c}{\nabla_{b}{V^{a}}}
        - \nabla_{b}{\nabla_{c}{V^{a}}}.                        # cdb (ex-0302.103, Vabc)

substitute (Vabc,deriv2)                                        # cdb (ex-0302.104, Vabc)
substitute (Vabc,deriv1)                                        # cdb (ex-0302.105, Vabc)

distribute     (Vabc)                                           # cdb (ex-0302.106, Vabc)
product_rule   (Vabc)                                           # cdb (ex-0302.107, Vabc)

sort_product   (Vabc)                                           # cdb (ex-0302.108, Vabc)
rename_dummies (Vabc)                                           # cdb (ex-0302.109, Vabc)
canonicalise   (Vabc)                                           # cdb (ex-0302.110, Vabc)
factor_out     (Vabc,$V^{a?}$)                                  # cdb (ex-0302.111, Vabc)
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\begin{aligned}
\nabla_c(\nabla_b V^a) - \nabla_b(\nabla_c V^a) &= \partial_c(\nabla_b V^a) + \Gamma^a{}_{dc}\nabla_b V^d - \Gamma^d{}_{bc}\nabla_d V^a - \partial_b(\nabla_c V^a) - \Gamma^a{}_{db}\nabla_c V^d + \Gamma^d{}_{cb}\nabla_d V^a && \texttt{(ex-0302.104)}\\
&= \partial_c\left(\partial_b V^a + \Gamma^a{}_{db}V^d\right) + \Gamma^a{}_{dc}\left(\partial_b V^d + \Gamma^d{}_{eb}V^e\right) - \Gamma^d{}_{bc}\left(\partial_d V^a + \Gamma^a{}_{ed}V^e\right) - \partial_b\left(\partial_c V^a + \Gamma^a{}_{dc}V^d\right) - \Gamma^a{}_{db}\left(\partial_c V^d + \Gamma^d{}_{ec}V^e\right)\\
&\quad + \Gamma^d{}_{cb}\left(\partial_d V^a + \Gamma^a{}_{ed}V^e\right) && \texttt{(ex-0302.105)}\\
&= \partial_{cb}V^a + \partial_c\left(\Gamma^a{}_{db}V^d\right) + \Gamma^a{}_{dc}\partial_b V^d + \Gamma^a{}_{dc}\Gamma^d{}_{eb}V^e - \Gamma^d{}_{bc}\partial_d V^a - \Gamma^d{}_{bc}\Gamma^a{}_{ed}V^e - \partial_{bc}V^a - \partial_b\left(\Gamma^a{}_{dc}V^d\right) - \Gamma^a{}_{db}\partial_c V^d - \Gamma^a{}_{db}\Gamma^d{}_{ec}V^e\\
&\quad + \Gamma^d{}_{cb}\partial_d V^a + \Gamma^d{}_{cb}\Gamma^a{}_{ed}V^e && \texttt{(ex-0302.106)}\\
&= \partial_{cb}V^a + \partial_c\Gamma^a{}_{db}V^d + \Gamma^a{}_{dc}\Gamma^d{}_{eb}V^e - \Gamma^d{}_{bc}\partial_d V^a - \Gamma^d{}_{bc}\Gamma^a{}_{ed}V^e - \partial_{bc}V^a - \partial_b\Gamma^a{}_{dc}V^d - \Gamma^a{}_{db}\Gamma^d{}_{ec}V^e + \Gamma^d{}_{cb}\partial_d V^a + \Gamma^d{}_{cb}\Gamma^a{}_{ed}V^e && \texttt{(ex-0302.107)}\\
&= \partial_{cb}V^a + V^d\partial_c\Gamma^a{}_{db} + V^e\Gamma^a{}_{dc}\Gamma^d{}_{eb} - \Gamma^d{}_{bc}\partial_d V^a - V^e\Gamma^a{}_{ed}\Gamma^d{}_{bc} - \partial_{bc}V^a - V^d\partial_b\Gamma^a{}_{dc} - V^e\Gamma^a{}_{db}\Gamma^d{}_{ec} + \Gamma^d{}_{cb}\partial_d V^a + V^e\Gamma^a{}_{ed}\Gamma^d{}_{cb} && \texttt{(ex-0302.108)}\\
&= \partial_{cb}V^a + V^d\partial_c\Gamma^a{}_{db} + V^d\Gamma^a{}_{ec}\Gamma^e{}_{db} - \Gamma^d{}_{bc}\partial_d V^a - V^d\Gamma^a{}_{de}\Gamma^e{}_{bc} - \partial_{bc}V^a - V^d\partial_b\Gamma^a{}_{dc} - V^d\Gamma^a{}_{eb}\Gamma^e{}_{dc} + \Gamma^d{}_{cb}\partial_d V^a + V^d\Gamma^a{}_{de}\Gamma^e{}_{cb} && \texttt{(ex-0302.109)}\\
&= V^d\partial_c\Gamma^a{}_{bd} + V^d\Gamma^a{}_{ce}\Gamma^e{}_{bd} - V^d\partial_b\Gamma^a{}_{cd} - V^d\Gamma^a{}_{be}\Gamma^e{}_{cd} && \texttt{(ex-0302.110)}\\
&= V^d\left(\partial_c\Gamma^a{}_{bd} + \Gamma^a{}_{ce}\Gamma^e{}_{bd} - \partial_b\Gamma^a{}_{cd} - \Gamma^a{}_{be}\Gamma^e{}_{cd}\right) && \texttt{(ex-0302.111)}\\
&= -R^a{}_{dbc}V^d
\end{aligned}
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This result agrees with Misner, Thorne and Wheeler. pg. 266.