

Exercise 3.5 Commutation of covariant derivatives

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1 {a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w#}::Indices(position=independent).
2
3 \nabla{#}::Derivative.
4
5 expr := \nabla_{d}{\nabla_{c}{A_{a} B_{b}}}
6         - \nabla_{c}{\nabla_{d}{A_{a} B_{b}}}. # cdb(ex-0305.100,expr)
7
8 product_rule (expr) # cdb(ex-0305.101,expr)
9 distribute (expr) # cdb(ex-0305.102,expr)
10 product_rule (expr) # cdb(ex-0305.103,expr)
11 factor_out (expr,$A_{a?},B_{b?}$) # cdb(ex-0305.104,expr)

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$$\nabla_d (\nabla_c (A_a B_b)) - \nabla_c (\nabla_d (A_a B_b)) = \nabla_d (\nabla_c A_a B_b + A_a \nabla_c B_b) - \nabla_c (\nabla_d A_a B_b + A_a \nabla_d B_b) \quad (\text{ex-0305.101})$$

$$= \nabla_d (\nabla_c A_a B_b) + \nabla_d (A_a \nabla_c B_b) - \nabla_c (\nabla_d A_a B_b) - \nabla_c (A_a \nabla_d B_b) \quad (\text{ex-0305.102})$$

$$= \nabla_d (\nabla_c A_a) B_b + A_a \nabla_d (\nabla_c B_b) - \nabla_c (\nabla_d A_a) B_b - A_a \nabla_c (\nabla_d B_b) \quad (\text{ex-0305.103})$$

$$= B_b (\nabla_d (\nabla_c A_a) - \nabla_c (\nabla_d A_a)) + A_a (\nabla_d (\nabla_c B_b) - \nabla_c (\nabla_d B_b)) \quad (\text{ex-0305.104})$$