

Exercise 6.4 Scalar curavture of a 2-sphere

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1  {\theta, \varphi}::Coordinate.
2  {a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).
3
4  \partial{#}::PartialDerivative.
5
6  g^{a b}::InverseMetric. # essential when using complete (gab, $g^{a b}$)
7
8  Gamma := \Gamma^{a}_{b c} -> 1/2 g^{a d} ( \partial_{b}{g_{d c}}
9                                     + \partial_{c}{g_{b d}}
10                                    - \partial_{d}{g_{b c}}).
11
12  Rabcd := R^{a}_{b c d} -> \partial_{c}{\Gamma^{a}_{b d}}
13                        - \partial_{d}{\Gamma^{a}_{b c}}
14                        + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
15                        - \Gamma^{e}_{b c} \Gamma^{a}_{d e}.
16
17  Rab := R_{a b} -> R^{c}_{c}_{a b}.
18
19  R := R -> R_{a b} g^{a b}.
20
21  gab := { g_{\theta\theta} = r**2,
22          g_{\varphi\varphi} = r**2 \sin(\theta)**2 }. # cdb(ex-0604.101,gab)
23
24  complete (gab, $g^{a b}$) # cdb(ex-0604.102,gab)
25
26  substitute (Rabcd, Gamma)
27  substitute (Rab, Rabcd)
28  substitute (R, Rab)
29
30  evaluate (Gamma, gab, rhsonly=True) # cdb(ex-0604.103,Gamma)
31  evaluate (Rabcd, gab, rhsonly=True) # cdb(ex-0604.104,Rabcd)
32  evaluate (Rab, gab, rhsonly=True) # cdb(ex-0604.105,Rab)
33  evaluate (R, gab, rhsonly=True) # cdb(ex-0604.106,R)

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$$[g_{\theta\theta} = r^2, \ g_{\varphi\varphi} = r^2(\sin \theta)^2] \quad (\text{ex-0604.101})$$

$$\left[g_{\theta\theta} = r^2, \ g_{\varphi\varphi} = r^2(\sin \theta)^2, \ g^{\theta\theta} = r^{-2}, \ g^{\varphi\varphi} = (r^2(\sin \theta)^2)^{-1} \right] \quad (\text{ex-0604.102})$$

$$\Gamma^a_{bc} \rightarrow \square_{cb}{}^a \begin{cases} \square_{\varphi\theta}{}^\varphi = (\tan \theta)^{-1} \\ \square_{\theta\varphi}{}^\varphi = (\tan \theta)^{-1} \\ \square_{\varphi\varphi}{}^\theta = -\frac{1}{2} \sin(2\theta) \end{cases} \quad (\text{ex-0604.103})$$

$$R^a_{bcd} \rightarrow \square_{db}{}^a{}_c \begin{cases} \square_{\varphi\varphi}{}^\theta{}_\theta = (\sin \theta)^2 \\ \square_{\varphi\theta}{}^\varphi{}_\theta = -1 \\ \square_{\theta\varphi}{}^\theta{}_\varphi = -(\sin \theta)^2 \\ \square_{\theta\theta}{}^\varphi{}_\varphi = 1 \end{cases} \quad (\text{ex-0604.104})$$

$$R_{ab} \rightarrow \square_{ba} \begin{cases} \square_{\varphi\varphi} = (\sin \theta)^2 \\ \square_{\theta\theta} = 1 \end{cases} \quad (\text{ex-0604.105})$$

$$R \rightarrow 2r^{-2} \quad (\text{ex-0604.106})$$