

Exercise 6.7 Schwarzschild spacetime in isotropic coordinates

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1 {t, r, \theta, \varphi}::Coordinate.
2 {a,b,c,d,e,f,g,h#}::Indices(values={t, r, \theta, \varphi}, position=independent).
3
4 \partial{#}::PartialDerivative.
5
6 g^{a b}::InverseMetric. # essential when using complete (gab, $g^{a b}$)
7
8 Gamma := \Gamma^{a}_{f g} -> 1/2 g^{a b} ( \partial_{g}\{g_{b f}\}
9                                     + \partial_{f}\{g_{b g}\}
10                                    - \partial_{b}\{g_{f g}\} ).
11
12 Rabcd := R^{d}_{e f g} -> \partial_{f}\{\Gamma^{d}_{e g}\}
13                      - \partial_{g}\{\Gamma^{d}_{e f}\}
14                      + \Gamma^{d}_{b f} \Gamma^{b}_{e g}
15                      - \Gamma^{d}_{b g} \Gamma^{b}_{e f}.
16
17 Rab := R_{a b} -> R^{c}_{c} _{a b}.
18
19 gab := { g_{t t}          = -((2*r-m)/(2*r+m))**2,
20         g_{r r}          = (1+m/(2*r))**4,
21         g_{\theta\theta}   = r**2 (1+m/(2*r))**4,
22         g_{\varphi\varphi} = r**2 \sin(\theta)**2 (1+m/(2*r))**4}. # cdb(ex-0607.101,gab)
23
24 complete (gab, $g^{a b}$) # cdb(ex-0607.102,gab)
25
26 substitute (Rabcd, Gamma)
27 substitute (Rab, Rabcd)
28
29 evaluate (Gamma, gab, rhsonly=True) # cdb(ex-0607.103,Gamma)
30 evaluate (Rabcd, gab, rhsonly=True) # cdb(ex-0607.104,Rabcd)
31 evaluate (Rab, gab, rhsonly=True) # cdb(ex-0607.105,Rab)

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$$\left[g_{tt} = -((2r - m)(2r + m)^{-1})^2, \quad g_{rr} = \left(1 + \frac{1}{2}mr^{-1}\right)^4, \quad g_{\theta\theta} = r^2\left(1 + \frac{1}{2}mr^{-1}\right)^4, \quad g_{\varphi\varphi} = r^2(\sin\theta)^2\left(1 + \frac{1}{2}mr^{-1}\right)^4 \right] \quad (\text{ex-0607.101})$$

$$\left[g_{tt} = -((2r - m)(2r + m)^{-1})^2, \quad g_{rr} = \left(1 + \frac{1}{2}mr^{-1}\right)^4, \quad g_{\theta\theta} = r^2\left(1 + \frac{1}{2}mr^{-1}\right)^4, \quad g_{\varphi\varphi} = r^2(\sin\theta)^2\left(1 + \frac{1}{2}mr^{-1}\right)^4, \quad g^{tt} = - (m + 2r)^2(-m + 2r)^{-2}, \quad g^{rr} = \left(\frac{1}{2}mr^{-1} + 1\right)^{-4}, \quad g^{\theta\theta} = \left(r^2\left(\frac{1}{2}mr^{-1} + 1\right)^4\right)^{-1}, \quad g^{\varphi\varphi} = \left(r^2\left(\frac{1}{2}mr^{-1} + 1\right)^4(\sin\theta)^2\right)^{-1} \right] \quad (\text{ex-0607.102})$$

$$\Gamma_{fg}^a \rightarrow \square_{fg}^a \left\{ \begin{array}{l} \square_{\varphi r}^\varphi = (-m + 2r)(r(m + 2r))^{-1} \\ \square_{\varphi\theta}^\varphi = (\tan\theta)^{-1} \\ \square_{\theta r}^\theta = (-m + 2r)(r(m + 2r))^{-1} \\ \square_{rr}^r = -2m(r(m + 2r))^{-1} \\ \square_{tr}^t = 4m(-m^2 + 4r^2)^{-1} \\ \square_{r\varphi}^\varphi = (-m + 2r)(r(m + 2r))^{-1} \\ \square_{\theta\varphi}^\varphi = (\tan\theta)^{-1} \\ \square_{r\theta}^\theta = (-m + 2r)(r(m + 2r))^{-1} \\ \square_{rt}^t = 4m(-m^2 + 4r^2)^{-1} \\ \square_{\varphi\varphi}^r = r(m - 2r)(\sin\theta)^2(m + 2r)^{-1} \\ \square_{\varphi\varphi}^\theta = -\frac{1}{2}\sin(2\theta) \\ \square_{\theta\theta}^r = r(m - 2r)(m + 2r)^{-1} \\ \square_{tt}^r = -64mr^4(m - 2r)(m + 2r)^{-7} \end{array} \right. \quad (\text{ex-0607.103})$$

$$R^d_{efg} \rightarrow \square_{eg}^d{}_f \left\{ \begin{array}{l} \square_{tt}{}^r{}_r = -128m^3r^3(m+2r)^{-8} + 512m^2r^4(m+2r)^{-8} - 512mr^5(m+2r)^{-8} \\ \square_{\theta\theta}{}^r{}_r = -4mr(m^2+4mr+4r^2)^{-1} \\ \square_{\varphi\varphi}{}^\theta{}_\theta = 8mr(\sin\theta)^2(m+2r)^{-2} \\ \square_{\varphi\varphi}{}^r{}_r = -4mr(\sin\theta)^2(m^2+4mr+4r^2)^{-1} \\ \square_{rt}{}^t{}_r = -8m(r(m^2+4mr+4r^2))^{-1} \\ \square_{r\theta}{}^\theta{}_r = 4m(r(m^2+4mr+4r^2))^{-1} \\ \square_{\theta\varphi}{}^\varphi{}_\theta = (m-2r)^2(m+2r)^{-2} - 1 \\ \square_{r\varphi}{}^\varphi{}_r = 4m(r(m^2+4mr+4r^2))^{-1} \\ \square_{tr}{}^r{}_t = 128m^3r^3(m+2r)^{-8} - 512m^2r^4(m+2r)^{-8} + 512mr^5(m+2r)^{-8} \\ \square_{\theta r}{}^r{}_\theta = 4mr(m^2+4mr+4r^2)^{-1} \\ \square_{\varphi\theta}{}^\theta{}_\varphi = (m-2r)^2(\sin\theta)^2(m+2r)^{-2} - (\sin\theta)^2 \\ \square_{\varphi r}{}^r{}_\varphi = 4mr(\sin\theta)^2(m^2+4mr+4r^2)^{-1} \\ \square_{rr}{}^t{}_t = 8m(r(m^2+4mr+4r^2))^{-1} \\ \square_{rr}{}^\theta{}_\theta = -4m(r(m^2+4mr+4r^2))^{-1} \\ \square_{\theta\theta}{}^\varphi{}_\varphi = 8mr(m+2r)^{-2} \\ \square_{rr}{}^\varphi{}_\varphi = -4m(r(m^2+4mr+4r^2))^{-1} \\ \square_{\varphi\varphi}{}^t{}_t = -4mr(\sin\theta)^2(m+2r)^{-2} \\ \square_{\theta\theta}{}^t{}_t = -4mr(m+2r)^{-2} \\ \square_{tt}{}^\varphi{}_\varphi = 64mr^3(m-2r)^2(m+2r)^{-8} \\ \square_{tt}{}^\theta{}_\theta = 64mr^3(m-2r)^2(m+2r)^{-8} \\ \square_{\varphi t}{}^t{}_\varphi = 4mr(\sin\theta)^2(m+2r)^{-2} \\ \square_{\theta t}{}^t{}_\theta = 4mr(m+2r)^{-2} \\ \square_{t\varphi}{}^\varphi{}_t = -64mr^3(m-2r)^2(m+2r)^{-8} \\ \square_{t\theta}{}^\theta{}_t = -64mr^3(m-2r)^2(m+2r)^{-8} \end{array} \right. \quad (\text{ex-0607.104})$$

$$R_{ab} \rightarrow 0 \quad (\text{ex-0607.105})$$