

## Example 6-01 Evaluating components

```
1  {\theta, \varphi}::Coordinate.  
2  {a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).  
3  
4  \partial{#}::PartialDerivative.  
5  
6  V := { V_{\theta} = \varphi, V_{\varphi} = \sin(\theta) }.      # cdb(ex-06.100,V)  
7  dV := \partial_{b}{V_{a}} - \partial_{a}{V_{b}}.             # cdb(ex-06.101,dV)  
8  
9  evaluate (dV, V)    # cdb(ex-06.102,dV)
```

$$V_a = [V_\theta = \varphi, V_\varphi = \sin \theta] \quad (\text{ex-06.100})$$

$$\partial_b V_a - \partial_a V_b = \square_{ab} \begin{cases} \square_{\varphi\theta} = \cos \theta - 1 \\ \square_{\theta\varphi} = -\cos \theta + 1 \end{cases} \quad (\text{ex-06.102})$$

## Example 6-02 Riemann tensor of a 2-sphere

```

1  {\theta, \varphi}::Coordinate.
2  {a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).
3
4  \partial{#}::PartialDerivative.
5
6  Gamma := \Gamma^{a}_{b c} -> 1/2 g^{a d} ( \partial_{b}{g_{d c}}
7                                     + \partial_{c}{g_{b d}}
8                                     - \partial_{d}{g_{b c}}).
9
10 Rabcd := R^{a}_{b c d} -> \partial_{c}{\Gamma^{a}_{b d}}
11                        - \partial_{d}{\Gamma^{a}_{b c}}
12                        + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
13                        - \Gamma^{e}_{b c} \Gamma^{a}_{d e}.
14
15 gab := { g_{\theta\theta} = r**2,
16          g_{\varphi\varphi} = r**2 \sin(\theta)**2 }. # cdb(ex-06.201,gab)
17
18 iab := { g^{\theta\theta} = 1/r**2,
19          g^{\varphi\varphi} = 1/(r**2 \sin(\theta)**2) }. # cdb(ex-06.202,iab)
20
21 substitute (Rabcd, Gamma) # cdb(ex-06.203,Gamma)
22
23 evaluate (Gamma, gab+iab, rhsonly=True) # cdb(ex-06.204,Gamma)
24 evaluate (Rabcd, gab+iab, rhsonly=True) # cdb(ex-06.205,Rabcd)
25
26 # convert from a rule to a simple expression
27 Riem := R^{a}_{b c d}.
28 substitute (Riem, Rabcd) # cdb(ex-06.206,Riem)
29
30 from cdb.core.component import *
31
32 RiemCompt = get_component (Riem, $\theta, \varphi, \theta, \varphi$) # cdb(ex-06.207,RiemCompt)

```

$$[g_{\theta\theta} = r^2, \ g_{\varphi\varphi} = r^2(\sin \theta)^2] \quad (\text{ex-06.201})$$

$$[g^{\theta\theta} = r^{-2}, \ g^{\varphi\varphi} = (r^2(\sin \theta)^2)^{-1}] \quad (\text{ex-06.202})$$

$$\Gamma^a_{bc} \rightarrow \square_{cb}^a \begin{cases} \square_{\varphi\theta}^{\varphi} = (\tan \theta)^{-1} \\ \square_{\theta\varphi}^{\varphi} = (\tan \theta)^{-1} \\ \square_{\varphi\varphi}^{\theta} = -\frac{1}{2} \sin(2\theta) \end{cases} \quad (\text{ex-06.204})$$

$$R^a_{bcd} \rightarrow \square_{db}^a{}_c \begin{cases} \square_{\varphi\varphi}^{\theta}{}_{\theta} = (\sin \theta)^2 \\ \square_{\varphi\theta}^{\varphi}{}_{\theta} = -1 \\ \square_{\theta\varphi}^{\theta}{}_{\varphi} = -(\sin \theta)^2 \\ \square_{\theta\theta}^{\varphi}{}_{\varphi} = 1 \end{cases} \quad (\text{ex-06.205})$$

$$\square_{db}^a{}_c \begin{cases} \square_{\varphi\varphi}^{\theta}{}_{\theta} = (\sin \theta)^2 \\ \square_{\varphi\theta}^{\varphi}{}_{\theta} = -1 \\ \square_{\theta\varphi}^{\theta}{}_{\varphi} = -(\sin \theta)^2 \\ \square_{\theta\theta}^{\varphi}{}_{\varphi} = 1 \end{cases} \quad (\text{ex-06.206})$$

$$R^{\theta}{}_{\varphi\theta\varphi} = -(\sin \theta)^2 \quad (\text{ex-06.207})$$

## Example 6-03 Using complete to compute the inverse metric

This version uses `complete` to compute the inverse metric.

```
1  {\theta, \varphi}::Coordinate.
2  {a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).
3
4  \partial{#}::PartialDerivative.
5
6  g^{a b}::InverseMetric.  # essential when using complete (gab, $g^{a b}$)
7
8  Gamma := \Gamma^{a}_{b c} -> 1/2 g^{a d} ( \partial_{b}{g_{d c}}
9                                         + \partial_{c}{g_{b d}}
10                                         - \partial_{d}{g_{b c}}).
11
12  Rabcd := R^{a}_{b c d} -> \partial_{c}{\Gamma^{a}_{b d}}
13                        - \partial_{d}{\Gamma^{a}_{b c}}
14                        + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
15                        - \Gamma^{e}_{b c} \Gamma^{a}_{d e}.
16
17  gab := { g_{\theta\theta} = r**2,
18          g_{\varphi\varphi} = r**2 \sin(\theta)**2 }.      # cdb(ex-06.301,gab)
19
20  complete (gab, $g^{a b}$)                                # cdb(ex-06.302,gab)
21
22  substitute (Rabcd, Gamma)
23
24  evaluate (Gamma, gab, rhsonly=True)                       # cdb(ex-06.303,Gamma)
25  evaluate (Rabcd, gab, rhsonly=True)                       # cdb(ex-06.304,Rabcd)
```

$$[g_{\theta\theta} = r^2, \ g_{\varphi\varphi} = r^2(\sin \theta)^2] \quad (\text{ex-06.301})$$

$$\left[ g_{\theta\theta} = r^2, \ g_{\varphi\varphi} = r^2(\sin \theta)^2, \ g^{\theta\theta} = r^{-2}, \ g^{\varphi\varphi} = (r^2(\sin \theta)^2)^{-1} \right] \quad (\text{ex-06.302})$$

$$\Gamma^a_{bc} \rightarrow \square_{cb}^a \begin{cases} \square_{\varphi\theta}^{\varphi} = (\tan \theta)^{-1} \\ \square_{\theta\varphi}^{\varphi} = (\tan \theta)^{-1} \\ \square_{\varphi\varphi}^{\theta} = -\frac{1}{2} \sin(2\theta) \end{cases} \quad (\text{ex-06.303})$$

$$R^a_{bcd} \rightarrow \square_{db}^a{}_c \begin{cases} \square_{\varphi\varphi}^{\theta} = (\sin \theta)^2 \\ \square_{\varphi\theta}^{\varphi} = -1 \\ \square_{\theta\varphi}^{\theta} = -(\sin \theta)^2 \\ \square_{\theta\theta}^{\varphi} = 1 \end{cases} \quad (\text{ex-06.304})$$

## Example 6-04 Components by scalar projection

This example shows how one component of the Riemann tensor can be computed using a scalar projection.

```
1  {\theta, \varphi}::Coordinate.
2  {a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).
3
4  theta{#}::LaTeXForm{"\theta"}.
5  varphi{#}::LaTeXForm{"\varphi"}.
6
7  # usual definitions for the connection and Riemann tensor
8
9  Gamma := \Gamma^{a}_{b c} -> 1/2 g^{a d} ( \partial_{b}{g_{d c}}
10                                     + \partial_{c}{g_{b d}}
11                                     - \partial_{d}{g_{b c}}).
12
13  Rabcd := R^{a}_{b c d} -> \partial_{c}{\Gamma^{a}_{b d}}
14                                     - \partial_{d}{\Gamma^{a}_{b c}}
15                                     + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
16                                     - \Gamma^{e}_{b c} \Gamma^{a}_{d e}.
17
18  substitute (Rabcd, Gamma)
19
20  gab := { g_{\theta \theta} = r**2,
21           g_{\varphi \varphi} = r**2 \sin(\theta)**2 }. # cdb(ex-06.400,gab)
22
23  iab := { g^{\theta \theta} = 1/r**2,
24           g^{\varphi \varphi} = 1/(r**2 \sin(\theta)**2) }.
25
26  # complete (gab, $g^{a b}$)
27  # evaluate (Rabcd, gab+iab, rhsonly=True)
28
29  # define the basis for vectors and dual vectors
30
31  basis := {theta^{\theta} = 1, varphi^{\varphi} = 1}.
32  dual := {theta_{\theta} = 1, varphi_{\varphi} = 1}.
33
34  # obtain components by contracting with basis
```

```

35
36 compt := R^{a}_{[b c d]} theta_{a} varphi^{b} theta^{c} varphi^{d}. # cdb(ex-06.401,compt)
37 substitute (compt,Rabcd)
38
39 evaluate (compt,gab+iab+basis+dual) # cdb(ex-06.402,compt)
40
41 compt_sympy = compt._sympy_()
42
43 # cdbBeg(print.ex-06.04)
44 print ('type compt = ' + str(type(compt))) # shows that compt is a Cadabra object
45 print ('type ghiphi = ' + str(type(compt_sympy))) # shows that ghiphi is a Python object
46 print ('      compt = ' + str(compt)) # will contain LaTeX markup
47 print ('      ghiphi = ' + str(compt_sympy)) # will be pure Python/SymPy
48 # cdbEnd(print.ex-06.04)
49
50 checkpoint.append (compt)

```

$$R^{\theta}_{\varphi\theta\varphi} = R^a_{bcd}\theta_a\varphi^b\theta^c\varphi^d \quad (\text{ex-06.401})$$

$$= (\sin \theta)^2 \quad (\text{ex-06.402})$$

```

1 type compt = <class 'cadabra2.Ex'>
2 type ghiphi = <class 'sympy.core.power.Pow'>
3     compt = (\sin(\theta))**2
4     ghiphi = sin(theta)**2

```

## Example 6-05 Components by selection

This example shows how one component of the metric tensor can be computed by indexing the result of a call to `evaluate`.

```
1  {\theta, \varphi}::Coordinate.
2  {a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).
3
4  theta{#}::LaTeXForm{"\theta"}.
5  varphi{#}::LaTeXForm{"\varphi"}.
6
7  gab := { g_{\theta \theta}    = r**2,
8           g_{\varphi \varphi} = r**2 \sin(\theta)**2 }.  # cdb(ex-06.500,gab)
9
10 metric := g_{a b}.
11
12 evaluate (metric,gab)
13
14 indcs = metric[2][1][0]          # cdb(ex-06.501,indcs)
15 compt = metric[2][1][1]         # cdb(ex-06.502,compt)
16
17 # cdbBeg(print.ex-06.05)
18 print ('metric = ' + str(metric.input_form())+'\n')  # reveals Cadabra's internal structure for storing metric
19
20 print ('metric[0] = ' + str(metric[0]))
21 print ('metric[1] = ' + str(metric[1]))
22 print ('metric[2] = ' + str(metric[2])+'\n')
23
24 print ('metric[2][1] = '+ str(metric[2][1]))
25 print ('metric[2][1][0] = '+ str(metric[2][1][0]))
26 print ('metric[2][1][1] = '+ str(metric[2][1][1]))
27 # cdbEnd(print.ex-06.05)
28
29 checkpoint.append (indcs)
30 checkpoint.append (compt)
```



$$g_{\varphi\varphi} = g_{[\varphi, \varphi]} \quad (\text{ex-06.501})$$

$$= r^2 (\sin \theta)^2 \quad (\text{ex-06.502})$$

```

1  metric = \components_{a b}({{\theta}, \theta} = (r)**2, {\varphi, \varphi} = (r)**2 (\sin(\theta))**2)
2
3  metric[0] = a
4  metric[1] = b
5  metric[2] = {{\theta}, \theta} = (r)**2, {\varphi, \varphi} = (r)**2 (\sin(\theta))**2}
6
7  metric[2][1] = {\varphi, \varphi} = (r)**2 (\sin(\theta))**2
8  metric[2][1][0] = {\varphi, \varphi}
9  metric[2][1][1] = (r)**2 (\sin(\theta))**2

```