Example 2 Covariant derivatives

```
1    {a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices.
2
3    \nabla{#}::Derivative.
4    \partial{#}::PartialDerivative.
5
6    # rule for covariant derivative of v^{a}
7
8    deriv := \nabla_{a}{v^{b}} -> \partial_{a}{v^{b}} + \Gamma^{b}_{c a} v^{c}.
9
10   # create an expression
11
12   foo := \nabla_{a}{v^{b}}.                    # cdb (ex-02.101,foo)
13
14   # apply the rule, then simplify
15
16   substitute   (foo,deriv)                     # cdb (ex-02.102,foo)
17   canonicalise (foo)                           # cdb (ex-02.103,foo)
18
19   json.append (foo)
```

$$\nabla_a v^b = \partial_a v^b + \Gamma^b{}_{ca} v^c \tag{ex-02.102}$$
$$= \partial_a v^b + \Gamma^{bc}{}_a v_c \tag{ex-02.103}$$

Example 2 Covariant derivatives using "position=independent"

```
1    {a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices(position=independent).
2
3    \nabla{#}::Derivative.
4    \partial{#}::PartialDerivative.
5
6    # rule for covariant derivative of v^{a}
7
8    deriv := \nabla_{a}{v^{b}} -> \partial_{a}{v^{b}} + \Gamma^{b}_{c a} v^{c}.
9
10   # create an expression
11
12   foo := \nabla_{a}{v^{b}}.                    # cdb (ex-02.201,foo)
13
14   # apply the rule, then simplify
15
16   substitute   (foo,deriv)                     # cdb (ex-02.202,foo)
17   canonicalise (foo)                           # cdb (ex-02.203,foo)
18
19   json.append (foo)
```

$$\nabla_a v^b = \partial_a v^b + \Gamma^b{}_{ca} v^c \qquad \text{(ex-02.202)}$$
$$= \partial_a v^b + \Gamma^b{}_{ca} v^c \qquad \text{(ex-02.203)}$$

Example 2 Covariant derivatives using generic rule for deriv

```
1   {a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices(position=independent).
2
3   \nabla{#}::Derivative.
4   \partial{#}::PartialDerivative.
5
6   # template for covariant derivative of a vector
7
8   deriv := \nabla_{a}{A?^{b}} -> \partial_{a}{A?^{b}} + \Gamma^{b}_{c a} A?^{c}.
9
10  # create an expression
11
12  foo := \nabla_{a}{u^{b}} + \nabla_{a}{v^{b}}. # cdb (ex-02.301,foo)
13
14  # apply the rule, then simplify
15
16  substitute   (foo,deriv)                    # cdb (ex-02.302,foo)
17  canonicalise (foo)                          # cdb (ex-02.303,foo)
18
19  json.append (foo)
```

$$\nabla_a u^b + \nabla_a v^b = \partial_a u^b + \Gamma^b{}_{ca} u^c + \partial_a v^b + \Gamma^b{}_{ca} v^c \qquad \text{(ex-02.302)}$$
$$= \partial_a u^b + \Gamma^b{}_{ca} u^c + \partial_a v^b + \Gamma^b{}_{ca} v^c \qquad \text{(ex-02.303)}$$