## Exercise 2.2 Covariant derivative of $v_{ab}$

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\{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u\#\}::Indices(position=independent).
    \nabla{#}::Derivative.
     \partial{#}::PartialDerivative.
     # template for covariant derivative of a vector
     derivU := \nabla_{a}^{A?^{b}} -> \partial_{a}^{A?^{b}} + \Gamma^{b}_{c} a A?^{c}.
     derivD := \nabla_{a}{A?_{b}} -> \partial_{a}{A?_{b}} - \Gamma^{c}_{b} \ A?_{c}.
10
     vab := v_{a b} -> A_{a} B_{b}.
     iab := A_{a} B_{b} -> v_{a} b.
12
13
     pab := \frac{a}{A_{b}} B_{c} -> \frac{a}{A_{b}} B_{c}.
14
15
     # create an object
16
17
     Dvab := \lambda_{a}{v_{b c}}.
                                    # cdb (ex-0202.101,Dvab)
19
     # apply the rule, then simplify
20
21
                    (Dvab, vab)
     substitute
                                      # cdb (ex-0202.102, Dvab)
22
                    (Dvab)
     product_rule
                                      # cdb (ex-0202.103, Dvab)
     substitute
                    (Dvab,derivD)
                                     # cdb (ex-0202.104, Dvab)
                    (Dvab,derivU)
     substitute
                                      # cdb (ex-0202.105, Dvab)
                    (Dvab)
                                      # cdb (ex-0202.106, Dvab)
     distribute
26
                    (Dvab,pab)
                                      # cdb (ex-0202.107, Dvab)
     substitute
27
                    (Dvab)
                                      # cdb (ex-0202.108,Dvab)
     canonicalise
28
                    (Dvab, iab)
                                      # cdb (ex-0202.109,Dvab)
     substitute
29
                                      # cdb (ex-0202.110, Dvab)
     sort_product
                    (Dvab)
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$\nabla_a v_{bc} = \nabla_a \left( A_b B_c \right)$	(ex-0202.102)
$= \nabla_a A_b B_c + A_b \nabla_a B_c$	(ex-0202.103)
$= \left(\partial_a A_b - \Gamma^d{}_{ba} A_d\right) B_c + A_b \left(\partial_a B_c - \Gamma^d{}_{ca} B_d\right)$	(ex-0202.104)
$= \left(\partial_a A_b - \Gamma^d{}_{ba} A_d\right) B_c + A_b \left(\partial_a B_c - \Gamma^d{}_{ca} B_d\right)$	(ex-0202.105)
$= \partial_a A_b B_c - \Gamma^d{}_{ba} A_d B_c + A_b \partial_a B_c - A_b \Gamma^d{}_{ca} B_d$	(ex-0202.106)
$= \partial_a \left( A_b B_c \right) - \Gamma^d{}_{ba} A_d B_c - A_b \Gamma^d{}_{ca} B_d$	(ex-0202.107)
$= \partial_a \left( A_b B_c \right) - \Gamma^d{}_{ba} A_d B_c - A_b \Gamma^d{}_{ca} B_d$	(ex-0202.108)
$= \partial_a v_{bc} - \Gamma^d{}_{ba} v_{dc} - v_{bd} \Gamma^d{}_{ca}$	(ex-0202.109)
$= \partial_a v_{bc} - \Gamma^d{}_{ba} v_{dc} - \Gamma^d{}_{ca} v_{bd}$	(ex-0202.110)