Exercise 6.4 Scalar curavture of a 2-sphere

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{\theta, \varphi}::Coordinate.
    {a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).
    \partial{#}::PartialDerivative.
    g^{a b}::InverseMetric. # essential when using complete (gab, $g^{a b}$)
    Gamma := Gamma^{a}_{b c} -> 1/2 g^{a d} ( partial_{b}_{g_{d c}})
                                              + \partial_{c}{g_{b d}}
                                              - \partial_{d}{g_{b c}}).
10
11
    12
                              - \partial_{d}{\Gamma^{a}_{b c}}
13
                              + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
14
                              - \Gamma^{e}_{b c} \Gamma^{a}_{d e}.
15
16
    Rab := R_{a b} -> R^{c}_{a c b}.
17
18
    R := R -> R_{a b} g^{a b}.
19
20
    gab := { g_{\text{theta}} = r**2,
21
             g_{\text{varphi}} = r**2 \cdot (\theta)**2 .
                                                              # cdb(ex-0604.101,gab)
22
23
    complete (gab, $g^{a b}$)
                                                              # cdb(ex-0604.102,gab)
24
25
    substitute (Rabcd, Gamma)
26
    substitute (Rab, Rabcd)
27
    substitute (R, Rab)
28
29
               (Gamma, gab, rhsonly=True)
                                                              # cdb(ex-0604.103, Gamma)
    evaluate
               (Rabcd, gab, rhsonly=True)
                                                              # cdb(ex-0604.104, Rabcd)
     evaluate
31
                                                              # cdb(ex-0604.105,Rab)
     evaluate
               (Rab,
                       gab, rhsonly=True)
32
                       gab, rhsonly=True)
                                                              # cdb(ex-0604.106,R)
    evaluate
               (R,
33
```

$$[g_{\theta\theta} = r^2, g_{\varphi\varphi} = r^2(\sin\theta)^2]$$
 (ex-0604.101)

$$\left[g_{\theta\theta} = r^2, \ g_{\varphi\varphi} = r^2 (\sin \theta)^2, \ g^{\theta\theta} = r^{-2}, \ g^{\varphi\varphi} = \left(r^2 (\sin \theta)^2 \right)^{-1} \right]$$
 (ex-0604.102)

$$\Gamma^{a}{}_{bc} \to \Box_{cb}{}^{a} \begin{cases} \Box_{\varphi\theta}{}^{\varphi} = (\tan\theta)^{-1} \\ \Box_{\theta\varphi}{}^{\varphi} = (\tan\theta)^{-1} \\ \Box_{\varphi\varphi}{}^{\theta} = -\frac{1}{2}\sin(2\theta) \end{cases}$$
 (ex-0604.103)

$$R^{a}_{bcd} \to \Box_{db}{}^{a}_{c} \begin{cases} \Box_{\varphi\varphi}{}^{\theta}_{\theta} = (\sin\theta)^{2} \\ \Box_{\varphi\theta}{}^{\varphi}_{\theta} = -1 \\ \Box_{\theta\varphi}{}^{\theta}_{\varphi} = -(\sin\theta)^{2} \\ \Box_{\theta\theta}{}^{\varphi}_{\varphi} = 1 \end{cases}$$
 (ex-0604.104)

$$R_{ab} \to \Box_{ba} \begin{cases} \Box_{\varphi\varphi} = (\sin \theta)^2 \\ \Box_{\theta\theta} = 1 \end{cases}$$
 (ex-0604.105)

$$R \to 2r^{-2}$$
 (ex-0604.106)