Exercise 6.4 Scalar curavture of a 2-sphere

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1   {\theta, \varphi}::Coordinate.
2   {a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).
3
4   \partial{#}::PartialDerivative.
5
6   g^{a b}::InverseMetric.   # essential when using complete (gab, $g^{a b}$)
7
8   Gamma := \Gamma^{a}_{b c} -> 1/2 g^{a d} (   \partial_{b}{g_{d c}}
9                                             + \partial_{c}{g_{b d}}
10                                             - \partial_{d}{g_{b c}}).
11
12  Rabcd := R^{a}_{b c d} ->   \partial_{c}{\Gamma^{a}_{b d}}
13                            - \partial_{d}{\Gamma^{a}_{b c}}
14                            + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
15                            - \Gamma^{e}_{b c} \Gamma^{a}_{d e}.
16
17  Rab := R_{a b} -> R^{c}_{a c b}.
18
19  R  := R -> R_{a b} g^{a b}.
20
21  gab := { g_{\theta\theta}   = r**2,
22           g_{\varphi\varphi} = r**2 \sin(\theta)**2 }.      # cdb(ex-0604.101,gab)
23
24  complete   (gab, $g^{a b}$)                                # cdb(ex-0604.102,gab)
25
26  substitute (Rabcd, Gamma)
27  substitute (Rab, Rabcd)
28  substitute (R, Rab)
29
30  evaluate   (Gamma, gab, rhsonly=True)                      # cdb(ex-0604.103,Gamma)
31  evaluate   (Rabcd, gab, rhsonly=True)                      # cdb(ex-0604.104,Rabcd)
32  evaluate   (Rab,   gab, rhsonly=True)                      # cdb(ex-0604.105,Rab)
33  evaluate   (R,     gab, rhsonly=True)                      # cdb(ex-0604.106,R)
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$$\left[g_{\theta\theta} = r^2, g_{\varphi\varphi} = r^2(\sin\theta)^2\right] \tag{ex-0604.101}$$

$$\left[g_{\theta\theta} = r^2, g_{\varphi\varphi} = r^2(\sin\theta)^2, g^{\theta\theta} = r^{-2}, g^{\varphi\varphi} = \left(r^2(\sin\theta)^2\right)^{-1}\right] \tag{ex-0604.102}$$

$$\Gamma^a{}_{bc} \to \Box_{cb}{}^a \begin{cases} \Box_{\varphi\theta}{}^{\varphi} = (\tan\theta)^{-1} \\ \Box_{\theta\varphi}{}^{\varphi} = (\tan\theta)^{-1} \\ \Box_{\varphi\varphi}{}^{\theta} = -\dfrac{1}{2}\sin(2\theta) \end{cases} \tag{ex-0604.103}$$

$$R^a{}_{bcd} \to \Box_{db}{}^a{}_c \begin{cases} \Box_{\varphi\varphi}{}^{\theta}{}_{\theta} = (\sin\theta)^2 \\ \Box_{\varphi\theta}{}^{\varphi}{}_{\theta} = -1 \\ \Box_{\theta\varphi}{}^{\theta}{}_{\varphi} = -(\sin\theta)^2 \\ \Box_{\theta\theta}{}^{\varphi}{}_{\varphi} = 1 \end{cases} \tag{ex-0604.104}$$

$$R_{ab} \to \Box_{ba} \begin{cases} \Box_{\varphi\varphi} = (\sin\theta)^2 \\ \Box_{\theta\theta} = 1 \end{cases} \tag{ex-0604.105}$$

$$R \to 2r^{-2} \tag{ex-0604.106}$$