Exercise 5.5 Commuting covariant derivatives

```
\{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u\#\}::Indices(position=independent).
     ;::Symbol.
     def add_tags (obj,tag):
        n = 0
        ans = Ex('0')
        for i in obj.top().terms():
           foo = obj[i]
           bah = Ex(tag+'_{i-1}'' + str(n) + ')'
10
           ans := @(ans) + @(bah) @(foo).
11
           n = n + 1
12
        return ans
13
14
     def clear_tags (obj,tag):
15
        ans := @(obj).
16
        foo = Ex(tag+'_{a?} -> 1')
17
        substitute (ans,foo)
        return ans
19
     rule := V^{a}_{; b ; c} \rightarrow V^{a}_{; c ; b} - R^{a}_{d b c} V^{d}.
21
22
     expr := V^{a}_{; b ; c} - V^{a}_{; c ; b}. # cdb (ex-0505.100,expr)
23
24
     expr = add_tags (expr,'\\mu')
                                                    # cdb (ex-0505.101,expr)
26
                (expr, $\mu_{0} Q??$)
                                                    # cdb (ex-0505.102,expr)
     ZOOM
27
     substitute (expr, rule)
                                                    # cdb (ex-0505.103,expr)
28
                 (expr)
                                                    # cdb (ex-0505.104,expr)
     unzoom
29
30
     expr = clear_tags (expr,'\\mu')
                                                    # cdb (ex-0505.105,expr)
```

$$\begin{split} V^a{}_{;b;c} - V^a{}_{;c;b} &= \mu_0 V^a{}_{;b;c} - \mu_1 V^a{}_{;c;b} & (\text{ex-0505.101}) \\ &= \mu_0 V^a{}_{;b;c} - \mu_1 V^a{}_{;c;b} & (\text{ex-0505.101}) \\ &= \mu_0 V^a{}_{;b;c} + \dots & (\text{ex-0505.102}) \\ &= \mu_0 \left(V^a{}_{;c;b} - R^a{}_{dbc} V^d \right) + \dots & (\text{ex-0505.103}) \\ &= \mu_0 \left(V^a{}_{;c;b} - R^a{}_{dbc} V^d \right) - \mu_1 V^a{}_{;c;b} & (\text{ex-0505.104}) \\ &= -R^a{}_{dbc} V^d & (\text{ex-0505.105}) \end{split}$$