Exercise 1.2 Christoffel symbol and dg

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1    {a,b,c,d,e,f,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices.
2
3    g_{a b}::Metric.
4    g_{a}^{b}::KroneckerDelta.
5
6    \partial{#}::PartialDerivative.
7
8    GammaU := \Gamma^{a}_{b c} -> (1/2) g^{a d} (  \partial_{b}{g_{d c}}
9                                         + \partial_{c}{g_{b d}}
10                                        - \partial_{d}{g_{b c}} ).
11
12   GammaD := \Gamma_{a b c} -> g_{a d} \Gamma^{d}_{b c}.
13
14   expr := \Gamma_{a b c} + \Gamma_{b a c} - \partial_{c}{g_{a b}}.   # cdb (ex-0102.101,expr)
15
16   substitute          (expr, GammaD)                               # cdb (ex-0102.102,expr)
17   substitute          (expr, GammaU)                               # cdb (ex-0102.103,expr)
18   distribute          (expr)                                       # cdb (ex-0102.104,expr)
19   eliminate_metric    (expr)                                       # cdb (ex-0102.105,expr)
20   eliminate_kronecker (expr)                                       # cdb (ex-0102.106,expr)
21   canonicalise        (expr)                                       # cdb (ex-0102.107,expr)
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$$\Gamma_{abc} + \Gamma_{bac} - \partial_c g_{ab} = g_{ad}\Gamma^d{}_{bc} + g_{bd}\Gamma^d{}_{ac} - \partial_c g_{ab}$$

$$= \frac{1}{2}g_{ad}g^{de}\left(\partial_b g_{ec} + \partial_c g_{be} - \partial_e g_{bc}\right) + \frac{1}{2}g_{bd}g^{de}\left(\partial_a g_{ec} + \partial_c g_{ae} - \partial_e g_{ac}\right) - \partial_c g_{ab}$$

$$= \frac{1}{2}g_{ad}g^{de}\partial_b g_{ec} + \frac{1}{2}g_{ad}g^{de}\partial_c g_{be} - \frac{1}{2}g_{ad}g^{de}\partial_e g_{bc} + \frac{1}{2}g_{bd}g^{de}\partial_a g_{ec} + \frac{1}{2}g_{bd}g^{de}\partial_c g_{ae} - \frac{1}{2}g_{bd}g^{de}\partial_e g_{ac} - \partial_c g_{ab}$$

$$= \frac{1}{2}g_a{}^e\partial_b g_{ec} + \frac{1}{2}g_a{}^e\partial_c g_{be} - \frac{1}{2}g_a{}^e\partial_e g_{bc} + \frac{1}{2}g_b{}^e\partial_a g_{ec} + \frac{1}{2}g_b{}^e\partial_c g_{ae} - \frac{1}{2}g_b{}^e\partial_e g_{ac} - \partial_c g_{ab}$$

$$= \frac{1}{2}\partial_c g_{ba} - \frac{1}{2}\partial_c g_{ab}$$

$$= 0$$