

## Exercise 2.3 Covariant derivative of $v^a_b$

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1 {a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices(position=independent).
2
3 \nabla{#}::Derivative.
4 \partial{#}::PartialDerivative.
5
6 # template for covariant derivative of a vector
7
8 derivU := \nabla_{a}{A^{b}} -> \partial_{a}{A^{b}} + \Gamma^{b}_{c a} A^{c}.
9 derivD := \nabla_{a}{A_{b}} -> \partial_{a}{A_{b}} - \Gamma^{c}_{c b} A_{c}.
10
11 vab := v^{a}_{b} -> A^{a} B_{b}.
12 iab := A^{a} B_{b} -> v^{a}_{b}.
13
14 pab := \partial_{a}{A^{b}} B_{c} -> \partial_{a}{A^{b} B_{c}} - A^{b} \partial_{a}{B_{c}}.
15
16 # create an object
17
18 Dvab := \nabla_{a}{v^{b}_{c}}. # cdb (ex-0203.101,Dvab)
19
20 # apply the rule, then simplify
21
22 substitute (Dvab,vab) # cdb (ex-0203.102,Dvab)
23 product_rule (Dvab) # cdb (ex-0203.103,Dvab)
24 substitute (Dvab,derivD) # cdb (ex-0203.104,Dvab)
25 substitute (Dvab,derivU) # cdb (ex-0203.105,Dvab)
26 distribute (Dvab) # cdb (ex-0203.106,Dvab)
27 substitute (Dvab,pab) # cdb (ex-0203.107,Dvab)
28 canonicalise (Dvab) # cdb (ex-0203.108,Dvab)
29 substitute (Dvab,iab) # cdb (ex-0203.109,Dvab)
30 sort_product (Dvab) # cdb (ex-0203.110,Dvab)
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$$\nabla_a v^b{}_c = \nabla_a (A^b B_c) \quad (\text{ex-0203.102})$$

$$= \nabla_a A^b B_c + A^b \nabla_a B_c \quad (\text{ex-0203.103})$$

$$= \nabla_a A^b B_c + A^b (\partial_a B_c - \Gamma^d{}_{ca} B_d) \quad (\text{ex-0203.104})$$

$$= (\partial_a A^b + \Gamma^b{}_{da} A^d) B_c + A^b (\partial_a B_c - \Gamma^d{}_{ca} B_d) \quad (\text{ex-0203.105})$$

$$= \partial_a A^b B_c + \Gamma^b{}_{da} A^d B_c + A^b \partial_a B_c - A^b \Gamma^d{}_{ca} B_d \quad (\text{ex-0203.106})$$

$$= \partial_a (A^b B_c) + \Gamma^b{}_{da} A^d B_c - A^b \Gamma^d{}_{ca} B_d \quad (\text{ex-0203.107})$$

$$= \partial_a (A^b B_c) + \Gamma^b{}_{da} A^d B_c - A^b \Gamma^d{}_{ca} B_d \quad (\text{ex-0203.108})$$

$$= \partial_a v^b{}_c + \Gamma^b{}_{da} v^d{}_c - v^b{}_d \Gamma^d{}_{ca} \quad (\text{ex-0203.109})$$

$$= \partial_a v^b{}_c + \Gamma^b{}_{da} v^d{}_c - \Gamma^d{}_{ca} v^b{}_d \quad (\text{ex-0203.110})$$