Example 6-01 Evaluating components

```
{\theta, \varphi}::Coordinate.
{a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).

\partial{#}::PartialDerivative.

V  := { V_{\theta} = \varphi, V_{\varphi} = \sin(\theta) }.   # cdb(ex-06.100,V)
dV := \partial_{b}{V_{a}} - \partial_{a}{V_{b}}.              # cdb(ex-06.101,dV)

evaluate (dV, V)   # cdb(ex-06.102,dV)
```

$$V_a = [V_\theta = \varphi,\ V_\varphi = \sin\theta] \tag{ex-06.100}$$

$$\partial_b V_a - \partial_a V_b = \square_{ab} \begin{cases} \square_{\varphi\theta} = \cos\theta - 1 \\ \square_{\theta\varphi} = -\cos\theta + 1 \end{cases} \tag{ex-06.102}$$

Example 6-02 Riemann tensor of a 2-sphere

```
{\theta, \varphi}::Coordinate.
{a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).

\partial{#}::PartialDerivative.

Gamma := \Gamma^{a}_{b c} -> 1/2 g^{a d} (   \partial_{b}{g_{d c}}
                                           + \partial_{c}{g_{b d}}
                                           - \partial_{d}{g_{b c}}).

Rabcd := R^{a}_{b c d} ->   \partial_{c}{\Gamma^{a}_{b d}}
                          - \partial_{d}{\Gamma^{a}_{b c}}
                          + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
                          - \Gamma^{e}_{b c} \Gamma^{a}_{d e}.

gab := { g_{\theta\theta}   = r**2,
         g_{\varphi\varphi} = r**2 \sin(\theta)**2 }.      # cdb(ex-06.201,gab)

iab := { g^{\theta\theta}   = 1/r**2,
         g^{\varphi\varphi} = 1/(r**2 \sin(\theta)**2) }.  # cdb(ex-06.202,iab)

substitute (Rabcd, Gamma)                                  # cdb(ex-06.203,Gamma)

evaluate   (Gamma, gab+iab, rhsonly=True)                  # cdb(ex-06.204,Gamma)
evaluate   (Rabcd, gab+iab, rhsonly=True)                  # cdb(ex-06.205,Rabcd)

# convert from a rule to a simple expression
Riem := R^{a}_{b c d}.
substitute (Riem, Rabcd)                                   # cdb(ex-06.206,Riem)

from cdb.core.component import *

RiemCompt = get_component (Riem, $\theta, \varphi, \theta, \varphi$)   # cdb(ex-06.207,RiemCompt)
```

$$\left[g_{\theta\theta} = r^2,\ g_{\varphi\varphi} = r^2(\sin\theta)^2\right] \tag{ex-06.201}$$

$$\left[g^{\theta\theta} = r^{-2},\ g^{\varphi\varphi} = \left(r^2(\sin\theta)^2\right)^{-1}\right] \tag{ex-06.202}$$

$$\Gamma^a{}_{bc} \to \square_{cb}{}^a \begin{cases} \square_{\varphi\theta}{}^{\varphi} = (\tan\theta)^{-1} \\ \square_{\theta\varphi}{}^{\varphi} = (\tan\theta)^{-1} \\ \square_{\varphi\varphi}{}^{\theta} = -\dfrac{1}{2}\sin(2\theta) \end{cases} \tag{ex-06.204}$$

$$R^a{}_{bcd} \to \square_{db}{}^a{}_c \begin{cases} \square_{\varphi\varphi}{}^{\theta}{}_{\theta} = (\sin\theta)^2 \\ \square_{\varphi\theta}{}^{\varphi}{}_{\theta} = -1 \\ \square_{\theta\varphi}{}^{\theta}{}_{\varphi} = -(\sin\theta)^2 \\ \square_{\theta\theta}{}^{\varphi}{}_{\varphi} = 1 \end{cases} \tag{ex-06.205}$$

$$\square_{db}{}^a{}_c \begin{cases} \square_{\varphi\varphi}{}^{\theta}{}_{\theta} = (\sin\theta)^2 \\ \square_{\varphi\theta}{}^{\varphi}{}_{\theta} = -1 \\ \square_{\theta\varphi}{}^{\theta}{}_{\varphi} = -(\sin\theta)^2 \\ \square_{\theta\theta}{}^{\varphi}{}_{\varphi} = 1 \end{cases} \tag{ex-06.206}$$

$$R^{\theta}{}_{\varphi\varphi\theta} = -(\sin\theta)^2 \tag{ex-06.207}$$

Example 6-03 Using `complete` to compute the inverse metric

This version uses `complete` to compute the inverse metric.

```
{\theta, \varphi}::Coordinate.
{a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).

\partial{#}::PartialDerivative.

g^{a b}::InverseMetric.  # essential when using complete (gab, $g^{a b}$)

Gamma := \Gamma^{a}_{b c} -> 1/2 g^{a d} (   \partial_{b}{g_{d c}}
                                           + \partial_{c}{g_{b d}}
                                           - \partial_{d}{g_{b c}}).

Rabcd := R^{a}_{b c d} ->   \partial_{c}{\Gamma^{a}_{b d}}
                          - \partial_{d}{\Gamma^{a}_{b c}}
                          + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
                          - \Gamma^{e}_{b c} \Gamma^{a}_{d e}.

gab := { g_{\theta\theta}   = r**2,
         g_{\varphi\varphi} = r**2 \sin(\theta)**2 }.      # cdb(ex-06.301,gab)

complete   (gab, $g^{a b}$)                                # cdb(ex-06.302,gab)

substitute (Rabcd, Gamma)

evaluate   (Gamma, gab, rhsonly=True)                      # cdb(ex-06.303,Gamma)
evaluate   (Rabcd, gab, rhsonly=True)                      # cdb(ex-06.304,Rabcd)
```

$$\left[g_{\theta\theta} = r^2,\ g_{\varphi\varphi} = r^2(\sin\theta)^2\right] \tag{ex-06.301}$$

$$\left[g_{\theta\theta} = r^2,\ g_{\varphi\varphi} = r^2(\sin\theta)^2,\ g^{\theta\theta} = r^{-2},\ g^{\varphi\varphi} = \left(r^2(\sin\theta)^2\right)^{-1}\right] \tag{ex-06.302}$$

$$\Gamma^{a}{}_{bc} \rightarrow \Box_{cb}{}^{a} \begin{cases} \Box_{\varphi\theta}{}^{\varphi} = (\tan\theta)^{-1} \\ \Box_{\theta\varphi}{}^{\varphi} = (\tan\theta)^{-1} \\ \Box_{\varphi\varphi}{}^{\theta} = -\dfrac{1}{2}\sin(2\theta) \end{cases} \tag{ex-06.303}$$

$$R^{a}{}_{bcd} \rightarrow \Box_{db}{}^{a}{}_{c} \begin{cases} \Box_{\varphi\varphi}{}^{\theta}{}_{\theta} = (\sin\theta)^2 \\ \Box_{\varphi\theta}{}^{\varphi}{}_{\theta} = -1 \\ \Box_{\theta\varphi}{}^{\theta}{}_{\varphi} = -(\sin\theta)^2 \\ \Box_{\theta\theta}{}^{\varphi}{}_{\varphi} = 1 \end{cases} \tag{ex-06.304}$$

Example 6-04 Components by scalar projection

This example shows how one component of the Riemann tensor can be computed using a scalar projection.

```
{\theta, \varphi}::Coordinate.
{a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).

theta{#}::LaTeXForm{"\theta"}.
varphi{#}::LaTeXForm{"\varphi"}.

# usual definitions for the connection and Riemann tensor

Gamma := \Gamma^{a}_{b c} -> 1/2 g^{a d} (   \partial_{b}{g_{d c}}
                                           + \partial_{c}{g_{b d}}
                                           - \partial_{d}{g_{b c}}).

Rabcd := R^{a}_{b c d} ->   \partial_{c}{\Gamma^{a}_{b d}}
                          - \partial_{d}{\Gamma^{a}_{b c}}
                          + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
                          - \Gamma^{e}_{b c} \Gamma^{a}_{d e}.

gab := { g_{\theta \theta}   = r**2,
         g_{\varphi \varphi} = r**2 \sin(\theta)**2 }.   # cdb(ex-06.400,gab)

iab := { g^{\theta\theta}   = 1/r**2,
         g^{\varphi\varphi} = 1/(r**2 \sin(\theta)**2) }.

substitute (Rabcd, Gamma)
evaluate   (Rabcd, gab+iab, rhsonly=True)

# above code just to compute Rabcd
# following code is all that is needed for the scalar projection method

# define the basis for vectors and dual vectors

basis := {theta^{\theta} = 1, varphi^{\varphi} = 1}.
dual  := {theta_{\theta} = 1, varphi_{\varphi} = 1}.

```

```
# obtain components by contracting with basis

compt := R^{a}_{b c d} theta_{a} varphi^{b} theta^{c} varphi^{d}.   # cdb(ex-06.401,compt)
substitute (compt,Rabcd)

evaluate (compt,basis+dual)                                         # cdb(ex-06.402,compt)

compt_sympy = compt._sympy_()

# cdbBeg(print.ex-06.04)
print ('type compt   = ' + str(type(compt)))         # shows that compt is a Cadabra object
print ('type gphiphi = ' + str(type(compt_sympy)))   # shows that gphiphi is a Python object
print ('     compt   = ' + str(compt))               # will contain LaTeX markup
print ('     gphiphi = ' + str(compt_sympy))         # will be pure Python/SymPy
# cdbEnd(print.ex-06.04)

checkpoint.append (compt)
```

$$R^{\theta}{}_{\varphi\theta\varphi} = R^{a}{}_{bcd}\theta_{a}\varphi^{b}\theta^{c}\varphi^{d} \tag{ex-06.401}$$

$$= (\sin\theta)^{2} \tag{ex-06.402}$$

```
type compt   = <class 'cadabra2.Ex'>
type gphiphi = <class 'sympy.core.power.Pow'>
     compt   = (\sin(\theta))**2
     gphiphi = sin(theta)**2
```

Example 6-05 Components by selection

This example shows how one component of the metric tensor can be computed by indexing the result of a call to `evaluate`.

```
{\theta, \varphi}::Coordinate.
{a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).

theta{#}::LaTeXForm{"\theta"}.
varphi{#}::LaTeXForm{"\varphi"}.

gab := { g_{\theta \theta}   = r**2,
         g_{\varphi \varphi} = r**2 \sin(\theta)**2 }.   # cdb(ex-06.500,gab)

metric := g_{a b}.

evaluate (metric,gab)

indcs = metric[2][1][0]                                  # cdb(ex-06.501,indcs)
compt = metric[2][1][1]                                  # cdb(ex-06.502,compt)

# cdbBeg(print.ex-06.05)
print ('metric = ' + str(metric.input_form())+'\n')  # reveals Cadabra's internal structure for storing metric

print ('metric[0] = ' + str(metric[0]))
print ('metric[1] = ' + str(metric[1]))
print ('metric[2] = ' + str(metric[2])+'\n')

print ('metric[2][1] = '+ str(metric[2][1]))
print ('metric[2][1][0] = '+ str(metric[2][1][0]))
print ('metric[2][1][1] = '+ str(metric[2][1][1]))
# cdbEnd(print.ex-06.05)

checkpoint.append (indcs)
checkpoint.append (compt)
```

$$g_{\varphi\varphi} = g_{[\varphi,\ \varphi]} \qquad \text{(ex-06.501)}$$
$$= r^2(\sin\theta)^2 \qquad \text{(ex-06.502)}$$

```
metric = \components_{a b}({{\theta, \theta} = (r)**2, {\varphi, \varphi} = (r)**2 (\sin(\theta))**2})

metric[0] = a
metric[1] = b
metric[2] = {{\theta, \theta} = (r)**2, {\varphi, \varphi} = (r)**2 (\sin(\theta))**2}

metric[2][1] = {\varphi, \varphi} = (r)**2 (\sin(\theta))**2
metric[2][1][0] = {\varphi, \varphi}
metric[2][1][1] = (r)**2 (\sin(\theta))**2
```