Example 14 The Weyl tensor is conformally invariant

This example shows that the Weyl tensor is conformally invariant. That is, for a pair of metrics g and \overline{g} related by a conformal transformation, $\overline{g}_{ab} = \phi g_{ab}$ then $\overline{C}^a_{bcd} = C^a_{bcd}$ or equally $\overline{C}_{abcd} = \phi C_{abcd}$.

```
{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,u,v,w#}::Indices (position=independent).
               \partial{#}::PartialDerivative.
               g_{a b}::Metric.
               g^{a b}::InverseMetric.
               g_{a}^{b}::KroneckerDelta.
               GammaU := Gamma^{a}_{b c} \rightarrow 1/2 g^{a d} ( partial_{b}_{g_{d c}})
                                                                                                                                                         + \partial_{c}{g_{b d}}
10
                                                                                                                                                           - \partial_{d}{g_{b c}}).
11
12
               GammaD := \Gamma_{a b c} -> 1/2 ( \partial_{b}_{g_{a c}})
13
                                                                                                                           + \partial_{c}{g_{b a}}
14
                                                                                                                            - \partial_{a}{g_{b c}}).
15
16
               Rabcd := R_{a b c d} \rightarrow \operatorname{partial}_{c}{\operatorname{Gamma}_{a b d}}
17
                                                                                         - \partial_{d}{\Gamma_{a b c}}
18
                                                                                         + \Gamma_{e a d} \Gamma^{e}_{b c}
19
                                                                                          - \Gamma_{e a c} \Gamma^{e}_{b d}.
20
21
                                       := R_{a b} -> g^{c d} R_{a c b d}.
               Rab
22
23
               Rscalar := R \rightarrow g^{a} b R_{a} b.
24
^{25}
               # Weyl in 4-dimensions
26
27
               Cabcd := R_{a b c d} - (1/2) (R_{a c} g_{b d} - R_{a d} g_{b c})
28
                                                                                -(1/2) (g_{a c} R_{b d} - g_{a d} R_{b c})
29
                                                                               + (R/6) (g_{a c} g_{b d} - g_{a d} g_{b c}).
30
31
               {\partial_{a b}{\phi},\partial_{a}{\phi},\phi}::SortOrder.
32
               {\hat{a}}_{g_{b_{1}},p_{a_{1}}}(a_{a_{1}},p_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_{1}},a_{a_
```

```
34
     substitute (Cabcd, Rscalar)
     substitute (Cabcd,Rab)
     substitute (Cabcd, Rabcd)
37
     substitute (Cabcd, GammaU)
38
     substitute (Cabcd, GammaD)
39
40
                     (Cabcd)
     distribute
41
42
     sort_product
                     (Cabcd)
43
     rename_dummies (Cabcd)
44
     canonicalise
                     (Cabcd)
45
46
     # this is the Weyl tensor on the base metric
47
     baseC := @(Cabcd).
48
49
     conformal := \{g_{a b} \rightarrow \phi_{a b}, g^{a b} \rightarrow (1/phi) g^{a b}\}.
50
51
     substitute
                     (Cabcd, conformal)
     product_rule (Cabcd)
     distribute
                     (Cabcd)
54
     product_rule (Cabcd)
55
     distribute
                     (Cabcd)
56
57
                     (Cabcd, "simplify")
     map_sympy
58
     sort_product
                     (Cabcd)
60
     rename_dummies (Cabcd)
61
     canonicalise
                     (Cabcd)
62
63
     # this is the Weyl tensor on the conformal metric
64
     confC := @(Cabcd).
66
     # their difference, should be zero
67
     diff := @(confC) - \phi @(baseC). # cdb (ex-14.diff.100,diff)
68
69
     distribute
                     (diff)
70
     sort_product (diff)
```

```
rename dummies (diff)
                    (diff) # cdb (ex-14.diff.101,diff)
     canonicalise
74
     # this trick is not essential but it does reduce the number of terms in diff
75
                    (diff, \alpha_{a}{\beta_{a}}) = (diff, \alpha_{a}) -> g_{c} d b a
     substitute
76
                    (diff, \alpha_{a}\{g_{b c}\} \rightarrow 0)
     substitute
77
                    (diff, g_{c d}) + cdb (ex-14.diff.102, diff)
     substitute
79
     # standard expressions in 4-d
                    (diff, g_{a b} g^{a b} -> 4, repeat=True)
                                                                       # cdb (ex-14.diff.201,diff)
     substitute
81
                    (diff, g_{a b} g^{c b} -> g_{a}^{c}, repeat=True) # cdb (ex-14.diff.202, diff)
     substitute
82
                    (diff, g_{b a} g^{b c} -> g_{a}^{c}, repeat=True) # cdb (ex-14.diff.203, diff)
     substitute
                    (diff, g_{a}^{a} -> 4, repeat=True)
                                                                       # cdb (ex-14.diff.204,diff)
     substitute
                    (diff, $g^{a}_{a} -> 4$, repeat=True)
                                                                       # cdb (ex-14.diff.205,diff)
     substitute
     eliminate_kronecker (diff)
                                                                       # cdb (ex-14.diff.206,diff)
87
     # need a second round since the above block introduces new terms that match those just eliminated
88
                    (diff, g_{a b} g^{a b} \rightarrow 4, repeat=True)
                                                                       # cdb (ex-14.diff.301,diff)
     substitute
                    (diff, g_{a b} g^{c b} -> g_{a}^{c}, repeat=True) # cdb (ex-14.diff.302, diff)
     substitute
                    (diff, $g_{b a} g^{b c} -> g_{a}^{c}$, repeat=True) # cdb (ex-14.diff.303, diff)
     substitute
                    (diff, g_{a}^{a} -> 4, repeat=True)
                                                                       # cdb (ex-14.diff.304,diff)
     substitute
92
                    (diff, $g^{a}_{a} -> 4$, repeat=True)
                                                                       # cdb (ex-14.diff.305,diff)
     substitute
93
     eliminate_kronecker (diff)
                                                                       # cdb (ex-14.diff.306,diff)
94
95
     sort_product
                    (diff)
     rename_dummies (diff)
                    (diff) # cdb (ex-14.diff.400,diff)
     canonicalise
98
99
     checkpoint.append (baseC)
100
     checkpoint.append (confC)
```

$$\Delta = \frac{1}{2} \partial_{bc} \varphi g_{ad} - \frac{1}{2} \partial_{bc} \varphi g_{bd} - \frac{1}{2} \partial_{bd} \varphi g_{ac} + \frac{1}{2} \partial_{ad} \varphi g_{bc} + \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{bc} g_{df} g^{ef} - \frac{1}{4} \partial_{b} \varphi \partial_{d} \varphi \varphi^{-1} g_{ac} g_{ef} g^{ef} - \frac{1}{4} \partial_{c} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} - \frac{1}{4} \partial_{c} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} - \frac{1}{4} \partial_{c} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} - \frac{1}{4} \partial_{c} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} - \frac{1}{4} \partial_{c} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} - \frac{1}{4} \partial_{c} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} + \frac{1}{4} \partial_{c} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} - \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ac} g_{ef} g^{ef} - \frac{1}{4} \partial_{c} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} + \frac{1}{4} \partial_{c} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} - \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} - \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} - \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} - \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} - \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} - \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} + \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} - \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} + \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} - \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} + \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} - \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} + \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} - \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} - \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} - \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} g^{ef} + \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} g^{ef} + \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} g^{ef} + \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} g^{ef} - \frac{1}{4} \partial_{c} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} g^{ef} + \frac{1}{4} \partial_{a} \varphi \partial_{c} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} g^{ef} g^{ef} - \frac{1}{4} \partial_{c} \varphi \partial_{e} \varphi \varphi^{-1} g_{ad} g_{ef} g^{ef} g^{ef} g^{ef} + \frac{1}{4} \partial_{c} \varphi \partial_{e} \varphi \varphi^{-1} g_{ad$$

$$\Delta = -\frac{1}{2}\partial_{bc}\phi g_{ad} + \frac{1}{2}\partial_{ac}\phi g_{bd} + \frac{1}{2}\partial_{bd}\phi g_{ac} - \frac{1}{2}\partial_{ad}\phi g_{bc} + \frac{1}{4}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{bc}g_{df}g^{ef} + \frac{1}{4}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{ac}g_{cf}g^{ef} + \frac{1}{4}\partial_{d}\phi\partial_{c}\phi\phi^{-1}g_{af}g_{bc}g^{ef} + \frac{1}{4}\partial_{c}\phi\partial_{c}\phi\phi^{-1}g_{ad}g_{bf}g^{ef} - \frac{1}{12}\partial_{c}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g^{ef} - \frac{1}{4}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{ac}g_{df}g^{ef} - \frac{1}{4}\partial_{c}\phi\partial_{c}\phi\phi^{-1}g_{ad}g_{bf}g^{ef} - \frac{1}{4}\partial_{c}\phi\partial_{c}\phi\phi^{-1}g_{ad}g_{bf}g^{ef} - \frac{1}{4}\partial_{c}\phi\partial_{c}\phi\phi^{-1}g_{ac}g_{df}g^{ef} - \frac{1}{4}\partial_{c}\phi\partial_{c}\phi\phi^{-1}g_{ac}g_{df}g_{df}g^{ef} - \frac{1}{4}\partial_{c}\phi\partial_{c}\phi\phi^{-1}g_{ac}g_{df}g_{df}g^{ef} - \frac{1}{4}\partial_{c}\phi\partial_{c}\phi\phi^{-1}g_{ac}g_{df}g_{df}g^{ef} + \frac{1}{4}\partial_{c}\phi\partial_{c}\phi\phi^{-1}g_{ac}g_{df}g_{df}g^{ef} + \frac{1}{4}\partial_{c}\phi\partial_{c}\phi\phi^{-1}g_{ac}g_{df}g_{df}g^{ef} - \frac{1}{4}\partial_{c}\phi\partial_{c}\phi\phi^{-1}g_{a$$

$$\Delta = -\frac{1}{2}\partial_{bc}\phi g_{ad} + \frac{1}{2}\partial_{ac}\phi g_{bd} + \frac{1}{2}\partial_{bd}\phi g_{ac} - \frac{1}{2}\partial_{ad}\phi g_{bc} + \frac{1}{4}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{be}g_{d}^{e} + \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{bc}g_{d}^{e} + \frac{1}{4}\partial_{b}\phi\partial_{d}\phi\phi^{-1}g_{ae}g_{c}^{e} + \frac{1}{4}\partial_{d}\phi\partial_{e}\phi\phi^{-1}g_{ae}^{e}g_{bc} \\ + \frac{1}{4}\partial_{b}\phi\partial_{e}\phi\phi^{-1}g_{ad}g_{c}^{e} + \frac{1}{4}\partial_{c}\phi\partial_{e}\phi\phi^{-1}g_{ad}g_{b}^{e} - \frac{1}{12}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g^{ef} - \frac{1}{4}\partial_{a}\phi\partial_{d}\phi\phi^{-1}g_{be}g_{c}^{e} - \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{bd}g_{c}^{e} - \frac{1}{4}\partial_{b}\phi\partial_{c}\phi\phi^{-1}g_{ae}g_{d}^{e} \\ - \frac{1}{4}\partial_{c}\phi\partial_{e}\phi\phi^{-1}g_{a}^{e}g_{bd} - \frac{1}{4}\partial_{b}\phi\partial_{e}\phi\phi^{-1}g_{ae}g_{d}^{e} - \frac{1}{4}\partial_{d}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{b}^{e} + \frac{1}{12}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{c}\phi g_{a}^{e}g_{bd} - \frac{1}{6}\partial_{ef}\phi g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{ae}\phi g_{bg}g^{e} \\ - \frac{1}{8}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{bd}g_{ef}g_{g}^{f}g^{eg} - \frac{1}{4}\partial_{c}\phi\partial_{f}\phi\phi^{-1}g_{ag}g_{bd}g_{c}^{e}g^{ef} + \frac{1}{4}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{a}^{e}g_{bd}g_{c}^{ef} + \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{bd}g_{ef}g_{g}^{f}g^{eg} + \frac{1}{4}\partial_{c}\phi\partial_{e}\phi\phi^{-1}g_{ag}g_{bd}g_{e}^{ef}g^{ef}g^{ef}g_{e$$

$$\Delta = -\frac{1}{2}\partial_{bc}\phi g_{ad} + \frac{1}{2}\partial_{ac}\phi g_{bd} + \frac{1}{2}\partial_{bd}\phi g_{ac} - \frac{1}{2}\partial_{ad}\phi g_{bc} + \frac{1}{4}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{be}g_{d}^{e} + \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{bc}g_{d}^{e} + \frac{1}{4}\partial_{b}\phi\partial_{d}\phi\phi^{-1}g_{ae}g_{c}^{e} + \frac{1}{4}\partial_{d}\phi\partial_{e}\phi\phi^{-1}g_{ae}g_{bc}^{e} \\ + \frac{1}{4}\partial_{b}\phi\partial_{e}\phi\phi^{-1}g_{ad}g_{c}^{e} + \frac{1}{4}\partial_{c}\phi\partial_{e}\phi\phi^{-1}g_{ad}g_{b}^{e} - \frac{1}{12}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g^{ef} - \frac{1}{4}\partial_{a}\phi\partial_{d}\phi\phi^{-1}g_{be}g_{c}^{e} - \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{bd}g_{c}^{e} - \frac{1}{4}\partial_{b}\phi\partial_{c}\phi\phi^{-1}g_{ae}g_{d}^{e} \\ - \frac{1}{4}\partial_{c}\phi\partial_{e}\phi\phi^{-1}g_{a}^{e}g_{bd} - \frac{1}{4}\partial_{b}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{d}^{e} - \frac{1}{4}\partial_{d}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{b}^{e} + \frac{1}{12}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{c}\phi}g_{a}^{e}g_{bd} - \frac{1}{6}\partial_{e}f\phi g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{ae}g_{bd}g^{ef} - \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bg}g^{ef} - \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bg}g^{ef} - \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bg}g^{ef} - \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bg}g^{ef} - \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bg}g^{ef} - \frac{1}{4}\partial_{e}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bg}g^{ef} - \frac{1}{4}\partial_{e}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bg}g^{ef} - \frac{1}{4}\partial_{e}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bg}g^{ef} - \frac{1}{4}\partial_{e}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bg}g^{ef} - \frac{1}{4}\partial_{e}\phi\partial_{e$$

$$\Delta = -\frac{1}{2} \partial_{bc} \phi g_{ad} + \frac{1}{2} \partial_{ac} \phi g_{bd} + \frac{1}{2} \partial_{bd} \phi g_{ac} - \frac{1}{2} \partial_{ad} \phi g_{bc} + \frac{1}{4} \partial_{a} \phi \partial_{c} \phi \phi^{-1} g_{be} g_{d}^{e} + \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{c}^{e} + \frac{1}{4} \partial_{b} \phi \partial_{d} \phi \phi^{-1} g_{ae} g_{c}^{e} + \frac{1}{4} \partial_{d} \phi \partial_{e} \phi \phi^{-1} g_{ae}^{e} g_{bc} \\ + \frac{1}{4} \partial_{b} \phi \partial_{e} \phi \phi^{-1} g_{ad} g_{c}^{e} + \frac{1}{4} \partial_{c} \phi \partial_{e} \phi \phi^{-1} g_{ad} g_{b}^{e} - \frac{1}{12} \partial_{e} \phi \partial_{f} \phi \phi^{-1} g_{ae} g_{bc}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{d} \phi \phi^{-1} g_{be} g_{c}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{bd} g_{c}^{e} - \frac{1}{4} \partial_{b} \phi \partial_{c} \phi \phi^{-1} g_{ae} g_{d}^{e} \\ - \frac{1}{4} \partial_{c} \phi \partial_{e} \phi \phi^{-1} g_{a}^{e} g_{bd} - \frac{1}{4} \partial_{b} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{d}^{e} - \frac{1}{4} \partial_{d} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{b}^{e} + \frac{1}{12} \partial_{e} \phi \partial_{f} \phi \phi^{-1} g_{ae} g_{bd} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{bd} g^{e}^{e} - \frac{1}{4} \partial_{b} \phi \partial_{c} \phi \phi^{-1} g_{ae} g_{b}^{e} g_{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{bd} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{bd} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{bd} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{bd} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{bd} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{bd} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{bd} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{bd} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{bd} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{bd} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{bd} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{bd} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{bd} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{bd} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{be} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{be} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{be} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{be} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^{-1} g_{ae} g_{be} g^{e}^{e} - \frac{1}{4} \partial_{a} \phi \partial_{e} \phi \phi^$$

$$\Delta = -\frac{1}{2}\partial_{bc}\phi g_{ad} + \frac{1}{2}\partial_{ac}\phi g_{bd} + \frac{1}{2}\partial_{bd}\phi g_{ac} - \frac{1}{2}\partial_{ad}\phi g_{bc} + \frac{1}{4}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{be}g_{d}^{e} + \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{c}^{e} + \frac{1}{4}\partial_{b}\phi\partial_{d}\phi\phi^{-1}g_{ae}g_{c}^{e} + \frac{1}{4}\partial_{d}\phi\partial_{c}\phi\phi^{-1}g_{ad}^{e}g_{bc} + \frac{1}{4}\partial_{b}\phi\partial_{c}\phi\phi^{-1}g_{ad}g_{c}^{e} - \frac{1}{4}\partial_{b}\phi\partial_{c}\phi\phi^{-1}g_{ad}g_{c}^{e} - \frac{1}{4}\partial_{b}\phi\partial_{c}\phi\phi^{-1}g_{ae}g_{d}^{e} - \frac{1}{4}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{ae}g_{d}^{e}g_{d}^{e} - \frac{1}{4}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{ae}g_{d}^{e}g_{d}^{e} - \frac{1}{4}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{ae}g_{d}^{e}g_{d}^{e} - \frac{1}{4}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{ae}g_{d}^{e}g_{d}^{e} - \frac{1}{4}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{ae}g_{d}^{e}g_$$

$$\Delta = -\frac{1}{4}\partial_{bc}\phi g_{ad} + \frac{1}{4}\partial_{ac}\phi g_{bd} + \frac{1}{4}\partial_{bd}\phi g_{ac} - \frac{1}{4}\partial_{ad}\phi g_{bc} + \frac{1}{4}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{bd} - \frac{1}{4}\partial_{a}\phi\partial_{d}\phi\phi^{-1}g_{bc} + \frac{1}{4}\partial_{b}\phi\partial_{d}\phi\phi^{-1}g_{ac} + \frac{1}{4}\partial_{d}\phi\partial_{a}\phi\phi^{-1}g_{bc} - \frac{1}{4}\partial_{b}\phi\partial_{c}\phi\phi^{-1}g_{ad}$$

$$+ \frac{1}{4}\partial_{c}\phi\partial_{b}\phi\phi^{-1}g_{ad} + \frac{5}{12}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g^{ef} - \frac{1}{4}\partial_{c}\phi\partial_{a}\phi\phi^{-1}g_{bd} - \frac{1}{4}\partial_{d}\phi\partial_{b}\phi\phi^{-1}g_{ac} - \frac{5}{12}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{ca}\phi g_{bd} - \frac{1}{6}\partial_{ef}\phi g_{ac}g_{bd}g^{ef}$$

$$- \frac{1}{8}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{bd}g_{f}^{f} + \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{bd}g_{cg}g^{eg} + \frac{1}{4}\partial_{c}\phi\partial_{e}\phi\phi^{-1}g_{ag}g_{bd}g^{eg} + \frac{1}{6}\partial_{e}\phi\partial_{g}\phi\phi^{-1}g_{ac}g_{bd}g^{eg} + \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bd}g^{ef}$$

$$- \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{bc}g_{dg}g^{eg} - \frac{1}{4}\partial_{d}\phi\partial_{e}\phi\phi^{-1}g_{ag}g_{bc}g^{eg} - \frac{1}{6}\partial_{e}\phi\partial_{g}\phi\phi^{-1}g_{ad}g_{bc}g^{eg} - \frac{1}{4}\partial_{d}\phi\phi_{ac} - \frac{1}{8}\partial_{b}\phi\partial_{d}\phi\phi^{-1}g_{ac}g_{f}^{f} + \frac{1}{4}\partial_{b}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{dg}g^{eg}$$

$$+ \frac{1}{4}\partial_{d}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bg}g^{eg} + \frac{1}{4}\partial_{c}\phi\phi_{ad} + \frac{1}{8}\partial_{b}\phi\partial_{c}\phi\phi^{-1}g_{ad}g_{f}^{f} - \frac{1}{4}\partial_{b}\phi\partial_{e}\phi\phi^{-1}g_{ad}g_{bg}g^{eg} - \frac{1}{4}\partial_{c}\phi\partial_{e}\phi\phi^{-1}g_{ad}g_{bg}g^{eg} - \frac{1}{6}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g^{eg}$$

$$- \frac{1}{4}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ac}g_{bd}g_{gi}g^{eg}g^{fi} + \frac{1}{4}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g_{gi}g^{eg}g^{fi} + \frac{1}{8}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ac}g_{bd}g_{h}^{h}g^{ef} - \frac{1}{8}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g_{h}^{h}g^{ef}$$

$$(ex-14.diff.206)$$

$$\Delta = -\frac{1}{4}\partial_{bc}\phi g_{ad} + \frac{1}{4}\partial_{ac}\phi g_{bd} + \frac{1}{4}\partial_{bd}\phi g_{ac} - \frac{1}{4}\partial_{ad}\phi g_{bc} + \frac{1}{4}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{bd} - \frac{1}{4}\partial_{a}\phi\partial_{d}\phi\phi^{-1}g_{bc} + \frac{1}{4}\partial_{b}\phi\partial_{d}\phi\phi^{-1}g_{ac} + \frac{1}{4}\partial_{d}\phi\partial_{a}\phi\phi^{-1}g_{bc} - \frac{1}{4}\partial_{b}\phi\partial_{c}\phi\phi^{-1}g_{ad} + \frac{1}{4}\partial_{c}\phi\partial_{b}\phi\phi^{-1}g_{ad} + \frac{5}{12}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g^{ef} - \frac{1}{4}\partial_{c}\phi\partial_{a}\phi\phi^{-1}g_{bd} - \frac{1}{4}\partial_{d}\phi\partial_{b}\phi\phi^{-1}g_{ac} - \frac{5}{12}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{c}\phi\partial_{c}\phi\phi^{-1}g_{bd}g_{ef}g^{ef} - \frac{1}{4}\partial_{c}\phi\partial_{a}\phi\phi^{-1}g_{bd}g_{eg}g^{eg} + \frac{1}{4}\partial_{c}\phi\partial_{e}\phi\phi^{-1}g_{ag}g_{bd}g^{eg} + \frac{1}{6}\partial_{e}\phi\partial_{g}\phi\phi^{-1}g_{ac}g_{bd}g^{eg} + \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{bd}g_{bc}g^{ef} + \frac{1}{8}\partial_{a}\phi\partial_{d}\phi\phi^{-1}g_{ac}g_{bd}g^{eg} + \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bd}g^{eg} + \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bd}g^{eg} + \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bd}g^{eg} - \frac{1}{4}\partial_{d}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bd}g^{eg} - \frac{1}{4}\partial_{b}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bd}g^{eg} - \frac{1}{4}\partial_{b}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bd}g^{eg} - \frac{1}{4}\partial_{b}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bd}g^{eg} - \frac{1}{4}\partial_{c}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bd}g^{eg} - \frac{1}{4}\partial_{c}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bd}g^{eg} - \frac{1}{4}\partial_{c}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bd}g^{eg} - \frac{1}{4}\partial_{c}\phi\partial_{e}\phi\phi^{-1}g_{ad}g_{bc}g^{eg} - \frac{1}{6}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g^{eg} - \frac{1}{6}\partial_{e}\phi$$

$$\Delta = -\frac{1}{4}\partial_{bc}\phi g_{ad} + \frac{1}{4}\partial_{ac}\phi g_{bd} + \frac{1}{4}\partial_{bd}\phi g_{ac} - \frac{1}{4}\partial_{ad}\phi g_{bc} + \frac{1}{4}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{bd} - \frac{1}{4}\partial_{a}\phi\partial_{d}\phi\phi^{-1}g_{bc} + \frac{1}{4}\partial_{b}\phi\partial_{d}\phi\phi^{-1}g_{ac} + \frac{1}{4}\partial_{d}\phi\partial_{a}\phi\phi^{-1}g_{bc} - \frac{1}{4}\partial_{b}\phi\partial_{c}\phi\phi^{-1}g_{ad}$$

$$+ \frac{1}{4}\partial_{c}\phi\partial_{b}\phi\phi^{-1}g_{ad} + \frac{5}{12}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g^{ef} - \frac{1}{4}\partial_{c}\phi\partial_{a}\phi\phi^{-1}g_{bd} - \frac{1}{4}\partial_{d}\phi\partial_{b}\phi\phi^{-1}g_{ac} - \frac{5}{12}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{ca}\phi g_{bd} - \frac{1}{6}\partial_{ef}\phi g_{ac}g_{bd}g^{ef}$$

$$- \frac{1}{8}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{bd}g_{f}^{f} + \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{bd}g_{c}^{e} + \frac{1}{4}\partial_{c}\phi\partial_{e}\phi\phi^{-1}g_{a}^{e}g_{bd} + \frac{1}{6}\partial_{e}\phi\partial_{g}\phi\phi^{-1}g_{ac}g_{bd}g^{eg} + \frac{1}{4}\partial_{a}\phi\phi_{bc}g^{ef} + \frac{1}{8}\partial_{a}\phi\partial_{d}\phi\phi^{-1}g_{bc}g_{f}^{f}$$

$$- \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{bc}g_{e}^{e} - \frac{1}{4}\partial_{d}\phi\partial_{e}\phi\phi^{-1}g_{a}^{e}g_{bc} - \frac{1}{6}\partial_{e}\phi\partial_{g}\phi\phi^{-1}g_{ad}g_{bc}g^{eg} - \frac{1}{4}\partial_{b}\phi\phi_{ac}g_{ac} - \frac{1}{8}\partial_{b}\phi\partial_{d}\phi\phi^{-1}g_{ac}g_{f}^{f} + \frac{1}{4}\partial_{b}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bd}^{e}g^{ef}$$

$$+ \frac{1}{4}\partial_{cb}\phi g_{ad} + \frac{1}{8}\partial_{b}\phi\partial_{c}\phi\phi^{-1}g_{ad}g_{f}^{f} - \frac{1}{4}\partial_{b}\phi\partial_{e}\phi\phi^{-1}g_{ad}g_{c}^{e} - \frac{1}{4}\partial_{c}\phi\partial_{e}\phi\phi^{-1}g_{ad}g_{bc}^{e}g^{ef} - \frac{1}{8}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}^{e}g^{ef}$$

$$+ \frac{1}{4}\partial_{c}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g_{f}^{f}g^{eg} + \frac{1}{8}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ac}g_{bd}g_{h}^{h}g^{ef} - \frac{1}{8}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g_{h}^{h}g^{ef}$$

$$+ \frac{1}{4}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g_{g}^{f}g^{eg} + \frac{1}{8}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ac}g_{bd}g_{h}^{h}g^{ef} - \frac{1}{8}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g_{h}^{h}g^{ef}$$

$$+ \frac{1}{4}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g_{g}^{f}g^{eg} + \frac{1}{8}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ac}g_{bd}g_{h}^{h}g^{ef} - \frac{1}{8}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g_{h}^{h}g^{ef}$$

$$+ (ex-14.diff.302)$$

$$\Delta = -\frac{1}{4}\partial_{bc}\phi g_{ad} + \frac{1}{4}\partial_{ac}\phi g_{bd} + \frac{1}{4}\partial_{bd}\phi g_{ac} - \frac{1}{4}\partial_{ad}\phi g_{bc} + \frac{1}{4}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{bd} - \frac{1}{4}\partial_{a}\phi\partial_{d}\phi\phi^{-1}g_{bc} + \frac{1}{4}\partial_{b}\phi\partial_{d}\phi\phi^{-1}g_{ac} + \frac{1}{4}\partial_{d}\phi\partial_{a}\phi\phi^{-1}g_{bc} - \frac{1}{4}\partial_{b}\phi\partial_{c}\phi\phi^{-1}g_{ad}$$

$$+ \frac{1}{4}\partial_{c}\phi\partial_{b}\phi\phi^{-1}g_{ad} + \frac{5}{12}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g^{ef} - \frac{1}{4}\partial_{c}\phi\partial_{a}\phi\phi^{-1}g_{bd} - \frac{1}{4}\partial_{d}\phi\partial_{b}\phi\phi^{-1}g_{ac} - \frac{5}{12}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{ca}\phi g_{bd} - \frac{1}{6}\partial_{ef}\phi g_{ac}g_{bd}g^{ef}$$

$$- \frac{1}{8}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{bd}g_{f}^{f} + \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{bd}g_{c}^{e} + \frac{1}{4}\partial_{c}\phi\partial_{e}\phi\phi^{-1}g_{a}^{e}g_{bd} + \frac{1}{6}\partial_{e}\phi\partial_{g}\phi\phi^{-1}g_{ac}g_{bd}g^{eg} + \frac{1}{4}\partial_{da}\phi g_{bc} + \frac{1}{6}\partial_{ef}\phi g_{ad}g_{bc}g^{ef} + \frac{1}{8}\partial_{a}\phi\partial_{d}\phi\phi^{-1}g_{bc}g_{f}^{f}$$

$$- \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{bc}g_{d}^{e} - \frac{1}{4}\partial_{d}\phi\partial_{e}\phi\phi^{-1}g_{a}^{e}g_{bc} - \frac{1}{6}\partial_{e}\phi\partial_{g}\phi\phi^{-1}g_{ad}g_{bc}g^{eg} - \frac{1}{4}\partial_{d}\phi\phi g_{ac} - \frac{1}{8}\partial_{b}\phi\partial_{d}\phi\phi^{-1}g_{ac}g_{f}^{f} + \frac{1}{4}\partial_{b}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bd}g^{e}$$

$$+ \frac{1}{4}\partial_{cb}\phi g_{ad} + \frac{1}{8}\partial_{b}\phi\partial_{c}\phi\phi^{-1}g_{ad}g_{f}^{f} - \frac{1}{4}\partial_{b}\phi\partial_{e}\phi\phi^{-1}g_{ad}g_{c}^{e} - \frac{1}{4}\partial_{c}\phi\partial_{e}\phi\phi^{-1}g_{ad}g_{b}^{e} + \frac{1}{6}\partial_{eg}\phi g_{ac}g_{bd}g^{eg} - \frac{1}{6}\partial_{eg}\phi g_{ac}g_{bd}g^{eg} - \frac{1}{6}\partial_{eg}\phi g_{ac}g_{bd}g^{eg} - \frac{1}{6}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ac}g_{bd}g_{f}^{f}g^{eg}$$

$$+ \frac{1}{4}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g_{g}^{f}g^{eg} + \frac{1}{8}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ac}g_{bd}g_{h}^{h}g^{ef} - \frac{1}{8}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g_{h}^{h}g^{ef}$$

$$+ (ex-14.diff.303)$$

$$\Delta = -\frac{1}{4}\partial_{bc}\phi g_{ad} + \frac{1}{4}\partial_{ac}\phi g_{bd} + \frac{1}{4}\partial_{bd}\phi g_{ac} - \frac{1}{4}\partial_{ad}\phi g_{bc} - \frac{1}{4}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{bd} + \frac{1}{4}\partial_{a}\phi\partial_{d}\phi\phi^{-1}g_{bc} - \frac{1}{4}\partial_{b}\phi\partial_{d}\phi\phi^{-1}g_{ac} + \frac{1}{4}\partial_{d}\phi\partial_{a}\phi\phi^{-1}g_{bc} + \frac{1}{4}\partial_{b}\phi\partial_{c}\phi\phi^{-1}g_{ad} + \frac{1}{4}\partial_{b}\phi\partial_{c}\phi\phi^{-1}g_{ac} + \frac{1}{4}\partial_{a}\phi\partial_{b}\phi\phi^{-1}g_{ac} + \frac{1}{4}\partial_{a}\phi\partial_{d}\phi\phi^{-1}g_{ac} + \frac{1}{4}\partial_{c}\phi\partial_{a}\phi\phi^{-1}g_{ac} + \frac{1}{4}\partial_{c}\phi\partial_{c}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{c}\phi\partial_{a}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} + \frac{1}{4}\partial_{a}\phi\partial_{b}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{c}\phi\partial_{a}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} + \frac{1}{4}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g$$

$$\Delta = -\frac{1}{4}\partial_{bc}\phi g_{ad} + \frac{1}{4}\partial_{ac}\phi g_{bd} + \frac{1}{4}\partial_{bd}\phi g_{ac} - \frac{1}{4}\partial_{ad}\phi g_{bc} - \frac{1}{4}\partial_{a}\phi\partial_{c}\phi\phi^{-1}g_{bd} + \frac{1}{4}\partial_{a}\phi\partial_{d}\phi\phi^{-1}g_{bc} - \frac{1}{4}\partial_{b}\phi\partial_{d}\phi\phi^{-1}g_{ac} + \frac{1}{4}\partial_{d}\phi\partial_{a}\phi\phi^{-1}g_{bc} + \frac{1}{4}\partial_{b}\phi\partial_{c}\phi\phi^{-1}g_{ad} + \frac{1}{4}\partial_{b}\phi\partial_{c}\phi\phi^{-1}g_{ac} + \frac{1}{12}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{c}\phi\partial_{a}\phi\phi^{-1}g_{bd} - \frac{1}{4}\partial_{d}\phi\partial_{b}\phi\phi^{-1}g_{ac} + \frac{1}{12}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{c}\phi\partial_{g}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} + \frac{1}{4}\partial_{a}\phi\partial_{b}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} + \frac{1}{4}\partial_{a}\phi\partial_{e}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{a}\phi$$

$$\begin{split} \Delta &= -\frac{1}{4}\partial_{bc}\phi g_{ad} + \frac{1}{4}\partial_{ac}\phi g_{bd} + \frac{1}{4}\partial_{bd}\phi g_{ac} - \frac{1}{4}\partial_{ad}\phi g_{bc} - \frac{1}{12}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ad}g_{bc}g^{ef} + \frac{1}{12}\partial_{e}\phi\partial_{f}\phi\phi^{-1}g_{ac}g_{bd}g^{ef} - \frac{1}{4}\partial_{ca}\phi g_{bd} - \frac{1}{6}\partial_{ef}\phi g_{ac}g_{bd}g^{ef} \\ &- \frac{1}{12}\partial_{e}\phi\partial_{g}\phi\phi^{-1}g_{ac}g_{bd}g^{eg} + \frac{1}{4}\partial_{da}\phi g_{bc} + \frac{1}{6}\partial_{ef}\phi g_{ad}g_{bc}g^{ef} + \frac{1}{12}\partial_{e}\phi\partial_{g}\phi\phi^{-1}g_{ad}g_{bc}g^{eg} - \frac{1}{4}\partial_{db}\phi g_{ac} + \frac{1}{4}\partial_{cb}\phi g_{ad} + \frac{1}{6}\partial_{eg}\phi g_{ac}g_{bd}g^{eg} \\ &- \frac{1}{6}\partial_{eg}\phi g_{ad}g_{bc}g^{eg} \end{split}$$
 (ex-14.diff.306)

$$\Delta = 0 \tag{ex-14.diff.400}$$