

Exercise 6.4 Scalar curavture of a 2-sphere

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1  {\theta, \varphi}::Coordinate.
2  {a,b,c,d,e,f,g,h#}::Indices(values={\theta, \varphi}, position=independent).
3
4  \partial{#}::PartialDerivative.
5
6  g^{a b}::InverseMetric.  # essential when using complete (gab, $g^{a b}$)
7
8  Gamma := \Gamma^{a}_{f g} -> 1/2 g^{a b} ( \partial_{g}\Gamma_{b f}
9                                         + \partial_{f}\Gamma_{b g}
10                                         - \partial_{b}\Gamma_{f g} ).
11
12  Rabcd := R^{d}_{e f g} -> \partial_{f}\Gamma^{d}_{e g}
13                        - \partial_{g}\Gamma^{d}_{e f}
14                        + \Gamma^{d}_{b f} \Gamma^{b}_{e g}
15                        - \Gamma^{d}_{b g} \Gamma^{b}_{e f}.
16
17  Rab := R_{a b} -> R^{c}_{c} _{a b}.
18
19  R := R -> R_{e g} g^{e g}.
20
21  gab := { g_{\theta\theta} = r**2,
22          g_{\varphi\varphi} = r**2 \sin(\theta)**2 }.  # cdb(ex-0604.101,gab)
23
24  complete (gab, $g^{a b}$)  # cdb(ex-0604.102,gab)
25
26  substitute (Rabcd, Gamma)
27  substitute (Rab, Rabcd)
28  substitute (R, Rab)
29
30  evaluate (Gamma, gab, rhsonly=True)  # cdb(ex-0604.103,Gamma)
31  evaluate (Rabcd, gab, rhsonly=True)  # cdb(ex-0604.104,Rabcd)
32  evaluate (Rab, gab, rhsonly=True)  # cdb(ex-0604.105,Rab)
33  evaluate (R, gab, rhsonly=True)  # cdb(ex-0604.106,R)

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$$[g_{\theta\theta} = r^2, \ g_{\varphi\varphi} = r^2(\sin \theta)^2] \quad (\text{ex-0604.101})$$

$$\left[g_{\theta\theta} = r^2, \ g_{\varphi\varphi} = r^2(\sin \theta)^2, \ g^{\theta\theta} = r^{-2}, \ g^{\varphi\varphi} = (r^2(\sin \theta)^2)^{-1} \right] \quad (\text{ex-0604.102})$$

$$\Gamma^a_{fg} \rightarrow \square_{fg}^a \begin{cases} \square_{\varphi\theta}^{\varphi} = (\tan \theta)^{-1} \\ \square_{\theta\varphi}^{\varphi} = (\tan \theta)^{-1} \\ \square_{\varphi\varphi}^{\theta} = -\frac{1}{2} \sin(2\theta) \end{cases} \quad (\text{ex-0604.103})$$

$$R^d_{efg} \rightarrow \square_{eg}^d \begin{cases} \square_{\varphi\varphi}^{\theta} = (\sin \theta)^2 \\ \square_{\theta\varphi}^{\varphi} = -1 \\ \square_{\varphi\theta}^{\theta} = -(\sin \theta)^2 \\ \square_{\theta\theta}^{\varphi} = 1 \end{cases} \quad (\text{ex-0604.104})$$

$$R_{ab} \rightarrow \square_{ab} \begin{cases} \square_{\varphi\varphi} = (\sin \theta)^2 \\ \square_{\theta\theta} = 1 \end{cases} \quad (\text{ex-0604.105})$$

$$R \rightarrow 2r^{-2} \quad (\text{ex-0604.106})$$