```
{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w#}::Indices(position=independent).

\nabla{#}::Derivative.
\partial{#}::PartialDerivative.

g_{a b}::Metric.
g^{a b}::InverseMetric.
g_{a}^{b}::KroneckerDelta.
g^{a}_{b}::KroneckerDelta.

R_{a b c d}::RiemannTensor.
R^{a}_{b c d}::RiemannTensor.

\Gamma^{a}_{b c}::TableauSymmetry(shape={2}, indices={1,2}).
\Gamma_{a b c}::TableauSymmetry(shape={2}, indices={1,2}).

g_{a b}::Depends(\partial{#}).
g^{a b}::Depends(\partial{#}).
\Gamma^{a}_{b c}::Depends(\partial{#}).
\Gamma^{a b c}::Depends(\partial{#}).

dgab := \partial_{c}{g_{a b}} ->   \Gamma^{d}_{a c} g_{d b}
                                 + \Gamma^{d}_{b c} g_{a d}.           # cdb(dgab.000,dgab)

diab := \partial_{c}{g^{a b}} -> - \Gamma^{a}_{d c} g^{d b}
                                 - \Gamma^{b}_{d c} g^{a d}.           # cdb(diab.000,diab)

GammaU := \Gamma^{a}_{b c} ->  1/2 g^{a d} (   \partial_{b}{g_{d c}}
                                             + \partial_{c}{g_{b d}}
                                             - \partial_{d}{g_{b c}}). # cdb(Gamma.000,GammaU)

GammaD := \Gamma_{a b c} ->  1/2 (   \partial_{b}{g_{a c}}
                                   + \partial_{c}{g_{b a}}
                                   - \partial_{a}{g_{b c}}).           # cdb(Gamma.010,GammaD)

RabcdU := R^{a}_{b c d} ->   \partial_{c}{\Gamma^{a}_{b d}}
                           - \partial_{d}{\Gamma^{a}_{b c}}
                           + \Gamma^{e}_{b d} \Gamma^{a}_{c e}
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                          - \Gamma^{e}_{b c} \Gamma^{a}_{d e}.        # cdb(Rabcd.000,RabcdU)

RabcdD := R_{a b c d} ->   \partial_{c}{\Gamma_{a b d}}
                         - \partial_{d}{\Gamma_{a b c}}
                         + \Gamma_{e a d} \Gamma^{e}_{b c}
                         - \Gamma_{e a c} \Gamma^{e}_{b d}.           # cdb(Rabcd.010,RabcdD)

Rab := R_{a b} -> R^{c}_{a c b}.                                       # cdb(Rab.000,Rab)

Rscalar := R -> g^{a b} R_{a b}.                                       # cdb(Rscalar.000,Rscalar)

# Weyl in 4 dimensions
Cabcd := C_{a b c d} -> R_{a b c d} - (1/2) (R_{a c} g_{b d} - R_{a d} g_{b c})
                                    - (1/2) (g_{a c} R_{b d} - g_{a d} R_{b c})
                                    + (R/6) (g_{a c} g_{b d} - g_{a d} g_{b c}).     # cdb(Cabcd.000,Cabcd)

expr := \partial_{c}{g_{a b}}.   # cdb(libdg.dgab.000,expr)
substitute (expr,dgab)           # cdb(libdg.dgab.001,expr)

expr := \partial_{c}{g^{a b}}.   # cdb(libdg.diab.000,expr)
substitute (expr,diab)           # cdb(libdg.diab.001,expr)

expr := \Gamma^{a}_{b c}.        # cdb(libdg.Gamma.000,expr)
substitute (expr,GammaU)         # cdb(libdg.Gamma.001,expr)

expr := \Gamma_{a b c}.          # cdb(libdg.Gamma.010,expr)
substitute (expr,GammaD)         # cdb(libdg.Gamma.011,expr)

expr := R^{a}_{b c d}.           # cdb(libdg.Rabcd.000,expr)
substitute (expr,RabcdU)         # cdb(libdg.Rabcd.001,expr)

expr := R_{a b c d}.             # cdb(libdg.Rabcd.010,expr)
substitute (expr,RabcdD)         # cdb(libdg.Rabcd.011,expr)

expr := R_{a b}.                 # cdb(libdg.Rab.000,expr)
substitute (expr,Rab)            # cdb(libdg.Rab.001,expr)

expr := R.                       # cdb(libdg.Rscalar.000,expr)
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```
substitute (expr,Rscalar)          # cdb(libdg.Rscalar.001,expr)

expr := C_{a b c d}.               # cdb(libdg.Cabcd.000,expr)
substitute (expr,Cabcd)            # cdb(libdg.Cabcd.001,expr)
```

$$\partial_c g_{ab} \to \Gamma^{d}{}_{ac} g_{db} + \Gamma^{d}{}_{bc} g_{ad} \tag{1}$$

$$\partial_c g^{ab} \to -\Gamma^{a}{}_{dc} g^{db} - \Gamma^{b}{}_{dc} g^{ad} \tag{2}$$

$$\Gamma^{a}{}_{bc} \to \frac{1}{2}\, g^{ad} \left(\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc}\right) \tag{3}$$

$$\Gamma_{abc} \to \frac{1}{2}\, \partial_b g_{ac} + \frac{1}{2}\, \partial_c g_{ba} - \frac{1}{2}\, \partial_a g_{bc} \tag{4}$$

$$R^{a}{}_{bcd} \to \partial_c \Gamma^{a}{}_{bd} - \partial_d \Gamma^{a}{}_{bc} + \Gamma^{e}{}_{bd} \Gamma^{a}{}_{ce} - \Gamma^{e}{}_{bc} \Gamma^{a}{}_{de} \tag{5}$$

$$R_{abcd} \to \partial_c \Gamma_{abd} - \partial_d \Gamma_{abc} + \Gamma_{ead} \Gamma^{e}{}_{bc} - \Gamma_{eac} \Gamma^{e}{}_{bd} \tag{6}$$

$$R_{ab} \to R^{c}{}_{acb} \tag{7}$$

$$R \to g^{ab} R_{ab} \tag{8}$$

$$C_{abcd} \to R_{abcd} - \frac{1}{2}\, R_{ac} g_{bd} + \frac{1}{2}\, R_{ad} g_{bc} - \frac{1}{2}\, g_{ac} R_{bd} + \frac{1}{2}\, g_{ad} R_{bc} + \frac{1}{6}\, R \left(g_{ac} g_{bd} - g_{ad} g_{bc}\right) \tag{9}$$

$$\partial_c g_{ab} = \Gamma^{d}{}_{ac} g_{db} + \Gamma^{d}{}_{bc} g_{ad} \tag{10}$$

$$\partial_c g^{ab} = -\Gamma^{a}{}_{dc} g^{db} - \Gamma^{b}{}_{dc} g^{ad} \tag{11}$$

$$\Gamma^{a}{}_{bc} = \frac{1}{2}\, g^{ad} \left(\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc}\right) \tag{12}$$

$$\Gamma_{abc} = \frac{1}{2}\, \partial_b g_{ac} + \frac{1}{2}\, \partial_c g_{ba} - \frac{1}{2}\, \partial_a g_{bc} \tag{13}$$

$$R^{a}{}_{bcd} = \partial_c \Gamma^{a}{}_{bd} - \partial_d \Gamma^{a}{}_{bc} + \Gamma^{e}{}_{bd} \Gamma^{a}{}_{ce} - \Gamma^{e}{}_{bc} \Gamma^{a}{}_{de} \tag{14}$$

$$R_{abcd} = \partial_c \Gamma_{abd} - \partial_d \Gamma_{abc} + \Gamma_{ead} \Gamma^{e}{}_{bc} - \Gamma_{eac} \Gamma^{e}{}_{bd} \tag{15}$$

$$R_{ab} = R^{c}{}_{acb} \tag{16}$$

$$R = g^{ab} R_{ab} \tag{17}$$

$$C_{abcd} = R_{abcd} - \frac{1}{2}\, R_{ac} g_{bd} + \frac{1}{2}\, R_{ad} g_{bc} - \frac{1}{2}\, g_{ac} R_{bd} + \frac{1}{2}\, g_{ad} R_{bc} + \frac{1}{6}\, R \left(g_{ac} g_{bd} - g_{ad} g_{bc}\right) \tag{18}$$

```python
import cdblib

cdblib.create ('dgeom.json')

cdblib.put ('dgab',     dgab,    'dgeom.json')
cdblib.put ('diab',     diab,    'dgeom.json')
cdblib.put ('GammaU',   GammaU,  'dgeom.json')
cdblib.put ('GammaD',   GammaD,  'dgeom.json')
cdblib.put ('RabcdU',   RabcdU,  'dgeom.json')
cdblib.put ('RabcdD',   RabcdD,  'dgeom.json')
cdblib.put ('Rab',      Rab,     'dgeom.json')
cdblib.put ('Rscalara', Rscalar, 'dgeom.json')
cdblib.put ('Cabcd',    Cabcd,   'dgeom.json')
```