Exercise 2.3 Covariant derivative of v^{a}_{b}

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\{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u\#\}::Indices(position=independent).
     \nabla{#}::Derivative.
     \partial{#}::PartialDerivative.
     # template for covariant derivative of a vector
     derivU := \nabla_{a}^{A?^{b}} -> \partial_{a}^{A?^{b}} + \Gamma^{b}_{c} a A?^{c}.
     derivD := \nabla_{a}{A?_{b}} -> \partial_{a}{A?_{b}} - \Gamma^{c}_{b} \ A?_{c}.
10
     vab := v^{a}_{b} -> A^{a}_{b}.
     iab := A^{a} B_{b} -> v^{a}_{b}.
12
13
     pab := \hat{A}_{a}_{a}^{b} B_{c} -> \hat{A}_{b} B_{c} -> \hat{A}_{b} B_{c}.
14
15
     # create an object
16
17
     Dvab := \frac{a}{v^{b}_{c}}. # cdb (ex-0203.101,Dvab)
19
     # apply the rule, then simplify
21
                    (Dvab, vab)
     substitute
                                      # cdb (ex-0203.102, Dvab)
22
                    (Dvab)
     product_rule
                                      # cdb (ex-0203.103, Dvab)
     substitute
                    (Dvab,derivD)
                                      # cdb (ex-0203.104, Dvab)
                    (Dvab,derivU)
     substitute
                                      # cdb (ex-0203.105, Dvab)
                    (Dvab)
                                      # cdb (ex-0203.106,Dvab)
     distribute
26
                    (Dvab,pab)
                                      # cdb (ex-0203.107, Dvab)
     substitute
27
                    (Dvab)
                                      # cdb (ex-0203.108,Dvab)
     canonicalise
28
     substitute
                    (Dvab, iab)
                                      # cdb (ex-0203.109,Dvab)
29
                                      # cdb (ex-0203.110,Dvab)
     sort_product
                    (Dvab)
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$$\begin{split} \nabla_{a}v^{b}{}_{c} &= \nabla_{a}\left(A^{b}B_{c}\right) & (\text{ex-0203.102}) \\ &= \nabla_{a}A^{b}B_{c} + A^{b}\nabla_{a}B_{c} & (\text{ex-0203.103}) \\ &= \nabla_{a}A^{b}B_{c} + A^{b}\left(\partial_{a}B_{c} - \Gamma^{d}{}_{ca}B_{d}\right) & (\text{ex-0203.104}) \\ &= \left(\partial_{a}A^{b} + \Gamma^{b}{}_{da}A^{d}\right)B_{c} + A^{b}\left(\partial_{a}B_{c} - \Gamma^{d}{}_{ca}B_{d}\right) & (\text{ex-0203.105}) \\ &= \partial_{a}A^{b}B_{c} + \Gamma^{b}{}_{da}A^{d}B_{c} + A^{b}\partial_{a}B_{c} - A^{b}\Gamma^{d}{}_{ca}B_{d} & (\text{ex-0203.106}) \\ &= \partial_{a}\left(A^{b}B_{c}\right) + \Gamma^{b}{}_{da}A^{d}B_{c} - A^{b}\Gamma^{d}{}_{ca}B_{d} & (\text{ex-0203.107}) \\ &= \partial_{a}\left(A^{b}B_{c}\right) + \Gamma^{b}{}_{da}A^{d}B_{c} - A^{b}\Gamma^{d}{}_{ca}B_{d} & (\text{ex-0203.108}) \\ &= \partial_{a}v^{b}{}_{c} + \Gamma^{b}{}_{da}v^{d}{}_{c} - v^{b}{}_{d}\Gamma^{d}{}_{ca} & (\text{ex-0203.109}) \\ &= \partial_{a}v^{b}{}_{c} + \Gamma^{b}{}_{da}v^{d}{}_{c} - \Gamma^{d}{}_{ca}v^{b}{}_{d} & (\text{ex-0203.110}) \end{split}$$