The metric connection

This is a very standard computation that shows if

$$\Gamma^a_{bc} = \frac{1}{2} g^{ad} (\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc}) \tag{1}$$

then

$$g_{ab;c} = 0. \tag{2}$$

This example might well be regarded as the Cadabra counterpart to the familiar *Hello World* program of undergraduate programming classes.

```
{a,b,c,d,e,f,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices.

g_{a b}::Metric.
g_{a}^{b}::KroneckerDelta.

\partial_{#}::PartialDerivative.

cderiv:=\partial_{c}{g_{a b}} - g_{a d}\Gamma^{d}_{b c}
                             - g_{d b}\Gamma^{d}_{a c}.            # cdb (term31,cderiv)

Gamma:=\Gamma^{a}_{b c} -> (1/2) g^{a d} (  \partial_{b}{g_{d c}}
                                          + \partial_{c}{g_{b d}}
                                          - \partial_{d}{g_{b c}} ). # cdb (term32,Gamma)

substitute          (cderiv,Gamma);     # cdb (term33,cderiv)
distribute          (cderiv)            # cdb (term34,cderiv)
eliminate_metric    (cderiv)            # cdb (term35,cderiv)
eliminate_kronecker (cderiv)            # cdb (term36,cderiv)
canonicalise        (cderiv)            # cdb (term37,cderiv)
```

The metric connection

$$\texttt{term31} := \partial_{c}g_{ab} - g_{ad}\Gamma^{d}{}_{bc} - g_{db}\Gamma^{d}{}_{ac}$$
$$\texttt{term32} := \Gamma^{a}{}_{bc} \rightarrow \frac{1}{2}g^{ad}\left(\partial_{b}g_{dc} + \partial_{c}g_{bd} - \partial_{d}g_{bc}\right)$$
$$\texttt{term33} := \partial_{c}g_{ab} - \frac{1}{2}g_{ad}g^{de}\left(\partial_{b}g_{ec} + \partial_{c}g_{be} - \partial_{e}g_{bc}\right) - \frac{1}{2}g_{db}g^{de}\left(\partial_{a}g_{ec} + \partial_{c}g_{ae} - \partial_{e}g_{ac}\right)$$
$$\texttt{term34} := \partial_{c}g_{ab} - \frac{1}{2}g_{ad}g^{de}\partial_{b}g_{ec} - \frac{1}{2}g_{ad}g^{de}\partial_{c}g_{be} + \frac{1}{2}g_{ad}g^{de}\partial_{e}g_{bc} - \frac{1}{2}g_{db}g^{de}\partial_{a}g_{ec} - \frac{1}{2}g_{db}g^{de}\partial_{c}g_{ae} + \frac{1}{2}g_{db}g^{de}\partial_{e}g_{ac}$$
$$\texttt{term35} := \partial_{c}g_{ab} - \frac{1}{2}g_{a}{}^{e}\partial_{b}g_{ec} - \frac{1}{2}g_{a}{}^{e}\partial_{c}g_{be} + \frac{1}{2}g_{a}{}^{e}\partial_{e}g_{bc} - \frac{1}{2}g_{b}{}^{e}\partial_{a}g_{ec} - \frac{1}{2}g_{b}{}^{e}\partial_{c}g_{ae} + \frac{1}{2}g_{b}{}^{e}\partial_{e}g_{ac}$$
$$\texttt{term36} := \frac{1}{2}\partial_{c}g_{ab} - \frac{1}{2}\partial_{c}g_{ba}$$
$$\texttt{term37} := 0$$

```
\begin{align*}
   &\cdb*{term31}\\
   &\cdb*{term32}\\
   &\cdb*{term33}\\
   &\cdb*{term34}\\
   &\cdb*{term35}\\
   &\cdb*{term36}\\
   &\cdb*{term37}
\end{align*}
```