Elementary maths

This example is based on a similar example in the Python collection. Its purpose is to show that Cadabra is fluent in Python – which is not surprisng since the Cadabra language is based on Python (and a subset of LaTeX).

```
from sympy import *
x, y, z = symbols('x y z')
a, b, c = symbols('a b c')
ans = expand((a+b)**3)                                  # cdb (ans.101,ans)
ans = factor(-2*x+2*x+a*x-x**2+a*x**2-x**3)              # cdb (ans.102,ans)
ans = solve(x**2-4, x)                                  # cdb (ans.103,ans)
ans = solve([2*a-b - 3, a+b+c - 1,-b+c - 6],[a,b,c])    # cdb (ans.104,ans)
ans = N(pi,50)                                          # cdb (ans.105,ans)
ans = apart(1/((1 + x)*(5 + x)))                        # cdb (ans.106,ans)
ans = together((1/(1 + x) - 1/(5 + x))/4)               # cdb (ans.107,ans)
ans = simplify(tanh(log(x)))                            # cdb (rhs.108,ans)
ans = simplify(tanh(I*x))                               # cdb (rhs.109,ans)
ans = simplify(sinh(3*x) - 3*sinh(x) - 4*(sinh(x))**3)  # cdb (rhs.110,ans)
ans = tanh(log(x))                                      # cdb (lhs.108,ans)
ans = tanh(UnevaluatedExpr(I*x))                        # cdb (lhs.109,ans)
ans = sinh(3*x) - 3*sinh(x) - 4*(sinh(x))**3            # cdb (lhs.110,ans)
```

```
\begin{align*}
   &\cdb*{ans.101}\\
   &\cdb*{ans.102}\\
   &\cdb*{ans.103}\\
   &\cdb*{ans.104}\\
   &\cdb*{ans.105}\\
   &\cdb*{ans.106}\\
   &\cdb*{ans.107}\\
   \cdb{lhs.108} &= \Cdb{rhs.108}\\
   \cdb{lhs.109} &= \Cdb{rhs.109}\\
   \cdb{lhs.110} &= \Cdb{rhs.110}
\end{align*}
```

$$\texttt{ans.101} := a^3 + 3a^2b + 3ab^2 + b^3$$

$$\texttt{ans.102} := -x\left(-a + x\right)\left(x + 1\right)$$

$$\texttt{ans.103} := \left[-2,\ 2\right]$$

$$\texttt{ans.104} := \left\{a : \frac{1}{5},\ b : -\frac{13}{5},\ c : \frac{17}{5}\right\}$$

$$\texttt{ans.105} := 3.1415926535897932384626433832795028841971693993751$$

$$\texttt{ans.106} := -\frac{1}{4\left(x + 5\right)} + \frac{1}{4\left(x + 1\right)}$$

$$\texttt{ans.107} := \frac{1}{\left(x + 1\right)\left(x + 5\right)}$$

$$\tanh\left(\log\left(x\right)\right) = \tanh\left(\log\left(x\right)\right) \qquad (\texttt{rhs.108})$$

$$\tanh\left(ix\right) = i\tan\left(x\right) \qquad (\texttt{rhs.109})$$

$$-4\sinh^3\left(x\right) - 3\sinh\left(x\right) + \sinh\left(3x\right) = 0 \qquad (\texttt{rhs.110})$$

Linear Algebra

```
from sympy import linsolve
lamda = Symbol('lamda')
mat   = Matrix([[2,3], [5,4]])                    # cdb (ans.201,mat)
eig1  = mat.eigenvects()[0][0]                    # 1st eigenvalue
eig2  = mat.eigenvects()[1][0]                    # 2nd eigenvalue
v1    = mat.eigenvects()[0][2][0]                 # 1st eigenvector
v2    = mat.eigenvects()[1][2][0]                 # 2nd eigenvector
eig   = simplify(Matrix([eig1,eig2]))             # cdb (ans.202,eig)
vec   = simplify(5*Matrix([]).col_insert(0,v1)
                             .col_insert(1,v2))   # cdb (ans.203,vec)
det   = expand((mat - lamda * eye(2)).det())      # cdb (ans.204,det)
rhs   = Matrix([[3],[7]])                         # cdb (ans.205,rhs)
ans   = list(linsolve((mat,rhs),x,y))[0]          # cdb (ans.206,ans)
```

```
\begin{align*}
   &\cdb*{ans.201}\\
   &\cdb*{ans.202}\\
   &\cdb*{ans.203}\\
   &\cdb*{ans.204}\\
   &\cdb*{ans.205}\\
   &\cdb*{ans.206}
\end{align*}
```

$$\texttt{ans.201} := \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$$

$$\texttt{ans.202} := \begin{bmatrix} -1 \\ 7 \end{bmatrix}$$

$$\texttt{ans.203} := \begin{bmatrix} -5 & 3 \\ 5 & 5 \end{bmatrix}$$

$$\texttt{ans.204} := \lambda^2 - 6\lambda - 7$$

$$\texttt{ans.205} := \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$\texttt{ans.206} := \left(\frac{9}{7}, \frac{1}{7}\right)$$

Limits

```
n, dx = symbols('n dx')
ans = limit(sin(4*x)/x,x,0)                    # cdb (ans.301,ans)
ans = limit(2**x/x,x,oo)                       # cdb (ans.302,ans)
ans = limit(((x+dx)**2 - x**2)/dx, dx,0)       # cdb (ans.303,ans)
ans = limit((4*n + 1)/(3*n - 1),n,oo)          # cdb (ans.304,ans)
ans = limit((1+(a/n))**n,n,oo)                 # cdb (ans.305,ans)
```

```
\begin{align*}
   &\cdb*{ans.301}\\
   &\cdb*{ans.302}\\
   &\cdb*{ans.303}\\
   &\cdb*{ans.304}\\
   &\cdb*{ans.305}
\end{align*}
```

$$\texttt{ans.301} := 4$$
$$\texttt{ans.302} := \infty$$
$$\texttt{ans.303} := 2x$$
$$\texttt{ans.304} := \frac{4}{3}$$
$$\texttt{ans.305} := e^{a}$$

Series

```
ans = series((1 + x)**(-2), x, 1, 6)           # cdb (ans.401,ans)
ans = series(exp(x), x, 0, 6)                  # cdb (ans.402,ans)
ans = Sum(1/n**2, (n,1,50)).doit()             # cdb (ans.403,ans)
ans = Sum(1/n**4, (n,1,oo)).doit()             # cdb (ans.404,ans)
```

```
\begin{align*}
   &\cdb*{ans.401}\\
   &\cdb*{ans.402}\\
   &\cdb*{ans.403}\\
   &\cdb*{ans.404}
\end{align*}
```

$$\texttt{ans.401} := \frac{1}{2} + \frac{3\left(x-1\right)^{2}}{16} - \frac{\left(x-1\right)^{3}}{8} + \frac{5\left(x-1\right)^{4}}{64} - \frac{3\left(x-1\right)^{5}}{64} - \frac{x}{4} + O\left(\left(x-1\right)^{6}; x \to 1\right)$$
$$\texttt{ans.402} := 1 + x + \frac{x^{2}}{2} + \frac{x^{3}}{6} + \frac{x^{4}}{24} + \frac{x^{5}}{120} + O\left(x^{6}\right)$$
$$\texttt{ans.403} := \frac{312157992955169267846963566083562620966170 9}{192081536785946309960051152615192956019200 0}$$
$$\texttt{ans.404} := \frac{\pi^{4}}{90}$$

Calculus

This example shows how `\Cdb` can be used to set the equation tag on the far right hand side.

```
ans = diff(x*sin(x),x)                                  # cdb (ans.501,ans)
ans = diff(x*sin(x),x).subs(x,pi/4)                     # cdb (ans.502,ans)
ans = integrate(2*sin(x)**2, (x,a,b))                   # cdb (ans.503,ans)
ans = Integral(2*exp(-x**2), (x,0,oo))                  # cdb (lhs.504,ans)
ans = ans.doit()                                        # cdb (ans.504,ans)
ans = Integral(Integral(x**2 + y**2, (y,0,x)), (x,0,1)) # cdb (lhs.505,ans)
ans = ans.doit()                                        # cdb (ans.505,ans)
```

```
\begin{align*}
   &\cdb*{ans.501}\\
   &\cdb*{ans.502}\\
   &\cdb*{ans.503}\\
    \cdb{lhs.504}&=\Cdb{ans.504}\\
    \cdb{lhs.505}&=\Cdb{ans.505}
\end{align*}
```

$$\texttt{ans.501} := x\cos\left(x\right) + \sin\left(x\right)$$

$$\texttt{ans.502} := \frac{\sqrt{2}\pi}{8} + \frac{\sqrt{2}}{2}$$

$$\texttt{ans.503} := -a + b + \sin\left(a\right)\cos\left(a\right) - \sin\left(b\right)\cos\left(b\right)$$

$$\int_0^{\infty} 2e^{-x^2}\,dx = \sqrt{\pi} \tag{ans.504}$$

$$\int_0^1 \int_0^x \left(x^2 + y^2\right)\,dy\,dx = \frac{1}{3} \tag{ans.505}$$

Differential equations

```
y = Function('y')
C1, C2 = symbols('C1 C2')

ode = Eq(y(x).diff(x) + y(x), 2*a*sin(x))
sol = expand(dsolve(ode,y(x)).rhs)                                  # cdb (ans.601,sol)
cst = solve([sol.subs(x,0)],dict=True)
sol = sol.subs(cst[0])                                              # cdb (ans.602,sol)

ode = Eq(y(x).diff(x,2) + y(x), 0)
sol = expand(dsolve(ode,y(x)).rhs)                                  # cdb (ans.603,sol)
cst = solve([sol.subs(x,0),sol.diff(x).subs(x,0)-1],dict=True)
sol = sol.subs(cst[0])                                              # cdb (ans.604,sol)

ode = Eq(y(x).diff(x,2) + 5*y(x).diff(x) - 6*y(x), 0)
sol = expand(dsolve(ode,y(x)).rhs)                                  # cdb (ans.605,sol)
sol = sol.subs({C1:2,C2:3})                                         # cdb (ans.606,sol)
```

```
\begin{align*}
   &\cdb*{ans.601}\\
   &\cdb*{ans.602}\\
   &\cdb*{ans.603}\\
   &\cdb*{ans.604}\\
   &\cdb*{ans.605}\\
   &\cdb*{ans.606}
\end{align*}
```

$$\texttt{ans.601} := C_1 e^{-x} + a\sin(x) - a\cos(x)$$
$$\texttt{ans.602} := a\sin(x) - a\cos(x) + ae^{-x}$$
$$\texttt{ans.603} := C_1\sin(x) + C_2\cos(x)$$
$$\texttt{ans.604} := \sin(x)$$
$$\texttt{ans.605} := C_1 e^{-6x} + C_2 e^{x}$$
$$\texttt{ans.606} := 3e^{x} + 2e^{-6x}$$