

The metric connection

This is a very standard computation that shows if

$$\Gamma_{bc}^a = \frac{1}{2} g^{ad} (\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc}) \quad (1)$$

then

$$g_{ab;c} = 0. \quad (2)$$

This example might well be regarded as the Cadabra counterpart to the familiar *Hello World* program of undergraduate programming classes.

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{a,b,c,d,e,f,h,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices.

g_{a b}::Metric.
g_{a}^{b}::KroneckerDelta.

\partial_{#}::PartialDerivative.

cderiv:=\partial_{c}{g_{a b}} - g_{a d}\Gamma^{d}_{b c}
        - g_{d b}\Gamma^{d}_{a c}. # cdb (term31,cderiv)

Gamma:=\Gamma^{a}_{b c} -> (1/2) g^{a d} ( \partial_{b}{g_{d c}}
        + \partial_{c}{g_{b d}}
        - \partial_{d}{g_{b c}} ). # cdb (term32,Gamma)

substitute      (cderiv,Gamma); # cdb (term33,cderiv)
distribute      (cderiv) # cdb (term34,cderiv)
eliminate_metric (cderiv) # cdb (term35,cderiv)
eliminate_kronecker (cderiv) # cdb (term36,cderiv)
canonicalise    (cderiv) # cdb (term37,cderiv)
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The metric connection

$$\begin{aligned}
 \text{term31} &:= \partial_c g_{ab} - g_{ad} \Gamma^d{}_{bc} - g_{db} \Gamma^d{}_{ac} \\
 \text{term32} &:= \Gamma^a{}_{bc} \rightarrow \frac{1}{2} g^{ad} (\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc}) \\
 \text{term33} &:= \partial_c g_{ab} - \frac{1}{2} g_{ad} g^{de} (\partial_b g_{ec} + \partial_c g_{be} - \partial_e g_{bc}) - \frac{1}{2} g_{db} g^{de} (\partial_a g_{ec} + \partial_c g_{ae} - \partial_e g_{ac}) \\
 \text{term34} &:= \partial_c g_{ab} - \frac{1}{2} g_{ad} g^{de} \partial_b g_{ec} - \frac{1}{2} g_{ad} g^{de} \partial_c g_{be} + \frac{1}{2} g_{ad} g^{de} \partial_e g_{bc} - \frac{1}{2} g_{db} g^{de} \partial_a g_{ec} - \frac{1}{2} g_{db} g^{de} \partial_c g_{ae} + \frac{1}{2} g_{db} g^{de} \partial_e g_{ac} \\
 \text{term35} &:= \partial_c g_{ab} - \frac{1}{2} g_a{}^e \partial_b g_{ec} - \frac{1}{2} g_a{}^e \partial_c g_{be} + \frac{1}{2} g_a{}^e \partial_e g_{bc} - \frac{1}{2} g_b{}^e \partial_a g_{ec} - \frac{1}{2} g_b{}^e \partial_c g_{ae} + \frac{1}{2} g_b{}^e \partial_e g_{ac} \\
 \text{term36} &:= \frac{1}{2} \partial_c g_{ab} - \frac{1}{2} \partial_c g_{ba} \\
 \text{term37} &:= 0
 \end{aligned}$$

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\begin{align*}
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\end{align*}

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