

Curvature of a 2-sphere

This examples uses standard methods to compute the scalar curvature of a 2-sphere.

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{\theta, \varphi}::Coordinate.
{\alpha, \beta, \gamma, \delta, \rho, \sigma, \mu, \nu, \lambda}::Indices(values={\varphi, \theta}, position=independent).

\partial{#}::PartialDerivative.

g_{\alpha\beta}::Metric.
g^{\alpha\beta}::InverseMetric.

Chr := \Gamma^{\alpha}_{\mu\nu} -> 1/2 g^{\alpha\beta} ( \partial_{\nu}\{g_{\beta\mu}\}
                                         + \partial_{\mu}\{g_{\beta\nu}\}
                                         - \partial_{\beta}\{g_{\mu\nu}\} ).

Rabcd := R^{\rho}_{\sigma\mu\nu} -> \partial_{\mu}\{\Gamma^{\rho}_{\sigma\nu}\}
- \partial_{\nu}\{\Gamma^{\rho}_{\sigma\mu}\}
+ \Gamma^{\rho}_{\beta\mu} \Gamma^{\beta}_{\sigma\nu} - \Gamma^{\rho}_{\beta\nu} \Gamma^{\beta}_{\sigma\mu}.

Rab := R_{\sigma\nu} -> R^{\rho}_{\sigma\rho\nu}.

R := R -> R_{\sigma\nu} g^{\sigma\nu}.

gab:={ g_{\theta\theta} = r**2,
      g_{\varphi\varphi} = r**2 \sin(\theta)**2 }. # cdb(gab,gab)

complete (gab, $g^{\alpha\beta}$) # cdb(iab,gab)

evaluate (Chr, gab, rhsonly=True) # cdb(Chr,Chr)

substitute (Rabcd, Chr)
evaluate (Rabcd, gab, rhsonly=True) # cdb(Rabcd,Rabcd)

substitute (Rab, Rabcd)
evaluate (Rab, gab, rhsonly=True) # cdb(Rab,Rab)
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substitute (R, Rab)

evaluate (R, gab, rhsonly=True)

cdb(R,R)

$$\left[g_{\theta\theta} = r^2, g_{\varphi\varphi} = r^2 (\sin \theta)^2, g^{\varphi\varphi} = (r^2 (\sin \theta)^2)^{-1}, g^{\theta\theta} = r^{-2} \right]$$

$$\Gamma_{\mu\nu}^{\alpha} \rightarrow \square_{\mu\nu}^{\alpha} \begin{cases} \square_{\varphi\theta}^{\varphi} = (\tan \theta)^{-1} \\ \square_{\theta\varphi}^{\varphi} = (\tan \theta)^{-1} \\ \square_{\varphi\varphi}^{\theta} = -\frac{1}{2} \sin(2\theta) \end{cases}$$

$$R_{\sigma\mu\nu}^{\rho} \rightarrow \square_{\sigma\nu}^{\rho\mu} \begin{cases} \square_{\varphi\varphi}^{\theta\theta} = \frac{1}{2} \sin(2\theta) (\tan \theta)^{-1} - \cos(2\theta) \\ \square_{\theta\varphi}^{\varphi\theta} = -1 \\ \square_{\varphi\theta}^{\theta\varphi} = -\frac{1}{2} \sin(2\theta) (\tan \theta)^{-1} + \cos(2\theta) \\ \square_{\theta\theta}^{\varphi\varphi} = 1 \end{cases}$$

$$R_{\sigma\nu} \rightarrow \square_{\sigma\nu} \begin{cases} \square_{\varphi\varphi} = \frac{1}{2} \sin(2\theta) (\tan \theta)^{-1} - \cos(2\theta) \\ \square_{\theta\theta} = 1 \end{cases}$$

$$R \rightarrow 2r^{-2}$$

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\begin{align*}
&\&\text{cdb{iab}}\&\&[10pt]
&\&\text{cdb{Chr}}\&\&[10pt]
&\&\text{cdb{Rabcd}}\&\&[10pt]
&\&\text{cdb{Rab}}\&\&[10pt]
&\&\text{cdb{R}}
\end{align*}
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