

Step-by-step integration

This is another nice example drawn from the Pythontex gallery, see <https://github.com/gpoore/pythontex>.

It shows the step-by-step computations of a simple triple integral.

```
1  xmax = 2;  ymax = 3;  zmax = 4;
2  xmin = 0;  ymin = 0;  zmin = 0;
3
4  fun = f[x,y,z];
5
6  mytmp = HoldForm[Integrate[#1, {z, #6, #7},
7                      {y, #4, #5},
8                      {x, #2, #3}]] & @@ {fun, xmin, xmax, ymin, ymax, zmin, zmax}; (* mma(lhs.01,mytmp) *)
9
10 fun = x y + y Sin[z] + Cos[x+y];
11
12 myint = HoldForm[Integrate[#1, {z, #6, #7},
13                      {y, #4, #5},
14                      {x, #2, #3}]] & @@ {fun, xmin, xmax, ymin, ymax, zmin, zmax}; (* mma(rhs.01,myint) *)
15
16 myansx =      Integrate[#1, {x, #2, #3}] & @@ {fun, xmin, xmax};
17 myintx = HoldForm[Integrate[#1, {z, #4, #5},
18                      {y, #2, #3}]] & @@ {myansx, ymin, ymax, zmin, zmax}; (* mma(rhs.02,myintx) *)
19
20 myansxy =      Integrate[#1, {y, #2, #3}] & @@ {myansx, ymin, ymax};
21 myintxy = HoldForm[Integrate[#1, {z, #2, #3}]] & @@ {myansxy, zmin, zmax}; (* mma(rhs.03,myintxy) *)
22
23 myansxyz = Integrate[#1, {z, #2, #3}] & @@ {myansxy, zmin, zmax}; (* mma(rhs.04,myansxyz) *)
24
25 myapprox = N[myansxyz,15]; (* mma(rhs.05,myapprox) *)
```

$$\begin{aligned}
\int_0^4 \int_0^3 \int_0^2 f(x, y, z) dx dy dz &= \int_0^4 \int_0^3 \int_0^2 (xy + \cos(x + y) + y \sin(z)) dx dy dz \\
&= \int_0^4 \int_0^3 2(y + \cos(1 + y) \sin(1) + y \sin(z)) dy dz \\
&= \int_0^4 (8 + \cos(2) + \cos(3) - \cos(5) + 9 \sin(z)) dz \\
&= 41 + 4 \cos(2) + 4 \cos(3) - 9 \cos(4) - 4 \cos(5) \\
&\approx 40.1235865133293
\end{aligned}$$

```

\begin{align*}
\mma{lhs.01} &= \mma{rhs.01} \\
&= \mma{rhs.02} \\
&= \mma{rhs.03} \\
&= \mma{rhs.04} \\
&\approx \mma{rhs.05}
\end{align*}

```