

# Step-by-step integration

This is another nice example drawn from the Pythontex gallery, see <https://github.com/gpoore/pythontex>.

It shows the step-by-step computations of a simple triple integral.

```
# Define limits of integration
x_max := 2:   y_max := 3:   z_max := 4:
x_min := 0:   y_min := 0:   z_min := 0:

ans := int(f(x,y,z), [x=x_min..x_max, y=y_min..y_max, z=z_min..z_max]):           # mpl(lhs.01,ans)

f := (x,y,z) -> x*y + y*sin(z) + cos(x+y):

ans := ''int''(''int''(''int''(f(x,y,z), x=x_min..x_max), y=y_min..y_max), z=z_min..z_max):   # mpl(rhs.01,ans)
ans := ''int''(''int''(int(f(x,y,z), x=x_min..x_max), y=y_min..y_max), z=z_min..z_max):       # mpl(rhs.02,ans)
ans := ''int''(int(int(f(x,y,z), x=x_min..x_max), y=y_min..y_max), z=z_min..z_max):         # mpl(rhs.03,ans)
ans := int(int(int(f(x,y,z), x=x_min..x_max), y=y_min..y_max), z=z_min..z_max):           # mpl(rhs.04,ans)

# And now, a numerical approximation
ans := evalf[15](ans):                                                                # mpl(rhs.05,ans)
```

$$\begin{aligned}\int_0^4 \int_0^3 \int_0^2 f(x,y,z) \, dx \, dy \, dz &= \int_0^4 \int_0^3 \int_0^2 xy + y \sin(z) + \cos(x+y) \, dx \, dy \, dz \\ &= \int_0^4 \int_0^3 -\sin(y) + 2y + 2y \sin(z) + \sin(2+y) \, dy \, dz \\ &= \int_0^4 8 + \cos(2) + 9 \sin(z) + \cos(3) - \cos(5) \, dz \\ &= 41 - 9 \cos(4) + 4 \cos(3) - 4 \cos(5) + 4 \cos(2) \\ &\approx 40.1235865133292\end{aligned}$$

```
\begin{align*}
\mpl{lhs.01} &= \mpl{rhs.01}\\
&= \mpl{rhs.02}\\
&= \mpl{rhs.03}\\
&= \mpl{rhs.04}\\
&\approx \mpl{rhs.05}
\end{align*}
```