Curvature of a 2-sphere

This examples uses standard methods to compute the scalar curvature of a 2-sphere.

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{\theta, \varphi}::Coordinate.
{\alpha, \beta, \gamma, \delta, \rho, \sigma, \mu, \nu, \lambda}::Indices(values={\varphi, \theta}, position=independent).

\partial{#}::PartialDerivative.

g_{\alpha\beta}::Metric.
g^{\alpha\beta}::InverseMetric.

Chr := \Gamma^{\alpha}_{\mu\nu} -> 1/2 g^{\alpha\beta} (  \partial_{\nu}{g_{\beta\mu}}
                                                        + \partial_{\mu}{g_{\beta\nu}}
                                                        - \partial_{\beta}{g_{\mu\nu}} ).

Rabcd := R^{\rho}_{\sigma\mu\nu} ->   \partial_{\mu}{\Gamma^{\rho}_{\sigma\nu}}
                                    - \partial_{\nu}{\Gamma^{\rho}_{\sigma\mu}}
                                    + \Gamma^{\rho}_{\beta\mu} \Gamma^{\beta}_{\sigma\nu}
                                    - \Gamma^{\rho}_{\beta\nu} \Gamma^{\beta}_{\sigma\mu}.

Rab := R_{\sigma\nu} -> R^{\rho}_{\sigma\rho\nu}.

R := R -> R_{\sigma\nu} g^{\sigma\nu}.

gab:={ g_{\theta\theta}   = r**2,
       g_{\varphi\varphi} = r**2 \sin(\theta)**2 }.   # cdb(gab,gab)

complete   (gab, $g^{\alpha\beta}$)                   # cdb(iab,gab)

evaluate   (Chr, gab, rhsonly=True)                   # cdb(Chr,Chr)

substitute (Rabcd, Chr)
evaluate   (Rabcd, gab, rhsonly=True)                 # cdb(Rabcd,Rabcd)

substitute (Rab, Rabcd)
evaluate   (Rab, gab, rhsonly=True)                   # cdb(Rab,Rab)
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substitute (R, Rab)
evaluate   (R, gab, rhsonly=True)                    # cdb(R,R)
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$$\left[g_{\theta\theta} = r^{2},\; g_{\varphi\varphi} = r^{2}(\sin\theta)^{2},\; g^{\varphi\varphi} = \left(r^{2}(\sin\theta)^{2}\right)^{-1},\; g^{\theta\theta} = r^{-2}\right]$$

$$\Gamma^{\alpha}{}_{\mu\nu} \rightarrow \square_{\mu\nu}{}^{\alpha} \begin{cases} \square_{\varphi\theta}{}^{\varphi} = (\tan\theta)^{-1} \\ \square_{\theta\varphi}{}^{\varphi} = (\tan\theta)^{-1} \\ \square_{\varphi\varphi}{}^{\theta} = -\frac{1}{2}\sin(2\theta) \end{cases}$$

$$R^{\rho}{}_{\sigma\mu\nu} \rightarrow \square_{\sigma\nu}{}^{\rho}{}_{\mu} \begin{cases} \square_{\varphi\varphi}{}^{\theta}{}_{\theta} = \frac{1}{2}\sin(2\theta)(\tan\theta)^{-1} - \cos(2\theta) \\ \square_{\theta\varphi}{}^{\varphi}{}_{\theta} = -1 \\ \square_{\varphi\theta}{}^{\theta}{}_{\varphi} = -\frac{1}{2}\sin(2\theta)(\tan\theta)^{-1} + \cos(2\theta) \\ \square_{\theta\theta}{}^{\varphi}{}_{\varphi} = 1 \end{cases}$$

$$R_{\sigma\nu} \rightarrow \square_{\sigma\nu} \begin{cases} \square_{\varphi\varphi} = \frac{1}{2}\sin(2\theta)(\tan\theta)^{-1} - \cos(2\theta) \\ \square_{\theta\theta} = 1 \end{cases}$$

$$R \rightarrow 2r^{-2}$$

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\begin{align*}
   &\cdb{iab}\\[10pt]
   &\cdb{Chr}\\[10pt]
   &\cdb{Rabcd}\\[10pt]
   &\cdb{Rab}\\[10pt]
   &\cdb{R}
\end{align*}
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