## Step-by-step integration

This is another nice example drawn from the Pythontex gallery, see https://github.com/gpoore/pythontex.

It shows the step-by-step computations of a simple triple integral.

```
from sympy import *

x, y, z = symbols('x,y,z')
f = Symbol('f(x,y,z)')

# Define limits of integration
x_max = 2;   y_max = 3;   z_max = 4;
x_min = 0;   y_min = 0;   z_min = 0;

lhs = Integral(f, (x, x_min, x_max),
                  (y, y_min, y_max),
                  (z, z_min, z_max))                    # py(lhs.01,lhs)

f = x*y + y*sin(z) + cos(x+y)

rhs = Integral(f, (x, x_min, x_max),
                  (y, y_min, y_max),
                  (z, z_min, z_max))                    # py(rhs.01,rhs)
rhs = Integral(Integral(f, (x, x_min, x_max)).doit(),
                           (y, y_min, y_max),
                           (z, z_min, z_max))           # py(rhs.02,rhs)
rhs = Integral(Integral(f, (x, x_min, x_max),
                           (y, y_min, y_max)).doit(),
                           (z, z_min, z_max))           # py(rhs.03,rhs)
rhs = Integral(f, (x, x_min, x_max),
                  (y, y_min, y_max),
                  (z, z_min, z_max)).doit()             # py(rhs.04,rhs)

# And now, a numerical approximation
rhs = N(rhs)                                            # py(rhs.05,rhs)
```

$$
\begin{aligned}
\int_0^4 \int_0^3 \int_0^2 f(x,y,z)\,dx\,dy\,dz &= \int_0^4 \int_0^3 \int_0^2 \left(xy + y\sin\left(z\right) + \cos\left(x+y\right)\right)\,dx\,dy\,dz \\
&= \int_0^4 \int_0^3 \left(2y\sin\left(z\right) + 2y - \sin\left(y\right) + \sin\left(y+2\right)\right)\,dy\,dz \\
&= \int_0^4 \left(9\sin\left(z\right) + \cos\left(3\right) + \cos\left(2\right) - \cos\left(5\right) + 8\right)\,dz \\
&= 4\cos\left(3\right) + 4\cos\left(2\right) - 4\cos\left(5\right) - 9\cos\left(4\right) + 41 \\
&\approx 40.1235865133293
\end{aligned}
$$

```
\begin{align*}
   \py{lhs.01} &= \py{rhs.01}\\
               &= \py{rhs.02}\\
               &= \py{rhs.03}\\
               &= \py{rhs.04}\\[8pt]
               &\approx \py{rhs.05}
\end{align*}
```