

Curvature of a 2-sphere

This examples uses standard methods to compute the scalar curvature of a 2-sphere.

```
{\theta, \varphi}::Coordinate.
{\alpha, \beta, \gamma, \delta, \rho, \sigma, \mu, \nu, \lambda}::Indices(values={\varphi, \theta}, position=independent).

\partial{#}::PartialDerivative.

g_{\alpha\beta}::Metric.
g^{\alpha\beta}::InverseMetric.

Chr := \Gamma^{\alpha}_{\mu\nu} -> 1/2 g^{\alpha\beta} ( \partial_{\nu}{g_{\beta\mu}}
                                         + \partial_{\mu}{g_{\beta\nu}}
                                         - \partial_{\beta}{g_{\mu\nu}} ).

Rabcd := R^{\rho}_{\sigma\mu\nu} -> \partial_{\mu}{\Gamma^{\rho}_{\sigma\nu}}
- \partial_{\nu}{\Gamma^{\rho}_{\sigma\mu}}
+ \Gamma^{\rho}_{\beta\mu} \Gamma^{\beta}_{\sigma\nu}
- \Gamma^{\rho}_{\beta\nu} \Gamma^{\beta}_{\sigma\mu}.

Rab := R_{\sigma\nu} -> R^{\rho}_{\sigma\rho\nu}.

R := R -> R_{\sigma\nu} g^{\sigma\nu}.

gab:={ g_{\theta\theta} = r**2,
      g_{\varphi\varphi} = r**2 \sin(\theta)**2 }. # cdb(gab,gab)

complete (gab, $g^{\alpha\beta}$) # cdb(iab,gab)

evaluate (Chr, gab, rhsonly=True) # cdb(Chr,Chr)

substitute (Rabcd, Chr)
evaluate (Rabcd, gab, rhsonly=True) # cdb(Rabcd,Rabcd)

substitute (Rab, Rabcd)
evaluate (Rab, gab, rhsonly=True) # cdb(Rab,Rab)
```

substitute (R, Rab)

evaluate (R, gab, rhsonly=True)

cdb(R,R)

$$\left[g_{\theta\theta} = r^2, \quad g_{\varphi\varphi} = r^2(\sin\theta)^2, \quad g^{\varphi\varphi} = (r^2(\sin\theta)^2)^{-1}, \quad g^{\theta\theta} = r^{-2} \right]$$

$$\Gamma^\alpha{}_{\mu\nu} \rightarrow \square_{\mu\nu}{}^\alpha \begin{cases} \square_{\varphi\theta}{}^\varphi = (\tan\theta)^{-1} \\ \square_{\theta\varphi}{}^\varphi = (\tan\theta)^{-1} \\ \square_{\varphi\varphi}{}^\theta = -\frac{1}{2}\sin 2\theta \end{cases}$$

$$R^\rho{}_{\sigma\mu\nu} \rightarrow \square_{\sigma\nu}{}^\rho{}_\mu \begin{cases} \square_{\varphi\varphi}{}^\theta{}_\theta = \frac{1}{2}\sin 2\theta(\tan\theta)^{-1} - \cos 2\theta \\ \square_{\theta\varphi}{}^\varphi{}_\theta = -1 \\ \square_{\varphi\theta}{}^\theta{}_\varphi = -\frac{1}{2}\sin 2\theta(\tan\theta)^{-1} + \cos 2\theta \\ \square_{\theta\theta}{}^\varphi{}_\varphi = 1 \end{cases}$$

$$R_{\sigma\nu} \rightarrow \square_{\sigma\nu} \begin{cases} \square_{\varphi\varphi} = \frac{1}{2}\sin 2\theta(\tan\theta)^{-1} - \cos 2\theta \\ \square_{\theta\theta} = 1 \end{cases}$$

$$R \rightarrow 2r^{-2}$$

```
\begin{align*}
&\&\text{cdb}\{\text{iab}\}\&\&[10\text{pt}]
&\&\text{cdb}\{\text{Chr}\}\&\&[10\text{pt}]
&\&\text{cdb}\{\text{Rabcd}\}\&\&[10\text{pt}]
&\&\text{cdb}\{\text{Rab}\}\&\&[10\text{pt}]
&\&\text{cdb}\{\text{R}\}
\end{align*}
```