The Gauss relation for the curvature of a hypersurface

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{a,b,c,d,e,f,g,i,j,k,l,m,n,o,p,q,r,s,t,u\#}::Indices.
\nabla_{#}::Derivative.
K_{a b}::Symmetric.
g^{a}_{b}::KroneckerDelta.
# Define the projection operator
hab:=h^{a}_{b} -> g^{a}_{b} - n^{a} n_{b}.
# 3-covariant derivative obtained by projection on 4-covariant derivative
vpq:=v_{p q} \rightarrow h^{a}_{p} h^{b}_{q} \nabla_{b}{v_{a}}.
# Compute 3-curvature by commutation of covariant derivatives
 vpqr:= h^{a}_{p} h^{b}_{q} h^{c}_{r} ( \abla_{c}_{v_{a} b}) - \abla_{b}_{v_{a} c}). 
substitute (vpq,hab)
substitute (vpqr,vpq)
distribute (vpqr)
product_rule (vpqr)
distribute (vpqr)
eliminate_kronecker(vpqr)
# Standard substitutions
substitute (vpqr,$h^{a}_{b} n^{b} -> 0$)
substitute (vpqr,h^{a}_{b} = 0)
substitute (vpqr,\alpha_{a}{g^{b}_{c}} -> 0)
substitute (vpqr,h^{p}_{a} h^{q}_{b} \quad (p_{q}) -> K_{a b})
substitute (vpqr,h^{p}_{a} h^{q}_{b} \quad (p_{n^{b}} -> K_{a}^{q}_{b})
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# Tidy up and display the results
{h^{a}_{b}, nabla_{a}{v_{b}}::SortOrder.}

sort_product (vpqr)
rename_dummies (vpqr)
canonicalise (vpqr)
factor_out (vpqr,$h^{a?}_{b?}$)
factor_out (vpqr,$v_{a?}$) # cdb(gauss,vpqr)
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$$D_r(D_q v_p) - D_q(D_r v_p) = h^a{}_p h^b{}_q h^c{}_r \left(\nabla_c \left(\nabla_b v_a \right) - \nabla_b \left(\nabla_c v_a \right) \right) + v_a \left(K_{pr} K_q{}^a - K_{pq} K_r{}^a \right)$$

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\label{eq:continuous} $$D_{r}(D_{q}v_p) - D_{q}(D_{r}v_p) = \cdb{gauss} \end{align*}
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