

Elementary maths

This example is based on a similar example in the [Python collection](#). Its purpose is to show that Cadabra is fluent in Python – which is not surprising since the Cadabra language is based on Python (and a subset of LaTeX).

```
from sympy import *
x, y, z = symbols('x y z')
a, b, c = symbols('a b c')
ans = expand((a+b)**3)
ans = factor(-2*x+2*x+a*x-x**2+a*x**2-x**3)
ans = solve(x**2-4, x)
ans = solve([2*a-b - 3, a+b+c - 1, -b+c - 6], [a,b,c])
ans = N(pi,50)
ans = apart(1/((1 + x)*(5 + x)))
ans = together((1/(1 + x) - 1/(5 + x))/4)
ans = simplify(tanh(log(x)))
ans = simplify(tanh(I*x))
ans = simplify(sinh(3*x) - 3*sinh(x) - 4*(sinh(x))**3)
ans = tanh(log(x))
ans = tanh(UnevaluatedExpr(I*x))
ans = sinh(3*x) - 3*sinh(x) - 4*(sinh(x))**3
```

```
\begin{align*}
&\&\backslash\text{cdb}\{ans.101\}\\
&\&\backslash\text{cdb}\{ans.102\}\\
&\&\backslash\text{cdb}\{ans.103\}\\
&\&\backslash\text{cdb}\{ans.104\}\\
&\&\backslash\text{cdb}\{ans.105\}\\
&\&\backslash\text{cdb}\{ans.106\}\\
&\&\backslash\text{cdb}\{ans.107\}\\
&\backslash\text{cdb}\{lhs.108\} \&= \backslash\text{Cdb}\{rhs.108\}\\
&\backslash\text{cdb}\{lhs.109\} \&= \backslash\text{Cdb}\{rhs.109\}\\
&\backslash\text{cdb}\{lhs.110\} \&= \backslash\text{Cdb}\{rhs.110\}
\end{align*}
```

$$\text{ans.101} := a^3 + 3a^2b + 3ab^2 + b^3$$

$$\text{ans.102} := -x(-a+x)(x+1)$$

$$\text{ans.103} := [-2, \quad 2]$$

$$\text{ans.104} := \left\{ a : \frac{1}{5}, \quad b : -\frac{13}{5}, \quad c : \frac{17}{5} \right\}$$

$$\text{ans.105} := 3.1415926535897932384626433832795028841971693993751$$

$$\text{ans.106} := -\frac{1}{4(x+5)} + \frac{1}{4(x+1)}$$

$$\text{ans.107} := \frac{1}{(x+1)(x+5)}$$

$$\tanh(\log(x)) = \tanh(\log(x)) \quad (\text{rhs.108})$$

$$\tanh(ix) = i \tan(x) \quad (\text{rhs.109})$$

$$-4 \sinh^3(x) - 3 \sinh(x) + \sinh(3x) = 0 \quad (\text{rhs.110})$$

Linear Algebra

```

from sympy import linsolve
lamda = Symbol('lamda')
mat = Matrix([[2,3], [5,4]])
eig1 = mat.eigenvecs()[0][0]
eig2 = mat.eigenvecs()[1][0]
v1 = mat.eigenvecs()[0][2][0]
v2 = mat.eigenvecs()[1][2][0]
eig = simplify(Matrix([eig1,eig2]))
vec = simplify(5*Matrix([]).col_insert(0,v1)
               .col_insert(1,v2))
det = expand((mat - lamda * eye(2)).det())
rhs = Matrix([[3], [7]])
ans = list(linsolve((mat,rhs),x,y))[0]

```

cdb (ans.201,mat)
1st eigenvalue
2nd eigenvalue
1st eigenvector
2nd eigenvector
cdb (ans.202,eig)
cdb (ans.203,vec)
cdb (ans.204,det)
cdb (ans.205,rhs)
cdb (ans.206,ans)

```

\begin{align*}
&\&\text{\texttt{ans.201}}\\
&\&\text{\texttt{ans.202}}\\
&\&\text{\texttt{ans.203}}\\
&\&\text{\texttt{ans.204}}\\
&\&\text{\texttt{ans.205}}\\
&\&\text{\texttt{ans.206}}
\end{align*}

```

$$\begin{aligned}
 \text{ans.201} &:= \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix} \\
 \text{ans.202} &:= \begin{bmatrix} -1 \\ 7 \end{bmatrix} \\
 \text{ans.203} &:= \begin{bmatrix} -5 & 3 \\ 5 & 5 \end{bmatrix} \\
 \text{ans.204} &:= \lambda^2 - 6\lambda - 7 \\
 \text{ans.205} &:= \begin{bmatrix} 3 \\ 7 \end{bmatrix} \\
 \text{ans.206} &:= \left(\frac{9}{7}, \frac{1}{7} \right)
 \end{aligned}$$

Limits

```
n, dx = symbols('n dx')
ans = limit(sin(4*x)/x,x,0)           # cdb (ans.301,ans)
ans = limit(2**x/x,x,oo)              # cdb (ans.302,ans)
ans = limit(((x+dx)**2 - x**2)/dx, dx,0) # cdb (ans.303,ans)
ans = limit((4*n + 1)/(3*n - 1),n,oo)  # cdb (ans.304,ans)
ans = limit((1+(a/n))**n,n,oo)        # cdb (ans.305,ans)
```

```
\begin{align*}
&\&\backslash\text{cdb}\{ans.301\}\\
&\&\backslash\text{cdb}\{ans.302\}\\
&\&\backslash\text{cdb}\{ans.303\}\\
&\&\backslash\text{cdb}\{ans.304\}\\
&\&\backslash\text{cdb}\{ans.305\}
\end{align*}
```

```
ans.301 := 4
ans.302 := ∞
ans.303 := 2x
ans.304 :=  $\frac{4}{3}$ 
ans.305 :=  $e^a$ 
```

Series

```
ans = series((1 + x)**(-2), x, 1, 6)   # cdb (ans.401,ans)
ans = series(exp(x), x, 0, 6)          # cdb (ans.402,ans)
ans = Sum(1/n**2, (n,1,50)).doit()     # cdb (ans.403,ans)
ans = Sum(1/n**4, (n,1,oo)).doit()     # cdb (ans.404,ans)
```

```
\begin{align*}
&\&\backslash\text{cdb}\{ans.401\}\\
&\&\backslash\text{cdb}\{ans.402\}\\
&\&\backslash\text{cdb}\{ans.403\}\\
&\&\backslash\text{cdb}\{ans.404\}
\end{align*}
```

```
ans.401 :=  $\frac{1}{2} + \frac{3(x-1)^2}{16} - \frac{(x-1)^3}{8} + \frac{5(x-1)^4}{64} - \frac{3(x-1)^5}{64} - \frac{x}{4} + O((x-1)^6; x \rightarrow 1)$ 
ans.402 :=  $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + O(x^6)$ 
ans.403 :=  $\frac{3121579929551692678469635660835626209661709}{1920815367859463099600511526151929560192000}$ 
ans.404 :=  $\frac{\pi^4}{90}$ 
```

Calculus

This example shows how `\Cdb` can be used to set the equation tag on the far right hand side.

```
ans = diff(x*sin(x),x) # cdb (ans.501,ans)
ans = diff(x*sin(x),x).subs(x,pi/4) # cdb (ans.502,ans)
ans = integrate(2*sin(x)**2, (x,a,b)) # cdb (ans.503,ans)
ans = Integral(2*exp(-x**2), (x,0,oo)) # cdb (lhs.504,ans)
ans = ans.doit() # cdb (ans.504,ans)
ans = Integral(Integral(x**2 + y**2, (y,0,x)), (x,0,1)) # cdb (lhs.505,ans)
ans = ans.doit() # cdb (ans.505,ans)
```

```
\begin{align*}
&\&\cdb*{ans.501}\\
&\&\cdb*{ans.502}\\
&\&\cdb*{ans.503}\\
&\&\cdb{lhs.504}\&=\Cdb{ans.504}\\
&\&\cdb{lhs.505}\&=\Cdb{ans.505}
\end{align*}
```

$$\text{ans.501} := x \cos(x) + \sin(x)$$

$$\text{ans.502} := \frac{\sqrt{2}\pi}{8} + \frac{\sqrt{2}}{2}$$

$$\text{ans.503} := -a + b + \sin(a) \cos(a) - \sin(b) \cos(b)$$

$$\int_0^\infty 2e^{-x^2} dx = \sqrt{\pi} \quad (\text{ans.504})$$

$$\int_0^1 \int_0^x (x^2 + y^2) dy dx = \frac{1}{3} \quad (\text{ans.505})$$

Differential equations

```

y = Function('y')
C1, C2 = symbols('C1 C2')

ode = Eq(y(x).diff(x) + y(x), 2*a*sin(x))
sol = expand(dsolve(ode,y(x)).rhs) # cdb (ans.601,sol)
cst = solve([sol.subs(x,0)],dict=True)
sol = sol.subs(cst[0]) # cdb (ans.602,sol)

ode = Eq(y(x).diff(x,2) + y(x), 0)
sol = expand(dsolve(ode,y(x)).rhs) # cdb (ans.603,sol)
cst = solve([sol.subs(x,0),sol.diff(x).subs(x,0)-1],dict=True)
sol = sol.subs(cst[0]) # cdb (ans.604,sol)

ode = Eq(y(x).diff(x,2) + 5*y(x).diff(x) - 6*y(x), 0)
sol = expand(dsolve(ode,y(x)).rhs) # cdb (ans.605,sol)
sol = sol.subs({C1:2,C2:3}) # cdb (ans.606,sol)

```

```

\begin{align*}
&\&\backslash\text{cdb}\{ans.601\}\\
&\&\backslash\text{cdb}\{ans.602\}\\
&\&\backslash\text{cdb}\{ans.603\}\\
&\&\backslash\text{cdb}\{ans.604\}\\
&\&\backslash\text{cdb}\{ans.605\}\\
&\&\backslash\text{cdb}\{ans.606\}
\end{align*}

```

$$\text{ans.601} := C_1 e^{-x} + a \sin(x) - a \cos(x)$$

$$\text{ans.602} := a \sin(x) - a \cos(x) + a e^{-x}$$

$$\text{ans.603} := C_1 \sin(x) + C_2 \cos(x)$$

$$\text{ans.604} := \sin(x)$$

$$\text{ans.605} := C_1 e^{-6x} + C_2 e^x$$

$$\text{ans.606} := 3e^x + 2e^{-6x}$$