The Gauss relation for the curvature of a hypersurface

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{a,b,c,d,e,f,g,i,j,k,l,m,n,o,p,q,r,s,t,u#}::Indices.

\nabla_{#}::Derivative.

K_{a b}::Symmetric.
g^{a}_{b}::KroneckerDelta.

# Define the projection operator

hab:=h^{a}_{b} -> g^{a}_{b} - n^{a} n_{b}.

# 3-covariant derivative obtained by projection on 4-covariant derivative

vpq:=v_{p q} -> h^{a}_{p} h^{b}_{q} \nabla_{b}{v_{a}}.

# Compute 3-curvature by commutation of covariant derivatives

vpqr:= h^{a}_{p} h^{b}_{q} h^{c}_{r} ( \nabla_{c}{v_{a b}} - \nabla_{b}{v_{a c}} ).

substitute (vpq,hab)
substitute (vpqr,vpq)

distribute   (vpqr)
product_rule (vpqr)
distribute   (vpqr)
eliminate_kronecker(vpqr)

# Standard substitutions

substitute (vpqr,$h^{a}_{b} n^{b} -> 0$)
substitute (vpqr,$h^{a}_{b} n_{a} -> 0$)
substitute (vpqr,$\nabla_{a}{g^{b}_{c}} -> 0$)
substitute (vpqr,$n^{a} \nabla_{b}{v_{a}} -> -v_{a} \nabla_{b}{n^{a}}$)
substitute (vpqr,$v_{a} \nabla_{b}{n^{a}} -> v_{p} h^{p}_{a}\nabla_{b}{n^{a}}$)
substitute (vpqr,$h^{p}_{a} h^{q}_{b} \nabla_{p}{n_{q}} -> K_{a b}$)
substitute (vpqr,$h^{p}_{a} h^{q}_{b} \nabla_{p}{n^{b}} -> K_{a}^{q}$)
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# Tidy up and display the results

{h^{a}_{b},\nabla_{a}{v_{b}}}::SortOrder.

sort_product    (vpqr)
rename_dummies  (vpqr)
canonicalise    (vpqr)
factor_out      (vpqr,$h^{a?}_{b?}$)
factor_out      (vpqr,$v_{a?}$)       # cdb(gauss,vpqr)
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$$D_r(D_q v_p) - D_q(D_r v_p) = h^{a}{}_{p} h^{b}{}_{q} h^{c}{}_{r} \left(\nabla_c(\nabla_b v_a) - \nabla_b(\nabla_c v_a)\right) + v_a \left(K_{pr} K_{q}{}^{a} - K_{pq} K_{r}{}^{a}\right)$$

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\begin{align*}
   D_{r}(D_{q}v_p) - D_{q}(D_{r}v_p) = \cdb{gauss}
\end{align*}
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